$U_{\text{PMNS}} = U_\ell^\dagger U_\nu$

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Abstract

We consider corrections to vanishing U_{e3} and maximal atmospheric neutrino mixing originating from the relation $U = U_\ell^\dagger U_\nu$, where U is the PMNS mixing matrix and U_ℓ (U_ν) is associated with the diagonalization of the charged lepton (neutrino) mass matrix. We assume that in the limit of U_ℓ or U_ν being the unit matrix, one has $U_{e3} = 0$ and $\theta_{23} = \pi/4$, while the solar neutrino mixing angle is a free parameter. Well-known special cases of the indicated scenario are the bimaximal and tri-bimaximal mixing schemes. If $U_{e3} \neq 0$ and $\theta_{23} \neq \pi/4$ due to corrections from the charged leptons, $|U_{e3}|$ can be sizable (close to the existing upper limit) and we find that the value of the solar neutrino mixing angle is linked to the magnitude of CP violation in neutrino oscillations. In the alternative case of the neutrino sector correcting $U_{e3} = 0$ and $\theta_{23} = \pi/4$, we obtain a generically smaller $|U_{e3}|$ than in the first case. Now the magnitude of CP violation in neutrino oscillations is connected to the value of the atmospheric neutrino mixing angle θ_{23}. We find that both cases are in agreement with present observations. We also introduce parametrization independent "sum-rules" for the oscillation parameters.

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1. Introduction

The low energy neutrino mixing implied by the neutrino oscillation data can be described by the Lagrangian (see, e.g., [1])

$$\mathcal{L} = -\frac{g}{\sqrt{2}} \overline{\ell_L} \gamma^\mu \nu_L W_\mu - \frac{1}{2} \overline{\nu_R^c} m_\nu \nu_L - \overline{\ell_R} m_\ell \ell_L + \text{h.c.,} \tag{1}$$

which includes charged lepton and Majorana neutrino mass terms. When diagonalizing the neutrino and charged lepton mass matrices via $m_\nu = U_\nu^* m_\nu^{\text{diag}} U_\nu^\dagger$ and $m_\ell = V_\ell m_\ell^{\text{diag}} U_\ell^\dagger$, we obtain the lepton mixing (PMNS) matrix in the weak charged lepton current

$$U = U_\ell^\dagger U_\nu. \tag{2}$$

From the analyzes of the currently existing neutrino oscillation data it was found [2] that the present best-fit values of the CHOOZ and atmospheric neutrino mixing angles, θ_{13} and θ_{23}, correspond to $|U_{e3}| = \sin\theta_{13} = 0$ and $\theta_{23} = \pi/4$, i.e., to $|U_{\mu3}| = |U_{\tau3}|$.

Accordingly, the "best-fit" PMNS matrix is given by

$$U = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\frac{\sin\theta_{12}}{\sqrt{2}} & \frac{\cos\theta_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{\sin\theta_{12}}{\sqrt{2}} & \frac{\cos\theta_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix},$$

(3)

where we have not written the Majorana phases [3,4] and used $\theta_{23} = -\pi/4$ in the usual Particle Data Group (PDG) parametrization of the PMNS matrix. One well-known possibility to construct this "phenomenological" mixing matrix is to require a μ–τ exchange symmetry for the neutrino mass matrix in the basis in which the charged lepton mass matrix is diagonal [5]. Well-known examples of neutrino mixing with μ–τ symmetry are the bimaximal [6] and tri-bimaximal [7] mixing matrices

$$U_{\text{bi}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad \text{and} \quad U_{\text{tri}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}.$$

(4)

A common feature of these two mixing matrices is $\theta_{23} = \pm\pi/4$ and $\theta_{13} = 0$, which is perfectly compatible with the current data. However, they differ in their prediction for the value of the solar neutrino mixing angle, namely, $\sin^2\theta_{12} = 1/2$ and $1/3$, respectively. The best-fit value of $\sin^2\theta_{12}$ determined from the neutrino oscillation data is $\sin^2\theta_{12} = 0.30$. Actually, $\sin^2\theta_{12} = 1/2$ is ruled out by the data at more than 6σ [8].

A natural possibility to obtain a phenomenologically viable PMNS neutrino mixing matrix, and to generate non-zero $|U_{e3}|$ and non-maximal θ_{23}, is to assume that one of the two matrices in $U = U_\ell^\dagger U_\nu$ corresponds to Eq. (3) or (4), and is "perturbed" by the second matrix leading to the required PMNS matrix. Following this assumption, corrections to bimaximal [9–13] and tri-bimaximal [13–15] mixing have previously been analyzed. For instance, scenarios in which the CKM quark mixing matrix corrects the bimaximal mixing pattern are important for models incorporating Quark–Lepton Complementarity (QLC) [16–18] (for earlier reference see [19]). Corrections to mixing scenarios with $\theta_{12} = \pi/4$ and $\theta_{13} = 0$ were considered in [20] (motivated by the L_e–L_μ–L_τ flavor symmetry [21]) and in [12]. The case with $\theta_{23} = \pi/4$ and $\theta_{13} = 0$ has been investigated in Refs. [13,22–24].

Up to now in most analyzes it has been assumed that U_ν possesses a form which leads to $\sin^2\theta_{23} = 1/2$ and $\theta_{13} = 0$. However, the alternative possibility of $\theta_{23} = \pi/4$ and $\theta_{13} = 0$ originating from U_ℓ is phenomenologically equally viable. We are aware of only few papers in which that option is discussed [11,12,25,26]. A detailed study is still lacking in the literature. In the present article we perform, in particular, a comprehensive analysis of this possibility. We also revisit the case of $U_{e3} \neq 0$ and $\theta_{23} \neq \pi/4$ due to corrections from $U_\ell^\dagger$ and derive parametrization independent sum-rules for the relevant oscillation parameters. We point out certain "subtleties" in the identification of the relevant phases governing CP violation in neutrino oscillations with the Dirac phase of the standard parametrization of the PMNS matrix.

Our Letter is organized as follows: Section 2 briefly summarizes the formalism and the relevant matrices from which the neutrino mixing observables can be reconstructed. We analyze the possibility of U_ν leading to $\sin^2\theta_{23} = 1/2$ and $\theta_{13} = 0$ and being corrected by a non-trivial U_ℓ in Section 3. In Section 4 the alternative case of U_ℓ causing $\sin^2\theta_{23} = 1/2$ and $\theta_{13} = 0$ and being modified by a non-trivial U_ν is discussed. Section 5 contains our conclusions.

2. Formalism and definitions

We will use the following parametrization of the PMNS matrix:

$$U = V \text{diag}\left(1, e^{i\alpha}, e^{i(\beta+\delta)}\right) = O_{23}(\theta_{23}) U_{13}(\theta_{13}, \delta) O_{12}(\theta_{12}) \text{diag}\left(1, e^{i\alpha}, e^{i(\beta+\delta)}\right)$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \text{diag}\left(1, e^{i\alpha}, e^{i(\beta+\delta)}\right),$$

(5)

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$ and $O_{ij}(\theta_{ij})$ is a 3×3 orthogonal matrix of rotations on angle θ_{ij} in the ij-plane. We have also defined

$$U_{13}(\theta_{13}, \delta) = \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}.$$

(6)

