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Low-scale seesaw and the CP violation in neutrino oscillations

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Abstract

We consider a version of the low-scale type I seesaw mechanism for generating small neutrino masses, as an alternative to the standard seesaw scenario. It involves two right-handed (RH) neutrinos ν_{1R} and ν_{2R} having a Majorana mass term with mass M, which conserves the lepton charge L. The RH neutrino ν_{2R} has lepton-charge conserving Yukawa couplings $g_{\ell 2}$ to the lepton and Higgs doublet fields, while small lepton-charge breaking effects are assumed to induce tiny lepton-charge violating Yukawa couplings $g_{\ell 1}$ for ν_{1R}, $l = e, \mu, \tau$. In this approach the smallness of neutrino masses is related to the smallness of the Yukawa coupling of ν_{1R} and not to the large value of M: the RH neutrinos can have masses in the few GeV to a few TeV range. The Yukawa couplings $|g_{\ell 2}|$ can be much larger than $|g_{\ell 1}|$, of the order $|g_{\ell 2}| \sim 10^{-4}$–$10^{-2}$, leading to interesting low-energy phenomenology. We consider a specific realisation of this scenario within the Froggatt–Nielsen approach to fermion masses. In this model the Dirac CP violation phase δ is predicted to have approximately one of the values $\delta \simeq \pi/4$, $3\pi/4$, or $5\pi/4$, $7\pi/4$, or to lie in a narrow interval around one of these values. The low-energy phenomenology of the considered low-scale seesaw scenario of neutrino mass generation is also briefly discussed.

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1. Introduction

The seesaw mechanism [1] of neutrino mass generation is a very attractive mechanism which explains naturally the small masses of the neutrinos. According to the standard seesaw scenario the smallness of neutrino masses has its origin from large lepton-number violating Majorana masses of right-handed (RH) neutrinos. A very appealing aspect of the seesaw scenario is that we can relate the existence of large Majorana masses of the RH neutrinos to a spontaneous breaking of some high scale symmetry, for example, GUT symmetry. However, direct tests of the standard seesaw mechanism are almost impossible due to the exceedingly large masses of the RH neutrinos.

In the present article we consider an alternative mechanism for generating small neutrino masses. It involves two RH neutrinos ν_{1R} and ν_{2R} which have a Majorana mass $M\,\nu_{1R}^T\,C^{-1}\,\nu_{2R}$, where C is the charge conjugation matrix. Assuming that ν_{1R} and ν_{2R} carry total lepton charges $L(\nu_{1R}) = -1$ and $L(\nu_{2R}) = +1$, respectively, this mass term conserves L. This implies that, as long as L is conserved, ν_{1R} and ν_{2R} (more precisely, ν_{1R} and $\nu_{2L}^C \equiv C\,\overline{\nu_{2R}}^T$) form a heavy Dirac neutrino. Since $L(\nu_{2R}) = +1$, ν_{2R} can have lepton-charge conserving Yukawa couplings, $-\mathcal{L} \supset g_{\ell 2}\,\overline{\nu_{2R}}\,H^{c\dagger}\,L_\ell$, where $\ell = e, \mu, \tau$, $L_\ell(x) = (\nu_{\ell L}(x) \;\; \ell_L(x))^T$ and $H^c = i\sigma_2 H^*$, $H = (H^+ \;\; H^0)^T$ being the Higgs doublet field whose neutral component acquires a vacuum expectation value (VEV). On the other hand, the RH neutrino ν_{1R} cannot have a neutrino Yukawa coupling as long as lepton charge L is conserved.

We assume further that some small lepton-charge breaking effects induce tiny lepton-charge violating Yukawa couplings for ν_{1R}, namely $-\mathcal{L} \supset g_{\ell 1}\,\overline{\nu_{1R}}\,H^{c\dagger}\,L_\ell$, $\ell = e, \mu, \tau$, with $|g_{\ell 1}| \ll |g_{\ell' 2}|$, $\ell, \ell' = e, \mu, \tau$. Our setup will imply that the lepton-charge breaking diagonal Majorana mass terms are either forbidden or suppressed. In this case ν_{1R} and ν_{2R} (i.e., ν_{1R} and ν_{2L}^C) form a pseudo-Dirac pair. In this scenario the smallness of neutrino masses is due to the small Yukawa coupling $|g_{\ell 1}| \ll 1$ and hence we do not have to introduce the large Majorana mass M of the standard seesaw scenario. The mass M of the $\nu_{1R}^T\,C^{-1}\,\nu_{2R}$ mass term can be at the weak scale.

The strong hierarchy $|g_{\ell 1}| \ll |g_{\ell' 2}|$ between the two sets of Yukawa couplings can be realised rather naturally, for example, within the Froggatt–Nielsen (FN) scenario [2]. Employing this scenario we will additionally consider that the Yukawa couplings $g_{\ell 2}$ obey a standard FN hierarchy [3], $|g_{e2}| : |g_{\mu 2}| : |g_{\tau 2}| \sim \epsilon : 1 : 1$, $\epsilon \sim 0.2$. The magnitude of the Yukawa couplings of ν_{1R} should be completely different from that of the Yukawa coupling of ν_{2R}. However, due to the usual $\mathcal{O}(1)$ ambiguity in the FN approach, it is impossible to predict unambiguously the flavour dependence of $g_{\ell 1}$ and thus the ratios $|g_{e1}| : |g_{\mu 1}| : |g_{\tau 1}|$.

We show in the present article, in particular, that in the model of neutrino mass generation with two RH neutrinos with the hierarchy and flavour structure of their Yukawa couplings and the mass term outlined above the Dirac CP-violating (CPV) phase is predicted to have one of the values $\delta \simeq \pi/4,\ 3\pi/4$, or $5\pi/4,\ 7\pi/4$.

2. General setup

We minimally extend the Standard Model (SM) by adding two RH neutrinos, i.e., two chiral fields $\nu_{aR}(x)$, $a = 1, 2$, which are singlets under the SM gauge symmetry group. Following the notations of Refs. [4–7], the relevant low-energy Lagrangian is

$$\mathcal{L}_\nu = -\overline{\nu_{aR}}\,(M_D^T)_{a\ell}\,\nu_{\ell L} - \frac{1}{2}\,\overline{\nu_{aR}}\,(M_N)_{ab}\,\nu_{bL}^C + \text{h.c.}\,, \tag{2.1}$$

with $\nu_{aL}^C \equiv (\nu_{aR})^C \equiv C\,\overline{\nu_{aR}}^T$, C being the charge conjugation matrix. $M_N = (M_N)^T$ is the 2×2 Majorana mass matrix of RH neutrinos, while M_D denotes the 3×2 neutrino Dirac mass matrix, generated from the Yukawa couplings of neutrinos following the breaking of electroweak (EW) symmetry. These Yukawa interactions read

$$\mathcal{L}_Y = -\overline{\nu_{aR}}\,(Y_D^T)_{a\ell}\,H^{c\dagger}\,L_\ell + \text{h.c.}\,, \qquad M_D = v\,Y_D\,, \tag{2.2}$$

where $L_\ell(x) = (\nu_{\ell L}(x)\ \ \ell_L(x))^T$ and $H^c = i\sigma_2 H^*$, $H = (H^+\ \ H^0)^T$ being the Higgs doublet field whose neutral component acquires a VEV $v = \langle H^0\rangle = 174$ GeV. The matrix of neutrino Yukawa couplings has the form

$$Y_D \equiv \begin{pmatrix} g_{e1} & g_{e2} \\ g_{\mu 1} & g_{\mu 2} \\ g_{\tau 1} & g_{\tau 2} \end{pmatrix}, \tag{2.3}$$

where $g_{\ell a}$ denotes the coupling of $L_\ell(x)$ to $\nu_{aR}(x)$, $\ell = e,\mu,\tau$, $a = 1,2$.

The full 5×5 neutrino Dirac–Majorana mass matrix, given below in the (ν_L, ν_L^C) basis, can be made block-diagonal by use of a unitary matrix Ω,

$$\Omega^T \begin{pmatrix} 0 & M_D \\ M_D^T & M_N \end{pmatrix} \Omega = \begin{pmatrix} U^*\hat{m}U^\dagger & 0 \\ 0 & V^*\hat{M}V^\dagger \end{pmatrix}, \tag{2.4}$$

where $\hat{m} \equiv \text{diag}(m_1, m_2, m_3)$ contains the masses m_i of the light Majorana neutrino mass eigenstates χ_i, while $\hat{M} \equiv \text{diag}(M_1, M_2)$ contains the masses $M_{1,2}$ of the heavy Majorana neutrinos, $N_{1,2}$. Here, U and V are 3×3 and 2×2 unitary matrices, respectively. The matrix Ω can be parametrised as [4,8]:

$$\Omega = \exp\begin{pmatrix} 0 & R \\ -R^\dagger & 0 \end{pmatrix} = \begin{pmatrix} 1 - \frac{1}{2}RR^\dagger & R \\ -R^\dagger & 1 - \frac{1}{2}R^\dagger R \end{pmatrix} + \mathcal{O}(R^3)\,, \tag{2.5}$$

under the assumption that the elements of the 3×2 complex matrix R are small, which will be justified later. At leading order in R, the following relations hold [4]:

$$R^* \simeq M_D\,M_N^{-1}\,, \tag{2.6}$$

$$m_\nu \equiv U^*\hat{m}U^\dagger \simeq R^*M_N R^\dagger - R^*M_D^T - M_D R^\dagger = -\,R^*M_N R^\dagger\,, \tag{2.7}$$

$$V^*\hat{M}V^\dagger \simeq M_N + \frac{1}{2}R^T R^* M_N + \frac{1}{2}M_N R^\dagger R \simeq M_N\,, \tag{2.8}$$

where[3] we have used eq. (2.6) to get the last equality in eq. (2.7). From the first two we recover the well-known seesaw formula for the light neutrino mass matrix,

$$m_\nu = -M_D\,M_N^{-1}\,M_D^T\,. \tag{2.9}$$

We are interested in the case where only the L-conserving Majorana mass term of $\nu_{1R}(x)$ and $\nu_{2R}(x)$, $M\,\nu_{1R}^T\,C^{-1}\,\nu_{2R}$, with $M > 0$ and, e.g., $L(\nu_{1R}) = -1$ and $L(\nu_{2R}) = +1$, L being the total lepton charge, is present in the Lagrangian. In this case the Majorana mass matrix of RH neutrinos $\nu_{1R}(x)$ and $\nu_{2R}(x)$ reads:

