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Supersymmetric Grand Unification

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Contents

1. Introduction

This thesis stems from the context of supersymmetric grand unified theories. In this introduction I shortly present the contents that will be discussed in the following chapters. I will also discuss the motivations for this class of models.

1.1. The present knowledge: the Standard Model

The Standard Model (SM) [1] has been successfully tested in the past years, and the experimental results are so well in accord with the predictions that it is quite a challenge to build a "beyond the SM" scenario. Let us remember that the SM is a renormalisable theory, and it is entirely consistent as such; it has lead us to the discovery of neutral currents, and more recently of the vector bosons responsible for the effective current-current interactions of quarks and leptons. The Higgs sector is very simple, and we now know that substantial modifications, *e.g.* the introduction of an Higgs triplet, are severely constrained by low energy phenomenology. No flavour changing neutral current are present at the tree level, and the observed CP violation can be fully described within the experimentally tested three families scenario. Among the tests still lacking is the discovery of the top quark, to complete the three family picture, and of the neutral Higgs boson, the only scalar degree of freedom present in the model, remnant of the electroweak spontaneous breaking.

1.1.1. *The top quark*

There is no serious doubt that the top should exist: the bottom weak isospin is today measured to be $-1/2$ up to 10% by the forward-backward asymmetry measurements [2], pointing to the fact that the bottom quark is in fact a member of a doublet; the $B^0 - \bar{B}^0$ mixing magnitude requires the presence of the virtual top exchange, unless other exotic particles contribute; finally the top is needed for the anomaly cancellation. But more important are the precision electroweak measurements, that require a SM top with $m_t = 136^{+26}_{-28}\ GeV$ to fit the data [3]. We have also to recall the CDF bound $m_t > 89\ GeV$ in the SM (and $> 55\ GeV$ independently from the model) [4]; the CDF Collaboration at Fermilab may actually find the top in the next months, since some candidate events have already been found. At any rate, the top mass is surely much higher than the mass of the other observed fermions.

1.1.2. The Higgs particle

What about the standard Higgs boson? The previously recalled electroweak precision measurements suggest a SM Higgs in the range from 6 to 300 GeV [5]. But these figures are 1σ limits, and the error is not on m_H, but on its logarithm! This means that at 2σ accuracy we need only $m_H \in [2, 1200] GeV$; that is, no effective information is today available from indirect Higgs effects. On the other hand, from experimental direct searches at LEP we have: $m_H > 59\ GeV$ [6].

From the theoretical point of view, a scalar sector is needed for the spontaneous breaking of the gauge symmetry, which is the only way we know to have a renormalisable theory of massive gauge vector bosons. However the presence of fundamental scalars in the low energy theory gives rise to a theoretical problem, since there is no symmetry that "protects" the "lightness" of these scalars in the perturbative framework. This leads us to the so called naturalness problem.

1.1.3. The naturalness problem

G. 't Hooft [7] argued that the standard model is not natural because of the presence of quadratic divergencies.

However, in my opinion the naturalness problem in the standard model is a fake problem unless we assume the existence of finite theory of which the SM is the low energy remnant. In fact, if we suppose a finite theory exists which describes the physics above the energy scale Λ, the standard model beeing just a low energy manifestation of this theory, we would obtain for the Higgs mass m_H

$$m_H^2 = a\Lambda^2 + m_{bare}^2 \tag{1.1}$$

where the first term in the right hand side is the effect of the quadratic divergencies ($a \sim 1$ in the SM), while the second parameter is given by the fundamental theory; notice that even if m_{bare} is a bare mass, there are no infinities involved, because the theory is finite. The cutoff Λ acquires a physical meaning, forcing the scalar masses to be naturally of $O(\Lambda)$ or larger.

What do we know about the Higgs mass in the SM? It cannot be very far from the Fermi scale (unitarity bounds from longitudinal W-W scattering require $m_H < 1.2\ TeV$ [8]); so, if $\Lambda \gg 1\ TeV$, a huge cancellation must occur in the previous mass formula; if this cancellation is not related to any dynamical reason, to be concrete is not "protected" by any symmetry of the SM lagrangian, we consider it non-natural, after 't Hooft.

We may avoid this "problem" by assuming that in the TeV region the standard model is replaced by a theory *which does not exhibit quadratic divergencies.* This is in fact the feature characterizing the class of low energy softly broken supersymmetric theories that we discuss in this report.

Actually, if we were to trust this argument we could use it to advocate the nearness of the supersymmetric scales to the Fermi scale.

The above conclusion assumes that supersymmetry has something to do with nature, but other possibilties may work as well. A non supersymmetric solution would be for instance represented by the TeV-scale technicolor models [9], in which there appear no fundamental scalar and therefore there are no quadratic divergencies. However, it is very difficult to make this class of model consistent with low energy phenomenology, in particular because of the potentially large flavour changing neutral current effects related to the presence of a large effective scalar sector [10].

1.2. Advocating supersymmetry

Supersymmetry by definition relates bosons and fermions, putting them in the same representation. In the following we will deal with a specific class of supersymmetric lagrangians models; specifically the supergravity derived models, that offer the possibility of realistic low energy scenarios. Supersymmetry at our energy scale must be broken. This is realized in this class of models by explicit breaking terms, parametrized by massive coefficients, that however do not change the UV behaviour of the theory, *i.e.* the absence of quadratic divergencies (soft breaking). The size of the soft breaking parameters characterizes the scale at which we should expect to find evidence of supersymmetric particles.

From the phenomenological side, supersymmetry has proven to be a safe way to go beyond the standard model, at variance with many alternative proposals. For instance, although the supersymmetric Higgs sector requires at least the presence of two doublets, there are no flavour changing neutral currents at the tree level, since supersymmetry itself does not allow more than one Higgs doublet to couple to a given isospin type fermion.

Theoreticallly appealing motivations can be: *i*) supersymmetry is a step in the direction of unifying particles of different spins; *ii*) local supersymmetry theories contain naturally the einsteinian gravity, and the present attempts to quantize gravity seem to require it. However, one must realize the fact that these motivations do not necessarily require supersymmetry in the nearby energy scales; only the naturalness argument mentioned above can be phenomenologically relevant for our energy scales, and, together with the apparent success of supersymmetric grand unification (that we are going to study in great detail), motivates the experimental effort in the search for SUSY signals.

1.3. Grand unified theories

Grand unified theories (GUT) provide us with an unification in some sense orthogonal to the supersymmetric one: unifying particles with different gauge quantum numbers but same spin.

The grand unified theories are needed if we want *i*) to explain in terms of a single, and more fundamental interaction, the strong and the electroweak interactions, and similarly *ii*) to *predict* the many free parameters of the standard model, instead of determining them phenomenologically.

GUTs energies will probably never be testable in accelerators, but GUTs may lead to testable and interesting consequences in the low energy world; for instance the instability of the proton, whose non-observation at the level of the present experimental sensitivity has lead to the rejection of the Georgi-Glashow $SU(5)$ GUT [11].

1.3.1. Gauge hierarchy in grand unified theories

In a non supersymmetric GUT there are usually heavy Higgs scalars ($m_H \sim M_{GUT}$) which are related to the light Higgs fields ($\sim M_{Fermi}$) by quantum fluctuations, requiring an unnatural fine-tuning of vacuum expectation values. This scale hierarchy problem is similar in origin to that for the standard Higgs mass, although it refers to hierarchies between renormalized parameters. Again, supersymmetry provides us with cancellations between the radiative contribution of a particle and the contribution of its supersymmetric partner, that keeps the scale hierarchies natural. Once supersymmetry is introduced, we have technically "stabilized", although not explained, the presence of large mass scale differences in the theory.

1.3.2. Scaling down to low energies

Once the GUT lagrangian is given, in order to study its phenomenological implications we have to perform a renormalisation group analysis. The scaling of all the parameters of the theory to the energy scale that we want to study is needed since we have to minimize the error that we make by truncating at a given order (usually 1-loop) the perturbative expansion of our effective lagrangian; in other words the RGE tells us how to resum and keep under control (by a variation of the value of the renormalized parameters of the theory) the large logarithms that would otherwise appear at any order of the perturbative series.

This leads us to investigate the structure of the the renormalisation group equations for a general softly broken supersymmetric theory and will allow us to study in details the problem of gauge coupling unification and the spectrum of the low energy theory.

1.4. Layout of the thesis

The present introduction draws the general context of the SUSY GUTs. I discuss in details in the next chapter the SUSY gauge lagrangians with soft breaking terms. An effort is done to derive consistently all the various terms of the SUSY lagrangians from the superfield formulation, following the notations of Appendix A; we use the resulting lagrangians to illustrate the phenomenological necessity of the soft breaking terms.

In the third chapter I discuss the GUT predictions for the gauge coupling constant unification, at the 1-loop level. The experimental data both on the proton lifetime and on the gauge coupling constants allow us to discard the Georgi-Glashow $SU(5)$ model in its non supersymmetric form. The remarkable success of its SUSY counterpart is proved to depend critically on the minimality of the Higgs sector.

In the last chapter I quantify and discuss in details the impact of the two-loop refinement of the analysis, and the uncertainties due to the presence of threshold effects, both at the low energy scale (SUSY particles+top) and at the GUT scale. The effect of supersymmetric particles thresholds is parametrized by physically meaningful *effective* thresholds; we discuss finally the interplay between the GUT and the low-energy threshold effects on the predictions.

In Appendix C, I derive the SUSY renormalisation group equations, in a form valid for for any lagrangian with soft breaking terms. I apply this formalism to the case of the minimal SUSY extension of the standard model. I would like to emphasize that these equations have a wide domain of applicability, not yet fully exploited; in particular they are needed: *i*) for the analysis of the SUSY Higgs sector, which is crucial for grand unification; *ii*) to scale, from the GUT energies down to the Fermi scale, the predictions on the Yukawa couplings of a given grand unified model, with the aim to study mass spectra and CP violation.

2. Supersymmetric lagrangians

Supersymmetry is a symmetry that transforms bosons into fermions, that is, an extension of the Poincarè group, unificating different representations of this group. The aims of this chapter are to write down the supersymmetric lagrangians that are renormalisable and gauge invariant, to recall briefly the theoretical concepts relevant for our analysis (we use as a general reference the book of Wess and Bagger [12]), and finally to discuss the kind of supersymmetric models that are considered in best standing from the phenomenological point of view: the models with soft breaking of supersymmetry. We will follow for the lagrangian the conventions of Haber and Kane [13], that are, up to minor changes, the same conventions used by Wess and Bagger.

2.1. Supermultiplets

We shall present in this section the particle content of a supersymmetric theory, whose aim is to describe the phenomenology of the low energy world.

Trying to mimic the usual infinitesimal transformation setting for continuous symmetries, we see that, if we want to maintain the commuting (anticommuting) character of the boson (fermion) fields, the parameters of supersymmetric transformations (ϵ and $\bar{\epsilon}$ in the next formula) are necessarily fermionic in character. These parameter are connected with fermionic generators, that enlarge the Poincarè algebra; the resulting mathematical structures are called graded Poincarè algebras, or Poincarè *super*algebras; the associated transformations are:

$$\delta_{SUSY} = \epsilon\, Q + \bar{\epsilon}\, \bar{Q} \tag{2.1}$$

We can conveniently study the representations of supersymmetry relying on the so called *superspace* formalism. In fact the ordinary 4-dimensional coordinates x_μ can be regarded as the coordinates of the coset space obtained from the Poincarè group (thought as a topological group, that is a group whose elements are in a one-to-one correspondence with coordinates) once the Minkowsky subgroup is factored out. The space spanned by the anticommuting coordinates θ and $\bar{\theta}$, and by the usual 4 parameter of the translations is what we obtain using the same factorization on the Poincarè *super*group; this explain the name superspace.

The functions defined on the superspace carry an obvious SUSY representation, obtained transforming the superspace coordinates; they are called superfields. Once expanded in series of the anticommuting parameters θ and $\bar{\theta}$ these functions are shown to be equivalent to a finite set of ordinary fields of different spin; these sets are called *supermultiplets.* In fact, only the simplest superalgebra, with a minimum number of fermionic generators, can be relevant for the low energy phenomenology, because only in this case the supermultiplets can have a chiral structure, needed to describe properly the weak interactions. More specifically only two kind of superfields bear importance for low energy phenomenology: the vector and the chiral superfields. The vector superfield contains a Majorana fermion $\lambda_M(x)$ and a real vector field $V_\mu(x)$, and is obtained from the most general superfield imposing reality:

$$V(x,\theta,\bar{\theta}) = C(x) + [\theta\xi(x) + \theta^2 M(x) + \bar{\theta}^2\theta(-i\lambda(x)) + h.c.] + \theta\sigma^\mu\bar{\theta}V_\mu(x) + \frac{1}{2}D(x) \quad (2.2)$$

(the field λ in the previous equation is in fact a Weyl field, but also $\bar{\lambda}$ is present, and a Majorana field can be reconstructed). The chiral superfield contains a Weyl fermion $\psi(x)$ and a charged scalar $z(x)$:

$$\Phi(x,\theta) = z(x) + \sqrt{2}\,\theta\psi(x) + \theta^2\, F(x) \quad (2.3)$$

Due to the fact that the fields will appear in the generating functional, we have to represent supersymmetry also off-shell, that is without the constraints of the equations of motion. This requires the presence of the so called *auxiliary fields*, that is fields that are needed to represent supersymmetry, but that have no dynamic role (that is they do not propagate). In particular let us note that the mass dimension of the fields $F(x)$ and $D(x)$ is two, not the canonical one; for instance in eq. (2.3), if $z(x)$ is the canonical scalar field (with mass dimension 1) and $\psi(x)$ is fermionic (with dimension 3/2) we conclude that the coordinate θ has dimension $-1/2$, while $F(x)$ has dimension 2. Similar considerations apply to the auxiliary field $D(x)$ in eq. (2.2).

2.1.1. *Gauge assignments; the "SM" supermultiplets*

The gauge transformations must commute with the fermionic generators in any low energy relevant context; otherwise a local gauge transformation, acting on the fermionic generators would imply the presence in the theory of local fermionic transformations. Considering the fact that the anticommutator of two fermionic generators in supersymmetry yield a translation, that would imply a local translation, i.e. we would be lead to consider gravity (this second part of the argument can be considered an interesting argument by itself; we will discuss the point at the end of this chapter). This observation implies that the gauge assignations are given on supermultiplets; that is every ordinary particle must have a superpartner with the same gauge assignment.

Now let us set the notations. Let us consider the case of the up-quarks. The left component belong to the superfields

$$U_L(x,\theta) = \tilde{u}_L(x) + \sqrt{2}\,\theta u_L(x) + \theta^2\, F_{u_L}(x) \tag{2.4}$$

(we have implicitely defined θ to be a left bispinor). The right component of the up-quark (with the same charge) cannot be used together with θ to form a chiral superfield; it is instead a component of an antichiral superfield $U_R(x,\bar{\theta})$:

$$U_R(x,\bar{\theta}) = \tilde{u}_R(x) + \sqrt{2}\,\bar{\theta} u_R(x) + \bar{\theta}^2\, F_{u_R}(x) \tag{2.5}$$

It is useful to define another superfield $U_L^c(x,\theta)$ whose fermionic component is the left bispinor $u_L^c(x)$:

$$U_L^c(x,\theta) = \tilde{u}_L^c(x) + \sqrt{2}\,\theta u_L^c(x) + \theta^2\, F_{u_L^c}(x) \tag{2.6}$$

and then define its conjugate (see Appendix A) so that

$$U_L^c(x,\theta) = \overline{U_R(x,\bar{\theta})} \tag{2.7}$$

This relation in components reads:

$$\begin{cases} \overline{u_R} & = & u_L^c \\ \overline{\tilde{u}_R} & = & \tilde{u}_L^c \\ \overline{F_{u_R}} & = & F_{u_L^c} \end{cases} \tag{2.8}$$

Finally, let us note that one can write

$$u_L^c = (u^c)_L \tag{2.9}$$

where in the left-hand-side we have the left-component of the charge-conjugate up-quark four component spinor, written in the chiral representation of the gamma matrices, and the bispinor is seen as a four-spinor with zero right-handed part (with the notations of Appendix A).

We use the same notations for the down-quarks and for the leptons; the non-standard particle, following Haber and Kane, have a tilde. I do not use the Haber and Kane notation for the gauginos, but a simplified notation, well suited for my purposes.

$$\begin{array}{lcll} u_L, u_L^c & \longrightarrow & \tilde{u}_L, \tilde{u}_L^c & \text{scalar-quark} \\ d_L, d_L^c & \longrightarrow & \tilde{d}_L, \tilde{d}_L^c & \text{scalar-quark} \\ e_L, e_L^c & \longrightarrow & \tilde{e}_L, \tilde{e}_L^c & \text{scalar-lepton} \\ \nu_L & \longrightarrow & \tilde{\nu}_L & \text{scalar-neutrino} \\ H_1^+ & \longrightarrow & \tilde{H}_1^+ & \text{higgsino} \\ H_1^0 & \longrightarrow & \tilde{H}_1^0 & \text{higgsino} \\ H_2^0 & \longrightarrow & \tilde{H}_2^0 & \text{higgsino} \\ H_2^- & \longrightarrow & \tilde{H}_2^- & \text{higgsino} \\ V_\mu & \longrightarrow & \lambda & \text{gaugino} \end{array} \tag{2.10}$$

Notice that all the fermionic fields are left fields.

We need an even number of Higgs superfield doublets with opposite hypercharge: in fact each higgsino is a fermion and contributes to the anomaly. Morover, we will see in the following that we need at least two Higgs superfields to build a realistic *supersymmetric* lagrangian.

We may notice that the second Higgs doublet H_2 (a bosonic field) does have the same gauge numbers as the leptonic doublet $L_L = (\nu_L, e_L)$ (a fermionic field); but the attempts to reconstruct a supermultiplet using this two doublets of "ordinary" fields were not successful, firstly because the electroweak breaking would imply a violation of the lepton number as well.

2.2. Gauge interactions

In this sector of the lagrangian supersymmetry generates, besides the ordinary interactions, every possible "SUSY related" term; *e.g.* the usual quark-quark-gauge field interaction term is related to a gaugino-quark-squark interaction, and in the pure gauge sector appear a gaugino-gaugino-IVB interaction term.

We will refer for our discussion to the superfield formulation, because it allows us to treat the supermultiplets in a compact way. To generalize the concept of local (gauge) on matter fields, we have simply to translate "matter field" in "chiral superfield", and replace the real functions $\alpha^a(x)$ that parametrize the gauge transformation with an analogous set of chiral superfields.

2.2.1. *Gauge interactions of matter fields*

To find the interaction between matter and gauge fields we have to render invariant the kinetic term of the chiral superfields; finally we have to expand the supersymmetric expressions in term of the ordinary fields. I will always work in the Wess-Zumino gauge; this is a choice of the chiral superfields used as gauge parameters such that the auxiliary fields C, ξ, M of eq. (2.2) disappear, even if the usual gauge invariance is left. The price to pay is that neither supersymmetry or generalized gauge invariance are separately present.

