Scuola Internazionale Superiore di Studi Avanzati

International School for Advanced Studies

Weak interactions of heavy quarks in the Standard Model

Thesis submitted for the degree of

"Magister Philosophiæ"

CANDIDATE

Ugo Aglietti

SUPERVISOR

Prof. Guido Martinelli

October 1991

Index

Introduction

1. Quark weak interactions in the Standard Model

1.1. Introduction

1.2. Quark mixing in the Standard Model

1.3. Parametrizations of the CKM matrix

1.4. Experimental determinations of mixing angles

2. Particle-antiparticle mixing

2.1. General theory

2.2. $K^0\overline{K}^0$ mixing

2.3. $B^0\overline{B}^0$ mixing

3. Effective Theories

3.1. Introduction

3.2. Construction of an effective theory

3.3. Non relativistic QED

3.4. $\Delta I = 1/2$ rule

4. Heavy quark bound states in QCD

4.1. Introduction

4.2. Static theory for heavy quarks

4.3. Relativistic infinite mass theory

4.4. Correlation functions

4.5. Symmetry relations

4.6. Renormalization

4.7. Large Energy Effective Theory

4.8. Factorization

Conclusions

Acknowledgements

References

Introduction

This thesis is a survey on weak interactions of quarks at low energy in the framework of the Standard Model. By low energy we mean those phenomena in which the propagation of the W vector boson is unimportant.

The first half of the thesis deals with phenomenology. In chapter one we expose the elements of the Standard model relative to the quark mixing, and describe the most recent determinations of the CKM matrix real angles. A fundamental test of the Standard Model can be realized in this sector by overconstraining the parameters of the quark mixing matrix. As it is stressed, non perturbative effects of strong interactions play generally an important role in quark weak processes. The theoretical task is that of evaluating the matrix elements of the weak Hamiltonian between hadronic states. An improvement of the theoretical techniques is shown to be necessary.

In chapter two we discuss particle-antiparticle mixing. This kind of processes allows a determination of the CP violating phase in the CKM matrix. We limit ourselves to a combined analysis of ϵ_K and x_d; the first parameter describes CP violation in the superposition of the neutral kaon states while the second determines beauty oscillations in the B system. The experimental value of ϵ', the parameter describing CP violation in kaon decays, is still controversial and we do not report the theoretical estimates. The computation of perturbative QCD corrections to mixing amplitudes is a very technical and complicated topic and we give an outline of the present status. The main uncertainties in the theoretical predictions do not come however from corrections at short distances but from the unknown value of the top quark mass and of the hadronic matrix elements of kaons and B-mesons.

The second half of the thesis deals with the most promising techniques available at present for the understanding of strong dynamic in quark weak processes.

In chapter three we expose the general theory of effective Hamiltonians, together with some popular applications. This approach constitutes in effect a renormalization group strategy in that it separates the effects of interactions acting at different mass scales.

In chapter four we apply the theory of effective Hamiltonians to strong interactions, in particular to bound state dynamic, in the limit in which the mass of the heavy quark is very large. The contributions to the amplitudes of high energy strong interactions can be then systematically computed in perturbation theory. The effects of low energy strong interactions can be isolated in the matrix elements of a series of operators generated by the effective theory. There are symmetries in the effective theory that are not present in the full theory and that lead to many interesting physical predictions.

Chapter 1

Quark weak interactions in the Standard Model

1.1 Introduction

The Standard Model (SM) of electroweak interactions formulated by Glashow, Weinberg and Salam in the late sixties [1] describes in an excellent way the phenomenology of weak interactions up to the higher energies so far explored, $E \sim 100\ GeV$. It is a renormalizable chiral gauge theory based on the $SU(2)_I \otimes U(1)_Y$ Lie group. The structure of the SM, in particular the fermion spectrum, is mainly motivated by phenomenological considerations. In order to reproduce the correct low energy weak interaction phenomenology, left handed quarks and leptons are assigned to weak isospin doublets of the fundamental representation of the $SU(2)_I$ group:

$$\begin{pmatrix} u_L \\ d'_L \end{pmatrix}, \begin{pmatrix} c_L \\ s'_L \end{pmatrix}, \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} \cdots \tag{1}$$

where d'_L, s'_L... denote superpositions of the mass eigenstates (see sect.1.2). Charged weak interactions are then produced by exchange of gauge field quanta (W bosons) associated to the off-diagonal generators of the $SU(2)_I$ group. Right handed fermions like u_R, d_R, $e_R \ldots$ are singlets with respect to the weak isospin.

Hypercharges Y are assigned to fermions f using an analogue of the Gell-Mann-Nishijima formula:

$$Y(f) \;=\; Q(f) - I_3(f) \tag{2}$$

where $Q(f)$ is the observed electric charge of fermion f. It is possible including QED in the gauge interactions requiring the combination of generators $Q = I_3 + Y$ to not suffer spontaneous symmetry breaking. The vector field coupled to the current Q is identified with the electromagnetic field A_μ, and it is a linear combination of the neutral $SU(2)_I$ and $U(1)_Y$ gauge fields. The orthogonal combination is the neutral Z^0 field, and generates neutral weak current processes.

In the minimal Standard Model spontaneous symmetry breaking (SSB) of the $SU(2)_I \otimes U(1)_Y$ group down to $U(1)_{em}$ is realised with an isodoublet of scalars in order to account both for the short range of weak interactions ($\sim 10^{-16} cm$) and for fermion masses (the generation of quark masses in the SM is discussed in detail in the next section). With SSB 3 degrees of freedom of the scalar doublet are transformed in the longitudinal polarization of the massive vector fields. We are then left with a real scalar field, which is identified with a physical boson called the Higgs particle. The existence of the Higgs is a genuine prediction of the theory.

SM incorporated successful models of particle interactions like the old V-A theory, QED, the quark model and current algebra and led to the successful predictions of the existence of neutral current processes and of the W and Z vector bosons. The masses and the properties of these particle were predicted in terms of low energy parameters only. In view of its theoretical consistency and experimental reliability it is naturally assumed as the standard to which compare experimental results.

A sector of the SM that still needs a detailed verification is that of weak

interactions of quarks. It is necessary to measure the parameters of the quark mixing matrix with a high precision in order to make decisive consistency checks. There is also a further physical interest. The origin of fermion masses and of mixing angles is not explained in the SM and is a fundamental open problem in particle physics. Weak decays and especially particle-antiparticle mixing are very sensitive to new interactions up to fairly high mass scales scales and constitute the only manifestation of the CP-symmetry violation. Let us mention some examples.
i) The more stringent lower bound on the mass of additional W bosons present in left-right symmetric models

$$M_{W_R} > 1.6\ TeV \tag{3}$$

comes from the mass difference of neutral kaons ΔM_K.
ii) The first indication of a massive top quark was suggested by the observation of a relatively large amount of $B - \overline{B}$ mixing [2].
iii) Historically one of the first dynamical evidences of the charm quark, together with an estimate of its mass $M_c \sim 1\ GeV$, came from the observed small value of ΔM_K (GIM mechanism [3]).

With respect to purely leptonic processes, as for example μ decay, the study of weak process involving quarks is relatively involved due to the interplay between weak and strong interactions. We assume strong interactions are described by Quantum Chromodynamics (QCD). Quarks interact strongly and are never observed as asymptotic states (particles); theoretical predictions of weak transitions amplitudes can be obtained only if one is able to compute matrix elements of the weak Hamiltonian between physical hadronic states. Quark weak processes are generally much affected by strong interactions acting at every mass scale present in the theory, like the inverse of the confinement radius R^{-1}, quark masses, W mass etc... The main theoretical tool developed until now for dealing with strong interactions is the concept of effective theories. The basic idea is that of separating the effects of strong interactions acting at different mass scales. Since QCD is asymptotically free, high energy strong interactions are relatively weak and can reliably computed with perturbation theory (α_S goes to zero only logarithmically and it is a too crude approximation to simply neglect them).

Low energy strong interactions, acting at mass scales of order R^{-1}, where R is the confinement radius, down to zero momenta, need true non perturbative techniques, like lattice QCD simulation, chiral perturbation theory, 1/N expansion or QCD sum rules. Alternatively one can use suitable models, like constituent quark models or bag models. In the latter case however, it is not clear how these models could be derived from QCD. It is practically impossible to give a realistic estimate of the systematic errors of the computations. The only techniques based on first principles to compute the dynamic of strong interactions are lattice QCD, $1/N$ expansion and QCD sum rules. In the last two cases it is very complicated to go beyond the lowest order approximation. Statistical and systematic errors involved in lattice QCD computations can instead be progressively reduced with increasing power calculus.

The strategy to combine the effects of high energy and low energy strong interactions in the evaluation of the hadronic matrix elements is the following. The natural scale of electroweak interactions is the W mass $M_W \sim 80\ GeV$. The operators entering in the weak Hamiltonian are usually renormalized at this scale

$$\mu \sim M_W. \tag{4}$$

We scale the renormalization point μ from the value of eq.(4) down to a low but still perturbative mass scales $\overline{\mu} \gg R^{-1}$. The evolution of the operators is controlled by renormalization group equations and the coefficients are computed with perturbation theory. The effects of perturbative strong interactions, then, are contained in the normalization of the operators of the weak Hamiltonian. The main contributions generally amount to large logarithms of the heavy particle masses like $\ln(M_W/\mu)$, $\ln(M_t/\mu)$, etc... Their physical origin can be understood considering that the heavy masses of the SM act as an ultraviolet cut-off for radiative corrections of low-energy processes.

The quantitative effect of low energy strong interactions is contained in the values of the hadronic matrix elements of the operators entering in the effective weak hamiltonian, defined at a low but still perturbative renormalization point μ (or, in the language of bare operators, at a low but still perturbative ultraviolet cut-off Λ_{UV}). With a non perturbative technique we then compute the effects of strong interactions from the scale $\overline{\mu}$ up to zero momenta.

The general theory underlying effective Hamiltonians is exposed in detail in chapter 3 and the applications to strong interactions in chapter (4).

1.2 Quark mixing in the Standard Model

In the SM quark current masses are generated through spontaneous symmetry breaking (SSB) of the $SU(2)_L \otimes U(1)_Y$ gauge group down to $U(1)_{em}$. Renormalizable gauge-invariant Yukawa couplings are introduced between quark fields and the complex scalar isodoublet field Φ in the lagrangian. The field Φ acquires a non zero vacuum expectation value (VEV)

$$< 0 \mid \Phi(x) \mid 0 > = v \tag{1}$$

after SSB. Denoting by L the relevant part of the SM lagrangian we have:

$$L = h_{ij}^{D} \, \overline{d}_{Ri} \Phi^{\dagger} \cdot q_{Lj} + h_{ij}^{U} \, \overline{q}_{Li} \cdot \Phi^{C} u_{Rj} + h.c. \tag{2}$$

where:

q_{Lj} denotes the lefthanded quark doublet of the j-th generation:

$$q_{Lj} = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix} \ldots \tag{3}$$

$u_{Ri} = (u_R, c_R, t_R)$, $d_{Ri} = (d_R, s_R, b_R)$ are vectors in generation space containing the righthanded up, down quarks, which are assigned to isosinglets.

$\Phi^C = i\tau_2 \Phi^{\dagger}$ is the charge conjugate field of Φ.

h^U, h^D are adimensional complex matrices of Yukawa couplings, acting in generation space.

The SM for the real world is build up repeating the same multiplet scheme for every generation observed in nature or assumed to exist. Within the Minimal Standard Model there is not any physical principle which protects h^U and h^D matrices from being off-diagonal, or forces them to have a particular form. As a consequence, many arbitrary parameters are introduced in the Yukawa sector. These parameters correspond to the quark masses and to the quark mixing matrix (10 real parameters for the case of 3 generations, see later).

After SSB, the field Φ is usually parametrized as:

$$\Phi(x) = e^{\frac{i}{v}\xi_k(x)\tau_k/2} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \tag{4}$$

(expanding up to first order the exponential in eq.(4) we get the usual cartesian expression of a doublet of complex scalars).

$H(x)$ is a real scalar field with zero VEV associated to radial excitations of the Φ field. $\xi_k(x)$ (for $k = 1, 2, 3$) are gauge dependent real scalar fields determining the orientation of the Φ field in isospin space.

Performing an $SU(2)_L \otimes U(1)_Y$ gauge transformation we can set to zero the functions $\xi_k(x)$ in all the space-time (we go in the so called unitary gauge [4]) and we can identify directly the particle content and the spectrum of the theory. $H(x)$ is clearly to be identified with the Higgs field.
Substituting in eq.(2)

$$\Phi(x) = \begin{pmatrix} 0 \\ v \end{pmatrix} \tag{5}$$

we get the terms for quark Dirac masses:

$$L_{m_Q} = \overline{d}_{Ri} M^D_{ij} d_{Lj} + \overline{u}_{Li} M^U_{ij} u_{Rj} \ + \ h.c. \tag{6}$$

where we have defined the up and down quark mass matrices as

$$\begin{aligned} M^U_{ij} &= v h^U_{ij} \\ M^D_{ij} &= v h^D_{ij} \end{aligned} \tag{7}$$

Quark masses, like vector boson and lepton masses, are all proportional to the symmetry breaking mass scale $v \simeq 250\ GeV$ present in the theory. The extraordinary dispersion of observed fermion mass values, which go from $m_\nu \sim$ few eV up to $M_t \sim 100\ GeV$, is accommodated by introducing different dimensionless Yukawa couplings. That is clearly a weak point of the model: there is no physical reason why a neutrino may have a small mass as few eV in comparison with the symmetry breaking mass scale.

In order to get the spectrum of the theory, it is necessary to diagonalize the harmonic part of the lagrangian, i.e. the lagrangian part containing quark mass terms. We can perform an independent unitary transformation in generation space on the left-handed and right-handed components of the up and down quark fields:

$$\begin{aligned} u^0_{R,L} &= U_{R,L} u_{R,L} \\ d^0_{R,L} &= D_{R,L} d_{R,L} \end{aligned} \tag{8}$$

where the '0' fields are the gauge eigenstates considered up to now, and the fields without superscript are the physical ones.
Expressing the quark mass lagrangian in the physical fields we get:

$$L_{m_Q} = \overline{d}_R D_R^\dagger M^D D_L d_L + \overline{u}_L U_L^\dagger M^U U_R u_R + \ h.c. \tag{9}$$

Every complex matrix C can be diagonalized and put in a real form C_{diag} by means of a biunitary transformation of the form (for the proof see [4]):

$$C_{diag} \ = \ U \ C \ V \tag{10}$$

where U and V are two independent unitary matrices. We can then choose $D_{L,R}$, $U_{L,R}$ so as to diagonalize the mass matrices:

$$D_R^\dagger M^D D_L = M_{diag}^D = \begin{pmatrix} m_d & 0 & 0 & \dots \\ 0 & m_s & 0 & \dots \\ 0 & 0 & m_b & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix} \tag{11}$$

$$U_R^\dagger M^U U_L = M_{diag}^U = \begin{pmatrix} m_u & 0 & 0 & \dots \\ 0 & m_c & 0 & \dots \\ 0 & 0 & m_t & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix} \tag{12}$$

Let us see the effect of transformations (8) in the expression of the currents of the SM.

All the neutral currents J_N can be expressed as linear combinations of bilinears of righthanded or lefthanded quark fields:

$$\overline{q}_R^0 \gamma_\mu q_R^0, \ \ \overline{q}_L^0 \gamma_\mu q_L^0 \tag{13}$$

where $q = u, d$. The field rotations (8) have no effect on the form of the J_N of the SM, that remain diagonal in flavour (GIM mechanism at the tree level).

For the charged currents J_C we have instead:

$$J_C = \overline{u}^0_L \gamma_\mu d^0_L = \overline{u}_L \gamma_\mu \; V \; d_L \tag{14}$$

where $V = U^\dagger_V D_L$ is a general $n \times n$ unitary matrix, and n is the number of generations.
We have thus a quark mixing in the charged current sector as a consequence of the 'primordial' relative orientation of the left-handed up and down quarks in generation space.

If neutrinos are massless, no analog mixing occurs in the leptonic sector, since an appropriate unitary rotation can be applied to the left-handed neutrino fields, to transform the mixing matrix V into the identity (the neutrino mass matrix of course remains diagonal because it is identically zero). In other words, we see that conservation of separate leptonic flavors in the SM is a consequence of massless neutrinos.

In quantum field theory, particles and antiparticles are destroyed (and created) respectively by complex fields and their hermitian conjugates, such as ϕ, $\phi^\dagger$, ψ, $\overline{\psi}$, *etc.* Let us define antiparticles with the CP operation. The theory is then CP-conserving if it doesn't make any distinction between particles and antiparticles. A necessary condition for CP violation is then the presence of complex couplings in the lagrangian. In order to see if the quark mixing in the SM is CP conserving or not, we have to establish if the matrix V can be put or not in a real form with an appropriate redefinition of the quark field phases. We are faced with a phase counting problem.

The number of observable CP-violating phases f is given by:

$$f \;=\; d - a - u \tag{15}$$

where:

$d = n^2$ is the dimension of a unitary $n \times n$ matrix.

$a = n \cdot (n-1)/2$ (16) is the number of real angles of a unitary $n \times n$ matrix, i.e. the number of parameters of an orthogonal $n \times n$ matrix.

u is the number of unobservable phases. Performing an appropriate $U(1)$ rotation of the up/down quark fields we can make real an arbitrary row/column of matrix V. Since any row and column of a matrix intersect at one element, we can make real only $2n-1$ entries of V. It follows that $u = 2n-1$.

Substituting in eq.(15) we get:

$$f = \frac{(n-1) \cdot (n-2)}{2} > 0 \tag{17}$$

We arrive then at the fundamental result of Kobayashi and Maskawa [5] that the Standard Model is CP-violating if 3 or more generations are included in the particle spectrum .

Employing formulas (16) and (17) for $n = 2$ generations, we recover the famous Cabibbo result quark mixing is controlled by only one real angle, the Cabibbo angle θ_C.

For $n \geq 3$ generations the KM matrix is determined by $n \cdot (n-1)/2$ real angles and $(n-1) \cdot (n-2)/2$ phases.

In the case of three generations, as indicated by LEP data, mixing between quarks is determined by three real angles and all CP-violating effects are to be ascribed to a single phase.

1.3 Parametrisations of the CKM matrix

A parametrization of the CKM matrix can be realised composing 3 independent orthogonal rotations in the generation space spanned by the gauge eigenstates $\mid d>, \mid s>, \mid b>$ and inserting a single phase factor $e^{i\phi}$ in arbitrary entries of the matrix; the only requirement is that $e^{i\phi}$ cannot be removed by phase redefinition of the up and down quark fields.

In this thesis I will follow the parametrization introduced by Maiani [6] and adopted by the Particle Data Group. According to Maiani, we can perform first an orthogonal rotation by an angle θ in the d-s plane:

$$R_{ds}(\theta) = \begin{pmatrix} c_\theta & s_\theta & 0 \\ -s_\theta & c_\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{1}$$

where $c_\theta = \cos_\theta$, $s_\theta = \sin_\theta$.

Limiting to the $\mid d>, \mid s>$ subspace, $R_{ds}(\theta)$ is the mixing matrix for the case of 2 quark generations and θ is the Cabibbo angle θ_C.

After that, we rotate by a real angle β in the d-b plane the transformed basis and insert the phase factor $e^{i\phi}$ imposing unitarity:

$$R_{db}(\beta,\phi) = \begin{pmatrix} c_\beta & 0 & s_\beta e^{i\phi} \\ 0 & 1 & 0 \\ -s_\beta e^{-i\phi} & 0 & c_\beta \end{pmatrix} \tag{2}$$

The phase factor cannot be removed by field redefinition because it is inserted in the intermediate rotation.

Finally, we perform an orthogonal rotation in the s-b plane by an angle γ:

$$R_{sb}(\gamma) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\gamma & s_\gamma \\ 0 & -s_\gamma & c_\gamma \end{pmatrix} \tag{3}$$

Making the matrix product we derive the parametrization of the CKM matrix for the case of 3 generations:

$$\begin{aligned} V &= R_{sb}(\gamma)\, R_{db}(\beta,\phi)\, R_{ds}(\theta) = \\ &= \begin{pmatrix} c_\beta\, c_\theta & c_\beta\, s_\theta & s_\beta\, e^{i\phi} \\ -c_\gamma\, s_\theta - s_\beta\, s_\gamma\, c_\theta\, e^{-i\phi} & c_\theta\, c_\gamma - s_\beta\, s_\gamma\, s_\theta\, e^{-i\phi} & s_\gamma\, c_\beta \\ s_\gamma\, s_\theta - s_\beta\, c_\gamma\, c_\theta\, e^{-i\phi} & -s_\gamma\, c_\theta - s_\beta\, s_\theta\, c_\gamma\, e^{-i\phi} & c_\gamma\, c_\beta \end{pmatrix} \end{aligned} \tag{4}$$

In the Maiani parametrization the ratio of $u-s$ to $u-d$ couplings is given by $\tan\theta$, like in Cabibbo theory. The ratio of $u-d$ coupling to $\nu_\mu - \mu$ coupling is given by $c_\beta c_\theta$, and involves a second mixing angle in addition to θ, that, in principle, can be measured by a very accurate determination of V_{ud}, V_{us}.