[3] The factors 1/2 in the two terms $\propto R^T R^* M_N$ and $\propto M_N R^\dagger R$ in eq. (2.8) are missing in the corresponding expression in Ref. [4]. These two terms provide a sub-leading correction to the leading term M_N and have been neglected in the discussion of the phenomenology in Ref. [4]. We will also neglect them in the phenomenological analysis we will perform.

$$M_N = \begin{pmatrix} 0 & M \\ M & 0 \end{pmatrix} . \tag{2.10}$$

Using eqs. (2.2), (2.3) and eq. (2.9), we get the following expression for the light neutrino Majorana mass matrix m_ν:

$$m_\nu = -\frac{v^2}{M} \begin{pmatrix} 2\, g_{e1}\, g_{e2} & g_{\mu 1}\, g_{e2} + g_{e1}\, g_{\mu 2} & g_{\tau 1}\, g_{e2} + g_{e1}\, g_{\tau 2} \\ g_{\mu 1}\, g_{e2} + g_{e1}\, g_{\mu 2} & 2\, g_{\mu 1}\, g_{\mu 2} & g_{\tau 1}\, g_{\mu 2} + g_{\mu 1}\, g_{\tau 2} \\ g_{\tau 1}\, g_{e2} + g_{e1}\, g_{\tau 2} & g_{\tau 1}\, g_{\mu 2} + g_{\mu 1}\, g_{\tau 2} & 2\, g_{\tau 1}\, g_{\tau 2} \end{pmatrix} . \tag{2.11}$$

With the assignments $L(\nu_{1R}) = -1$ and $L(\nu_{2R}) = +1$ made, the requirement of conservation of the total lepton charge L leads to $g_{\ell 1} = 0$, $\ell = e, \mu, \tau$. In this limit of $g_{\ell 1} = 0$, we have $m_\nu = 0$, the light neutrino masses vanish and ν_{1R} and ν^C_{2L} combine to form a Dirac fermion N_D of mass $\tilde{M} \equiv \sqrt{M^2 + v^2 \sum_\ell |g_{\ell 2}|^2}$,[4]

$$N_D = \frac{N_1 \pm i\, N_2}{\sqrt{2}} = \nu_{1R} + \nu^C_{2L} , \tag{2.12}$$

with $N_k = N_{kL} + N_{kR} \equiv N_{kL} + (N_{kL})^C = C\, \overline{N_k}^T$, $k = 1, 2$, and $\nu_{1R} = (N_{1R} \pm i\, N_{2R})/\sqrt{2}$, $\nu^C_{2L} = (N_{1L} \pm i\, N_{2L})/\sqrt{2}$.

Thus, the massive fields $N_k(x)$ are related to the fields $\nu_{aR}(x)$ by $\nu_{aR}(x) \simeq V^*_{ak}\, N_{kR}(x)$, where

$$V = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & \mp i \\ 1 & \pm i \end{pmatrix} , \tag{2.13}$$

where the upper (lower) signs correspond to the case with the upper (lower) signs in eq. (2.12) and in the expressions for ν_{1R} and ν^C_{2L} given after it.

Small L-violating couplings $g_{\ell 1} \neq 0$ split the Dirac fermion N_D into the two Majorana fermions N_1 and N_2 which have very close but different masses, $M_1 \neq M_2$, $|M_2 - M_1| \ll M_{1,2}$. As a consequence, N_D becomes a pseudo-Dirac particle [10,11]. Of the three light massive neutrinos one remains massless (at tree level), while the other two acquire non-zero and different masses. The splitting between the masses of N_1 and N_2 is of the order of one of the light neutrino mass differences and thus is extremely difficult to observe in practice.

More specifically, in the case of a neutrino mass spectrum with normal ordering (see, e.g., [12]) we have (at tree level) keeping terms up to 4th power in the Yukawa couplings $g_{\ell 1}$ and $g_{\ell 2}$ and taking $g_{\ell a}$ to be real for simplicity:

$$m_1 = 0 , \quad m_{2,3} \simeq \frac{1}{M} \left[\sqrt{\Delta} \left(1 - \frac{D(A^2 + \Delta)}{2M^2 \Delta} \right) \mp A \left(1 - \frac{D}{M^2} \right) \right] + \mathcal{O}(g^6_{\ell a}) , \tag{2.14}$$

where

$$D \equiv v^2 \left(g^2_{e1} + g^2_{\mu 1} + g^2_{\tau 1} + g^2_{e2} + g^2_{\mu 2} + g^2_{\tau 2} \right) \tag{2.15}$$

$$\Delta \equiv v^4 \left(g^2_{e1} + g^2_{\mu 1} + g^2_{\tau 1} \right) \left(g^2_{e2} + g^2_{\mu 2} + g^2_{\tau 2} \right) , \tag{2.16}$$

$$A \equiv v^2 \left(g_{e1}\, g_{e2} + g_{\mu 1}\, g_{\mu 2} + g_{\tau 1}\, g_{\tau 2} \right) . \tag{2.17}$$

[4] These general results can be inferred just from the form of the conserved "non-standard" lepton charge L' [9] which is expressed in terms of the individual lepton charges L_ℓ, $\ell = e, \mu, \tau$, and $L_a(\nu_{bR}) = -\delta_{ab}$, $a, b = 1, 2$: $L' = L_e + L_\mu + L_\tau + L_1 - L_2$ ($L'(\nu_{1R}) = L_1(\nu_{1R}) = -1$ and $L'(\nu_{2R}) = -L_2(\nu_{2R}) = +1$). Then $\min(n_+, n_-)$ and $|n_+ - n_-|$ are the numbers of massive Dirac and massless neutrinos, respectively, n_+ (n_-) being the number of charges entering into the expression for L' with positive (negative) sign.

The heavy neutrino mass spectrum is given by:

$$M_{1,2} \simeq M\left[1+\frac{D}{2M^2}-\frac{1}{2M^4}\left(\Delta+2A^2+\frac{D^2}{4}\right)\right] \mp \frac{A}{M}\left(1-\frac{D}{M^2}\right)+\mathcal{O}(g_{\ell a}^6)\,. \quad (2.18)$$

The values of $m_{2,3}$ and $M_{1,2}$ given in eqs. (2.14) and (2.18) can be obtained as approximate solutions of the *exact* mass-eigenvalue equation:

$$\lambda^4-\lambda^2\left(M^2+D\right)-2\lambda M A-\left(\Delta-A^2\right)=0\,. \quad (2.19)$$

Note that, as it follows from eqs. (2.14) and (2.18), we have [4]: $M_2-M_1 \simeq 2(A/M)(1-D/M^2)=m_3-m_2$. Therefore, the splitting between M_2 and M_1, as we have already noted, is exceedingly small. Indeed, for a neutrino mass spectrum with normal ordering (NO) and $m_1=0$, we have $m_2=\sqrt{\Delta m^2_{21}}\simeq 8.6\times 10^{-3}$ eV, $m_3=\sqrt{\Delta m^2_{31}}\simeq 0.051$ eV, and

$$M_2-M_1=m_3-m_2\simeq 0.042\,\text{eV}\,, \quad (2.20)$$

where we have used the best fit values of Δm^2_{21} and Δm^2_{31} determined in the recent global analysis of the neutrino oscillation data [13] (see also Table 2). The corrections to the matrix V which diagonalises M_N are of the order of AD/M^4 and are negligible, as was noticed also in [4].

To leading order in (real) $g_{\ell 1}$ and $g_{\ell 2}$, the expressions in eqs. (2.14) and (2.18) simplify significantly [4]:

$$m_1=0\,,\quad m_2\simeq\frac{1}{M}\left(\sqrt{\Delta}-A\right)\,,\quad m_3\simeq\frac{1}{M}\left(\sqrt{\Delta}+A\right)\,, \quad (2.21)$$

$$M_1\simeq M\left(1+\frac{D}{2M^2}\right)-\frac{A}{M}\,,\quad M_2\simeq M\left(1+\frac{D}{2M^2}\right)+\frac{A}{M}\,. \quad (2.22)$$

The low-energy phenomenology involving the pseudo-Dirac neutrino N_D, or equivalently the Majorana neutrinos N_1 and N_2, is controlled by the matrix RV of couplings of N_1 and N_2 to the charged leptons in the weak charged lepton current (see Section 6). When both $g_{\ell 1}$ and $g_{\ell 2}$ couplings are present, this matrix is given by:

$$RV\simeq\frac{1}{\sqrt{2}}\frac{v}{M}\begin{pmatrix} g^*_{e1}+g^*_{e2} & i\,(g^*_{e1}-g^*_{e2}) \\ g^*_{\mu 1}+g^*_{\mu 2} & i\,(g^*_{\mu 1}-g^*_{\mu 2}) \\ g^*_{\tau 1}+g^*_{\tau 2} & i\,(g^*_{\tau 1}-g^*_{\tau 2}) \end{pmatrix}\,, \quad (2.23)$$

where we have used the expression for the matrix V in eq. (2.13) with the upper signs. We will adhere to this convention further on.

It follows from the preceding discussion that the generation of non-zero light neutrino masses may be directly related to the generation of the L-non-conserving neutrino Yukawa couplings $g_{\ell 1}\neq 0$, $\ell=e,\mu,\tau$. Among the many possible mechanisms leading to $g_{\ell 1}\neq 0$ there is at least one we will discuss further, that could lead to exceedingly small $g_{\ell 1}$, say $|g_{\ell 1}|\sim 10^{-12}$–$10^{-8}$. In this case the RH neutrinos can have masses in the few GeV to a few TeV range and the neutrino Yukawa couplings $|g_{\ell 2}|$ can be much larger than $|g_{\ell 1}|$, of the order $|g_{\ell 2}|\sim 10^{-4}$–10^{-2}, leading to interesting low-energy phenomenology. For these ranges of $|g_{\ell 2}|$ and M, the approximations $D/M^2\ll 1$ and $\tilde{M}\simeq M$ are valid and will be used in what follows, i.e., we will use eqs. (2.21) and (2.22).

Thus, in the scenario we are interested in with two RH neutrinos possessing a Majorana mass term which conserves the total lepton charge L, the smallness of the light Majorana neutrino masses is related to the smallness of the L-non-conserving neutrino Yukawa couplings $g_{\ell 1}$ and

Table 1
Charge assignments of lepton superfields under the $U(1)_{FN}$ symmetry group.