The component lagrangian for the fields in eq. (2.3) can be found applying the rules in Appendix A:

$$\begin{aligned}&|\mathcal{D}_\mu z_a|^2 + i\bar{\psi}_a(\hat{\mathcal{D}}\psi)_a + F_a^* F_a \\ &+i\sqrt{2}g(z_a^*\bar{\lambda}_{ab}\psi_b - \bar{\psi}_a\lambda_{ab}z_b) + g z_a^* D_{ab} z_b\end{aligned} \tag{2.11}$$

where the index a spans the gauge multiplet. I wrote only one coupling constant and I have included the group generator in the definition of the gauge superfields; that is in the vector field, in the gaugino λ and in the auxiliary gauge field D (*e.g.* : $D_{ab} = D^\alpha\ T^\alpha_{ab}$). Let us recall that the field D (like F) is an auxiliary field with mass dimension 2. As such it can enter a renormalisable lagrangian only quadratically (like F in eq. (2.11)) or linearly, eventually multiplied by scalars with canonical dimension (like D in the previous lagrangian). We can form actually an invariant term by picking the D component of a U(1) vector superfield:

$$\mathcal{L}_\xi = \xi\ D(x) \tag{2.12}$$

We can translate eq. (2.11) in 4-component notations by introducing a Majorana and a Dirac spinor fields λ_M, Ψ_D respectively; the only non-obvious part is the gaugino interaction; using $\lambda_M = (\lambda^A, \bar{\lambda}_{\dot{A}})$ we have:

$$i\sqrt{2}g(z^*\bar{\lambda}_M\frac{1-\gamma_5}{2}\Psi_D - \bar{\Psi}_D\frac{1+\gamma_5}{2}\lambda_M z) \tag{2.13}$$

while using $\lambda'_A = -i\lambda^A$ and the Majorana spinor $\lambda'_M = (\lambda^{A\prime}, \bar{\lambda}'_{\dot{A}})$ we find:

$$-\sqrt{2}g(z^*\bar{\lambda}'_M\frac{1-\gamma_5}{2}\Psi_D + \bar{\Psi}_D\frac{1+\gamma_5}{2}\lambda'_M z) \tag{2.14}$$

The gaugino interactions of eq. (2.11) provide a contribution to the generalized mass matrix of the fermions, and mixes the gauginos with the "matter" spinors (higgsinos included):

$$\mathcal{L}_{\psi\lambda} = \sqrt{2}(g^\alpha z_a^* T^\alpha_{ab})(i\bar{\lambda}_\alpha)\psi_b + \sqrt{2}(g^\alpha T^\alpha_{ab}z_b)\bar{\psi}_a(-i\lambda_\alpha) \tag{2.15}$$

The use of the spinor $-i\lambda$ instead of λ itself is due to the fact that, following the conventions of Wess and Bagger $-i\lambda$ appears in the vector supefield of eq. (2.2).

The vector fields get their mass from the first term in eq. (2.11), namely

$$\mathcal{L}_{VV} = (g^\alpha T^\alpha_{ac} z_c)(g^\beta z^*_b T^\beta_{bc}) V_{\alpha\mu} V^\mu_\beta \tag{2.16}$$

2.2.2. *Interactions of gauge fields*

The derivation of the gauge field lagrangian in this formalism is more tricky, requiring the use of the covariant derivatives* to build the supersymmetric and gauge invariant term $W^A\ W_A$. Nevertheless the lagrangian that result from the same kind of analysis is quite obvious to guess; the gauge fields and the gauginos propagate in the usual way, the field D appears in a covariant fashion:

$$-\frac{G^2_\alpha}{4} + i\bar{\lambda}_\alpha \gamma^\mu (\mathcal{D}_\mu \lambda)_\alpha + \frac{D^2_\alpha}{2} \tag{2.17}$$

We can eliminate the auxiliary field D^α from the theory by a gaussian functional integration; doing this way, and using eq. (2.11) and eq. (2.17), the resulting term in the lagrangian is

$$\mathcal{L}_D = -\frac{D^2_\alpha}{2} \tag{2.18}$$

$$D^\alpha = g^\alpha z^*_a T^\alpha_{ab} z_b \tag{2.19}$$

This is a quartic contribute to the Higgs scalar potential, with a coefficient that is the square of the coupling constant; this is mostly interesting because in the SM the quartic terms in the Higgs lagrangian appear with an arbitrary coefficient λ. If the term of eq. (2.12) is present, we are left, up to a constant, with an additional term in the lagrangian: $-\xi^\alpha\ D_\alpha$; remember that to have such a term we need a U(1) factor in the gauge group.

2.3. The SUSY flavour sector

If we focus our attention on the auxiliary field $F(x)$ in eq. (2.3) we understand how to build other SUSY invariant terms; under a supersymmetry transformation, the variation

*To be honest covariant derivatives are needed also to obtain the chiral superfields by imposing a SUSY covariant constraint to the most general superfield; they may be viewed as differential operators connected with right multiplication on the left coset space.

of $F(x)$ must include, besides the supersymmetric parameter ϵ, also a *total derivative* of the field $\psi(x)$ for dimensional reasons; so we can build invariant actions just taking the product of chiral superfields, picking the highest dimension component as lagrangian and integrating the result on the usual 4 space. These terms extends the usual Yukawa sector in a SUSY fashion.

As for any lagrangian, we need just to insure the global invariance of our SUSY flavour interactions to obtain also the local invariance, since no derivatives enter this part of the lagrangian. In general, we must build a polynomial of chiral superfields (the matter fields, Higgs included), that is a chiral (composite) superfield by itself, called the *superpotential*:

$$f(\Phi) = l^a \Phi_a + \frac{1}{2}\mu^{ab}\Phi_a\Phi_b + \frac{1}{6}f^{abc}\Phi_a\Phi_b\Phi_c \tag{2.20}$$

where I considered only monomials leading to operators in the lagrangian of dimension 4 or less. The superfields Φ_a are the chiral (commuting) superfields that describe the theory: they form a reducible representation that contains all the "matter" fields (quarks, leptons, Higgs) The indices a, b, c identify the field components within this multiplet, *e.g.* $a = H_1^\sigma$; the coefficients f^{abc}, μ^{ab} are symmetric functions by definition.

Using the formulae in Appendix A it is easy to derive the formula for the F-component of the superpotential:

$$\begin{aligned} f(\Phi) &= \ldots + \theta^2 \left(\sum_a F_a \frac{\partial}{\partial z_a} - \frac{1}{2}\sum_{a,b} \psi_a\psi_b \frac{\partial^2}{\partial z_a \partial z_b} \right) f(z) \\ &= \ldots + \theta^2 \left(\sum_a F_a f^a(z) - \frac{1}{2}\sum_{a,b} \psi_a\psi_b \, f^{ab}(z) \right) \end{aligned} \tag{2.21}$$

where I used the notations

$$f^a \equiv \frac{\partial f}{\partial z_a}, \qquad f^{ab} \equiv \frac{\partial^2 f}{\partial z_a \partial z_b}. \tag{2.22}$$

The resulting terms in the lagrangian are, besides Yukawa terms, interactions of scalars:

$$\mathcal{L}_Y = F_a \, f^a - \frac{1}{2} \, \psi_a\psi_b \, f^{ab} + h.c. \tag{2.23}$$

It is a simple task the elimination of the auxiliary fields $F(x)$ from the total lagrangian; using eq. (2.23) and eq. (2.11) one obtains $F_a(x) = -f_a(x)$, and therefore the contribution to the scalar potential is given by

$$\mathcal{L}_F = - \mid f^a \mid^2 \tag{2.24}$$

Finally we have the generalized mass term for the chiral fields:

$$\mathcal{L}_{\psi\psi} = -\frac{1}{2}\ \psi_a\psi_b\ f^{ab} + h.c. \tag{2.25}$$

2.3.1. *The SM SUSY case*

We can give at this point another explanation why we need two Higgs doublets to extend the SM in a supersymmetric way. The up and down type of quarks need to be coupled to two doublets with opposite hypercharge (the coupling is used to give mass to both up and down quarks respectively†); but we cannot use, as in the SM, one Higgs and its conjugate, because the conjugation transforms a chiral superfield in a antichiral superfield (a different representation of the supersymmetry group), that would spoil the supersymmetric character of the superpotential.

It is easy to build all the possible low energy SUSY invariants with the fundamental superfields. In components we understand a priori that we can obtain more invariants than in the SM, because we have an additional Higgs doublets, and also because the standard spectrum is doubled. Using however the compact superfield notation the welcome invariants are (an antysimmetric $SL(2)$-covariant matrix where needed is intended):

$$H_1LE^c, H_1QD^c, H_2QU^c, H_1H_2 \tag{2.26}$$

The first three terms are just the superfield extension of the SM Yukawa terms, while the third is "new" and forbids a trivial global invariance in the Higgs sector. In principle there are other invariants

$$L^2E^c, H_1^2E^c; \epsilon_{\alpha\beta\gamma}U^{c\alpha}D^{c\beta}D^{c\gamma}, LQD^c; LH_2 \tag{2.27}$$

We see that the first two terms arise like the first term in eq. (2.26) because L and H_1 have the same quantum numbers; similarly for the last two terms. The fact is that these terms lead to lepton and/or baryon number violation at the tree level. The most economical way to exclude this terms is just to impose a *matter parity*, under which only the matter superfields transform (not the Higgs); in fact in all of the unwanted terms the number of matter fields is odd, while is even in the others.

We can also exclude them using a continuos $U(1)$ symmetry, called R symmetry, in which the supersymmetric fields (squarks, gauginos, etc.) transform differently from

†Some authors use for this reason the notation H_u, H_d instead then H_1, H_2

their "standard" counterpart. A way to introduce readily this kind of transformations, that are compatible with the supersymmetric ones, is to consider transformations on the superfields in which the anticommuting parameters pick a phase, and the component fields rotate in such a way that the superfield is simply multiplied by a phase.

The superpotential for the SUSY SM reads:

$$f_{SM} = \epsilon_{\sigma\tau}[\Gamma^E_{ij}\ H_1^\sigma L_i^\tau E_j^c + \Gamma^D_{ij}\ H_1^\sigma Q_i^\tau D_j^c + \Gamma^U_{ij}\ H_2^\sigma Q_i^\tau U_j^c + \mu\ H_1^\sigma H_2^\tau] \tag{2.28}$$

where I have written the $SU(2)_L$ indices, and left the $SU(3)_c$ ones implicit.

With the notations of eq. (2.20) one write:

$$\begin{aligned} f_{H_1^\sigma L_i^\tau E_j^c} &= \Gamma^E_{ij}\ \epsilon_{\sigma\tau} \\ f_{H_1^\sigma Q_i^{\tau\alpha} D_j^{c\beta}} &= \Gamma^D_{ij}\ \epsilon_{\sigma\tau}\ \delta_{\alpha\beta} \\ f_{H_2^\sigma Q_i^{\tau\alpha} U_j^{c\beta}} &= \Gamma^U_{ij}\ \epsilon_{\sigma\tau}\ \delta_{\alpha\beta} \\ \mu_{H_1^\sigma, H_2^\tau} &= \mu\ \epsilon_{\sigma\tau} \end{aligned} \tag{2.29}$$

The matrices Γ have indices on flavour space and also on colour and isospin-weak space.

2.4. The low energy supergravity model

In the context of model building the main question is simply: do the supersymmetry models provide us with extensions of the standard model lagrangian that are phenomenologically acceptable? In the low energy world supersymmetry appears to be broken, and we have therefore to justify how that breaking comes about. I will first discuss how the SUSY lagrangians written in the previous sections have to be modified in order to be realistic. This will be achieved via the introduction of soft breaking. Spontaneously broken low energy supergravity is the natural framework in which to implement such scenario. I will illustrate in details why phenomenology requires soft breaking terms, discussing the supertrace of the mass matrix in a general SUSY theory, setting also useful notations; at the end I will briefly mention the theoretical context in which these results arise.

2.4.1. Soft breaking terms

A phenomenological problem one encounters in supersymmetric model building is the tree level mass sum rule first proved by Ferrara, Girardello and Palumbo [14]. Defining the supertrace of the mass matrix to be the sum of the traces of the mass matrices of spin zero, spin 1/2, spin 1, weighted by the corresponding spin factor $1, -2, 3$ respectively one can prove, using equations (2.25), (2.24), (2.16) that $\mathrm{Str}(M^2) = 0$ if supersymmetry is exact or spontaneously broken; this is obviously a limitation for the model building. In fact we can say that it is because of that that theorists were lead to consider explicit breaking of supersymmetry.

The explicit breaking that has been considered is in fact not the most general, but the so called soft breaking, in order not to spoil the cancellation of the quadratic divergencies, characteristic of the supersymmetric theories. The soft terms have been catalogued by Girardello and Grisaru [15] and are of the form

$$m^2 zz^* \quad m^3 z \quad m^2 z^2 \quad mz^3 \quad \mu\lambda\lambda \tag{2.30}$$

where z is a scalar from a chiral superfield and λ is a gaugino, and I used m and μ to emphasize the fact that the couplings are dimensional. We can also notice that this terms are the lowest dimension component of the monomial of superfield used in the derivation of the supersymmetric lagrangian; the first comes from the kinetic term $\Phi^*\Phi$ of the chiral superfields, the last from the kinetic term $W^2 + \bar{W}^2$ of the vector superfields, the others from the "flavour" part (cfr. eq. (2.20)). When we consider a realistic model building an important requisite is to have soft terms that are gauge invariant; with the previous observation we know that the gauge invariance of the soft terms is assured by the gauge invariance of the corresponding *supersymmetric* monomials. We will use:

$$\begin{aligned} \mathcal{L}_{soft} &= \mathcal{L}_{\lambda\lambda} - V_{soft} && (2.31)\\ \mathcal{L}_{\lambda\lambda} &= +\frac{1}{2}\,\mu_\alpha\lambda_\alpha\lambda_\alpha + h.c. && (2.32)\\ V_{soft} &= [\eta(z) + h.c] + m^2_{ab} z^*_a z_b && (2.33)\\ \eta(z) &= L^a z_a + \frac{1}{2}M^{ab} z_a z_b + \frac{1}{6}\eta^{abc} z_a z_b z_c && (2.34) \end{aligned}$$

where I defined the dimensionful couplings in η in analogy with the terms in eq. (2.20); notice also the apparently unconventional sign $+$ in the gaugino mass term (this is due to the fact that we use conventionaly $-i\lambda$ in the vector superfield; see for instance eq. (2.15)).

The objection of arbitrariness of this kind of breaking of supersymmetry is in fact not so severe because soft terms may arise in the flat limit of supergravity theories (for a review see ref [16]). Spontaneously broken $N = 1$ supergravity, in its minimal formulation, allows also to reduce the number of couplings that appear in the previous equations;

making reference to eq. (2.20) we have:

$$m_{ab}^2 = m_{3/2}^2 \delta_{ab} \quad L^a = m_{3/2} C l^a \quad M^{ab} = m_{3/2} B \mu^{ab} \quad \eta^{abc} = m_{3/2} A f^{abc} \tag{2.35}$$

where $m_{3/2}$ is the gravitino mass; this form of the soft breaking terms hold after the flat limit $M_{Pl} \to \infty$ is taken.

We are now ready to write the general form of the supersymmetric lagrangian with soft breaking terms:

$$\mathcal{L} = \mathcal{L}_g + \mathcal{L}_{g.m.} + \frac{1}{2}\left[\mu_\alpha \lambda_\alpha \lambda_\alpha - \psi_a \psi_b \, f^{ab} + h.c.\right] - V \tag{2.36}$$

where $\mathcal{L}_g$ is the pure gauge lagrangian of eq. (2.17), without the D-term; $\mathcal{L}_{g.m.}$ is the gauge-matter lagrangian of eq. (2.11), without the terms in which F or D appear; V is the scalar potential, including the terms resulting from auxiliary field elimination and the soft terms. The explicit form of V is written in Appendix C, formula (C.14).

In order to make contact with experiments the parameters in the lagrangian have to be scaled with a renormalisation group analysis down to the low energy region. This kind of analysis has been performed by different authors[17]; I will discuss a very beautiful and simple method to obtain the 1-loop renormalisation group equations based on the effective potential in Appendix C.

2.4.2. *Mass matrices*

Let us set some useful notation: we will distinguish a field from its conjugate with the position of the representation indices:

$$z^a \equiv (z_a)^*, \qquad \xi^a \equiv (\xi_a)^*, \qquad \lambda^a \equiv (\lambda_a)^*, \qquad f_a(z) \equiv f^{a*}(z^*) \quad \ldots \tag{2.37}$$

in the last example I recalled the notation of eq. (2.22) for the partial derivative of the superpotential $f(z)$, that I will use widely in the following.

Now we separate out the terms in the lagrangian that give rise to the mass matrices for the different fields, as follows

$$\mathcal{L}_{\text{"mass"}} = \frac{1}{2}\left\{ V_\mu^t \cdot M_1^2 \cdot V^\mu - \left[\xi^t \cdot M_{1/2} \cdot \xi + h.c.\right] - Z^t \cdot M_0^2 \cdot Z\right\} \tag{2.38}$$

I wrote "mass" with quotation marks because I am referring to the generalized mass matrices, as it is apparent in the next paragraph; the usual mass term and mass matrices are obtained simply taking the expectation value of the scalar fields that appear in the matrices.

For the intermediate vector bosons we use equation (2.16) to write

$$M_1^2 = D^{\alpha a} D^{\beta}_a + D^{\alpha}_a D^{\beta a} \tag{2.39}$$

Whereas eqs.(2.15), (2.25), (2.32) lead to the following fermionic mass matrix:

$$M_{1/2} = \begin{pmatrix} f^{ab} & \sqrt{2} D^{\beta a} \\ \sqrt{2} D^{\alpha b} & \mu^{\alpha} \delta^{\alpha\beta} \end{pmatrix}, \qquad \xi = \begin{pmatrix} \psi_a \\ -i\lambda_\alpha \end{pmatrix} \tag{2.40}$$

Finally, using eqs. (2.18), (2.24) and (2.33) we can write the scalar mass matrix as:

$$M_0^2 = \begin{pmatrix} f^{ac} f_{cb} + m_b^{2a} + D^{\alpha a} D^{\alpha}_b + D^{\alpha a}_b D^{\alpha} & f_{abc} f^c + \eta_{ab} + D^{\alpha}_a D^{\alpha}_b \\ f^{abc} f_c + \eta^{ab} + D^{\alpha a} D^{\alpha b} & f_{ac} f^{cb} + m_a^{2b} + D^{\alpha}_a D^{\alpha b} + D^{\alpha b}_a D^{\alpha} \end{pmatrix} \tag{2.41}$$

where the field $Z = (z^a, z_a)$ contains all the scalar fields in the theory.