Hereby we have included the Dirac CP violating phase δ and the two Majorana CP violating phases α and β [3,4]. In general, all phases and mixing angles of U are functions of the parameters characterizing U_ν and U_ℓ. It can be shown that [10,27] after eliminating the unphysical phases, U can be written as $U = \tilde{U}_\ell^\dagger U_\nu$, where in the most general case U_ν and $\tilde{U}_\ell$ are given by

$$U_\nu = P\tilde{U}_\nu Q = \mathrm{diag}\big(1, e^{i\phi}, e^{i\omega}\big)\tilde{U}_\nu \mathrm{diag}\big(1, e^{i\sigma}, e^{i\tau}\big) = P\, O_{23}\big(\theta_{23}^\nu\big)U_{13}\big(\theta_{13}^\nu, \xi\big)O_{12}\big(\theta_{12}^\nu\big)Q$$

$$= P\begin{pmatrix} c_{12}^\nu c_{13}^\nu & s_{12}^\nu c_{13}^\nu & s_{13}^\nu e^{-i\xi} \\ -s_{12}^\nu c_{23}^\nu - c_{12}^\nu s_{23}^\nu s_{13}^\nu e^{i\xi} & c_{12}^\nu c_{23}^\nu - s_{12}^\nu s_{23}^\nu s_{13}^\nu e^{i\xi} & s_{23}^\nu c_{13}^\nu \\ s_{12}^\nu s_{23}^\nu - c_{12}^\nu c_{23}^\nu s_{13}^\nu e^{i\xi} & -c_{12}^\nu s_{23}^\nu - s_{12}^\nu c_{23}^\nu s_{13}^\nu e^{i\xi} & c_{23}^\nu c_{13}^\nu \end{pmatrix}Q, \tag{7}$$

where $P = \mathrm{diag}(1, e^{i\phi}, e^{i\omega})$, $Q = \mathrm{diag}(1, e^{i\sigma}, e^{i\tau})$ are rather important for the results to be obtained, and

$$\tilde{U}_\ell = O_{23}\big(\theta_{23}^\ell\big)U_{13}\big(\theta_{13}^\ell, \psi\big)O_{12}\big(\theta_{12}^\ell\big)$$

$$= \begin{pmatrix} c_{12}^\ell c_{13}^\ell & s_{12}^\ell c_{13}^\ell & s_{13}^\ell e^{-i\psi} \\ -s_{12}^\ell c_{23}^\ell - c_{12}^\ell s_{23}^\ell s_{13}^\ell e^{i\psi} & c_{12}^\ell c_{23}^\ell - s_{12}^\ell s_{23}^\ell s_{13}^\ell e^{i\psi} & s_{23}^\ell c_{13}^\ell \\ s_{12}^\ell s_{23}^\ell - c_{12}^\ell c_{23}^\ell s_{13}^\ell e^{i\psi} & -c_{12}^\ell s_{23}^\ell - s_{12}^\ell c_{23}^\ell s_{13}^\ell e^{i\psi} & c_{23}^\ell c_{13}^\ell \end{pmatrix}. \tag{8}$$

Here we have defined $c_{ij}^{\ell,\nu} = \cos\theta_{ij}^{\ell,\nu}$ and $s_{ij}^{\ell,\nu} = \sin\theta_{ij}^{\ell,\nu}$. Thus, $\tilde{U}_\nu$ and $\tilde{U}_\ell$ contain one physical CP violating phase each.[2] The remaining four phases are located in the diagonal matrices P and Q. Note that Q is "Majorana-like" [10], i.e., the phases σ and τ contribute only to the low energy observables related to the Majorana nature of the neutrinos with definite mass. Typically that are specific observables associated with $|\Delta L| = 2$ processes, like neutrinoless double beta decay $(A, Z) \to (A, Z+2)+e^- +e^-$ (see, e.g., [28,29]). In the following we will be interested in models and the phenomenological consequences that result if $\tilde{U}_\nu$ corresponds to Eq. (3), while $\tilde{U}_\ell$ contains comparatively small angles, and vice versa. It proves convenient to introduce the abbreviations $\sin\theta_{ij}^{\ell,\nu} = \lambda_{ij} > 0$ for the small quantities we will use as expansion parameters in our further analysis.

Turning to the observables, the sines of the three mixing angles of the PMNS matrix U are given by

$$\sin^2\theta_{13} = |U_{e3}|^2, \qquad \sin^2\theta_{12} = \frac{|U_{e2}|^2}{1 - |U_{e3}|^2}, \qquad \sin^2\theta_{23} = \frac{|U_{\mu3}|^2}{1 - |U_{e3}|^2}. \tag{9}$$

The expressions quoted above are in terms of the absolute values of the elements of U, which emphasizes the independence of parametrization. In the case of 3ν mixing under discussion there are, in principle, three independent CP violation rephasing invariants, associated with the three CP violating phases of the PMNS matrix. The invariant related to the Dirac phase δ is given as

$$J_{CP} = \mathrm{Im}\big\{U_{e1}^* U_{\mu3}^* U_{e3} U_{\mu1}\big\}, \tag{10}$$

which controls the magnitude of CP violation effects in neutrino oscillations and is a directly observable quantity [30]. It is analogous to the rephasing invariant associated with the Dirac phase in the Cabibbo–Kobayashi–Maskawa quark mixing matrix, introduced in Ref. [31]. In addition to J_{CP}, there are two rephasing invariants associated with the two Majorana phases in the PMNS matrix, which can be chosen as[3] [32,33] (see also [29]):

$$S_1 = \mathrm{Im}\big\{U_{e1}U_{e3}^*\big\}, \qquad S_2 = \mathrm{Im}\big\{U_{e2}U_{e3}^*\big\}. \tag{11}$$

The rephasing invariants associated with the Majorana phases are not uniquely determined. Instead of S_1 defined above we could also have chosen $S_1' = \mathrm{Im}\{U_{\tau1}^* U_{\tau2}\}$ or $S_1'' = \mathrm{Im}\{U_{\mu1}U_{\mu2}^*\}$, while instead of S_2 we could have used $S_2' = \mathrm{Im}\{U_{\tau2}^* U_{\tau3}\}$ or $S_2'' = \mathrm{Im}\{U_{\mu2}U_{\mu3}^*\}$. The Majorana phases α and β, or β and $(\beta - \alpha)$, can be expressed in terms of the rephasing invariants in this way introduced [29], for instance via $\cos\beta = 1 - S_1^2/|U_{e1}U_{e3}|^2$. The expression for, e.g., $\cos\alpha$ in terms of S_1' is somewhat more cumbersome (it involves also J_{CP}) and we will not give it here. Note that CP violation due to the Majorana phase β requires that both $S_1 = \mathrm{Im}\{U_{e1}U_{e3}^*\} \neq 0$ and $\mathrm{Re}\{U_{e1}U_{e3}^*\} \neq 0$. Similarly, $S_2 = \mathrm{Im}\{U_{e2}^* U_{e3}\} \neq 0$ would imply violation of the CP symmetry only if in addition $\mathrm{Re}\{U_{e2}^* U_{e3}\} \neq 0$.

Finally, let us quote the current data on the neutrino mixing angles [2,8]:

$$\sin^2\theta_{12} = 0.30^{+0.02,0.10}_{-0.03,0.06}, \qquad \sin^2\theta_{23} = 0.50^{+0.08,0.18}_{-0.07,0.16}, \qquad |U_{e3}|^2 = 0^{+0.012,0.041}_{-0.000},$$

where we have given the best-fit values as well as the 1σ and 3σ allowed ranges.

3. Maximal atmospheric neutrino mixing and $U_{e3} = 0$ from the neutrino mass matrix

In this section we assume that maximal atmospheric neutrino mixing and vanishing $|U_{e3}|$ are realized in the limiting case, where U_ℓ corresponds to the unit matrix. We can obtain $\theta_{23}^\nu = -\pi/4$ and $\theta_{13}^\nu = 0$ by requiring $\mu-\tau$ exchange symmetry [5,23] of the neutrino mass matrix in the basis in that the charged lepton mass matrix is diagonal. Under this condition we have

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[2] In Section 4 it will be convenient to define instead of U_ℓ its transposed matrix as $U_\ell^T = O_{23}(\theta_{23}^\ell)U_{13}(\theta_{13}^\ell, \psi)O_{12}(\theta_{12}^\ell)$. In addition, $U_\nu^\dagger = P\tilde{U}_\nu Q$ will be used there.