	$\hat{S}$	$\hat{N}_1$	$\hat{N}_2$	$\hat{H}_u$	$\hat{L}_e$	$\hat{L}_\mu$	$\hat{L}_\tau$	$\hat{e}^c$	$\hat{\mu}^c$	$\hat{\tau}^c$
Q_{FN}	-1	n	-1	0	2	1	1	4	2	0

not to the RH neutrinos having large Majorana masses in the range of $\sim(10^{10}$–$10^{14})$ GeV. Moreover, in contrast to the standard seesaw scenario, the heavy Majorana neutrinos of the scenario of interest can have masses at the TeV or lower scale, which makes them directly observable, in principle, in collider (LHC, future e^+–e^- and p–p) experiments.

The low-scale type I seesaw scenario of interest with two RH neutrinos ν_{1R} and ν_{2R} with L-conserving Majorana mass term and L-conserving (L-non-conserving) neutrino Yukawa couplings $g_{\ell 2}$ ($g_{\ell 1}$) of ν_{2R} (of ν_{1R}) was considered in [4] on purely phenomenological grounds (see also, e.g., [14]). It was pointed out in [4], in particular, that the strong hierarchy $|g_{\ell 1}| \ll |g_{\ell' 2}|$, $\ell, \ell' = e, \mu, \tau$, is a perfectly viable possibility from the point of view of generation of the light Majorana neutrino masses and that in this case the L-non-conserving effects would be hardly observable. In the present article we provide a possible theoretical justification of the strong hierarchy between the L-conserving and L-non-conserving neutrino Yukawa couplings based on the Froggatt–Nielsen approach to the flavour problem. We also investigate the phenomenology of this specific version of the low-scale type I seesaw model of neutrino mass generation, including the predictions for Dirac and Majorana leptonic CP violation.

3. Froggatt–Nielsen scenario

We work in a supersymmetric (SUSY) framework and consider a global broken $U(1)_{FN}$ Froggatt–Nielsen flavour symmetry, whose charge assignments we motivate below. We will show how an approximate $U(1)_L$ symmetry, related to the L-conservation, may arise in such a model, with $g_{\ell 1} \neq 0$ as the leading L-breaking effect responsible for neutrino masses.

In our setup, one of the RH neutrino chiral superfields has a negative charge under $U(1)_{FN}$, namely $Q_{FN}(\hat{N}_2) = -1$, while the other carries a positive FN charge, $Q_{FN}(\hat{N}_1) \equiv n > 0$. The FN mechanism is realised thanks to the VEV of the lowest component S of a chiral superfield $\hat{S}$, which is a singlet under the SM gauge symmetry group and carries negative FN charge, $Q_{FN}(\hat{S}) = -1$. Charges for the $\hat{L}_\ell$ superfields follow a standard lopsided assignment [3], namely $Q_{FN}(\hat{L}_e) = 2$, $Q_{FN}(\hat{L}_\mu) = 1$, and $Q_{FN}(\hat{L}_\tau) = 1$, which allows for large ν_μ–ν_τ mixing. For definiteness we take $Q_{FN}(\hat{H}_u) = 0$, $Q_{FN}(\hat{e}^c) = 4$, $Q_{FN}(\hat{\mu}^c) = 2$, and $Q_{FN}(\hat{\tau}_c) = 0$. The FN suppression parameter $\epsilon \equiv \langle S \rangle / \Lambda$ is thus chosen to be close to the sine of the Cabibbo angle λ_C, specifically $\epsilon = 0.2$, in order to reproduce the hierarchies between charged lepton masses (see also [15,16]). Here, Λ is the FN flavour dynamics scale. The charge assignments under $U(1)_{FN}$ relevant to the present study are summarised in Table 1.

The effective superpotential[5] for the neutrino sector reads

$$W_\nu \sim M_0 \, (\epsilon^{2n} \, \hat{N}_1 \, \hat{N}_1 + \epsilon^{n-1} \, \hat{N}_1 \, \hat{N}_2) + (\epsilon \, \hat{L}_e + \hat{L}_\mu + \hat{L}_\tau) \, (\epsilon^{n+1} \, \hat{N}_1 + g_2 \, \hat{N}_2) \, \hat{H}_u \, , \qquad (3.1)$$

where $M_0 \sim \Lambda$ and g_2 is an *a priori* $\mathcal{O}(1)$ coupling. Due to the condition of holomorphicity of the superpotential, no quadratic term for $\hat{N}_2$ is allowed, justifying the absence of the Majorana

[5] The presence of an R-parity preventing the usual L- and B-violating terms in the MSSM superpotential is assumed.

mass term $M\,\nu_{2R}^T\,C^{-1}\,\nu_{2R}$. This framework may naturally arrange for the suppression $(M_N)_{11} \ll (M_N)_{12}$, as well as for a hierarchy between RH masses and the FN scale, $M \sim \epsilon^{n-1}\,\Lambda \ll \Lambda$, provided the charge n is sufficiently large.

The limit of a large $\hat{N}_1$ charge, $n \gg 1$, is quite interesting. In this limit, one finds an accidental (approximate) $\mathrm{U(1)_L}$ symmetry, with assignments $L(\hat{N}_{1,2}) = \pm 1$. Furthermore, the desired hierarchy between (would-be) L-breaking and (would-be) L-conserving Yukawa couplings, $|g_{\ell 1}| \sim \epsilon^{n+1} \ll |g_{\ell' 2}|$, is manifestly achieved. Finally, the mass term for $\hat{N}_1$ is suppressed with respect to Λ by the FN parameter to the power of $2n \gg 1$. This observation and the holomorphicity of the superpotential justify the absence of diagonal Majorana mass terms $M\,\nu_{aR}^T\,C^{-1}\,\nu_{aR}$, $a = 1, 2$, in eq. (2.10) which could push up the light neutrino masses to unwanted heavy scales. We will focus on the case of a sufficiently large charge n in what follows.

The lopsided choice of FN charges for the lepton doublets is responsible for the structure $|g_{e2}| : |g_{\mu 2}| : |g_{\tau 2}| \simeq \epsilon : 1 : 1$ of Yukawa couplings of ν_{2R}. However, due to the large FN charge of ν_{1R}, such FN flavour structure might be diluted in the L-violating Yukawa couplings. Indeed, for each insertion of $\hat{S}$, a factor of ϵ is in principle accompanied by an $\mathcal{O}(1)$ factor. This uncertainty makes it impossible to have an unambiguous prediction for the ratios $|g_{e1}| : |g_{\mu 1}| : |g_{\tau 1}|$ in the model under discussion. This is in contrast to the case of the $g_{\ell 2}$ couplings.

Thus, in the present setup, the Yukawa matrix Y_D obeys the following structure (up to phases):

$$Y_D \sim \begin{pmatrix} g_{e1} & \epsilon\, g_2 \\ g_{\mu 1} & g_2 \\ g_{\tau 1} & g_2 \end{pmatrix} \sin\beta\,, \tag{3.2}$$

with $\sin\beta = \langle H_u^0 \rangle / v$, and where $g_{\ell 1}, g_2 > 0$, and the hierarchy $g_{\ell 1} \ll g_2 \lesssim 1$ is naturally realised. We see from eq. (2.11) that the scale of light neutrino masses depends on the size of the product $g_{\ell 1}\, g_2$, namely

$$(m_\nu)_{\ell\ell'} \sim \frac{v^2\,\sin^2\beta}{M}\,(g_{\ell 1} + g_{\ell' 1})\,g_2\,. \tag{3.3}$$

Despite being suppressed, the quadratic term for $\hat{N}_1$, and thus the Majorana mass term $\mu\,\nu_{1R}^T\,C^{-1}\,\nu_{1R}$, may still play a non-negligible role, for instance, in studies of leptogenesis [17]. A complete suppression of μ can be achieved through the modification of our setup which we summarise in the following. Consider $(4+1)$ dimensions where the extra dimension is compactified on an S^1/Z_2 orbifold. This extra dimension has two fixed points, y_1 and y_2. We localize all SM fields on y_1, a new chiral superfield $\hat{\Phi}$ (with lowest component Φ) on y_2, and allow the FN field S and both RH neutrino fields to propagate in the bulk. We impose, aside from the aforementioned FN symmetry ($Q_{\mathrm{FN}}(\Phi) = 0$), an $\mathrm{U(1)}_{B-\hat{L}}$ symmetry with the charge assignments $(B - \hat{L})(\nu_{1,2R}) = -1$ and $(B - \hat{L})(\Phi) = +2$. Notice that $\hat{L}$ does not coincide with the standard (total) lepton charge L.[6] Then, interactions of the type $\Phi\,\nu_{aR}^T\,C^{-1}\,\nu_{bR}$ $(a, b = 1, 2)$ are allowed, provided a sufficient number of insertions of S are considered. They generate mass terms for the RH neutrinos once Φ develops a non-zero VEV, $\langle\Phi\rangle \neq 0$. The Yukawa couplings $g_{\ell a}$ are allowed as before and retain their FN hierarchy. Assuming an enhanced $\mathrm{U(1)}_L$ symmetry at y_2 with charges $L(\nu_{1R}) = -1$, $L(\nu_{2R}) = +1$ and $L(\Phi) = 0$, diagonal Majorana mass terms for $\nu_{1,2R}$ are thus forbidden.

[6] Indeed, we have $\hat{L}(\nu_{1R}) = \hat{L}(\nu_{2R}) = +1$ while $L(\nu_{1R}) = -L(\nu_{2R}) = -1$ (see Section 2).

4. Neutrino mixing

The addition of the terms of eq. (2.1) to the SM Lagrangian leads to a Pontecorvo–Maki–Nakagawa–Sakata (PMNS) neutrino mixing matrix, $U_{\rm PMNS}$, which is not unitary. Indeed, the charged and neutral current weak interactions involving the light Majorana neutrinos χ_i read:

$$\mathcal{L}^{\nu}_{\rm CC} = -\frac{g}{\sqrt{2}}\,\bar{\ell}\,\gamma_\alpha\left(U_l^\dagger(1+\eta)U\right)_{\ell i}\,\chi_{iL}\,W^\alpha + \text{h.c.}\,, \tag{4.1}$$

$$\mathcal{L}^{\nu}_{\rm NC} = -\frac{g}{2c_w}\,\overline{\chi_{iL}}\,\gamma_\alpha\left(U^\dagger(1+2\eta)U\right)_{ij}\,\chi_{jL}\,Z^\alpha\,, \tag{4.2}$$

where $\ell = e, \mu, \tau$ and U_l is a unitary matrix which originates from the diagonalisation of the charged lepton mass matrix and $\eta \equiv -\,R\,R^\dagger/2$. The transformation U_l does not affect the power counting in the structure of eq. (3.2), though it may provide a source of deviations. We then choose to work in the charged lepton mass basis, in which $U_l = 1$. In this basis the PMNS neutrino mixing matrix is given by: $U_{\rm PMNS} = (1+\eta)U$, where U is the unitary matrix diagonalising the Majorana neutrino mass matrix generated by the seesaw mechanism and η describes the deviation from unitarity of the PMNS matrix. As we will see further, the experimental constraints on the elements of η imply $|\eta_{\ell\ell'}| \lesssim 10^{-3}$, $\ell, \ell' = e, \mu, \tau$.