Notice that the presence of a ξ term, eq. (2.12), leads to an additional constant contribute to the *scalar* mass matrix of the form:

$$\delta M_0^2 = \xi^{\alpha'} \begin{pmatrix} D^{\alpha' a}_b & 0 \\ 0 & D^{\alpha' b}_a \end{pmatrix} \tag{2.42}$$

where the index α' runs over the U(1) generators $Y^{\alpha'}$.

In terms of the generalized mass matrices, previously introduced, the supertrace reads:

$$\mathrm{Str}(M^2) = tr(M_0^2) - 2\ tr(M_{1/2}^{+} M_{1/2}) + 3\ tr(M_1^2) \tag{2.43}$$

and can be easily computed to find

$$\mathrm{Str}(M^2) = 2\left[\xi^{\alpha'} g^{\alpha'} tr(Y^{\alpha'}) + tr(m^2) - \sum_{\alpha} \mid \mu_\alpha \mid^2\right] \tag{2.44}$$

We can now prove the previously mentioned sum rule; in the case in which the soft breaking terms m_b^a and μ_α and η are zero, and the trace of the U(1) generators is zero (it is the case of the standard model hypercharge, and it is necessary if we want to embed the group in a simple GUT) also the supertrace of the square of the mass matrix is zero; in this case it is easy to convince oneself that a realistic spectrum cannot be reproduced, if one would suppose the ordinary particles and their SUSY partners to be the spectrum of the theory. It is clear that, in a softly broken supersymmetric theory, the phenomenological requests on the SUSY spectrum can be taken into account easily. Let us stress in any case the fact that this sum rule is valid only at the tree level. The alternative attempts to use the ξ terms to evade the phenomenological bound lead to a new U(1) with non zero trace; and one finds that the condition of absence of anomalies requires an enormous number of particles [18]; that is why this kind of models are not pursued.

From now on we will limit ourselves with the study of softly broken SUSY models as derived from spontaneously broken $N = 1$ supergravity. In this framework, the scale of soft breaking $m_{3/2}$ is related to the scale M_S of spontaneous supergravity breaking, and to the Planck mass M_{Pl} by a see-saw like formula:

$$m_{3/2}^2 = O(1)\frac{M_S^2}{M_{Pl}} \tag{2.45}$$

As a consequence, if $M_S = O(10^{11}\ GeV)$ we obtain $m_{3/2} = O(1\ TeV)$, providing a suggestive connection between heaven and earth. For the following we will assume the flat limit $M_{Pl} \to \infty$, $m_{3/2}$ fixed, that is we will assume that at the grand unified scale $M_{GUT} \simeq 10^{16}\ GeV$ local supersymmetry is already broken and we have effectively a global supersymmetry theory with explicit soft breaking characterized by the TeV scale.

3. Grand unification

We are going to study the predictions of the minimal group of grand unification, namely the Georgi $SU(5)$ GUT, for $\sin^2\theta_W$ and for unification mass (for a review see [19]). The choice of the group is not very restrictive, if no exotic light particle is present; in fact in $SO(10)$ and in E_6 we would find the same results, modulo GUT threshold effects. We will see that the SUSY $SU(5)$ model is fairly in agreement with the experimental data, whereas its non-SUSY counterpart fails.

First we shall discuss the one-loop predictions, then we will investigate two-loop and threshold effects, which are needed to confront properly the theory with the precise experimental data that are presently available.

3.1. One-loop analysis

In this section we will discuss the general philosophy of the renormalisation group analysis of GUT theories; then we will give the formulae for the "one-step-GUT" predictions, we will use these formulae to confront the SUSY with the non-SUSY unifications. The outcome is that the "SM" unification, in contrast with the SUSY one, is not successful. At the end we will employ all the available experimental informations on the couplings to perform a slightly non standard analysis, in which we leave the "one-step-GUT" context, and find a prediction for the scale at which the supersymmetric particle should be found. This last item can give a useful hint for model building and experiment; the possibility of a nonstandard spectrum of "light" SUSY Higgs supermultiplets is considered. Finally we will state our conclusions, stressing the need of the 2-loop improvement.

3.1.1. How a single coupling leads to more couplings

If the physics is described by a simple unification group the gauge interactions are parameterized by a single coupling constant. If we are studying processes in which the typical momenta M are larger than M_{GUT}, where M_{GUT} (M_G for short) is the grand unification scale, the theory appears in this region symmetric‡, and our coupling constant

‡This happens when M_G is larger than the various physical masses.

runs quietly according to:

$$\frac{1}{\alpha_G(M)} = \frac{1}{\alpha_G(M_G)} - \frac{b_G}{2\pi} \cdot \ln\left(\frac{M}{M_G}\right) \qquad M > M_G \tag{3.1}$$

Suppose that we are instead studying the other regime, $M \ll M_G$, and suppose that a spontaneous symmetry breaking of the group G down to $H = \prod_i G_i$ has taken place at a scale $\sim M_G$. When we write the residual gauge group H as a direct product of G_i we are obviously thinking to the standard model $SU(3)_c \times SU(2)_L \times U(1)_Y$. We concentrate first on the gauge coupling of the simple subgroup G_i, that is on the light vector bosons relative to this subgroup, secondly we split b_G in its two parts: the first is due to the heavy particles (with masses of order M_G), and the second one to the light particles running in the loops:

$$b_G = b_i^h + b_i^l \tag{3.2}$$

If the decoupling theorem applies only the second contribution has to be retained; this is the so called *step approximation* for the beta function. The crucial point is that the splitting of the universal constant b_G is i-dependent, and so the low energy physics is described by many coupling constant (as much as the number of simple subgroups):

$$\frac{1}{\alpha_i(M)} = \frac{1}{\alpha_G(M_G)} - \frac{b_i^l}{2\pi} \cdot \ln\left(\frac{M}{M_G}\right) \qquad M < M_G \tag{3.3}$$

Let us change slightly the perspective, and investigate the relation between the couplings above and below the threshold. We can look at eq. (3.1) as the definining equation for $\alpha_G(M)$ even for $M \lesssim M_G$, and we could try to describe also the low energy regime with this single "effective" coupling constant. Now let us confront it with eq. (3.3). We simply observe that latter equation is what we would write in a theory without heavy fields; or better, the equation we would write in the *effective theory* obtained integrating out the heavy fields in a generating functional; that is the equation one correctely use in this regime. We see that the relation between the coupling is

$$\frac{1}{\alpha_i(M)} = \frac{1}{\alpha_G(M)} + \frac{b_i^h}{2\pi} \cdot \ln\left(\frac{M}{M_G}\right) \tag{3.4}$$

We conclude that if we compute a Green functions of light vector bosons using $\alpha_G(M)$ as the parameter of perturbative expansion, the couplings $\alpha_i(M)$ must appear as result of the resummation of non negligible higher order effects of heavy particles (in fact a "resubtraction" of the heavy contributes, that appear in eq. (3.1)); that is $\alpha_G(M)$ reveals simply to be not reliable at low perturbative orders in the low energy regime. By the same argument we can understand that one can interchangeably use $\alpha_i(M)$ and $\alpha_G(M)$

in the region in which higher order effects are small (for instance we can use α_G slightly below the unification mass).

We are allowed to use the step approximation in a momentum dependent renormalisation scheme, (as an example see [20]) but the connection between the unified theory and the low energy one must be studied from the point of view of effective theories if we are working in a momentum independent renormalisation scheme. The crucial point is not that the decoupling theorem does not apply, but simply that the concept of decoupling, momentum-dependent, does not apply: it is clear in fact that the ϵ poles from heavy particle exchange are present above and below the threshold; as we have seen, we have to consider in fact two distinct couplings and two distinct theories. This point of view has been introduced by S.Weinberg,[21] and will be formalized when discussing the 2-loop running. We will use here the result that approximatively the running coupling constants of the low energy theory and of the high energy theory meet exactly at the threshold, so that a posteriori one can naively speak of a single coupling that at the threshold changes its behaviour, inasmuch one speaks of decoupled degrees of freedom in the momentum-dependent schemes.

3.1.2. Normalization of the generators

We see a possible manifestation of this grand unified world in the ordinary one: we can fix the value of the unified coupling constant and of the unification mass M_G looking at the crossing point of two standard running coupling constants, and then predict the third.

But if we want to confront properly the coupling constants of different subgroups we must be careful about the normalization of the generators in each representation of the grand unified group; in fact the generators that survive the spontaneous symmetry breaking are the same as in the unified theory, and the universality of the normalization is one-to-one with the universality of the charge, because the generators enter the lagrangian multiplied by the corresponding charges.

Let us recall some group theoretical notions. For a given group G the index $T_2(G, R)$ of a representation is defined according to

$$\mathrm{Tr}(R^{\alpha}(G)R^{\beta}(G)) = T_2(G, R) \cdot \delta^{\alpha\beta}. \tag{3.5}$$

Summing on $\alpha = \beta$ we obtain the relation with the eigenvalue of the Casimir operator:

$$T_2(R) = \frac{\mathrm{Dim}(R)}{\mathrm{Dim}(G)}\, C_2(R) \tag{3.6}$$

Notice that we are dealing with compact and simple unification groups; this fact allows us to use hermitian generators, so that the T_2 is a positive coefficient; the price to pay is an i in the commutation relations. In the group $SU(N)$ we define T_2 to be equal to 1/2 for the fundamental representation, and as a consequence we find $T_2(\mathrm{Adj}) = N \ (= C_2(\mathrm{Adj}))$. If a representation R is the direct sum of some representations R^α, it follows that $T_2(R)$ is just $\sum_\alpha T_2(R^\alpha)$; for instance, considering the hypercharge generator, the representation of the Higgs doublet H is counted as 2 times the same $U(1)_Y$ (one dimensional) representation, since its SM numbers are $(1, 2, 1/2)$: $T_2(U(1)_Y, H) = 1/4 + 1/4 = 1/2$.

If we restrict ourselves to the $SU(5)$ GUT, knowing that the down-type of antiquarks, with SM quantum numbers $(\bar{3}, 1, 1/3)$ and the leptonic doublet, $(1, 2, -1/2)$ form the fundamental representation $\bar{5}$, the computation of the T_2 for the subgroups $SU(3)_c$,$SU(2)_L$, $U(1)_Y$ must give the same result. The generators of the first two subgroup are normalized to 1/2, in accord with the fact that the representation is the fundamental, while the hypercharge gives: $3 \cdot 1/9 + 2 \cdot 1/4 = 5/6$. This means simply that the generator to be considered in an $SU(5)$ unification context is not Y_{SM} but

$$Y_{GUT} = \sqrt{3/5} \cdot Y_{SM} \tag{3.7}$$

and in the same time we must replace the corresponding charge by

$$g'_{GUT} = \sqrt{5/3} \cdot g'_{SM} \tag{3.8}$$

In $SO(10)$ and in E_6, larger unification groups, the normalization of the generators is the same as in $SU(5)$; this means that we would perform the same analysis, if the light spectrum would stay the same.

3.1.3. One-loop β coefficients

Let us recall that the coefficients b_i of eq. (3.3), which determine the running of the gauge coupling constants at one-loop relative to the groups G_i, are:

$$\begin{cases} b_i^S = \frac{1}{6} \cdot T_{2i}^S & \text{real scalars (charged: } \times 2) \\ b_i^D = \frac{4}{3} \cdot T_{2i}^D & \text{Dirac fermions (Weyl or Majorana: } \div 2) \\ b_i^G = -\frac{11}{3} \cdot T_{2i}^G & \text{massless IVB (}e.g.\text{ gluons)} \\ b_i^M = -\frac{7}{2} \cdot T_{2i}^M & \text{massive IVB (with its Goldstone boson)} \end{cases} \tag{3.9}$$

Since we shall consider supersymmetric theories, it is convenient to list the corresponding coefficients for supermultiplets:

$$\begin{cases} b_i^C = 1 \cdot T_{2i}^C & \text{chiral superfields} \\ b_i^V = -3 \cdot T_{2i}^V & \text{vector superfields} \\ b_i^H = -2 \cdot T_{2i}^H & \text{massive vector superfields} \end{cases} \tag{3.10}$$

Notice that the would be Goldstone boson in the SUSY case is a chiral superfield (without gauge fixing, supersymmetry implies a Goldstone fermion). The indices T_2 are assigned once we give the spectrum of the theory. It is easy to compute the coefficients relevant to our analysis using the group theoretical informations given in the previous paragraph; we list them in the Appendix B.

3.1.4. *Formulae for the 1-loop predictions*

Let us suppose a *great desert* picture, that is: the world between M_Z and M_G is described completely by the SM or by its minimal SUSY extension. We can apply in this case the following 1-loop equations for the running of the gauge coupling constants:

$$\frac{1}{\alpha_i} = \frac{1}{\alpha_G} + b_i t \qquad\qquad i = 1, 2, 3 \tag{3.11}$$

On the left-hand side we input the experimental values of the couplings, that is $\alpha_i(M_Z)$, computed in the $\overline{MS}$ scheme. It is common use to set $t = (1/2\pi)\ln(M_G/M_Z)$. Then let us use $\sin^2\theta_W(M)\mid_{\overline{MS}}$ – a running quantity by definition – and α, the electromagnetic coupling constant in place of α_1,α_2:

$$\begin{cases} \frac{1}{\alpha_1} = \frac{3}{5}\frac{1-\sin^2\theta_W}{\alpha} \\ \frac{1}{\alpha_2} = \frac{\sin^2\theta_W}{\alpha} \end{cases} \tag{3.12}$$

The factor 3/5 is due to the normalization of the $U(1)_Y$ coupling constant, consistently with the $SU(5)$ embedding (eq. (3.8)).

Now we can use the three GUT equations, in which $\sin^2\theta_W, \alpha, \alpha_3, \alpha_G$ and t appear and two experimental inputs to obtain a prediction for the remaining quantities. A different approach is to predict an intermediate threshold, for instance the SUSY one, using the three experimental data as an input; this will be discussed in a following section.

Historically we have to remember that $\sin^2\theta_W$ has been poorly known until the discovery of intermediate vector bosons, therefore older analysis used to present it as a predicted quantity. Today the situation is different, and the main experimental uncertainty is on the strong coupling constant. As a consequence it is worthwhile to use $\sin^2\theta_W$ as input, and quote the prediction for α_3. We will call *scheme a* the first way of proceeding, *scheme b* the second one.

From the mathematical point of view, we have to solve the following two linear systems

$$\textit{scheme a}: \qquad \begin{pmatrix} \frac{3}{5}\frac{1}{\alpha} \\ 0 \\ \frac{1}{\alpha_3} \end{pmatrix} = \begin{pmatrix} 1 & b_1 & \frac{3}{5} \\ 1 & b_2 & -1 \\ 1 & b_3 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\alpha_G} \\ t \\ \frac{\sin^2\theta_W}{\alpha} \end{pmatrix} \tag{3.13}$$

$$\textit{scheme b}: \qquad \begin{pmatrix} \frac{3}{5}\frac{1-\sin^2\theta_W}{\alpha} \\ \frac{\sin^2\theta_W}{\alpha} \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & b_1 & 0 \\ 1 & b_2 & 0 \\ 1 & b_3 & -1 \end{pmatrix} \begin{pmatrix} \frac{1}{\alpha_G} \\ t \\ \frac{1}{\alpha} \end{pmatrix} \tag{3.14}$$

After inverting the two matrices we obtain:

$$\textit{scheme a}: \qquad \begin{pmatrix} \frac{1}{\alpha_G} \\ t \\ \frac{\sin^2\theta_W}{\alpha} \end{pmatrix} = \frac{1}{d} \begin{pmatrix} -b_3 & -\frac{3}{5}b_3 & b_1+\frac{3}{5}b_2 \\ 1 & \frac{3}{5} & -\frac{8}{5} \\ b_2-b_3 & b_3-b_1 & b_1-b_2 \end{pmatrix} \begin{pmatrix} \frac{3}{5}\frac{1}{\alpha} \\ 0 \\ \frac{1}{\alpha_3} \end{pmatrix} \tag{3.15}$$

$$\textit{scheme b}: \qquad \begin{pmatrix} \frac{1}{\alpha_G} \\ t \\ \frac{1}{\alpha} \end{pmatrix} = \frac{1}{b_1-b_2} \begin{pmatrix} -b_2 & b_1 & 0 \\ 1 & -1 & 0 \\ b_3-b_2 & b_1-b_3 & b_2-b_1 \end{pmatrix} \begin{pmatrix} \frac{3}{5}\frac{1-\sin^2\theta_W}{\alpha} \\ \frac{\sin^2\theta_W}{\alpha} \\ 0 \end{pmatrix} \tag{3.16}$$

where $d = 3/5(b_2-b_3)+(b_1-b_3)$, and at the end we can write the 1-loop result as follows:

$$\textit{scheme a}: \qquad \begin{cases} \frac{1}{\alpha_G} &= \frac{1}{d}\left[(b_1+\frac{3}{5}b_2)\frac{1}{\alpha_3} - \frac{3}{5}b_3\frac{1}{\alpha}\right] \\ t &= \frac{1}{d}\left[\frac{3}{5}\frac{1}{\alpha} - \frac{8}{5}\frac{1}{\alpha_3}\right] \\ \frac{\sin^2\theta_W}{\alpha} &= \frac{1}{d}\left[(b_2-b_3)\frac{3}{5}\frac{1}{\alpha} + (b_1-b_2)\frac{1}{\alpha_3}\right] \end{cases} \tag{3.17}$$

$$\textit{scheme b}: \qquad \begin{cases} \frac{1}{\alpha_G} &= \frac{1}{b_1-b_2}\left[(b_1+\frac{3}{5}b_2)\frac{\sin^2\theta_W}{\alpha} - \frac{3}{5}b_2\frac{1}{\alpha}\right] \\ t &= \frac{1}{b_1-b_2}\left[\frac{3}{5}\frac{1}{\alpha} - \frac{8}{5}\frac{\sin^2\theta_W}{\alpha}\right] \\ \frac{1}{\alpha_3} &= \frac{1}{b_1-b_2}\frac{1}{\alpha}\left[\frac{3}{5}(b_2-b_3)+d\right] \end{cases} \tag{3.18}$$

We have been quite detailed in this analysis because it will be also useful when computing the two-loop contributions to the running and the threshold effects as corrections to the previous framework; this is in fact the case for SUSY $SU(5)$. In fact the couplings $\alpha_i(M_Z)$ are different from the one-loop values by little corrections Δ_i, which can be thought as modifying the input values in the right hand side of eqs. (3.15–3.16). The procedure will be detailed in the following.

