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T E S I

DIPLOMA DI PERFEZIONAMENTO

"DOCTOR PHILOSOPHIAE"

QUENCHING APPROACHES IN LATTICE GAUGE THEORIES

CANDIDATO:
Zhang Yi-Cheng

RELATORE:
Prof. G. Parisi

Anno Accademico 1983/84

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CHAPTER I

GENERAL SURVEY OF LATTICE GAUGE THEORIES.

1. Introduction.

Nowadays particle physics is formulated by quantized field theories. The procedures of quantization can be classified into canonical operator formalism, Feynman path integral formalism and stocastical formalism. Hereafter we will be concentrated in Feynman path formulation, which we think to provide a natural bridge between field theories and statistical mechanism. It is well known that non-trivial field theories in the continuum are plagued with infinities. These infinities are unphysical and can be avoided by the usual renormalization procedures. Moreover, a field theory defined in continuum can only be tackled properly by perturbations, which is rather limited in its applications. Many crucial questions and practical problems like confinement and hadron mass spectra cannot be even hoped to be answered in perturbative regions. Many such motivations force us to find some alternatives to solve these problem.

Wilson proposed (1) to put a field theory on a lattice. For a finite volume we are dealing with only finite number degrees of freedom. The lattice spacing provides a natural cut off of momenta in the sense that distances

smaller than the lattice spacing have no meaning, and so are the momenta larger than π/a. Therefore by definition there is no ultraviolet divergencies in the lattice theory. The continuum theory is got by sending the lattice spacing to zero at the end of calculations. However, one may take another point of view that our space time is fundamentally discrete, that is to say that the continuum approach is but an approximation to this fundamental lattice, when one goes into the sub-Planck length scale one should see this lattice structure, as recently advocated by T.D. Lee and others (2,3).

2. Lattice Gauge Theories.

Once we have a lattice, we may assign our fields (scalers, vectors, tensors) to different lattice topologies. The basic cubic lattice structures are called r-cells. A point is called a 0-cell, a bond is a 1-cell, a plaquette is a 2-cell and so on. The continuum analogs are differential forms. Thus it is natural to put the scaler on the points, vector on the links, tensor on the plaquettes.

If we want to put photons on the lattice, we would put our basic variables on links call it

$$U_{n,\mu} = \exp\{iagA_\mu(n)\} \tag{1.1}$$

which is a complex number modula unity, or a abelian phase.

Out of these link variables we may define a action of the theory

$$S = \beta \sum_{p} (1 - U_p) \tag{1.2}$$

and the partition function

$$Z = \int \mathcal{D}U \exp\{-S\}$$

where Up is a product around a basic plaquette in the lattice. We see that such action is local gauge invariant. At this stage the only criterion for the choice of action is that it reduces to the continuum action when lattice spacing goes to zero.

$$S \underset{a \to 0}{\simeq} -\frac{1}{4} \int d^D x \, F_{\mu\nu}^2 \qquad \beta = \frac{1}{2g^2} \tag{1.3}$$

This limit sometimes is called the naive continuum limit because we have not included the quantum fluctuations which are implied in the Feynman path integral eq. (12)

Now we have a well defined theory. One immediate advantage is that we can do strong coupling expansions. Take the Wilson loop for example

$$\langle W(C) \rangle = \int \mathcal{D}U \prod_{l \in C} U_l \exp\{-S\} \Big/ Z \tag{1.4}$$

which represents the worldline of a quark pair created at

some moment and annihilated some time later. If this expectation value behaves like

$$\langle W(C) \rangle \sim \exp\{-\mathrm{Area}(C)\} \tag{1.5}$$

We may conclude that there is always a linear potential acting between the two quarks, it would cost us infinite energy if we want to separate them infinitely long apart, so is the confinement picture. On the other hand if

$$\langle W(C) \rangle \sim \exp\{-\mathrm{Perimeter}(C)\} \tag{1.6}$$

then to separate two quarks costs virtually no energy. Thus we have free quarks around and confinement is lost.

The nonabelian generalization is straightforward if we want to put QCD on the lattice. Instead of an abelian phase in eq. (1.1) we now have a SU (N) matrix and instead of Up in eq. (1.2) we now have TrUp to ensure the local gauge invariance.

We shall argue later that whereas the nonabelian gauge theories have always confinement the abelian lattice gauge theories have always a phase transition separating the confining phase and deconfining phase.

The model proposed by Wilson eqs. (1.1,2) is called compact action as constracted with

$$S = \frac{-1}{4} \sum_{x,\mu,\nu} (\nabla_\mu A_\nu(x) - \nabla_\nu A_\mu(x))^2 \tag{1.7}$$

which is also a lattice gauge theory enjoys also local gauge invariance and reduces to the same continuum limit eq. (1.3). For this latter model we cannot perform strong coupling expansion. The crucial difference lies in that in eq. (1.1) we can freely add $2\pi/ag$ to A_μ without changing anything whereas it is not true for the action in eq. (1.7).

3. Dual Meissner- Higgs effect and confinement.

One of the central aims in lattice gauge theories is to understand why quarks are confined. Now people believe that the confinement mechanism for abelian lattice gauge theories is understood properly, at least qualitatively. The basic argument in arriving at this conclusion is by recognizing the crucial role played by the topological defects such as monopoles in the compact models. Confinement is realized in these models by the so-called dual Meissner -Higgs effect as opposed to the superconductors, as advocated by Mandelstom and 'tHooft (4.5). In an abelian theory one can argue it quantitatively. Take the Wilson action for U(1) group.

$$Z = \int \mathcal{D}A \exp\{-S\}$$

$$S = \beta \sum_{n,\mu,\nu} \cos(\nabla_\mu A_\nu(n) - \nabla_\nu A_\mu(n)) \qquad (1.8)$$

where

$$\nabla_\mu A_\nu(n) = A_\nu(n+\mu) - A_\nu(n)$$

To see the dual picture clearly one has to invoke an approximation due to Villain (6). It says that for β large one can expand the Boltzmann weight around the periodic gaussian forms,

$$e^{-\beta\cos(\nabla_\mu A_\nu(n) - \nabla_\nu A_\mu(n))} \underset{(\beta\gg1)}{\simeq} \sum_{n_{\mu\nu}(n)=-\infty}^{\infty} e^{-\frac{\beta}{2}(\nabla_\mu A_\nu(n) - \nabla_\nu A_\mu(n) + 2\pi n_{\mu\nu}(n))^2} \qquad (1.9)$$

where the integer field $n_{\mu\nu}(n)$ is associated with each plaquette. It is convenient to introduce the dual field $f_{\mu\nu}(n)$ as

$$Z = \prod_{n,\mu\neq\nu} \sum_{n_{\mu\nu}(n)=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{df_{\mu\nu}(n)}{\sqrt{2\pi\beta}} \int_{-\pi}^{\pi} \frac{dA_\mu(n)}{2\pi} \times$$

$$\times \exp\left\{-\frac{1}{2\beta}\sum_{n,\mu\neq\nu} f^2_{\mu\nu}(n) + i f_{\mu\nu}(n)(\nabla_\mu A_\nu - \nabla_\mu A_\nu + 2\pi n_{\mu\nu})\right\} \qquad (1.10)$$

Now we may integrate over fields A_μ and it renders $f_{\mu\nu}$ divergenceless and summing over $n_{\mu\nu}$ restricts $f_{\mu\nu}$ to be integer-valued and we have

$$Z = \prod_{n,\mu\neq\nu} \sum_{f_{\mu\nu}=-\infty}^{\infty} \frac{1}{\sqrt{2\pi\beta}} \delta(\bar\nabla_\mu f_{\mu\nu}) \exp\left\{-\frac{1}{2\beta}\sum_{n,\mu\neq\nu} f^2_{\mu\nu}\right\} \qquad (1.11)$$

where we have been sloppy in gauge fixing factors, they should be inserted where appropriate, and

$$\bar{\nabla}_\mu f(n) = f(n) - f(n-\mu) \tag{1.12}$$

the constraint

$$\bar{\nabla}_n f_{\mu\nu}(n) = 0$$

can be solved by the ansatz

$$\begin{aligned} &a)\quad f_{\mu\nu} = \varepsilon_{\mu\nu\sigma}\bar{\nabla}_\sigma \varphi \quad \text{in 3 dimensions and} \\ &b)\quad f_{\mu\nu} = \varepsilon_{\mu\nu\sigma\rho}\bar{\nabla}_\sigma \bar{A}_\rho \end{aligned} \tag{1.14}$$

in four dimensions.

Let us first look at the three dimensional problem. To extend the $f_{\mu\nu}$'s from integer values to the whole range we can again introduce an integer valued field $m(n)$ and

since $\quad f_{\mu\nu}^2 = (\bar{\nabla}_\mu \varphi)^2$

we have

$$Z = \prod_n \int_{-\infty}^{\infty} \frac{d\varphi}{\sqrt{2\pi\beta}} \sum_{m(n)=-\infty}^{\infty} \exp\left\{-\frac{1}{2\beta}\sum_n (\nabla_\mu\varphi)^2 + 2\pi i m(n)\varphi(n)\right\} \tag{1.15}$$

We may interpret $m(n)$ as monopole density since we can relate it as

$$m(n) = \varepsilon_{\mu\nu\sigma}\bar{\nabla}_\nu A_\mu = \bar{\nabla}_\sigma B_\sigma \tag{1.16}$$

obviously the A_μ field is not integrable and we have defects in the original model and such defects are called monopoles.

These monopoles interact via a scaler field $\varphi(n)$ In fact, we can carry out the φ integral to get

$$Z = \prod_n \sum_{m(n)=-\infty}^{\infty} \exp\left\{-2\beta\sum_{\langle ij\rangle} m(i)V(i-j)m(j)\right\} \tag{1.17}$$

where $V(r)$ is the lattice Comlomb potential and has the form

$$V(r) = \int_{-\pi}^{\pi} \frac{d^3k}{(2\pi)^3} \frac{e^{irk}}{6-2\sum_{\mu=1}^{3}\cos k_\mu} \tag{1.18}$$

We see that 3-dimensional lattice QED is equivalent to an ensemble of monopoles interacting through a Comlomb potential. When β is large we expect that the density of monopoles is very small. However, since they interact via a long range force and we do expect Debye screening for the charged particles take place and confinement is always there for charged particles for any finite low temperature. This conclusion was first reached by Polyakov (7) and the lattice formulation was done in ref. (8).

Now we go to four dimensions and we shall see that confinement is lost at low temperature.

In four dimensions instead of eq. (1.14a) using the anstaz eq. (1.14b)

$$Z = \prod_{n,\mu} \int_{-\infty}^{\infty} \frac{d\tilde{A}_\mu}{\sqrt{2\pi\beta}} \sum_{\ell_\mu=-\infty}^{\infty} \delta(\bar{\nabla}_\mu \ell_\mu) \exp\left\{ -\frac{1}{2\beta} \sum_{n,\mu\neq\nu} (\nabla_\mu \tilde{A}_\nu - \nabla_\nu \tilde{A}_\mu)^2 + 2\pi i \sum_{n,\mu} \ell_\mu \tilde{A}_\mu \right\} \tag{1.19}$$

where we have imposed $\tilde{\nabla}_\mu \ell_\mu = 0$ to ensure the gauge invariance. The ℓ_μ field can be naturally interpreted as monopole loops which describe the monopole pair worldlines in the space-time. We can also integrate out $\tilde{A}_\mu$ fields and get instead of eq. (1.17)

$$Z = \prod_{n,\mu} \sum_{\ell_\mu=-\infty}^{\infty} \delta(\bar{\nabla}_\mu \ell_\mu) \exp\left\{ -2\beta \sum_{\langle ij \rangle} \ell_\mu(i) V(i-j) \ell_\mu(j) \right\} \tag{1.20}$$

$$V(r) = \int_{-\pi}^{\pi} \frac{d^4k}{(2\pi)^4} \frac{e^{ikr}}{2\sum^4 (1-\cos k_\mu)}$$

We have now an ensemble of monopole loops. At the low temperature, we expect that the density and the lengths of these monopole loops to be small. Unlike the monopoles themselves in three dimensions, the small and sparse monopole loops have only dipole type interaction at large distances which are much weaker in character. So they cannot screen the charged forces at sufficiently low temperatures and we don't have confinement there. However, due to their much larger entropy associated with larger loops, we expect that as we increase the temperature, say $T > T_c$, we would find infinite long loops everywhere. They have finite density in space-time and they are energetically

strong enough to squeeze the charged force lines and we thereby get confinement. Therefore, four dimensional lattice QED is a two-phase theory.

In later chapters we want to show that when external random disorders are properly introduced, the theories can be made manifestly confining. The idea for abelian lattice gauge theory is most simple and we outline it below.

Instead of the Wilson action we may define our theory like

$$S = \sum_p \beta_p U_p \tag{1.21}$$

where we assign a complex coupling to each plaquette

$$\beta_p = \beta_0 \exp(2\pi i \omega_p)$$

and we assume ω_p to be quenched variables. That is, the free energy of the theory is given by

$$F(\beta_0) = \prod_p \int_{-1}^{1} d\omega_p \log Z(\omega) \tag{1.22}$$

$$Z(\omega) = \int \mathcal{D}A \exp\{\beta_0 \sum \cos(\nabla_\mu A_\nu - \nabla_\nu A_\mu + 2\pi\omega_{\mu\nu})\} \qquad \omega_{\mu\nu} \equiv \omega_p$$

If now we go through all the steps leading to eq. (1.20) we would get

$$Z[\omega] = \prod_{n,\mu} \sum_{l_\mu=-\infty}^{\infty} \delta(\bar{\nabla}_\mu l_\mu)\,\delta(\bar{\nabla}_\mu l'_\mu) \tag{1.23}$$
$$\times \exp\left\{-\beta/2 \sum_{\langle i,j \rangle} (l_\mu(i) + l'_\mu(i))\, v(i-j)\,(l_\mu(j) + l'_\mu(j))\right\}$$

where

$$l'_\mu = \varepsilon_{\mu\nu\sigma\rho}\bar{\nabla}_\nu \omega_{\sigma\rho}$$

is the loop bit due to the quenched random disorder variables. At low temperatures, the dynamic loops l_μ are suppressed and we are still left with random loops frozen in. Their density can be controlled at will and we expect that their population can also squeeze the charged force lines and give rise to the confinement. This is the main motivation for introducing random disorders and we shall elaborate further this idea in some later parts of this work also for nonabelian lattice gauge theories.