Though expression (4) is exact, a natural approximation on V can be introduced, by noting that, empirically:

$$\begin{aligned} s_\theta &= 0.221 \pm .002 \\ s_\gamma &\sim s_\theta^2 \\ s_\beta &\sim s_\theta^3 \end{aligned} \tag{5}$$

In the small angle approximation V reduces to:

$$V \cong \begin{pmatrix} c_\theta & s_\theta & s_\beta e^{i\phi} \\ -s_\theta & c_\theta & s_\gamma \\ s_\gamma s_\theta - s_\beta e^{-i\phi} & -s_\gamma & 1 \end{pmatrix} \tag{6}$$

Apart from V_{td}, every matrix element depends only on a real mixing angle. In particular:

$$| V_{12} | = s_\theta \qquad | V_{23} | = s_\gamma \qquad | V_{13} | = s_\beta \tag{7}$$

Maiani parametrization is so simple because it consists of rotations about the 3 different axis, that are commuting for infinitesimal angles; parametrizations employing for instance Euler angles θ_z, θ_y, θ'_z are less practical since rotations about the x axis are obtained as commutators of rotations about y and z axis.

We can switch to an even more convenient parametrization setting, according to Wolfenstein [7]:

$$\begin{aligned} s_\theta &= \lambda \\ s_\gamma &= A\lambda^2 \\ s_\beta &= A\rho\lambda^3 \end{aligned} \tag{8}$$

where A and ρ are real constants of order one.

Including terms up to order $\lambda^3(\lambda^5)$ for the real (complex) parts, the CKM matrix reads:

$$V = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\rho\lambda^3 e^{i\phi} \\ -\lambda(1+A^2\lambda^4\rho e^{-i\phi}) & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-(1-\lambda^2/2)\rho e^{-i\phi}) & -A\lambda^2(1+\rho\lambda^2 e^{-i\phi}) & 1 \end{pmatrix} \quad (9)$$

The parameter ρ entering in eq.(9) is the analog of $\sqrt{\rho^2+\eta^2}$ of the original parametrization of Wolfenstein. We see explicitly that the coupling between 2^0 and 3^0 generation is of order λ^2 and between 1^0 and 3^0 generation of order λ^3. Since $\lambda \ll 1$, $A \sim 1$ and $\rho \sim 0.5$ the CKM matrix is almost diagonal. That is the way the quark mixing scheme accommodates the old experimental observation that strange particles and, after, beauty particles have lifetimes relatively large with respect to the naive dimensional estimate of the decay width, $\Gamma \propto G_F M^5$. These particles, the lightest in the respective doublet, are unstable only by virtue of small non zero intergenerational couplings.

1.4 Experimental determinations of mixing angles

The normalization of the CKM matrix elements is needed to make consistency checks and requires the knowledge of the Fermi constant G_F, the coupling characterizing the strength of charged weak processes at low energy. μ decay is the cleanest weak process observed in nature since it consists of a single decay channel and doesn't involve any strong radiative correction up to 1 loop included. G_F (called in this case G_μ) is then best measured by comparing the experimental

μ lifetime

$$\tau_\mu = \frac{1}{\Gamma_{exp}} = (2.197035 \pm .000040) \times 10^{-6} sec \qquad (1)$$

with its theoretical decay rate.

Since the momentum transfer to the $e^-\overline{\nu}_e\ q$ is negligible $q^2 \leq m_\mu^2/2 \ll M_W^2$, we can drop the W boson propagation and express the Hamiltonian for μ decay as a local 4 fermion operator H_{eff}:

$$H_{eff} = \frac{G_F}{\sqrt{2}} \overline{\nu}_\mu \gamma_\mu (1-\gamma_5)\mu \cdot \overline{e}_\mu \gamma_\mu (1-\gamma_5)\nu_e \qquad (2)$$

The theoretical decay rate is given by:

$$\begin{aligned} \Gamma_{th} &= \Gamma(\mu \to e\nu_e\nu_\mu) = \\ &= \frac{G_\mu^2 m_\mu^5}{192\pi^3} f(\frac{m_e^2}{m_\mu^2})\ (1 + \frac{3m_\mu^2}{5M_W^2})\ [1 + \frac{\alpha(m_\mu)}{2\pi}(\frac{25}{4} - \pi^2)] \end{aligned} \qquad (3)$$

where $f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x$ and $\alpha^{-1}(m_\mu) = \alpha^{-1} - \frac{1}{3\pi}\ln(\frac{m_\mu^2}{m_e^2}) + \frac{1}{6\pi} =$ 136.

To match the high experimental precision on μ lifetime determination, in the expression for the theoretical decay rate (3) 1 loop electromagnetic radiative corrections have been included, together with corrections due to finite values of m_e^2 and $1/M_W^2$. Since $m_\mu \gg m_e$ the whole series of e.m. leading logs, of the form $\alpha^n \ln^n(m_\mu^2/m_e^2)$ has been summed up (see reference [8] for a more convenient renormalization scheme).

Real mixing angles $\theta,\ \beta,\ \gamma$ of the Maiani parametrization, or the constants $\lambda,\ A,\ \rho$ in the Wolfenstein one, are fixed from the absolute values of the CKM matrix elements $\mid V_{ij} \mid$ contained in the first 2 row [8,9]. The latter, in turn, can be quite well determined from semileptonic decays of various flavored hadrons:

$$H \to H' + l + v_l \qquad (4)$$

where H and H' are any two mesons or barons coupled by the charged quark weak current. Semileptonic decays are selected for the analysis because they are much simpler to compute than hadronic or leptonic ones. In many cases they have small renormalization effects by strong interactions. Non leptonic decays involve generally many interfering amplitudes and final state interactions. They may have huge renormalization effects; the most obvious example is the $\Delta I = 1/2$ rule in $K \to \pi\pi$ decay, where strong interactions reduce by about two orders of magnitude the rate of $\Delta I = 3/2$ transitions. Their computation is problematic also with lattice QCD. Factorization of the amplitude simplify the task. It can be proved in the decays of heavy quarks at the leading order of a special effective theory (the topic is discusses in detail in section 4.8 of chapter 4).

Leptonic decays such as

$$P \to l + \nu, \tag{5}$$

where P is a pseudoscalar meson, involve basic low energy effects of strong interactions. Thee amplitude is proportional to the meson annihilation constant f_P defined by:

$$< 0 \mid A_\mu(0) \mid P > = \; i p_\mu f_P \tag{6}$$

f_P is a completely non perturbative quantity because it depends on the meson wave function.

The determination of the phase ϕ that makes complex the CKM matrix requires the analysis of particle-antiparticle mixing processes, that have a much more rich dynamical structure than semileptonic decays. Apart from the construction of the formalism describing a 2 state quantum mechanical dissipative system, considerable more control on strong interaction effects is required for the evaluation of the relevant transition amplitudes. For this reason, they will be discussed in a separate chapter. Here we limit ourselves to discuss the measures of the real mixing angles.

Let us consider then in detail hadron semileptonic decays. They are produced by the action of an effective hamiltonian H_{eff} consisting of the product of a quark and a lepton charged weak currents:

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{ij} J^H_{\mu ij} J^{L\mu\dagger} + h.c. \tag{7}$$

where $J^H_{\mu ij} = \overline{u}_i \gamma_\mu (1-\gamma_5) d_j$ and $J^L_\mu = \overline{\nu}_\mu \gamma_\mu (1-\gamma_5) l$.
Semileptonic decay rates for $j \to i$ transition are proportional to $| V_{ij} |^2$. Also in this case we can drop the W propagation since, in any case:

$$q^2 \leq M_b^2/2 \ll M_W^2 \tag{8}$$

where q is the momentum carried by the W. With present accuracy, except in nuclear β decay it is sufficient to consider the weak and electromagnetic interactions at lowest order in perturbation theory. Strong interactions need to be computed at every order in α_S. Since they do not couple J^L with J^H, the transition amplitude A for semileptonic decay (4) is expressed as the product of a leptonic and a hadronic matrix elements:

$$A = L_\mu \cdot H^\mu \tag{9}$$

where $L_\mu =< l, \nu_l \mid J^L_\mu(0) \mid 0 >$ and $H_\mu =< H' \mid J^H_\mu(0) \mid H >$. The computational problem of process (4) is then turned to that of evaluating the hadronic matrix element H_μ.

The fundamental property of semileptonic decays is that, as anticipated, in the limit of very light or very heavy quark masses, the computation of the rate does not require the evaluation of the renormalization effects of low energy strong interactions.

Light hadron exclusive semileptonic decays like pion or superallowed nuclear β decays can be computed by means of isotopic spin symmetry. In the limit of equal current masses of up and down quarks

$$m_u = m_d, \tag{10}$$

that is a good approximation since

$$| m_u - m_d | \ll \Lambda_{QCD}, \tag{11}$$

the weak vector current becomes a Noether current for the strong interactions. The symmetry is realised in the quantum theory and fixes the value of the hadronic matrix elements of the decays:

$$\begin{aligned} < \pi^0 \mid \int d^3x V_0^{u\to d}(x) \mid \pi^+ > &= \sqrt{2} \\ < Z \mid \int d^3x V_0^{u\to d}(x) \mid Z+1 > &= 1 \end{aligned} \tag{12}$$

where $\mid Z >$, $\mid Z+1 >$ denote the states of two specular nuclei. We can figure the decays as a $u(\overline{d})$ quark that is transformed by the weak current into a $d(\overline{u})$ quark with a negligible momentum release with respect to typical bound state momentum transfers in the hadron. Since the strong force is flavour independent, in the limit of equal u-d masses nothing happens during decay with respect to hadron dynamics. In addition, symmetry breaking corrections are of second order in the symmetry breaking parameter (Ademollo-Gatto theorem [10]).

Extending the isotopic spin symmetry into the $SU(3)$ flavour symmetry under the assumption $m_u = m_d = m_s$ allows to extract directly, with analogous considerations, $\mid V_{us} \mid$ from kaon or hyperion semileptonic decays. $SU(3)_f$ symmetry violations are however much larger than in the $SU(2)_I$ case, of the order

$\sim 20 \div 30\%$. A detailed knowledge of $| V_{us} |$ requires to compute with chiral perturbation theory symmetry breaking effects.

In the limit of very massive quarks Q

$$M_Q \gg \Lambda_{QCD} \tag{13}$$

perturbative QCD becomes appropriate for computing totally inclusive semileptonic decay widths. We can implement a short distance approach analog to the parton model of hadrons [11]. The b quark lies probably close to the asymptotic region (13).

Semileptonic decays of charmed particles are the most difficult to compute because the charm quark is in the border zone between the two limiting cases:

$$m_q \ll \Lambda_{QCD}, \qquad m_q \gg \Lambda_{QCD}. \tag{14}$$

There is not any simple symmetry or dynamical property that prevents low energy strong interactions from playing an important role. Neither the parton model nor chiral perturbation theory can be applied because the mass breaking effect is substantial.

The usual approach for computing hadronic matrix elements is that of choosing a particularly simple exclusive decay channel. In the case the initial and final hadron are 2 pseudoscalar mesons H and H', only the vector current V_μ contributes to decay, due to parity conservation of strong interactions. Its matrix elements can be parametrized in terms of two relativistic form factors:

$$< H' \mid V_\mu(0) \mid H > = f^+(q^2)(p_H + p_{H'})_\mu + f^-(q^2) q_\mu \tag{15}$$

where $q = p_H - p_{H'}$.

The electronic (muonic) current is conserved up to small terms of order m_e^2/m_H^2 (m_μ^2/m_H^2). In the case of electronic (muonic) decays then only $f^+(q^2)$ gives a non zero contribution on contraction of H_μ with L^μ. The problem is that of evaluating $f^+(q^2)$.

A traditional technique employs quark model wave functions for the initial and final mesons. The free parameters of the model are fixed comparing with the experimental mass spectrum. $f^+(q^2)$ is computed at $q^2 = q^2_{MAX} = (M_H - M_{H'})^2$ by an overlap integral of the initial and final meson wave functions. For determining the form factor in the full kinematic range it is necessary a further dynamical assumption on the behaviour of $f^+(q^2)$. It is generally assumed nearest pole dominance. A first principle evaluation of form factors is possible with numerical simulation of QCD. Matrix elements (15) for $K \to \pi$, $D \to K$, $D \to \pi$ semileptonic decays have been already computed with lattice QCD simulations .

After these general considerations, we pass to discuss the most recent determinations of individual CKM matrix elements.

$| V_{ud} |$ is determined by means of well tested nuclear technique from $0^+ \to 0^+$ superallowed β nuclear decay rates, that are measured with a precision of 10^{-3} or better. The main uncertainty comes from the computation of Coulomb isospin breaking effects in nuclear wave functions. A careful analysis including also weak and electromagnetic radiative corrections leads to:

$$| V_{ud} | = 0.9736 \pm 0.0010 \ . \tag{16}$$

$| V_{ud} |$ is by far the best determined CKM matrix element.

To control eventual systematic errors, two other independent measures of $| V_{ud} |$ have been performed. The first one comes from neutron β decay:

$$n \to p + e^- + \overline{\nu}_e \tag{17}$$

Coulomb corrections to (17) are much less important but the transition is of a mixed type and it is required an accurate determination of g_A, via β asymmetry or $e-\nu$ angular correlation:

$$g_A = 1.222 \pm 0.005 \tag{18}$$

This method yields:

$$\mid V_{ud} \mid = 0.9778 \pm 0.0029 \tag{19}$$

a value about 1 standard deviation greater than the preceeding one.

Finally, pion β decay

$$\pi^- \longrightarrow \pi^0 + e^- + \overline{\nu}_e \tag{20}$$

gives:

$$\mid V_{ud} \mid = 0.968 \pm 0.018 \tag{21}$$

Experimentally, partial decay rates Γ_c are determined by measuring particle lifetimes $\tau = 1/\Gamma$ and branching ratio B_c:

$$\Gamma_c = B_c\,\Gamma \tag{22}$$

This measure is less precise of an order of magnitude than the first two because the branching ratio $B(\pi^- \longrightarrow \pi^0 e^- \nu_e)$ is of order 10^{-8}. The quoted error is a statistical error. On the theoretical side decay (20) is by far the simplest one to compute.

$\mid V_{us} \mid$ is measured by K_{e3} decay rates:

$$K^+ \longrightarrow \pi^0 + e^+ + \nu_e \tag{23}$$

plus isospin related and charged conjugate reactions. For the extraction of $\mid V_{us} \mid$ from data the knowledge of $f^+(q^2)$ is necessary. Assuming nearest pole dominance

$$f^+(q^2) = f^+(0)\frac{M_{K^*}^2}{M_{K^*}^2 - q^2}, \tag{24}$$

that is supported by Dalitz plot analysis, and employing for $f^+(0)$ the values

$$\begin{aligned} f^+_{K^+}(0) &= 0.982 \pm 0.008 \\ f^+_{K^0}(0) &= 0.961 \pm 0.008 \end{aligned} \tag{25}$$

we quote:

$$\mid V_{us} \mid = \ 0.2196 \pm 0.0023 \tag{26}$$

An independent determination of $\mid Vus \mid$ comes from semileptonic hyperion decays:

$$\mid V_{us} \mid = \ 0.222 \pm 0.003 \tag{27}$$

Symmetry breaking effects are best computed in decay (23).

Assuming that it is correct, combining measure (26) with (27) we obtain:

$$\mid V_{us} \mid = \ 0.2205 \pm 0.0018 \tag{28}$$

$\mid V_{cd} \mid$ is best determined from deep-inelastic nucleon-neutrino scattering experiments, through inclusive cross section measurement of the reaction:

$$\nu_\mu + d \ = \ c + \mu^- \qquad (+\text{conjugate reaction.}) \tag{29}$$

Processes (29) are detected tagging the muon produced in semileptonic decays of charmed particles. One selects high energy, high transverse momentum muon pairs (so called dimuon events). The knowledge of the distribution of quark d inside the nucleon is required to extract $\mid V_{cd} \mid$. Averaging Cern and Fermilab results we quote:

$$\mid V_{cd} \mid = \ 0.215 \pm 0.016 \tag{30}$$

A much less precise value, due to uncertainties on the form factor and limited statistics, is determined by D meson exclusive semileptonic decays

$$D \rightarrow \pi/\rho + l + \nu_l \tag{31}$$

The independent determination of $| V_{cd} |$ from the scalar and vector channel decays allows a consistency check of the theoretical methods employed, which are essentially lattice QCD simulations [12] and quark model wave function computations. It has been estimated:

$$| V_{cd} | = 0.23^{+0.14}_{-0.06} \tag{32}$$

$| V_{cs} |$ is determined from D exclusive semileptonic decays to charmed particles:

$$D \rightarrow K/K^* + l + \nu_l \tag{33}$$

Statistical errors are reduced of an order of magnitude with respect to decays (31) since (33) are Cabibbo favorite. The six relevant form factors of decays (33) have been computed both with quark models and lattice QCD, showing a better agreement of the latter technique with the measures.

A less precise determination, essentially because of uncertainties in strange quark sea structure functions, comes from the reaction:

$$\nu_\mu + s \rightarrow c + \mu^- \tag{34}$$

Combining the 2 kind of determinations we get:

$$| V_{cs} | = 0.98 \pm 0.12 \tag{35}$$

$| V_{cb} |$ and $| V_{ub} |$ are determined with a model dependent analysis from B meson semileptonic decays:

$$B \rightarrow X + l + \nu \tag{36}$$

where X is any hadronic final state. Their knowledge is essential for determining the angles γ and β in the Maiani parametrization or the constants A and ρ in the Wolfenstein one.

The traditional method of analysis is inclusive and considers the energy spectrum of the charged lepton l produced in the decay (36), while the most recent one is exclusive and refers directly to the partial decay rates.

Let us consider the oldest approach. The differential decay rate width for the reaction in eq.(36) is given by:

$$\frac{d\Gamma}{dE_l}(B \rightarrow X + l + \nu) = | V_{cb} |^2 \frac{d\Gamma}{dE_l}(B \rightarrow X_c + l + \nu) + | V_{ub} |^2 \frac{d\Gamma}{dE_l}(B \rightarrow X_u + l + \nu) \tag{37}$$

where E_l is the lepton energy in the B meson rest frame and we have factored out explicitly the CKM matrix elements out of the currents. X_c, X_u denote any hadronic state containing the c/u quark coming from b decay. From experiments and unitarity we know that:

$$| V_{cb} | \gg | V_{ub} | \, . \tag{38}$$

$| V_{cb} |$ is determined comparing the theoretical and experimental distributions of leptons coming from inclusive $b \rightarrow c$ semileptonic decays.
$| V_{ub} |$ is determined by looking at inclusive leptons from $b \rightarrow u + l + \nu$ transitions. To avoid the huge background from $b \rightarrow c$ transitions, only the spectrum above the kinematical endpoint for $B \rightarrow D$ transitions is analysed. The kinematic interval in E_l is $2.2 \div 2.6 \; GeV$.

The second method deals with the following exclusive decay channels:

$$B \rightarrow D/D^* + l + \nu \tag{39}$$

for the extraction of $| V_{cb} |$ and

$$B \rightarrow \pi/\rho + l + \nu. \tag{40}$$

for the extraction of $| V_{ub} |$. The branching ratio for the vector channel in eq.(40) has been already measured by the ARGUS collaboration.

Let us briefly discuss two of the most popular models used in the experimental analysis of semileptonic B decay.

For the computation of the charged lepton spectrum

$$\frac{d\Gamma}{dE_l}(b \to c + l + \nu,\ E_l). \tag{41}$$

an improved partonic model approach has been followed by Altarelli et al. [13] The parton model is applicable if the energy available to the hadronic degrees of freedom during the process is sufficiently large. It is then possible to decouple high and low energy strong interaction effects. High energy strong interactions can be computed with perturbative QCD. Low energy strong interactions act after the partonic process takes place. They create from vacuum soft $q\bar{q}$ pairs and gluons needed to recombine quarks in colour singlets, in the variety of the exclusive hadronic final states.