Notice that the number of families has no influence on the 1-loop prediction of $\sin^2\theta_W$ (or $1/\alpha_3$), because the number of generations cancels in the differences of the $b_i - b_j$. At the root of this observation there is the fact that the light matter particles fill a complete $SU(5)$ multiplet, and the contributions to the different b_i are, as a consequence, the same. Even in the prediction of M_G the number of families is irrelevant, but the proton lifetime depends on this number at the 1-loop level via the unificated coupling. The number of Higgs doublets is instead relevant, as I will show in the following, just because they *do not* form a complete $SU(5)$ multiplet[§].

3.2. SM versus SUSY grand unification

It is a simple application of the previous formulae to use the predictions of coupling unification to establish the running determined by the SM or by its SUSY extension. Using the b_i given in Appendix B we get the following predictions for $\sin^2\theta_W$ (we stay in *scheme a* for the purpose of illustration):

$$\begin{aligned} \sin^2\theta_W|_{SM} &= \frac{\alpha}{22+n_d/3}\left[\left(\frac{11}{3}+\frac{n_d}{6}\right)\frac{1}{\alpha}+\left(\frac{110}{9}-\frac{n_d}{9}\right)\frac{1}{\alpha_3}\right] \\ \sin^2\theta_W|_{SUSY} &= \frac{\alpha}{18+n_d}\left[\left(3+\frac{n_d}{2}\right)\frac{1}{\alpha}+\left(10-\frac{n_d}{3}\right)\frac{1}{\alpha_3}\right] \end{aligned} \tag{3.19}$$

Notice that the two formulae coincide if $n_d \to \infty$ or $n_d = 0$. The fact that in supersymmetry we have fermions and scalars in the same supermultiplet implies the stronger dependence on n_d in the SUSY case. Using the experimental inputs[22]:

$$\begin{aligned} \frac{1}{\alpha} &= 127.9 \pm 0.1 \\ \alpha_3 &= 0.12 \pm 0.01 \end{aligned} \tag{3.20}$$

eq. (3.19) gives us:

$$\begin{aligned} &\sin^2\theta_W|_{SM} = 0.207, 0.211, 0.215, ...0.232 \pm 0.004 \quad \text{for } n_d = 1,2,3,...8 \\ &\sin^2\theta_W|_{SUSY} = 0.230, 0.252, ... \pm 0.003 \quad \text{for } n_d = 2,4,... \end{aligned} \tag{3.21}$$

where the errors come from α_3; recall that the number of Higgs doublets in the minimal supersymmetric model is two but in any case is an even number. It is clear that the SM prediction, in which $n_d = 1$ is not in accord with the experimental data, whereas the SUSY SM with $n_d = 2$ is successful[¶].

[§] It is not possible to have light Higgs quintuplets because the triplet component mediate proton decay.

[¶] It is curious to notice that the experimental value of $\sin^2\theta_W$ has been drifting from ~ 0.23 before 1980, due to tree level analysis of the data, to ~ 0.21 after considering 1-loop electroweak corrections, value in accord with *non* SUSY GUT predictions, and finally to the present value, ~ 0.23.

Also the unification mass in the SM$\hookrightarrow$ $SU(5)$ GUT scheme (at variance with the SUSY $SU(5)$) is lower then the experimental bound obtained by the searches of proton decay;

$$\begin{aligned} t\mid_{SM} &= \frac{1}{22+n_d/3}\left[\frac{1}{\alpha}-\frac{8}{3}\frac{1}{\alpha_3}\right] \\ t\mid_{SUSY} &= \frac{1}{18+n_d}\left[\frac{1}{\alpha}-\frac{8}{3}\frac{1}{\alpha_3}\right] \end{aligned} \tag{3.22}$$

numerically

$$\begin{aligned} &M_G\mid_{SM}= 4.8\times 10^{14}, 2.1\times 10^{14},...,4.5\times 10^{13} \quad \text{for } n_d = 2,4,...,8 \\ &M_G\mid_{SUSY}= 2.4\times 10^{16}, 1.2\times 10^{15},... \quad \text{for } n_d = 2,4,... \end{aligned} \tag{3.23}$$

This results are obtained using the central values of eq. (3.20). The inclusion of the error in α_3 can double or halve the prediction on M_G (the exponentiation makes the error multiplicative in nature), even if the second term in the square brackets is 5 times smaller then $1/\alpha$.

3.2.1. *"Prediction" of M_{SUSY}*

From the fact that the three coupling constants are experimentally known we can try to test for an intermediate threshold M_S, that is to consider a possible threshold between M_Z and M_G as an output of our three evolution equations (togheter with the GUT scale and the GUT unificated coupling). When we will study the running with more detail (threshold effects and 2-loop effects) we will see that higher order corrections will shed some light on the physical interpretation of the parameter M_S. For the time being we will use M_S as a rough parametrization of the SUSY threshold.

We are studying a two-step-GUT ‖; the equation are modified as follows:

$$\frac{1}{\alpha_i} = \frac{1}{\alpha_G} + b_i t + B_i T \qquad \text{i=1,2,3} \tag{3.24}$$

where $t = 1/(2\pi)\ln(M_S/M_Z), T = 1/(2\pi)\ln(M_G/M_S)$. Inverting the matrix of the linear system, in which the three couplings are the known terms, and using the Appendix for the coefficients (b_i=standard model with n_l doublets, B_i=SUSY SM with $n_d = n_l + n_h$ doublets) we find:

$$M_S = M_Z \exp\left\{\frac{5\pi}{11n_d - 3n_l}\left[\left(\frac{3}{\alpha_1}-\frac{9}{\alpha_2}+\frac{6}{\alpha_3}\right) + n_d\cdot\left(\frac{1}{2\alpha_1}-\frac{3}{10\alpha_2}-\frac{1}{5\alpha_3}\right)\right]\right\} \tag{3.25}$$

‖We can analyze with the same formalism other two-step-GUT, *e.g.* an extension of the SM in which the number of heavy Higgs bosons is large.

$$M_G = M_Z \exp\left\{\frac{5\pi}{11n_d - 3n_l}\left[\left(-\frac{2}{3\alpha_1} + \frac{2}{\alpha_2} - \frac{4}{3\alpha_3}\right) + n_d \cdot \left(\frac{1}{2\alpha_1} - \frac{3}{10\alpha_2} - \frac{1}{5\alpha_3}\right) + n_l \cdot \left(-\frac{1}{6\alpha_1} + \frac{1}{10\alpha_2} + \frac{1}{15\alpha_3}\right)\right]\right\} \quad (3.26)$$

A numerical analysis of eq. (3.25) gives us the following result. Within the experimental errors of the coupling we obtain, in the case $n_d = 2, n_l = 1$, M_S in the range between $\sim 500\ GeV$ to $M_S \sim 0.2GeV$ which shows a huge instability of the prediction. In any case the fact to be noticed is the numerical cancellation between the linear combination of the inverses of the couplings in the first of eq. (3.25); in fact we could a priori expect a number of order $1/\alpha = 128$ instead then the number that we find using the central values, ~ -2. This cancellation is exactly what leads to the successful prediction of eq. (3.21) in the SUSY case when $n_d = 2$ and $n_l = 1$; it is just a different way to state the agreement of the SUSY GUT with the experimental data.

The cancellation does not hold in the case $n_d \geq 4$; for instance if $n_d = 4$ we have $M_S \sim 5 \cdot 10^7$, very different from the unification and from the weak scales. The conclusion of this analysis is that the successful prediction in the SUSY SM of $\sin^2\theta_W$ depends in an essential way on the minimality of the SUSY Higgs sector.

3.2.2. Conclusions; the need of the two-loop analysis

We can conclude that it is worth pursuing the SUSY grand unification but not the *non* SUSY one, if we suppose a great desert scenario.

The errors in eq. (3.21) are quite large; we could believe that they are large enough to hide two-loop effects; but we must remember that we can use $\sin^2\theta_W$ as an input parameter, and get in this way a precise prediction for α_s. What is the estimate of the two-loop effect (to be justified a posteriori)? Remembering that the quantity we predict is $\sin^2\theta/\alpha$ and supposing that the 2-loop corrections to the running do not upset the 1-loop prediction (we can suppose that they add a number of the order of the unity, small with respect to the 1-loop prediction, to the right hand sides of eqs. (3.17)) we find from eq. (3.19), in the case $n_d = 2$

$$\sin^2\theta_W = \frac{1}{5} + \frac{7}{15}\frac{\alpha}{\alpha_3} + O(\alpha) \quad (3.27)$$

this in fact will turn to be an overestimate of a factor ~ 3, but it is an important effect, confronted with the experimental uncertainty on $\sin^2\theta_W$. We find that the two-loop effect on the GUT scale is even more important, even if a variation of the experimental determination of α_s leads to a multiplicative uncertainty factor in the prediction $\sim e^{7\Delta\alpha_s/\alpha_s}$;

$$M_G = 2.4 \cdot 10^{16} \cdot e^{O(1)} \tag{3.28}$$

Finally I should stress that the two-loop context implies not only a numerical improvement of our predictions, but also a conceptual improvement; I am here refering to the one-step approximation used for the threshold effects, and also to the physical meaning of the parameter M_{SUSY}, that we used implicitly (but, as I will show, also in a deceiving way) as "the mass at which SUSY begins"; notice that the last issue has been thouroughly addressed only very recently (see for instance ref. [22]).

4. Beyond the 1-loop predictions

With the knowledge that the 1-loop SUSY GUT predictions are substantially correct we are going to consider the running at the two-loop level. We shall also in a sense refine the step approximation, clarifying its meaning in the momentum independent renormalisation schemes. Then we will treat the threshold effects, due both to the SUSY particles and to the heavy $SU(5)$ particles as corrections to the 1-loop picture.

The unification equations ruling the running, eq. (3.11), are to be changed with the following equations (recall that $t = \frac{1}{2\pi} \ln \frac{M_G}{M_Z}$):

$$\frac{1}{\alpha_i} - \theta_i^{2loop} + \Delta_i^{conv} + \Delta_i^{low} + \Delta_i^{high} = \frac{1}{\alpha_G} + b_i t \qquad i = 1, 2, 3 \tag{4.1}$$

where we have given a name to the previously mentioned corrections; which will be precisely defined in the following, and at the end we will use them to extract the predictions in the very same way we did in the 1-loop context. The coefficients b_i, without any other specification, are SUSY SM coefficients of Appendix B in the rest of the chapter.

4.1. Refining the 1-loop running

First we will consider a consistent approximation for the 2-loop running θ_i, and discuss then some renormalisation scheme dependent features that must be taken in account in a 2-loop SUSY context; we will use this opportunity to discuss the meaning of the step approximation. Then we will distinguish two parts in the low energy threshold corrections, that is $\Delta_i^{low} = \Delta_i^{SUSY} + \Delta_i^{SM}$. The first is the SUSY particles contributions, that can be considered as a modification of the running, and will be treated introducing a convenient parametrization of the mass spectrum; the second is the contribution of heavy SM particles: $\Delta_i^{top} + \Delta_i^{Higgs}$; in fact the top mass enter $\sin^2 \theta_W$ in a peculiar way (in a sense modifies the input parameters). Finally we will recall briefly the necessary informations about the SUSY $SU(5)$ Georgi model in order to write the heavy threshold corrections in this model.

4.1.1. Two loop renormalisation group equation

The renormalisation group equation (RGE) in a gauge theory take the form

$$\dot{\alpha}_i(t) = 4\pi\alpha_i^2(t)\left[\frac{b_i}{4\pi} + \sum_j \frac{b_{ij}}{(4\pi)^2}\alpha_j(t) + \sum_{jk}\frac{b_{ijk}}{(4\pi)^3}\alpha_j(t)\alpha_k(t) + ...\right] \qquad (4.2)$$

where the first is the one-loop term, the second is the two-loop term, *etc.* , and the initial condition on $\alpha_i(t)$ is α_i; we call n^{th} order RGE the equation obtained putting all the $(n+1)^{th}$ terms to zero (the common factor 4π is consistent with the use of $t = \frac{1}{2\pi}\ln\frac{M}{M_0}$, and with the coefficients b_i of eq. (3.9) and b_{ij}, listed in Appendix B; we neglect the Yukawa contributions).

We can develop a systematic approximation procedure by writing this equation in a different form; namely, we define

$$\begin{aligned} x_{ij,\dots k} &= \frac{\alpha_i\alpha_j\dots\alpha_k}{(4\pi)^n} b_{i,j,\dots k} \\ z_i(t) &= \frac{\alpha_i}{\alpha_i(t)} \end{aligned} \qquad (4.3)$$

and we write the renormalisation group equation in an integral form, better suitable to show the approximation procedure:

$$z_i(t) = 1 - 4\pi\sum_{j,\dots k} x_{i,j,\dots k}\int_0^t \left[\frac{1}{z_j(\tau)}\cdots\frac{1}{z_k(\tau)}\right] d\tau \qquad (4.4)$$

The first step of the approximation is done retaining x_i only:

$$z_i^{(1)}(t) = 1 - 4\pi x_i t \qquad (4.5)$$

that is equivalent to the 1-loop result:

$$\frac{1}{\alpha_i^{(1)}(t)} = \frac{1}{\alpha_i} - \frac{b_i}{2\pi}t \qquad (4.6)$$

We will assume that the two-loop solution $z_i^{(2)}(t)$, in the interval in which we are interested, is not very different from the one-loop value, and we will justify this hypothesis a posteriori. Instead, it is not correct to suppose that the one-loop effects are always small; from the 1-loop we estimate at the grand unification scale $z_3^{(1)} \sim 3$ and therefore $4\pi x_3 t = O(1)$.

The hypothesis of two-loop smallness implies that we can use $z_i^{(1)}(t)$ consistently in the integral of eq. (4.4) to improve the approximation. The approximate two-loop RGE solution we get reads:

$$z_i^{(2)}(t) = z_i^{(1)}(t) + \sum_j \frac{x_{ij}}{x_j} \cdot \ln[z_j^{(1)}(t)] \tag{4.7}$$

that we readily translate into an expression for the couplings:

$$\frac{1}{\alpha_i^{(2)}(t)} = \frac{1}{\alpha_i} - \frac{b_i}{2\pi}t + \frac{b_{ij}}{4\pi b_j} \ln \frac{\alpha_j}{\alpha_i^{(1)}(t)} \tag{4.8}$$

in which the well-known log(log) terms are apparent. Using the fact that $b_{ij}/(4\pi b_j) = O(1)$ (see the Appendix B) or equivalently $x_{ij}/x_j = O(\alpha_i)$ we conclude that the two-loop corrections are indeed small, and the procedure is consistent.

We could again improve the solution above replacing in the two-loop integral $z_i^{(1)}(t)$ with $z_i^{(2)}(t)$. But, after some work, one concludes that the additional correction obtained with this further improvement can be comparable to a three-loop effect.

By comparing eq. (4.1) and eq. (4.8), we can write

$$\theta_i = \sum_j \frac{b_{ij}}{4\pi b_j} \ln \left(\frac{\alpha_G}{\alpha_j} \right) \tag{4.9}$$

4.1.2. *Thresholds effects in $\overline{MS}$ and $\overline{DR}$*

As we have discussed in the 1-loop context we can invoke the decoupling theorem in a momentum dependent renormalisation scheme (see ref. [23]), but in a momentum independent scheme the threshold effects have to be treated in a different manner.

Following S.Weinberg [24] we consider the effective theory that we obtain integrating out the heavy degrees of freedom of the high energy (more fundamental) lagrangian. This procedure defines a relation between the coupling constants that we must use above the mass scale of the heavy degrees of freedom and the coupling constants below this mass scale; that is defines a boundary condition between two theories that should be considered different. This boundary condition can be interpreted eventually as the change in the running of a single coupling constant.

Going into details we can perform a gauge fixing of the fundamental gauge theory that does not spoil the gauge invariance under the remaining group $H = \prod_i G_i$; this is a slight modification of the R_ξ gauge fixing for the complete gauge group G, in which the ordinary derivative is replaced by the covariant derivative with respect to the group H (the heavy fields belong to a given representation of the residual group; see the section on high energy threshold). Using this gauge fixing we see that the effect of the functional integration is to provide new gauge invariant contributions to the low energy lagrangian (we generate an invariant effective lagrangian); in particular the term in the bare lagrangian for the propagation of light gauge fields get modified simply according to:

$$-\frac{1}{4}F^i_{B\mu\nu}F^{i\mu\nu}_B \longrightarrow -\frac{1}{4}Z_i\ F^i_{B\mu\nu}F^{i\mu\nu}_B \tag{4.10}$$

Notice that even if the left hand side member is effectively i-independent (it is a term of the GUT lagrangian) the r.h.s. is i-dependent, because of Z_i; this is at the root of the diversity of the couplings. It is easy to see that concentrating our attention on the wavefunction renormalisation of the light vector fields we can get a relation between the GUT bare coupling constant above threshold and the bare couplings below threshold. This relation can be readily converted in a relation between renormalised couplings, when we consider that in the region in the nearby of the threshold we can use both the fundamental and the low energy effective theory.

The result of the analysis can be stated in this way: if $g_>$ ($g_<$) the coupling constant above (below) the M_X scale

$$g_<(M) = g_>(M) - \frac{g^3_>(M)}{(4\pi)^2}\left[\delta b_X \ln\left(\frac{M}{M_X}\right) + \delta b_X C_X\right] \qquad M \simeq M_X \tag{4.11}$$

where I emphasized that the constant C_X are renormalisation scheme artifacts; for instance if we follow the often used convention $\mathrm{Tr}(\gamma_\mu\gamma_\nu) = 4g_{\mu\nu}$ in $D-$dimensions, $C_X\mid_{\overline{MS}}$ is different from zero only for IVB, and its value is 1/21. Multiplying it by the "massive" vector coefficient of eq. (3.9) one obtains

$$\delta b_{\mathrm{IVB}} C_{\mathrm{IVB}} = -\frac{1}{6}\ T_2(\mathrm{IVB}) \tag{4.12}$$

If we are interested to the heavy IVB corresponding to the generator of the coset G/H we can also use the equation

$$T_2(\mathrm{IVB}) = T_2(G) - T_2(H) = C_2(G) - C_2(H) \tag{4.13}$$

We can appreciate explicitly from eq. (4.11) the fact that the couplings in the low energy effective theory incorporate implicitly a resummation of effects related to the high energy scales. What prevent us from using the *unified* coupling in the low energy regime is just the unreliability of the truncated perturbative expansion at low scales. The regime of validity of eq. (4.12) is in fact

$$\left| \frac{g_>^2(M)}{(4\pi)^2} \delta b_X \ln\left(\frac{M}{M_X}\right) \right| < 1 \tag{4.14}$$

At each threshold eq. (4.12) tells us how to connect the coupling constant below threshold with that above threshold. We may however write eq. (4.12) in the suggestive form

$$\frac{1}{\alpha(M)} = \frac{1}{\alpha(M_0)} - \frac{1}{2\pi} \begin{cases} b \ln\left(\frac{M}{M_0}\right) & \text{if } M < M_X \\ b \ln\left(\frac{M}{M_0}\right) + \delta b_X \ln\left(\frac{M}{M_X}\right) + \delta b_X C_X & \text{if } M > M_X \end{cases} \tag{4.15}$$

where $M_0 \ll M, M_X$. Eq. (4.15) defines a *single* coupling constant with a discontinuity at $M = M_X$ (one can also devise a subtraction procedure to obtain this result [25]). This framework can be useful, but we must keep in mind that the coupling constants of the "fundamental" and of the "low energy" effective theories are different objects, and that eq. (4.12) is just the matching condition between the two theories.