Due to the structure of the matrix of Yukawa couplings Y_D given in eq. (3.2), in the scheme we are considering the normal ordering (NO) of the light neutrino mass spectrum, $m_1 < m_2 < m_3$, is favoured over the spectrum with inverted ordering (IO), $m_3 < m_1 < m_2$. We henceforth consider the NO case, for which, as we have already commented, we have $m_1 = 0$, $m_2 = \sqrt{\Delta m^2_{21}}$, and $m_3 = \sqrt{\Delta m^2_{31}}$. Working in the basis of diagonal charged lepton mass term and neglecting the deviations from unitarity, which are parametrised by η, we identify the PMNS mixing matrix with the unitary matrix U which diagonalises m_ν, $U_{\rm PMNS} \simeq U$. Given that one neutrino is massless (at tree level), the neutrino mixing matrix U can be parametrised as:

$$U_{\rm PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \text{diag}(1, e^{i\alpha/2}, 1), \tag{4.3}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$, with $\theta_{ij} \in [0, \pi/2]$, while δ and α denote the Dirac and Majorana [18] CP violation (CPV) phases, respectively, $\delta, \alpha \in [0, 2\pi]$. The current best fit values and 3σ allowed ranges for the neutrino mixing parameters and mass squared differences for NO spectrum are summarised in Table 2.

5. Predictions for the CPV phases

It proves convenient for our further analysis to use the Casas–Ibarra parametrisation [19] of the Dirac mass matrix M_D (neutrino Yukawa matrix Y_D):

$$M_D \;=\; v\,Y_D \;=\; i\,U^*_{\rm PMNS}\,\sqrt{\hat{m}}\,O\,\sqrt{\hat{M}}\,V^\dagger\,, \tag{5.1}$$

where $\hat{m} = \text{diag}(m_1, m_2, m_3)$ and O is a complex orthogonal matrix. In the scheme with two heavy RH Majorana neutrinos the matrix O has the form [20]:

Table 2
Best fit values and 3σ ranges of the neutrino oscillation parameters for neutrino mass spectrum with normal ordering (NO), obtained in the global analysis of Ref. [13].

Parameter	Best fit value	3σ range
$\Delta m^2_{21}/10^{-5}$ eV^2	7.37	6.93–7.96
$\Delta m^2_{31}/10^{-3}$ eV^2	2.56	2.45–2.69
$\sin^2\theta_{12}/10^{-1}$	2.97	2.50–3.54
$\sin^2\theta_{13}/10^{-2}$	2.15	1.90–2.40
$\sin^2\theta_{23}/10^{-1}$	4.25	3.81–6.15
δ/π	1.38	$[0, 0.17] \cup [0.76, 2]$

$$O \equiv \begin{pmatrix} 0 & 0 \\ \cos\hat{\theta} & \pm\sin\hat{\theta} \\ -\sin\hat{\theta} & \pm\cos\hat{\theta} \end{pmatrix}, \quad \text{for NO mass spectrum,} \tag{5.2}$$

$$O \equiv \begin{pmatrix} \cos\hat{\theta} & \pm\sin\hat{\theta} \\ -\sin\hat{\theta} & \pm\cos\hat{\theta} \\ 0 & 0 \end{pmatrix}, \quad \text{for IO mass spectrum,} \tag{5.3}$$

where $\hat{\theta} \equiv \omega - i\xi$. The O-matrix in the case of NO spectrum of interest can be decomposed as follows[7]:

$$O = \frac{e^{i\hat{\theta}}}{2}\begin{pmatrix} 0 & 0 \\ 1 & \mp i \\ i & \pm 1 \end{pmatrix} + \frac{e^{-i\hat{\theta}}}{2}\begin{pmatrix} 0 & 0 \\ 1 & \pm i \\ -i & \pm 1 \end{pmatrix} = O_+ + O_- \,. \tag{5.4}$$

The Dirac neutrino mass matrix can be presented accordingly as $M_D = M_{D+} + M_{D-}$, with obvious notation. For the elements of $M_{D+} = v\, Y_{D+}$ and $M_{D-} = v\, Y_{D-}$ we get:

$$v\,(Y_D)_{\ell a} = v\,(Y_{D+})_{\ell a} + v\,(Y_{D-})_{\ell a} = v\, g^{(+)}_{\ell a} + v\, g^{(-)}_{\ell a}, \quad \ell = e, \mu, \tau,\ a = 1, 2\,, \tag{5.5}$$

where

$$v\, g^{(+)}_{\ell 1} \simeq i\,\frac{e^{i\omega}e^{\xi}}{2\sqrt{2}}\left(\sqrt{M_1} \pm \sqrt{M_2}\right)\left(\sqrt{m_2}\,U^*_{\ell 2} + i\,\sqrt{m_3}\,U^*_{\ell 3}\right), \tag{5.6}$$

$$v\, g^{(+)}_{\ell 2} \simeq i\,\frac{e^{i\omega}e^{\xi}}{2\sqrt{2}}\left(\sqrt{M_1} \mp \sqrt{M_2}\right)\left(\sqrt{m_2}\,U^*_{\ell 2} + i\,\sqrt{m_3}\,U^*_{\ell 3}\right), \tag{5.7}$$

$$v\, g^{(-)}_{\ell 1} \simeq i\,\frac{e^{-i\omega}e^{-\xi}}{2\sqrt{2}}\left(\sqrt{M_1} \mp \sqrt{M_2}\right)\left(\sqrt{m_2}\,U^*_{\ell 2} - i\,\sqrt{m_3}\,U^*_{\ell 3}\right), \tag{5.8}$$

$$v\, g^{(-)}_{\ell 2} \simeq i\,\frac{e^{-i\omega}e^{-\xi}}{2\sqrt{2}}\left(\sqrt{M_1} \pm \sqrt{M_2}\right)\left(\sqrt{m_2}\,U^*_{\ell 2} - i\,\sqrt{m_3}\,U^*_{\ell 3}\right). \tag{5.9}$$

Given the fact that $(\sqrt{M_2} - \sqrt{M_1})/(\sqrt{M_2} + \sqrt{M_1}) \simeq (m_3 - m_2)/(4M) \ll 1$ and, e.g., for $M =$ 10 (100) GeV, $(m_3 - m_2)/(4M) \simeq 10^{-12}$ (10^{-13}), it is clear from eqs. (5.6)–(5.9) that for $\xi = 0$ we have (barring accidental cancellations): $|g^{(-)}_{\ell 1}| \ll |g^{(+)}_{\ell' 1}|$, $|g^{(+)}_{\ell 2}| \ll |g^{(-)}_{\ell' 2}|$, $|g^{(+)}_{\ell 1}| \sim |g^{(-)}_{\ell' 2}|$, and

[7] A similar decomposition exists for the IO spectrum [5].

thus $|g_{\ell 1}| \sim |g_{\ell' 2}|$, where we have used the upper signs in the expressions for $g^{(\pm)}_{\ell 1}$ and $g^{(\pm)}_{\ell 2}$. Unless otherwise stated we will employ this sign choice in the discussion which follows.

Taking for definiteness $\xi < 0$, it follows from eqs. (5.6)–(5.9) that $|g^{(-)}_{\ell a}|$ ($|g^{(+)}_{\ell a}|$) grows (decreases) exponentially with $|\xi|$.[8] Therefore, for sufficiently large $|\xi|$ we will have

$$\frac{|g^{(+)}_{\ell 1}|}{|g^{(-)}_{\ell' 2}|} = e^{-2|\xi|}\, r_{\ell\ell'} \ll 1\,, \quad r_{\ell\ell'} \equiv \frac{\left|\sqrt{m_2}\, U^*_{\ell 2} + i\,\sqrt{m_3}\, U^*_{\ell 3}\right|}{\left|\sqrt{m_2}\, U^*_{\ell' 2} - i\,\sqrt{m_3}\, U^*_{\ell' 3}\right|}\,, \quad \ell, \ell' = e, \mu, \tau\,. \tag{5.10}$$

Using the 3σ allowed ranges of the neutrino oscillation parameters found in the global analysis of the neutrino oscillation data in [13] and given in Table 2 and varying the CP violation phases in the PMNS matrix in their defining intervals it is not difficult to show that the ratios r in eq. (5.10) vary in the interval $r_{\ell\ell'} = (0.04\text{–}22.5)$.

Therefore even for the maximal cited value of $r_{\ell\ell'}$ we would have $|g^{(+)}_{\ell 1}| \ll |g^{(-)}_{\ell' 2}|$ for a sufficiently large value of $|\xi|$. At the same time the inequalities $|g^{(-)}_{\ell 1}|/|g^{(-)}_{\ell' 2}| \ll 1$, and $|g^{(+)}_{\ell 2}|/|g^{(-)}_{\ell' 2}| \ll 1$, $\ell, \ell' = e, \mu, \tau$, always hold. Thus, for $\xi < 0$ and sufficiently large $|\xi|$ we get the requisite hierarchy of Yukawa couplings: $|g_{\ell 1}| \simeq |g^{(+)}_{\ell 1}| \ll |g_{\ell' 2}| \simeq |g^{(-)}_{\ell' 2}|$. For $|\xi| = 9$, for example, we find for $r_{\ell\ell'} \simeq 1$: $|g_{\ell 1}|/|g_{\ell' 2}| \simeq |g^{(+)}_{\ell 1}|/|g^{(-)}_{\ell' 2}| \simeq 1.5 \times 10^{-8}$, which is in the range of values relevant for our discussion. We get the same hierarchy of Yukawa couplings, $|g_{\ell 1}| \ll |g_{\ell' 2}|$, $\ell, \ell' = e, \mu, \tau$, in the case of the lower signs in the expressions in eqs. (5.6)–(5.9) for sufficiently large $\xi > 0$. In this case $|g_{\ell 1}| \simeq |g^{(-)}_{\ell 1}| \ll |g_{\ell' 2}| \simeq |g^{(+)}_{\ell' 2}|$.

We will show next that, given the present neutrino oscillation data, enforcing the flavour pattern specified in eq. (3.2) results in a prediction for the Dirac phase δ close to $\pi/4$, $3\pi/4$, $5\pi/4$, $7\pi/4$, and for the Majorana phase α close to zero.