Altarelli et al. make some assumptions about the dynamic of the decay. The meson decay rate is computed by building a model of B meson dynamic. The spectator quark q is assigned a constituent mass $M_q = 150 \div 450\ MeV$ and a gaussian Fermi momentum distribution $f(p_q)$, whose width p_F, of order Λ_{QCD}, is fixed by fitting the lepton spectrum. Kinematic constraints are satisfied by treating the spectator quark as a on-shell particle and assigning to the b-quark the proper virtuality, i.e. an invariant mass $W = W(p_q)$ such that:

$$W^2 \ = \ M_B^2 \ + \ M_q^2 \ - \ 2M_B\sqrt{p_q^2 + M_q^2} \tag{42}$$

The physical decay rate $d\Gamma/dE_l(B \to X_c/X_u + l + \nu)$, is then evaluated by convolving the parton decay rate $d\Gamma/dE_l(b \to c/u + l + \nu,\ W)$ of a b quark with mass W with the spectator quark momentum distribution $f(p_q)$:

$$\frac{d\Gamma}{dE_l}(B \to X_c/X_u + l + \nu) = \int dp_q\ p_q^2\ f(p_q)\ \frac{d\Gamma}{dE_l}(b \to c/u + l + \nu,\ W) \tag{43}$$

An a-posteriori criterion for a proper applicability of the parton model is the experimental observation of many multihadronic final states in the process. It is experimentally established that $B \rightarrow D/D^* + l + \nu$ almost saturate the $b \rightarrow c$ semileptonic width, indicating still significant nonperturbative effects of strong interactions. The parton model is probably more justified for semileptonic $b \rightarrow u$ decays.

A complementary approach that privileges bound state properties of strong dynamic in the decay has been employed by Isgur and Wise [14]. The authors compute the following differential exclusive widths:

$$\frac{d\Gamma}{dE_l}(B \rightarrow M_c/M_u + l + \nu) \tag{44}$$

where M_c/M_u are low-lying final mesons containing the c/u quark coming from b decay.

Relativistic form factors for various channels are evaluated at high q^2 as overlap integrals of nonrelativistic meson wave functions computed with the Cornell potential [15]:

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + kr + C \tag{45}$$

where α_S is the QCD coupling constant computed at the relevant scale, k is the string tension and C is an additive constant determined by fitting the mass spectrum.

Inclusive decay widths are computed by summing over all the low-mass meson exclusive decay channels. Multihadronic decays are then assumed to proceed only via resonance decays.

The most recent determinations of $\mid V_{cb} \mid$ according to the Altarelli et al. model

(ACM) and the Isgur and Wise one (ISGW) are:

$$\begin{aligned} \mid V_{cb} \mid &= 0.046 \pm .003 \qquad ACM \\ \mid V_{cb} \mid &= 0.045 \pm .004 \qquad ISGW \end{aligned} \tag{46}$$

whilst for $\mid V_{ub} \mid$ we have:

$$\begin{aligned} \frac{\mid V_{ub} \mid}{\mid V_{cb} \mid} &= 0.11 \pm 0.01 \qquad ACM \\ &= 0.20 \pm 0.02 \qquad ISGW \end{aligned} \tag{47}$$

As it stems from the values of $\mid V_{cb} \mid$, and even more of $\mid V_{ub} \mid$, an improvement of the theoretical computations is essential for a precise test of the CKM scheme. The research we have developed this year aims to compute the form factors for transitions (39) and (40) within the framework of lattice QCD. As will be discussed in chapter (4) the computation is relatively involved. The basic problem is that with present computers it is not possible to simulate the dynamic for the b quark. We have then decided to implement an effective theory for the b quark (static theory, Eichten [4.6]). A detailed understanding of the renormalization properties of the static theory is essential. Using renormalization group techniques, it is possible to obtain physical quantities, such as the pseudoscalar decay constant f_B, the decay form factors $f^+(q^2)^{B\to D}$, $f^+(q^2)^{B\to\rho}$, etc..., from the calculation of the corresponding quantities in the effective theory, on the lattice. On overview of the static theory is presented in chapter (4).

Chapter 2

Particle-antiparticle mixing

2.1 General theory

In quantum field theory particles p and antiparticles $\overline{p}$ are distinguished by opposite eigenvalues $\pm q_i \neq 0$ of a set of charge operators Q_i associated to internal $U(1)$ symmetries $S_i = e^{i\alpha_i Q_i}$ of the theory:

$$\begin{aligned} Q_i \mid p> &= +q_i \mid p> \\ Q_i \mid \overline{p}> &= -q_i \mid \overline{p}> \qquad i = 1 \ldots n \end{aligned} \tag{1}$$

and:

$$[H, Q] = 0 \tag{2}$$

where H is the hamiltonian of the interacting fields.

If for a given field Φ no such internal symmetry exists, particles are neutral, i.e. they coincide with their antiparticles

$$p = \overline{p} \tag{3}$$

as is the case for γ, π^0, Φ, Z^0, *etc*...

$\{Q_i\}$ may be a single operator as baryonic number B in the case of neutron systems, or various operators , as lepton number L and electric charge Q_{el} for electrons.

An 'intermediate' case between neutral and charged particles presents in nature when the greatest part of forces acting on a system, described by an Hamiltonian H_0, do verifye an internal symmetry $S = e^{i\alpha Q}$

$$[H_0, Q] = 0 \tag{4}$$

that is violated by the remaining ones, contained in H_I:

$$[H_I, Q] \neq 0 \tag{5}$$

In the latter case a rich variety of mixing and interference phenomena occurs in the dynamic of the system, due to degeneracy of the unperturbed spectrum [1].

Q_{el}, B, L are conserved by any interaction contained in the standard model, while flavors like strangeness S, charm C, beauty B, etc.. are conserved by strong and electromagnetic forces but violated by weak forces. Neutral flavored mesons like K^0, D^0, B_s, B_d, *etc...* can then undergo to a transition with the corresponding antiparticle. We may set:

$$\begin{aligned} H_0 &= H_{st} + H_{em} \\ H_I &= H_{wk} \end{aligned} \tag{6}$$

Neglecting H_I we can define particle-antiparticle states distinguished by a flavour operator F: F^0, $\overline{F}^0$, related to each other by C (CP) transformation [2]:

$$\begin{aligned} C \, : \quad F^0 &\to \overline{F}^0 \\ \overline{F}^0 &\to F^0 \end{aligned} \tag{7}$$

Of course, particle and antiparticle states of H_0 are only introduced for their utility; at the end of the analysis of $K^0 - \overline{K}^0$ mixing, for instance, we will end with quite distinct K_L, K_S states, representing simply independent particles just like π^0 and η mesons.

Weak interaction Hamiltonian H_{wk} can be treated as a small perturbation of H_0, that connects F^0 with $\overline{F}^0$ and with various continuum states $c = \pi^+\pi^-$, $\pi^0\pi^0$, $\pi^- l^+ \nu_l$, $\pi^+\pi^-\pi^0$, *etc...* (see fig.1). The complete system of evolution equations for the state of the system $\mid \psi(t) >$

$$\mid \psi(t) > = a_{F^0}(t) \mid F^0 > + a_{\overline{F}^0}(t) \mid \overline{F}^0 > + a_c(t) \mid c > + a_{c'}(t) \mid c' > + \cdots \quad (8)$$

are:

$$i\frac{d}{dt}a_i(t) = h_{ij}a_j(t) \quad (9)$$

where: $a_1 = a_{F^0}$, $a_2 = a_{\overline{F}^0}$, $a_3 = a_c$... and $h_{ij} = m_i\delta_{ij} + O(H_{wk})$.

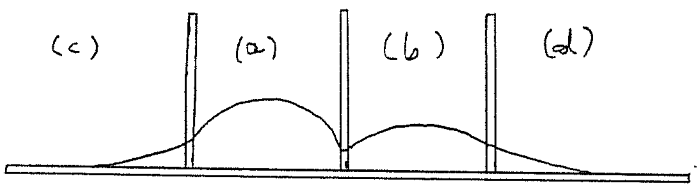


Fig.1: quantum mechanical analog of particle-antiparticle mixing. The amplitude of the wave function in regions $a(b)$ and $c(d)$, is the analog of the amplitude of the particle(antiparticle) state and of the decay channels of the particle(antiparticle).

Continuum amplitudes $a_c(t)$, $a_{c'}(t)$... can be eliminated as state variables solving the equations (9) and substituting the known $a_c(t)$ values. By the superposition principle the resulting (integro-differential) equations are linear and in a certain approximation they reduce to the following differential equations [2,3]:

$$i\frac{d}{dt}\begin{pmatrix} a_{F^0}(t) \\ a_{\overline{F}^0}(t) \end{pmatrix} = H \begin{pmatrix} a_{F^0}(t) \\ a_{\overline{F}^0}(t) \end{pmatrix} \quad (10)$$

where:

$$H = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \tag{11}$$

is an effective hamiltonian describing the time evolution of $| \psi >$ restricted to the $F^0 - \overline{F}^0$ subspace:

$$| \psi(t) > = a_{F^0}(t) \mid F^0 > + a_{\overline{F}^0}(t) \mid \overline{F}^0 > \tag{12}$$

If CPT symmetry holds for both H_0 and H_I: $H_{11} = H_{22}$.

We can decompose H into an hermitian M and antihermitian $-i\Gamma/2$ matrices (mass and decay matrices respectively):

$$H = M - i\Gamma/2 \tag{13}$$

Equation (13) is the generalization for a 2 state system of the effective hamiltonian for an unstable particle: $H = m - i\gamma/2$ which gives $\psi(t) = \psi(0)e^{-imt-\gamma t/2}$, that is valid for $\gamma \ll m$ (quasi stationary states [4]).

The physical content of M, Γ matrices is derived expressing the effective Hamiltonian matrix elements H_{ij} with ordinary perturbation theory and separating dispersive and absorbitive part of the amplitude:

$$\begin{aligned} H_{ij} &= m\delta_{ij} + < i \mid H_{wk} \mid j > + \\ &+ \sum_{n \neq i,j} \frac{< i \mid H_{wk} \mid n >< n \mid H_{wk} \mid j >}{m - E_n + i\epsilon} + O(H_{wk}^3) \end{aligned} \tag{14}$$

where $i, j = F^0, \overline{F}^0$ and $m = m_{F^0} = m_{\overline{F}^0}$ is the unperturbed F^0 mass ($m \gg \Gamma_{ij}, M_{ij}$ for $i \neq j$). Substituting in (14) the identity:

$$\frac{1}{m - E_n + i\epsilon} = P\frac{1}{m - E_n} - i\pi\delta(m - E_n) \tag{15}$$

we get:

$$M_{ij} = m\delta_{ij} + < i \mid H_{wk} \mid j > + P \sum_{n \neq i,j} \frac{< i \mid H_{wk} \mid n >< n \mid H_{wk} \mid j >}{m - E_n} \quad (16)$$

and:

$$\Gamma_{ij} = 2\pi \sum_{n \neq i,j} \delta(m - E_n) < i \mid H_{wk} \mid n >< n \mid H_{wk} \mid j > \quad (17)$$

Then, M_{12}, Γ_{12} are respectively the dispersive/absorbitive part of the transition amplitude $\overline{F}^0 \rightarrow F^0$, generated by virtual/real intermediate states in the spectrum of H_0. In other words, Γ_{12} is the coupling of $F^0 - \overline{F}^0$ states through common decay channels. In covariant perturbation theory Γ_{12} is computed using Cutkowsky rule, cutting in all possible ways Feynman diagrams for $\overline{F}^0 \rightarrow F^0$ amplitude and substituting in place of propagators:

$$\frac{1}{p^2 - m^2 + i\epsilon} \rightarrow 2\pi\delta^+(p^2 - m^2) \quad (18)$$

Relation (18) is the covariant analogue of (15). For instance, in the case of $B - \overline{B}$ mixing $\Gamma_{12}(B)$ is given by the cut Feynman diagrams of fig.(2):

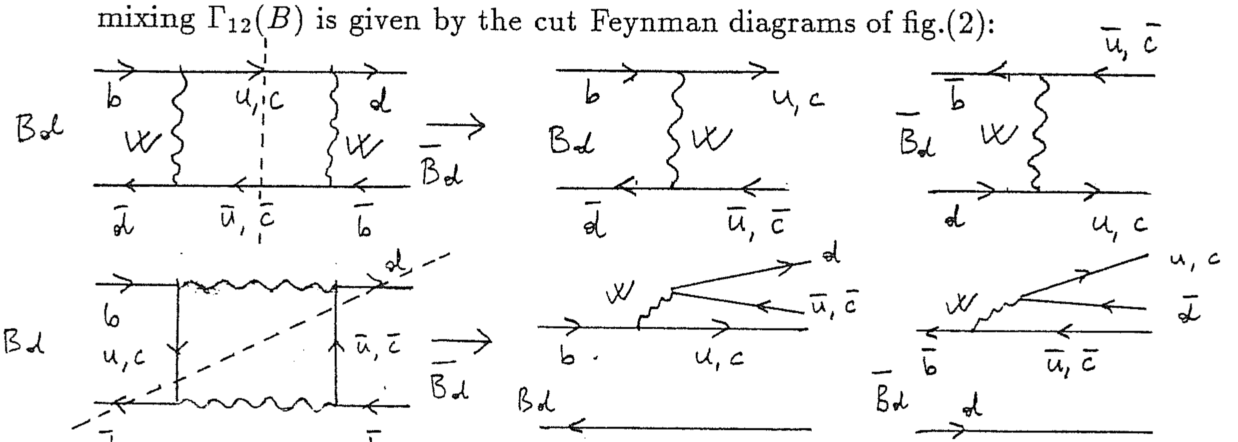


fig.2

As it stems from eq.(16,17), the theoretical task in the study of particle antiparticle mixing is the computation of $\Delta F = 2$ weak interaction matrix elements between eigenstates of $H_0 = H_{st} + H_{em}$.

The eigenvectors $|+>$, $|->$ of H are associated to quasi stationary states of the system and represent the physical particles F_+, F_- who evolve in time independently transferring amplitude only in continuum states. Masses M_+, M_- and widths $\Gamma_+/2$, $\Gamma_-/2$ are respectively the real and imaginary parts of the eigenvalues $\lambda_{+,-}$ of H.

Imposing:

$$det(H - \lambda I) = 0 \tag{20}$$

we derive:

$$\lambda_{+,-} = M_{11} - i\Gamma_{11}/2 \pm Q \tag{21}$$

where

$$Q = \sqrt{(M_{12}^* - i\Gamma_{12}^*/2)(M_{12} - i\Gamma_{12}/2)} \tag{22}$$

then:

$$\begin{aligned} M_{phys}^{\pm} &= M_{11} \pm ReQ \\ \Gamma_{phys}^{\pm}/2 &= \Gamma_{11}/2 \mp ImQ \end{aligned} \tag{23}$$

Masses and decay width differences are functions of off diagonal elements of H only, being an effect of weak interactions:

$$\begin{aligned} \Delta M_{phys} &= M_+ - M_- = 2ReQ \\ \Delta\Gamma_{phys}/2 &= \Gamma_+/2 - \Gamma_-/2 = -2ImQ \end{aligned} \tag{24}$$

The eingenstates in the F^0, $\overline{F}^0$ basis are given by:

$$\begin{aligned} |+> &= N\begin{pmatrix} 1 \\ +\eta \end{pmatrix} \\ |-> &= N\begin{pmatrix} 1 \\ -\eta \end{pmatrix} \end{aligned} \tag{25}$$

where:

$$\eta = \sqrt{\frac{M_{12}^* - i\Gamma_{12}^*/2}{M_{12} - i\Gamma_{12}/2}} \tag{26}$$

and N is a normalization factor.

We have now to see how CP violation is incorporated in the effective Hamiltonian.

The following basic theorem holds: If M_{12} and Γ_{12} have equal phases:

$$arg(M_{12}) = arg(\Gamma_{12}) \tag{27}$$

there is not CP violation in $F^0 - \overline{F}^0$ mixing.

Proof: If eq.(27) is true, we may set $M_{12} =| M_{12} | e^{i\phi}$, $| \Gamma_{12} | e^{i\phi}$. η equals a phase factor

$$\eta = e^{-i\phi} \tag{28}$$

and the eigenvectors can be written as:

$$\begin{aligned} | + > &= \frac{1}{\sqrt{2}}(| F^0 > + e^{-i\phi} | \overline{F}^0 >) \\ | - > &= \frac{1}{\sqrt{2}}(| F^0 > - e^{-i\phi} | \overline{F}^0 >) \end{aligned} \tag{29}$$

The relative phase between F^0 and $\overline{F}^0$ can be redefined by letting:

$$| \overline{F}^0 > \rightarrow e^{i\phi} | \overline{F}^0 > \tag{30}$$

Eigenstates now become:

$$\begin{aligned} | + > &= \frac{1}{\sqrt{2}}(| F^0 > + | \overline{F}^0 >) \\ | - > &= \frac{1}{\sqrt{2}}(| F^0 > - | \overline{F}^0 >) \end{aligned} \tag{31}$$

Defining CP as in KM convention: $CP | \overline{F}^0 >=| F^0 >$ we see that Hamiltonian eigenstates can be made coincident with CP eigenstates, i.e. CP is conserved. c.v.d.

In the case M_{12} and Γ_{12} have the same phase, the expressions (23) and (24) for the masses and widths simplify to:

$$\begin{aligned} M^{\pm}_{phys} &= M_{11} \pm \mid M_{12} \mid \\ \Gamma^{\pm}_{phys} &= \Gamma_{11}/2 \pm \mid \Gamma_{12} \mid /2 \end{aligned} \tag{32}$$

and

$$\begin{aligned} \Delta M_{phys} &= 2 \mid M_{12} \mid \\ \Delta \Gamma_{phys} &= 2 \mid \Gamma_{12} \mid \end{aligned} \tag{33}$$

It is common in the literature to express the above theorem in terms of a complex mixing parameter ϵ, defined by:

$$\frac{1-\epsilon}{1+\epsilon} = \eta \tag{34}$$

If CP is conserved, $\eta = e^{-i\phi}$ and $\epsilon = i\tan\phi/2$, i.e. it is a purely imaginary number; also, the eigenstates can be put in form (29), implying that if CP is conserved we can always transform ϵ to zero. In terms of ϵ the normalized eigenstates are given by:

$$\begin{aligned} \mid + > &= N(\epsilon) \begin{pmatrix} 1+\epsilon \\ 1-\epsilon \end{pmatrix} \\ \mid - > &= N(\epsilon) \begin{pmatrix} 1+\epsilon \\ -(1-\epsilon) \end{pmatrix} \end{aligned} \tag{35}$$

where:

$$N(\epsilon) = \frac{1}{\sqrt{2(1+\mid \epsilon \mid^2)}} \tag{36}$$

The analysis of particle-antiparticle mixing developed until now has been completely general and we assumed only CPT symmetry of all the interactions. We did not specify neither the physical system nor the dynamical laws. Before ending this section, however, let us briefly mention the main concrete differences between the two most important cases: $K^0 - \overline{K}^0$ and $B^0\text{-}\overline{B}^0$ systems (they are discussed in detail in sections (2.2) and (2.3)).

In the framework of the SM, as discussed in chapter (1), flavor dynamic is described by the CKM matrix. V_{ud}, V_{us} are real and V_{cd}, V_{cs} have very small imaginary parts in the Maiani or Wolfenstein parametrizations.

By eq.(17) $\Gamma_{12}(K)$ is the coupling of K^0 and $\overline{K}^0$ states through common decay channels (analog graphs of those in fig.(2)). Since $M_K \sim 0.5\ GeV$ only light quarks give the dominant contribution to $\Gamma_{12}(K)$, which is then practically real.

In the dispersive box diagram amplitude for $K^0 - \overline{K}^0$ mixing a significant imaginary part to the amplitude is generated by top exchange contribution, that is highly Cabibbo suppressed. As a consequence, M_{12} has a small imaginary versus real part. CP violation effects are then small in the K system.

For B mesons it's quite different since b quarks appear as external states. Assuming all quarks massless except b and t, $\Gamma_{12}(B)$ is proportional to M_b^2:

$$\Gamma_{12}(B) \propto (V_{ub}V_{ud}^* + V_{cb}V_{cd}^*)^2 M_b^2 = (V_{tb}V_{td}^*)^2 M_b^2 \tag{37}$$

The imaginary parts of $M_{12}(B)$, $\Gamma_{12}(B)$ are not small anymore, but the phases are very similar by virtue of eq.(37) and (2.3.11) , giving again a small CP violating effect.

