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The Effect of Radiation Pressure on Spherical Accretion

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Summary. Spherical accretion is studied taking into account the effect of radiation pressure on the flow of the gas. The gas pressure is neglected in the dynamic equations and the opacity is described using an approximate Eddington factor. The paper corrects and extends a previous work on the same subject (Maraschi et al., 1974).

Key words: accretion — neutron stars — black holes

1. Introduction

Spherical accretion onto a neutron star for luminosities near the Eddington limit L_E was first studied by Shakura (1974). His equations included the exchange of momentum between the infalling matter and the outgoing radiation but neglected the convection of radiation. His results are therefore valid at large distances from the star's surface.

The importance of radiation convection at $L \simeq L_E$ was pointed out by Davidson (1973) in a plane geometry and by Maraschi et al. (1974) (M.R.T.) who treated the spherical case neglecting the variation of opacity with depth. Kafka and Meszaros (1976) noted that the M.R.T. equations were incorrect by a factor 4/3 in the convective term and applied the corrected equations to the case of an accreting black hole.

Here we come back to the problem of the accreting neutron star deriving in detail the radiative transfer equations to the first order in v/c and solving the system numerically with an approximate treatment of the opacity variation.

2. Derivation of the Equations

We shall assume, as in the works previously quoted, that the infalling gas is completely ionized hydrogen and that its pressure can be neglected with respect to radiation pressure. The cross section for a photon interaction is then the Thomson cross section σ_T, so that the opacity is frequency independent. A rigorous treatment of the radiative transfer in a spherically symmetric flow to the first order in v/c was given by Cassinelli and Castor (1973) to whom we refer for further references. With the assumptions stated above the relevant equations read:

$$\frac{d}{dr}(\rho v r^2)=0 \qquad \text{(Mass conservation)}, \qquad (1)$$

$$v\frac{dv}{dr}+\frac{GM}{r^2}=\frac{4\pi}{c}\frac{\sigma_T}{m_p}H \qquad \text{(Euler equation)}, \qquad (2)$$

$$\frac{d}{dr}\left(16\pi^2 r^2\left(H+\frac{v}{c}(J+K)\right)\right) = -\frac{\dot{m}}{c}\frac{4\pi\sigma}{m_p}H \qquad \text{(Energy conservation)}, \qquad (3)$$

$$\frac{dK}{dr}+\frac{3K-J}{r}+\frac{v}{c}\frac{dH}{dr}-2\frac{v}{c}\frac{H}{r} -2\frac{v}{c}\frac{1}{\rho}\frac{d\rho}{dr}H=-\frac{\sigma}{m_p}\rho H \qquad \text{(transfer equation)}, \qquad (4)$$

where ρ is the gas density, v the velocity, H is the first moment of the radiation intensity, $F=4\pi H$ is the radiation flux, $L=4\pi r^2 F$ is the luminosity at a given r, J is the angle mean intensity, $u=4\pi J/c$ is the radiation energy density and K is the second moment of the intensity. H, J and K are integrated over the frequency and refer to the frame comoving with the fluid.

Multiplying Equation (4) by $(v/c)\,4\pi r^2$, its right hand side becomes of the same order as that of Equation (3). Neglecting terms $O(v^2/c^2)$ one has

$$4\pi r^2\frac{v}{c}\left(\frac{dK}{dr}+\frac{3K-J}{r}\right)=-\frac{\sigma}{m_p c}\dot{m}H. \qquad (4')$$

In order to have a closed system one must have a relation between K and J which implies a model for the opacity. Following Tamazawa et al. (1975) who treated a similar but more complicated problem for an accreting

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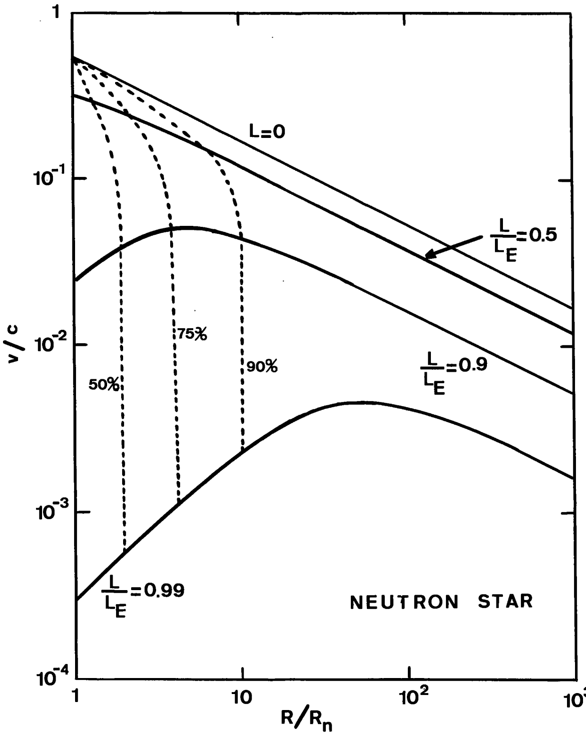