As we have seen, the matrix of neutrino Yukawa couplings Y_D can be reconstructed up to normalization, a complex parameter, and a sign using eqs. (5.1) and (5.4) (for NO spectrum). For the cases of interest, with sufficiently large values of $|\xi|$, necessary to ensure the requisite hierarchy of Yukawa couplings $|g_{\ell 1}| \ll |g_{\ell' 2}|$, $\ell, \ell' = e, \mu, \tau$, the ratios of (absolute values of) Yukawa couplings read:

$$R^{(1)}_{\ell\ell'} \equiv \frac{|g_{\ell 1}|}{|g_{\ell' 1}|} \simeq \frac{\left|\sqrt{m_2}\, U^*_{\ell 2} \pm i\,\sqrt{m_3}\, U^*_{\ell 3}\right|}{\left|\sqrt{m_2}\, U^*_{\ell' 2} \pm i\,\sqrt{m_3}\, U^*_{\ell' 3}\right|}\,, \tag{5.11}$$

$$R^{(2)}_{\ell\ell'} \equiv \frac{|g_{\ell 2}|}{|g_{\ell' 2}|} \simeq \frac{\left|\sqrt{m_2}\, U^*_{\ell 2} \mp i\,\sqrt{m_3}\, U^*_{\ell 3}\right|}{\left|\sqrt{m_2}\, U^*_{\ell' 2} \mp i\,\sqrt{m_3}\, U^*_{\ell' 3}\right|}\,, \tag{5.12}$$

where the upper and lower signs correspond to the case with $\xi < 0$ and upper signs in eq. (5.4) and to the case with $\xi > 0$ and lower signs in eq. (5.4), respectively. Recall that $|g_{\ell 1}| \simeq |g^{(+)}_{\ell 1}|$, $|g_{\ell 2}| \simeq |g^{(-)}_{\ell 2}|$ in the former case ($\xi < 0$), and $|g_{\ell 1}| \simeq |g^{(-)}_{\ell 1}|$, $|g_{\ell 2}| \simeq |g^{(+)}_{\ell 2}|$ in the latter ($\xi > 0$).

One sees that the dependence on the complex parameter $\hat{\theta}$ drops out in the ratios $R^{(1,2)}_{\ell\ell'}$, which are determined by the light neutrino masses m_2 and m_3 and by neutrino mixing parameters only, once the sign in O in eq. (5.4) (or equivalently in eqs. (5.6)–(5.9)) is fixed. In particular, the flavour structure depends on the elements $U_{\ell 2}$ and $U_{\ell 3}$ of the PMNS matrix. Given the fact that $m_2 = \sqrt{\Delta m^2_{21}}$, $m_3 = \sqrt{\Delta m^2_{31}}$, and that Δm^2_{21}, Δm^2_{31} and the three neutrino mixing angles θ_{12},

[8] Obviously, if $\xi > 0$, $|g^{(+)}_{\ell a}|$ ($|g^{(-)}_{\ell a}|$) will grow (decrease) exponentially with ξ.

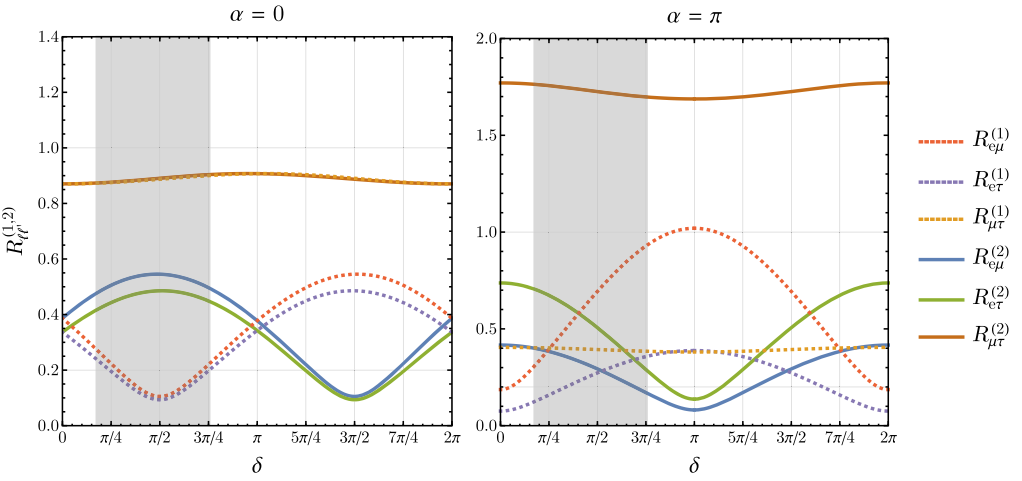

Fig. 1. Ratios $R^{(1,2)}_{\ell\ell'}$ of (absolute values of) Yukawa couplings for a NO neutrino spectrum as a function of the CPV phase δ for $\alpha = 0$ (left panel) and $\alpha = \pi$ (right panel), in the case $\xi < 0$. The figure is obtained using the best fit values of $\Delta m^2_{21,31}$ and $\sin^2\theta_{ij}$ quoted in Table 2. The vertical grey band indicates values of δ which are disfavoured at 3σ. The case $\xi > 0$ is obtained by exchanging $R^{(1)}_{\ell\ell'}$ and $R^{(2)}_{\ell\ell'}$. (For interpretation of the colours in the figure(s), the reader is referred to the web version of this article.)

θ_{23} and θ_{13} have been determined in neutrino oscillation experiments with a rather high precision, the quantities $R^{(1)}_{\ell\ell'}$ and $R^{(2)}_{\ell\ell'}$ depend only on the CPV phases δ and α once the sign of ξ is fixed. This means that knowing any two of the ratios $|g_{\ell 1}|/|g_{\ell' 1}|$ or $|g_{\ell 2}|/|g_{\ell' 2}|$, $\ell \neq \ell' = e, \mu, \tau$ allows to determine both δ and α.

In Figs. 1 and 2 we present the ratios $R^{(1,2)}_{\ell\ell'}$ as a function of δ for the case $\xi < 0$ and two representative values of α. Fig. 1 is obtained using the best fit values of $\Delta m^2_{21,31}$ and $\sin^2\theta_{ij}$ taken from Table 2. In Fig. 2 we show the ranges in which $R^{(1,2)}_{\ell\ell'}$ vary when $\Delta m^2_{21,31}$ and the $\sin^2\theta_{ij}$ are varied in their respective 3σ allowed intervals given in Table 2. In Table 3 we report the respective intervals in which each of the six ratios can lie. As Table 3 indicates, certain specific simple patterns cannot be realised within the scheme considered. Among those are, for example, the patterns $|g_{e1}| : |g_{\mu 1}| : |g_{\tau 1}| \simeq 1 : 1 : 1$ and $|g_{e2}| : |g_{\mu 2}| : |g_{\tau 2}| \simeq 1 : 1 : 1$.

The flavour structure of eq. (3.2), which is naturally realised in the model of Section 3, corresponds to the pattern $|g_{e2}| : |g_{\mu 2}| : |g_{\tau 2}| \simeq \epsilon : 1 : 1$, and thus to $R^{(2)}_{e\mu} \simeq R^{(2)}_{e\tau} \simeq \epsilon$ and $R^{(2)}_{\mu\tau} \simeq 1$. The requirement of having $R^{(2)}_{\mu\tau} \simeq 1$ favours α close to zero.[9] As can be inferred from Fig. 1, given the current best fit values of neutrino mass squared differences and mixing parameters, the requirement of $R^{(2)}_{e\mu} \simeq R^{(2)}_{e\tau} \simeq \epsilon = 0.2$ leads, for $\xi < 0$, to the prediction of $\delta \simeq 5\pi/4,\ 7\pi/4$.[10] Taking into account the 3σ allowed ranges of $\Delta m^2_{21,31}$ and $\sin^2\theta_{ij}$ leads, as Fig. 2 shows, to δ ly-

[9] Marginalizing over δ (either in its defining or in its 3σ range) and varying $\Delta m^2_{21,31}$ and the $\sin^2\theta_{ij}$ in their respective 3σ allowed ranges, the requirement that $|R^{(2)}_{\mu\tau} - 1| < 0.1$ implies $\alpha < 0.36\pi \vee \alpha > 1.64\pi$, independently of the sign of ξ. However, if we require that the relative probability of α having a given value in the indicated intervals is not less than 0.15, then we have $\alpha < 0.2\pi$ or $\alpha > 1.8\pi$. For these values of α and $\epsilon = 0.2$, the predictions for δ can be read off from the plots where $\alpha = 0$.

[10] Similar predictions for the δ and α were obtained in a different context in Ref. [21].

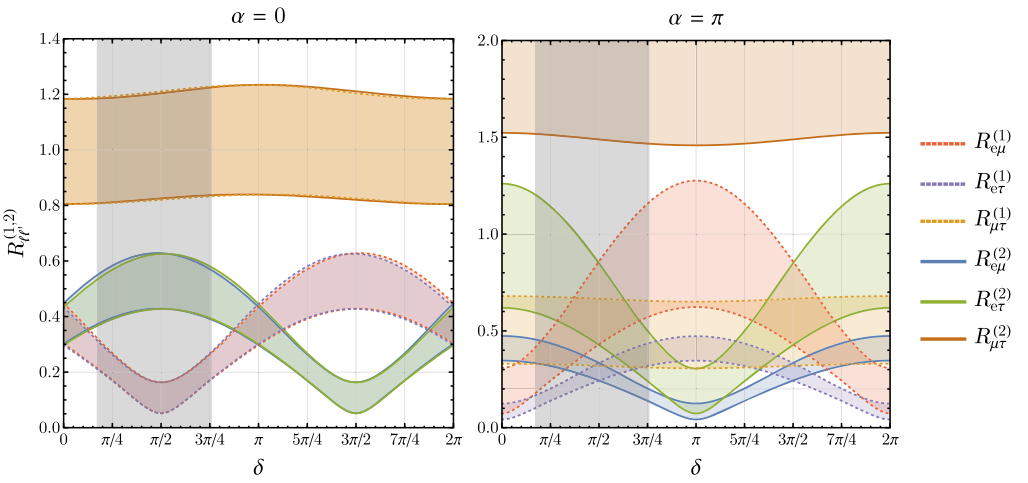

Fig. 2. Ratios $R^{(1,2)}_{\ell\ell'}$ of (absolute values of) Yukawa couplings for a NO neutrino spectrum as a function of the CPV phase δ for $\alpha = 0$ (left panel) and $\alpha = \pi$ (right panel), in the case $\xi < 0$. Bands are obtained by varying $\Delta m^2_{21,31}$ and the $\sin^2\theta_{ij}$ in their respective 3σ allowed ranges given in Table 2. In the case $\alpha = \pi$, the upper boundary of the $R^{(2)}_{\mu\tau}$ band (not shown) is located at $R^{(2)}_{\mu\tau} \simeq 3.0$–$3.2$. The vertical grey band indicates values of δ which are disfavoured at 3σ. The case $\xi > 0$ is obtained by exchanging $R^{(1)}_{\ell\ell'}$ and $R^{(2)}_{\ell\ell'}$.