2.2 $K^0 - \overline{K}^0$ mixing

2.2.1 Interference phenomena

Consider the time evolution of a coherent $K^0 - \overline{K}^0$ beam, described by a state $\mid \tau >$, with τ the proper time:

$$\mid \tau >= a_S(\tau) \mid S > + a_L(\tau) \mid L > \tag{1}$$

where $| S >$, $| L >$ are the neutral kaon eigenstates and

$$\begin{aligned} a_S(\tau) =& a_S(0) e^{-iM_S \tau - \gamma_S \tau/2} \\ a_L(\tau) =& a_L(0) e^{-iM_L \tau - \gamma_L \tau/2} \end{aligned} \tag{2}$$

The state $| \tau = 0 >$ has to be fixed by an initial condition; it is supposed to be known and is related to the specific production mechanism. For instance, $| \tau = 0 >$ may be a pure K^0 state, produced in the reaction:

$$\pi^- + p \to \Lambda^0 + K^0 \tag{3}$$

The threshold for $\overline{K}^0$ production is indeed sensibly higher. In this case then:

$$a_S(0) = a_L(0) = \frac{\sqrt{1 + | \epsilon |^2}}{\sqrt{2}(1 + \epsilon)} \tag{4}$$

The decay rate Γ_c of state $| \tau >$ to a given channel $| c >$ is easily computed as:

$$\begin{aligned} \Gamma_c(\tau) \propto & |< c | \tau >|^2 = | a_S(0) |^2 |< c | S >|^2 e^{-\gamma_S \tau} + | a_L(0) |^2 |< c | L >|^2 e^{-\gamma_L \tau} \\ & + 2 a_S(0) a_L^*(0) |< c | S >||< c | L >| \times \\ & \times \exp\{-i(M_S - M_L)\tau + \phi_L(c) - \phi_S(c) - (\gamma_S + \gamma_L)\tau/2)\} + c.c. \end{aligned} \tag{5}$$

where we have factorized decay amplitude phases:

$$< c | K_{S,L} > = |< c | K_{S,L} >| e^{i\phi_{S,L}(c)} \tag{6}$$

In K system, since lifetimes are well separated and large, $\tau_S, \tau_L \sim 10^{-8}, 10^{-10} sec$, $\Gamma_c(\tau)$ can be measured at definite τ values disposing along the beam a series of detectors, at coordinates x, x', x''..., and measuring the intensity of c states in each of them. We only need the relativistic transformations:

$$t = \gamma\tau, \qquad x = vt = v\gamma\tau \tag{7}$$

where v is the 3-velocity and γ the Lorentz factor of the kaons.

The third term in eq.(5) is an interference term and originates in K system because it is possible to create superposition of $| S >$, $| L >$ states with strong and electromagnetic interactions that decay weakly in the same channel c. It produces in $\Gamma_c(\tau)$ an oscillating behaviour of decay product intensity on time τ (or x).

Fitting measured intensity $\Gamma_c(\tau)$ with expression (5) we can determine the K_L, K_S mass difference and both modulus and phase of the various decay amplitudes:

$$< c \,|\, K_L >, \qquad < c \,|\, K_S > . \tag{8}$$

The experimental confirmation of behaviour (5) is a clean verification of the superposition principle of quantum mechanics, since any non linearity inevitably leads to additional frequencies in time evolution. Also, with kaon systems, we are able to observe directly the time dependence of quantum amplitudes, that, according to wave-particle duality is given by$\sim e^{-iET}$.

CP violation has been discovered in K system detecting 2π decay modes ($CP = 1$) at long times τ, $\tau \gg \tau_S$, when only the long lived component $| L >$ is present in the beam. With careful measurement and analysis, going at intermediate times $\tau^{-1} \sim (\gamma_S + \gamma_L)/2$, it is possible to extract from the interference term in (5) the relative phase of $< \pi\pi \,|\, K_S >$ and $< \pi\pi \,|\, K_L >$ transition amplitudes (see sec.2.2.3 for implications).

An interesting consequence of equation (5) is the phenomenon of strangeness oscillations (we derive it in a simpler way than setting $| c\rangle =| \pi^- l^+ \nu_l\rangle$, $| \pi^+ l^- \overline{\nu}_l\rangle$ in (5) and using the selection rule $\Delta S = \Delta Q$). The strangeness content $a_{S=1}(\tau)$ in the beam at time τ is computed expressing $| \tau >$ as superposition of flavour eigenstates $K^0, \overline{K}^0$:

$$| \tau >= \frac{1}{\sqrt{2(1+ | \epsilon |^2}} \{(1+\epsilon)[a_S(0)e^{-iM_S\tau-\gamma_S\tau/2} + a_L(0)e^{-iM_L\tau-\gamma_L\tau/2}] \mid K^0 > \\ + (1-\epsilon)[a_S(0)e^{-iM_S\tau-\gamma_S\tau/2} - a_L(0)e^{-iM_L\tau-\gamma_L\tau/2}] \mid \overline{K}^0 >\} \tag{9}$$

The coefficients of $K^0, \overline{K}^0$ in eq.(9) give the amplitude of strangeness $S = \pm 1$.

Starting at $\tau = 0$ with a pure K^0 state (eq.(10)), strangeness intensities $I_{S=1}(\tau)$, $I_{S=-1}(\tau)$ are given by:

$$\begin{aligned} I_{S=+1} &= | a_{S=+1} |^2 = \frac{1}{4} [e^{-\gamma_S\tau} + e^{-\gamma_L\tau} + 2\cos\Delta M e^{-(\gamma_S+\gamma_L)\tau/2}] \\ I_{S=-1} &= | a_{S=-1} |^2 = \left| \frac{1-\epsilon}{1+\epsilon} \right|^2 \frac{1}{4}[e^{-\gamma_S\tau} + e^{-\gamma_L\tau} - 2\cos\Delta M e^{-(\gamma_S+\gamma_L)\tau/2}] \end{aligned} \tag{10}$$

Since empirically $\Delta M \sim \gamma_S \gg \gamma_L$, eq.(10) shows an effective oscillation in the strangeness content of the beam, before intensity decay occurs.

Since down quarks/antiquarks decay semileptonically only into negative/positive leptons ($\Delta S = \Delta Q$ rule), strangeness oscillations appearing in eq.(10) are observable, measuring the rate of negative and positive leptons produced in the beam as a function of τ (or x):

$$\begin{aligned} \Gamma_{l^-X^+}(\tau) &\propto I_{S=+1}(\tau) \\ \Gamma_{l^+X^-}(\tau) &\propto I_{S=-1}(\tau) \end{aligned} \tag{11}$$

A simple mechanical analog of strangeness oscillations is offered by a pair of classical oscillators with the same unperturbed frequency ω_0 coupled to each other by a tiny spring (see fig.1). Dropping dissipation effects, the equations of motion are:

$$\begin{aligned} \frac{d^2}{dt^2}a(t) &= -\omega_0^2 a(t) + \epsilon(a(t) - \overline{a}(t)) \\ \frac{d^2}{dt^2}\overline{a}(t) &= -\omega_0^2 \overline{a}(t) + \epsilon(a(t) - \overline{a}(t)) \end{aligned} \tag{12}$$

giving:

$$\begin{aligned} a(t) &= C \cos \epsilon t e^{-i\omega t} \\ \bar{a}(t) &= iC \sin \epsilon t e^{-i\omega t} \end{aligned} \tag{13}$$

The analogies are the following:

$$\begin{aligned} \text{unperturbed frequencies } \omega_0 &\leftrightarrow \text{strong eigenstate masses } M_{K^0} = M_{\overline{K}^0} \\ \text{tiny spring} &\leftrightarrow \text{weak forces} \\ \text{oscillation amplitudes} &\leftrightarrow \text{strangeness amplitudes} \\ \text{beats} &\leftrightarrow \text{strangeness oscillations} \end{aligned} \tag{14}$$

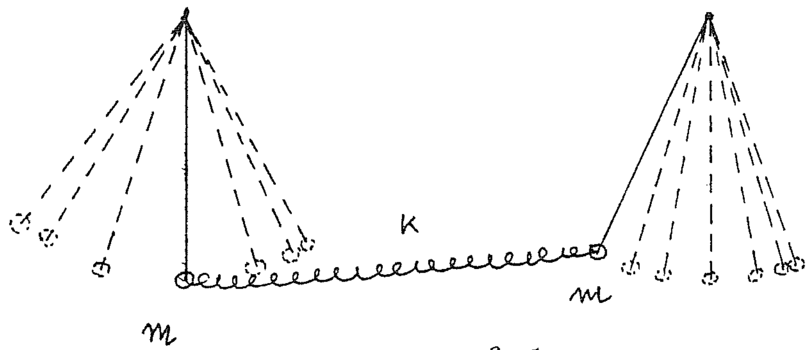


fig.1

2.2.2 ϵ parameter

In the case of K mesons it holds:

$$\begin{aligned} \mid ImM_{12} \mid &\ll \mid ReM_{12} \mid \\ \mid Im\Gamma_{12} \mid &\ll \mid Re\Gamma_{12} \mid \end{aligned} \tag{15}$$

and the mixing parameter can be expanded in ImM_{12}, $Im\Gamma_{12}$, keeping only the first order terms. We have from eq.(26) and eq.(34):

$$\epsilon_k \cong i\frac{1}{2}\frac{ImM_{12} + Im\Gamma_{12}/2}{ReM_{12} - iRe\Gamma_{12}/2} \tag{16}$$

Since we are interested in comparing theory with experiments only with respect to CP violation, we can express ReM_{12}, $Re\Gamma_{12}$ in terms of mass differences and widths determined experimentally. In strangeness oscillations we can surely drop CP violation effects, and using eq.(33) we get:

$$\Delta M \simeq 2ReM_{12} \tag{17}$$

and

$$Re\Gamma_{12} \simeq \mid \Gamma_{12} \mid \simeq \frac{\Delta\Gamma}{2} = \frac{\Gamma_L - \Gamma_S}{2} \simeq -\frac{\Gamma_S}{2} \tag{18}$$

then:

$$\epsilon_k \cong \frac{iImM_{12}}{\Delta M_k + i\Gamma_S/2} \tag{19}$$

where we have neglected $\mid Im\Gamma_{12} \mid \ll \mid ImM_{12} \mid$. Experimentally $\Gamma_S/2 \simeq \Delta M_k$ and then:

$$\epsilon_k \cong \frac{iImM_{12}}{(1+i)\Delta M_k} = \frac{e^{+i\pi/4}}{\sqrt{2}} \cdot \frac{ImM_{12}}{\Delta M_k} \tag{20}$$

Then ϵ_k phase is known as a consequence of $\mid Im\Gamma_{12} \mid \ll \mid ImM_{12} \mid$ and the theoretical task reduce to computing the imaginary part of the dispersive box diagram amplitude ImM_{12}.

Let us now discuss the computation of M_{12} in the framework of the SM. As discussed in chapter (1), in principle, the knowledge of ϵ fixes the complex phase in the CKM matrix. M_{12} is computed by means of an effective $\mid \Delta S \mid = 2$ weak Hamiltonian $H_{eff}^{|\Delta S|=2}$:

$$M_{12} = \langle \overline{K}^0 \mid H_{eff}^{|\Delta S|=2} \mid K^0 \rangle \tag{21}$$

The inclusion of perturbative QCD corrections in $H_{eff}^{|\Delta S|=2}$ is essential for a quantitative understanding of the mixing. For simplicity, however, we expose the computation neglecting in a first step the effect of high energy strong interactions. We will take them into account later.

$H_{eff}^{|\Delta S|=2}(x)$ is given by the dispersive part of the $|\Delta S|= 2$ box diagrams having as external states $\overline{K}^0$ and K^0 valence quarks with zero momenta (see fig.2).

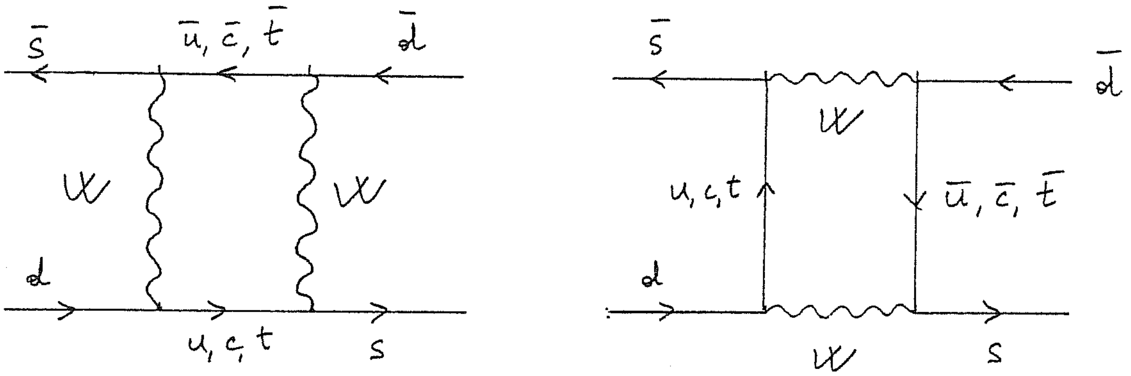


fig.2

The transition requires two W emissions and absorptions, with a total of four weak vertices that generate a double W scattering or annihilation of the initial d, $\bar{s}$ quarks. Computing box diagrams (a) and (b) with arbitrary masses of the up quarks M_i, M_j circulating in the loop, we have (Inami and Lim [5]):

$$H_{eff}^{|\Delta S|=2} = \frac{G_F^2}{16\pi^2} M_W^2 (\bar{d}\gamma_\mu(1-\gamma_5)s)^2 \sum_{i,j}^{u,c,t} \lambda_i \lambda_j E(x_i, x_j) \tag{22}$$

where

$$\begin{aligned} \lambda_i &= V_{is} V_{id}^* \\ x_i &= \frac{M_i^2}{M_W^2} \end{aligned} \tag{23}$$

and $E(x_i, x_j)$ (Inami-Lim functions) are given by:

$$E(x_i, x_i) = E(x_i) = x_i[\frac{1}{4} + \frac{9}{4}\frac{1}{(1-x_i)^2} - \frac{3}{2}\frac{1}{(1-x_i)^2}] + \frac{3}{2}\frac{x_i}{x_i - 1}\ln x_i \tag{24}$$

for $i = j$ and:

$$\begin{aligned} E(x_i, x_j) =& x_i x_j \{[\frac{1}{4} + \frac{3}{2}\frac{1}{1-x_j} - \frac{3}{4}\frac{1}{(1-x_j)^2}]\frac{\ln x_j}{x_j - x_i} + \\ &+ (x_j \leftrightarrow x_i) - \frac{3}{4}\frac{1}{(1-x_i)(1-x_j)}\} \end{aligned} \tag{25}$$

for $x_j \neq x_i$. For $x_i \ll 1$ we have:

$$E(x_i) \simeq x_i \tag{26}$$

implying that a given quark i has a contribution to mixing $\propto M_i^2$. For $x_j \ll x_i \ll 1$:

$$E(x_i, x_j) \simeq x_j \ln \frac{x_i}{x_j} \tag{27}$$

Inserting eq.(22) into eq.(21) we get:

$$M_{12} = \frac{G_F^2}{16\pi^2} M_W^2 (\sum_{i,j}^{u,c,t} \lambda_i \lambda_j E(x_i, x_j))\ S \tag{28}$$

where:

$$S = < \overline{K}^0 \mid (\overline{d}\gamma_\mu(1-\gamma_5)s)^2 \mid K^0 > \tag{29}$$

Low energy strong interactions acting at scales $\sim M_K$ up to zero momenta enter in the computation of the matrix element S, that involves non perturbative physics. A reference value for S is obtained inserting vacuum state only as intermediate state between $\overline{d}s$ quark bilinears. We set:

$$S = \frac{4}{3} B_k f_k^2 M_k \tag{30}$$

where f_k is the kaon decay constant

$$< 0 \mid \overline{s}\gamma_\mu(1-\gamma_5)d \mid K^0 > = \frac{i p_\mu f_k}{\sqrt{2E}} \tag{31}$$

and B_k is a factor parametrizing deviations from vacuum saturation approximation in S. A factor 2 is included in eq.(30) to take into account that each of the currents $\overline{s}\gamma_\mu(1-\gamma_5)d$ can annihilate (create) $K^0(\overline{K}^0)$ meson. A further factor $\frac{4}{3} = 1 + \frac{1}{3}$

arises because $K^0(\overline{K}^0)$ can be annihilated (created) by $\bar{s}, d$ operators in the same or in different colour singlet bilinears. Using eq.(30) we get:

$$M_{12} = \frac{G_F^2}{12\pi^2} M_W^2 B_k f_k^2 M_k \sum_{i,j}^{u,c,t} \lambda_i \lambda_j E(x_i, x_j) \tag{32}$$

Taking the real part of M_{12}, we compute the neutral kaon mass difference

$$\Delta M_k \simeq 2ReM_{12} \tag{33}$$

while ImM_{12} determines the mixing parameter $|\ \epsilon\ |$ ($t_0 = ImA_0/ReA_0$ can safely be neglected).

Let us discuss the computation of ϵ.

By unitarity:

$$\lambda_u + \lambda_c + \lambda_t = \sum_i V_{di}^\dagger V_{is} = 0 \tag{34}$$

and by convention:

$$Im\lambda_u = 0 \tag{35}$$

then:

$$Im\lambda_c = -Im\lambda_t \tag{36}$$

For ϵ we have to compute:

$$\begin{aligned} Im\lambda_c^2 &= 2Re\lambda_c Im\lambda_t \simeq -2\lambda Im\lambda_c = 2\lambda Im\lambda_t \\ Im\lambda_t^2 &= 2Re\lambda_t Im\lambda_t \simeq -2A^2\lambda^5(-1+\rho\cos\delta)Im\lambda_t \\ Im(\lambda_c\lambda_t) &\simeq Re\lambda_c Im\lambda_t \simeq -\lambda Im\lambda_t \end{aligned} \tag{37}$$

Neglecting the small contribution of $u-c$, $u-t$ exchange we derive for the imaginary part:

$$\begin{aligned} &Im(\lambda_c^2 E(x_c) + \lambda_t^2 E(x_t) + 2\lambda_c\lambda_t E(x_c, x_t)) = \\ &= -2A^2\lambda^6\rho\sin\delta[E(x_c, x_t) + A^2\lambda^4(1-\rho\cos\delta)E(x_t) - E(x_c)] \end{aligned} \tag{38}$$

and then:

$$
\begin{aligned}
|\,\epsilon\,| = \frac{1}{\sqrt{2}\Delta M_k}\frac{G_F^2}{12\pi^2}M_W^2 B_k f_k^2 M_k 2A^2\lambda^6\rho\sin\delta\,\times \\
\times[E(x_c,x_t) + A^2\lambda^4(1-\rho\cos\delta)E(x_t) - E(x_c)]
\end{aligned}
\tag{39}
$$

Let us now consider briefly perturbative corrections to the effective Hamiltonian. One loop QCD corrections to $H_{eff}^{|\Delta S|=2}$ consist in one gluon exchange between all pairs of quark legs (both internal and external) in the box diagram; they have been computed in LLA in the two limiting cases:

1) $M_t \ll M_W$ that is now ruled out by CDF experimental lower bound 89 $GeV < M_t$ [6,7,8].

2) $M_t \gg M_W$ [6,9].

In general, corrections to the bare result (39) are of order $\sim 150 \div 300\%$. We sketch only the discussion (for a much deeper analysis see ref. [6]). In the first case, the effect of strong interactions amounts simply to a rescaling of factors $\eta_1,\ \eta_2,\ \eta_3$ of the bare Inami-Lim functions:

$$
\begin{aligned}
F(x_c) &= \eta_1 E(x_c) \\
F(x_t) &= \eta_2 E(x_t) \\
F(x_c,x_t) &= \eta_3 E(x_c,x_t)
\end{aligned}
\tag{40}
$$

where we called F_i the corrected functions. In the second case, the functional dependence on M_c, M_t is different but there is a good agreement in the intermediate top mass region: $M_t \sim M_W$ [6]. In general, the computation of case 1) can be extrapolated to region 2) while viceversa is not true. In table (1) we give a tabulation of the Inami-Lim functions taken from reference [6] for the cases: 1) no strong interactions, 2) $M_t \ll M_W$, 3) $M_t \gg M_W$.

Remarks:

1) It is interesting to note that $|\,\epsilon\,|$ depends on many parameter of the SM: the

masses M_W, M_c, M_t, M_s, all the angles of the CKM matrix λ, A, ρ, δ and Λ_{QCD}, through strong radiative corrections.

2) At the present stage of knowledge two essentially unknowns appear in eq.(39): the M_t value, from which $\mid \epsilon \mid$ is highly dependent and that gives $\sim 90\%$ of the contribution, and $\sin\delta$, to which $\mid \epsilon \mid$ is roughly proportional.

3) The quantities ReM_{12}, ImM_{12} have a different theoretical reliability. Apart from the hadronic matrix element S, $\mid \epsilon \mid$ is a short distance quantity since it involves only heavy quark c and t exchange.