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PRELUDE TO CHAPTER II

As we shall be concentrated on a special model in Chapter II, we want to explain briefly in this prelude how the problem arises.

We have introduced a discretized manifold (lattice) for field theories in Chapter I, so we can proceed to put the relevant problem on a computer to simulate it. For example Creutz was the first to measure SU(2) lattice gauge theory phase diagram by Monte Carlo simulation, where it was found that the internal energy follows a rather smooth curve as the temperature varies from the high end to the low end without showing sign of latent heat, in contrast to the U(1) lattice gauge theory case. This kind of simulations gives indication that the nonabelian lattice gauge theories are always in confining phase, a long sought property by Physicists.

However later simulations using more complicated actions than Wilson action revealed a quite complicated phase diagram, i.e. a multiphase picture. Thus it would suggest that nonabelian lattice gauge theories might have decomfining phases. But later analytic work by Bachos and Dashan suggest that the phase transitions observed by Monte Carlo simulations are initiated by some artificial lattice defects whose effects would not have any consequence in the continuum limit. The Chapter II is devoted to a model proposed by the author (Zhang Y.C., Phys. Lett. 124B (1983) 394 and Nucl. Phys. B220 (1983) 302) to show how would one systematically avoid these artificial phase transitions.

Also in the prelude we want to give a schematic prescription of quenching procedure which is used extensively in all later chapters in various versions.

Suppose our system is described by two kinds of fields collectively denoted by U and V, the action is

$$S[U,V]$$

The usual quantum field theory is said to be annealed because the free energy is given by

$$F = \log \int \mathcal{D}U \mathcal{D}V \exp\{-S[U,V]\}$$

However, we can take part of the variables to be quenched, say U, in a sense they are taken to be slow variables and their fluctuation can be neglected. The free energy in the quenched prescription is

$$F_{V} = \int \mathcal{D}U F(U)$$

$$F(U) = \log \int \mathcal{D}V \exp\{-S[U,V]\}$$

and similarly we have expectation values in the quenched prescription as

$$Z[U] = \int \mathcal{D}V \exp\{-S[U,V]\}$$

$$\langle O \rangle = \int \mathcal{D}U \left\{ \frac{\int \mathcal{D}V\, O(U,V) \exp[-S[U,V]]}{Z[U]} \right\}$$

Although in this paper we shall be only interested in applying quenched approaches to guage systems, we would like to recall some extensive applications in statistical mechanics and some intuitive interpretation of the realization of quenching disorders in physical reality.

For this purpose let us mention the spinglasses and localization problems, which are still the focuses in the field and undergo rapid progresses.

Let us start with the Ising model

$$H = \sum_{\langle ij \rangle} J_{ij} S_i S_j$$

$$Z = \sum_{\{S\}} \exp[-\beta H]$$

$$J_{ij} = \begin{cases} -1 & \text{when } i,j \text{ are nearest neighbors} \\ 0 & \text{otherwise} \end{cases}$$

with the above choice of J_{ij}, the model describes pure

ferromagnetic spin in the equilibrium at the temperature $T = 1/\beta$. However in reality we may have situation in which many background defects are present. They are random in character and influence the Ising spins in the original model but they receive no feedback reaction from the configurations of Ising spins. To simulate such a situation using a mathematic model it was suggested (there is a vast literature in this subject, e.g. ref. (5.1))

$$F = -\overline{\log \sum_{\{S\}} \exp\{-\beta H\}}$$

where the bar denotes average over random links

$$\overline{J_{ij}} = 0$$

$$\overline{J_{ij} J_{i'j'}} = \frac{h}{2} \delta_{i,i'} \delta_{j,j'}$$

where the links are assumed to be gaussian distributed. In the language of the preceding general discussion ($S[U,V]$) the U variables are just the random interacting links and the V variables are just Ising spins. We may say Ising spins are the fast variables while the random links are slow variables.

Another popular model using quenching ideas is localization in which one studies the effective Hamiltonian of the Schrödinger equation of a noninteracting electron

$$(\Delta_x + V(x))\psi = E\psi$$

where V(x) is again assumed gaussian distributed

$$P_v = \exp\left\{-\lambda \int d^dx\, V^2(x)\right\}$$

with λ measures the strength of disorders. Localization is a phenomenon that when the disorders are sufficiently strong, the solution of the effective Schrödinger equation may be fast decaying function, which means we do not have macroscopic conductivity. We stop here this digression and return to gauge systems.

CHAPTER II

A QUENCHED NONABELIAN LATTICE GAUGE MODEL.

1. Introduction.

QCD is now a generally accepted theory describing the hadronic world. It is fashionable to put the theory on a lattice to go beyond the perturbative regions. In the strong coupling limit confinement is manifest since the Wilson loop decays according to an area law. For a nonabelian gauge theory like pure gluonic QCD one expects that the confining phase characterized by the area law should persist to the weak coupling region without encountering essential singularities. Such naive expectation was obscured by the sharp crossover in the lattice SU(2) gauge theory via Monte Carlo simulations (1) and subsequently by the peak in the specific heat (2). The situation may be understood using a generalized action with both fundamental and adjoint representations first studied numerically by Bhanot and Creutz (3), they found a rich phase structure of the SU(2)-SO(3) lattice gauge model, there are first order phase transition lines in the (β_f, β_A) plane and the sharp crossover of the Wilson action is believed to be the shadow of the transition line. People have since argued that these first order phase transitions are not deconfining; one can get rid of these singularities by either going around them or by choosing some suitable actions. There are many existing explanations for the origin of these first order phase transitions; Instantons (4), fluxons (5) or monopole condensates (6). Most recently Bachas and Dashen proposed a criterion for these first order phase transitions (7). They observed that the transition lines discovered by Monte Carlo simulations lie always in regions in the coupling plane where the plaquette actions possess non-trivial minima.

However we will refer to these local topological objects generically as lattice defects, since it is questionable how relevant these lattice effects are to the continuum physics, where the correlation length in terms of lattice spacing is infinite. Moreover these lattice local topological objects are model dependent while we assume the continuum limit is unique. The presence of these lattice defects can initiate the first order phase transitions in the region separating the strong and weak coupling limits, just like the gas-liquid transition as the density of the local lattice defects varies. Such first order phase transitions are responsible for the singular structures in the strong coupling approaches. They stop us from extrapolating continuum physics smoothly from the strong coupling end, and also obscure the in-

formations obtained in the scaling regions in the Monte Carlo simulations (8).

It would be nice to find a systematic way to avoid the first order phase transitions caused by the condensation of those lattice defects. Motivated by these ge-geral considerations we proposed a quenching scenario in ref. (9) and we argued the quenched lattice gauge models (QLGM) have smoother crossover behaviors by just avoiding the first order phase transitions.

In this chapter we spell out further the quenching scenario in following steps. In sec. 2 we discuss the local minima in lattice gauge theories and the connection with topological objects, in sec. 3 we describe the QLGM and discuss its weak and scaling regions, where we show theories scale universally, sec. 4 is devoted to solving QLGM in two dimensions and at large N limit analytically and showing the absence of Gross-Witten transition (10), in sec. 5 we use the mean field approach to show the absence of the first order transitions for generic QLGM and discuss how the second order phase transitions can possibly survive, in sec. 6 we point out some interesting connections between the random fluxes and confinement problem.

2. Local minima, monopoles and fluxes.

We start with the lattice gauge models with both fundamental and adjoint representations which have been studied extensively recently. We parameterize the theory in the following way

$$Z = \int \mathcal{D}U \exp\left\{\sum_p \left[(\beta_0 - \beta_A) N (\mathrm{tr}U_p - N + h.c.) + \beta_A (|\mathrm{tr}U_p|^2 - N^2)\right]\right\} \tag{2.1}$$

This parameterization is essentially the same as that used in the literature but has the virtue that the procedure of going to the continuum is done only by $\beta_0 (\beta_0 \to \infty)$ β_A in a sense is orthogonal to β_0 and plays the role of an auxiliary parameter or soft parameter, as its physical meaning will be clear in the next sections.

Bachas and Dashen have studied the local minima of the action and found they are responsible for the first order phase transitions discovered by Monte Carlo simulations. However, for our later purpose, we will go to the random complex coupling planes by rewriting Eq. (1) as

$$Z = \prod_p \left(\frac{N^2}{\pi\beta_A} \int d^2\beta_p\right) \exp\left(-\sum_p N^2 |\beta_p - \beta_0|^2 / \beta_A\right) \int \mathcal{D}U \exp\left[\sum_p \beta_p N (\mathrm{tr}U_p - N) + h.c.\right] \tag{2.2}$$

where β_p is the random complex coupling assigned to each unoriented plaquette and is gaussian distributed with the common mean and variance with

$$\int d^2\beta_p = \int_{-\infty}^{\infty} d(\mathrm{Re}\beta_p) \int_{-\infty}^{\infty} d(\mathrm{Im}\beta_p) \tag{2.3}$$

The mapping from β_p planes to the (β_0, β_A) plane is valid for $\beta_A > 0$. When β_A approaches zero from above the gaussian measure becomes a delta function and the action reduces precisely to the Wilson action.

It was discussed in ref. (9) that the plaquette action in Eq. (2.2) may have minima when the plaquette variable U_p (and thus also the link variables U_ℓ) sit at the element of their abelian subgroup $Z(N)$ in the case of $SU(N)$ gauge group and $u(1)$ in the case of $U(N)$ gauge group. The corresponding abelian matrix

$$U_p^{(0)} = I_N e^{i\alpha_p} \qquad U_p^{(0)} \to U_p^{(0)} e^{i\omega_p} \tag{2.4}$$

is the minimum solution to the plaquette action (where ω_p is a small hermitian matrix)

$$-S_p = \beta_p N (\mathrm{tr}U_p - N) + h.c. \tag{2.5}$$

only if the requirement

$$\delta^2 S = N \mathrm{tr}(\omega_p)^2 (\mathrm{Re}\beta_p \cos\alpha_p - \mathrm{Im}\beta_p \sin\alpha_p) > 0$$

is satisfied. Therefore we see on the half β_p plane ($\delta^2 S_p > 0$) divided by the line through the origin

$$\frac{\mathrm{Re}\beta_p}{\mathrm{Im}\beta_p} = \tan\alpha_p$$

(2.6)

the minima can be formed.

These configurations were first studied by Yoneya (5) for the Wilson action as fluxons and their relevance to the phase transition and confinement was pointed out there.

Now we want to study the partition function Eq. (2.2) with the gaussian random coupling measure taken out

$$Z = \prod_p \left(\frac{N^2}{\pi\beta_A}\int d^2\beta_p\right) \exp\left(-\sum_p N^2 |\beta_p-\beta_0|^2/\beta_A\right) Z[\beta_p] \tag{2.7}$$

$$Z[\beta_p] = \int \mathcal{D}U \exp\left\{\sum_p \beta_p N(\mathrm{tr}U_p - N) + h.c.\right\} \tag{2.8}$$

denote

$$\beta_p = \lambda_p e^{i\alpha_p} \qquad \lambda_p = |\beta_p| \tag{2.9}$$

we note in the expression

$$Z[\lambda_p,\alpha_p] \equiv Z[\beta_p] \tag{2.10}$$

if we make a change for a particular plaquette

$$\alpha_p \to \alpha_p + \gamma_p \tag{2.11}$$

and let the phase shift γ_p go to the variable tr U_p, then we see the change of the system is, for the variable

$$\prod_{p\in C} \mathrm{tr}U_p \to e^{\pm i\gamma_p} \prod_{p\in C} \mathrm{tr}U_p \tag{2.12}$$

the product is around the cubes bordered at the plaquette P, in three dimensions there is a pair of cubes with opposite phase changes and in general there are (D-2) pairs of them.

The choice of cubes is such because these variables are invarieant under the so-called third kind gauge transformations (11) (of course they are ordinary gauge invariants)

$$U_\ell \to \omega_\ell U_\ell \tag{2.13}$$

where U_ℓ is link variable and ω_ℓ is the element of the corresponding abelian subgroup.

Now we can say that under the change of Eq. (2.11) the system is added with D-2 paires of abelian monopole-anti-monopole with magnetic fluxes going through plaquette P. The energy of such monopole pairs is carried by the fluxes and can be measured by the difference in the free energies

$$f(\gamma_p) = \log Z[\lambda_p, \alpha_p + \gamma_p] - \log Z[\lambda_p, \alpha_p] \quad . \tag{2.14}$$

In the trivial two dimensional case (with free boundary conditions) the fluxes are invisible since $f(\gamma_p) = 0$.

The three dimensional case is particular since there are same number fo plaquettes and links. At first sight it would seem that all the abelian phases in Eq. (2.9) can be absorbed by the third kind gauge transformation Eq. (2.14). But this is not true because the presence of the abelian monopoles, presumably it can be achieved by some singular gauge transformations like the ones in the continuum case discussed by Wu and Yang (12) to make the fluxes invisible (i.e. $f(\gamma_p) = 0$). However we do not know how to realize this on a lattice. In higher than three dimensions fluxes carry energy in whatsoever sense since there are more plaquettes than links.

These kind of monopoles were early studied by Brower Kessler and Levine (6) for the SU(2) Wilson action where they found numerically that at the crossover region the monopole density has a sudden drop. Thus they concluded the sharp crossover for the SU(2) lattice gauge theory is caused by transition in monopole phases.

In the next sections we will argue that a quenching scenario in random couplings is able to prevent there topological objects to have phase transitions.