[3] The expressions for the invariants $S_{1,2}$ that we give here and will use further in the discussion correspond to Majorana conditions for the fields of neutrinos with definite mass ν_j that do not contain phase factors, see, e.g., [29].

$$m_\nu = \begin{pmatrix} A_\nu & B_\nu & B_\nu \\ \cdot & D_\nu + E_\nu & E_\nu - D_\nu \\ \cdot & \cdot & D_\nu + E_\nu \end{pmatrix},$$

with $\quad A_\nu \equiv m_1 c_{12}^2 + e^{-2i\alpha} m_2 s_{12}^2, \quad B_\nu \equiv (e^{-2i\alpha} m_2 - m_1) c_{12} s_{12}/\sqrt{2},$

$$D_\nu \equiv e^{-2i\beta} m_3/2, \quad E_\nu \equiv \frac{1}{2}(e^{-2i\alpha} m_2 c_{12}^2 + m_1 s_{12}^2), \tag{12}$$

where $m_{1,2,3}$ are the neutrino masses. The indicated symmetry is assumed to hold in the charged lepton mass basis, although the charged lepton masses are obviously not μ–τ symmetric. However, such a scenario can, for example, be easily realized in models with different Higgs doublets generating the up- and down-like particle masses.

For the sines of the "small" angles in the matrix U_ℓ we introduce the convenient notation $\sin \theta_{ij}^\ell = \lambda_{ij} > 0$ with $ij = 12, 13, 23$. We obtain the following expressions for the observables relevant for neutrino oscillation in the case under consideration:

$$\sin^2 \theta_{12} \simeq \sin^2 \theta_{12}^\nu - \frac{1}{\sqrt{2}} \sin 2\theta_{12}^\nu (\lambda_{12} \cos\phi + \lambda_{13} \cos(\omega - \psi)),$$

$$|U_{e3}| \simeq \frac{1}{\sqrt{2}} |\lambda_{12} e^{i\phi} - \lambda_{13} e^{i(\omega-\psi)}|,$$

$$\sin^2 \theta_{23} \simeq \frac{1}{2} + \lambda_{23} \cos(\omega - \phi) - \frac{1}{4}(\lambda_{12}^2 - \lambda_{13}^2) + \frac{1}{2} \cos(\omega - \phi - \psi)\lambda_{12}\lambda_{13},$$

$$J_{CP} \simeq \frac{1}{4\sqrt{2}} \sin 2\theta_{12}^\nu (\lambda_{12} \sin\phi - \lambda_{13} \sin(\omega - \psi)). \tag{13}$$

Setting in these equations θ_{12}^ν to $\pi/4$ (to $\sin^{-1}\sqrt{1/3}$) reproduces the formulas from [10] (from [15]).

A comment on the CP phases is in order. The relevant Dirac CP violating phase(s) can be identified from the expression for the rephasing invariant J_{CP}: these are ϕ or $(\omega - \psi)$, depending on the relative magnitude of λ_{12} and λ_{13}. However, within the approach we are employing, a Dirac CP violating phase appearing in J_{CP} does not necessarily coincide with the Dirac phase in the standard parametrization of the PMNS matrix. For illustration it is sufficient to consider the simple case of $\lambda_{12} \neq 0$ and $\lambda_{13} = \lambda_{23} = 0$. Working to leading order in λ_{12}, it is easy to find that in this case the PMNS matrix can be written as

$$U \simeq \tilde{P} \begin{pmatrix} c_{12}^\nu e^{-i\phi} + \frac{\lambda_{12} s_{12}^\nu}{\sqrt{2}} & s_{12}^\nu e^{-i\phi} - \frac{\lambda_{12} c_{12}^\nu}{\sqrt{2}} & \frac{\lambda_{12}}{\sqrt{2}} \\ \lambda_{12} c_{12}^\nu e^{-i\phi} - \frac{s_{12}^\nu}{\sqrt{2}} & \lambda_{12} s_{12}^\nu e^{-i\phi} + \frac{c_{12}^\nu}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{s_{12}^\nu}{\sqrt{2}} & \frac{c_{12}^\nu}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \tilde{Q}, \tag{14}$$

where $\tilde{P} = \mathrm{diag}(e^{i\phi}, e^{i\phi}, e^{i\omega})$ and $\tilde{Q} = \mathrm{diag}(1, e^{i\sigma}, e^{i\tau})$. The phase matrix $\tilde{P}$ can be eliminated from U by a redefinition of the phases of the charged lepton fields. The Majorana phases α and $\beta' \equiv (\beta + \delta)$ can be directly identified (modulo 2π) with σ and τ. It is clear from the expressions (5) and (14) for U, however, that the phase ϕ does not coincide with the Dirac phase δ of the standard parametrization of U. Actually, the phase ϕ could be directly identified with the Dirac CP violating phase of a different parametrization of the PMNS matrix, namely, the parametrization in which $\tilde{U}$ in Eq. (14) is given by

$$\tilde{U} = O_{12}(\tilde\theta_{12}) \, \mathrm{diag}(e^{-i\delta'}, 1, 1) O_{23}(\tilde\theta_{23}) O_{12}(\theta_{12}')$$

$$= \begin{pmatrix} c_{12}' \tilde{c}_{12} e^{-i\delta'} - \tilde{c}_{23} s_{12}' \tilde{s}_{12} & \tilde{c}_{12} s_{12}' e^{-i\delta'} + c_{12}' \tilde{c}_{23} \tilde{s}_{12} & \tilde{s}_{12} \tilde{s}_{23} \\ -\tilde{c}_{12} \tilde{c}_{23} s_{12}' - c_{12}' \tilde{s}_{12} e^{-i\delta'} & c_{12}' \tilde{c}_{12} \tilde{c}_{23} - s_{12}' \tilde{s}_{12} e^{-i\delta'} & \tilde{c}_{12} \tilde{s}_{23} \\ s_{12}' \tilde{s}_{23} & -c_{12}' \tilde{s}_{23} & \tilde{c}_{23} \end{pmatrix}. \tag{15}$$

From this parametrization it would follow (using $|U_{e3}| = \sin \tilde\theta_{23} \sin \tilde\theta_{12}$ and $|U_{\mu3}/U_{\tau3}|^2 = \cos^2 \tilde\theta_{12} \tan^2 \tilde\theta_{23}$) that $\tilde\theta_{12}$ should be small and that atmospheric neutrino mixing was governed in leading order by $\tilde\theta_{23}$. In the limit of $\tilde\theta_{23} = \pm\pi/4$ and $\tilde\theta_{12} = 0$ one would have $|U_{e2}/U_{e1}|^2 = \tan^2 \theta_{12}'$. Hence, to leading order the solar neutrino mixing would be governed by θ_{12}' and leptonic CP violation in neutrino oscillations would be described by $J_{CP} = -\frac{1}{8} \sin 2\theta_{12}' \sin 2\tilde\theta_{12} \sin 2\tilde\theta_{23} \sin \tilde\theta_{23} \sin \delta'$. We would recover Eq. (14) from Eq. (15) if we identified $\tilde\theta_{23} = -\pi/4$, $\tilde{s}_{12} = -\lambda_{12}$, $c_{12}' = c_{12}^\nu$, $s_{12}' = s_{12}^\nu$, and $\delta' = \phi$.