The computation of ReM_{12} is instead uncertain because it depends on long distance effects of strong interactions, for the following facts:

1) Double top quark exchange in box diagram gives a negligible contribution to ReM_{12} for every realistic value of $M_t < M_c/\lambda^4 \sim 500 \div 600\ GeV$, because of Cabibbo suppression. In Wolfenstein parametrization we have:

$$
\begin{aligned}
\lambda_u &= +(1-\lambda^2/2)\lambda + \ldots = \quad +\lambda + \ldots \\
\lambda_c &= -\lambda(1-\lambda^2/2) + \ldots = \quad -\lambda \\
\lambda_t &= -A^2\lambda^5(-1+\rho e^{i\delta}) = -A^2\lambda^5(-1+\rho\cos\delta) - iA^2\lambda^5\rho\sin\delta
\end{aligned}
\tag{41}
$$

then

$$
\begin{aligned}
\lambda_c^2 &\sim \lambda^2 \\
\lambda_t &\sim \lambda^{10}
\end{aligned}
\tag{42}
$$

and then:

$$
\frac{(\text{weight of top})}{(\text{weight of c})} = \frac{E(x_t)\lambda_t^2}{E(x_c)\lambda_c^2} \sim \left(\frac{M_t}{M_c/\lambda^4}\right)^2 \ll 1
\tag{43}
$$

Also top-charm exchange gives a negligible contribution since:

$$
\frac{E(x_c, x_t)\lambda_c\lambda_t}{E(x_c)\lambda_c^2} \sim \ln\left(\frac{M_t}{M_c}\right)\lambda^4 \ll 1
\tag{44}
$$

2) The contribution of *up* quark exchange is important.

In a convergent loop integral, the region of momenta which gives the main contribution to the amplitude, are the momenta of the order of the external momenta:

$$| p_{loop} | \sim | p_{ext} | \sim M_K \qquad (45).$$

In loops involving t or c quarks, it is then justified setting to zero external momenta (that is the same as shrinking the box diagram to a point), since $M_t, M_c \gg M_k$. On the contrary, for an up quark circulating in the loop the non local and non perturbative effects corresponding to soft internal momenta are very important. The leading contribution to ΔM_k coming from double charm exchange alone , is not sufficient to explain the observed experimental value:

$$\frac{\Delta M_k(box)}{\Delta M_k(exp)} = 0.7(\frac{M_c}{1.5})B_k \qquad (46)$$

and, from lattice computations, $B_k = 0.94 \pm 0.02$.

2.2.3 ϵ' parameter

By means of interference effects (see sec.2.2.1) it is possible in neutral kaon system to measure both modulus and phase of the CP violating amplitudes:

$$\begin{aligned} \eta_{+-} &= \frac{< \pi^+\pi^- \mid K_L >}{< \pi^+\pi^- \mid K_S >} \\ \eta_{00} &= \frac{< \pi^0\pi^0 \mid K_L >}{< \pi^0\pi^0 \mid K_S >} \end{aligned} \qquad (47)$$

The observable (47) are expressed in terms of the physical states $\mid K_S >, \mid K_L >$ and $\mid \pi^+\pi^- >, \mid \pi^0\pi^0 >$. As stressed above, hadronic matrix elements are computed with perturbation theory in the basis of the eigenstates of the strong interaction Hamiltonian H_0, composed of flavour eigenstates $\mid K^0 >, \mid \overline{K}^0 >$ and isospin eigenstates $\mid \pi\pi(I=0)>$, $\mid \pi\pi(I=2)>$. To compare predictions with measures it is necessary to connect the two set of states:

$$\begin{array}{ccc} K_S,\ K_L & \rightarrow & K^0,\ \overline{K}^0 \\ \pi^+\pi^-, \pi^0\ \pi^0 & & I=0,\ I=2 \end{array} \tag{48}$$

Let us divide the computation in two steps:

1) $\pi^+\pi^-, \pi^0\pi^0 \rightarrow \pi\pi(I=0,2)$

Since pions are emitted in S wave, by Bose symmetry:

$$\begin{aligned} \mid \pi^0\pi^0 > &= \sqrt{\frac{2}{3}} \mid I=2> \ - \ \frac{1}{\sqrt{3}} \mid I=0> \\ \mid \pi^+\pi^- > &= \frac{1}{\sqrt{3}} \mid I=2> \ + \ \sqrt{\frac{2}{3}} \mid I=0> \end{aligned} \tag{49}$$

Expressing η_{+-}, η_{00} in terms of amplitudes for pions with definite isospin, we have:

$$\begin{aligned} \eta_{+-} &= \frac{a_{0L} + 1/\sqrt{2}a_{2L}}{a_{0S} + 1/\sqrt{2}a_{2S}} \\ \eta_{00} &= \frac{a_{0L} - \sqrt{2}a_{2L}}{a_{0S} - \sqrt{2}a_{2S}} \end{aligned} \tag{50}$$

where we have defined the 4 relevant transition amplitudes:

$$\begin{array}{lllll} a_{0S} & = & <0 \mid S> & \Delta I = 1/2 & CP\ cons \\ a_{0L} & = & <0 \mid L> & = 1/2 & viol \\ a_{2S} & = & <2 \mid S> & = 3/2 & cons \\ a_{2L} & = & <2 \mid L> & = 3/2 & viol \end{array} \tag{51}$$

a_{0S} is clearly the largest amplitude since it is $\Delta I = 1/2$ and it is CP conserving. a_{0L}/a_{2S} is suppressed because it is CP violating/$\Delta I = 3/2$. a_{2L} is doubly depressed being at the same time $\Delta I = 3/2$ and CP violating.

From eq.(50) we see that $\eta_{+-} \neq \eta_{00}$ as a consequence of small $\Delta I = 3/2$ weak transitions.

Expressing η_{+-} and η_{00} as sums of $\Delta I = 1/2$ and $\Delta I = 3/2$ terms we get:

$$\begin{aligned} \eta_{+-} &= \epsilon + \frac{\epsilon'}{1 + 1/\sqrt{2}\omega} \\ \eta_{00} &= \epsilon - 2\frac{\epsilon'}{1 - \sqrt{2}\omega} \end{aligned} \tag{52}$$

where:

$$\begin{aligned} \epsilon &= \frac{a_{0L}}{a_{0S}} \\ \omega &= \frac{a_{2S}}{a_{0S}} \\ \epsilon' &= \frac{1}{\sqrt{2}}(\frac{a_{2L}}{a_{0S}} - \omega\epsilon) \end{aligned} \tag{53}$$

ω is the ratio of CP conserving $\Delta I = 3/2$ and $\Delta I = 1/2$ amplitudes, and is known experimentally to be very small:

$$| \omega | \simeq 0.045 \tag{54}$$

Neglecting ω with respect to 1 we arrive at the well known expressions:

$$\begin{aligned} \eta_{+-} &\cong \epsilon + \epsilon' \\ \eta_{00} &\cong \epsilon - 2\epsilon' \end{aligned} \tag{55}$$

Note that simply neglect a_{2S} with respect to a_{0S} in eq.(50) results in a mistake: we loose the $\omega\epsilon$ term in eq.(53) that is of the same order as a_{2L}/a_{0S}. ϵ' is expected to be much smaller than ϵ_k since it originates from $\Delta I = 3/2$ and CP-violating interactions.

2) $K_L,\ K_S \rightarrow K^0,\ \overline{K}^0$

We have to express $| K_S >$, $| K_L >$ in terms of $| K^0 >$, $| \overline{K}^0 >$ and the mixing parameter ϵ_k:

$$\begin{aligned} | K_S > &= N(\epsilon)[\ (1 + \epsilon_k) \mid K^0 > \ + (1 - \epsilon_k) \mid \overline{K}^0 >] \\ | K_L > &= N(\epsilon)[\ (1 + \epsilon_k) \mid K^0 > \ - (1 - \epsilon_k) \mid \overline{K}^0 >] \end{aligned} \tag{56}$$

Substituting eq.(56) in eq.(51) and eq.(51) in eq.(53), we get:

$$\epsilon = \frac{\epsilon_k + it_0}{1 + i\epsilon_k t_0} \tag{57}$$

where we defined:

$$\begin{aligned} < 0 \mid K^0 > &= A_0 e^{i\delta_0} \\ < 0 \mid \overline{K}^0 > &= A_0^* e^{i\delta_0} \\ < 2 \mid K^0 > &= A_2 e^{i\delta_2} \\ < 2 \mid \overline{K}^0 > &= A_2^* e^{i\delta_2} \end{aligned} \tag{58}$$

where $t_0 = ImA_0/ReA_0$ and we have factored out strong interaction phases. Since strong interactions are CP conserving, their phases do not change sign under CP transformation that brings the first/third equation into the second/fourth.

For ω we have:

$$\omega = e^{i(\delta_2-\delta_0)} \frac{\frac{ReA_2}{ReA_0} + i\epsilon_k \frac{ImA_2}{ReA_0}}{1 + i\epsilon t_0} \tag{59}$$

The exact expression of ϵ' is finally given by:

$$\epsilon' = \frac{e^{i(\delta_2-\delta_0+\pi/2)}}{\sqrt{2}} \frac{1-\epsilon_k^2}{(1+i\epsilon_k t_0)^2} [\frac{ImA_2}{ReA_0} - \frac{ImA_0}{ReA_0}\frac{ReA_2}{ReA_0}] \tag{60}$$

Since $\mid \epsilon_k \mid \sim 10^{-3}$ and $\mid t_0 \mid \ll 1$ (since decay amplitudes have small imaginary parts) we may safely approximate eq.(60) as:

$$\epsilon' \cong \frac{e^{i(\delta_2-\delta_0+\pi/2)}}{\sqrt{2}} [\frac{ImA_2}{ReA_0} - \frac{ImA_0}{ReA_0}\frac{ReA_2}{ReA_0}] \tag{61}$$

ϵ' phase is then determined uniquely in terms of strong interaction phases. From partial wave analysis of $\pi\pi$ diffusion it is known that:

$$\delta_2 - \delta_0 + \frac{\pi}{2} = (48 \pm 8)^o \tag{62}$$

ϵ' and ϵ phases are then approximately equal and the theory has only to compute the ratio of their moduli:

$$\left| \frac{\epsilon'}{\epsilon} \right| \tag{63}$$

At present the best experimental determinations os $|\epsilon'/\epsilon|$ are:

$$\begin{aligned} \left| \frac{\epsilon'}{\epsilon} \right| &= (+3.3 \pm 1.1) \cdot 10^{-3} \qquad \text{(CERN)} \\ &= (-0.5 \pm 1.5) \cdot 10^{-3} \qquad \text{(FERMILAB)} \end{aligned} \tag{64}$$

2.3 $B - \overline{B}$ mixing

The study of $B - \overline{B}$ mixing gives independent informations on the CKM matrix in addition to neutral kaons. A combined analysis of $K - \overline{K}$ and $B - \overline{B}$ systems can then impose interesting constraints on the value of the complex phase ϕ. Up to now only beauty oscillations have been measured with B mesons. We then limit ourselves to discuss the experimental measure and the theoretical computation of $B - \overline{B}$ mixing.

2.3.1 Time integrated beauty oscillations

Observables in $B - \overline{B}$ mixing are not the same as in kaon system because of concrete experimental differences: lifetimes are much shorter and relativistic time dilation factors, at the same energies, are reduced of an order of magnitude:

$$\tau_B \sim 10^{-12}\, sec \ll \tau_K \quad \text{and} \quad \frac{M_K}{M_B} \sim \frac{1}{10} \tag{1}$$

As a consequence, up to now, only time-integrated mixing parameters have been measured. Just as in the case of strangeness oscillations (sec.2.2.1), also beauty oscillations can be studied considering semileptonic decays of B mesons. A classical measure of mixing in $B - \overline{B}$ events is the ratio ρ of the numbers of events with same sign dileptons to that of unlike sign:

$$\rho = \frac{N(l^+l^+) + N(l^-l^-)}{N(l^+l^-)} \tag{2}$$

If beauty particles are produced incoherently, as in hadronic collisions $\pi + p$, $\pi +$ target, $p + \overline{p}$, etc., B and $\overline{B}$ evolve independently. Starting with a pure B^0 state at $t = 0$, the ratio r of positive l^+ to negative leptons l^- produced in the whole story of the event is given by the time integrated intensities:

$$\begin{aligned} r = & \frac{N_{l^+}}{N_{l^-}} = \frac{\int_0^\infty I_{B=-1}(\tau)d\tau}{\int_0^\infty I_{B=+1}(\tau)d\tau} \\ & = \left| \frac{1-\epsilon}{1+\epsilon} \right|^2 \frac{x^2+y^2}{2+x^2-y^2} \end{aligned} \tag{3}$$

where we used equation (2.2.10) and we defined:

$$x = \frac{\Delta M}{\overline{\Gamma}}, \qquad y = \frac{\Delta\Gamma}{2\overline{\Gamma}} \tag{4}$$

$\overline{\Gamma}$ is a mean B meson width. Starting with a pure $\overline{B}$ state we have instead:

$$\begin{aligned} \overline{r} = & \frac{N_{l^-}}{N_{l^+}} = \frac{\int_0^\infty \overline{I}_{B=+1}(\tau)d\tau}{\int_0^\infty \overline{I}_{B=-1}(\tau)d\tau} \\ & = \left| \frac{1+\epsilon}{1-\epsilon} \right|^2 \frac{x^2+y^2}{2+x^2-y^2} \end{aligned} \tag{5}$$

Neglecting CP violation in beauty oscillations, we may set:

$$r = \overline{r}. \tag{6}$$

We may also neglect y with respect to x since by eq.(2.1.19):

$$\begin{aligned} \Delta\Gamma \cong 2 \mid \Gamma_{12} \mid \sim M_b^2 \\ \Delta M \cong 2 \mid M_{12} \mid \sim M_t^2 \end{aligned} \tag{7}$$

then:

$$r = \overline{r} = \frac{x^2}{2 + x^2} \tag{8}$$

As it is intuitively clear, mixing increases with ΔM and decreases with small lifetimes.

In the incoherent case ρ is then given by:

$$\rho = \frac{2r}{1 + r^2} \qquad \text{(incoherence)} \tag{9}$$

In the case of coherent production, as is the case in timelike γ, Y or Z^0 decays, $B\overline{B}$ do not evolve independently, because of Bose symmetry. For $J = odd$ in the initial state, we have [3]:

$$\rho = r \qquad \text{(J = odd)} \tag{10}$$

2.3.2 $B - \overline{B}$ mixing in the SM

M_{12} in $B - \overline{B}$ mixing is computed as in K system with $\Delta B = 2$ box diagrams. Since external quarks are $b\overline{d}(\overline{s})$ instead of $s\overline{d}$, top-top exchange contribution is by far the dominant one, both for ReM_{12} and ImM_{12} (fig.1).

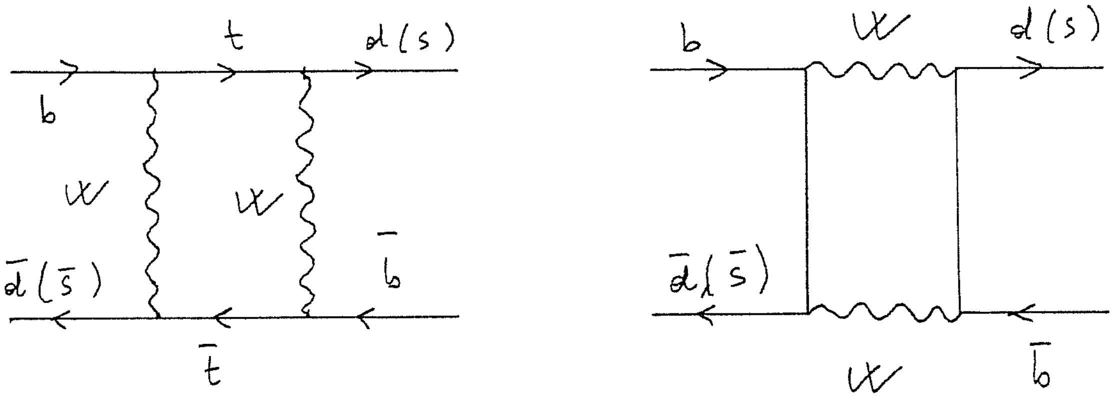


fig.1

$B - \overline{B}$ mixing is then a short distance phenomenon. Perturbative QCD corrections to the effective Hamiltonian $H_{eff}^{|\Delta B|=2}$ are the same as in K system, since external momenta are set to zero. We have then:

$$\begin{aligned} x_{B_d} = & \frac{\Delta M_{B_d}}{\Gamma_{B_d}} = 2 \mid M_{12} \mid \tau_{B_d} = \\ = & \frac{G_F^2 M_W^2 M_{B_d}}{6\pi^2} A^2 \lambda^6 (1 + \rho^2 - 2\rho\cos\delta) F(x_t) f_{B_d}^2 B_{B_d} \tau_{B_d} \end{aligned} \tag{11}$$

The main uncertainties in eq.(11) come from the unknown value of the top quark mass M_t which enters in the argument of the function $F(x_t)$, and from the hadronic matrix element of the effective $\Delta B = 2$ Hamiltonian between B meson states. As in the case of $K^0\overline{K}^0$ mixing (eq.1.2.19), the hadronic matrix element is parametrized in terms of the coefficient B_{B_d} and the B meson decay constant f_{B_d}. There is a general agreement that $B_{B_d} = 1$. The main justifications come from lattice computations [10] and from the Zweig rule. For the value of f_{B_d} to insert in eq.(11) the situation is still controversial. In the case of kaons, f_K is measured through leptonic deacay: $K^{\pm} \to l^{\pm} + \nu_l$. For B mesons instead, the branching ratio for $B \to l + \nu_l$ is extremely small and it has not yet been measured or bounded; it is then necessary refer to a theoretical value. We report two determinations of f_{B_d}. On the one side, assuming the validity of the decay constant scaling law of the static theory (sec.4.2) and assuming for f_D the value from lattice computations and QCD sum rules [3]

$$f_D \cong 180\ Mev \tag{13}$$

we get:

$$f_B^{scal} \cong\ 110\ MeVv \tag{14}$$

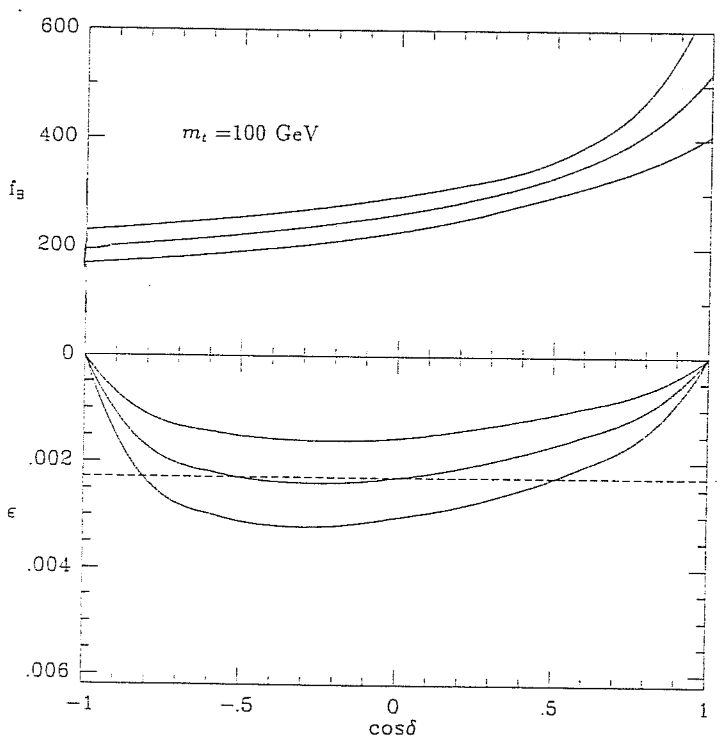
On the either side, assuming for f_B the value computed by two groups with lattice QCD simulations of the static b theory (see sec.(4.2)), we have [10];

$$f_B^{stat} = (310 \pm 25 \pm 50)\ MeV \tag{15}$$

2.3.3 Combined analysis of $K^0 - \overline{K}^0$ and $B^0 - \overline{B}^0$ mixing

Without making a choice between values (13) and (15) for f_{B_d}, we report the plot of ref.[6] of f_B and ϵ_k as functions of $\cos\delta$ at given M_t values (fig.2). Let us give some qualitative comment. Increasing the M_t value, GIM mechanism becomes less efficient and ϵ_k increases. At given ϵ_k value, then, the range of allowed $\sin\delta$ values moves toward zero with increasing M_t. It turns out that, for large M_t values, imposing the experimental ϵ_K value, two allowed regions of $\cos\delta$ emerge.