3. The quenching scenario.

In the preceeding section we have seen that in certain regions of coupling planes the lattice topological objects are energetically favorable (being local action minima); we assumed that their condensation can cause phase transitions of the whole system. An analog with condensed matter physics can be drawn here. The motion of electrons in the back ground random magnetic impurities is conventionally described by coupling the quenched random external magnetic fields to the pure system (13). In our case the monopole pairs discussed above act as random magnets, and since our assumption is that they are merely impurities in the lattice gauge system we are led naturally to study the theory with these impurities quenched.

The quenched lattice gauge models (QLGM) can be described by saying that one averages over the random coup-- plings on physical observables rather than on the pratition function. For example, the free energy

$$-F(\beta_0, \beta_A) = \prod_p \left(\frac{N^2}{\pi \beta_A} \int d^2\beta_p \right) \exp\left(-\sum_p N^2 |\beta_p - \beta_c|^2 / \beta_A\right) \times \log Z[\beta_p] \tag{2.15}$$

and the correlation function

$$\langle f \rangle = \prod_p \left(\frac{N^2}{\pi \beta_A} \int d^2\beta_p \right) \exp\left(-\sum_p N^2 |\beta_p - \beta_c|^2 / \beta_A\right) \times \langle f \rangle_0 \tag{2.16}$$

where $\langle f \rangle_0$ is calculated using the action Eq. (8).

For the QLGM to have any relevance to the continuum physics they have to have the same continuum limit as in the annealed case, the conventional two parameter lattice gauge models expressed in Eq. (1). We will argue that it is the case.

In the weak coupling region ($\beta_0 >> 1$) we will parametrize the link variables as

$$U_{\mu,n} = e^{ia A_\mu(n)} \tag{2.17}$$

instead of

$$U_{\mu,n} = e^{iag A_\mu(n)} \tag{2.18}$$

since there is an ambiguity in assigning a coupling to a link. Therefore, we will express the ordinary expansion in coupling constant as expansion in loops; the two approaches are known to be formally equivalent.

We will first look at the annealed case Eq.(2.2) which is simply a rewriting of Eq. (2.1), the expection value is

$$\langle f \rangle = \frac{1}{Z} \prod_p \left(\frac{N^2}{\pi \beta_A} \int d^2\beta_p \right) \exp\left(-\sum_p N^2 |\beta_p|^2 / \beta_A \right) \int \mathcal{D}U\, f(A) \exp\Big\{ \beta_0 \sum_p 2N\, \mathrm{Re}(\mathrm{tr}U_p - N) + \sum_p \left[\beta_p N (\mathrm{tr}U_p - N) + \mathrm{h.c.} \right] \Big\} \tag{2.19}$$

This theory is known to have naive continuum limit

$$\beta_0 = \frac{1}{g_0^2} \tag{2.20}$$

and we suppose it is also true at the quantum level as usual, although a rigorous proof like the one for QED by Sharatchandra (26) is lacking.

Comparison of the diagrams between Eq. (2.19) and continuum theory like QCD reveals that there are new vertices thus new diagrams in the lattice theory. However the naive power counting in lattice spacing a tells us that they are less divergent or finite. Therefore the parameters defining the theory such as the first two efficients of β-function should be the same as in the continuum case, while the parameter in general differs from one subtraction scheme to another. This is the general expectation toward the annealed case which should also hold for the rewriting Eq. (2.19).

In Eq. (2.19) we see that besides the Feynman rules in Wilson action, we have also the propagator,

$$\overset{k}{- - - \rightarrow - - -} \qquad \frac{a^4 \beta_A}{N^2}$$

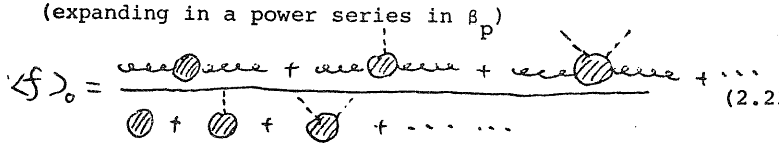
and new vertices proportional to β_p. After calculating

$$\int \mathcal{D}U\, f(A) \exp\{-S(\beta_0, \beta_p)\} \tag{2.21}$$

to some loop order with β_p as external fields, then the average over β_p selects the diagrams with an even number of legs. Now we shall see the difference in diagramatic language between the annealed case and the quenched case. In the average

$$\langle f\rangle_0 = \frac{\int DU f(A)\exp\{-S(\beta_0,\beta_A)\}}{Z[\beta_p]} \tag{2.22}$$

the vacuum diagram cancellation is incomplete due to the presence of the external fields β_p. Taking a two point Green's function for example symbolically we can write (expanding in a power series in β_p)

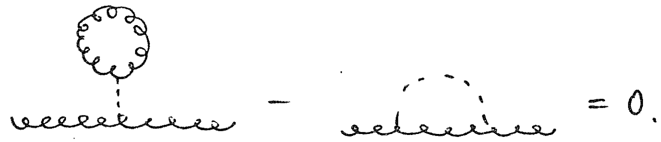


(2.23)

where the dotted lines are β_p legs. Carrying out the average over β_p, we see, in the quenched case, there are new diagrams coming from combination of diagrams in the numerator and in the denominator (which is again expanded in a power series in β_p). These new diagrams are necessarily proportional to β_∇. However, there is no new diagram in character arising in the quenched case since the propagators and vertices are the same. The difference is in the weights of the finite and less divergent contributions therefore we expect that the quenched case can also produce the same kind of continuum physics as in the annealed

case when the lattice spacing goes to zero.

It is interesting to see how the Λ parameter changes in the quenching scenario. Using the perscription Eq. (2.23) it is easily seen that in the quenched case the tadpole diagrams proportional to β_A exactly cancel at least to the orders required to calculate the Λ parameter. Specifically



(2.24)

On very general grounds we would expect that in the annealed case the local topological objects which act as megnets would be free to line up thus lowering the energy of teh system, while in quenched case such a tendency is minimized. When equating the energies of the lattice systems with that of continuum theory, the annealed case requires a smaller Λ parameter than the quenched case to compensate.

This argument can be readily seen using the background field method of Dashen and Gross (15). In place of Eq. (2.13) of Ref. (15) we have

$$-\log\langle Z_a[\beta_p]\rangle + \log Z_m = \left[\frac{1}{4g^2(a)} - \frac{1}{4g^2(m)} + d_A(ma)\right]\int d^4x\,(F_{\mu\nu})^2 \tag{2.25}$$

$$-\langle \log Z_a[\beta_p]\rangle + \log Z_m = \left[\frac{1}{4g^2(a)} - \frac{1}{4g^2(m)} + d_Q(ma)\right]\int d^4x\,(F_{\mu\nu})^2$$

$$ \tag{2.26} $$

for the annealed case and quenched case, respectively. Where A stands for annealed, Q stands for quenched, $\langle \ldots \rangle$ denotes the quenching average, $g^2(a)$ is the bare lattice coupling approriate for a lattice spacing a and is the <u>same</u> for both annealed and quenched cases, $g^2(m)$ is the coupling of the corresponding continuum theory using Pauli-Villars regularization. The only scheme dependence is contained in d_A (ma) and d_Q (ma), which are espressed (15)

$$ d_{A,Q}(ma) = \frac{11N}{96\pi^2}\left(\log ma + C(N)_{A,Q}\right) \tag{2.27} $$

where we have used implicitly the assumption that the two schemes have the same divergent structures as for the continuum theory. Applying Jensen's inequality (the quenching measure is normalized properly to 1)

$$ \langle \log Z_a[\beta_p] \rangle \leq \log \langle Z_a[\beta_p] \rangle \tag{2.28} $$

to Eq. (2.25) and (2.26) we get

$$ C(N)_Q \geq C(N)_A \tag{2.29} $$

The Λ parameters are related as

$$ \frac{\Lambda^L_{A,Q}}{\Lambda_{PV}} = e^{C(N)_{A,Q}} \tag{2.30} $$

which in turn gives

$$ \Lambda^L_Q \geq \Lambda^L_A \tag{2.31} $$

In arriving at this conclusion we have inplicitly assumed that the inequality holds also at the leading loop order.

Let us calculate quantitatively Λ^L_Q . In the light of Eq. (2.24) we would expect that the tadpole diagram proportional to β_A does not appear in the quenched case. We knew previously (8,27) that the β_A dependence of the Λ parameter in the annealed case to one loop order arises solely from this tadpole diagram. Thus in the quenched case Λ^L_Q should be β_A independent to that order.

Now we show the above assertion using the background field method and we follow the notations in Ref. (15). We omit the almost identical steps in our calculation, for details we refer to Ref. (15). In place of the Eq. (3.8) in Ref. (15) we have now

$$ \begin{aligned} -S_p + S_p^0 &= 2N\beta_0 \operatorname{tr}(U_p - U_p^0) + \mathrm{Re}\,\beta_p 2N \,\mathrm{Re}\operatorname{tr}(U_p - 1) \\ &\quad - \mathrm{Im}\,\beta_p 2N \,\mathrm{Im}\operatorname{tr} U_p \\ &= -N\beta_0 \operatorname{tr}(D^0_\mu \alpha^\nu - D^0_\nu \alpha^\mu)^2 - \frac{\beta_0}{2}\operatorname{tr}(\nabla_\mu \alpha^\nu - \nabla_\nu \alpha^\mu)^2 \operatorname{tr}(G^0_p - 2) \\ &\quad + N\beta_0 \operatorname{tr} i E_p H^0_p + \mathrm{Re}\,\beta_p N\left[\operatorname{tr}(G^0_p - 2) - \operatorname{tr}(\nabla_\mu \alpha^\nu - \nabla_\nu \alpha^\mu)^2\right] \end{aligned} \tag{2.32} $$

where the slight difference in notations is due to our parameterization Eq. (1) and we have used further condensed notation $f_p \equiv f_{\mu\nu}(x)$. In arriving at the last expression we have dropped all the eventual high order contributions in α's and background fields. Only the last piece is new. Note that the new term does not get mixed with any other term to the leading order when we carry out the α integration.

The contribution to $C(N)_Q$ is proportional to

$$\left\langle \log\left\{1 + L + \mathrm{Re}\beta_p N\left[\mathrm{tr}(G_p^0 - 2) - A\right] - (\mathrm{Re}\beta_p N)^2 A\,\mathrm{tr}(G_p^0 - 2)\right\}\right\rangle \quad (2.33)$$

where the α integration is done, L denotes collectively the old contributions ($\beta_A = 0$) and A is a constant proportional to $1/\beta_o$.

If the quenching average were inside the logarithm, as in the annealed case, only the last piece inside the parenthesis survives.; Expanding the logarithm we would obtain a contribution proportional to β_A/β_o to $C(N)_A$ (we have in mind that $\beta_A \sim \beta_o$ for finite N). We write down that result explicitly for the SU(N) gauge theory (which is known to the authers of Ref. (8,27) using different parameterizations)

$$\frac{\Lambda_A^L(\beta_A,\beta_o)}{\Lambda_{PV}} = \frac{\Lambda_A^L(0,\beta_o)}{\Lambda_{PV}} \cdot e^{-\frac{(N^2-1)3\pi^2}{11N^2}\frac{\beta_A}{\beta_o}} \quad (2.34)$$

However, in the quenched case, we have first to expand the

logarithm and then do the quenching average. It is readily seen that when expanding the logarithm, the coefficient of $(\mathrm{Re}\beta_p)^2$ contains only higher order terms (i.e. terms proportional to $1/\beta_o^2$ or $[\mathrm{tr}(G_p^0-2)]^2$). The lowest order terms (proportional to $\mathrm{tr}(G_p^0-2)/\beta_o$) are cancelled out exactly. Just as we have anticipated from Eq. (2.24), we have done an explicit calculation to show Λ_Q^L is β_A independent to the leading order

$$\Lambda_Q^L(\beta_o,\beta_A) = \Lambda_Q^L(\beta_o,0) = \Lambda_W^L(\beta_o) \quad (2.35)$$

where $\Lambda_W^L(\beta_o)$ stands for the Λ parameter of the corresponding Wilson action. We have seen that in our QLGM the Λ parameter does not depend on β_A, therefore a rather board class of theories with different positive β_A's will scale universally just as the Wilson action, thus giving comforting support to the current Monte Carlo result (28). The theories with finite positive β_A in the QLGM, which presumably are not plagued with the first order phase transitions, will scale just like the Wilson action. Thus if we don't have to worry about the first order phase transitions, we can just use the Wilson action to do the calculations. However, the Monte Carlo comparisons with this expectation are strongly suggested.

4. Absence of the Gross-Witten transition in QLGM.

Gauge theories in two dimensions are essentially a one plaquette problem when a proper gauge is chosen. In the large N limit for the gauge group U(N), the saddle point approximation is expected to be exact, thus rendering these models soluble. Gross andWitten solved this problem for the Wilson action and they found that there is a third order phase transition separating the strong and weak coupling phases (10). Such singular behaviour was also found for the extended action in the annealed case (17). However, it can be avoided by choosing proper actions (18). Here we will see that the quenching scenario is a systematic way to eliminate such phase transitions.