A very important observation is that there is a favourite renormalisation scheme for supersymmetric theories, the $\overline{DR}$ scheme. First we regularize dimensionally, keeping the γ matrices *and* the metric tensor that appear in the Feynman rules in 4 dimensions (differently from the 't Hooft-Veltman regularization scheme), while the momenta in the loops are dimensionally extended [26]; then we subtract only the $\overline{MS}$ pole from the Green functions. The reason why this renormalisation procedure is suited for SUSY is that SUSY is not well defined in $4-\epsilon$ dimensions (the authors of [27] proved that, at two loop level, the Ward identities are in effect preserved if $\overline{DR}$ is used).

The relation between the coupling constant in the $\overline{MS}$ and in the $\overline{DR}$ schemes is [28]:

$$\frac{1}{\alpha^i_{\overline{MS}}} = \frac{1}{\alpha^i_{\overline{DR}}} + \frac{C_2(G_i)}{12\pi} \tag{4.16}$$

Notice that, comparing eq. (4.11), eq. (4.12) and eq. (4.13) this means that $\alpha_{\overline{DR}}(M)$ have no constant term in the matching condition. From the point of view of a single coupling, there are no discontinuities at threshold when we renormalise according to $\overline{DR}$ (in passing let me mention that the constant terms of eq. (4.12) - in $\overline{MS}$ - arises from $D-$dimensional traces of the metric tensor present in the vector bosons vertices).

The standard model coupling constants are usually obtained in the $\overline{MS}$ scheme, but we have to use the $\overline{DR}$ scheme in the high energy regime, where the theory is supersymmetric. This implies that we have to convert the three couplings α_i, before reaching the grand unified scale. The conversion factors Δ_i^{conv} in eq. (4.1) are those eq. (4.16):

$$\Delta_i^{conv} = -\frac{C_2(G_i)}{12\pi} \tag{4.17}$$

In the following, we will implicitly assume, unless otherwise specified, that the low energy parameters are defined in $\overline{MS}$, whereas the quantities at the GUT scale are given in the $\overline{DR}$ scheme.

4.1.3. Δ_i^{SUSY}: *effctive parametrization of the SUSY thresholds*

Let us suppose that the supersymmetric particle masses $M_a, a = 1, 2, ...n$ are above M_Z (that we call now M_0), and let us treat them by the step approximation; the running of the coupling constants well above this SUSY threshold will be modified as follows:

$$\frac{2\pi}{\alpha_i(M)} = \frac{2\pi}{\alpha_i(M_0)} - \left[b_i^{SM}\ln\left(\frac{M}{M_0}\right) + \delta b_i^1 \ln\left(\frac{M}{M_1}\right) + ... + \delta b_i^n \ln\left(\frac{M}{M_n}\right)\right] \tag{4.18}$$

where b_i^{SM} are the β-function coefficients of the unbroken standard model for α_i, before the first supersymmetric threshold; δb_i^a is the a-th sparticle contribute; we can refer to eq. (4.18) as a smooth knee description of the running (notice that we are assigning a given mass to each interaction eigenstate).

Now let introduce b_i, the "total" beta coefficient at the end of the SUSY thresholds:

$$b_i = b_i^{SM} + \sum_{a=1}^{n} \delta b_i^a \tag{4.19}$$

$$\begin{aligned} \frac{2\pi}{\alpha_i(M)} &= \frac{2\pi}{\alpha_i(M_0)} - \left[(b_i - \sum_{a=1}^{n} \delta b_i^a)\ \ln\left(\frac{M}{M_0}\right) + \sum_{a=1}^{n} \delta b_i^a\ \ln\left(\frac{M}{M_a}\right)\right] \\ &= \frac{2\pi}{\alpha_i(M_0)} - b_i\ \ln\left(\frac{M}{M_0}\right) + \sum_{a=1}^{n} \delta b_i^a\ \ln\left(\frac{M_a}{M_0}\right) \end{aligned} \tag{4.20}$$

The threshold corrections are lumped in the last term. We can usefully define an intermediate scale M_i, depending on the gauge group associated with α_i, such to lump in it the threshold effects (a steep knee equivalent description of the running), that is we define M_i in this way:

$$\frac{2\pi}{\alpha_i(M)} = \frac{2\pi}{\alpha_i(M_0)} - b_i^{SM}\ \ln\left(\frac{M_i}{M_0}\right) - b_i\ \ln\left(\frac{M}{M_i}\right) \tag{4.21}$$

It is quite easy to find, using the previous definitions that

$$\begin{aligned} \frac{2\pi}{\alpha_i(M)} &= \frac{2\pi}{\alpha_i(M_0)} + (b_i - b_i^{SM}) \ln\left(\frac{M_i}{M_0}\right) - b_i \ln\left(\frac{M}{M_0}\right) \\ &= \frac{2\pi}{\alpha_i(M_0)} + \sum_{a=1}^{n} \delta b_i^a \ln\left(\frac{M_i}{M_0}\right) - b_i \ln\left(\frac{M}{M_0}\right) \end{aligned} \tag{4.22}$$

That is, comparing eq. (4.20) and eq. (4.22) we see that the phenomenological mass M_i is just a geometric average of the physical masses, weighted with a factor of relevance to the running:

$$M_i = \prod_{a=1}^{n} (M_a)^{\omega_i^a} \qquad \omega_i^a = \frac{\delta b_i^a}{\sum_{a=1}^{n} \delta b_i^a} \tag{4.23}$$

Notice that we are supposing that $\sum_{a=1}^{n} \delta b_i^a \neq 0$, that is $b_i^{SM} \neq b_i$; otherwise we can proceed as follows: go back to the last term of eq. (4.20), the threshold corrections; notice that it is independent from M_0; choose M_0 equal to one physical mass, so that this mass is "out of the game" and the remaining b_i^a have a sum $\neq 0$; employ the previous formulae (4.20), second line, and (4.23), excluding from the sums and from the product the chosen mass.

In fact this formalism can be employed to describe the effect of every particle threshold on the running. The "effective" masses M_i have been recently introduced by P.Langacker and N.Polonsky[22] to achieve a model independent description of the low energy threshold effects to the unification prediction.

Comparing eq. (4.22) and eq. (4.1)

$$\Delta_i^{SUSY} = \frac{1}{2\pi}(b_i - b_i^{SM}) \ln\left(\frac{M_i}{M_Z}\right) \tag{4.24}$$

This equation has an obvious physical; it accounts for the "SUSY deficit" in the range between M_Z and M_i.

4.1.4. Top and Higgs thresholds

In the 1-loop analysis of chapt. 3 we have implicitly considered the top and the Higgs mass, as well the masses of the supersymmetric particles, at M_Z. Now, one may naively believe that the realistic case $m_t > M_Z$ is simply taken into account by subtracting the top contribution to the running in the range $M_Z \div m_t$, that is (with the definitions of eq.

(4.1); $b_i^{top} = 2/3$)

$$\Delta_i^{top} \stackrel{?}{=} \frac{b_i^{top}}{2\pi} \ln\left(\frac{m_t}{M_Z}\right) \tag{4.25}$$

This is correct only for α_3 since α_1 and α_2 depend on $\sin\theta_W$, and the value of $\sin\theta_W$ extracted from experiment depends on the top mass in a nontrivial way.

The value of $\sin^2\theta_W(M_Z)$ reported in eq. (4.69), together with $m_t = 138\ GeV$, is the outcome of a 2 parameter fit of the experimental data ($\nu - hadron$, $\nu_\mu - e^-$ deep inelastic scattering, μ lifetime, M_W and M_Z, *etc.* ; $m_{h^0} = M_Z$ is assumed) performed by the authors of ref. ([29]). If the top mass is different from $\bar{m}_t = 138\ GeV$ the effect on $\sin^2\theta_W$ is well parametrized by [22]

$$\Delta_{s^2}^{top} \approx -1.96 \times 10^{-3} \left[\left(\frac{m_t}{\bar{m}_t}\right)^2 - 1\right] \tag{4.26}$$

where only the corrections quadratic in the top mass, that are the leading effects, are considered.

Finally, let us mention that if we use the value of $\alpha(M_Z)$ quoted in eq. (4.67) we must be aware that it is obtained assuming $m_t = 138\ GeV$. As a consequence, we include the effect of $m_t \neq \bar{m}_t$ on α by writing

$$\frac{1}{\alpha(M_Z)} \to \frac{1}{\alpha(M_Z)} + \Delta_\alpha^{top} \qquad \Delta_\alpha^{top} = \frac{1}{2\pi}\,\frac{4}{3}\,3\left(\frac{2}{3}\right)^2 \ln\left(\frac{m_t}{\bar{m}_t}\right) = \frac{8}{9\pi} \ln\left(\frac{m_t}{\bar{m}_t}\right) \tag{4.27}$$

where 4/3 is the factor for Dirac fermions and 3 is color counting (we can find the same result using $5/3\ b_1 + b_2$ as coefficient of the running).

If we recall the expressions α_1 and α_2 in terms of α and $\sin^2\theta_W$

$$\begin{cases} \frac{1}{\alpha_1} = \frac{3}{5}\frac{1-\sin^2\theta_W}{\alpha} \\ \frac{1}{\alpha_2} = \frac{\sin^2\theta_W}{\alpha} \end{cases} \tag{4.28}$$

we readily translate eq. (4.25), eq. (4.26) and eq. (4.27) into the following corrections terms:

$$\begin{cases} \frac{1}{\alpha_1} + \Delta_1^{top} & : & \Delta_1^{top} = \frac{3}{5}\left[\frac{8}{9\pi}\left(\ln\frac{m_t}{\bar{m}_t}\right)\cos^2\theta_W - \frac{1}{\alpha}\Delta_{s^2}^{top}\right] \\ \frac{1}{\alpha_2} + \Delta_2^{top} & : & \Delta_2^{top} = \frac{8}{9\pi}\left(\ln\frac{m_t}{\bar{m}_t}\right)\sin^2\theta_W + \frac{1}{\alpha}\Delta_{s^2}^{top} \\ \frac{1}{\alpha_3} + \Delta_3^{top} & : & \Delta_3^{top} = \frac{1}{3\pi}\left(\ln\frac{\bar{m}_t}{M_Z}\right) + \frac{1}{3\pi}\left(\ln\frac{m_t}{\bar{m}_t}\right) \end{cases} \tag{4.29}$$

The effect of the SM Higgs on the running is negligible, that is $\Delta_i^{h_0} \ll \Delta_i^{top}$. This is mostly due to the fact that $\Delta_{s^2}^{h_0} \ll \Delta_{s^2}^{top}$, but also to the fact that the Higgs carries neither color or charge ($\Rightarrow \Delta_3^{h_0} = \Delta_\alpha^{h_0} = 0$). The value of $\sin^2\theta_W = 0.2324 \pm 0.0003$ assumes $m_{h^0} = M_Z$, but the effect of varying m_{h^0} up to 1 TeV is within the quoted error.

In conclusion, recalling eq. (4.1) we can write

$$\Delta_i^{low} = \Delta_i^{SUSY} + \Delta_i^{top} \tag{4.30}$$

4.1.5. *Thresholds at the GUT scale*

We will confine our discussion to the minimal $SU(5)$ supersymmetric model, the SUSY extension of the Georgi model, in which the Higgs sector comprehends a chiral superfield in the 24, needed to break $SU(5)$, and two quintuplets, containing the Higgs doublets. Our formalism can be simply extended to more complicated Higgs sectors or groups.

The representations of the grand unification group are obviously representations also of the remaining group, called the stability group because it leaves the vacuum invariant; and the mass matrices must be constant along the irreducible subrepresentations because of the symmetry. Let us analyze closely the adjoint representation, built on the generators T^α. This representation decomposes in the unbroken generators, say T^a, that are associated with the massless IVB V_μ^a, and in the broken generators say T^A, associated with the massive IVB V_μ^A. The heavy vector bosons are in a given representation of the stability group; in fact, thanks to the normalization of the group, eq. (3.5), the structure constants $f^{\alpha\beta\gamma}$ are completely antisymmetric, and from the equation $f^{abC} = 0$, which states that the stability group is a group, we can conclude

$$[T^a, T^B] = i \cdot f^{aB\gamma}T^\gamma = i \cdot (f^{aBc}T^c + f^{aBC}T^C) = i \cdot f^{aBC}T^C \tag{4.31}$$

The heavy spectrum of $SU(5)$ can be divided in three groups of particles. The vector part contains two charged vectors, $SU(3)_c$ triplets, called X and Y, or collectively V, with charges $\pm 4/3$ and $\pm 1/3$ respectively; they appear in the decomposition of the adjoint representation:

$$24 = (3,2,5/6) + (\bar{3},2,-5/6) + (8,1,0) + (1,3,0) + (1,1,0) \tag{4.32}$$

The corresponding scalar fields (chiral superfield in the SUSY case) in the same representation are the would be Goldstone bosons; the remaining 12 fields are physical, and we call them collectively Φ. Finally there is a color triplet T appearing with the light Higgs doublet in

$$5 = (3, 1, 1/3) + (1, 2, -1/2) \tag{4.33}$$

together with the antitriplet $\bar{T}$ present in $\bar{5}$. Notice that from the fact that 5 is the fundamental, and $5 \times \bar{5} = 24 + 1$ we can easily deduce the decomposition of the 24; similarly for the 15 and the 10, that are the symmetric and the antisymmetric parts of 5×5.

Now let discuss the coefficients for the running induced by the heavy particles. It is simple to compute the values of the $T_{2,i}$ for these heavy fields; they depend on i because the heavy fields we are considering are not complete $SU(5)$ multiplets.

	$SU(3)_c$	$SU(2)_L$	$U(1)_{Y_{gut}}$
$X, Y, \bar{X}, \bar{Y}$	2	3	5
$T, \bar{T}$	1	0	2/5
Φ	3	2	0

Table 1: $T_{2,i}$; $i = 1, 2, 3$

We will suppose that inside a given kind of superfield the mass is degenerate; that is, for instance, that the scalar and the fermionic fields in the triplets have the same mass. Employing the formulae (1) we find

	$SU(3)_c$	$SU(2)_L$	$U(1)_{Y_{gut}}$
$X, Y, \bar{X}, \bar{Y}$	-4	-6	-10
$T, \bar{T}$	1	0	2/5
Φ	3	2	0

Table 2: GUT δb_i^a; $i = 1, 2, 3$, $a = V, T, \Phi$

The relation between the sum of the heavy contribute and the SUSY standard model b_i is;

$$b_i + \sum_a \delta b_i^a = b_G \tag{4.34}$$

This imply the curious fact that $b_3 = b_G$ in the minimal $SU(5)$ SUSY model.

These coefficients can be again employed, in the step approximation, in the same kind of analysis performed for the low energy SUSY thresholds. First we should define M_G;

we will identify M_G with the highest of the heavy masses, so that above it the unification is complete.

We could think to simply make use of eq. (4.24), with the changes $M_Z \longrightarrow M_G$, $M_i \longrightarrow M_i'$, $b_i^{SM} \longrightarrow b_i$ and $b_i \longrightarrow b_G$:

$$\Delta_i^{high} = \tfrac{1}{2\pi} \cdot (b_G - b_i) \cdot \ln\left(\tfrac{M_i'}{M_G}\right) \tag{4.35}$$

$$M_i' = \prod_{a=1}^{n} (M_a)^{\omega_i^a} \qquad \omega_i^a = \frac{\delta b_i^a}{\sum_{a=heavy} \delta b_i^a} \tag{4.36}$$

Unfortunately we cannot use eq. (4.36) since $\sum_{a=heavy} \delta b_3^a = 0$; nonetheless we could use these formulae excluding for instance the vectors V from the sums, as discussed in the comment following eq. (4.23). In this case it is probably simpler to use directly the three masses M_V, M_T, M_Φ, keeping in mind eq. (4.34); in fact the low energy parameters M_i' are to be eventually connected to the soft breaking masses anyhow, while the M_a are directly the fundamental parameters. We then have:

$$\Delta_i^{high} = \sum_{a=heavy} \Delta_i^a$$

$$\Delta_i^a = \frac{1}{2\pi} \cdot \delta b_i^a \cdot \ln\left(\frac{M_a}{M_G}\right) \tag{4.37}$$

Let us finally discuss an interesting physical issue related to the triplet and to the proton lifetime. If its major role would be to mediate proton decay via a tree-level diagram, in much the same way as the IVB at the GUT scale do, the bound on its mass from the present bounds on the proton lifetime ($\tau_P > 10^{31}$ years, [30]) would be $M_T \gtrsim 10^{11}$ GeV (the diagram is suppressed with respect to gauge boson exchange by the Yukawa couplings of light particles); but the triplet can induce proton decay at the 1-loop level (via dimension five operators) with third-family sparticles running in the loop and therefore with big Yukawa couplings. While the effective dimension 6 operators exhibit a quadratic suppression in the mass of the heavy particle exchanged, the one loop contributions are only linearly suppressed and therefore, depending on the details of the light SUSY particle spectrum, they may be competitive with the former or even be the dominant contribution. In the literature the bound $M_T \gtrsim 3 \cdot 10^{16}$ GeV is sometimes used [22], but it involves assumptions on the low-energy spectrum, that are sensitively model dependent.

4.2. 2-loop SUSY $SU(5)$ predictions

In this part we will show how the results of the previous analysis on the influence of thresholds and of two-loop effects on the running modify the 1-loop predictions; the thresholds are a source of uncertainty, while the two-loop terms are to be treated as corrections terms. We will also specify the meaning of the widely used SUSY threshold parameter, namely M_{SUSY}, and finally give the numerical values of the two-loop SUSY GUT predictions.

4.2.1. *Evaluation of the two-loop corrections*

Let us show the way in which the two-loop term θ_i, defined in eq. (4.9) can be used to improve the 1-loop predictions (this part of the analysis had already been performed by M.B. Einhorn and D.R.T. Jones in 1982 [31]); the procedure we will describe for θ_i must be repeated independently for each of the correction terms $-\Delta_i$ of eq. (4.1), given in in eq. (4.17), eq. (4.24) and eq. (4.37), so that the equations (4.42) and (4.43) that we obtain in the present section hold for Δ_i as well, with the replacement $\theta_i \to -\Delta_i$.