Table 3
Ranges for the ratios of absolute values of Yukawa couplings, obtained by varying $\Delta m^2_{21,31}$, the $\sin^2\theta_{ij}$, and δ in their respective 3σ allowed ranges and α in its defining range, for $\xi < 0$. The case $\xi > 0$ is obtained by exchanging $R^{(1)}_{\ell\ell'}$ and $R^{(2)}_{\ell\ell'}$.

Ratio	Allowed range
$R^{(1)}_{e\mu}$	0.05–1.28
$R^{(1)}_{e\tau}$	0.04–0.63
$R^{(1)}_{\mu\tau}$	0.31–1.23
$R^{(2)}_{e\mu}$	0.04–0.63
$R^{(2)}_{e\tau}$	0.05–1.26
$R^{(2)}_{\mu\tau}$	0.80–3.21

ing in narrow intervals around the values $5\pi/4$ and $7\pi/4$. Allowing for a somewhat smaller value of ϵ, e.g., $\epsilon = 0.15$, we find that δ should lie in the interval $\delta \simeq [5\pi/4,\ 7\pi/4]$ which includes the value $3\pi/2$ (see Fig. 2).

For $\delta \simeq 5\pi/4,\ 7\pi/4$, $\alpha = 0$ and the best fit values of $\Delta m^2_{21,31}$ and the $\sin^2\theta_{ij}$ we get the following pattern of the Yukawa couplings of ν_{1R}: $|g_{e1}| : |g_{\mu 1}| : |g_{\tau 1}| \sim 0.5 : 1 : 1$.

For $\xi > 0$, using the same arguments we obtain instead $\delta \simeq \pi/4, 3\pi/4$, or $\delta \simeq [\pi/4, 3\pi/4]$. According to the global analyses [13,22], however, these values of δ are strongly disfavoured (if not ruled out) by the current data.

In a more phenomenological approach, we get $\delta \simeq 3\pi/2$ provided, e.g., $|g_{e2}| : |g_{\mu 2}| : |g_{\tau 2}| \simeq 0.14 : 1 : 1$ and $\alpha \simeq \pi/5$. In this case, the remaining ratios read $|g_{e1}| : |g_{\mu 1}| : |g_{\tau 1}| \simeq 0.5 : 0.7 : 1$. In the GUT-inspired scenario of Ref. [23], a different FN charge assignment leads to $\epsilon = 0.06$, in which case $\delta \simeq 3\pi/2$ is favoured.

6. Phenomenology

The low-energy phenomenology of the model of interest resembles that of the model with two heavy Majorana neutrinos $N_{1,2}$ forming a pseudo-Dirac pair considered in [4–6], in which the splitting between the masses of $N_{1,2}$ is exceedingly small. For this model direct and indirect constraints on the model's parameters, which do not depend on the splitting between the masses of N_1 and N_2, as well as expected sensitivities of future lepton colliders have been analysed, e.g., in Refs. [4–6,24,25] (see also [26,27]).

Due to the mixing of LH and RH neutrino fields, i) the PMNS neutrino mixing matrix, $U_{\rm PMNS}$, as we have already noticed, is not unitary, as also the expressions for the charged and neutral current weak interaction of the light Majorana neutrinos χ_i given in eqs. (4.1) and (4.2) show, and ii) the heavy Majorana neutrinos $N_{1,2}$ also participate in charged and neutral current weak interactions with the $W^\pm$ and Z^0 bosons:

$$\mathcal{L}^N_{\rm CC} = -\frac{g}{\sqrt{2}}\,\bar{\ell}\,\gamma_\alpha\left(RV\right)_{\ell k}\,N_{kL}\,W^\alpha + \text{h.c.}\,, \tag{6.1}$$

$$\mathcal{L}^N_{\rm NC} = -\frac{g}{2c_w}\,\overline{\nu_{\ell L}}\,\gamma_\alpha\left(RV\right)_{\ell k}\,N_{kL}\,Z^\alpha + \text{h.c.}\,. \tag{6.2}$$

Due to the Yukawa interactions, cf. eq. (2.2), there are interactions of the heavy Majorana neutrinos $N_{1,2}$ with the SM Higgs boson h as well (see [7]):

$$\mathcal{L}^N_{\rm H} = -\frac{M_k}{v}\,\overline{\nu_{\ell L}}\left(RV\right)_{\ell k}\,N_{kR}\,h + \text{h.c.}\,. \tag{6.3}$$

6.1. Neutrino mass matrix and non-unitarity bounds

The first constraint on the RV elements follows from the fact that the elements of the light neutrino Majorana mass matrix, $(m_\nu)_{\ell\ell'}$, have rather small maximal values. Indeed, as it follows from eq. (2.7), we have [4]:

$$|(m_\nu)_{\ell\ell'}| = |U^*_{\ell j}\,m_j\,U^*_{\ell' j}| \simeq \left|\sum_a (RV)^*_{\ell a}\,M_a\,(RV)^*_{\ell' a}\right|\,,\ \ell,\ell' = e,\mu,\tau\,, \tag{6.4}$$

where the sum is effectively over $j = 2, 3$ since in the model considered $m_1 = 0$.[11] The elements of the neutrino Majorana mass matrix $(m_\nu)_{\ell\ell'}$ depend, apart from $m_2 = \sqrt{\Delta m^2_{21}} \simeq 8.6\times 10^{-3}$ eV, $m_3 = \sqrt{\Delta m^2_{31}} \simeq 0.051$ eV, θ_{12}, θ_{23}, θ_{13}, on the CPV phases δ and α. The maximal value a given element of m_ν can have depends on its flavour indices ℓ and ℓ'. It is not difficult to derive these maximal values using the results reported in Table 2. We have:

[11] Strictly speaking, we have $m_1 = 0$ only at tree level. Higher order corrections lead to a non-zero value of m_1, which is however negligibly small.

i) $|(m_\nu)_{ee}| \lesssim 4.3 \times 10^{-3}$ eV ($\alpha + 2\delta = 0$);
ii) $|(m_\nu)_{e\mu}| \lesssim 9.2 \times 10^{-3}$ eV ($\delta = \pi$, $\alpha = \pi$);
iii) $|(m_\nu)_{e\tau}| \lesssim 9.2 \times 10^{-3}$ eV ($\delta = 0$, $\alpha = \pi$);
iv) $|(m_\nu)_{\mu\mu}| \lesssim 3.4 \times 10^{-2}$ eV ($\delta = \pi$, $\alpha = 0$);
v) $|(m_\nu)_{\mu\tau}| \lesssim 2.9 \times 10^{-2}$ eV ($\delta = 3\pi/2$, $\alpha = \pi$);
vi) $|(m_\nu)_{\tau\tau}| \lesssim 3.5 \times 10^{-2}$ eV ($\delta = 0$, $\alpha = 0$).

The quoted maximal values are reached for the values of the CPV phases given in the brackets. It should be added that the dependence of $\max(|(m_\nu)_{\ell\ell'}|)$, $\ell, \ell' = \mu, \tau$, on δ and α is rather weak since the terms involving δ always include the suppressing factor $\sin\theta_{13}$, while the term $\propto m_2$ is considerably smaller (typically by a factor of 10) than the term $\propto m_3$ as $m_2/m_3 \simeq 0.17$. We will consider $|(m_\nu)_{ee}| \lesssim 4 \times 10^{-3}$ eV, $|(m_\nu)_{e\mu}|$, $|(m_\nu)_{e\tau}| \lesssim 9 \times 10^{-3}$ eV, and $|(m_\nu)_{\ell\ell'}| \lesssim 3 \times 10^{-2}$ eV, $\ell, \ell' = \mu, \tau$, as reference maximal values in the numerical analysis which follows.

From the expression for RV given in eq. (2.23) and eq. (6.4), and taking into account the mass splitting between N_1 and N_2, we get to leading order in $|g_{\ell 1}|$, $|g_{\ell' 2}|$ and $|g_{\ell 1} g_{\ell' 2}|$:

$$|(m_\nu)_{\ell\ell'}| \simeq \frac{v^2}{M} \, |g_{\ell 1} g_{\ell' 2} + g_{\ell 2} g_{\ell' 1}| + \mathcal{O}(g_{\ell 1} g_{\ell' 1}) \,, \tag{6.5}$$

which coincides (up to higher order corrections) with the form given in eq. (2.11). Thus, for a given value of M, the upper bounds on $|(m_\nu)_{\ell\ell'}|$ lead via eq. (6.4) to upper bounds on the magnitude of the product of the neutrino Yukawa couplings of ν_{1R} and ν_{2R}, $g_{\ell 1}$ and $g_{\ell' 2}$. As we have seen, these bounds depend on the flavour of the lepton doublet to which ν_{1R} and ν_{2R} are coupled.

For $M = 100$ GeV (1 TeV), for example, the constraint of interest $|(m_\nu)_{ee}| \lesssim 4 \times 10^{-3}$ eV implies $2|g_{e1} g_{e2}| \lesssim 1.3 \times 10^{-14}$ (1.3×10^{-13}). This upper limit can be satisfied for, e.g., $|g_{e1}| \sim 0.65 \times 10^{-12}$ (0.65×10^{-11}) and $|g_{\ell' 2}| \sim 10^{-2}$. The upper bounds on $|g_{e1} g_{\ell 2} + g_{\ell 1} g_{e2}|$, $\ell = \mu, \tau$, is approximately by a factor of 2 larger than the quoted upper bound on $2|g_{e1} g_{e2}|$, while those on $|g_{\ell 1} g_{\ell' 2} + g_{\ell 2} g_{\ell' 1}|$, $\ell, \ell' = \mu, \tau$ are larger approximately by a factor of 8.