Fig. 1. Infall velocity (v/c) vs radial distance r/R_N for various values of $\lambda = L_\infty/L_E$. The dotted curves give the radius at which the luminosity in the rest frame is 50%, 75%, 90% of L_∞ for different values of λ. The curves refer to a neutron star of radius $R_N = 10^6$ cm and mass 1 $M_\odot$

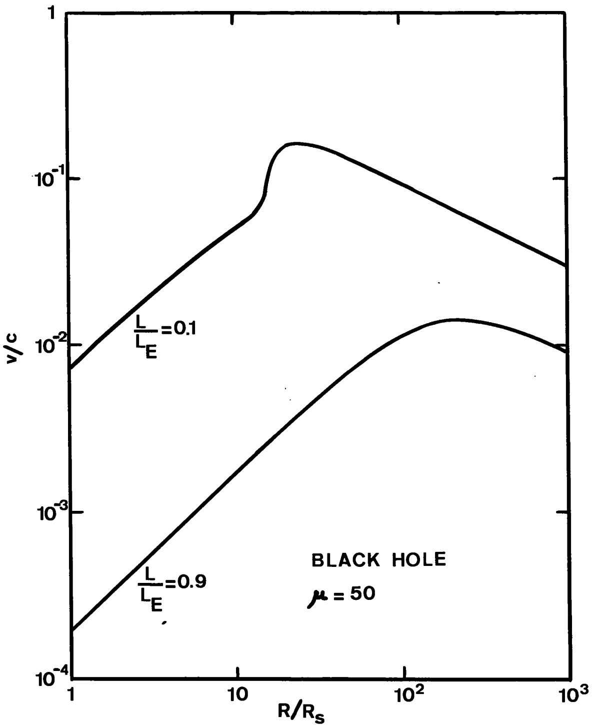

Fig. 2. Infall velocity (v/c) vs radial distance r/R_s for a black hole. The accretion rate is $\mu = 50$ and the luminosity is $L_\infty = 0.1$, 0.9 L_E. The values of μ and L_∞ were chosen for making a direct comparison with the results of Kafka and Meszaros (1976)

black hole we put $K = fJ$ where f is an approximate Eddington factor given by

$$f = (1+\tau)/(1+3\tau), \tag{5}$$

where, according to our initial assumption τ is the Thomson opacity $\tau = \int_r^\infty \rho\sigma_T dr$. With the position

$$E = \frac{1}{2}v^2 - \frac{GM}{r} \tag{6}$$

and recalling the definitions of F and u, the system of equations becomes

$$4\pi r^2 \rho v = \dot{m}, \tag{1a}$$

$$dE/dr = \sigma_T/(\mathrm{cm}_p) F, \tag{2a}$$

$$\frac{d}{dr}(4\pi r^2(F + (f+1)uv)) = -\frac{\dot{m}}{c}\frac{\sigma_T}{m_p}F, \tag{3a}$$

$$\frac{d(fu)}{dr} + (3f-1)\frac{u}{r} = -\frac{\sigma_T}{cm_p}\rho F, \tag{4a}$$

$$df/dr = \frac{1}{2}(3f-1)^2\frac{\sigma_T}{m_p}\rho, \tag{5a}$$

$$v = \frac{\dot{m}}{|\dot{m}|}\left(2\left(E + \frac{GM}{r}\right)\right)^{1/2}, \tag{6a}$$

where $\dot{m}$ and v are negative in the case of accretion. For $f = 1/3$ this system coincides with that of Kafka and Meszaros (1976). From Equations (2a) and (3a) one has

$$4\pi r^2(F + (f+1)uv) = -\dot{m}E + \mathrm{const}. \tag{3b}$$

Since for $r \to \infty$ $E \to 0$, $uv \to 0$ the constant must be the luminosity at infinity L_∞. For an accreting neutron star $L_\infty = -\dot{m}(GM/R_N)$ where R_N is the radius of the star. If the accreting object is a black hole the relation between L_∞ and $\dot{m}$ cannot be specified at this stage and L_∞ can be taken as a parameter provided that $L_\infty < L_E$ (see Kafka and Meszaros, 1976). For $r \to \infty$ the stationary and comoving quantities coincide and the first three equations yield the solution obtained by Shakura (1974), which is therefore an asymptotic solution of our system:

$$E_\infty = +\frac{L_\infty}{\dot{m}}\left(1 - \exp\left(\frac{GM\dot{m}}{L_E}\frac{1}{r}\right)\right). \tag{7}$$

3. Solution of the Equations

We introduce the following adimensional quantities

$x = r/R,$

$\beta = v/c,$

$$\varepsilon = \frac{E}{(GM/R)},$$

$$\mu = \dot{m}\frac{GM}{R}\frac{1}{L_E},$$

$$\rho_a = \rho\frac{\sigma_T}{m_p}R,$$

$$u_a = u 4\pi R^2 c/L_E,$$

where $L_E = 4\pi GMm_p c/\sigma_T$ and the scale length R will be taken as the radius of the star R_N if the accreting object is a neutron star or as the Schwarzschild radius R_S in the case of an accreting black hole.