As stems from eq.(11), x_{B_d} decreases with increasing $\cos\delta$. At given x_{B_d} value, then, $\cos\delta$ increases with increasing f_{B_d}. The small value of f_B (13) selects the region with $\cos\delta < 0$ while the large one (15) the region with $\cos\delta > 0$.



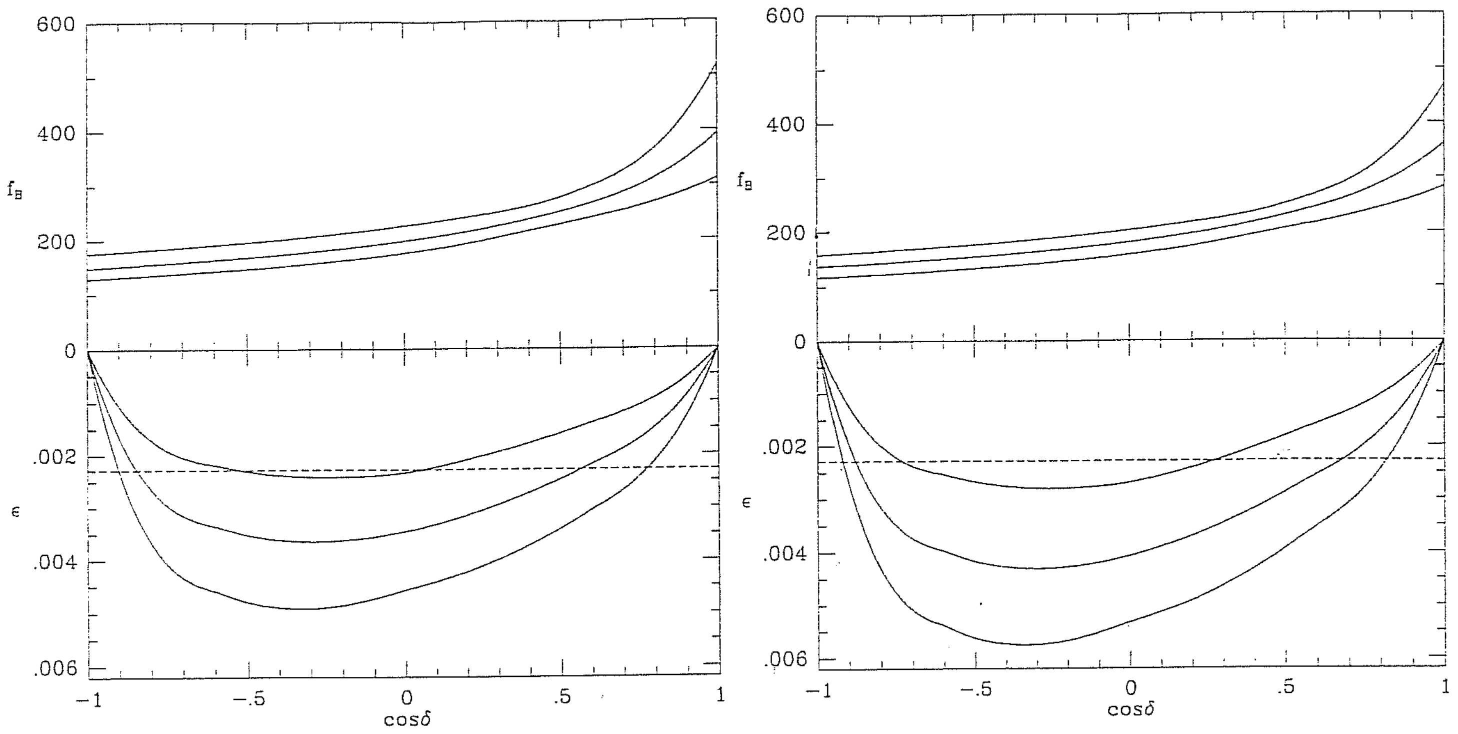


fig. 2

m_t	$E_I(x_t)$	$E_G(x_t)$	$E_D(x_t)$
40	0.221	0.138	0.118
50	0.331	0.197	0.180
60	0.456	0.267	0.252
70	0.595	0.342	0.328
80	0.745	0.424	0.411
90	0.906	0.515	0.503
100	1.075	0.607	0.597
110	1.253	0.709	0.700
120	1.439	0.811	0.802
130	1.632	0.916	0.908
140	1.832	1.025	1.017
150	2.038	1.137	1.130
160	2.251	1.252	1.245
170	2.471	1.381	1.375
180	2.696	1.504	1.498
190	2.928	1.629	1.623
200	3.166	1.757	1.752
210	3.411	1.896	1.891
220	3.661	2.030	2.026
230	3.918	2.168	2.164
240	4.181	2.314	2.310
250	4.450	2.464	2.460

$E_I(x_t)$, $E_G(x_t)$ and $E_D(x_t)$ for different values of m_t

m_t	$E_I(x_c, x_t)$	$E_G(x_c, x_t)$	$E_D(x_c, x_t)$
40	2.00 e-03	6.86 e-04	6.84 e-04
50	2.14 e-03	7.27 e-04	7.25 e-04
60	2.25 e-03	7.50 e-04	7.49 e-04
70	2.33 e-03	7.67 e-04	7.67 e-04
80	2.40 e-03	7.81 e-04	7.81 e-04
90	2.46 e-03	7.93 e-04	7.93 e-04
100	2.51 e-03	8.03 e-04	8.03 e-04
110	2.56 e-03	8.11 e-04	8.12 e-04
120	2.60 e-03	8.18 e-04	8.19 e-04
130	2.63 e-03	8.32 e-04	8.33 e-04
140	2.66 e-03	8.38 e-04	8.39 e-04
150	2.69 e-03	8.44 e-04	8.45 e-04
160	2.72 e-03	8.49 e-04	8.50 e-04
170	2.74 e-03	8.53 e-04	8.54 e-04
180	2.77 e-03	8.57 e-04	8.58 e-04
190	2.79 e-03	8.60 e-04	8.62 e-04
200	2.80 e-03	8.64 e-04	8.65 e-04
210	2.82 e-03	8.67 e-04	8.68 e-04
220	2.84 e-03	8.70 e-04	8.71 e-04
230	2.85 e-03	8.72 e-04	8.74 e-04
240	2.87 e-03	8.75 e-04	8.76 e-04
250	2.88 e-03	8.77 e-04	8.79 e-04

$E_I(x_c, x_t)$, $E_G(x_c, x_t)$ and $E_D(x_c, x_t)$ for different values of m_t

Table 1

Chapter 3

Effective theories

3.1 Introduction

An effective theory [1,2] is a tool for computing low energy processes, with a prescribed accuracy. It may be generated from a complete theory (describing processes at every mass scale) as an expansion for small momenta. The main advantage is that of being simpler than the original theory.

The reasons that lead to the construction of effective theories are related to the following characteristics of quantum phenomena. The amplitude of a process occurring at an energy scale E receives contributions from virtual states of every energy $E' \neq E$. In many important cases the relevant contributions come from high energy states. This implies that when studying an elementary process at the scale E we cannot simply forget about the different energy scales E' present in nature.

A clear example of this aspect of quantum theory is neutron beta decay,

$$n \rightarrow p + e + \nu. \tag{1}$$

It is generated by the exchange of the W boson that has a mass $M_W \sim 80\ GeV$, while the scale of the process is $E \sim M_n \sim 1\ GeV$. The decay occurs because of the existence of the W boson, that enters in an intermediate state with a high virtuality. The contribution of particles with mass $M > E$ is just the origin of the decay. The decay amplitude is a function of the W mass $A = A(M_W)$ and at the tree level can be expanded as:

$$A(M_W) = \frac{A_1}{M_W^2} + \frac{A_2}{(M_W^2)^2} + \dots \tag{2}$$

where A_1, $A_2, \dots$ are functions of the external momenta only.
Perturbative corrections modify the behaviour of eq.(2) only by logarithmic factors of the form $\ln(M_W/\mu)$. There is then a complete decoupling only in the limit $M_W \to \infty$. Since the process is depressed by the high virtuality of the intermediate state, we are interested only in the lowest order (or at most in the first few terms) in the $1/M_W^2$ expansion. The systematic method for isolating the leading terms in the inverse of heavy particle masses involves the construction of an effective theory.

The idea is that we can neglect the W field for the description of the neutron decay and replace it's effects by new local interactions between the particles appearing as asymptotic states: n, p, e, ν. At the tree level, it corresponds to the following expansion for the W propagator:

$$\frac{1}{q^2 - M_W^2 + i\epsilon} = -\frac{1}{M_W^2} - \frac{1}{M_W^2}\frac{q^2}{M_W^2} + \dots \tag{3}$$

On the right hand side of eq.(3) the degrees of freedom of the W field do not appear any more and new interactions take their place. In general, with effective theories we truncate expansion (3) at a given order, determined by the precision required. Since $q^2 \sim (M_n - M_p)^2 \ll M_W^2$ the series is rapidly convergent. At lowest order in $1/M_W^2$ we recover the old Fermi theory of beta decay.

Other important examples are given by the effective Hamiltonians for proton decay or neutron-antineutron oscillations in the framework of Grand Unified Theories.

These examples generalize to light particles with low energy E whose dynamic is generated or largely modified by heavy particle effects. By 'heavy' we mean particles with mass $M > E$, that cannot appear as physical asymptotic states.

In the real world many different mass scales are present: the masses of the observed particles, the QCD fundamental mass scale, the Plank mass etc... Every elementary process receives dynamical contributions related to the existence of these mass scale. It is possible to build up an effective theory for low energy processes eliminating explicitly the particles with mass M greater than the energy scale E:

$$M > E \tag{4}$$

The fundamental property of the these effective theories is that they are simpler than the original theory and at the same time contain the effects of the heavy particles. They can be thought as an expansion of the heavy particle 4-momentum around the null vector

$$p_{null} = (0, \vec{0}). \tag{5}$$

The construction of the effective Hamiltonians is outlined in section (3.2) and a well-known example is discussed in section (3.4).

Another interesting example is the anomalous magnetic moment of the electron μ_e. As we shall see in section (3.2), a proper generalization and formalisation of this example leads to the construction of the so called non relativistic QED. The anomalous part of μ_e is computed in QED by loop corrections to the basic electron-photon vertex (see fig.1) and is then a consequence of virtual electron-photon states.

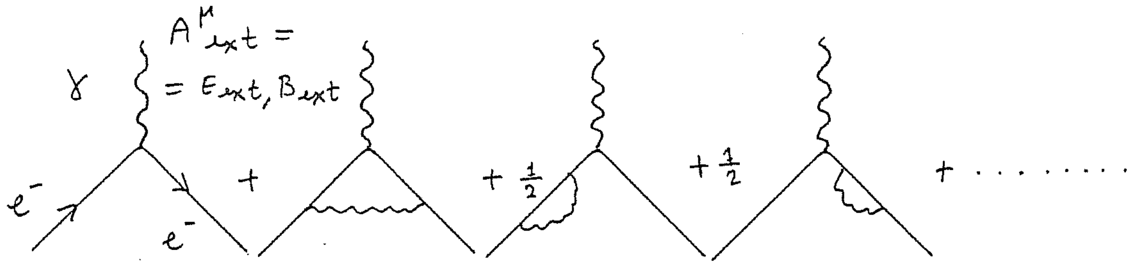


fig.1

An important contribution is given by relativistic virtual electrons, with energies $E \gtrsim m_e$, because they feel an increased external magnetic field B_{ext} induced by the Lorentz transformation:

$$B'_{ext} \sim \gamma_L \; B_{ext} \tag{6}$$

where γ_L is the Lorentz factor of the boost from the laboratory frame to the rest frame for the virtual electron.

Relativity is then necessary and the computation of the same graphs with the Pauli Hamiltonian would result in a quite different correction value for μ_e.

The experiment for the measure of μ_e can be realised with very slow electrons and a weak external magnetic field B_{ext}; the only mass scale involved is m_e. Then, we arrive at the conclusion that with an accurate measure at the scale m_e one can observe effects related to much higher scales $E \gg m_e$. In other words, with experiments involving only slow electrons we may have an indirect verification of relativity, that refers to states with $E \gtrsim m_e$.

Now it comes the fundamental observation. Even though there is a coupling between low energy and high energy states in quantum theory, we can separate the two with the following strategy. If we limit ourselves to measure slow electrons and soft photons ($\lambda \gg 1/m_e$) we can take into account the anomalous magnetic moment correction of QED, by simply adding to the Pauli Hamiltonian a local magnetic dipole term. Since, in this particular case, there is already a dipole magnetic term, we are renormalizing its coupling:

$$\frac{e}{2m_e} 2\vec{\sigma} \cdot \vec{B}_{ext} \to Z \frac{e}{2m_e} 2\vec{\sigma} \cdot \vec{B}_{ext} \tag{7}$$

where:

$$Z = 1 + \frac{\alpha}{\pi} + \cdots \tag{8}$$

It is to be understood that the renormalized magnetic dipole interaction has to be computed at tree level in the low energy theory, because radiative corrections have already been included. In certain applications however we are interested in computing matrix elements of the low energy Hamiltonian at an higher order, for instance at one loop level. In this case consistency requires subtracting from the renormalization constant Z the non relativistic vertex correction contribution:

$$Z = 1 + (\text{full theory corrections}) - (\text{low energy theory corrections}) \tag{9}$$

As we shall see in section (3.2), with replacements of the form (7) we transform the Pauli Hamiltonian in a quantum effective Hamiltonian for low energy electromagnetic processes. It is essential to note that going to the effective theory we do not loose the quantum effects of the high energy theory.

This example generalizes to processes involving real particles that suffer small momentum transfer q_μ with respect to their mass M:

$$q^2 \ll M^2 \tag{10}$$

These particles can be considered 'heavy'.

In these phenomena the 'heavy' particle neither decays nor is generated in the dynamic. Its large rest mass M is not created by or annihilated in momenta of light degrees of freedom. It always appears in the initial and final states of the reaction. In the language of diagrams there is an high energy flow along the heavy particle line. We can build up an effective theory for such particles that is basically an expansion for small momenta around the on-shell momentum:

$$p_{on\ shell} = (M, \vec{0}). \tag{11}$$

We cannot eliminate the heavy particle in the effective theory because it appears in the external states; we remove the heavy particle degrees of freedom that decouple in the $M \to \infty$. This operation generates new interactions between the remaining degrees of freedom. This effective theory can be generated as an expansion in the inverse of the heavy particle mass, $1/M$. In section (3.2) we apply it to atoms with one electron. Chapter (4) deals with the application of the $1/M$ expansion to heavy quark bound states.

The two examples discussed, and the related effective theories, have the basic concepts in common, but there is an important difference. In the first case the energy and the spatial momentum of the heavy particle E, $\vec{p}$ are much less than its mass M:

$$E, \mid \vec{p} \mid \ll M. \tag{12}$$

The effective theory can be thought as an expansion of the heavy particle 4-momentum around a null 4-momentum (eq.5). In the second case, instead, the heavy particle is real and its spatial momenta $\mid \vec{p} \mid$ are much less than M:

$$\begin{aligned} E &\sim M \\ \mid \vec{p} \mid &\ll M. \end{aligned} \tag{13}$$

This second kind of effective theories can be regarded as an expansion of the heavy particle 4-momentum around an on-shell value (eq.11).

3.2 Construction of an effective theory

We can identify the following steps in the construction of an effective hamiltonian describing processes up to an energy scale E.

1) Removal of the heavy degrees of freedom.

The effective Hamiltonian is constructed by eliminating from the original theory all the fields with mass $M > E$. The physical degrees of freedom are then identified through the fields which correspond to external states.

2) Construction of all the possible interactions, allowed by the symmetries of the original theory, between light fields.

The effect of virtual heavy particles is mimicked in the effective theory adding to the original vertices all the possible interactions between light fields. It is only required that new interactions satisfy the fundamental laws of nature, like CPT symmetry, Lorentz invariance, hermiticity, etc... In contrast to the complete high energy theories we do not require the new interactions to be renormalizable. That is quite in agreement with the idea that effective theories do not describe correctly the physics at small scales. Their S matrix elements have to be computed at tree level or at most at a fixed number of loops. Each new interaction O_i introduced in the effective Hamiltonian has a strength that is characterised by an undetermined coupling constant c_i. In general the number of possible new interactions is infinite, because the theory is "effective" and does not require to be renormalizable. The dimension of c_i is M^{4-D}, where D is the dimension of the operator O_i. Usually D is negative and is related to the inverse of an heavy particle mass present in the original theory.

3) Matching of the 2 theories.

This is a consistency requirement. We have to impose that the original and the effective theories generate the same physics at low energies. The procedure is straightforward:

i) Expand the matrix elements of the full theory up to a given order in the inverse of the heavy particle masses.

ii) Compute a sufficient number of amplitudes in both theories at a given order in perturbation theory, in order to determine all effective couplings c_i of the effective theory.

iii) Equate the expanded matrix elements of the full theory with the corresponding matrix elements of the effective theory.

In this way all the couplings of the effective theory are determined in terms of the parameters of the high energy theory and of the heavy particle masses.

3.3 Non relativistic QED

Non relativistic QED is an effective theory for low energy electromagnetic interactions. It has been formulated for high level computations of bound states properties of atoms like muonium ($\mu^+ e^-$) and positronium ($e^+ e^-$) (Caswell and Lepage [3]). The essential problem of the computations of the hyperfine structures is that relativistic corrections play a crucial role, for the reasons exposed in section (3.1). The most direct way of taking them into account is to employ a fully relativistic formalism, based on Bethe-Salpeter equation [4]. On the either side, positronium or muonium are weakly bounded states, with small momentum transfers among the constituents. The particles have a non relativistic motion

$$\frac{v}{c} \sim \alpha, \tag{1}$$

that needs only the first few terms in the relativistic corrections. With an effective theory we can separate low energy and high energy electromagnetic interactions, that are coupled in a relativistic theory. They can be treated with different formalisms and gauges. According to the procedure described in section (3.1) we write down the most general nonrelativistic Lagrangian involving only the fields of the particles building up the atom. The electromagnetic field of course has the same description in QED and in NRQED. The differences lie in the interaction with the matter and in the description of the matter itself. We impose only general constraints on the structure of the interaction terms, such as hermiticity, Galilean invariance, etc. For muonium we have:

$$\begin{aligned} L_{eff} \;=\; & -\frac{1}{2}(E^2 - B^2) + \psi_e^\dagger(i\partial_t - e\phi + \vec{D}^2/2m)\psi_e + \\ & + \psi_e^\dagger[c_1\vec{D}^4/8m^3 + c_2(e/2m)\vec{\sigma}\cdot\vec{B} \\ & + c_3(e/8m^2)\vec{\partial}\cdot\vec{E} + c_4(e/8m^2)\{i\vec{D}\cdot\vec{E}\times\vec{\sigma}\}]\psi_e \\ & + \psi_e^\dagger[d_1(e/8m^3)\{\vec{D}^2, \vec{\sigma}\cdot\vec{B}\}]\psi_e \\ & - (d_2/m_e m_\mu)(\psi_e^\dagger\vec{\sigma}\psi_e)\cdot(\psi_\mu^\dagger\vec{\sigma}\psi_\mu) + \ldots \end{aligned} \tag{2}$$

where $\vec{D} = \vec{\partial} + ie\vec{A}$ is the gauge covariant derivative and ψ_e, ψ_μ are two component spinor fields. The effective lagrangian can be thought as an expansion in $1/m$ or in v/c since any power of $1/m$ brings a factor v/c.

The coefficients c_i and d_i are determined imposing the equality of a adequate number of scattering amplitudes computed at a given order in α both in full QED and in non relativistic QED (NRQED):

$$< f \mid S \mid i >_{QED} = < f \mid S \mid i >_{NRQED} \tag{3}$$

up to a given order in the inverse of the particles masses.

Relativistic quantum effects are then incorporated in the values of the coefficients c_i and d_i, up to a given order in α. They are easily computed in QED with covariant perturbation theory and a covariant gauge. At the tree level $c_1 ... c_4, d_1, d_2 = 1$. In general we have:

$$c_i,\ d_i \ = \ 1 + (\text{QED corrections}) \ - \ (\text{NRQED corrections}) \qquad (4)$$

With non relativistic QED we can compute every low energy process. The results agree with those of QED up to the higher order in the inverse of the particle masses considered in the effective interactions. Bound state properties are then computed using the Schroedinger equation with ordinary (non covariant) perturbation theory and the Coulomb gauge.

3.4 $\Delta I = 1/2$ rule

It is experimentally established that nonleptonic kaon decays obey an approximate $\Delta I = 1/2$ selection rule. $\Delta I = 3/2$ amplitudes are suppressed by about an order of magnitude with respect to $\Delta I = 1/2$ ones.