The intensive free energy in this case is

$$-F = \int \mathcal{D}\beta \, \frac{1}{LTN^2} \log \int \mathcal{D}U \exp\Big[\sum_p \beta_p N(\mathrm{tr}U_p - N) + h.c.\Big] \qquad (2.36)$$

where $\displaystyle\int \mathcal{D}\beta = \prod_p \frac{N^2}{\pi \beta_A} \int d^2\beta_p$

(where LT is the two dimensional space time volume and putting the lattice spacing a to 1).

$$\int \mathcal{D}\beta \, \frac{1}{LTN^2} \sum_p \log \int \mathcal{D}U \exp\big[\beta_p N(\mathrm{tr}U - N) + h.c.\big] \qquad (2.37)$$

where the summation runs over LT terms, and now we have

$$-F = \frac{N^2}{\pi \beta_A} \int d^2\beta_p \exp\{-N^2|\beta_p - \beta_0|^2/\beta_A\} \frac{1}{N^2} \log \int dU \exp\big[\beta_p N(\mathrm{tr}U - N) + h.c.\big] \qquad (2.38)$$

A similar procedure can now be implemented for a rectangular Wilson loop (of size RI)

$$W = \int \mathcal{D}\beta \Big(\int \mathcal{D}U \, \mathrm{tr}U(C) \exp\Big[\sum_p \beta_p N(\mathrm{tr}U_p - N) + h.c.\Big] \Big/ Z[\beta_p] \Big)$$

$$= [\omega]^{RI}$$

$$\omega = \frac{N^2}{\pi \beta_A} \int d^2\beta_p \exp(-N^2|\beta_p - \beta_0|^2/\beta_A) \frac{\int dU \, \mathrm{tr}U \exp[\beta_p N(\mathrm{tr}U - N) + h.c.]}{\int dU \exp[\beta_p N(\mathrm{tr}U - N) + h.c.]} \qquad (2.39)$$

for a rectangular loop.

The expression

$$E(\beta_p) = \frac{1}{N^2} \log \int dU \exp\big[\beta_p N(\mathrm{tr}U - N) + h.c.\big] \qquad (2.40)$$

in Eq. (2.38) has a finite separate large N limit. Writing

$$\beta_p = \beta e^{i\alpha} \qquad \beta = |\beta_p| \qquad (2.41)$$

Eq. (2.40) becomes

$$E(\beta_p) = \begin{cases} \beta^2 - 2\cos\alpha & \beta \lesssim 2 \\ 2\beta - \frac{1}{2}\log 2\beta - \frac{3}{4} - 2\cos\alpha & \beta \gtrsim 2 \end{cases} \qquad (2.42)$$

which is exact Gross-Witten solution when $\alpha = 0$.

Finally the intensitive free energy is (with $\gamma = \beta_A / N^2$ kept finite (11,9))

$$F(\beta_0,\gamma) = \frac{2}{\gamma}\int_0^2 \beta^3 d\beta\, e^{-(\beta^2+\beta_0^2)/\gamma} I_0\left(\frac{2\beta_0\beta}{\gamma}\right) + \frac{2}{\gamma}\int_2^{\infty} \beta d\beta\, e^{-(\beta^2+\beta_0^2)/\gamma} I_0\left(\frac{2\beta_0\beta}{\gamma}\right)$$
$$\times\left(2\beta - \frac{1}{2}\log 2\beta - \frac{3}{4}\right) - \frac{4}{\gamma}\int_0^{\infty} \beta d\beta\, e^{-(\beta^2+\beta_0^2)/\gamma} I_1\left(\frac{2\beta_0\beta}{\gamma}\right) \quad (2.43)$$

which is absent of the Gross-Witten phase transition.

5. Mean field analysis and survival of second order phase transitions from the quenching scenario.

Mean field approximations were very successful in statistical mechanics. Naive applications to gauge theories appear to be in contrast with the Elitzur's theorem. However the mean field approximations in the present formalism can be phrased as the saddle point approximations (19), and they are very impressive in predicting the first order phase transitions. The comparison with Monte Carlo results shows that they are qualitatively correct. As discussed in the preceeding sections, the first order phase transitions arise from the local minima of the action where the correlation length is expected to be short, thus

giving rather sharp minima.

In the following we show that the first order phase transitions are absent in the QLGM via a mean field approximation. To see this we do not even have to go into much detail.

For the QLGM we use a modified method of Drouffe (19). Define a trial Hamiltonian

$$H = \sum_{\ell} \mathrm{tr}\left(M_\ell^{\dagger} U_\ell + h.c.\right) \quad (2.44)$$

where M_ℓ is an arbitrary N by N matrix.
We take the free energy in QLGM

$$-F = \int \mathcal{D}\beta \log Z[\beta_p] \quad (2.45)$$

Jensen's inequality gives a bound for $Z[\beta_p]$

$$\langle e^{-S+H} \rangle \geq e^{\langle -S+H \rangle} \quad (2.46)$$

where the expectation denotes

$$\langle A \rangle = \int \mathcal{D}U e^{-H} A / Z_H \quad (2.47)$$

$$Z_H = \int \mathcal{D}U e^{-H}$$

and S is the action in Eq.(2.12) and thus

$$\bar{F} \leq \int \mathcal{D}\beta \left(\langle S-H \rangle - \log Z_H \right) \tag{2.48}$$

We, assume the parameterization

$$M_\ell = m V_\ell \tag{2.49}$$

where V_ℓ is in SU(N) or U(N) gauge group.

The task of the mean field approximation is then reduced to find an m which optimizes the bound Eq. (2.48). The general feature is that the optimizing m is a functional of couplings (e.g. β_0, β_A in the annealed case and β_p in the quenched case) and we shall denote it as $m(\beta)$. In the coupling region $m(\beta)$ may be qualitatively different; it may have finite jumps when crossing some lines (e.g. it can be somewhere zero and somewhere nonzero). When such jumps occur the first order phase transitions are said to be 'located'.

In the quenching scenario, Eq. (2.48) means to sum over all the, optimizing points in the complex β_p planes, even though finite jumps for $m(\beta)$ may occur. The integral as a function of β_0 and β_A is a smooth function everywhere in the upper (β_0, β_A) plane.

We conclude that in the framework of the mean field approximation no first order phase transitions can occur, or, to be prudent, the mean field approximation cannot predict any phase transition in QLGM.

We have seen that with finite β_A, the quenching scenario can smear a singular function, while for $\beta_A = 0$ we know the Wilson action for $N > 4$ has a first order phase transition. Perhaps the best way to understand the transient regime is to draw an analog with heat conduction. The quenching measure is like a heat propagating kernel and β_A plays a role of the relaxation time. The possible singular thermodynamic function acts as a possible singular beat distribution imposed as initial conditions. After a finite instant β_A we always find a smooth distribution function.

It is interesting to note how can the quenching scenario manages to eliminate the first order phase transitions while keeping the second order ones intact. The reason is in general in Eq. (15). In the quenching measure

$$\prod_p \left(\frac{N^2}{\pi \beta_A} \int d^2\beta_p \right) \exp\left(-\sum_p N^2 |\beta_p - \beta_0|^2 / \beta_A \right) \tag{2.50}$$

the exponent has a volume factor, which would mean when we go to the infinite volume limit the gaussian measure is merely a delta function and the theory reduces just to the Wilson action. The quenching scenario would have no thermal dynamic consequences. Such a case indeed happens and an example can be found in ref. (20). However, such cases do not occur in the quenching scenario generally, when the interactions in gauge fields are short ranged. An extreme example is the two dimensional gauge theory discussed in the last section, where plaquettes decouple and β_p are independent variables. At first order phase transitions, the correlation lengths are finite, therefore the quenching scenario is sufficient to smear out the first order transitions.

The situation is quite different for the second order phase transitions, where the correlation length in gauge fields is divergent. Within such a correlation range, β_p are effectively coupled and the volume factor in the exponent Eq. (50) is important the fluctuations of β_p are suppressed and the quenching measure gives a delta function. We conclude that the quenched theories approach the same critical limit as in the annealed case. Note that the continuum limit is just an important example of this statement.

Strictly speaking, the quenched regulated functions with finite positive β_A can be best said to be in the class C^∞, but not necessarily analytic. Moreover, the quenching scenario by itself cannot exclude any long range order phase transitions (e.g. the second order phase transitions) if there are any in the theory. The leading order in strong coupling expansion of a Wilson loop shows that the quenched theory is also confining (being independent of β_A). However, if long range phase transitions take place in the intermediate coupling re-

gions, like the case of U(1) gauge group, the theory can still have deconfining phases. The present approach, combined with the Bachas-Dashen criterion (which did not predict the second order phase transition of the U(1) gauge group Ref. (7)), may suggest an explicit demonstration that the artificial first order phase transitions due to the choices of lattice actions (including those observed in the Monte Carlo simulations) could be generally avoided. However, in the present stage, we cannot claim there are no long range singularities in the intermediate coupling regions for the non-abelian lattice gauge theories, though we do not expect any.

In the quenching measure there is another factor proportional to D^2 (D is the dimensionality). No matter how local the theory is the D links are coupled together through $D(D-1)/2$ plaquettes by definition, thus we expect when $D \to \infty$ the theory also reduces to the annealed case and the first order phase transitions can still survive in large D limit.

We can support the above expectation by following closely the work of Drouffe, Parisi and Sourlas (29), which predicts first order phase transitions for all the lattice gauge theories with Wilson action when $D \to \infty$. Their observation is essentially when doing the strong coupling expansion, the dominant diagrams have a structure of three-dimensional cubes, arranged in a tree and each plaquette has to be used <u>at most once</u>. Therefore, in our present case (the expansion parameters are $\beta_0 + \beta_p$ or $\beta_0 + \beta_p$), no β_p^2 dependence would appear to this leading order ($D \to \infty$). We should get an identical free energy as for the Wilson action which in turn has a first order phase transition in the parameter

6. Discussion.

We conclude this chapter with some speculations. It is a well known fact that in condensed matter physics the random quenched external fields coupled with the pure system reduce effectively the space-time dimensionality by two (21). Parisi and Soulas were able to explain this fact by revealing that there is a hidden supersymmetry in the mixed system (22). Their observation is currently being followed vigorously by the MIT group (23) for the case of a gauge theory in the continuum. There is a recent claim that confinement is manifest when the gauge theory is coupled by hand with quenched random sources (24) since the resulting theory is effectively a 4-2=2 dimensional theory. The connection between confinement and random quenched fluxes was pointed out by Parisi (14) and Olesen (25). We may view the random complex quenched couplings as a natural realization of the Parisi-Olesen fluxes. We

wonder if confinement can be manifest without doing strong coupling expansion in QLGM.

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PRELUDE TO CHAPTER III

In the beginning of 1982, there was a great excitment among physicists by the discovery made by Eguchi and Kawai showing the so-called large N reduction. Which roughly is to say that when the number of internal degrees of freedom is sufficiently large, theory defined in the infinite volume limit approaches the one defined on a single point which greatly simplifies the original theory. However, as it was later shown both by analytic work and computer simulations that such a reduction works only at high temperatures, and there is a first order phase transition in the reduced theory which does not exist for the theory in the infinite volume. It was subsequently suggested by the Princeton group and Parisi that a quenching procedure may help to reduce the theory exactly from infinite volume to a single point. In such a procedure the extra first order phase transition might be avoided and it so leads to the reduction.

However there are many different prescriptions of the quenching procedure. Chapter III is devoted to a detailed analysis of the quenched periodic boundary condition approach and is based on published material in (G. Parisi and Zhang Y.-C., Phys. Lett. 114B (1982) 319 and Nucl. Phys. B216 (1983) 408).

In the studies of reduced model, the quenching idea plays only a partial role, the limit N going to infinity is crucial here. Let us recall the basic elements of 1/N expansion as a preparation for Chapter III.

Take a scalar field theory in which ϕ_a has N components and the action is designed so that it is invariant under an internal SU(N) or SO(N) rotation, an example can be

$$S = \int d^D x \left\{ \sum_{\mu, a} (\partial_\mu \phi_a)(\partial^\mu \phi_a) + \lambda \left(\sum_a \phi_a^2 \right)^2 \right\}$$

Now suppose N is very large, we see that the first term is of order N while the second term is of order N^2. It may seem that the whole action explodes. In order to have a nontrivial large N limit, one has necessarily to keep $\lambda \sim O(\frac{1}{N})$. This model is generally called large N vector model, in the large N limit mean field approximation is exact, therefore we expect it can be solved by mean field technics.

Gauge theory is highly nontrivial under large N limit. This is because that gauge fields are described by matrices not by vectors, large N limit imposes only constraint on the eigenvalues but not on the off-diagonal elements of the gauge fields. However, remarkable exception has been made in zero and one space-time dimensions [2]. Despite these pessimistic remarks, we shall see that significant simplification can be achieved in the large N limit.

Let us look at SU(N) gauge theory in the continuum.

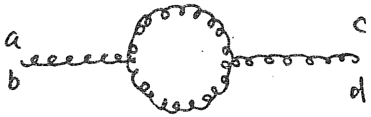
$$S = \frac{1}{4}\int d^Dx\, tr\left(\partial_\mu G_\nu - \partial_\nu G_\mu + ig[G_\mu, G_\nu]\right)^2$$

$$+ \text{(Ghost and gauge fixing)}$$

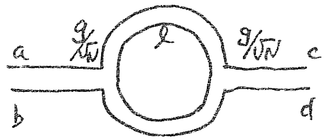
where G_μ is N x N matrix. As explained in the vector model, the coupling $\underline{g}$ should be order of $1/\sqrt{N}$ to have a meaningful large N limit. Now we look at perturbation diagram expansion, for simplicity we start with a two point function

$$\langle G_\mu^{ab}(x)\, G_\nu^{cd}(0)\rangle \qquad \begin{array}{l} tr G_\mu = 0 \\ a, b, c, d, = 1, \cdots, N \end{array}$$

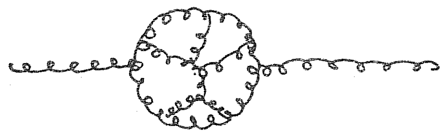
where we have used double indexes for the gauge fields instead of one which would have to run from 1 to $N^2 - 1$. This choice of bookkeeping greatly facilitates the counting rule of the leading Feynman diagram. Take the diagram



in double line representation of 'tHooft it would look like



although the coupling is very small $\sim g^2/N$ we still have a free index summation which gives a factor N thus the total contribution from this diagram is order one. Then we can proceed to inspect more complicated diagrams. For example



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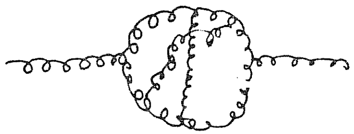
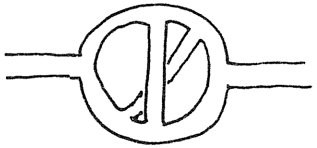
repeating the argument presented before we can be convinced that this diagram is also order one. In fact one can formulate the observation more rigorously using a topological theorem which says as long as the diagram can be drawn on a plane with no line overlaping each other, the vertexes and the index loops keep a fixed ratio, thus ensures the over all contribution is always order one.