In *scheme b* (see the section 3.1.4: *Formulae for the 1-loop predictions*) we consider a linear system

$$M\ x = c \tag{4.38}$$

where the inputs quantities are contained in the vector c:

$$c = (\alpha_1^{-1}, \alpha_2^{-1}, 0) \tag{4.39}$$

while the output vector x is

$$x = (\alpha_G^{-1}, t, \alpha_3^{-1}) \tag{4.40}$$

In the present case, the modifications in the running can be seen as a change in the input quantities; we have the equation:

$$x = M^{-1}(c + \theta) = M^{-1}c + M^{-1}\theta \tag{4.41}$$

the last term gives the corrections δx to the predicted quantities.

The formulae we obtain in this case (cfr. with eqs. (3.17), (3.18), and recall that

$d = (b_1 - b_3) + 3/5\ (b_2 - b_3))$ are:

$$\begin{cases} \delta\left(\frac{1}{\alpha_G}\right) & = & -\frac{1}{d}\left(b_1 + \frac{3}{5}b_2)\theta_3 - b_3(\theta_1 + \frac{3}{5}\theta_2)\right) \\ \delta(t) & = & -\frac{1}{d}\left(\theta_1 + \frac{3}{5}\theta_2 - \frac{8}{5}\theta_3\right) \\ \delta\frac{(\sin^2\theta_W)}{\alpha} & = & -\frac{1}{d}((b_2 - b_3)\theta_1 + (b_3 - b_1)\theta_2 + (b_1 - b_2)\theta_3) \end{cases} \tag{4.42}$$

$$\begin{cases} \delta\left(\frac{1}{\alpha_G}\right) & = & \frac{b_2\theta_1 - b_1\theta_2}{b_1 - b_2} \\ \delta(t) & = & \frac{\theta_2 - \theta_1}{b_1 - b_2} \\ \delta\left(\frac{1}{\alpha_3}\right) & = & \frac{1}{b_1 - b_2}((b_2 - b_3)\theta_1 + (b_3 - b_1)\theta_2 + (b_1 - b_2)\theta_3) \end{cases} \tag{4.43}$$

In solving these equations the quantities θ_i should be treated a little bit differently from the Δ_i, because of the functional dipendence of θ_i on the output parameter; that is eq. (4.41) should read:

$$x = M^{-1}(c + \theta(x)) \tag{4.44}$$

We can solve this equation using the following iterative procedure:

$$\begin{cases} x_0 & = & M^{-1}c \\ \epsilon_1 & = & M^{-1}\theta(x_0) \\ x_1 & = & x_0 + \epsilon_1 \\ \ldots & = & \ldots\ldots\ldots \\ \epsilon_{n+1} & = & M^{-1}\theta(x_n) \\ x_{n+1} & = & x_0 + \epsilon_{n+1} \end{cases} \tag{4.45}$$

we expect (and find) that the procedure is convergent, since $c \gg \theta$; (=the two loop effects are small). This procedure must be used if we consider accurate RGE solutions.

4.2.2. SUSY threshold

We will employ the effective mass parameters M_i to describe the modifications on the running due to the thresholds. We notice that the supersymmetric threshold are due to scalars and fermions only, so that all the individual masses contribute to the geometric average M_i with non-negative exponents (see eq. (4.23)); therefore the values of the masses M_i give us some indication about the mass distribution of the supersymmetric particles. As we will see we may relate M_i to the more common (although quite approximate) parameter M_{SUSY}. In this manner we hope to gain some insights on the physical significance of M_{SUSY}.

By making the replacements $\theta_i \to -\Delta_i$, as given in eq. (4.24), in the last equation of the systems (4.42) and (4.43) we see that the relevant combination of masses in both schemes is given by:

$$\sum_i d_i \frac{1}{2\pi} \ln(M_i/M_Z) \tag{4.46}$$

where (recall that the coefficients b_i are the supersymmetric ones)

$$\begin{aligned} d_1 &= (b_2 - b_3)(b_1 - b_1^{SM}) \\ d_2 &= (b_3 - b_1)(b_2 - b_2^{SM}) \\ d_3 &= (b_1 - b_2)(b_3 - b_3^{SM}) \end{aligned} \tag{4.47}$$

We may define M_{SUSY} by:

$$\sum_i d_i \ \ln(M_i/M_Z) = (\sum_i d_i) \ \ln(M_{SUSY}/M_Z) \tag{4.48}$$

that is equivalent to

$$M_{SUSY} = \prod_i (M_i)^{x_i} \qquad x_i = \frac{d_i}{\sum_i d_i} \tag{4.49}$$

This parameter enters eq. (4.42) and eq. (4.43) in the following way:

$$\frac{\delta(\sin^2\theta_W)}{\alpha} = \frac{1}{2\pi}\frac{\sum_i d_i}{d} \ln\left(\frac{M_{SUSY}}{M_Z}\right) \tag{4.50}$$

$$\delta\left(\frac{1}{\alpha_3}\right) = \frac{1}{2\pi}\frac{\sum_i d_i}{(b_2 - b_1)} \ln\left(\frac{M_{SUSY}}{M_Z}\right) \tag{4.51}$$

If we use the coefficients b_i in Appendix B, we can see that with 1 light (=standard) Higgs doublet and 2 heavy (=SUSY) Higgs doublets eq. (4.49) read:

$$M_{SUSY} = \frac{(M_2)^{\frac{100}{19}}}{(M_1)^{\frac{25}{19}} \cdot (M_3)^{\frac{56}{19}}} \tag{4.52}$$

this formula implies the striking conclusion that the value of the parameter M_{SUSY} is not necessarily of the same order of most of the mass spectrum. As an example, we can have $M_{SUSY} = 10\ GeV$, although having $M_1 = M_Z, M_2 = 2M_Z, M_3 = 664\ GeV$.

We therefore caution the reader against straightforward interpretations of many of the analysis that have appeared in the last years.

4.2.3. GUT thresholds

Similar conclusions are reached when we use eq. (4.37). Let me show first what kind of difficulties one encounters when trying to keep a strict analogy with discussion in the previous section. Referring to eqs. (4.35) and (4.36) we have:

$$\begin{aligned} d'_1 &= (b_2 - b_3)(b_G - b_1) \\ d'_2 &= (b_3 - b_1)(b_G - b_2) \\ d'_3 &= (b_1 - b_2)(b_G - b_3) \end{aligned} \tag{4.53}$$

However we *cannot* define

$$\sum_i d'_i \ \ln(M'_i/M_G) = (\sum_i d'_i) \ \ln(M_{SUSY-GUT}/M_G) \tag{4.54}$$

that is equivalent to

$$M_{SUSY-GUT} = \prod_i (M'_i)^{x'_i} \qquad x'_i = \frac{d'_i}{\sum_i d'_i} \tag{4.55}$$

because it is simple to show that, in all generality, the sum is zero. If we notice that the factor $\sum_i d'_i$ rules the dependence of $\sin^2\theta_W$ on M_G we realize that the reason is that $\sin^2\theta_W$ is a quantity independent of M_G. This mass scale was defined to be the largest physical mass, but it can be chosen larger without inconsistencies and this choice is irrelevant for the value of $\sin^2\theta_W$ that we predict.

Using eq. (4.37) we see that the contribution from the high energy thresholds to the predictions is:

$$\left.\begin{aligned} \frac{\delta(\sin^2\theta_W)}{\alpha} &= \frac{1}{2\pi}\frac{1}{d} \\ \delta\left(\frac{1}{\alpha_3}\right) &= \frac{1}{2\pi}\frac{1}{(b_2-b_1)} \end{aligned}\right\} \sum_{a=heavy} [\delta b_1^a \ (b_2 - b_3)\ln(\frac{M_a}{M_G})\text{+cyclic perm.}] \tag{4.56}$$

The common term can be recast in the form:

$$\ln\left(\prod_{a=heavy} (M_a^{\sigma_a})\right) \qquad \sigma_a = \delta b_1^a \ (b_2 - b_3) + \text{cyclic perm.} \tag{4.57}$$

where the coefficients σ_a sum up to zero because of the independence from M_G of $\sin^2\theta_W$ (or of $1/\alpha_3$) as we have discussed in the previous paragraph. Using Appendix B and the values of δb_i^a in eq. (2) we give explicitly this common term in the minimal Georgi $SU(5)$ SUSY model:

$$\frac{12}{5}\ln\left(\frac{M_T^3}{M_V^2 M_\Phi}\right) \tag{4.58}$$

The equivalence of the role of the high energy thresholds and of the low energy thresholds as sources of uncertainty in the predictions of $\sin^2\theta_W$ (or α_3 in *scheme b*) in SUSY GUT has been pointed by R.Barbieri and L.Hall [32]; in fact we get from the previous formulae in *scheme a*

$$\delta(\sin^2\theta_W) = \frac{\alpha}{60\pi}\,\delta_{thr} \tag{4.59}$$

where

$$\delta_{thr} = -19\ln\left(\frac{M_{SUSY}}{M_Z}\right) + 18\ln\left(\frac{M_T}{\sqrt[3]{M_V^2 M_\Phi}}\right) \tag{4.60}$$

whilst in *scheme b* we have that the 1-loop prediction $\alpha_3^{(1)}$ is modified by

$$\frac{1}{\alpha_3^{(2)}} = \frac{1}{\alpha_3^{(1)}} - \frac{1}{28\pi}\,\delta_{thr} \tag{4.61}$$

that implies, taking into account eq. (3.18)

$$\delta\alpha_3 = \frac{28\alpha^2}{(60\sin^2\theta_W - 12)^2\pi}\,\delta_{thr} \tag{4.62}$$

This observation is based simply on the fact that the size of the effects are numerically comparable, and explain why we cannot rely on the prediction of the supersymmetry scale done in the one-loop context using the experimental informations about the three standard model coupling constants.

In my opinion we have to wait for further experimental information about the supersymmetric parameters; subsequently, that is after having (eventually) quantified the SUSY threshold effects, we shall be able to obtain informations about the physics at unification scale using this kind of analysis.

4.2.4. Numerical predictions

Let us conclude this section giving the numerical results in the case of the Georgi $SU(5)$ SUSY model. The formulae we get collecting the 1-loop and the 2-loop results for the low-energy parameters $\sin^2\theta_W$ and α_3 are:

$$\begin{aligned}\sin^2\theta_W &= 0.2 + \frac{7}{15}\frac{\alpha}{\alpha_3}(1 \pm \delta\alpha_3 \pm \delta\alpha) \\ &\quad +0.0031 + \frac{\alpha}{60\pi}\,\Delta \end{aligned} \tag{4.63}$$

$$\begin{aligned}\alpha_3 &= \frac{7\alpha}{15\sin^2\theta_W - 3}(1 \pm \delta\alpha_3 \pm \delta\alpha) \\ &\quad +0.012 + \frac{28\alpha^2}{(60\sin^2\theta_W - 3)^2\pi}\,\Delta \end{aligned} \tag{4.64}$$

where in each case the first line stays for the 1-loop results; the first term in the second line is the effect of the "pure" two-loop term, and the other corrections have been summarized in Δ:

$$\Delta = 1 - 19\ln\left(\frac{M_{SUSY}}{M_Z}\right) + 18\ln\left(\frac{M_T}{\sqrt[3]{M_V^2 M_\Phi}}\right) + H\left(\frac{m_t}{\bar{m}_t}\right) \tag{4.65}$$

The unit constant is the effect of the conversion term of eq. (4.17), the rest are threshold effects; the function $H(x)$ includes the top effects:

$$H(x) = 47.3(x^2 - 1) + 7.60\ln x + 3.89 \tag{4.66}$$

We assume the top in the range $110 \div 160\ GeV$, that is in the range suggested by the electroweak precision measurements. Recall that we neglected the logarithmic corrections to $\sin^2\theta_W$ in eq. (4.26), since the quadratic corrections are leading. We now see that, in the range of top masses that we are considering, also the effect of the logarithmic contributions to α_3 and α we considered in eq. (4.29) is irrelevant with respect to the effect of the quadratic contributions. I believe that a reasonable estimate of the numerical impact of both logarithms in eq. (4.60) is a factor two, with sign $\pm$.

Using the experimental $\overline{MS}$ input

$$\frac{1}{\alpha(M_Z)} = 127.9 \pm 0.1 \tag{4.67}$$

together with (*scheme a*)

$$\alpha_3(M_Z) = 0.12 \pm 0.01 \tag{4.68}$$

or (*scheme b*)

$$\sin^2\theta_W(M_Z) = 0.2324 \pm 0.0003 \tag{4.69}$$

we conclude with the predictions (in *scheme a* or *scheme b* respectively)

$$\sin^2\theta_W = 0.2334 \pm 0.0025 + (\pm 0.0016 \pm 0.0006 \pm 0.0015) \tag{4.70}$$

$$\alpha_3 = 0.125 \pm 0.001 + (\pm 0.005 \pm 0.002 \pm 0.005) \tag{4.71}$$

where the central value is for $m_t = 138, M_{SUSY} = M_Z$ and no GUT threshold effects; the first uncertainty reflects the experimental uncertainty on α_3 (resp. on $\sin^2\theta_W$ in *scheme b*). Those in brackets are the theoretical uncertainties: low energy thresholds (sparticles+top) and GUT scale thresholds. The accord with the experimental determinations of $\sin^2\theta_W$ (α_3 respectively) is fairly good and may be interpreted as a success of SUSY grand unification.

In *scheme a* the total theoretical uncertainty, found adding (not in quadrature) the three bracketed terms, is of the same order of magnitude of the uncertainty due to the experimental determination of α_3. We reach the same conclusions in *scheme b* from a slightly different point of view; the uncertainty in the prediction due to experimental uncertainties is noticeably lower than the theoretical uncertainties, as promised; by converse the experimental determination of α_3 is comparable with the theoretical uncertainties. This means that we need both better experimental inputs (on α_3) and theoretical improvements to have a finer test of the SUSY $SU(5)$ theory.

Let us discuss finally the "trends" of the predictions, after observing that the central values predicted are somewhat higher then the central values of the experimentally measured quantities (even if one must stress again that the predictions of the coupling constant unification are in good agreement with the present data). The predicted values of $\sin^2\theta_W$ in *scheme a* (or α_3 in *scheme b*) decrease if *i)* $M_{SUSY} > M_Z$, *ii)* $m_t < 138\ GeV$, *iii)* $M_T < \sqrt[3]{M_V^2 M_\Phi}$. The last relation involves the mass of the Higgs triplet, which may play a crucial role for proton decay in SUSY GUT, through effective dimension 5 operators (see the discussion at the end of sect. 4.1), if lighter then $10^{16}\ GeV$. The second feature is at present the most interesting one, in view of the possibility of a forthcoming experimental discovery of the top at CDF (Fermilab).

Concerning the low energy supersymmetric spectrum we may hope that it is not dramatically larger than the Fermi scale, and that we may be able to detect it at the new generation of the high-energy colliders. In this case we would be able to test GUT scale physics, having quantifyied the SUSY thresholds effects. In order to perform a detailed study of the low energy SUSY spectrum in a given model we need the complete set of RGE for a SUSY-GUT theory. A convenient way to derive them, which uses the one-loop effective potential method, is presented in Appendix C. A complete listing of the RGE in the most general form is given.

APPENDICES

A. Conventions

Invariant antisymmetric tensors in $SL(2,C)$:

$$\begin{aligned} \epsilon^{12} &= -\epsilon_{12} = 1 \\ \epsilon^{\dot{1}\dot{2}} &= -\epsilon_{\dot{1}\dot{2}} = 1 \end{aligned} \tag{A.1}$$

Bispinorial representations of Lorentz group ($A, B = 1,2$ are $SL(2,C)$ indices):

$$\begin{aligned} \chi_A &= \epsilon_{AB} \cdot \chi^B \\ \chi^A &= \epsilon^{AB} \cdot \chi_B \\ \chi_{\dot{A}} &= \epsilon_{\dot{A}\dot{B}} \cdot \chi^{\dot{B}} \\ \chi^{\dot{A}} &= \epsilon^{\dot{A}\dot{B}} \cdot \chi_{\dot{B}} \end{aligned} \tag{A.2}$$

Conjugation on bispinors:

$$\begin{aligned} \overline{\chi^A} &= \bar{\chi}^{\dot{A}} \\ \overline{\chi_A} &= \bar{\chi}_{\dot{A}} \end{aligned} \tag{A.3}$$

Scalars in the tensor product:

$$\begin{aligned} \theta\psi &= \theta^A \psi_A \\ \bar{\theta}\bar{\psi} &= \bar{\theta}_{\dot{A}} \bar{\psi}^{\dot{A}} \end{aligned} \tag{A.4}$$

Fierzing:

$$\begin{aligned} \theta^A \theta^B &= -1/2 \cdot \theta^2 \\ (\theta\psi)(\theta\phi) &= -1/2 \cdot \theta^2 \cdot (\psi\phi) \end{aligned} \tag{A.5}$$

Vectors in the tensor product:

$$\begin{aligned} \sigma^\mu_{A\dot{B}} &= (1, \sigma)_{A\dot{B}} \\ \sigma_{\mu A\dot{B}} &= (1, -\sigma)_{A\dot{B}} \\ \bar{\sigma}^{\mu \dot{A}B} &= (1, -\sigma)^{\dot{A}B} \\ \bar{\sigma}_\mu^{\dot{A}B} &= (1, \sigma)^{\dot{A}B} \end{aligned} \tag{A.6}$$

Useful relations

$$\begin{aligned} \psi\theta &= \theta\psi \\ \bar{\psi}\bar{\theta} &= \bar{\theta}\bar{\psi} \\ \bar{\psi}\bar{\sigma}^\mu\phi &= -\phi\sigma^\mu\bar{\psi} \end{aligned} \tag{A.7}$$

Conjugation on bilinears:

$$\begin{aligned} \overline{\theta^A \psi^B} &= \bar{\psi}^{\dot{B}} \bar{\theta}^{\dot{A}} \\ \overline{\theta\psi} &= \bar{\theta}\bar{\psi} \\ \overline{\theta\sigma_\mu\bar{\psi}} &= \psi\sigma_\mu\bar{\theta} \end{aligned} \tag{A.8}$$

Gauge transformations (latin indices are gauge indices; ϵ is real):

$$\begin{aligned} \delta\psi^a &= i[\epsilon_\alpha(+T^\alpha)]^{ab}\psi^b \\ \delta\bar{\psi}^a &= i[\epsilon_\alpha(-\bar{T}^\alpha)]^{ab}\bar{\psi}^b \end{aligned} \tag{A.9}$$

Covariant derivatives:

$$\mathcal{D}_\mu = \partial_\mu + igV_\mu \tag{A.10}$$

Gauge lagrangians for the bispinors:

$$\begin{aligned} i\psi\hat{\mathcal{D}}\bar{\psi} &= i\psi\sigma^\mu\mathcal{D}_\mu\bar{\psi} \\ i\bar{\psi}\hat{\mathcal{D}}\psi &= i\bar{\psi}\bar{\sigma}^\mu\mathcal{D}_\mu\psi \end{aligned} \tag{A.11}$$

Four component spinors (greek indices are Lorentz indices):

$$\Psi^D_\alpha = \begin{pmatrix} \lambda_A \\ \bar{\chi}^{\dot{A}} \end{pmatrix} \tag{A.12}$$

$$\bar{\Psi}^D_\alpha = (\chi^A, \bar{\lambda}_{\dot{A}}) \tag{A.13}$$

Gamma matrices and bispinors:

$$\gamma^\mu_{\alpha\beta} = \begin{pmatrix} 0_A{}^B & \sigma^\mu{}_{A\dot{B}} \\ \bar{\sigma}^{\mu\dot{A}B} & 0^{\dot{A}}{}_{\dot{B}} \end{pmatrix} \tag{A.14}$$

Chirality matrix:

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \tag{A.15}$$

Chirality projectors:

$$\begin{aligned} P_L &= \tfrac{1-\gamma^5}{2} \\ P_R &= \tfrac{1+\gamma^5}{2} \end{aligned} \tag{A.16}$$

Relations between some "Weyl" bilinears and the "Dirac" bilinears:

$$\begin{aligned} \bar{\Psi}_D\gamma^\mu\Psi'_D &= \bar{\lambda}\bar{\sigma}^\mu\lambda' + \chi\sigma\bar{\chi}' \\ \bar{\Psi}_D P_L\Psi'_D &= \chi\lambda' \\ \bar{\Psi}_D P_R\Psi'_D &= \bar{\chi}\bar{\lambda}' \end{aligned} \tag{A.17}$$

Conjugation matrix:

$$C_{\alpha\beta} = \begin{pmatrix} \epsilon_{AB} & 0 \\ 0 & \epsilon^{\dot{A}\dot{B}} \end{pmatrix} \tag{A.18}$$

Majorana spinors:

$$C_{\alpha\beta}\ \bar{\Psi}^M_\beta = \Psi^M_\alpha \tag{A.19}$$

$$\Psi^M_\alpha = \begin{pmatrix} \chi_A \\ \bar{\chi}^{\dot{A}} \end{pmatrix} \tag{A.20}$$

B. Beta function coefficients

We list in this appendix the beta function coefficients relative to the gauge sector in the SM, in its supersymmetric extension, and some related quantities; the two-loop coefficients are taken from ref. [33].