Eliminating F through (3b), introducing the new variable $p = fu$ and writing again u and ρ for u_a and ρ_a the system takes the form

$$x^2\rho\beta = \mu, \tag{1c}$$

$$\frac{d\varepsilon}{dx} = \frac{\lambda}{x^2} - \mu\frac{\varepsilon}{x^2} - \beta(p+u), \tag{2c}$$

$$\frac{d(p)}{dx} = -\rho\frac{d\varepsilon}{dx} - (3p-u)\frac{1}{x}, \tag{4c}$$

$$\frac{du}{dx} = \frac{u}{p}\frac{dp}{dx} - \frac{1}{2}\frac{\rho}{p}(3p-u), \tag{5c}$$

$$\beta = \frac{\mu}{|\mu|}\left(\frac{2}{K}\left(\varepsilon + \frac{1}{x}\right)\right)^{1/2}, \tag{6c}$$

where $K = c^2R/GM$ and $\lambda = L_\infty/L_E$.

For a neutron star $\lambda = -\mu$ and with $M = 1\ M_\odot$, $R_N = 10^6$ cm, $K = 6.74$. For a black hole $K = 2$.

Since the integration makes use of the asymptotic condition (7) it is convenient to introduce the variable $t = 1/x$. Indicating the t derivatives with the subscript t the system reads

$$\rho\beta = t^2, \tag{1d}$$

$$\varepsilon_t = -\lambda + \mu\varepsilon + \beta t^{-2}(p+u), \tag{2d}$$

$$p_t = -\rho\varepsilon_t + t^{-1}(3p-u), \tag{4d}$$

$$u_t = \frac{u}{p}p_t + \frac{\rho}{2p}\frac{1}{t^2}(3p-u)^2, \tag{5d}$$

$$\beta = \frac{\mu}{|\mu|}\left(\frac{2}{K}(\varepsilon + t)\right)^{1/2}. \tag{6d}$$

The boundary conditions at $t = 0$ are

$$\varepsilon = 0, \quad u = 0, \quad p = 0, \quad u/p = 1. \tag{8}$$

The system (1d)–(6d) has been integrated numerically using a Runge Kutta routine.

4. Results

The results referring to a neutron star of radius $R_N = 10^6$ cm, mass $M = 1\,M_\odot$ are given in Figure 1 for $\lambda = -\mu = 0.5$, 0.9, 0.99. For $L \ll L_E$ the infall velocity increases monotonically towards the star surface, while for L approaching L_E there is a maximum at $r > R_N$. The deceleration of the gas at small radii implies that the radiation flux in the comoving frame is greater than the Eddington limit.

Another interesting feature of this family of curves is that there are no transitions from supersonic to subsonic flow. In all the cases examined if the flow is supersonic at infinity, $\beta_\infty > \beta_s \simeq (4u/9\rho)^{1/2}$, it remains so up to the surface of the star where a shock must form and no maximum occurs in the velocity profile. On the other hand when a maximum occurs the flow is subsonic everywhere. Since $(\beta/\beta_s)_\infty \simeq 3(1-\lambda)/K\lambda$ for $L > 0.6\,L_E$ neither at the surface nor at any radius a shock can occur, which could be important in affecting the spectrum of the emitted radiation.

In Figure 2 are shown the results pertaining to a black hole (of any mass) for $\mu = 50$ and $\lambda = 0.1$ and 0.9. These curves can be compared with those of Kafka and Meszaros (1976) who approximated the opacity variation taking the asymptotic solution (7) for $\tau < 2$ and $p = 1/3\,u$ for $\tau > 2$.

The similarity of our curves with theirs indicates that the results of the integration are not sensitive to the details of the opacity model. Again for $L > 0.8\ L_E$ the flow is everywhere subsonic. However in this case for $L < L_E$ a transition from a supersonic to a subsonic flow occurs at $r > R_s$ so that in every case the flow is subsonic at R_s. As noted by Kafka and Meszaros (1976) the fact that the flow is subsonic at the horizon indicates the inadequacy of the previous description in the case of accretion onto black holes.

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