We are not going to use the parametrization (15) in the following. Instead, the three neutrino mixing angles θ_{13}, θ_{12} and θ_{23} will be determined using the absolute values of the elements of the PMNS matrix, Eq. (9). Concerning the issue of CP violation in neutrino oscillations, we will work only with the CP violating rephasing invariant J_{CP}. However, it is still useful to keep in mind that, as the example discussed above illustrates, in the approach we are following the resulting Dirac CP violating phase, which is

the source of CP violation in neutrino oscillations, cannot always be directly identified[4] with the Dirac CP violating phase of the standard parametrization (5) of the neutrino mixing matrix.[5]

Returning to Eq. (13), we note that both $|U_{e3}|$ and $\sin^2\theta_{23}$ do not depend on the mixing angle θ_{12}^ν. The quantities λ_{12} and λ_{13} are crucial for the magnitudes of $|U_{e3}|$, $\sin^2\theta_{12}$ and J_{CP}, whereas they enter into the expression for $\sin^2\theta_{23}$ only quadratically. In fact, $\sin^2\theta_{23}$ receives first order corrections only from λ_{23}, which in turn contributes to the other observables only via terms proportional to λ_{23}^3. Unless there are accidental cancellations, $|U_{e3}|$ is lifted from its zero value due to non-zero λ_{12} and/or λ_{13}. Atmospheric neutrino mixing can be maximal, or very close to maximal, for instance if $\omega - \phi = \pi/2$. Note that λ_{12} and λ_{13} in the expressions for $\sin^2\theta_{12}$, $|U_{e3}|$ and J_{CP} are multiplied by cosines and/or sines of the same phases ϕ and $(\omega - \psi)$, respectively. This means that if the terms proportional to λ_{12} (to λ_{13}) dominate over the terms proportional to λ_{13} (to λ_{12})—we will refer to this possibility as λ_{12} (λ_{13})-dominance[6]—we have [10,13,23]:

$$\sin^2\theta_{12} = \sin^2\theta_{12}^\nu - \sin 2\theta_{12}^\nu |U_{e3}| \cos\gamma, \tag{16}$$

where $\gamma = \phi$ or $(\psi - \omega)$ is the CP violating phase (combination) appearing in the expression for J_{CP}, $J_{CP} \propto \sin\gamma$. The relation (16) implies a correlation of the initial 12-mixing in U_ν with $|U_{e3}|$ and the observable CP violation in neutrino oscillations. If $\tilde{U}_\nu$ is a bimaximal mixing matrix, we have $\sin^2\theta_{12}^\nu = 1/2$ and $\cos\gamma$ has to take a value close to one (while $|U_{e3}|$ has to be relatively large) in order to obtain sufficiently non-maximal solar neutrino mixing. Consequently, in the case of λ_{12} (λ_{13})-dominance, CP violation would be suppressed even though $|U_{e3}|$ can be sizable. On the other hand, if $\tilde{U}_\nu$ is a tri-bimaximal mixing matrix, we have $\sin^2\theta_{12}^\nu = 1/3$ which already is in good agreement with the present data. Hence, $|U_{e3}|\cos\gamma$ has to be relatively small. Consequently, CP violation can be sizable if $|U_{e3}|$ has a value close to the existing upper limit. This interesting feature has first been noticed in Ref. [15]. Generally, in the case of λ_{12} (λ_{13})-dominance we get from Eq. (13):

$$\sin^2\theta_{12} = \sin^2\theta_{12}^\nu - 4J_{CP}\cot\gamma, \tag{17}$$

where $\gamma = \phi$ ($\gamma = \psi - \omega$) for λ_{12}-dominance (λ_{13}-dominance). The following "sum-rule" holds as well:

$$\sin^2\theta_{12} = \sin^2\theta_{12}^\nu \pm \sqrt{|U_{e3}|^2 \sin^2 2\theta_{12}^\nu - 16J_{CP}^2}, \tag{18}$$

where the minus (plus) sign represents a positive (negative) cosine of the relevant Dirac CP violating phase. The sign ambiguity is unavoidable because the CP conserving quantity $\sin^2\theta_{12}$ can only depend on the cosine of a CP violating phase, whereas any CP violating quantity like J_{CP} can only depend on the sine of this phase. Knowing the cosine of a phase will never tell us the sign of the sine.[7] Note that since all parameters in Eq. (18) are rephasing invariant quantities, it can be applied to any parametrization of the PMNS matrix U and of the matrix $\tilde{U}_\nu$. If $\tilde{U}_\nu$ is a bimaximal (tri-bimaximal) mixing matrix, we get

$$\sin^2\theta_{12} = \frac{1}{2} \pm \sqrt{|U_{e3}|^2 - 16J_{CP}^2} \quad \text{and} \quad \sin^2\theta_{12} = \frac{1}{3}\left(1 \pm 2\sqrt{2}\sqrt{|U_{e3}|^2 - 6J_{CP}^2}\right), \tag{19}$$

respectively. The first relation has been obtained also in Ref. [18]. Obviously, one has to choose here the negative sign.

In Fig. 1 we show the allowed parameter space for the exact equations in the cases of $\sin^2\theta_{12}^\nu = 1/2$ (bimaximal mixing), $1/3$ (tri-bimaximal mixing) and 0.2. We have chosen the λ_{ij} to obey a CKM-like hierarchy: $0.1 \leqslant \lambda_{12} \leqslant 0.3$, $0.02 \leqslant \lambda_{23} \leqslant 0.08$ and $0 \leqslant \lambda_{13} \leqslant 0.01$. As $|U_{e3}|$ and $\sin^2\theta_{23}$ are independent of θ_{12}^ν we have plotted these observables only once. The chosen ranges of the λ_{ij} lead from Eq. (13) to a lower limit of $|U_{e3}| \gtrsim 0.09/\sqrt{2} \simeq 0.06$, as is seen in the figure. Improved future limits on the range of $\sin^2\theta_{12}$ and, in particular, on the magnitude of $|U_{e3}|$ can give us valuable information on the structure of U_ℓ. The allowed parameter space of $\sin^2\theta_{23}$ is roughly half of its allowed 3σ range. The interplay of θ_{12}^ν and leptonic CP violation in neutrino oscillations mentioned above results in the "falling donut" structure when J_{CP} is plotted against $\sin^2\theta_{12}$. We can also directly plot the sum-rule from Eq. (18), which is shown in Fig. 2. As a consequence of varying the observables in Eq. (18) we can extend the parameter space to smaller values of $|U_{e3}|$. In fact, if U_ν corresponds to tri-bimaximal mixing, U_{e3} is allowed to vanish. Eq. (13) can be used to understand the results in Fig. 2: if, for instance, we have $\sin^2\theta_{12}^\nu = 1/2$, the experimental upper limit of $(\sin^2\theta_{12})_{\max} = 0.4$ implies that $|U_{e3}| \geqslant 1/2 - (\sin^2\theta_{12})_{\max} \simeq 0.1$. On the other hand, for $\sin^2\theta_{12}^\nu = 0.2$, and therefore $\sin 2\theta_{12}^\nu = 0.8$, we have with $(\sin^2\theta_{12})_{\min} = 0.24$ that $|U_{e3}| \geqslant ((\sin^2\theta_{12})_{\min} - 0.20)/0.8 \simeq 0.05$, which is in agreement with the figure. A more stringent limit on, or a value of, $|U_{e3}|^2 \lesssim 0.01$ would strongly disfavor (or rule out) the simple bimaximal mixing scenario.

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[4] The same conclusion is valid, e.g., for the Dirac phase in the relation given in Eq. (1) of the third and fourth articles quoted in Ref. [13].

[5] The matrix V in the parametrization (5) of the PMNS matrix, and the matrix $\tilde{U}$ in the parametrization (15) are connected by a unitary matrix: $V = W\tilde{U}$. The latter reduces to the unit matrix (or to a diagonal phase matrix) only when the Dirac CP violating phases δ and δ', present in V and $\tilde{U}$, take CP conserving values: $\delta = k\pi$, $\delta' = k'\pi$, $k, k' = 0, 1, 2, \ldots$. In this case we can write $V = \tilde{U}$ and can express the angles of V in terms of the angles of $\tilde{U}$, and vice versa.

[6] More concretely, the conditions for, e.g., λ_{12}-dominance are: $|\lambda_{12}\cos\phi| \gg |\lambda_{13}\cos(\omega - \psi)|$ and $|\lambda_{12}\sin\phi| \gg |\lambda_{13}\sin(\omega - \psi)|$.