A partial explanation of this law is that it is induced by the strong interaction dynamic, as found with the use of the effective weak Hamiltonian (G. Altarelli and L. Maiani [5], M.K. Gaillard and B.W. Lee [6],[7,8]). Gluon exchanges are thought to be responsable for the effective strength of $\Delta I = 1/2$ interactions and suppression of the $\Delta I = 3/2$. The computation is simplified considering a Standard Model with only 2 quark generations:

$$\begin{pmatrix} u \\ d' \end{pmatrix}, \begin{pmatrix} c \\ s' \end{pmatrix} \qquad (1)$$

The effective Hamiltonian for strangeness changing decays $H_{eff}^{\Delta S=1}$ is given by:

$$H_{eff}^{\Delta S=1}(x) = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{us} \sum_i C_i(\mu, x) O_i(\mu, 0) \qquad (2)$$

where $O_i(\mu)$ are renormalized local four fermion operators and $C_i(\mu)$ are the relative coefficient functions. The matrix elements of the effective Hamiltonian must be independent of μ, because μ is simply the scale at which the renormalized operators have been defined. The dependence on μ of the coefficient functions C_i and of the operators O_i must compensate. The complete theory, including the W boson field, doesn't have other divergences than the usual of a renormalizable theory. In other words, one loop QCD corrections of the $\Delta S = 1$ vertex are finite in the complete theory. The physical reason is that the W mass acts as an ultraviolet cut-off in the high energy theory and it is not necessary to introduced any mass scale for the subtraction point.

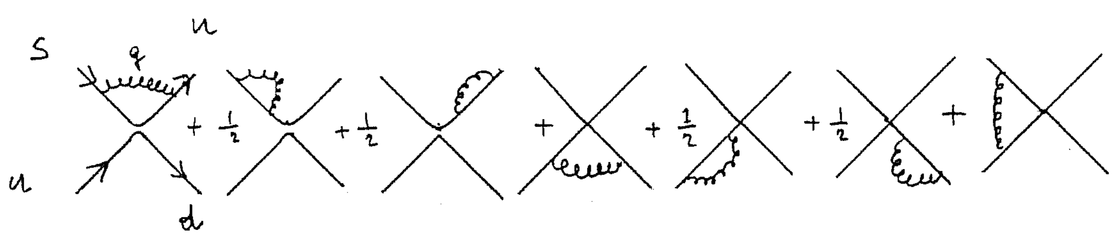
At the tree level in weak interactions and ignoring strong interactions, the effective Hamiltonian is given by:

$$H_{eff}^{\Delta S=1}(x) = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{us} O_1(x) + \ h.c. \tag{3}$$

where

$$O_1(x) = \overline{u}(x)\gamma_\mu(1-\gamma_5)s(x)\ \overline{d}(x)\gamma_\mu(1-\gamma_5)u(x) \tag{4}$$

We want to include the effect of strong interactions in the effective Hamiltonian in leading logarithmic approximation. Since μ is arbitrary, let us we choose $\mu \gg M_c$. Using standard renormalization group techniques, we need only to compute 1 loop Feynman diagrams with a gluon exchange between quarks legs of the operator O_1 (see fig.1).



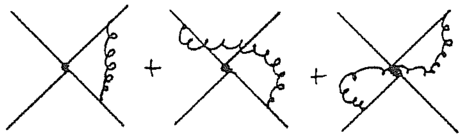


fig.1

Logarithmic divergences are found that impose the renormalization of O_1. They appear in graphs where colour is exchanged between quark bilinears, and the renormalization is then not multiplicative. The operator O_1 mixes with the following four fermion operator:

$$O_2 = \overline{u}(x)\gamma_\mu(1-\gamma_5)u(x)\ \overline{d}(x)\gamma_\mu(1-\gamma_5)s(x) \tag{5}$$

The linear combinations of O_1 and O_2 that are multiplicatively renormalized are:

$$\begin{aligned} O_+ &= \overline{u}\gamma_\mu(1-\gamma_5)s\ \overline{d}\gamma_\mu(1-\gamma_5)u + \overline{u}\gamma_\mu(1-\gamma_5)u\ \overline{d}\gamma_\mu(1-\gamma_5)s \\ O_- &= \overline{u}\gamma_\mu(1-\gamma_5)s\ \overline{d}\gamma_\mu(1-\gamma_5)u - \overline{u}\gamma_\mu(1-\gamma_5)u\ \overline{d}\gamma_\mu(1-\gamma_5)s \end{aligned} \tag{6}$$

O_- is antisymmetric under the exchange of $\overline{u}$ and $\overline{d}$ fields. Two of the three spinors carrying a nonzero isospin are then in an isosinglet state and the operator O_- can mediate only $\Delta I = 1/2$ transitions. O_+ is instead symmetric under the exchange of the $\overline{u}$ and $\overline{d}$ fields and then can produce both $\Delta I = 1/2$ and $\Delta I = 3/2$ transitions. After the inclusion of strong interactions the effective hamiltonian reads:

$$H_{eff}^{\Delta S=1} = \frac{G_F}{\sqrt{2}}[C_+(t,\alpha_S)O_+(\mu) + C_-(t,\alpha_S)O_-(\mu)] \tag{7}$$

for $M_c \ll \mu \ll M_W$. $t = \ln(M_W^2/\mu^2)$.

The requirement of μ independence of H_{eff} generates a renormalization group equation for the coefficient functions $C_\pm(t,\alpha_S)$:

$$\mu^2 \frac{d}{d\mu^2} C_\pm(t,\alpha_S)O_\pm(\mu) = 0 \tag{8}$$

The explicit form of eq.(8) is the following renormalization group equation:

$$\left(-\frac{\partial}{\partial t} + \beta(\alpha_S)\frac{\partial}{\partial \alpha_S} - \gamma^\pm(\alpha)\ \right)C_\pm(t,\alpha_S) = 0 \tag{9}$$

The solution is:

$$C_{\pm}(t,\alpha_S) = C_{\pm}(0,\overline{\alpha}_S(t,\alpha_S))\exp\left(\int_{\alpha_S(\mu)}^{\overline{\alpha}_S} d\alpha' \frac{\gamma_{\pm}(\alpha'_S)}{\beta(\alpha'_S)}\right) \tag{10}$$

We insert in eq.(10) the one loop expressions for the running coupling constant and the anomalous dimensions of the operators:

$$\begin{aligned} \mu\frac{d}{d\mu}\overline{\alpha}_S(\mu) &= -\,b\,\overline{\alpha}_S^2(\mu) \\ \gamma^{\pm}(\alpha_S) &= \gamma^{\pm(1)}\,\alpha_S \end{aligned} \tag{11}$$

where:

$$\gamma^{+(1)} = \frac{-1}{2\pi}, \qquad \gamma^{-(1)} = \frac{1}{\pi} \tag{12}$$

and

$$2b = (24\pi^2)^{-1}(33 - 2N_f) \tag{13}$$

The result of the integration is:

$$C_{\pm}(t,\alpha_S) = C_{\pm}(0,\overline{\alpha}_S(t,\alpha_S)\,)\left[\frac{\alpha_S}{\overline{\alpha}_S(t,\alpha_S)}\right]^{\gamma^{\pm(1)}/b} \tag{14}$$

We have now to impose the matching of the effective theory with the complete one, computing the same amplitude in the full theory and retaining only the leading term in $1/M_W$. If we impose the equality of the amplitudes at a particular value of μ, it will be true for every renormalization point, because of the μ independence. In a leading logarithmic computation the matching is greatly simplified and there is no need to compute any graph in the complete theory. We equate the renormalization point μ (or cut-off) in the effective theory to the W mass that appears in the full theory:

$$\mu = M_W \tag{15}$$

With this choice the difference of 1 loop matrix elements of the complete theory and the operators $O_\pm$ does not contain any leading log. We neglect subleading terms in our leading log philosophy, and the matching reduces to:

$$C_\pm(0, \overline{\alpha}_S(t, \alpha_S)\ ) = 1 \qquad (16)$$

The effective Hamiltonian then reads:

$$\begin{aligned} H_{eff}^{\Delta S=1} &= \frac{G_F}{\sqrt{2}} V_{ud}^* V_{us} \{ [\frac{\alpha_S}{\overline{\alpha}_S(t, \alpha_S)}]^{\gamma^{-(1)}/b} O_-(\mu) + \\ &+ [\frac{\alpha_S}{\overline{\alpha}_S(t, \alpha_S)}]^{\gamma^{+(1)}/b} O_+(\mu) \} + \ h.c. \end{aligned} \qquad (17)$$

for $M_c \ll \mu \ll M_W$.

Scaling the renormalization point μ from $\mu > M_c$ down to $\mu < M_c$ the charm quark field disappears and we go in a new effective theory containing only the flavors u, d, s as dynamical fields. There are new matching conditions to impose. We have to introduce in the 3 flavour effective theory new interactions between u, d, s fields representing at the lowest order in $1/M_c$ the effects of virtual c loops that are present in the 4 flavour theory. At one loop level, 4 quark operators containing $c\overline{c}$ fields do generate diagrams that can mediate $\Delta S = 1$ transitions (so called penguin diagrams [7,8], see fig.2).

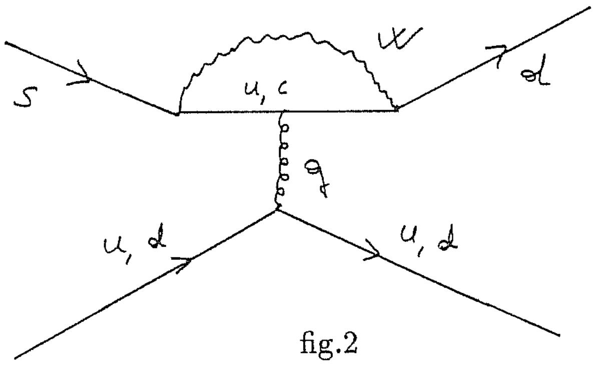


fig.2

New 4 quark operators, with a different chiral structure with respect to the original ones enter in the effective Hamiltonian. However, the charm mass scale is not far from the confinement radius R^{-1} and the perturbative theory rapidly becomes inadequate for the matching. We limit ourselves to an estimate of the enhancement effect and drop neglect penguin operators in the matching at the M_c threshold. We simply evolve the coefficient functions in eq.(15) down to μ in the region:

$$M_s < \mu < M_c \qquad (18).$$

We can try to minimize the differences in the (unknown) hadronic matrix elements of $O_-(\mu)$ and $O_+(\mu)$ by taking μ of the order of the kaon mass. With this choice there are not different mass scales in the matrix elements and no large logarithms can appear. Inserting the values $\Lambda_{QCD} = 300\ MeV$ and $\mu \sim 0.5\ GeV$ we get:

$$\begin{aligned} C_- &\simeq 3 \\ C_+ &\simeq 0.6 \end{aligned} \qquad (19)$$

giving an enhancement factor $\simeq$ 5. It is to note that scaling the renormalization point μ down to $\mu \sim 0.5 GeV$ is to a large extent arbitrary, because of fundamental non perturbative effects. Keeping μ in the perturbative region, $\mu \sim 1 \div 1.5\ GeV$, reduces considerably the enhancement.

Chapter 4

Heavy quark bound states in QCD

4.1 Introduction

The dynamic of a heavy quark Q in a hadron is an interesting problem in particle physics both for phenomenological and theoretical reasons.

As discussed in section (1.4), an accurate determination of the weak current matrix elements between mesons composed of heavy quarks is necessary for the extraction of $| V_{cb} |$, $| V_{ub} |$ from the experimental decay rates.

The dynamic of bound states containing heavy quarks contains also informations on the basic properties of strong interactions, as will be discussed in section (4.4).

The typical momentum exchange q among the constituents of a hadron is of the order of the inverse of the hadron size,

$$q \sim \Lambda_{QCD} \tag{1}$$

Light quarks q, whose mass $m_q \leq \Lambda_{QCD}$, are relativistic and subject to non perturbative effects of strong interactions. On the contrary, the motion of a heavy quark Q is slow, because exchange of high momentum gluons is suppressed by asymptotic freedom.

We can distinguish the following cases:

1) Mesons composed of a pair of light valence quarks, like the ρ or the K mesons. The dynamic is non perturbative. Since current lattices have an ultraviolet cut-off, $\Lambda_{UV} = 1/a \sim 2 \div 3\ GeV$, much greater than the typical momentum transfer q of quarks and gluons,

$$q^2 \ll \Lambda_{UV} \qquad (2),$$

processes involving light quarks can be safely computed with lattice QCD [1].

2) Mesons composed of a pair of heavy quarks $Q\overline{Q}$, like the J/ψ vector meson or the Y states. The motion of the constituents is non relativistic:

$$< v^2 > \sim \frac{1}{3}, \frac{1}{10} \qquad (4)$$

for $c\bar{c}$, $b\bar{b}$ systems respectively [2,3]. The dimensions of these mesons are much smaller with respect to the cases 1) and 3) (see below), implying larger momentum transfers. The dynamic can be described by Schroedinger equation, in analogy with the hydrogen atom. The effective potential represents the exchange of gluons between $Q\overline{Q}$ and creation of light pairs. An instantaneous "effective" potential model is a reasonable approximation of reality, if the dynamical degrees of freedom of the gluon field can be neglected. In physical terms, the motion of the constituents must be enough slow in order to not excite gluonic states [3].

3) Mesons composed of a heavy and a light quark $Q\overline{q}$, like the B's. It is an intermediate case between case 1) and 2). The light quark motion is relativistic and non perturbative while that of the heavy one is non relativistic. Momentum transfers are smaller than in case 2) and the mean velocity $< v >$ of the heavy

quarks, from quark model estimates, is given by:

$$< v > \sim \frac{1}{5}, \frac{1}{20} \tag{5}$$

for D and B mesons respectively [3]. In this chapter we are mainly interested in these kind of particles. An analogue classification holds for baryons.

We now introduce the basic idea for dealing with heavy quarks..

The basic idea, which corresponds to our "naive" physical intuition, is that in the limit of a very large mass, the heavy quark behaves as a static source of colour which screens the field of the light quark. It is then reasonable to expand the heavy quark Hamiltonian, H_Q, in powers of $1/M_Q$ (Eichten [2,3]). Heavy quark propagators, interaction vertices and finally physical observables like decay rates, mass splittings, etc... can then be computed like formal series in $1/M_Q$. The expected contribution of terms of a given order n is $\sim (\Lambda_{QCD}/M_Q)^n$.
The expansion in powers of $1/M_Q$ of the Dirac gauge-covariant Hamiltonian is a very old result (Foldy-Wouthuysen [4]). It originated from the idea of getting rid of the couplings between particle-antiparticle states with a proper unitary transformation S that diagonalizes in 2 by 2 blocks the Dirac Hamiltonian H in the standard representation:

$$S \; H \; S^{\dagger} = H_{diag}. \tag{6}$$

S is naturally organised as an expansion in powers of $1/M_Q$.

At the beginning of relativistic quantum mechanics [4], the nonrelativistic treatment of the Schroedinger equation has been discarded at the fundamental level, since it leads to a non local wave equation. Nevertheless it keeps it's validity at low energy as an effective theory.

The assumption that momentum transfers in a hadron are of order Λ_{QCD} is of fundamental importance for the validity of the $1/M_Q$ expansion. The main theoretical justification is that Quantum-Chromodynamic, being a non abelian gauge theory, is asymptotically free: the effective coupling constant α_S between quarks and gluons goes to zero with increasing momenta (though only logarithmically) and great momentum transfers among constituents in a hadron are suppressed.

These considerations should overcome to a large extent the theoretical difficulties related to the confinement of quarks and gluons inside hadrons. The $1/M$ expansion can be demonstrated only in the perturbative regime, as for example in the computation of electromagnetic bound states. In the latter case the force is weak, goes to zero as the distance r of the constituents increases, $r \rightarrow \infty$, and the binding energy is consequently small. In the case of strong forces one could imagine a model in which the constituents are inside a deep, negative, potential well. The observed value of the bound state mass M, for instance the proton mass $M_P \sim 1\ GeV$, doesn't imply in this case that momentum transfers q among the constituents are of the same order of magnitude of M:

$$q^2 \not\sim M^2. \tag{7}$$

In other words, the bound state mass can result as a small difference of large, opposite sign, kinetic and interaction energies.

There are two main applications of the $1/M_Q$ expansion.

1) Derivation of symmetry properties in heavy quark systems.

In the limit $M_Q \rightarrow \infty$, the effective Hamiltonian possesses new flavor-spin symmetries which allow to relate among themselves properties of different hadrons. They arise at a given order in $1/M_Q$ (for the moment computations have been mainly limited at lowest order): the main results concern asymptotic relations between hadron masses, form factors, decay constants, absolute normalization of hadronic matrix elements, etc...

2) Numerical simulation of heavy quark systems.

$1/M_Q$ expansion allows the numerical simulation on the lattice of the dynamic of quarks with mass $M_Q \geq 1/a$, where a is the lattice spacing. In lattice regularization of quantum field theories, the inverse of the lattice spacing $1/a$ acts as an ultraviolet cut-off in momentum:

$$\Lambda_{UV} \sim 1/a. \tag{8}$$

It is then impossible to simulate the dynamics of a particle with a mass

$$M \geq 1/a \ , \tag{9}$$

since it has a Compton wavelength λ_c smaller than a. With present computers it is not possible to simulate QCD with $1/a \geq 2 \div 3\ GeV$ [1,5], making questionable calculating charm quark dynamic and impossible the beauty one. Since however in many interesting processes involving heavy quarks, momentum transfers are much less than $1/a$, it is still possible to simulate them, giving up the description of structures with mass scales of order M_Q [3]. The systematic method for this task is just a appropriate discretization of the heavy quark $1/M_Q$ expansion. With $1/M_Q$ expansion we can then simulate the dynamic of c and b quarks.

4.2 Static theory for heavy quarks

In this section we discuss the static theory for a heavy quark, i.e. the effective Hamiltonian at lowest order in the $1/M_Q$ expansion [6,7].

The dynamic of a quark Q inside a given colour field $A_\mu(x) = A^a_\mu(x)t^a$ is determined by the Dirac lagrangian:

$$L(x) = \overline{Q}(x)(i\gamma^\mu D_\mu - M_Q)Q(x) \tag{1}$$

where: $D_\mu(x) = \partial_\mu + igA_\mu(x)$.

According to the previous ideas on the dynamic of hadronic bound states (section 4.1) it is natural to assume that a heavy quark Q in a meson or a baryon is nearly on shell and nearly at rest, because its momentum differs by $(M_Q, \vec{0})$ at most by terms of order Λ_{QCD}. It follows also that the heavy quark is subjected mainly to chromoelectric interactions and chromomagnetic effects can be neglected. At the lowest order we can drop terms related to the spatial motion of the heavy quark, of eq.(1), obtaining the following "effective" static theory lagrangian:

$$L_S = \overline{Q}(x)(i\gamma_0 D_0 - M_Q)Q(x) \tag{2}$$

The static theory (2) , unlike the high energy one (1), is no more Lorentz or even Galileo invariant, since we have set equal to zero the spatial components of the 4-vectors p_μ, A_μ. By abandoning the complete theory in favour of the static one we have done an operation analogous to the gauge fixing in quantizing gauge field theories, which notoriously breaks gauge symmetry.

In most applications of the static theory only heavy quarks or heavy anti-quarks are involved. We can decouple the corresponding fields [6] by writing $Q(x)$ as a combination of its upper and lower components:

$$Q(x) = \frac{(1+\gamma_0)}{2}H(x) + \frac{(1-\gamma_0)}{2}K(x) \tag{3}$$

In terms of quark and antiquark fields, H and K, $L_S(x)$ reads:

$$L_S(x) = H^\dagger(x)(iD_0 - M_Q)H(x) + K^\dagger(x)(-iD_0 - M_Q)K(x) \tag{4}$$

The number of degrees of freedom for a given orbital state is preserved since we have converted a 4 component theory in 2 independent 2-component theories. Such a transformation is impossible in the Lorentz invariant theory (as is well known from the first days of relativistic quantum theory [4]) , because of simultaneous particle-antiparticle creation. The static theory is a low energy effective theory and particles states with momenta comparable with the mass are absent from the spectrum: there is no way to excite the "Dirac sea".

