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Comments on "Improved algorithm for the discrete Fourier transform" [Rev. Sci. Instrum. 56, 2325 (1985)]

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S. Sorella

Comments on "Improved algorithm for the discrete Fourier transform" [Rev. Sci. Instrum. 56, 2325 (1985)]

S. Sorella

I.S.A.S., Str., Costiera n.11, 34014 Trieste, Italy

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It is shown that in the paper written by M. Froeyen and L. Hellemans [Rev. Sci. Instrum. **56**, 2325 (1985)], the algorithm POL2*FFT proposed by myself and S. K. Ghosh [S. Sorella and S. K. Ghosh, Rev. Sci. Instrum. **55**, 1348 (1984)] is used in a incorrect way, because the original paper was not very "explicit" in some steps. Therefore, the new algorithm represents only a different application of our general strategy.

In reference to the paper by M. Froeyen and L. Hellemans [Rev. Sci. Instrum. **56**, 2325 (1985)], I would like to clarify some points about my method POL2*FFT (Ref. 1) which has been misunderstood by these authors.

Indeed formula (8) of Ref. 1 refers to T-periodical functions that may be differentiated three times.

However, it can also be used for more general cases even if the discontinuities of the function lie on the extremes of the integration interval $(-\tau/2, T-\tau/2)$.

In this case formula (8) properly refers to

$$X(\omega) = \frac{1}{T}\int_{-\tau/2}^{T-\tau/2} x(t)e^{-i\omega t}dt \tag{1}$$

with

$$x^{(n)}\left(-\frac{\tau}{2}\right) \neq x^{(n)}\left(T-\frac{\tau}{2}\right)$$

that is Eq. (3) of my paper.

So, if we need the Fourier transform of a function $f(t)$ with $f^{(n)}(0) \neq f^{(n)}(T)$, $n = 0,1,2,3$, that is

$$X_f(\omega) = \frac{1}{T}\int_0^T f(t_1)e^{-i\omega t_1}dt_1 , \tag{2}$$

we can substitute

$$x(t) = f\left(t+\frac{\tau}{2}\right) \tag{3}$$

in Eq. (1) and by simple change of time: $t_1 = t + \tau/2$ in Eq. (1) we get trivially the following relation:

$$X(\omega) = e^{i(\omega\tau/2)}X_f(\omega) = e^{iu/N}X_f(\omega) , \tag{4}$$

where from now forward we use the same notation as in Ref. 1. Therefore,

$$X_f(\omega) = e^{-iu/N}X(\omega) \tag{5}$$

and $X(\omega)$ can be estimated with the algorithm POL2*FFT [Eq. (8) of Ref. 1] with the following sampling points [via relation no. (3)]:

$$x_m = f\left[\left(m+\frac{1}{2}\right)\tau\right] \quad m = 0,1,\ldots,N-1 . \tag{6}$$

It is to be stressed that following this scheme, we never interpolate a discontinuous function as asserted by the authors of Ref. 2.

All the calculations published in my paper were made keeping in mind the two simple Eqs. (5) and (6).

In order to avoid other misunderstandings, we write the complete expression for $X_f(\omega)$:

$$X_f(\omega) = e^{-iu/N}E(u)X_{\rm FFT}(\omega) + aY(u) - ibZ(u) . \tag{7}$$

To prove what I have just written, it is sufficient to rewrite Table II of Ref. 2 also with % errors, in order to get a matching with Table I(a), $N = 256$ (see table) of Ref. 1.