The above statement is no longer true when the diagram is nonplanar, for example



where the two internal lines necessarily cross each other. We translate this diagram into double line representation, i.e.



where we can see that from six verteces we get a factor g^6/N^3 while we have only one free index to sum and the total contribution is order of $1/N^2$, therefore in the large N limit this kind of nonplanar diagrams are to be neglected.

At least in perturbation theory, when N is large we can concentrate on a rather small portion of diagrams i.e. the planar diagrams. People have since tried to develop technics to sum planar diagrams which would mean solve the theory exactly in large N limit. People believe that even at infinite N the theory should still bear relevance to the theory with finite N, say SU(3)

QCD. For example, it is presumed that SU(∞) theory should be still confining, that one of reasons why people are working so hard in 1/N expansion studies.

In Chapter III we shall see that with some appropriate quenching procedure we can even set equivalence a single point theory and a infinite volume theory. We have to stress here that progresses in this field are still continuing and in present chapter we will emphasis only our point of view.

CHAPTER III

ON LARGE-N REDUCED MODEL USING QUENCHING IDEAS.

1. Introduction.

The 1/N expansion method has provided us with a valuable approach to the understanding of gauge theories and other field theory models in the non-perturbative regions. The main reason for this is that in the leading order of the 1/N approximation of the region where N is finite. For example, QCD in this leading order is the sum of all the planar diagrams, and confinement presumably persists to this level (1). People have since tried to construct this leading - order contribution and indeed results have been obtained in closed dorm for some notable models (2). It was advocated by Witten that for $N\to\infty$ there is a dominating configuration which he called the master field, and this idea was even stated explicitly by Coleman (3), claiming that the master fields should be formed in an appropriate gauge. Surprisingly such a hope has recently been partially realized by Eguchi and Kawai (4). They claimed the conventional U(N) Wilson lattice gauge theory defined in the infinite volume.

$$-S_W = \beta \sum_{x} \sum_{\mu\neq\nu} \mathrm{tr}\,(U_{x,\mu} U_{x+\mu,\nu} U^{\dagger}_{x+\nu,\mu} U^{\dagger}_{x,\nu}) \tag{3.1}$$

is equivalent to a much simpler model in the large-N limit,

$$-S_{EK} = \beta \sum_{\mu\neq\nu} \mathrm{tr}(U_\mu U_\nu U^{\dagger}_\mu U^{\dagger}_\nu) \tag{3.2}$$

They reduced the original model to a single point by identifying all the dynamical variables in the same directions; they have therefore implicitly employed the periodic boundary conditions

$$U_\mu(x+\hat{\mu}) = U_\mu(x) \tag{3.3}$$

However, as the authors of ref. (5) pointed out, their claim does not hold in the weak coupling regions, and they propose a quenched model to remedy the EK model. In ref. (6) a generalized periodic boundary condition was proposed and it was shown this approach gives the same set of planar diagrams as those in the infinite volume for the scalar matrix model

$$S = \int \left\{ \mathrm{tr}(\partial_\mu M \partial_\mu M^{\dagger}) + m^2 \mathrm{tr}(MM^{\dagger}) + \frac{g}{4N} \mathrm{tr}(MM^{\dagger}MM^{\dagger}) \right\} d^D x$$

The following rule was adopted to reduce the theory

$$M(x+\hat{\mu}) = e^{i\theta^\mu} M(x)\, e^{-i\theta^\mu} \tag{3.4}$$

$$(e^{i\theta^\mu})_{ij} = \delta_{ij}\, e^{i\theta^\mu_i} \qquad i,j = 1,\dots,N$$

This approach was subsequently applied to gauge theories in ref. (7). The main conclusion drawn there was for O(N) gauge theory, and it was shown that no spontaneous symmetry breaking can occur in any coupling region.

In the present chapter we intend to describe a more complete account of the approach. We will consistently follow the methods employed in ref. (6,7) using more examples and analysis; some points for U(N) gauge theory will be further clarified.

The layout of the chapter is as follows. In sect. 2 we interpret the generalized periodic boundary conditions with heuristic arguments which serve to explain the physical mechanism of ref. (6). Sect. 3 discusses the basic applications with the single model. Sect. 4 discusses the non-linear matrix model or the chiral spin model which serves to continue the approach. Sect. 5 contains a discussion of problems in gauge theories and Sect. 6 is devoted to various applications of the approach, especially for numerical simulations. In the conclusion we also try to argue the possible connections between gauge theory end the chiral spin model at the large -N limit.

2. The generalized periodic boundary conditions.

The quenched background gauge field method was sug-

gested in ref. (6) and was also employed in gauge theories (7). Here we explain its physical working by illustrations with intuitive argument.

It should be recognized that as the size of the system becomes smaller and smaller, the boundary effects become more and more important; in other words, an appropriate prescription of boundary conditions becomes more and more relevant. In order to resemble the physics of the infinite wolume on a smaller size space-time, the finite-size effects must be somehow maximally eliminated.

This goal can be achieved by using the method of quenched gauge fields. For the convenience of presentation, we take the continuum case and concentrate on one of the directions in space-time. The range of this direction is taken to be finite, with others finite.

We require for this direction

$$A^{\nu}_{ab}(x+\hat{\mu}) = A^{\nu}_{ab}(x)\, e^{i(\theta^{n}_{a}-\theta^{n}_{b})} \tag{3.5}$$

$$\psi_a(x+\hat{\mu}) = \psi_a(x)\, e^{i\theta^{n}_{a}} \tag{3.6}$$

The A and ψ fields can be thought of as gluon and quark fields on more generic fields in the regular and fundamental representations, respectively. The boundary conditions demand that the field variables experience the background gauge fields, the latter to be quenched.

The system is depicted in Fig. 1. We use the wave function and Feynman path integral language. Let us consider a particle or any other object at point A; we are interested in evaluating the total probability of arriving at point B. Suppose we have used the usual periodic boundary conditions where the left and right boundaries are identified. Typically the object would follow two kinds of path a and b in the infinite volume as shown in Fig. 1. However, due to the finiteness in that direction, we get some extra paths such as path c in Fig. 1. In order to correctly describe the physics of the infinite volume in a finite region, the contributions arising from such (like path c) must be eliminated. This can be achieved by averaging over the randomly distributed background gauge fields. In this way an object annihilated at the right-hand boundary and created again at the right-hand boundary along the path b in Fig. 1 will be associated with the opposite path; after averaging over θ, such a contribution survives, while an object following path c will necessarily have a non-trivial phase dependence and the contributions so obtained will vanish after integrating over θ.

However there is a hole in the above argument. Let us take for example a quark anti-quark combination $\overline{\psi}_a \psi_b$ or a gluon field A^{μ}_{ab}, there are N^2 of them. The above argument would not hold if the two indices coincide; in

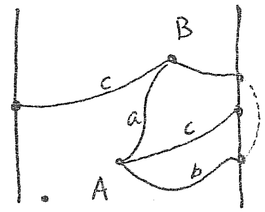


Fig. 1. The heuristic diagram.

such a case, that object would not have a θ-dependence and would still follow the "wrong paths" as before. But there are only N such objects, so we have to send N to infinity to suppress those unwanted contributions.

Moreover, sending N to infinity also serves to make the phase continuously distributed, since they would play the role of momenta. In this way we have maximally reduced the finite volume effects in the large-N limit.

3. The basic approach.

In ref. (6) it was shown that the exact set of planar diagrams can be obtained at a single point. The proof was via perturbation theory proceeding order-by-order. The proof could also be derived using non-perturbative means such as the Dyson-Schwinger equations. For the case at hand, we will use perturbation theory for the linear model and the D-S equations for its non-linear generalization in Sect. 4. For completeness, we summarize the results obtained in ref. (6). The model discussed there was a linear scalar matrix model in D-dimensional space-time.

$$S=\int d^Dx\left\{tr(\partial_\mu M\partial_\mu M^\dagger)+m^2 tr(MM^\dagger)+\frac{g}{4N}tr(MM^\dagger MM^\dagger)\right\} \tag{3.7}$$

where M is an arbitrary complex matrix. We use the double line representation for the Feynman diagrams, where we do not have to write down indices for Feynman values (Fig. 2).

The Feynman rules for the propagator and vertex are extremely simple (see Fig. 2),

$$\text{propagator} \quad \frac{1}{p^2+m^2} \qquad \text{vertex} \quad \frac{g}{N} \tag{3.8}$$

where there is no summation over repeated indices.

At a single point according to the generalized periodic boundary condition

$$M_{ij}(x+\hat{\mu})=M_{ij}(x)\,e^{i(\theta_i^\mu-\theta_j^\mu)} \tag{3.9}$$

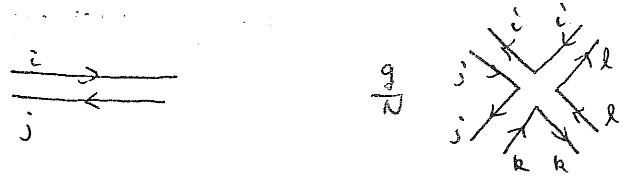


Fig. 2. The propagator and vertex in double-line representation.

we have the following action

$$S=\sum_{i,j}(\theta_i-\theta_j)^2|M_{ij}|^2+m^2 tr(MM^\dagger)+\frac{g}{4N}tr(MM^\dagger MM^\dagger) \tag{3.10}$$

Now the Feynman rules are almost the same as (3.8), (3.9) except that the propagator is replaced by

$$\frac{1}{(\theta_i-\theta_j)^2+m^2} \tag{3.11}$$

The identification between the correlation function in the infinite volume and at a single point was proposed, for example, for a two-point function as:

$$\langle M_{ij}(0)M^\dagger_{ij}(x)\rangle=\int\prod_{\mu,i}\frac{d\theta_i^\mu}{2\pi}e^{i(\theta_i-\theta_j)\cdot x}\langle M_{ij}M^\dagger_{ij}\rangle_\theta \tag{3.12}$$

where the left-hand side is evaluated using (3.7) and the right-hand side using (3.10). This relation is obvious in the free case.

It is easily seen that the above relation persists to higher orders of perturbation. Consider the graph shown in Fig. 3, the contribution is

$$\langle M_{ij} M^{\dagger}_{ij} \rangle_0 = \sum_{l,k} \left(\frac{g}{N}\right)^2 \frac{1}{[(\theta_i-\theta_j)^2+m^2]^2} \frac{1}{(\theta_i-\theta_k)^2+m^2} \tag{3.13}$$

$$\times \frac{1}{(\theta_l-\theta_j)^2+m^2} \frac{1}{(\theta_k-\theta_l)^2+m^2}$$

identifying

$$\theta_i-\theta_j = p \qquad \theta_i-\theta_k = k \qquad \theta_l-\theta_j = l$$

$$\theta_k-\theta_l = p-k-l \tag{3.14}$$

$$\frac{1}{N^2}\sum_{k,l} = \int \left(\frac{dl}{2\pi}\right)^D \left(\frac{dk}{2\pi}\right)^D$$

which is exactly the usual Feynman diagram in Fig. 3b.

It should be noted that the reult would not be true if the two fee indices k and l happened to coincide, but such events occure rarely as in higher orders of 1/N. Therefore in leading order of 1/N we do not have to bother.

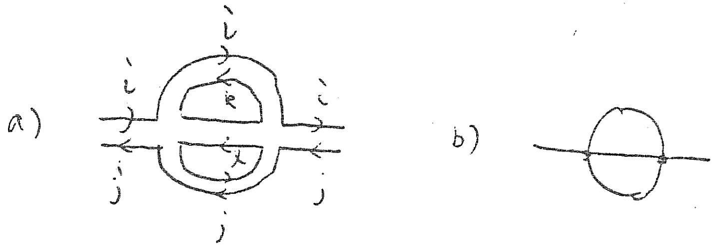


Fig. 3. Diagrams for the renormalization arguments.

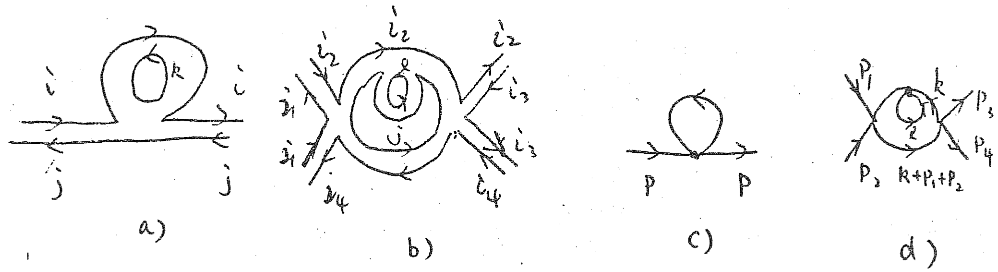


Fig. 4. Diagrams for the renormalization arguments.

There is a sharp cut-off in the theory; we can remove it by extending the limit of integration from $(-\pi,\pi)$ to $(-\infty,\infty)$. The usual renormalization can be discussed in a general way. Consider the two diagrams shown in Fig. 4, where mass renormalization is required. The contributions are respectively

$$\left(\frac{g}{N}\right) \sum_k \frac{1}{[(\theta_i-\theta_j)^2+m^2]^2} \frac{1}{(\theta_i-\theta_k)^2+m^2} \tag{3.15}$$

and

$$\left(\frac{g}{N}\right)^3 \frac{1}{(\theta_{i_1}-\theta_{i_2})^2+m^2} \frac{1}{(\theta_{i_4}-\theta_{i_1})^2} \frac{1}{(\theta_{i_2}-\theta_{i_3})^2+m^2} \frac{1}{(\theta_{i_3}-\theta_{i_4})^2+m^2} \tag{3.16}$$

$$\times \sum_{j,l} \frac{1}{[(\theta_{i_2}-\theta_j)^2+m^2]^2} \frac{1}{(\theta_j-\theta_l)^2+m^2} \frac{1}{(\theta_{i_4}-\theta_j)^2+m^2}$$

We make identifications similar to (3.14) which are shown in Fig. 4c and Fig. 4d. In this way all the references to particular θ's are only in the momentum forms, which are appropriate to the corresponding Feynman diagrams in the usual sense. If we subtract the quadratic divergent part in Fig. 4a, the subtraction will be the same for Fig. 4b. It is therefore renormalizable in the usual sense.