[7] The same ambiguity will show up if one identifies the phase ϕ with the phase δ of a given parametrization of the PMNS matrix, as done, e.g., in Ref. [13]. See also the comments given after Eq. (13).

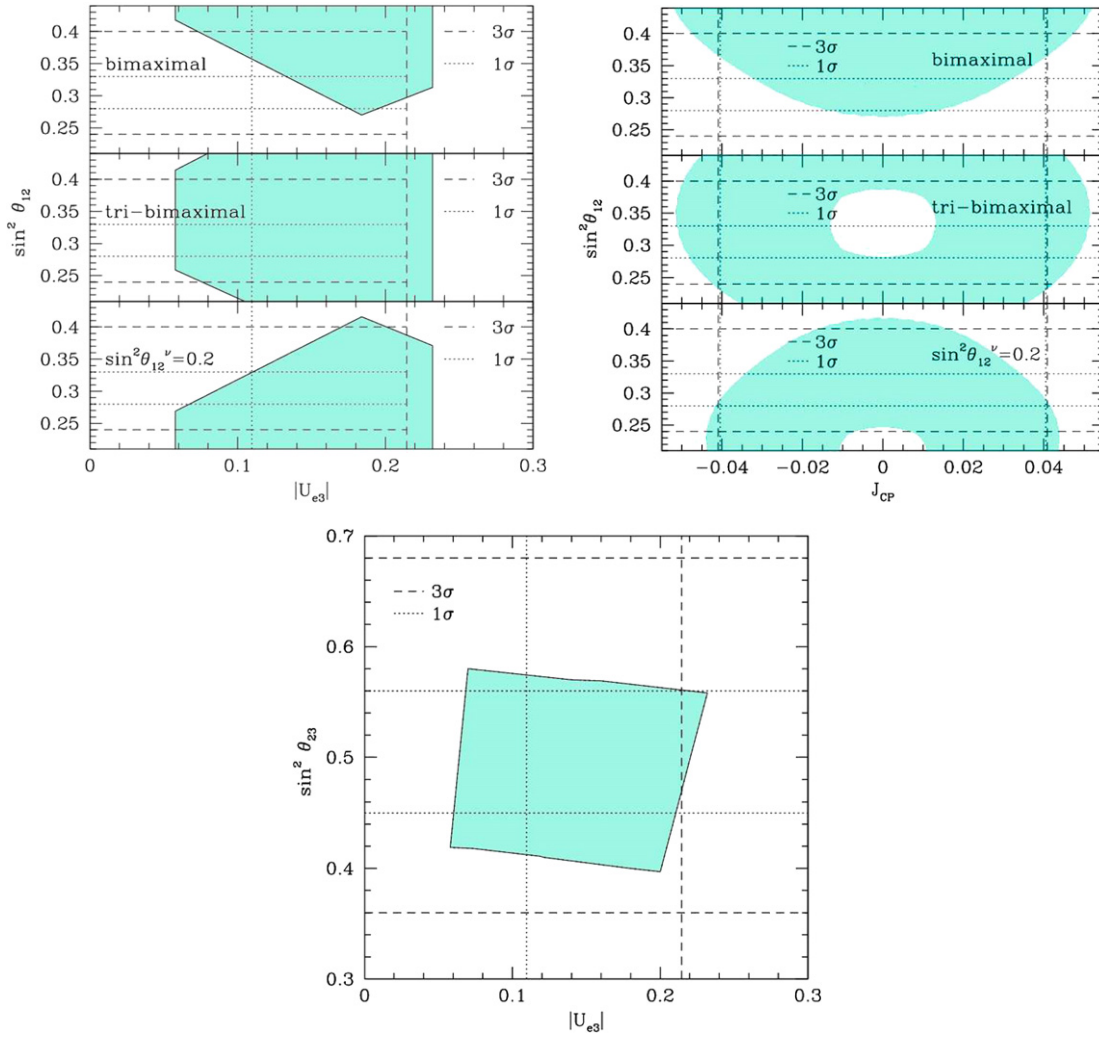Fig. 1. Correlations resulting from $U = U_\ell^\dagger U_\nu$ if U_ℓ is CKM-like and U_ν has maximal θ_{23}^ν, vanishing θ_{13}^ν, but free θ_{12}^ν, for three representative values of θ_{12}^ν (see text for details). The currently allowed 1σ and 3σ ranges of the observables are also indicated.
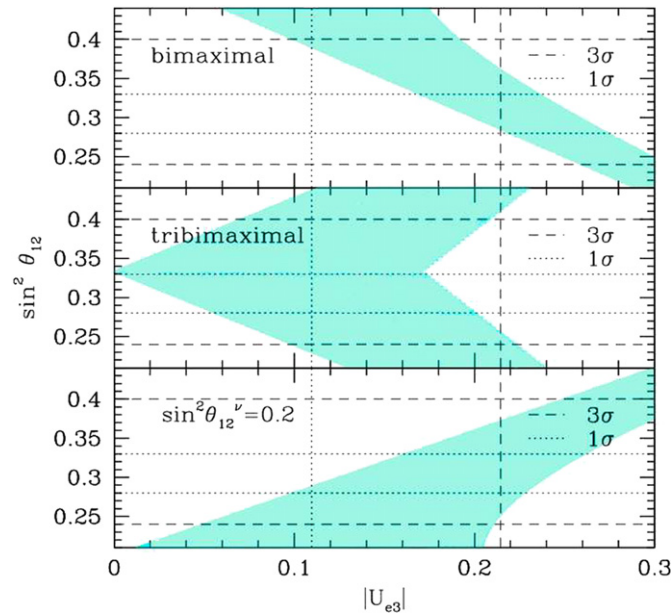

Fig. 2. The sum-rule from Eq. (18).

The equations given up to this point are also valid if neutrinos are Dirac particles. We will discuss now briefly the observables describing the CP violation associated with the Majorana nature of the massive neutrinos. We find that in the case under discussion

$$S_1 \simeq -\frac{1}{\sqrt{2}} \cos\theta_{12}^{\nu} \big(\lambda_{12} \sin(\phi + \tau) - \lambda_{13} \sin(\omega - \psi + \tau) \big),$$

$$S_2 \simeq \frac{1}{\sqrt{2}} \sin\theta_{12}^{\nu} \big(\lambda_{12} \sin(\sigma - \phi - \tau) - \lambda_{13} \sin(\sigma - (\omega - \psi + \tau)) \big). \tag{20}$$

According to the parameterization of Eq. (5), we have $S_1 = -c_{12}c_{13}s_{13}\sin\beta$ and $S_2 = s_{12}c_{13}s_{13}\sin(\alpha - \beta)$. Hence, we find that in the case of λ_{12}-dominance, β is associated[8] with $\phi + \tau$, while if the terms proportional to λ_{13} dominate over the terms proportional to λ_{12}, the phase β is associated with $\psi - \omega - \tau$. In both cases α is associated with σ.

Obviously, if $\sigma = 0$ we get in the case of λ_{12}- or λ_{13}-dominance that $S_1 \simeq S_2 \tan\theta_{12}^{\nu}$. We note also that, as it follows from Eqs. (13) and (20), for $\tau \simeq 0$ the Dirac CP violating phase δ will coincide with the Majorana CP violating phase β.