In [4,5] the constraint in eq. (6.4) is satisfied by finding a region, in the general parameter space of the model considered, in which to leading order $\sum_{a=1,2} (RV)^*_{\ell a} M_a (RV)^*_{\ell' a} = 0$, i.e., the two terms in the sum cancel. In the version of the low-scale type I seesaw model with two RH neutrinos we are considering the constraint in eq. (6.4) is satisfied due to smallness of the product of Yukawa couplings $|g_{\ell 1}|$ and $|g_{\ell' 2}|$. In the model under consideration one gets $\sum_{a=1,2} (RV)^*_{\ell a} M_a (RV)^*_{\ell' a} = 0$ in the limit of negligible couplings $g_{\ell 1}$. Indeed, setting $g_{\ell 1} = 0$ we get $M_1 = M_2$ and the expression for the matrix RV takes the form:

$$RV \simeq \frac{1}{\sqrt{2}} \frac{v}{M} \begin{pmatrix} g^*_{e2} & -i\, g^*_{e2} \\ g^*_{\mu 2} & -i\, g^*_{\mu 2} \\ g^*_{\tau 2} & -i\, g^*_{\tau 2} \end{pmatrix} . \tag{6.6}$$

This implies

$$(RV)_{\ell 1} = -i\, (RV)_{\ell 2} \,, \quad l = e, \mu, \tau \,, \tag{6.7}$$

which together with the equality $M_1 = M_2$ leads[12] to $\sum_{a=1,2} (RV)^*_{\ell a} M_a (RV)^*_{\ell' a} = 0$.

[12] The same relation (6.7) holds in the limit of zero splitting between the masses of N_1 and N_2 in the version of the TeV scale type I seesaw model considered in [5,6].

As we have already discussed, the matrix $\eta \equiv -R\,R^\dagger/2 = -(RV)(RV)^\dagger/2 = \eta^\dagger$ parametrises the deviations from unitary of the PMNS matrix. The elements of η are constrained by precision electroweak data and data on flavour observables. For heavy Majorana neutrino masses above the electroweak scale the most updated set of constraints on the absolute values of the elements of η at 2σ C.L. reads [28,29]:

$$|\eta| \;<\; \begin{pmatrix} 1.3\times 10^{-3} & 1.2\times 10^{-5} & 1.4\times 10^{-3} \\ 1.2\times 10^{-5} & 2.2\times 10^{-4} & 6.0\times 10^{-4} \\ 1.4\times 10^{-3} & 6.0\times 10^{-4} & 2.8\times 10^{-3} \end{pmatrix} . \tag{6.8}$$

The upper bound on the e–μ elements is relaxed to $|\eta_{e\mu}| < 3.4\times 10^{-4}$ for heavy Majorana neutrino masses below the electroweak scale (but still above the kaon mass, $M_k \gtrsim 500$ MeV) due to the restoration of a GIM cancellation [30]. The above constraints on η justify the assumption made in Section 2 regarding the smallness of the elements of R.

Using the expression for RV given in eq. (2.23) we find that, to leading order in $g_{\ell 1}$, $g_{\ell' 2}$, $|g_{\ell 1}| \ll |g_{\ell' 2}|$, we have:

$$|\eta_{\ell\ell'}| \simeq \frac{1}{2}\,\frac{v^2}{M^2}\;|g_{\ell 2}\,g_{\ell' 2}| + \mathcal{O}(g_{\ell 1}\,g_{\ell' 2}, g_{\ell' 1}\,g_{\ell 2})\,. \tag{6.9}$$

As a consequence, if M is given, the experimental limits on $|\eta|$ cited in eq. (6.8), in contrast to the limits on $|(m_\nu)_{\ell\ell'}|$, imply upper bounds on $|g_{\ell 2}\,g_{\ell' 2}|$, i.e., on the Yukawa couplings of ν_{2R}. For, e.g., $M = 100$ GeV we find, depending on the flavour indices, $|g_{\ell 2}\,g_{\ell' 2}|^{1/2} \lesssim (2.8\times 10^{-3}$–$4.3\times 10^{-2})$, i.e., $|g_{\ell 2}|$ can be relatively large. This can lead to interesting low-energy phenomenology involving the heavy Majorana neutrinos $N_{1,2}$.

6.2. *LFV observables and Higgs decays*

The predictions of the model under discussion for the rates of the lepton flavour violating (LFV) $\mu \to e\gamma$ and $\mu \to eee$ decays and μ–e conversion in nuclei, as can be shown, depend on $|(RV)^*_{\mu 1}(RV)_{e1} + (RV)^*_{\mu 2}(RV)_{e2}|^2 \simeq 4\,|(RV)^*_{\mu 2}(RV)_{e2}|^2$, where we have used eq. (6.7), and on the masses $M_1 \simeq M_2 \simeq M$ of the heavy Majorana neutrinos N_1 and N_2. The expressions for the $\mu \to e\gamma$ and $\mu \to eee$ decay branching ratios, $\mathrm{BR}(\mu \to e\gamma)$ and $\mathrm{BR}(\mu \to eee)$, and for the relative μ–e conversion in a nucleus X, $\mathrm{CR}(\mu X \to eX)$, coincide with those given in Refs. [5,6] and we are not going to reproduce them here. The best experimental limits on $\mathrm{BR}(\mu \to e\gamma)$, $\mathrm{BR}(\mu \to eee)$ and $\mathrm{CR}(\mu X \to eX)$ have been obtained by the MEG [31], SINDRUM [32] and SINDRUM II [33,34] Collaborations:

$$\mathrm{BR}(\mu \to e\gamma) \;<\; 4.2\times 10^{-13}\ (90\%\ \mathrm{C.L.})\,, \tag{6.10}$$

$$\mathrm{BR}(\mu \to eee) \;<\; 1.0\times 10^{-12}\ (90\%\ \mathrm{C.L.})\,, \tag{6.11}$$

$$\mathrm{CR}(\mu\,\mathrm{Ti} \to e\,\mathrm{Ti}) \;<\; 4.3\times 10^{-12}\ (90\%\ \mathrm{C.L.})\,, \tag{6.12}$$

$$\mathrm{CR}(\mu\,\mathrm{Au} \to e\,\mathrm{Au}) \;<\; 7\times 10^{-13}\ (90\%\ \mathrm{C.L.})\,. \tag{6.13}$$

The planned MEG II update of the MEG experiment [35] is expected to reach sensitivity to $\mathrm{BR}(\mu \to e\gamma) \simeq 4\times 10^{-14}$. The sensitivity to $\mathrm{BR}(\mu \to eee)$ is expected to experience a dramatic increase of up to four orders of magnitude with the realisation of the Mu3e Project [36], which aims at probing values down to $\mathrm{BR}(\mu \to eee) \sim 10^{-16}$ in its phase II of operation. Using an aluminium target, the Mu2e [37] and COMET [38] collaborations plan to ultimately be sensitive to $\mathrm{CR}(\mu\,\mathrm{Al} \to e\,\mathrm{Al}) \sim 6\times 10^{-17}$. The PRISM/PRIME project [39] aims at an impressive

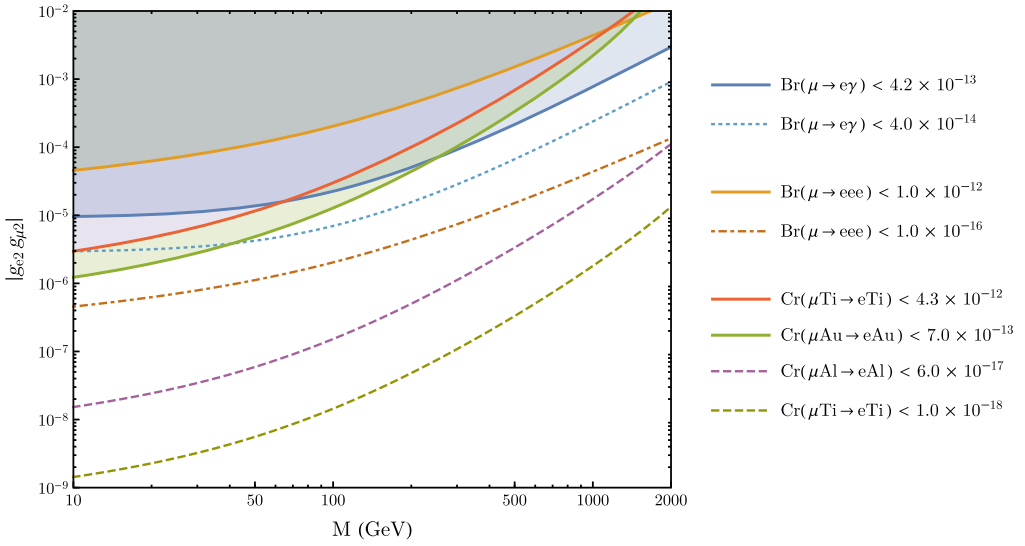

Fig. 3. Present limits (solid lines) and expected future sensitivities (dotted, dashed and dot-dashed lines) on $|g_{\mu 2}||g_{e2}|$ from data on muon LFV processes, as a function of the mass M of heavy Majorana neutrinos. See text for details.

increase of sensitivity to the μ–e conversion rate in titanium, planning to probe values down to $\mathrm{CR}(\mu\,\mathrm{Ti} \to e\,\mathrm{Ti}) \sim 10^{-18}$, an improvement of six orders of magnitude with respect to the bound of eq. (6.12).

We show in Fig. 3 the limits on $|g_{\mu 2}\, g_{e2}|$ implied by the experimental bounds in eqs. (6.10)–(6.13), as a function of the mass M, as well as the prospective sensitivity of the future planned experiments MEG II, Mu3e, Mu2e, COMET and PRISM/PRIME. The data from these experiments, as Fig. 3 indicates, will allow to test for values of $|g_{\mu 2}\, g_{e2}|$ significantly smaller than the existing limits, with a significant potential for a discovery.