In the infinite volume we have the general factorization property at the large N limit

$$\langle \mathrm{tr}A \, \mathrm{tr}B \rangle = \langle \mathrm{tr}A \rangle \langle \mathrm{tr}B \rangle \tag{3.17}$$

where A and B are products of M's and are not necessarily local objects. To be consistent, on a single point, we have to have the factorization property of the type

$$\int \prod_{\mu,a} \frac{d\theta_a^\mu}{2\pi} \langle \mathrm{tr}A(\theta)\,\mathrm{tr}B(\theta)\rangle_\theta = \int \prod_{\mu,a} \frac{d\theta_a^\mu}{2\pi} \langle \mathrm{tr}A(\theta)\rangle_\theta \times \int \prod_{\mu,a} \frac{d\theta_a^\mu}{2\pi} \langle \mathrm{tr}B(\theta)\rangle_\theta \tag{3.18}$$

which was implied in Ref. (6,7). This can be seen as the generalization of (3.17) at a single point in the presence of background gauge fields. Its validity can be checked using perturbation theory and can also be understood by inspection. Since in $(\mathrm{tr}A(\theta)\,\mathrm{tr}\,B(\theta))$ there is no free index summation joining two M's belonging separately to tr $A(\theta)$ and tr $B(\theta)$, there are no θ-phases being assigned

identically from $A(\theta)$ to tr $B(\theta)$. While the index summations run independently, the indices belonging to tr $A(\theta)$ and tr $B(\theta)$ can run across each other accidentally, which causes the connected correlations in θ's. But again this occurs at higher order of 1/N in accordance with the observations made in Sect. 2.

4. Non-linear matrix model.

The non-linear generalization of (3.7), discussed in this section, is called the chiral spin model in the literature (8). It can be reviewed as the non-linear generalization of (3.8) in the following way. Take the theory to be

$$S = \beta \int \mathrm{tr}(\partial_\mu M \partial_\mu M^+) d^D x \tag{3.19}$$

where M is an arbitrary complex matrix. The interaction is induced by the constraint on the integral measure in the same sense as is usually done for non-linear σ models

$$\delta(MM^+ - 1) \tag{3.20}$$

at every space-time point.

This model may be one of the simples to give planar diagrams at large N, as can be seen by introducing a dummy matrix field $\alpha(x)$. The partition function is now

$$Z = \int \mathcal{D}M\, \mathcal{D}\alpha \exp\left\{-\beta \int d^d x\, tr(\partial_\mu M \partial_\mu M^\dagger) + i\, tr\,\alpha(MM^\dagger - 1)\right\} \tag{3.21}$$

The planar diagram interpretation at large N follows obviously when we put M and into the double line representation (1).

At zero dimension, the planar diagrams are readily summed

$$Z \propto \int \mathcal{D}\alpha (\det\alpha)^{-N} e^{-i\, tr\,\alpha} = \int \mathcal{D}\alpha\, e^{-tr\,\alpha - N \log\det\alpha} \tag{3.22}$$

It is clear that the expression depends only on eigenvalues of the hermitian matrix α; the integral can be evaluated by following closely the method of Ref. (2). At one dimension on a lattice, the theory is exact two-dimensional lattice gauge theory (7).

Here we are mainly concerned with the lattice version, which is the U(N) X U(N) chiral spin model

$$-S = \beta \sum_\mu tr\, U(x)\, U^\dagger(x+\hat{\mu}) + h.c. \tag{3.23}$$

Using the generalized boundary conditions (3.19), we reduce the chiral spin model to a single point

$$-S = \beta \sum_\mu tr\, U e^{i\theta_\mu} U^\dagger e^{-i\theta_\mu} + h.c. \tag{3.24}$$

In the following we will show that the two theories are equivalent. We will proceed with the Dyson-Schwinger equations and show that these theories produce the same set of DS equations and assume that the DS equations of all kinds of correlation functions are sufficient to specify the theory (4). Thereby we establish the announced equivalence.

We start with a simple expression

$$\langle tr\, U(k)\, U^\dagger(\ell)\, T^j \rangle \tag{3.25}$$

where the T^j's are U(N) generators. We get a DS equation by making a shift at position ℓ, $U(\ell) \to (1 + i\varepsilon T^j) U(\ell)$

$$N\langle tr\, U(k) U^\dagger(\ell)\rangle + \beta \sum_{\hat{\rho}} \langle tr\, U(k) U^\dagger(\ell) U(\ell+\hat{\rho}) U^\dagger(\ell)\rangle \tag{3.26}$$
$$- \beta \sum_{\hat{\rho}} \langle tr\, U(k) U^\dagger(\ell+\hat{\rho})\rangle = 0$$

As in a gauge theory we have to consider correlations with more than one operator per site; for example

$$\langle tr\, U(k)\, U^\dagger(\ell)\, U(m)\, U^\dagger(\ell)\rangle, \tag{3.27}$$

is related to higher-order correlations via

$$N\langle tr\, U(k) U^\dagger(\ell) U(m) U^\dagger(\ell)\rangle + \langle tr\, U(k) U^\dagger(\ell)\rangle \langle tr\, U(m) U^\dagger(\ell)\rangle$$
$$+ \beta \sum_{\hat{\rho}} \langle tr\, U(k) U^\dagger(\ell) U(\ell+\hat{\rho}) U^\dagger(\ell) U(m) U^\dagger(\ell)\rangle$$
$$- \beta \sum_{\hat{\rho}} \langle tr\, U(k) U^\dagger(\ell+\hat{\rho}) U(m) U^\dagger(\ell)\rangle = 0 \tag{3.28}$$

Here we have used the factorization assumption for the second term at large N, which ensures the closedness of the infinite set of DS equations.

As in gauge theories we can go on to consider all complicated correlations by using the well-known flipping -switching methods (10). Note that since there is no local gauge invariance, we will have the DS equations of the type

$$\sum_{\hat{\rho}} \langle \mathrm{tr}\, U(k) U^{+}(l) U(m) U^{+}(m+\hat{\rho}) \rangle = \sum_{\hat{\rho}} \langle \mathrm{tr}\, U(k) U^{+}(l) U(m+\hat{\rho}) U^{+}(m) \rangle \tag{3.29}$$

by shifting a variable at an arbitrary position m instead of at two ends.

Next we work on a single point. The mapping between the theory defined in the infinite volume at a single point is proposed to be

$$\langle \mathrm{tr}\, U(k) U^{+}(l) \rangle = \int \prod_{\mu, a} \frac{d\theta_a^{\mu}}{2\pi} \langle \mathrm{tr}\, U e^{i\sum_{\mu} n_{\mu}\theta_{\mu}} U^{+} e^{-i\sum_{\mu} n_{\mu}\theta_{\mu}} \rangle_{\theta} \tag{3.30}$$

where n_{μ} specifies the minimal number of steps needed to go from point k to l in the μth direction, which can be negative in the anti-direction; $\langle\ \rangle_{\theta}$ denotes an average using the action (3.24).

Repeating the above steps, for example, we start with

$$\langle \mathrm{tr}\, U e^{i\sum_{\mu} n_{\mu}\theta_{\mu}} U^{+} T^{j} e^{-i\sum_{\mu} n_{\mu}\theta_{\mu}} \rangle_{\theta} , \tag{3.31}$$

and get the same equation as (3.26) except with one more term

$$\langle \mathrm{tr}\, e^{-i\sum_{\mu} n_{\mu}\theta_{\mu}}\, \mathrm{tr}\, U e^{i\sum_{\mu} n_{\mu}\theta_{\mu}} U^{+} \rangle_{\theta} \tag{3.32}$$

After averaging over the background gauge fields and using the factorization properly discussed in Sect. 3

$$\int \prod_{\mu} \frac{d\theta^{\mu}}{2\pi} \langle \mathrm{tr}A\, \mathrm{tr}B \rangle_{\theta} = \int \prod_{\mu} \frac{d\theta^{\mu}}{2\pi} \langle \mathrm{tr}A \rangle_{\theta} \int \prod_{\mu} \frac{d\theta^{\mu}}{2\pi} \langle \mathrm{tr}B \rangle_{\theta} ,$$

this contribution vanishes.

The phase θ's ensure that just the correct source terms, as the second term in (3.28) are kept intact. One cna proceed to convince oneself that one has the one-to -one correspondence of the DS equations between the theory defined at a single point and that in the infinite volume. For the equations of the type (3.29), we simply start with the expression

$$\langle \mathrm{tr}\, U e^{i\sum_{\mu} n_{\mu}\theta_{\mu}} U^{+} e^{-i\sum_{\mu} n_{\mu}\theta_{\mu}} e^{-i\sum_{\mu} m_{\mu}\theta_{\mu}} T^{j} e^{-i\sum_{\mu} m_{\mu}\theta_{\mu}} \rangle_{\theta} \tag{3.33}$$

Therefore, we have established the equivalence between the two theories in the large-N limit.

It is interesting to check whether there is any kind of spontaneous symmetry breaking in the weak coupling regions, where it is seen that the U matrix is fluctuating around a diagonal form

$$U = V e^{i\varphi} V^{+} \qquad V = e^{ia} \qquad a^{+} = a \sim 0 \tag{3.34}$$

where φ is another diagonal matrix with the eigenvalues of U.

Let us lock at the free energy

$$F = \int \prod_{\mu, a} \frac{d\theta_\mu^a}{2\pi} \log \int dV \prod_a d\varphi_a \prod_{a>b} \sin^2 \tfrac{1}{2}(\varphi_a - \varphi_b) \times e^{\beta \sum_\mu \mathrm{tr} V e^{i\varphi} V^{+}} \tag{3.35}$$

Carrying out the interaction over a, which when expanded to second order is gaussian, we notice that the repulsion between the a eigenvalues is just cancelled out by the attraction induced in the exponent; therefore, the eigenvalues are perfectly randomly distributed even in the weak coupling regions and there is no spontaneous symmetry breaking observed for gauge theory in Ref. (5).

5. Gauge theories.

In Ref. (7) we discussed gauge theories at large N and the main conclusion for the O(N) gauge group. Some

difficulty was pointed out for U(N); if we do not specify a gauge condition, when reduced to a single point the dependence can be transformed away completely, which is again the Eguchi-Kawai model. This fact suggests that the trouble with the gauge theories lies in the gauge freedom in the infinite volume. To be consistent, we have to strip its freedom by fixing the gauge condition before reducing to a single point; stated equivalently, we have to require that only physical variables can experience the background gauges at a single point. For simplicity, we choose the axial gauge where $U_1 = 1$.

The reduced action is now

$$-S = \beta \sum_{i \neq j} \mathrm{tr} U_i U_j U_i^{+} U_j^{+} + \beta \sum_j \mathrm{tr} U_j e^{i\theta_i} U_j^{+} e^{-\theta_i} + h.c. \tag{3.36}$$

where i and j run from 1 to 3.

We would first like to study the problematic weak coupling region, in which the U_i's fluctuate around the diagonal form (5). Note also that now there is no U(N) global gauge transformation which leaves the eigenvalues of the U_i's invariant, which may suggest that the eigenvalues of the U_i's do not play any particular role for the physical observables.

We calculate the eigenvalue distribution for large β following Ref. (5):

$$F \propto \int \prod_\alpha \frac{d\theta_\alpha^t}{2\pi} \log \int \prod_{i,\alpha} d\theta_\alpha^i \exp\Big(\sum_{\alpha>\beta} \sum_i \Big\{ \log \sin^2 \tfrac{1}{2}(\theta_t^\alpha - \theta_t^\beta) -$$

$$- \log \sum_\nu^{(i)} \sin^2 \tfrac{1}{2}(\theta_\nu^\alpha - \theta_\nu^\beta) \Big\} \Big)$$

(3.37)

where ν runs from 1 to D and the Dth θ is θ_μ quenched by assumption, $\Sigma_\nu^{(\neq i)}$ denotes summation with the ith term missing. It is easily seen that this is consistent with the observations made in Ref. (5) that for $D > 2$ the θ 's will attract each other in weak coupling regions and degeneracy in eigenvalues θ 's may occur.

To first order, we assume all θ_αs are equal to one value, α^i, as in Ref. (5).

$$U_i = e^{i\alpha^i} e^{ib_i} \qquad b_i = b_i^\dagger \quad tr\, b_i = 0 \tag{3.37}$$

where the b_i's are small. When expanding in b_i we see in the case that the first term of (3.36) gives a quartic term while the second term gives a quadratic one. In other words, in the weak, coupling region, where degeneracy in the eigenvalues occurs, the second term in (3.36) dominates; therefore the internal energy is correctly obtained in first order in $1/\beta$, $1-(1/2D)N/\beta$.

Since the second term in (3.36) dominates in the weak coupling region, the b eigenvalues are randomly distributed as in the case of the chiral spin; therefore, we expect that open Wilson loops in Ref. (4) will have a vanishing contribution even in the weak coupling. We have not shown this explicitly. It seems that only a Monte Carlo

check can determine whether (3.36) is identical to the Wilson theory in the intermediate coupling regions.

6. Monte Carlo simulations.

It is tempting to apply these results in simulating QCD on a lattice. The most naive proposal would be to take our formulation for the O(N) group (SO(N) and O(N) are equal at $N\to\infty$ limit) and implement for $N = 10\sim 20$. What could we gain from the numerical calculation point of view (we suppose 1/N corrections are negligible)?