The most natural possibility for the structure of U_ℓ is that it is "CKM-like", i.e., $\lambda_{23} = A\lambda_{12}^2$ and $\lambda_{13} = B\lambda_{12}^3$ with A and B of order one. The resulting equations are

$$\sin^2\theta_{12} \simeq \sin^2\theta_{12}^{\nu} - \frac{1}{\sqrt{2}}\cos\phi \sin 2\theta_{12}^{\nu}\lambda_{12} + \frac{1}{2}\cos 2\theta_{12}^{\nu}\lambda_{12}^2, \qquad |U_{e3}| \simeq \frac{\lambda_{12}}{\sqrt{2}},$$

$$\sin^2\theta_{23} \simeq \frac{1}{2} - \frac{1}{4}\big(1 - 4B\cos(\omega - \phi)\big)\lambda_{12}^2, \qquad J_{CP} \simeq \frac{1}{4\sqrt{2}}\lambda_{12}\sin 2\theta_{12}^{\nu}\sin\phi, \tag{21}$$

plus cubic terms. The sum-rule in Eq. (18) is of course valid. For the invariants describing the Majorana phases we have

$$S_1 \simeq -\frac{1}{\sqrt{2}}\big(\lambda_{12}\cos\theta_{12}^{\nu}\sin(\phi + \tau) + \lambda_{12}^2\sin\theta_{12}^{\nu}\sin\tau\big),$$

$$S_2 \simeq -\frac{1}{\sqrt{2}}\big(\lambda_{12}\sin\theta_{12}^{\nu}\sin(\phi - \sigma + \tau) + \lambda_{12}^2\cos\theta_{12}^{\nu}\sin(\sigma - \tau)\big). \tag{22}$$

4. Maximal atmospheric mixing and $U_{e3} = 0$ from the charged lepton mass matrix

Now we study the equally interesting possibility that maximal θ_{23} and vanishing $|U_{e3}|$ are realized in the limiting case, where U_ν is equivalent to the unit matrix. In this scenario we have

$$U_\ell^\dagger = U_\ell^T = \begin{pmatrix} c_{12}^\ell & s_{12}^\ell & 0 \\ -\frac{s_{12}^\ell}{\sqrt{2}} & \frac{c_{12}^\ell}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{s_{12}^\ell}{\sqrt{2}} & \frac{c_{12}^\ell}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{23}$$

where we have to define $U_\ell^T = O_{23}(\theta_{23}^\ell)U_{13}(\theta_{13}^\ell, \psi)O_{12}(\theta_{12}^\ell)$ in order to have the rotations in the correct order, cf. Eq. (8). Note that U_ℓ is real and therefore $m_\ell^\dagger m_\ell = U_\ell(m_\ell^{\text{diag}})^2 U_\ell^\dagger$ is symmetric. Reconstructing this matrix gives

$$m_\ell^\dagger m_\ell = \begin{pmatrix} m_e^2(c_{12}^\ell)^2 + \frac{1}{2}(s_{12}^\ell)^2(m_\mu^2 + m_\tau^2) & c_{12}^\ell s_{12}^\ell(m_e^2 - \frac{1}{2}m_\mu^2 - \frac{1}{2}m_\tau^2) & \frac{1}{2}s_{12}^\ell(m_\mu^2 - m_\tau^2) \\ \cdot & m_e^2(s_{12}^\ell)^2 + \frac{1}{2}(c_{12}^\ell)^2(m_\mu^2 + m_\tau^2) & \frac{1}{2}c_{12}^\ell(m_\mu^2 - m_\tau^2) \\ \cdot & \cdot & \frac{1}{2}(m_\mu^2 + m_\tau^2) \end{pmatrix}, \tag{24}$$

which does not obey a simple exchange symmetry as the neutrino mass matrix in Eq. (12). However, there are relations between the entries: for instance, if we additionally assume $\theta_{12}^\ell = -\pi/4$, we find

$$m_\ell^\dagger m_\ell = \begin{pmatrix} A_\ell + D_\ell & A_\ell - D_\ell & B_\ell \\ \cdot & A_\ell + D_\ell & B_\ell \\ \cdot & \cdot & 2A_\ell \end{pmatrix},$$

$$\text{with} \quad A_\ell \equiv \frac{1}{4}\big(m_\mu^2 + m_\tau^2\big), \quad B_\ell \equiv \big(m_\tau^2 - m_\mu^2\big)/\sqrt{8}, \quad D_\ell \equiv \frac{1}{2}m_e^2. \tag{25}$$

Discrete symmetries might be capable of generating such a texture. Another hint towards a possible origin of such a matrix can be obtained by noting that due to $m_\tau^2 \gg m_\mu^2 \gg m_e^2$ the entries are all of similar magnitude [11], and therefore $m_\ell^\dagger m_\ell$ resembles the mass matrices of the "flavor democratic" type.

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[8] Actually such an identification is always valid modulo 2π. For simplicity, we will omit stating this explicitly from here on.

We have to multiply $U_\ell^\dagger = U_\ell^T$ from Eq. (23) with the matrix U_ν to obtain the PMNS matrix. Let us first assume that U_ν is given by the hermitian adjoint of Eq. (7): $U_\nu^\dagger = P O_{23}(\theta_{23}^\nu) U_{13}(\theta_{13}^\nu, \xi) O_{12}(\theta_{12}^\nu) Q$. This will bring the 12-rotations of U_ℓ and U_ν directly together and, in absence of phases, would lead to $\theta_{12} = \theta_{12}^\ell - \theta_{12}^\nu$, a feature which makes this possibility interesting for Quark–Lepton Complementarity scenarios [16–18]. For the neutrino oscillation observables we get

$$\sin^2 \theta_{12} \simeq \sin^2 \theta_{12}^\ell - \lambda_{12} \sin 2\theta_{12}^\ell \cos \sigma + \frac{1}{4}(\lambda_{13}^2 - \lambda_{23}^2) \sin^2 2\theta_{12}^\ell + \lambda_{12}^2 \cos 2\theta_{12}^\ell,$$

$$|U_{e3}| \simeq |\lambda_{23} \sin \theta_{12}^\ell + \lambda_{13} \cos \theta_{12}^\ell e^{i(\xi-\sigma)}|,$$

$$\sin^2 \theta_{23} \simeq \frac{1}{2} + \lambda_{23} \cos \theta_{12}^\ell \cos(\xi - \sigma + \tau) - \lambda_{13} \sin \theta_{12}^\ell \cos \tau,$$

$$J_{CP} \simeq -\frac{1}{4} \sin 2\theta_{12}^\ell (\lambda_{23} \sin \theta_{12}^\ell \sin(\xi - \sigma + \tau) + \lambda_{13} \cos \theta_{12}^\ell \sin \tau). \tag{26}$$

The parameter λ_{12} is crucial for obtaining a sufficiently non-maximal angle θ_{12} in the case of a bimaximal $U_\ell^\dagger$. However, λ_{12} appears only in terms proportional to λ_{12}^3 in $|U_{e3}|$, $\sin^2 \theta_{23}$ and J_{CP}. In these latter observables λ_{13} and λ_{23} are multiplied by the sines or cosines of the same phases. As a consequence, we can write down a correlation analogous to the one given in Eq. (17). Namely, if the terms proportional to λ_{23} dominate over the terms proportional to λ_{13} ("λ_{23}-dominance"), we have

$$\sin^2 \theta_{23} \simeq \frac{1}{2} - 2 J_{CP} \frac{\cot(\xi - \sigma + \tau)}{\sin^2 \theta_{12}^\ell}. \tag{27}$$

The analogue of the sum-rule in Eq. (18) is

$$\sin^2 \theta_{23} \simeq \frac{1}{2} \pm \frac{1}{\sin^2 \theta_{12}^\ell} \sqrt{|U_{e3}|^2 \cos^2 \theta_{12}^\ell \sin^2 \theta_{12}^\ell - 4 J_{CP}^2}, \tag{28}$$

where the plus (minus) sign corresponds to $\cos(\xi - \sigma + \tau) > 0$ ($\cos(\xi - \sigma + \tau) < 0$). In this scenario the value of the atmospheric neutrino mixing angle is correlated with the magnitude of CP violation effects in neutrino oscillations. In the case of $\sin^2 \theta_{12}^\ell = 1/2$ or $1/3$ we find

$$\sin^2 \theta_{23} - \frac{1}{2} = \pm \sqrt{|U_{e3}|^2 - 16 J_{CP}^2} \quad \text{or} \quad \sin^2 \theta_{23} - \frac{1}{2} = \pm \sqrt{2} \sqrt{|U_{e3}|^2 - 18 J_{CP}^2}, \tag{29}$$

respectively. The first relation has been obtained also in Ref. [18]. A high precision measurement of $\sin^2 \theta_{23}$, combined with a sufficiently stringent limit on, or a relatively small measured value of, $|U_{e3}|^2$ might allow to discriminate between the simple bimaximal and tri-bimaximal mixing scenarios we are considering.