The interactions given in eq. (6.3) open up novel decay channels for the Higgs boson, provided the masses of the heavy neutrinos $N_{1,2}$ are below the Higgs boson mass. For $M_{1,2} < m_h = 125.1$ GeV, the new Higgs decay modes are those into one light and one heavy neutrino, $h \to \nu_{\ell L}\, N_k$, $\ell = e, \mu, \tau$, $k = 1, 2$. The phenomenology of the Higgs decays $h \to \nu_{\ell L}\, N_k$ in the model considered in the present article is similar to that of the same decay investigated in detail in [7] in the model discussed in [5]. The rate of the decay $h \to \nu_{\ell L}\, N_{1,2}$ to any $\nu_{\ell L}$ and N_1 or N_2 is given in Ref. [7] and in the limit of zero mass splitting of $N_{1,2}$ ($M_1 = M_2 = M$) reads:

$$\Gamma(h \to \nu\, N) = \frac{m_h}{16\pi}\left(1 - \frac{M^2}{m_h^2}\right)^2 \frac{M^2}{v^2} \sum_{\ell,k} \left|\left(RV\right)_{\ell k}\right|^2 , \tag{6.14}$$

where in the model considered by us

$$\frac{M^2}{v^2} \sum_{\ell,k} \left|\left(RV\right)_{\ell k}\right|^2 = |g_{e2}|^2 + |g_{\mu 2}|^2 + |g_{\tau 2}|^2 , \tag{6.15}$$

and we have used eqs. (6.6) and (6.7). The dominant decay mode of the SM Higgs boson is into bottom quark–antiquark pair, b–$\bar{b}$. The decay rate is given by:

$$\Gamma(h \to b\,\bar{b}) = \frac{3m_h}{16\pi}\left(\frac{m_b}{v}\right)^2\left(1-\frac{4m_b^2}{m_h^2}\right)^{3/2}, \tag{6.16}$$

$m_b \simeq 4.18$ GeV being the b-quark mass (in the $\overline{\rm MS}$ scheme). The SM branching ratio of this decay is 58.4% [40]. The total SM decay width of the Higgs boson is rather small [40]: $\Gamma^{\rm SM}_{\rm tot} \simeq 4.07 \times 10^{-3}$ GeV.

The upper bound on $(\sum_\ell |g_{\ell 2}|^2)$ is determined essentially by the upper bound on $|g_{\tau 2}|^2 = 2|\eta_{\tau\tau}|M^2/v^2$, which is less stringent than the upper bounds on $|g_{e2}|^2$ and $|g_{\mu 2}|^2$. Using the bound $|\eta_{\tau\tau}| < 2.8 \times 10^{-3}$ quoted in eq. (6.8), we get for $M = 100$ GeV the upper bound $|g_{\tau 2}|^2 < 1.8 \times 10^{-3}$. For the Higgs decay rate $\Gamma(h \to \nu N)$ in the case of $M = 100$ GeV and, e.g., $(\sum_\ell |g_{\ell 2}|^2) = 10^{-3}$, we get $\Gamma(h \to \nu N) = 3.2 \times 10^{-4}$ GeV. This decay rate would lead to an increase of the total SM decay width of the Higgs boson by approximately 8%. Thus, the presence of the $h \to \nu N$ decay would modify the SM prediction for the branching ratio for any generic (allowed in the SM) decay of the Higgs particle [7], decreasing it.

We finally comment on neutrinoless double beta ($(\beta\beta)_{0\nu}$-) decay (see, e.g., [12]). The relevant observable is the absolute value of the effective neutrino Majorana mass $|\langle m\rangle|$ (see, e.g., [41]), which receives an extra contribution from the exchange of heavy Majorana neutrinos N_1 and N_2. This contribution should be added to that due to the light Majorana neutrino exchange [42,43] (see also [4,44]). The sum of the two contributions can lead, in principle, to $|\langle m\rangle|$ that differs significantly from that due to the light Majorana neutrino exchange. The contribution due to the $N_{1,2}$ exchange in $|\langle m\rangle|$ in the model considered is proportional, in particular, to the difference between the masses of N_1 and N_2, which form a pseudo-Dirac pair. For $M \gtrsim 1$ GeV, as can be shown, it is strongly suppressed in the present setup due to the extremely small $N_1 - N_2$ mass difference, the stringent upper limit on $|g_{e2}|^2$, and the values of the relevant nuclear matrix elements (NME), which at $M = 1$ GeV are smaller approximately by a factor of 6×10^{-2} than the NME for the light neutrino exchange and scale with M as $(0.9\ \text{GeV}/M)^2$. As a consequence, the contribution to $|\langle m\rangle|$ due to the exchange of N_1 and N_2 is significantly smaller than the contribution from the exchange of light Majorana neutrinos χ_j.

7. Summary and conclusions

In the present paper we have explored a symmetry-protected scenario of neutrino mass generation, where two RH neutrinos are added to the SM. In the class of models considered, the main source of L-violation responsible for the neutrino masses are small lepton-charge violating Yukawa couplings $g_{\ell 1}$ ($\ell = e, \mu, \tau$) to one of the RH neutrinos, ν_{1R}. Thus, the smallness of the light Majorana neutrino masses is related to the smallness of the $g_{\ell 1}$ and not to the RH neutrinos having large Majorana masses in the range of $\sim(10^{10}$–$10^{14})$ GeV as in the standard seesaw scenario. We have considered heavy Majorana neutrinos forming a pseudo-Dirac pair with masses $M_{1,2} \simeq M$ at the TeV or lower scale, which are potentially observable in collider experiments.

The setup described above can be realised in a Froggatt–Nielsen (FN) scheme, as detailed in Section 3. In such a model, no $\mathrm{U}(1)_L$ symmetry is imposed, and instead the suppression of L-violating operators arises in the limit of a large FN charge for ν_{1R}. The FN charge assignments are partly motivated by large ν_μ–ν_τ mixing. The structure of the Yukawa couplings $g_{\ell a}$ ($a = 1, 2$) is then determined by the FN charges, and yields $|g_{e2}| : |g_{\mu 2}| : |g_{\tau 2}| \simeq \epsilon : 1 : 1$, where $\epsilon \simeq \lambda_C \simeq 0.2$ is the FN suppression parameter, while no unambiguous prediction may be extracted for the ratios $|g_{e1}| : |g_{\mu 1}| : |g_{\tau 1}|$.

It is interesting to point out that, given the exceedingly small splitting between heavy neutrinos, the dependence on the Casas–Ibarra complex parameter drops out in the ratios between absolute values of Yukawa couplings to the same RH neutrino. These ratios are then determined (up to the exchange of $g_{\ell 1}$ and $g_{\ell 2}$) by neutrino low-energy parameters alone, namely, by neutrino masses, mixing angles and CPV phases δ and α. Given the Yukawa structure of our model, $|g_{e2}| : |g_{\mu 2}| : |g_{\tau 2}| \simeq \epsilon : 1 : 1$ with $\epsilon \simeq \lambda_C \simeq 0.2$, the Dirac CPV phase δ is predicted to have approximately one of the values $\delta \simeq \pi/4$, $3\pi/4$, or $5\pi/4$, $7\pi/4$, or to lie in a narrow interval around one of these values, while a Majorana CPV phase $\alpha \simeq 0$ is preferred (Figs. 1 and 2).

In the considered scenario, the maximal values of the elements of the neutrino mass matrix lead to constraints on the combinations $|g_{\ell 1} g_{\ell' 2} + g_{\ell' 1} g_{\ell 2}|$, $\ell, \ell' = e, \mu, \tau$, which depend on products of L-conserving and L-violating Yukawa couplings (see Section 6.1). Deviations from unitarity of the PMNS matrix constrain instead the products $|g_{\ell 2} g_{\ell' 2}|$, $\ell, \ell' = e, \mu, \tau$, of L-conserving couplings alone. In particular, the product $|g_{\mu 2} g_{e 2}|$ is constrained by data on muon lepton flavour violating (LFV) processes. Data from future LFV experiments (MEG II, Mu3e, Mu2e, COMET, PRISM/PRIME) will allow to probe values of $|g_{\mu 2}\, g_{e 2}|$ significantly smaller than the existing limits (Fig. 3). The decay of the Higgs boson into one light and one heavy neutrino can have a rate $\Gamma(h \to \nu N)$ as large as 8% of the total SM Higgs decay width. This decay mode can lead to a change of the Higgs branching ratios with respect to the SM predictions. Concerning neutrinoless double beta decay in the considered model, the contribution due to $N_{1,2}$ exchange in the absolute value of the effective neutrino Majorana mass $|\langle m \rangle|$ is found to be negligible when compared to the contribution from the exchange of light Majorana neutrinos.

Finally, we comment on the issue of leptogenesis. For temperatures above the electroweak phase transition (EWPT), the Higgs VEV vanishes and thus, in the considered setup, the splitting between the masses of heavy neutrinos originates from the (suppressed) Majorana mass term $\mu\, \nu_{1R}^T\, C^{-1}\, \nu_{1R}$, with $\mu \sim \epsilon^{n+1} M \sim |g_{\ell 1}| M$. This component of the heavy neutrino mass matrix – which in our case presents a subleading contribution to neutrino masses – is then crucial for resonant leptogenesis to proceed (see, e.g., [45]). The resonant condition reads $\mu \simeq \Gamma/2$, where Γ denotes the average heavy neutrino decay width. However, the values of μ, Γ and neutrino masses are tightly connected in the FN model we analyse, which, together with the required smallness of μ, prevents reproducing the observed baryon asymmetry of the Universe (BAU), $\eta_B^{\rm obs} \simeq (6.09 \pm 0.06) \times 10^{-10}$ [46].

One may instead successfully generate the observed BAU through the mechanism of anti-leptogenesis [47] (also known as "neutrino assisted GUT baryogenesis"). In this case, an excess of both baryon number B and lepton number $\hat{L}$ (see Section 3) is produced at a high energy scale ($T > 10^{12}$ GeV, possibly related to grand unification), while conserving $B - \hat{L}$. If there are new $\hat{L}$-violating interactions in thermal equilibrium at such high temperature, they may erase the lepton number excess while leaving the baryon number excess untouched, since sphalerons are not efficient at these times. At later times, sphalerons are responsible for only a partial conversion of the baryon number excess into a lepton number excess, while some of the baryon excess remains. Unlike resonant leptogenesis, this mechanism relies on a suppression of the $\hat{L}$-violating heavy neutrino mass splitting above the EWPT, in order not to wash-out the asymmetry generated at a high scale. Modifying our setup as detailed in the end of Section 3, the Majorana mass term $\mu\, \nu_{1R}^T\, C^{-1}\, \nu_{1R}$ is forbidden and the heavy neutrinos are degenerate above the EWPT. One then adds a third RH neutrino in the bulk with $(B - \hat{L})(\nu_{3R}) = -1$ and vanishing $\mathrm{U}(1)_L$ charge, such that its Yukawa couplings, which violate lepton number, are allowed, and such that the mass term $M_3\, \nu_{3R}^T\, C^{-1}\, \nu_{3R}$ is generated, $M_3 \sim \langle \Phi \rangle$. Notice that only one such RH neutrino is needed to erase lepton number at high temperatures ($M_3 \sim (10^{12}$–$10^{13})$ GeV), and that there

is a large region of parameter space where the new contribution to the neutrino mass matrix is negligible [48]. Given these conditions, successful anti-leptogenesis may proceed.

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