Coefficients of the SM **1-loop** running, assuming N_g families and n_l light Higgs doublets (in square brackets the case $N_g = 3, n_l = 1$):

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -22/3 \\ -11 \end{pmatrix} + N_g \cdot \begin{pmatrix} 4/3 \\ 4/3 \\ 4/3 \end{pmatrix} + n_l \cdot \begin{pmatrix} 1/10 \\ 1/6 \\ 0 \end{pmatrix} \quad \left[= \begin{pmatrix} 41/10 \\ -19/6 \\ -7 \end{pmatrix} \right] \tag{B.1}$$

List of b-differences and the factor d (in square brackets the case $N_g = 3, n_l = 1$):

$$\begin{pmatrix} b_2 - b_3 \\ b_3 - b_1 \\ b_1 - b_2 \end{pmatrix} = \begin{pmatrix} 11/3 \\ -11 \\ 22/3 \end{pmatrix} + n_l \cdot \begin{pmatrix} 1/6 \\ -1/10 \\ -1/15 \end{pmatrix} \quad \left[= \begin{pmatrix} 23/6 \\ -111/10 \\ 109/15 \end{pmatrix} \right] \tag{B.2}$$

$$d = (b_1 - b_3) + \frac{3}{5}(b_2 - b_3) = \frac{3}{5}(22 + n_l/3) \quad [= 67/5] \tag{B.3}$$

Coefficients of the SM **2-loop** running, assuming N_g families and n_l light Higgs doublets (in square brackets the case $N_g = 3, n_l = 1$):

$$\begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -136/3 & 0 \\ 0 & 0 & -102 \end{pmatrix} + N_g \cdot \begin{pmatrix} 19/15 & 3/5 & 44/15 \\ 1/5 & 49/3 & 4 \\ 11/30 & 3/2 & 76/3 \end{pmatrix}$$
$$+ \; n_l \cdot \begin{pmatrix} 9/50 & 9/10 & 0 \\ 3/10 & 13/6 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \left[= \begin{pmatrix} 3.98 & 2.7 & 8.8 \\ 0.9 & 35/6 & 12 \\ 1.1 & 4.5 & -26 \end{pmatrix} \right] \tag{B.4}$$

Coefficients of the **SUSY SM 1-loop** running, assuming N_g families and n_d Higgs doublets; in n_d there are light and heavy doublets; $n_d = n_l + n_h$ (in square brackets the case $N_g = 3, n_d = 2$):

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + N_g \cdot \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} + 3 \cdot n_d \cdot \begin{pmatrix} 1/10 \\ 1/6 \\ 0 \end{pmatrix} \left[= \begin{pmatrix} 66/10 \\ 1 \\ -3 \end{pmatrix} \right] \tag{B.5}$$

List of b-differences and the factor d (in square brackets the case $N_g = 3, n_d = 2$):

$$\begin{pmatrix} b_2 - b_3 \\ b_3 - b_1 \\ b_1 - b_2 \end{pmatrix} = \begin{pmatrix} 3 \\ -9 \\ 6 \end{pmatrix} + 3 \cdot n_d \cdot \begin{pmatrix} 1/6 \\ -1/10 \\ -1/15 \end{pmatrix} \quad \left[= \begin{pmatrix} 4 \\ -48/5 \\ 28/5 \end{pmatrix} \right] \tag{B.6}$$

$$d = (b_1 - b_3) + \frac{3}{5}(b_2 - b_3) = \frac{3}{5}(18 + n_d) \quad [= 60/5] \tag{B.7}$$

Coefficients of the **SUSY SM 2-loop** running, assuming N_g families and n_d Higgs doublets (in square brackets the case $N_g = 3, n_d = 2$):

$$\begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -24 & 0 \\ 0 & 0 & -54 \end{pmatrix} + N_g \cdot \begin{pmatrix} 38/15 & 6/5 & 88/15 \\ 2/5 & 14 & 8 \\ 22/30 & 6/2 & 68/3 \end{pmatrix}$$

$$+ \; n_d \cdot \begin{pmatrix} 9/50 & 9/10 & 0 \\ 3/10 & 7/2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \left[= \begin{pmatrix} 7.96 & 5.4 & 17.6 \\ 1.8 & 25 & 24 \\ 2.2 & 9 & 14 \end{pmatrix} \right] \tag{B.8}$$

List of coefficients d_i of eq. (4.48), in the case $N_g = 3, n_d = 2, n_l = 1$:

$$\begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} = \frac{2}{5} \begin{pmatrix} 25 \\ -100 \\ 56 \end{pmatrix} \tag{B.9}$$

List of coefficients σ_a of eq. (4.57), in the case $N_g = 3, n_d = 2$ (minimal SUSY $SU(5)$):

$$\begin{pmatrix} \sigma_T \\ \sigma_V \\ \sigma_\Phi \end{pmatrix} = \frac{12}{5} \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix} \tag{B.10}$$

C. RGE for Softly Broken SUSY Gauge theories

For any renormalisable Yang-Mill theory with scalars, Weyl fermions and gauge vectors (not necessarily supersymmetric) we find, regularizing the theory with a momentum cutoff Λ, the following "structure of infinities" in the scalar sector at 1-loop[34] (using the Landau gauge)

$$V_\infty = \frac{1}{32\pi^2}\,\mathrm{Str}\left[\Lambda^4\left(\ln\Lambda - \frac{1}{4}\right) + \Lambda^2 M^2(z) - \ln\Lambda\; M^4(z)\right] \tag{C.1}$$

This formula allow us to extract a lot of informations about the dependence of the bare parameters of the scalar sector on the renormalised parameters, by performing elementary (but for the general case very long) algebraic computations of supertraces of mass matrices, without computing Feynman diagrams; in fact one lack only the informations about the wavefunction renormalisation. For the purpose of obtaining 1-loop the RGEs the best way to use this formula is the following. Consistently with the order of approximation used we write $V_\infty(z_R, ...g_R) = V_\infty(z_B, ...g_B)$, in terms of the bare and of the renormalised quantities respectively. Then we solve for the renormalised parameters the following equation

$$V(z^a_R, l^a_R, ...g^\alpha_R) = V(z^a_B, l^a_B, ...g^\alpha_B) + V_\infty(z^a_B, l^a_B, ...g^\alpha_B) \tag{C.2}$$

We will use an hat $(\hat{\ })$ to denote the renormalised quantities.

This method is particularly suited for the supersymmetric case, [35] since many parameters of the fermionic sector appear also as parameters of the scalar sector (for instance the Yukawa couplings appear in 4-scalar interactions). In fact, we can obtain all the SUSY RGE with this method (except for the gaugino masses) just feeding some information about the field anomalous dimensions.

When applying eq. (C.2) to SUSY theories, the fact that we have the same number of fermionic and of bosonic degrees of freedom implies that we have no quartic constant divergencies; that is the 0-point Green function, to be interpreted as a quantum-induced cosmological constant, is at most quadratically divergent. The tree level sum-rule of eq. (2.44) acquires in this context a very interesting meaning: any supersymmetric theory is *free from quadratic divergencies* [36] in the field dependent terms.

C.1. RGE for the Wess-Zumino model

Let us illustrate how the method works for a simple supersymmetric model, the Wess-Zumino model with N-chiral superfield (no vector superfield). We can compute the supertrace using the mass matrices of eq. (2.41) and eq. (2.40), in which we set to zero the gauge and the soft breaking parameters.

Referring to the superpotential f given in eq. (2.20) the result of the computation is:

$$V_\infty = -2k\ f_a X^a_b f^b \tag{C.3}$$

where I used $k = \ln \Lambda / 32\pi^2$, and defined the constants X as

$$X^a_b = f^{acd} f_{bcd}\ . \tag{C.4}$$

In this case eq. (C.2) reads,

$$\hat{f}_a \hat{f}^a = f_a f^a - 2k f^a X^b_a f_b + O(X^2) \tag{C.5}$$

which is solved by

$$\hat{f}_a = f_a - k X^b_a f_b + O(X^2) \tag{C.6}$$

that is, neglecting the $O(X^2)$ terms

$$\begin{cases} \hat{l}_a & = \quad l_a - k X^b_a l_b \\ \hat{\mu}_{ab} \hat{z}^b & = \quad \mu_{ab} z^b - k X^b_a \mu_{bc} z^c \\ \hat{f}_{abc} \hat{z}^b \hat{z}^c & = \quad f_{abc} z^b z^c - k X^b_a f_{bcd} z^c z^d\ . \end{cases} \tag{C.7}$$

At this point one needs some further information to solve without ambiguity for the renormalised parameters. We can prove, just looking at the relevant Feynman graphs, that the anomalous dimension of the scalar fields depends on f_{abc} only through X^b_a; more explicitly

$$\hat{z}^a = (\delta^a_{a'} + \alpha k X^a_{a'}) z^{a'} \tag{C.8}$$

where α is an unknown constant. With this supplementary information we get from eq. (C.7) the following equation for the parameter μ_{ab}:

$$\hat{\mu}_{ac} z^c = \mu_{ac} z^c - k X^b_a \mu_{bc} z^c - \alpha k \mu_{ab} X^b_c z^c \tag{C.9}$$

Using the symmetry $\mu_{ab} = \mu_{ba}$ we conclude that $\alpha = 1$. The same procedure for f_{abc} allow us to obtain

$$\left\{ \begin{array}{lcl} \hat{l}_a & = & (\delta_a^{a'} - kX_a^{a'})\; l_{a'} \\ \hat{\mu}_{ab} & = & (\delta_a^{a'} - kX_a^{a'})(\delta_b^{b'} - kX_b^{b'})\; \mu_{a'b'} \\ \hat{f}_{abc} & = & (\delta_a^{a'} - kX_a^{a'})(\delta_b^{b'} - kX_b^{b'})(\delta_c^{c'} - kX_c^{c'})\; f_{a'b'c'} \\ \hat{z}^a & = & (\delta_{a'}^a + kX_{a'}^a)\; z^{a'} \end{array} \right. \tag{C.10}$$

This equations are known as *non-renormalisation* of the superpotential, because they imply the equation $\hat{f}(\hat{z}) = f(z)$; we see that, in a sense, we have only the (logarithmic) wavefunction renormalisation.

We get the corresponding RGE deriving with respect to the logarithmic "divergence" factor $-2k$, that is with respect to the momentum scale $t = \frac{1}{(4\pi)^2} \ln Q$, ($Q$ appears implicitly in the logarithm together with Λ: $\ln(\Lambda/Q)$)

$$\frac{d}{dt} l_a = \frac{1}{2}\{X_a^{a'} l_{a'}\} \tag{C.11}$$

$$\frac{d}{dt} \mu_{ab} = \frac{1}{2}\{X_a^{a'} \mu_{a'b} + X_b^{b'} \mu_{ab'}\} \tag{C.12}$$

$$\frac{d}{dt} f_{abc} = \frac{1}{2}\{X_a^{a'} f_{a'bc} + X_b^{b'} \mu_{ab'c} + X_c^{c'} \mu_{abc'}\} \tag{C.13}$$

Notice that we use from now on just renormalised parameters; therefore we do not write the hats any more.

C.2. RGE in the general case

Let us recall the relevant functions of the scalar fields z_a, members of the chiral supermultiplet Φ_a, that define a supersymmetric theory with soft breaking ($z^a = z_a^*$)

$$\begin{array}{lcl} D^\alpha(z^a, z_a) & = & g^\alpha z^a T_a^{\alpha b} z_b \\ f(z_a) & = & l^a z_a + \frac{1}{2}\mu^{ab} z_a z_b + \frac{1}{6} f^{abc} z_a z_b z_c \\ \eta(z) & = & L^a z_a + \frac{1}{2} M^{ab} z_a z_b + \frac{1}{6}\eta^{abc} z_a z_b z_c \end{array}$$

In terms of these functions the most general SUSY scalar potential containing also soft breaking terms is given by ($f_a = \frac{\partial f}{\partial z^a} = (f^a)^*$):

$$\begin{array}{lcl} V & = & V_{SUSY}(z) + V_{soft}(z) \\ V_{SUSY} & = & f^a f_a + \frac{1}{2} D^\alpha D^\alpha \\ V_{soft} & = & m_b^{2a} z^b z_a + (\eta + \text{h.c.}) \end{array} \tag{C.14}$$

It is good policy in the general case to organize the computation of the supertrace of $M^4(z)$ using the "modularity" of the mass matrices; *i.e.* computing separately the effect of *i*) the parameters in the superpotential – as we have done explicitely in the example of the Wess-Zumino model –, *ii*) the gauge parameters, *iii*) the soft breaking parameters and *iv*) the "interference" terms (for instance those in which the gauge and the soft breaking parameters appear). The relevant mass matrices are listed in eq. (2.39), eq. (2.40) and eq. (2.41). In order to simplify the calculation it is very useful to take advantage also of the gauge covariance of the various terms in the scalar potential, as in the following example:

$$\delta_\alpha f(z) = \frac{\partial f}{\partial z^a}\delta_\alpha z = \frac{\partial f}{\partial z^a}T^{\alpha a}_b z^b = 0 \qquad \Rightarrow \qquad f_a D^{\alpha a} = 0 \tag{C.15}$$

Moreover, turning to the solution of eq. (C.2), it is convenient to compute first the effect of the supersymmetric parameters alone (gauge couplings+parameters in the superpotential), solving for the supersymmetric renormalised parameters and then compute the effect of the soft breaking parameters:

$$\hat{V}_{SUSY}(\hat{z}) = V_{SUSY} - k\,\mathrm{Str}\,M^4_{SUSY} \tag{C.16}$$

$$\hat{V}_{soft}(\hat{z}) = V_{soft} - k(\mathrm{Str}\,M^4 - \mathrm{Str}\,M^4_{SUSY}) \tag{C.17}$$

Notice that the soft breaking parameters are all massive; therefore they cannot change the renormalisation of z_a, g_α, f_{abc}; one can also prove that they cannot change the renormalisation of μ_{ab}; this implies that the previous splitting is correct when solving for these parameters. On the other hand it is worth mentioning that l_a plays only a formal role if the soft breaking parameters L_a and M_{ab} are also present. In fact, as one can prove by expanding in components eq. (C.14), l_a appears always in the combinations $C_a \equiv L_a + \mu_{ab}l^b$, $C_{ab} \equiv M_{ab} + f_{abc}l^c$ in the the scalar potential. This means that when these parameters are all present at once the splitting $V_{SUSY} - V_{soft}$ is somewhat arbitrary. However, since l_a, L_a and M_{ab} appear linearly in C_a and C_{ab} the superposition of the RGE's that we separately obtain from eq. (C.16) ($\rightarrow l_a$) and eq. (C.17) ($\rightarrow L_a, M_{ab}$) reproduce correctely and unambiguosly the RGE we would obtain for C_a and C_{ab} from eq. (C.2).

Let us finally list the RGE for the general SUSY theory with soft breakings. The variable t is defined in accord with eq. (C.34).

$$\frac{d}{dt}l_a = \frac{1}{2}X^{a'}_a l_{a'} \tag{C.18}$$

$$\frac{d}{dt}\mu_{ab} = \frac{1}{2}\{X^{a'}_a\mu_{a'b} + X^{b'}_b\mu_{ab'} - 8g^2_\alpha C_\alpha(a)\mu_{ab}\} \tag{C.19}$$

$$\frac{d}{dt}f_{abc} = \frac{1}{2}\{X_a^{a'}f_{a'bc} + X_b^{b'}f_{ab'c} + X_c^{c'}f_{abc'}$$
$$-4g_\alpha^2[C_\alpha(a) + C_\alpha(b) + C_\alpha(c)]f_{abc}\} \tag{C.20}$$

$$\frac{d}{dt}L_a = \frac{1}{2}\{X_a^{a'}L_{a'} + 2\eta_{acd}f^{bcd}l_b + 4m^{2b}_{\ c}f_{abd}\mu^{cd}$$
$$+2\mu_{ab}f^{bcd}M_{cd} + 2\eta_{abc}M^{bc}\} \tag{C.21}$$

$$\frac{d}{dt}M_{ab} = \frac{1}{2}\{X_a^{a'}M_{a'b} + X_b^{b'}M_{ab'}$$
$$-8g_\alpha^2C_\alpha(a)M_{ab} + 16\mu_{ab}\mu_\alpha g_\alpha^2C_\alpha(a) +$$
$$+2(f_{abc}M_{de} + \mu_{ac}\eta_{bde} + \mu_{bc}\eta_{ade})f^{cde}\} \tag{C.22}$$

$$\frac{d}{dt}\eta_{abc} = \frac{1}{2}\{X_a^{a'}\eta_{a'bc} + X_b^{b'}\eta_{ab'c} + X_c^{c'}\eta_{abc'}$$
$$4g_\alpha^2(2\mu_\alpha f_{abc} - \eta_{abc})[C_\alpha(a) + C_\alpha(b) + C_\alpha(c)]$$
$$+2(f_{abf}\eta_{cde} + f_{acf}\eta_{bde} + f_{bcf}\eta_{ade})f^{def}\} \tag{C.23}$$

$$\frac{d}{dt}m^{2a}_{\ \ b} = \frac{1}{2}\{X_a^{a'}m^{2b}_{\ \ a'} + X_{b'}^{b}m^{2b'}_{\ \ a} + 4g_\alpha D^{\alpha b}_{\ \ a}\,\mathrm{Tr}(T^\alpha m^2)$$
$$+4m^{2d}_{\ \ c}f_{ade}f^{bce} + 2\eta_{acd}\eta^{bcd} - 16g_\alpha^2 \mid \mu_\alpha \mid^2 C_\alpha(a)\delta^b_a\} \tag{C.24}$$

$$\frac{d}{dt}g_\alpha = \frac{1}{2}\{T_2(\alpha) - 3C_\alpha(\mathrm{Adj})\}g_\alpha^3 \tag{C.25}$$

$$\frac{d}{dt}\mu_\alpha = \{T_2(\alpha) - 3C_\alpha(\mathrm{Adj})\}g_\alpha^2\mu_\alpha \tag{C.26}$$

The last equation represents the gaugino mass renormalisation, and must be obtained with a different method, for instance computing the relevant Feynman graphs. The Dynkin index $T_2(\alpha)$ is computed for the reducible representation that contains *all* the scalar fields of the theory; the eigenvalue of the Casimir operator $C_\alpha(a)$ (resp. $C_\alpha(\mathrm{Adj})$ is computed for the representation of the scalar fields z_a (resp. for the adjoint representation); the index α labels the group.