The corresponding relations in the case of λ_{13}-dominance are

$$\sin^2 \theta_{23} \simeq \frac{1}{2} + 2 J_{CP} \frac{\cot \tau}{\cos^2 \theta_{12}^\ell}, \tag{30}$$

and

$$\sin^2 \theta_{23} \simeq \frac{1}{2} \mp \frac{1}{\cos^2 \theta_{12}^\ell} \sqrt{|U_{e3}|^2 \cos^2 \theta_{12}^\ell \sin^2 \theta_{12}^\ell - 4 J_{CP}^2}, \tag{31}$$

where the minus (plus) sign corresponds to $\cos \tau > 0$ ($\cos \tau < 0$). The results for $\sin^2 \theta_{12}^\ell = 1/2$ or $1/3$ can be easily obtained as

$$\sin^2 \theta_{23} - \frac{1}{2} = \mp \sqrt{|U_{e3}|^2 - 16 J_{CP}^2} \quad \text{or} \quad \sin^2 \theta_{23} - \frac{1}{2} = \mp \frac{1}{\sqrt{2}} \sqrt{|U_{e3}|^2 - 18 J_{CP}^2}. \tag{32}$$

In Fig. 3 we show the allowed parameter space for the exact equations in the cases of $\sin^2 \theta_{12}^\ell = 1/2$ (bimaximal), $1/3$ (tri-bimaximal) and 0.2. We have chosen again the λ_{ij} to follow a CKM-like hierarchy with $0.1 \leqslant \lambda_{12} \leqslant 0.3$, $0.02 \leqslant \lambda_{23} \leqslant 0.08$ and $0 \leqslant \lambda_{13} \leqslant 0.01$. Note that—in contrast to the first scenario—$|U_{e3}|$ is much smaller and can even vanish exactly not only when $\sin^2 \theta_{12}^\ell = 1/3$, but also for $\sin^2 \theta_{12}^\ell = 1/2$ or 0.2. Moreover, the range of the λ_{ij} and the dependence of $\sin^2 \theta_{23}$ on them lead to the absence of a characteristic donut-like structure as seen in Fig. 1. For a CKM-like U_ν, the importance of $\sin^2 \theta_{12}^\ell$ for $\sin^2 \theta_{23}$ and $|U_{e3}|$ is not as strong as it is in the first scenario considered in Section 3. As mentioned above, the value of $\sin^2 \theta_{12}^\ell$ is important mainly for the required magnitude of λ_{12} which is responsible only for subleading contributions to the other parameters. As in the first scenario, atmospheric neutrino mixing can be maximal. If $|U_{e3}|$ will be observed to be close to its current limit, scenarios in which a CKM-like U_ν corrects U_ℓ corresponding to $|U_{e3}| = 0$ and $\theta_{23} = \pi/4$ will be ruled out.

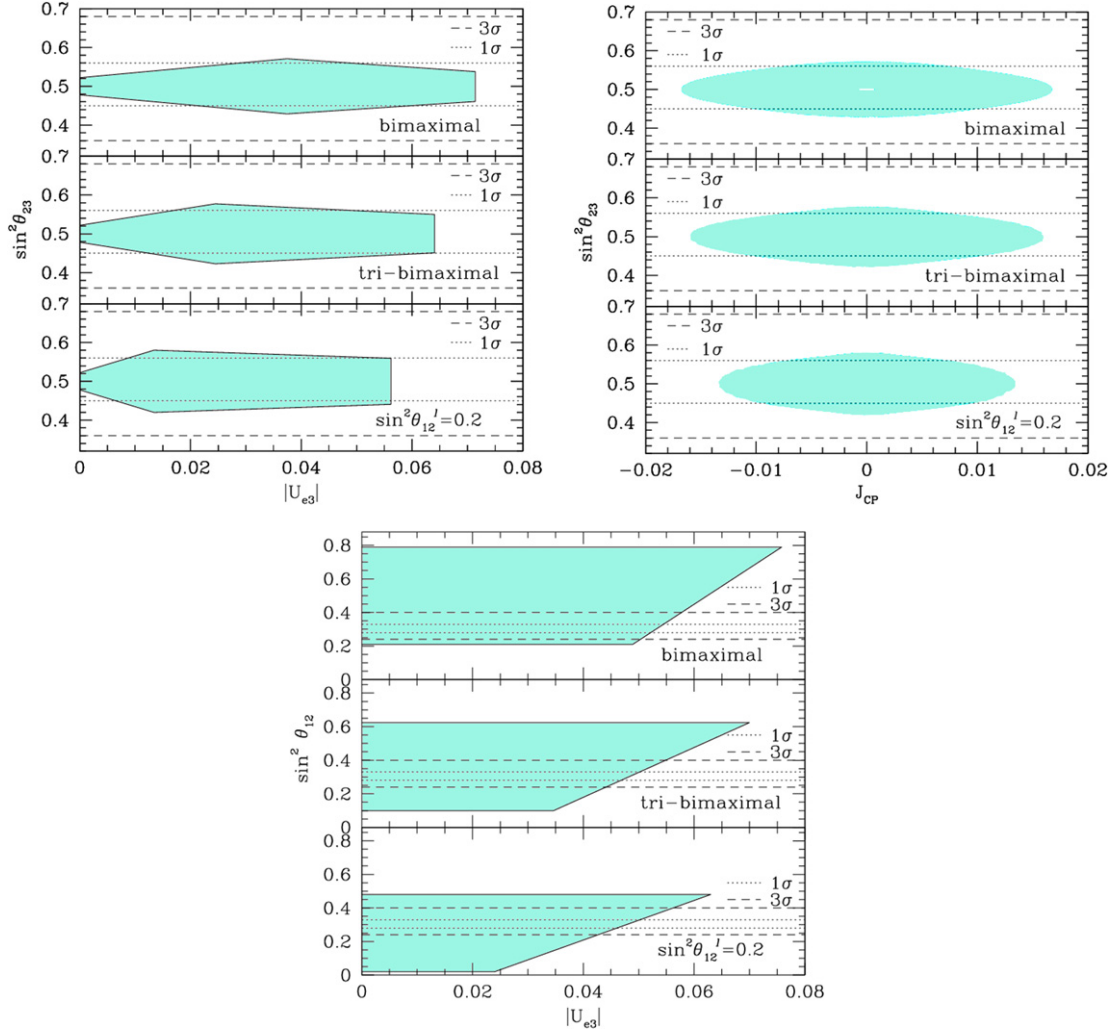The rephasing invariants associated with the Majorana CP violation are given by

Fig. 3. Correlations resulting from $U = U_\ell^\dagger U_\nu$ if U_ν is CKM-like and $U_\ell^\dagger$ has maximal θ_{23}^ℓ and vanishing θ_{13}^ℓ, but free θ_{12}^ℓ. The results shown correspond to three representative values of θ_{12}^ℓ. The currently allowed 1σ and 3σ ranges of the observables are also indicated.

$$S_1 \simeq -\cos\theta_{12}^\ell\big(\cos\theta_{12}^\ell \sin(\omega+\xi)\lambda_{13} + \sin\theta_{12}^\ell \sin(\omega+\sigma)\lambda_{23}\big),$$
$$S_2 \simeq -\sin\theta_{12}^\ell\big(\cos\theta_{12}^\ell \sin(\omega-\phi+\xi-\sigma)\lambda_{13} + \sin\theta_{12}^\ell \sin(\omega-\phi)\lambda_{23}\big). \tag{33}$$

In the case of λ_{23}-dominance (λ_{13}-dominance) we find that β is associated with $\omega+\sigma$ ($\omega+\xi$). In both cases α is associated with $\phi+\sigma$.