It is clear that different Monte Carlo runs must be made for different values of θ. However the number of θ's which are extracted (N_θ) seems to be the critical parameter. It is easy to understand that the single-point system (at $N \ll 1$) will simulate a system of volume proportional to N_θ.

This observation implies that at the end we cannot avoid doing a number of extractions proportional to the effective volume we consider.

The computation will become very long with increasing N.

If the main limitation is CPU time, nothing is gained. If the main limitation is memory (and this will be the case in the future), something important can be done.

Indeed in an SU(N) theory we expect that the finite-size effects should be reduced by quenching up to a factor $\alpha N/N = \alpha$. This means that in a typical Monte Carlo simulation for SU(3) we could work on a large lattice (say, 6^4) and reduce the finite volume effects by a factor of 10-20% by quenching the θ's. This operation would correspond to an increase in the volume by a small factor (2~3?), and therefore it seems that it would not be necessary to use many different choices of θ.

For this purpose we need an algorithm for SU(3); this is easy to implement on a computer. As discussed in the preceeding section, the main difficulty with gauge theories is that it is not so simple to superimpose a background gauge field. However if gauge fields are composite as in the CP^{n-1} model, this difficulty disappears. It was shown by Bars that SU(N) gauge theory can be written in terms of "corner operators" $W_\mu(x)$ on a lattice in such a way that

$$U_\mu(i) = W_\mu(i)\, W_\mu^\dagger(i+\hat{\mu})$$

and the action is the standard one in terms of the U's. Now the theory is obviously invariant under the transformation

$$W_\mu \to W_\mu R$$

R is an SU(N) rotation and

$$U_\mu' = W_\mu'(i)\, W_\mu'(i+\hat{\mu}) = U_\mu$$

We can now insert the background gauge fields acting on the W_μ's from the right-hand side; the argument of the preceeding sections should apply here too. The final theory is more complicated that O(N) theory which should be preferred for analytic computations; however, the former is more suited for numerical simulations. It is easy to see that this choice leads to the form of the action given in Ref. (5).

In practice the prescription on a lattice L^4 amounts to doing the standard computation for internal links and extracting the links which are responsible for the boundary conditions with a different measure, e.g.

$$U_1(L,y,z,t) = L^*(y,z,t)\, W(1,y,z,t)\, D_1(\theta)\, W^*(1,y,z,t),$$

where

$$L_1(y,z,t) = \prod_{1}^{L-1}{}_x\, U_1(x,y,z,t)$$

W is an SU(3) matrix extracted with the Haar measure and $D_x(\theta)$ is a diagonal U(3) matrix containing the θ's.

Another possibility, at least in the scalar case discussed before, would be to try to use the replica trick. We write

$$\langle \log Z \rangle_\theta = \frac{d}{dn} \langle Z^n \rangle_{\theta, n=0} \qquad Z = \int dM \exp(S(M,\theta)) ,$$

where $S(M,\theta)$ is given in (3.10), $\langle\ \rangle_\theta$ stands for

$$\int \prod_i \frac{d\theta_i}{2\pi} \exp\left(\frac{1}{2} \sum_i \frac{\theta_i^2}{\Lambda^2}\right)$$

and Λ plays the role of an ultraviolet cut-off.

For integer n, we have that

$$\langle Z^n \rangle_\theta = \int \prod_{\alpha=1}^{n} dM^\alpha \exp\left(\sum_{\alpha=1}^{n} S(M^\alpha, \theta)\right) ,$$

where the index α runs over the different replicas.

The integration over the θ is now trivial (gaussian) and gives

$$\langle Z^n \rangle_\theta = \int \prod_{\alpha=1}^{n} dM^\alpha \frac{1}{P(M)^{D/2}} \exp\left(\sum_{\alpha=1}^{n} S_R(M^\alpha)\right) ,$$

where $S_R(M) = m^2 \mathrm{tr}\, MM^+ + g/4N\ \mathrm{tr}\,(MM^+MM^+)$ and $P(M)$ is a polynomial of order 2N in the $M^\alpha_{a,b}$.

If we know the value of $(Z^n)_0$ for integral n, we can extrapolate it at $n=0$. This can be done analytically (that would solve the problem) in the limit $N\to\infty$ or numerically by Monte Carlo techniques. The advantage here would

be to bypass the problem of integrating over the θs numerically. It is clear that a lot of work has still to be done in this direction.

7. Discussion

We have followed a systematic approach to reduce some field theories from infinite volume to a single point. As a result, the original models now become few matrix problems; the linear and non-linear matrix models in D-dimensional space-time is now a problem of integrating one matrix coupled to d gauge phases. U(N) gauge theory in our description now is a problem if integrating D-1 U(N) matrices coupled to one gauge phase. A most interesting and challenging problem would be to find exact solutions. The relation between gauge theory and the chiral spin model, if there is any, could be discussed at a single point if we are only interested in the large-N limit. We discuss one plausible relation between the U(N) gauge theory and the chiral spin model in the following. Let us consider U(N) lattice gauge theory at a single point for D dimensions (3.36). Let us concentrate on the weak coupling region $\beta \gg 1$; we know that the U matrices are restricted to fluctuate around their diagonal form. The following parametrization might be possible; we choose one of the U matrices to be a fast cluctuating variable which is an N x N unitary matrix, while choosing the other D - 2 to be slow variables which are already in their diagonal forms (11). However, the fluctuations of these diagonal matrices can still react back to the "fast" variable, the N by N unitary matrix. In order to make the diagonal matrices really slow variables, we have to immobilize the eigenvalues of those D - 2 directions, i.e., quenching. The free energy obtained in this way is almost that of the chiral spin model in D - 1 dimensions, except for some extra measure factors for eigenvalues, which are to make them repel each other; since these variables are already quenched, these effects are not important.

After completing this chapter, several works came to our attention (13-17), in which various authors discuss the EK model, or modifications thereof, from various points of view. Many interesting questions arise.

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PRELUDE TO CHAPTER IV

Chapter IV is the continuation of Chapter III, which treats a more technical problem; since there exist two different quenching prescriptions (the one proposed by the Princeton group and the one proposed by Parisi), what is their relationship, and which is the correct prescription?

This chapter gives a partial answer to this question by showing the two are inequivalent and the former quenching prescription is trivial in the large N limit because their quenching measure becomes a delta function in large N limit therefore quenching integral can be trivially carried out. This chapter is based on a published note (Zhang Y.-C., Phys. Lett. 124B 506 (1983).

CHAPTER IV

TRIVIALITY OF A MODIFIED EK MODEL

Eguchi and Kawai (1 recentlty discovered that lattice gauge theory at large N has a drastical simplification; the theory defined on a single point, and that defined on the infinite volume are equivalent. It was soon pointed out by Bhanot et al. (2) that the EK model experiences symmetry breaking in weak coupling regions and they proposed a quenched EK model to rescue the EK model in weak coupling regions; they suggested to quench the eigen values of the unitary amtrices to present the eigenvalues getting degenerate, since such degeneracy can indeed happen in weak coupling regions for the EK model (2). We can phrase their model as "direct quenching" since they quench exactly the variables which could possibly go wrong.

Parisi subsequently suggested another quenching mechanism (3), his method is to assume non trivial boundary conditions to the theory and the phases of the boundary conditions are to be quenched. He showed that the planar diagrams can be generated at large N_1 and these boundary phases have the natural interpretation of lattice momenta.

His approach was followed and developed to some

depth in Ref. (4).

Here we have two apparent different quenching prescriptions; the one proposed in Ref. (2) and further elaborated in Ref. (5), the other is Parisi's model (3). It was claimed in the literature that these two quenching approaches are equivalent to each other (5) since they could interprete the quenched eigenvalues as lattice momenta (2,5). In this note we shall show that the two approached are essentially different and planar diagrams can not be generated if we want to interprete the quenched eigenvalues as lattice momenta, contrary to one's expectation (5).

We start with the EK model (1)

$$Z_{EK} = \int \prod_{\mu=1}^{D} dU_\mu \, \exp\left(S_{EK}(U_\mu)\right) \tag{4.1}$$

$$S_{EK}(U_\mu) = \beta N \sum_{\mu\neq\nu} tr\left[U_\mu U_\nu U_\mu^+ U_\nu^+\right]$$

since

$$U_\mu = V_\mu D_\mu V_\mu^+ \tag{4.2}$$

Eq. (4.1) can be rewritten as

$$Z_{EK} \propto \int \prod_{\mu, i} \frac{d\theta_\mu^i}{2\pi} \prod_\mu \prod_{i>j} \sin^2[\tfrac{1}{2}(\theta_\mu^i - \theta_\mu^j)] Z'(\theta_\mu^i) \tag{4.3}$$

$$Z'(\theta_\mu^i) = \int \prod_\mu dV_\mu \exp[S_{EK}(V_\mu, D_\mu)]$$

The QEK model (2) differs for the EK model in that the eigenvalue averages are not to be taken on the partition function but on the physical observables like free energy and correlation functions.

The free energy in QEK model is

$$F_{QEK} \propto \int \prod_{\mu, i} \frac{d\theta_\mu^i}{2\pi} \prod_\mu \prod_{i>j} \sin^2[\tfrac{1}{2}(\theta_\mu^i - \theta_\mu^j)] \times N^{-2} \log Z'(\theta_\mu^i) \tag{4.4}$$

and the correlation function

$$\langle O \rangle_Q \propto \int \prod_{\mu, i} \frac{d\theta_\mu^i}{2\pi} \prod_\mu \prod_{i>j} \sin^2[\tfrac{1}{2}(\theta_\mu^i - \theta_\mu^j)] \langle O \rangle_{D_\mu^i}$$

$$\langle O \rangle_{D_\mu} = \frac{1}{Z'(\theta_\mu^i)} \int \prod_\mu dV_\mu \exp[S_{EK}(V_\mu, D_\mu)] \times O(V_\mu, D_\mu) \tag{4.5}$$

Now we analyse the F_{QEK} defined in (4), $(1/N^2)$ X log Z^1 (θ_μ^i) is an intensive quantity in N, which means it has a non trivial large N limit itself and the limit can be taken separately on this part. The integral measure over the eigenvalues can be normalized in the following way as done in Refs. (6,7).

Define the normalized density function

$$N\rho(\alpha) = \sum_i \delta(\alpha - \theta_i) \tag{4.6}$$

the normalized measure for each direction is (up to a term having finite large N limit),

$$\frac{1}{N!} P_N \int \prod_\alpha N\, d\rho(\alpha) \exp[-N^2(\int d\alpha\, d\beta\, [\rho(\alpha) - 1/2\pi][\rho(\beta) - 1/2\pi] \times [-\log|\sin\tfrac{1}{2}(\alpha-\beta)|])] \tag{4.7}$$

where P_N denotes the permutation operator over the N eigenvalues of matrix U_μ. The term in the bracket of the exponent is exactly the vacuum energy E_0 obtained by Gross and Witten (7) for the case of $\lambda \to \infty$.

Note the limiting definition of delta function

$$\lim_{N \to \infty} N \exp(-N^2 x^2) = \delta(x) \tag{4.8}$$

the measure now becomes

$$\frac{1}{N!} P_N \int \prod_\alpha d\rho(\alpha) \prod_\alpha \delta(\rho(\alpha) - 1/2\pi) \tag{4.9}$$

this means that all the quenching integrals in the QEK model can be carried out with eigenvalues fixed at

$$\theta_\mu^i = 2\pi k_\mu^i / N \tag{4.10}$$

κ_μ^i is an arbitrary integer between 1 and N.

Now we are left with the permutations, however recall that there is always (many not be unique) a unitary matrix which does the reshuffling of the diagonal matrix.

$$D = \omega_\mu D_\mu \omega_\mu^\dagger \qquad (\text{no summation}) \tag{4.11}$$

where D_μ is the diagonal matrix for the μth direction and

$$D_{ij} = \delta_{ij} \frac{2\pi j}{N} \tag{4.12}$$

is independent on the directions, the unitary matrices ω_μ can be absorbed into V_μ, the permutations are just trivially reduced to 1.

Now we can rephase the QEK model defined by eqs. (4), (5), it is a special case of the EK model in which one fixes all the eigenvalues of all the link matrices to the uniform series of constants

$$\theta_\mu^i = \frac{2\pi i}{N} \tag{4.13}$$

and there is no quenching integrals or permutations left

to carry out. It is clear that the spontaneous symmetry breaking cannot occur for any coupling region, as the computer simulations also suggested (2,8).

Next we would like to see how the QEK model would behave at the weak coupling region, which means to consider the following parametrization

$$V_\mu = e^{i a_\mu} \omega_\mu \tag{4.14}$$

where a_μ is a small hermitean matrix and ω_μ is an arbitrary unitary permutation matrix acting of D, since any permutation gives equally minimum action. In $Z'(\theta_\mu^1)$ we keep only the terms $O(a^2)$, because only the coefficient of quadratic term can possibly become inverse propagators. The $Z'(\theta_\mu^i)$ in eq. (4.4) now is θ_μ^i independent

$$Z' \propto \int \prod_\mu d\omega_\mu \int \prod_\mu \prod_{i,j} d^2 a_{ij}^\mu \exp\Big[-32\beta \sum_\mu \sum_{i,j} |a_{ij}^\mu|^2 \times \sin^2\big[\tfrac{1}{2}(\theta_\mu^i - \theta_\mu^j)\big]\Big(\sum_\sigma \sin^2\big[\tfrac{1}{2}(\theta_\sigma^i - \theta_\sigma^j)\big]\Big)\Big] \tag{4.15}$$

Apart from a normalization constant, $\int \prod_\mu d\omega_\mu$ is the annealed averaging measure, in the restricted phase space (it takes only the uniformly distributed values of θ_μ^i's) of the original measure in the EK model

Thus, contrary to one's initial quenching motivation in the definition eqs.(4,4),(4,5), we have ended at a annealed prescription.