C.3. RGE in the SUSY SM

The previous set of RGE's can be applied to the case of the SUSY SM. We will identify the scalar fields in the SUSY SM using the index a of the multiplet z_a in the following way:

$$a \in \{H_1^\sigma, H_2^\sigma, L_i^\sigma, E_i^c, Q_i^{\alpha\sigma}, U_i^{c\alpha}, D_i^{c\alpha}\} \tag{C.27}$$

The field E_i^c (resp. $U_i^{c\alpha}; D_i^{c\alpha}$) is chosen to have charge $+1$ (resp $-2/3; +1/3$); see the discussion after eq. (2.4).

The function $f(z)$ for the SUSY SM is chosen as

$$f_{SM} = \epsilon_{\sigma\tau}[\Gamma^E_{ij}\ H^\sigma_1 L^\tau_i E^c_j + \Gamma^D_{ij}\ H^\sigma_1 Q^\tau_i D^c_j + \Gamma^U_{ij}\ H^\sigma_2 Q^\tau_i U^c_j + \mu\ H^\sigma_1 H^\tau_2] \tag{C.28}$$

where I have written only the $SU(2)_L$ indices. The soft terms contained in $\eta(z)$ are:

$$\eta_{SM} = \epsilon_{\sigma\tau}[\Gamma^E_{Aij}\ H^\sigma_1 L^\tau_i E^c_j + \Gamma^D_{Aij}\ H^\sigma_1 Q^\tau_i D^c_j + \Gamma^U_{Aij}\ H^\sigma_2 Q^\tau_i U^c_j + B\mu\ H^\sigma_1 H^\tau_2] \tag{C.29}$$

Considering the fact that soft breaking scalar mass terms have to be gauge invariant we write:

$$\begin{aligned} & m^2_{H_1}(H^\sigma_1)^* H^\sigma_1 + m^2_{H_2}(H^\sigma_2)^* H^\sigma_2 \\ + & \ m^2_{Lij}(L^\sigma_i)^* L^\sigma_j + m^2_{Qij}(Q^{\sigma\alpha}_i)^* Q^{\sigma\alpha}_j \\ + & \ m^2_{Eji}(E^c_i)^* E^c_j + m^2_{Uji}(U^{c\alpha}_i)^* U^{c\alpha}_j + m^2_{Dji}(D^{c\alpha}_i)^* D^{c\alpha}_j \end{aligned} \tag{C.30}$$

Notice that the fact that the mass matrices m^{2a}_b of the previous section commute with the gauge group generators does not implies that they are completely diagonal, since there are family replicas. I wrote m^2_{Eji} (resp. m^2_{Uji}, m^2_{Dji}), instead of m^2_{Eij} to refer to the mass matrix of the particle with charge -1** (resp. $+2/3, -1/3$).

By comparison with the notations of the previous section one writes for instance

$$\begin{aligned} f_{H^\sigma_1 Q^{\tau\alpha}_i D^{c\beta}_j} &= \Gamma^D_{ij}\ \epsilon_{\sigma\tau}\ \delta_{\alpha\beta} \\ \mu_{H^\sigma_1, H^\tau_2} &= \mu\ \epsilon_{\sigma\tau} \end{aligned} \tag{C.31}$$

The formulae for the terms in $\eta(z)$ are completely analogous. Some care is required when we consider the soft breaking scalar mass mass term; for instance

$$\begin{aligned} \{m^2\}^{E^c_j}_{E^c_i} &= m^2_{Eji} \\ \{m^2\}^{Q_{j\beta\tau}}_{Q_{i\alpha\sigma}} &= m^2_{Qij}\delta^\beta_\alpha \delta^\tau_\sigma \end{aligned} \tag{C.32}$$

The computation of the matrices X^a_b is a little more cumbersome; let us give an

**This point can be formulated more explicitly with the notation of eq. (2.4) and following, that is restoring the indices L, R: m^2_E is the mass matrix of E_R, that is $\mathcal{L} \ni E^*_R m^2_E E_R$. We choose to identify E^c_L as a component of the multiplet $\{z_a\}$; that is we are working with $E^c_L = E^*_R$; and $E^*_R m^2_E E_R = (E^c_L)^* (m^2_E)^T E^c_L$

example:

$$
\begin{aligned}
X_{H_1^\rho}^{H_1^\sigma} &= f^*_{H_1^\sigma, cd} f_{H_1^\sigma, cd} \\
&= f^*_{H_1^\sigma L_i^\tau E_j^c} f_{H_1^\rho L_i^\tau E_j^c} + f^*_{H_1^\sigma E_j^c L_i^\tau} f_{H_1^\rho E_j^c L_i^\tau} f^*_{H_1^\sigma Q_i^{\tau\alpha} D_j^{c\beta}} f_{H_1^\rho Q_i^{\tau\alpha} D_j^{c\beta}} + f^*_{H_1^\sigma D_j^{c\beta} Q_i^{\tau\alpha}} f_{H_1^\rho D_j^{c\beta} Q_i^{\tau\alpha}} \\
&= 2 f^*_{H_1^\sigma L_i^\tau E_j^c} f_{H_1^\rho L_i^\tau E_j^c} + 2 f^*_{H_1^\sigma Q_i^{\tau\alpha} D_j^{c\beta}} f_{H_1^\rho Q_i^{\tau\alpha} D_j^{c\beta}} \\
&= 2(\Gamma_{ij}^{E*} \epsilon_{\sigma\tau})(\Gamma_{ij}^{E} \epsilon_{\rho\tau}) + 2(\Gamma_{ij}^{D*} \epsilon_{\sigma\tau} \delta_\alpha^\beta)(\Gamma_{ij}^{D} \epsilon_{\rho\tau} \delta_\alpha^\beta) \\
&= 2 \,\mathrm{Tr}(\Gamma_E^\dagger \Gamma_E) \delta_\rho^\sigma + 2 \cdot 3 \,\mathrm{Tr}(\Gamma_D^\dagger \Gamma_D) \delta_\rho^\sigma
\end{aligned}
\tag{C.33}
$$

At this point we can apply the results of eqs. (C.18–C.26) to the model under consideration. We list in the following the whole set of the SUSY SM renormalisation group equations.

For notational convenience, the scale variable t in the following equations is defined as

$$t = \frac{1}{(4\pi)^2} \ln\left(\frac{Q}{Q_0}\right) \tag{C.34}$$

Yukawa couplings $\Gamma_E, \Gamma_U, \Gamma_D$ and μ-parameter:

$$\begin{aligned} \frac{d}{dt}\Gamma_E &= -3(g_2^2 + g_1^2)\Gamma_E \\ &\quad +3\Gamma_E\Gamma_E^\dagger\Gamma_E \\ &\quad + \mathrm{Tr}(\Gamma_E^\dagger\Gamma_E + 3\Gamma_D^\dagger\Gamma_D)\Gamma_E \end{aligned} \tag{C.35}$$

$$\begin{aligned} \frac{d}{dt}\Gamma_U &= -(\frac{16}{3}g_3^2 + 3g_2^2 + \frac{13}{9}g_1^2)\Gamma_U \\ &\quad +3\Gamma_U\Gamma_U^\dagger\Gamma_U + \Gamma_D\Gamma_D^\dagger\Gamma_D \\ &\quad +3\,\mathrm{Tr}(\Gamma_U^\dagger\Gamma_U)\Gamma_U \end{aligned} \tag{C.36}$$

$$\begin{aligned} \frac{d}{dt}\Gamma_D &= -(\frac{16}{3}g_3^2 + 3g_2^2 + \frac{7}{9}g_1^2)\Gamma_D \\ &\quad +3\Gamma_D\Gamma_D^\dagger\Gamma_D + \Gamma_U\Gamma_U^\dagger\Gamma_U \\ &\quad +3\,\mathrm{Tr}(\Gamma_D^\dagger\Gamma_D)\Gamma_D \end{aligned} \tag{C.37}$$

$$\begin{aligned} \frac{d}{dt}\mu &= [-(3g_2^2 + g_1^2) \\ &\quad + \mathrm{Tr}(\Gamma_E^\dagger\Gamma_E + 3\Gamma_U^\dagger\Gamma_U + 3\Gamma_D^\dagger\Gamma_D)]\mu \end{aligned} \tag{C.38}$$

Gauge coupling constants:

$$\frac{d}{dt}g_1 = [-9 + 2N_g]g_1^3 \tag{C.39}$$

$$\frac{d}{dt}g_2 = [-5 + 2N_g]g_2^3 \tag{C.40}$$

$$\frac{d}{dt}g_3 = [1 + \frac{10}{3}N_g]g_3^3 \tag{C.41}$$

Soft breaking parameters $\Gamma_E^A, \Gamma_U^A, \Gamma_D^A$ and B.

$$\begin{aligned}
\frac{d}{dt}\Gamma_E^A &= -3(g_2^2+g_1^2)\Gamma_E^A \\
&+2\cdot 3(g_2^2\mu_2+g_1^2\mu_1)\Gamma_E \\
&+5\Gamma_E\Gamma_E^\dagger\Gamma_E^A + 4\Gamma_E^A\Gamma_E^\dagger\Gamma_E \\
&+\mathrm{Tr}(\Gamma_E^\dagger\Gamma_E + 3\Gamma_D^\dagger\Gamma_D)\Gamma_E^A \\
&+2\,\mathrm{Tr}(\Gamma_E^\dagger\Gamma_E^A + 3\Gamma_D^\dagger\Gamma_D^A)\Gamma_E
\end{aligned} \tag{C.42}$$

$$\begin{aligned}
\frac{d}{dt}\Gamma_U^A &= -(\frac{16}{3}g_3^2+3g_2^2+\frac{13}{9}g_1^2)\Gamma_U^A \\
&+2(\frac{16}{3}g_3^2\mu_3+3g_2^2\mu_2+\frac{13}{9}g_1^2\mu_1)\Gamma_U \\
&+5\Gamma_U\Gamma_U^\dagger\Gamma_U^A + 4\Gamma_U^A\Gamma_U^\dagger\Gamma_U + 2\Gamma_D^A\Gamma_D^\dagger\Gamma_U + \Gamma_D\Gamma_D^\dagger\Gamma_U^A \\
&+3\,\mathrm{Tr}(\Gamma_U^\dagger\Gamma_U)\Gamma_U^A \\
&+2\cdot 3\,\mathrm{Tr}(\Gamma_U^\dagger\Gamma_U^A)\Gamma_U
\end{aligned} \tag{C.43}$$

$$\begin{aligned}
\frac{d}{dt}\Gamma_D^A &= -(\frac{16}{3}g_3^2+3g_2^2+\frac{7}{9}g_1^2)\Gamma_D^A \\
&+2(\frac{16}{3}g_3^2\mu_3+3g_2^2\mu_2+\frac{7}{9}g_1^2\mu_1)\Gamma_D \\
&+5\Gamma_D\Gamma_D^\dagger\Gamma_D^A + 4\Gamma_D^A\Gamma_D^\dagger\Gamma_D + 2\Gamma_U^A\Gamma_U^\dagger\Gamma_D + \Gamma_U\Gamma_U^\dagger\Gamma_D^A \\
&+\mathrm{Tr}(\Gamma_E^\dagger\Gamma_E + 3\,\mathrm{Tr}(\Gamma_D^\dagger\Gamma_D)\Gamma_D^A \\
&+2\,\mathrm{Tr}(\Gamma_E^\dagger\Gamma_E^A + 3\,\mathrm{Tr}(\Gamma_D^\dagger\Gamma_D^A)\Gamma_D
\end{aligned} \tag{C.44}$$

$$\begin{aligned}
\frac{d}{dt}B &= 2[3g_2^2\mu_2+g_1^2\mu_1 \\
&+\mathrm{Tr}(\Gamma_E^\dagger\Gamma_E^A + 3\Gamma_U^\dagger\Gamma_U^A + 3\Gamma_D^\dagger\Gamma_D^A)]
\end{aligned} \tag{C.45}$$

Gaugino masses:

$$\frac{d}{dt}\mu_1 = [-9+2N_g]g_1^2\mu_1 \tag{C.46}$$

$$\frac{d}{dt}\mu_2 = [-5+2N_g]g_2^2\mu_2 \tag{C.47}$$

$$\frac{d}{dt}\mu_3 = [1+\frac{10}{3}N_g]g_3^2\mu_3 \tag{C.48}$$

Soft breaking scalar masses:

$$\frac{d}{dt}m_{H_1}^2 = 2m_{H_1}^2 \, \mathrm{Tr}(\Gamma_E^\dagger \Gamma_E + 3\Gamma_D^\dagger \Gamma_D) + 2\,\mathrm{Tr}(\Gamma_E^\dagger m_L^2 \Gamma_E + 3\Gamma_D^\dagger m_Q^2 \Gamma_D + \Gamma_E m_E^2 \Gamma_E^\dagger + 3\Gamma_D m_D^2 \Gamma_D^\dagger) + 2\,\mathrm{Tr}(\Gamma_E^{A\dagger}\Gamma_E^A + 3\Gamma_D^{A\dagger}\Gamma_D^A) - 2(3g_2^2 \mid \mu_2 \mid^2 + g_1^2 \mid \mu_1 \mid^2) \tag{C.49}$$

$$\frac{d}{dt}m_{H_2}^2 = 2m_{H_2}^2 3\,\mathrm{Tr}(\Gamma_U^\dagger \Gamma_U) + 2\cdot 3\,\mathrm{Tr}(\Gamma_U^\dagger m_Q^2 \Gamma_U + \Gamma_U m_U^2 \Gamma_U^\dagger) + 2\cdot 3\,\mathrm{Tr}(\Gamma_U^{A\dagger}\Gamma_U^A) - 2(3g_2^2 \mid \mu_2 \mid^2 + g_1^2 \mid \mu_1 \mid^2) \tag{C.50}$$

$$\frac{d}{dt}m_L^2 = m_L^2 \Gamma_E \Gamma_E^\dagger + \Gamma_E \Gamma_E^\dagger m_L^2 + 2(\Gamma_E m_E^2 \Gamma_E^\dagger + m_{H_1}^2 \Gamma_E \Gamma_E^\dagger) + 2(\Gamma_E^A \Gamma_E^{A\dagger}) - 2(3g_2^2 \mid \mu_2 \mid^2 + g_1^2 \mid \mu_1 \mid^2)\cdot \mathrm{I} \tag{C.51}$$

$$\frac{d}{dt}m_E^2 = 2(m_E^2 \Gamma_E^\dagger \Gamma_E + \Gamma_E^\dagger \Gamma_E m_E^2) + 4(\Gamma_E^\dagger m_L^2 \Gamma_E + m_{H_1}^2 \Gamma_E^\dagger \Gamma_E) + 4\Gamma_E^{A\dagger}\Gamma_E^A - 2g_1^2 \mid \mu_1 \mid^2 \cdot \mathrm{I} \tag{C.52}$$

$$\frac{d}{dt}m_Q^2 = m_Q^2(\Gamma_U \Gamma_U^\dagger + \Gamma_D \Gamma_D^\dagger) + (\Gamma_U \Gamma_U^\dagger + \Gamma_D \Gamma_D^\dagger) m_Q^2 + 2(\Gamma_U m_U^2 \Gamma_U^\dagger + \Gamma_U \Gamma_U^\dagger m_{H_2}^2 + \Gamma_D m_D^2 \Gamma_D^\dagger + \Gamma_D \Gamma_D^\dagger m_{H_1}^2) + 2(\Gamma_D^A \Gamma_D^{A\dagger} + \Gamma_U^A \Gamma_U^{A\dagger}) - 2(\frac{16}{3}g_3^2 \mid \mu_3 \mid^2 + 3g_2^2 \mid \mu_2 \mid^2 + \frac{1}{9}g_1^2 \mid \mu_1 \mid^2)\cdot \mathrm{I} \tag{C.53}$$

$$\frac{d}{dt}m_U^2 = 2(m_U^2 \Gamma_U^\dagger \Gamma_U + \Gamma_U^\dagger \Gamma_U m_U^2) + 2\cdot 2(\Gamma_U^\dagger m_Q^2 \Gamma_U + m_{H_2}^2 \Gamma_U^\dagger \Gamma_U) + 2\cdot 2(\Gamma_U^{A\dagger}\Gamma_U^A) - 2(\frac{16}{3}g_3^2 \mid \mu_3 \mid^2 + + \frac{16}{9}g_1^2 \mid \mu_1 \mid^2)\cdot \mathrm{I} \tag{C.54}$$

$$\frac{d}{dt}m_D^2 = 2(m_D^2 \Gamma_D^\dagger \Gamma_D + \Gamma_D^\dagger \Gamma_D m_D^2) + 2\cdot 2(\Gamma_D^\dagger m_Q^2 \Gamma_D + m_{H_1}^2 \Gamma_D^\dagger \Gamma_D) + 2\cdot 2(\Gamma_D^{A\dagger}\Gamma_D^A) - 2(\frac{16}{3}g_3^2 \mid \mu_3 \mid^2 + + \frac{4}{9}g_1^2 \mid \mu_1 \mid^2)\cdot \mathrm{I} \tag{C.55}$$

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