Finally, we give the formulas for the case of a CKM-like U_ν, i.e., $\lambda_{23} = A\lambda_{12}^2$ and $\lambda_{13} = B\lambda_{12}^3$ with A and B of order one:

$$\sin^2\theta_{12} \simeq \sin^2\theta_{12}^\ell - \cos\sigma \sin 2\theta_{12}^\ell \lambda_{12} + \cos 2\theta_{12}^\ell \lambda_{12}^2, \qquad |U_{e3}| \simeq B\sin\theta_{12}^\ell \lambda_{12}^2,$$
$$\sin^2\theta_{23} \simeq \frac{1}{2} + B\cos\theta_{12}^\ell \cos(\xi-\sigma+\tau)\lambda_{12}^2,$$
$$J_{\mathrm{CP}} \simeq -\frac{1}{4}B\sin 2\theta_{12}^\ell \sin\theta_{12}^\ell \sin(\xi-\sigma+\tau)\lambda_{12}^2. \tag{34}$$

We note that for an identical in magnitude correction, $|U_{e3}|$ is smaller by one order in λ_{12}, i.e., $|U_{e3}| \propto \lambda_{12}^2$ if the correction comes from U_ν in contrast to $|U_{e3}| \propto \lambda_{12}$ if the correction comes from U_ℓ.

Consider next the case of U_ν (and not $U_\nu^\dagger$ as before) given by Eq. (7). For the neutrino oscillation observables we obtain

$$\sin^2\theta_{12} \simeq \sin^2\theta_{12}^\ell + \lambda_{12}\sin 2\theta_{12}^\ell \cos\phi + \frac{1}{4}\big(\lambda_{13}^2 - \lambda_{23}^2\big)\sin^2 2\theta_{12}^\ell + \lambda_{12}^2 \cos 2\theta_{12}^\ell,$$
$$|U_{e3}| \simeq \big|\lambda_{23}\sin\theta_{12}^\ell + \lambda_{13}\cos\theta_{12}^\ell e^{i(\phi+\xi)}\big|,$$
$$\sin^2\theta_{23} \simeq \frac{1}{2} - \lambda_{23}\cos\theta_{12}^\ell \cos(\omega-\phi) + \lambda_{13}\sin\theta_{12}^\ell \cos(\omega+\xi),$$

$$J_{\rm CP} \simeq -\frac{1}{4}\sin 2\theta_{12}^\ell \big(\lambda_{23}\sin\theta_{12}^\ell \sin(\omega-\phi) + \lambda_{13}\cos\theta_{12}^\ell \sin(\omega+\xi)\big). \tag{35}$$

The resulting formulas are very similar to those derived earlier: they can be obtained formally from Eq. (26) by simple changes of phases. Since in addition λ_{13} and λ_{23} in the expressions for $\sin^2\theta_{23}$ and $J_{\rm CP}$ in Eq. (35) are multiplied by the sines or cosines of the same phases, both the sum-rule corresponding to λ_{23}-dominance, Eq. (28), and the sum-rule associated with λ_{13}-dominance, Eq. (31), are valid in this case as well.

5. Summary

The results from various neutrino oscillation experiments indicate that θ_{23} is very close to $\pi/4$ and θ_{13} is very close to zero. It is natural to assume that at leading order these mixing angles take the quoted extreme values and some form of perturbation leads to non-zero θ_{13} and non-maximal θ_{23}. It is hoped that this perturbation is imprinted in correlations between various observables. Future precision experiments can tell us whether there are such correlations, which can then be used to identify the perturbation and to obtain thereby valuable hints on the flavor structure of the underlying theory. In this Letter we have studied one interesting class of perturbations: because the observable lepton mixing matrix is a product of the diagonalization matrices of the charged lepton and neutrino mass matrices, $U = U_\ell^\dagger U_\nu$, we assumed that in the limit of one of these matrices being the unit matrix, maximal θ_{23} and zero θ_{13} would result. When the second matrix deviates from being the unit matrix, i.e., has a CKM-like form, we investigated the effects on the CP conserving and CP violating observables. Free parameters are the small angles of the "correction matrix", the 12-mixing angle of the leading matrix, and various phases. Scenarios like bimaximal mixing, tri-bimaximal mixing or Quark–Lepton Complementarity are special cases of our analysis. We consistently worked only with rephasing invariants in order to avoid the subtleties of identifying CP phases within different parameterizations. We should stress here also that our analysis is independent of the neutrino mass ordering and hierarchy.

In the first scenario we have considered, the neutrino sector alone is responsible for zero θ_{13} and maximal θ_{23}. Requiring the neutrino mass matrix to obey a μ–τ symmetry can generate such a mixing pattern. Figs. 1 and 2 illustrate the results. We find that $|U_{e3}|$ will typically be non-zero, proportional to the sine of the largest angle in U_ℓ, and in most of the cases will be well within reach of up-coming experiments. If U_ν is bimaximal, $|U_{e3}|$ should satisfy $|U_{e3}| \gtrsim 0.1$ in order for $\sin^2\theta_{12}$ to be within the 3σ interval allowed by the current data. There is no similar constraint on $|U_{e3}|$ in the case of tri-bimaximal U_ν: even a vanishing value of $|U_{e3}|$ is allowed. Atmospheric neutrino mixing can be maximal. There is a correlation between the solar neutrino mixing, the magnitude of $|U_{e3}|$ and CP violation in neutrino oscillations, given by $\sin^2\theta_{12} = \sin^2\theta_{12}^\nu \pm \sqrt{|U_{e3}|^2 \sin^2 2\theta_{12}^\nu - 16 J_{\rm CP}^2}$, where θ_{12}^ν is the 12-rotation angle in U_ν. The magnitude of leptonic CP violation is rather sensitive to θ_{12}^ν. We have shown as well that in the approach we are following the resulting Dirac CP violating phase, which is the source of CP violation in neutrino oscillations, cannot always be directly identified with the Dirac CP violating phase of the standard PDG parametrization of the PMNS matrix. The identification of the Majorana CP violating phases is typically rather straightforward.

The alternative possibility corresponds to the charged lepton sector alone being responsible for zero θ_{13} and maximal θ_{23}. We have identified the required texture of the charged lepton mass matrix in Eq. (24) and plot the observables in Fig. 3. Typically, $|U_{e3}|$ is smaller than in the first scenario, being proportional to the sine of the second largest angle in U_ν. Another important difference with the first case is that now there exists a correlation between atmospheric neutrino mixing, the magnitude of $|U_{e3}|$ and CP violation in neutrino oscillations: with θ_{12}^ℓ being the 12-rotation angle in U_ℓ we find that $\sin^2\theta_{23} \simeq \frac{1}{2} \pm \frac{1}{\sin^2\theta_{12}^\ell}\sqrt{|U_{e3}|^2 \sin^2\theta_{12}^\ell \cos^2\theta_{12}^\ell - 4 J_{\rm CP}^2}$, or $\sin^2\theta_{23} \simeq \frac{1}{2} \mp \frac{1}{\cos^2\theta_{12}^\ell}\sqrt{|U_{e3}|^2 \cos^2\theta_{12}^\ell \sin^2\theta_{12}^\ell - 4 J_{\rm CP}^2}$, depending on whether the 23- or 13-rotation angle in U_ℓ dominates.

We find that both scenarios are in agreement with the existing neutrino oscillation data, have interesting phenomenology and testable differences. Future higher precision determinations of $\sin^2\theta_{12}$ and $\sin^2\theta_{23}$, and more stringent constraints on, or a measurement of, $|U_{e3}|$ can provide crucial tests of these simplest scenarios, shedding more light on whether any of the two scenarios is realized in Nature.

Acknowledgements

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