In the quenched approach of Parisi, there is no measure associated with the random boundary phases, after taking the large N limit, there are always non trivial quenching integrations to be carried out. The mechanism of this approach is not to quench the troublesome eigenvalues directly, but rather to quench the boundary phases, which was introduced in a natural way (3,4), to prevent the eigenvalues getting degenerate. It is known that in gauge systems more care is neede besides Parisi rule (4), the most clean confirmation may be the reduced U(N) chiral model, as recently computer simulation (8).

To end this chapter we summarize the results. In the QEK model (2,5) the quenching integrations can be just carried out, we can rewrite the model as a special case of EK model, with all eigenvalues fixed to a series uniform constants. The weak coupling behavior of QEK model is an annealed theory with the eigenvalues in the restricted phase space of EK model. The equivalence with Parisi's approach cannot be substantiated.

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PRELUDE TO CHAPTER V

This chapter can be viewed as a continuation of Chapter I and II. It is rather speculative because many open problems remain to be explored. It is devoted to show what kind of role the quenched disorder variables have in the nonabelian lattice gauge theory. In a model proposed in this chapter, we can show that nonabelian lattice theories coupled with abelian disorders could be made manifestly confining using duality arguements.

CHAPTER V

THE ROLE PLAYED BY RANDOM DISORDERS IN THE PROBLEM OF CONFINEMENT.

1 - Introduction.

It is generally believed that the nonabelian gauge theories are the only candidates which accommodate the asymptotic freedom and confinement peacefully. The complexity of these theories so far frustrates attempts of proving such coexistence rigorously. In this letter we will be concerned with lattice gauge theories. We will show when the sufficient random disorders are built into these theories, with the constraint that they should respect the asymptotic freedom (i.e. long distance behaviors must not be changed), the confinement problem may be facilatated.

We briefly review the various aspects of the random disorders. By random disorders we mean quenched variables, so even at very low temperatures like those of the ground states of the system the fluctuations are forced into the theory in contrast to the annealed variables that they have to find their equilibrium states at a given temperature according to a Boltzmann distribution. There is a vast leterature in this subject in the con-

densed matter Physics. For a general survey and references we refer to Ref. (11). To a field theorist we can classify the random disorders into three categories according to their influence on the infrared divergence structures (or long distance behaviors) of the corresponding 'pure' systems. Let us start with a scalar theory (e.g. the Landau-Ginzburg model) in which we couple the random disorders in the usual way

$$-S(h) = -S_{LG} + \int d^Dx\, h(x)\,\phi(x) \tag{5.1}$$

where $\underline{h}(\underline{x})$ is the quenched random variable and is taken as gaussian distributed

$$P[h] \sim e^{-\lambda \int h^2(x)\, d^Dx} \tag{5.2}$$

where λ is the strength that the random disorders couple to the system. A dimensionality analysis shows

$$[\phi] = \frac{D-2}{2} \qquad [h] = \frac{D+2}{2} \tag{5.3}$$

thus λ is bound to have dimension

$$[\lambda] = -2 \tag{5.4}$$

or, if the theory is lattice regulated

$$\lambda \sim a^2 \tag{5.5}$$

Therefore, the effective interactions induced by the random exchanges are always proportional to

$$\left(\frac{1}{a^2}\right)^n \tag{5.6}$$

with n being number of internal h lines.

We see that the random exchanges introduce new infrared divergence to the pure system. The original interactions of the pure system would appear to be 'irrelevant' compared to this effective interactions and to the leading order they are to be neglected. This fact leads to the well-known result that this kind of random disorders effectively reduce the dimensionality by two (2). Parisi and Sourlas found a supersymmetry which nicely cancels the 'irrelevant diagrams' and shows that dimensionality reduction (3). This is the first kind of random disorders whose importance overwhelms the interactions in the pure systems by a simple power counting. We will not be concerned with this kind of random disorders in this letter.

The second kind of disorders have competing importance with respect to the pure systems. For example in place of eq. (5.1) we can have

$$-S(h) = -S_{LG} + \int d^D x\, h(x)\, \phi^2(x) \tag{5.7}$$

if we take D = 4, then $[\lambda] = 0$. A simple power counting cannot exclude the logarithmic divergence which the pure system is supposed to have and it accounts the long-range order of the system (e.g. spin-wave phase).

The third kind of disorders have less divergent effects on the pure systems. They will not modify the lond distance behaviors of the system. An explicit example is found in Ref. (4), where it is argued that while the random disorders do not affect the correct continum limit of the theories like lattice QCD they play a significan role in the intermediate coupling ranges and help show the absence of a deconfining phase transition.

It is the purpose of this Letter to see to what extent we are allowed to introduce random disorders and how can they help prove the confinement problem while respecting the same continuum limit. In short, we will be interested in the third kind random disorders above mentioned.

2 - The models.

In Ref. (4) we have proposed a model and here we will use a modified version which extracts the essence (at least we believe) of that model. For the nonabelian gauge theory we will consider the model whose free energy is defined as

$$-F = \sum_{\{\sigma_p\}} \log \int \mathcal{D}U e^{\beta \sum_p \sigma_p \mathrm{tr} U_p} \tag{5.8}$$

where σ_p is an abelian phase associated with every plaquette°. It reduces to the Wilson model if we put all $\sigma_p = 1$. The annealed case of this model was studied by Halliday and Schwimmer (5).

The difference between the annealed version and the quenched version is only important in the low temperature region where we expect to get the continuum physics. In the annealed case, when $\beta \to \infty$, all the link variables as well as σ_ρ are forced to be close to the unity. While in the quenched case the σ_ρ have to run all possible values. We will examine the ground states of the quenched theory. To facilitate the analysis we will take the nonlinear σ-model instead of gauge theories, since the arguments are essentially the same. Given a configuration σ_ρ, the system is introduced with a distribution of frustrations (1,6) (for gauge theories they are called monopoles). The ground states will be those where the nonabelian site variables sit at their abelian

° which is an element of group Z(N) or U(1) if the gauge group is SU(N) or U(N).

counterparts (4,7). Take the SU(2) group for example, the ground states will be those like an Ising model with the random exchanges. If all the bond random exchanges adjacent to a spin take the trivial value 1, to flip that spin will cost an energy $2D\beta$, if one of the neighbour bonds is broken, that act will cost only an energy $2(D-1)\beta$, if D of them are broken, that spin can flip back and forth freely, thus we have a degeneracy. We refer to Ref. (6a) for a detailed analysis of the frustration energetics. In this way we see that with these frozen frustrations even at very low temperatures the spins are rather disordered, since the fluctuations cost less energy than in the pure system. The general conclusion about the energetics is that the random disorders activate fluctuations significantly also at low temperatures.

3. The confinement criterion.

It is by now the widely accepted notion that confinement is realized via a dual superconductivity mechanism (8,9). 'tHooft proposed a criterion of confinement (8) by saying that if the 'tHooft loop has a perimeter power low behavior the magnetic fluxes are said to be defocused thus the electric fluxes are therefore focused and that is the signature of the confinement. For discussions on a lattice we refer to Ref. (10, 12). It is known that if we use the Wilson action and take only into account of the abelian excitations, the 'tHooft loop has an area law instead of the desired perimeter law. One has to invoke the nonabelian excitations to prove the perimeter law and which appears to be very difficult (12,13).

The 'tHooft loop in our quenched model is defined as

$$\langle B^{\tau}(c)\rangle_Q = \sum_{\{\sigma_p\}} \frac{\int \mathcal{D}U \exp\{\beta \sum_p \sigma_p \tau^{S(p)} tr\, U_p\}}{Z[\sigma_p]} \tag{5.9}$$

where τ is some nontrivial abelian group element and S(p) = 1 for a set of plaquettes which satisfies ∂S = C, S(p) = 0 itgerwise. The 'tHooft loop measures the response of the theory to the introduction of an external magnetic flux. It is clear that in the annealed version of this model the response is trivial since $\langle B^{\tau}(c)\rangle_A = 1$ while the quenched model has nontrivial response eq. (5.9) analogous to that for the Wilson action.

$$\langle B^{\tau}(c)\rangle_W = \int \mathcal{D}U \exp\{\beta \sum_p \tau^{S(p)} tr\, U_p\}/Z \tag{5.10}$$

From now on we will concentrate on four dimensions. It is known (10-13) if we use only the abelian excitations

$$\langle B^{\tau}(c)\rangle_W \sim e^{-\alpha\, Area(S)} \tag{5.11}$$

since $\langle B^{\tau}(c)\rangle$ measures the energy difference respect to the ground states system that difference is proportional to the number of the flipped plaquettes.

Now we recall the so-called Anderson criterion for the spin glasses (14). He observed while the energy difference is proportional to the number of flipped bonds in a pure system that energy difference is proportional to the square root of that number in the random system. Note that Anderson's observation can be generalized to gauge systems for sufficient large 'tHooft loops. It is

$$\langle B^{\tau}(c)\rangle_Q \sim e^{-\alpha' \sqrt{Area(S)}} \tag{5.12}$$

for a simple geometry this implies

$$\langle B^{\tau}(c)\rangle_Q \sim e^{-\alpha'' Perimeter(S)}, \tag{5.13}$$

which gives the desired perimeter law for the 'tHooft loop. According to the current folklore eq. (5.13) implies the theory is confining.

It remains to be shown how the random disorders affect the long range behaviors of the theories. It turns out that the abelian disorders do not affect the long range behavior of the nonabelian gauge theories thus they belong to the third kind of random disorders while the abelian disorders do affect the long range behavior of the abelian gauge theories as they must be since the abelian gauge theories should not confine at weak couplings.

4. Compatibility of the abelian random disorders with nonabelian gauge theories.

There is an explicit example showing the quenched abelian random disorders do not effect the continuum limit of a nonabelian lattice gauge theory(4). Here for simplicity we will use a two dimensional O(n)xU(1) nonlinear σ-model ($n > 2$) since there is no essential difficulty to perform the same kind of calculations for gauge theories. On a lattice the free energy of the model is

$$-F = \sum_{\{\sigma_\ell\}} \log \int \mathcal{D}S \left\{ \beta \sum_{x,\mu} e^{i\theta_{x,\mu}} \vec{S}_x^{*} \cdot \vec{S}_{x+\mu} + h.c. \right\} \tag{5.14}$$

where $\sigma_\ell = e^{i\theta_{x,\mu}}$, $|\vec{S}_x|^2 = 1$. We may be interested to control the strength of the disorders by changing eq. (5.14) to

$$-F = \sum_{\{\sigma_\ell\}} e^{\lambda \sum_p \sigma(p)} \log \int \mathcal{D}S \exp\left\{ \beta \sum_{x,\mu} e^{i\theta_{x,\mu}} \vec{S}_x^{*} \cdot \vec{S}_{x+\mu} + h.c. \right\} \tag{5.15}$$

where $\sigma(p)$ is the product of four links around a plaquette, since frustration is a gauge invariant concept (15,6).

It is still simpler to analyse the continuum version of eq. (5.15)

$$-F = \int \mathcal{D}\theta\, e^{-\frac{\lambda}{4}\int d^2x F_{\mu\nu}^2} \log \int \mathcal{D}S\, e^{-\frac{1}{2g}\int d^2x |D_\mu \vec{S}|^2} \tag{5.16}$$

where

$$D_\mu \vec{S} = (\partial_\mu + i\theta_\mu)\vec{S}$$

$$F_{\mu\nu} = \partial_\mu \theta_\nu - \partial_\nu \theta_\mu$$

By a simple dimensionality counting we see.

$$[\vec{S}] = 0 \qquad [\theta] = 1 \qquad [g] = 0 \tag{5.17}$$

$$[\lambda] = -2 \,, \quad \text{or} \quad \lambda \sim a^2$$

thus one would conclude these are the first kind of random disorders discussed at the beginning since the random exchanges would induce more severe infrared divergences. However, it is not true for the abelian case where the random disorders θ_μ's decouple from the field variables $\vec{S}$. To see this let us choose a gauge so as to make $\vec{S}$ real then

$$(D_\mu \vec{S})^{+}(D_\mu \vec{S}) = (\partial_\mu \vec{S})^2 + (\theta_\mu \vec{S})^2 \tag{5.18}$$

For the abelian case θ_μ is a C-number then $(\theta_\mu \vec{S})^2 = \theta_\mu^2$.

Therefore, the abelian random disorders (it is true also for the annealed case) do not affect the continuum physics of the nonlinear model in the leading order and the theory is asymptotically free as Polyakov showed (16).

However the above statement is no longer true if we want to couple nonabelian random disorders to the same model where θ_μ is a $\underline{n}$ by $\underline{n}$ matrix. One can easily convince oneself that there is no way to get the random disorders and the field variables decoupled. Thus we learn (various dirty examples have been tried which will not be presented here) that the nonabelian disorders do affect the continuum physics of the nonlinear model and they belong to the first kind of random disorders discussed at the beginning, and presumably we shall lose the asymptotic freedom (it is also true for the annealed case). In thelight of this analysis we doubt the approaches using nonabelian frustration variables have the correct continuum physics (17).

We believe that the same analysis carries over to the gauge theories without any qualitative change in conclusion.

What happens if we couple the abelian random disorders to an abelian system? We believe these will be the first or second kind random disorders, since an analysis shows that these random disorders destroy the long range correlations (18). It is interesting to pursue further

how the random disorders for example affect the spin-wave phase and the Kosterlitz-Thouless transitions (19), which we shall not do it here.

5. Conclusions.

To end this chapter we summarize the main ideas and results. We have classified the random disorders into three kinds according to their influences to the long range orders with the emphasis on the usefulness of the third. It is known in lattice gauge theories that we can add to the Wilson action irrelevant pieces and the resulting theories stay in the same universal class. In this Letter we learn (also from Ref (4)) that the nonabelian lattice gauge theories coupled with the third kind random disorders defined in this Letter will also respect this universality. It appears that these third kind random disorders can be employed as a means of regularizing lattice theories, since they activate the possible fluctuations (uncer the constraint that long range behaviors are not altered) even at low temperatures. We have shown in our model the 'tHooft loop has a perimeter law, a suggestion of confinement.

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