TABLE I. Real part of the Fourier transform of the function $\exp(-t)$, obtained by POL2*FFT of Refs. 1 and 2.

r	Ref. Eq. (2)	POL2*FFT Ref. 1	POL2*FFT %	POL2*FFT corrected—Ref. 2	% error Eq. (10)—Ref. 2
0	1.44620062 − 01	1.44620060 − 01	2.00 − 06	1.4462 − 01	− 3.5 − 06
1	7.91422433 − 02	7.91422407 − 02	3.26 − 06	7.9142 − 02	− 7.46 − 05
33	1.60336743 − 04	1.60333963 − 04	1.78 − 03	1.6039 − 04	3.57 − 03
65	4.13610692 − 05	4.13569147 − 05	1.00 − 02	4.1411 − 05	1.03 − 02
97	1.85756252 − 05	1.85699828 − 05	3.04 − 02	1.8619 − 05	1.17 − 02

The total time T has been choosen with the condition $\exp(-T) = 10^{-3}$. The number of sampling points is 256. The values in the 4th and 5th columns are taken from Tables I and II of Ref. 2, while those of the 3rd refer to the Table I(a) of Ref. 1. The latter set of values is consistent with the 1st and the 2nd columns.

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In Table I we note that the column referred to as POL2*FFT is completely different from the one which appeared in Ref. 2, due to the bad interpretation of our method.

Furthermore, the row corresponding to $r = 97$, in Table II, is not consistent (our calculation is not affected by rounding-off errors in this limit because we work in double precision) with our reference value and the % error of POL2*FFT corrected (?) leads essentially to the same results as ours.

Finally, I claim that our method is quite easily programmable because $E(u)$, $Y(u)$, and $Z(u)$ are simple real functions, and it needs only $18 \times N$ real multiplications [$20 \times N$ if $f(t)$ is complex], or $6 \times N$ if one stores $Y(u)$, $Z(u)$, and $e^{-iu/N}E(u)$ [$8 \times N$ if $f(t)$ is complex], after the FFT program has been performed [$(N/2)\log_2 N$ complex multiplications].

[1]S. Sorella and S. K. Ghosh, Rev. Sci. Instrum. **55**, 1348 (1984).
[2]M. Froeyen and L. Hellemans, Rev. Sci. Instrum. **56**, 2325 (1985).

Reply to "Comments on 'Improved algorithm for the discrete Fourier transform' " [Rev. Sci. Instrum. 58, 714 (1987)]

M. Froeyen and L. Hellemans
Department of Chemistry, University of Leuven, Celestijnenlaan 200 D, B-3030 Heverlee, Belgium

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The difference between Sorella and Ghosh's approach [Rev. Sci. Instrum. **55**, 1348 (1984)] and ours [M. Froeyen and L. Hellemans, Rev. Sci. Instrum. **56**, 2325 (1985)] to obtain an expression for the Fourier transform of discrete data is highlighted. A practical advantage of our algorithm over Sorella's is stressed.

The common objective of the authors of both Refs. 1 and 2 was to obtain the best possible approximation of the Fourier transform

$$X(\omega) = 1/T \int_0^T x(t)\exp(-i\omega t)dt,$$

where $x(t)$ is a set of discrete experimental data recorded in the time window of width T.

As is apparent from Eqs. (2) and (3) of their paper,[1] Sorella and Ghosh have shifted the time window by an amount of $\tau/2$, leaving the time function $x(t)$ unchanged. Under these conditions their result [Eq. (8)] does not correspond to Eq. (2) nor to Eq. (3) of their paper. This is what we have demonstrated.[2]

In his present letter Sorella introduces a shift of the time window, as well as that of the signal. In this case his treatment leads essentially to the same results as ours. We have to note, however, that his derivation requires discrete data points which in real time lie at $t = \tau/2, 3\tau/2, 5\tau/2...$ as opposed to the usual $t = 0, \tau, 2\tau...$. In practice, this means that either such data must be found from interpolation, or that the sampling device must be delayed by precisely $\tau/2$ s. The former procedure requires more computing time, while the latter can easily lead to errors due to time misreferencing.[3] In this respect our algorithm is superior. Finally, we have to point out that the last column in the table of Sorella's letter is labeled erroneously. The values refer to the % error for the results obtained by our own algorithm and, consequently, do not correspond to the values in the fourth column.

[1]S. Sorella and S. K. Ghosh, Rev. Sci. Instrum. **55**, 1348 (1984).
[2]M. Froeyen and L. Hellemans, Rev. Sci. Instrum. **56**, 2325 (1985).
[3]D. Eadline and H. Leidheiser, Rev. Sci. Instrum. **57**, 898 (1986).

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