In the static theory the parameter M_Q can be removed because it does not represent any more a true, dynamical mass. In the free case it is equivalent to expanding in powers of $1/M_Q$ the relativistic energy-momentum relation

$$E = \sqrt{p^2 + M_Q^2} = M_Q + p^2/2M_Q + \ldots \tag{5}$$

keeping only the leading, momentum independent term

$$E = M_Q. \tag{6}$$

The parameter M_Q therefore doesn't control anymore the changes in energy related to a given change in momentum. In the lagrangian of eq.(4) it can be removed with a time dependent redefinition of the quark and antiquark fields:

$$\begin{aligned} H'(x) &= H(x)e^{+iM_Q t} \\ K'(x) &= K(x)e^{-iM_Q t} \end{aligned} \tag{7}$$

In classical field theories the propagation of waves in the space is described adding to the lagrangian bilinear terms in the field and it's spatial derivatives, such as

$$\partial_i\phi\partial_i\phi, \qquad \overline{\psi}\gamma_i\partial_i\psi \tag{8}$$

where ϕ/ψ is a bosonic/fermionic field (In euclidean theory terms (8) are associated to field diffusion). Dropping these terms, waves do not propagate any more and the field reduces to a continuum of independent oscillators, one for every point of the space. As a simple example, consider the Klein-Gordon lagrangian with spatial derivatives omitted:

$$L(\vec{x},t) = \frac{1}{2}\left(\frac{\partial\phi}{\partial t}\right)^2 - \frac{1}{2}m^2\phi^2 \tag{9}$$

The equations of motion are:

$$\frac{\partial^2}{\partial t^2}\phi(\vec{x},t) = -m^2\phi(\vec{x},t) \tag{10}$$

having as solutions:

$$\phi(\vec{x},t) = \phi(\vec{x},0)e^{-imt+i\delta(\vec{x})} + \phi(\vec{x},0)e^{+imt-i\delta(\vec{x})} \tag{11}$$

for every $\vec{x}$. The oscillation amplitudes $\phi(\vec{x})$ and phases $\delta(\vec{x})$ are completely arbitrary functions of $\vec{x}$. In quantizing the theory we get a spectrum of excitations consisting of particles created in various points (instead with a given momentum) $\vec{x}$, $\vec{x}'$ ... by different operators $a^\dagger_{\vec{x}}$, $a^\dagger_{\vec{x}'}$ ....

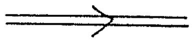
By dividing $L_S(x)$ of eq.(4), after the removal of the mass, into a free part

$$L_S(x) = \overline{H}(x)i\partial_0 H(x) + \overline{K}(x)(-i)\partial_0 K(x) \tag{12}$$

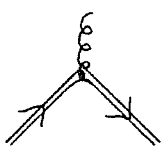
and an interacting one

$$L_I(x) = -g\overline{H}(x)A_0 H(x) + g\overline{K}(x)A_0(x)K(x), \tag{13}$$

it is straightforward to compute the Feynman rules for the static theory. For a quark we get:



$$= \frac{i}{p_0 + i\epsilon} \tag{14}$$



$$= -igt^a_{ij} \tag{15}$$

Taking the Fourier transform of the momentum space propagator of the 4-component theory

$$\frac{i}{\gamma_0 p_0 + i\epsilon} \tag{16}$$

and reinserting the mass term of eq.(2), we obtain:

$$S_F^0(x) = -i\delta^{(3)}(x)(\frac{1+\gamma_0}{2}\Theta(t)e^{-iM_Q t} + \frac{1-\gamma_0}{2}\Theta(-t)e^{+iM_Q t}) \tag{17}$$

As it stems from eq.(17), an infinite-mass quark is a classical particle: once created in a point it remains there forever. There is not contradiction with the uncertainty principle since $\delta x \delta v = \delta x \delta p/m \sim \hbar/m$ which goes to 0 as $m \to \infty$. The interacting propagator $S_F(x)$ is computed by noting that for an infinite-mass quark finite momentum transfers cannot change its motion nor rotate its spin. For a very heavy particle the sum over histories collapses in the classical one ($\vec{x}(t) = 0$ for each t). The interaction then generates only a phase factor in colour space. By gauge covariance the propagator can depend only on:

$$P(A) \;=\; P\exp(ig\int_0^t A_0(\vec{0},t')dt') \tag{18}$$

where P denotes path ordering, and then:

$$S_F(x) = P(A)\cdot S_F^0(x) \tag{19}$$

4.3 Relativistic infinite mass theory

4.3.1 General considerations

Consider the semileptonic decay of a B meson into a D or a D^* meson:

$$B \to D/D^* + l + \nu_l \tag{1}$$

where $l = e, \mu$ and ν_l is the corresponding neutrino. In the rest frame of the B meson, the constituting b quark, that is essentially at rest ($E_B = M_B + O(\Lambda_{QCD})$, $\vec{p}_B = O(\Lambda_{QCD})$), and is surrounded by the meson cloud, decays into a c quark and a $l + \nu_l$ couple. The charm quark emerges from the weak interaction vertex with a given velocity $\vec{v}$, which can go from 0 up to $\simeq 0.8c$. Since the c quark is heavy, $M_c \gg \Lambda$, it changes very little it's velocity due to the interaction with the meson cloud; in first approximation it behaves as a colour source moving with constant velocity $\vec{v}$, which is followed by the light colour-screening meson-cloud (see fig.1).

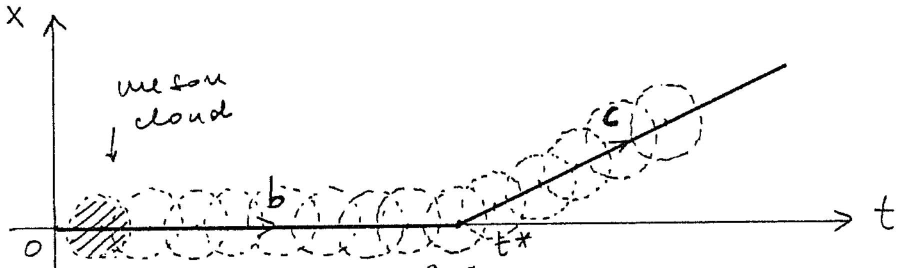


fig.1

Space-time diagram of $b \to c$ semileptonic decay. With respect to bound-state dynamics, the decay appears as a static colour source that starts moving at time t^* with constant velocity $\vec{v}$.

In the static theory B, D and D^* are composed of the same meson cloud. Since the cloud and the heavy quark spin are both conserved in time by hadron dynamic, spin flips are produced only at the weak vertex. The hadronic part of process (1) is then described as a b quark decaying into a c quark with the same/opposite spin orientation (fig.2).

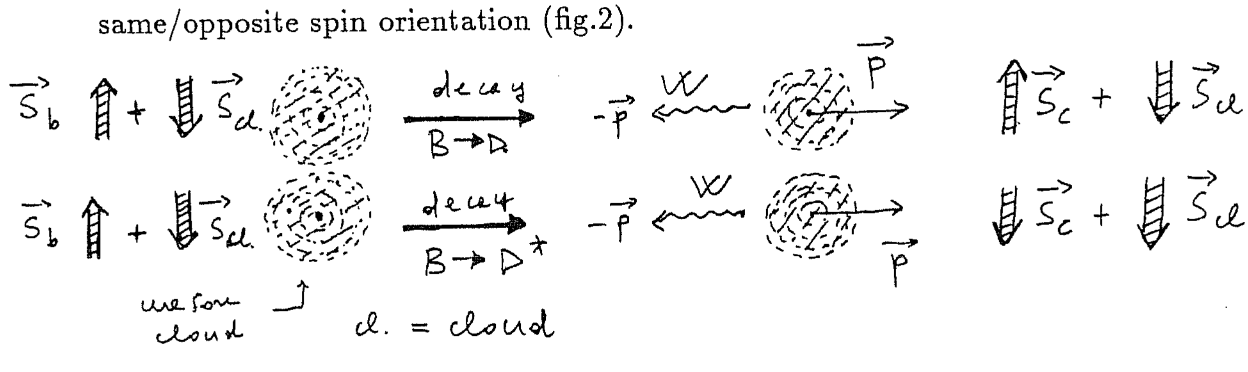


fig.2

The systematic errors in taking $M_c, M_b \rightarrow \infty$ are of order Λ_{QCD}/M_c. A more precise computation has to take into account $1/M_c$ correction terms to the static theory, which include the nonrelativistic term $\vec{p}^2/2M_c$, the chromomagnetic interaction due to the quark spin and of the orbital motion of the charm quark with the meson cloud.

From the above considerations it is clear that semileptonic $B \rightarrow D$, $B \rightarrow D^*$ transition amplitudes are related in the static theory. They equal the probability amplitude that the meson cloud doesn't excite when the colour source starts moving.

If $\vec{p}_D/\vec{p}_{D^*} = 0$ in decay (1) a b quark at rest is transformed into a c quark at rest, and nothing happens with respect to bound state dynamics: meson cloud doesn't feel any change and the broken line of fig.(1) turns into a straight line. These intuitive considerations indicate that there is an absolute normalization of the hadronic matrix elements of processes (1) in the static theory for both b and c quarks.

At this kinematic point the decay (1) resemblances a purely leptonic one, since a lepton and a neutrino emerge from the decay vertex with opposite spatial momenta $\vec{p}_\nu = -\vec{p}_l$ and fixed energies determined by $q^2 = (p_l + p_\nu)^2 = q^2_{MAX} = (M_B - M_D/M_{D^*})^2$.

In the framework of the $1/M_Q$ expansion it is easy to understand why various quark model predictions agree fairly well among themselves in the values of the hadronic form factors of processes (1) [8]: at lowest order in $1/M_Q$ the wave functions of the B and D/D^* mesons coincide and the overlap integral is independent of the parameters of a specific model.

The quantitative understanding of the properties related to processes (1) in the M_b, $M_c \to \infty$ limit requires the construction of a theory describing infinite mass quarks moving with various velocities. It is discussed in the following subsection.

4.3.2 Basic elements

The static theory for heavy quarks (section 4.2) is not relativistic because if we move from the laboratory system to an inertial frame with velocity $\vec{v}$, where a static quark is viewed as a particle with constant velocity $-\vec{v}$, this case is not described in the lagrangian (4.2.1). Relativistic invariance can be recovered by performing all the possible Lorentz transformations of the static lagrangian $L_S(x)$ (eq.4.2.1) and then summing the resulting expressions $L(x,v)$ with a relativistic invariant invariant measure (Georgi [9]).

In the rest frame S' of the infinite mass quark Q, the static equation of motion holds:

$$(i\gamma_0 D'_0 - M_Q)\psi'(x') = 0 \tag{2}$$

We have to express equation (2) in the variables of a reference frame S moving with velocity $-\vec{v}$ with respect to S'. The standard Lorentz coordinate transformation Λ from S to S' gives:

$$D'_0 = \Lambda^0_\mu D^\mu = \gamma D_0 - \gamma \vec{v} \cdot \vec{D} = v^\mu D_\mu \tag{3}$$

where $\gamma = 1/\sqrt{1-v^2}$ is the time dilation factor. The Dirac spinor ψ transforms according to:

$$\psi'(x') = S(\Lambda)\psi(x) \tag{4}$$

where $S(\Lambda)$ is the spinorial representation of Λ. Substituting eq.(3) and (4) into (2), left-multiplying the result by $S^{-1}(\Lambda) = S(\Lambda^{-1})$, and using the relation:

$$S^{-1}(\Lambda)\gamma_\mu S(\Lambda) = \Lambda_\mu^\nu \gamma_\nu \tag{5}$$

we arrive to the result:

$$(i\gamma_\mu v^\mu v \cdot D - M_Q)\psi(x) = 0 \tag{6}$$

The equation of motion (6) can be derived by a variation from the lagrangian:

$$L_v(x) = \overline{\psi}(i\gamma_\mu v^\mu v \cdot D - M_Q)\psi(x) = 0 \tag{7}$$

which is the lagrangian of an infinite mass quark moving with constant velocity $\vec{v}$. Summing $L_v(x)$ over all the velocities v^μ with the measure $\int d^4v\delta(v^2 - 1) = \int d^3v/2v^0$ we get the relativistic expression for the heavy quark $L(x)$:

$$L(x) = \int \frac{d^3v}{2v^0} L_v(x) \tag{8}$$

Since a static quark Q doesn't change spatial position by means of finite momentum transfer and an infinite mass quark with velocity $\vec{v}$ is simply a static quark observed from a moving frame, we derive a velocity superselection rule for the relativistic infinite mass theory (Georgi [9]), namely:

$$\Delta\vec{v} = 0 \tag{9}$$

where $\Delta\vec{v}$ is the velocity change in a collision.

The velocity superselection rule (9) can also be derived with the following observation. After a collision with momentum transfer k, the momentum of the meson containing the heavy quark is given by:

$$Mv' \; = \; Mv + k \tag{10}$$

where v and v' are the initial and final velocities and M is the meson mass, that coincides with the heavy quark mass M_Q up to order Λ_{QCD} terms. For finite k we have:

$$v' = v + \frac{k}{M} \longrightarrow v \tag{11}$$

for

$$M_Q \longrightarrow \infty. \tag{12}$$

The velocity superselection rule (9) implies that there is a separate field $\psi_v(x)$ for each velocity v, since no dynamical process can couple any two fields $\psi_v(x)$, $\psi_{v'}(x)$ with $v' \neq v$.

As in the case of the static theory, we can remove the parameter M_Q in the relativistic infinite mass theory lagrangian (8) by writing:

$$\psi'(x) = e^{+iM_Q v\cdot x}\psi(x)$$

We get (dropping for simplicity the prime):

$$L_v(x) = \overline{\psi} i\gamma_\mu v^\mu v \cdot D\psi(x) = 0 \tag{13}$$

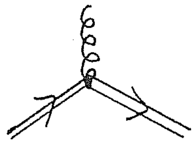
The Feynman propagator $S_F^{(0)}(k,v)$ of the relativistic infinite mass theory can be computed with a Lorentz transformation of the static one eq.(4.2.16). The same result could be obtained as follows. In the propagator of the Dirac theory:

$$S_F^{(0)}(p) = i\frac{\hat{p} + M_Q}{p^2 - M_Q^2 + i\epsilon} \tag{14}$$

set $p = M_Q v + k$ and keep only the leading term in the residual momentum k (small virtuality). The result is:

$$S_F^{(0)}(k,v) = i\frac{1+\hat{v}}{2} \cdot \frac{1}{v\cdot k + i\epsilon} \tag{15}$$

From the anarmonic part of (13) we get the vertex rule:



$$-ig\hat{v}v^{\mu}t^{a}_{ij} \tag{16}$$

The interacting propagator in configuration space $S_F(x,v)$ can be derived making the same Lorentz transformation Λ applied to the static lagrangian, on the static propagator $S_F(x)$ (eq.4.2.17).

The relevant transformation formulas are:

$$t' = v^{\mu}x_{\mu} \tag{17}$$

$$A'_0(x') = v_{\mu}A^{\mu}(x) \tag{18}$$

like eq.(3)

$$S^{-1}(\Lambda)\gamma_0 S(\Lambda) = \hat{v} \tag{19}$$

$$dt' = \frac{dt}{v^0} \tag{20}$$

where we have differentiated eq.(17) and we have used the equation of motion in the S system $x_i = (v_i/v_0)t$.

$$\Theta(t') = \Theta(t) \tag{21}$$

since proper Lorentz transformation do not change the sign of the time.

The transformation of the $\delta^{(3)}(x')$ is derived considering that the condition $\vec{x}' = 0$ in K' becomes in K $x_i = (v_i/v_0)t$. Then, it must hold the proportionality relation: $\delta^{(3)}(x') = a(v)\delta^{(3)}(\vec{x} - (\vec{v}/v_0)t)$.

Integrating both sides on $d^3x = d^3x'/\gamma$, because of Lorentz contraction, we get $a(v) = 1/\gamma$ and then:

$$\delta^{(3)}(x') = \frac{1}{\gamma}\delta^{(3)}(\vec{x} - \frac{\vec{v}}{v_0}t). \tag{22}$$

Using (17),(18),(19),(20),(21),(22) and (4.2.17), we derive:

$$S_F(x,v) = -iP(A) \cdot \frac{\delta^{(3)}(\vec{x} - \vec{u}t)}{v^0} \cdot \left(\frac{1+\hat{v}}{2}\Theta(t)e^{-iM_Q v \cdot x} + \frac{1-\hat{v}}{2}\Theta(-t)e^{+iM_Q v \cdot x}\right) \tag{23}$$

where:

$$P(A) = P\exp(ig\int_0^t A(\vec{u}t', t') \cdot v \; \frac{dt'}{v_0}) \tag{24}$$

Equation (23) is a generalization of the static case considered in eq.(4.2.19).

4.4 Correlation functions

In the infinite mass limit, dimensionful parameters disappear from the original theory and some interactions simplify so that new symmetries appear, relating properties of different hadrons and amplitudes of different processes.

A general technique for proving symmetry relations of the infinite mass theory is offered by the functional-integral formulation of quantum field theory, whose basic elements we review in this section.

4.4.1 2-Point correlation functions

The mass of the lightest particle P with given quantum numbers can be computed by the asymptotic values ($t \to \infty$) of the euclidean correlation functions $F_{AB}(x)$ of any two operators A,B having the same quantum numbers as the particle:

$$F_{AB}(x) =< 0 \mid T[A(\vec{x},t)B(0)] \mid 0 > \tag{1}$$

The operator $B(0)$, because of symmetry, can excite from the vacuum $\mid 0 >$ only those eigenstates of the hamiltonian, $\mid n >$, with the same quantum numbers of the particle B. In euclidean theory, where numerical simulations are performed, these states evolve in time as $e^{-E_n t}$. At long time separations between the operators A, B, because of decay, only the lightest states coupled to the sources, that are one particle states, generate the correlation and F_{AB} simplifies to:

$$F_{AB}(\vec{x},t) = \sum_{\alpha} \int \frac{d^3p}{(2\pi)^3 2E_p} \langle 0 \mid A(0) \mid P, \vec{p}, \alpha \rangle \langle P, \vec{p}, \alpha \mid B(0) \mid 0 \rangle e^{i\vec{p}\cdot\vec{x} - iE_P(\vec{p})t}$$
$$+\text{exponentially small terms} \tag{2}$$

where α denotes collectively all the particle quantum numbers (spin, isospin, etc.), $d^3p/2E_p$ is the Lorentz-invariant momentum measure for momentum eigenstates and we have used translation invariance. For simplicity we make the computation in Minkowsky space. Integrating over the space we arrive at:

$$\int d^3x F_{AB}(\vec{x},t) = \sum_{\alpha} \frac{1}{2M} < 0 \mid A(0) \mid P, \vec{0}, \alpha >< P, \vec{0}, \alpha \mid B(0) \mid 0 > e^{-iMt} \tag{3}$$

Computing the correlation function F_{AB} and, for instance, $F_{AA^\dagger}$ for various large t values we can determine the particle mass M and also the matrix elements of A and B between vacuum and 1-particle states.

According to Feynman path-integral formalism, Green functions like $F_{AB\ldots Z}(x, y \ldots, z)$ can be computed as expectation values of c-number operators like A(x)B(y)...Z(z) weighted over the distribution e^{iS}, where S is the action of the theory:

$$< 0 \mid TA(x)B(y)\ldots Z(z) \mid 0 > = < A(x)B(y)\ldots Z(z) > \tag{4}$$

where we have introduced the notation:

$$< A(x)B(y)\ldots Z(z) > = \frac{1}{Z} \int [d\phi] A(x)B(y)\ldots Z(z)\, e^{iS[\phi]} \tag{5}$$

where ϕ is the set of dynamical fields of the theory and $Z = \int [d\phi] e^{iS[\phi]}$ becomes in the euclidean theory $Z = \int [d\phi] e^{-S[\phi]_E}$ and can be viewed as the partition function at zero external field of a statistical system with hamiltonian $\beta H = S_E$.

Comparing eq.(3) with eq.(4) we conclude that masses and operator matrix elements can be computed with functional-integrations once the action S of the theory is given. In the case of QCD:

$$\sum_{\alpha} \frac{1}{2M} < 0 \mid A(0) \mid \alpha >< \alpha \mid B(0) \mid 0 > e^{-iMt} = \int < A(x)B(0) > d^3x \qquad (6)$$

The average in this case is over fermionic and gauge fields, and A, B are composite operators made out of quarks and gauge fields.

4.4.2 3-Point Correlation Functions

The matrix elements of an operator $O(x)$ between 1-particle states $\mid P_1 >$, $\mid P_2 >$

$$< P_1 \mid O(x) \mid P_2 > \qquad (7)$$

can be computed from the asymptotic values of an euclidean 3-point correlation function

$$F(x_1, x_2) =< 0 \mid T A_1(x_1) O(0) A_2(x_2) \mid 0 >=< 0 \mid A_1(x_1) O(0) A_2(x_2) \mid 0 > \quad \text{for } t_1 \to +\infty, \; t_2 \to -\infty \qquad (8)$$

where A_1, A_2 are operators with the same quantum numbers as particles P_1 and P_2 respectively. By the same reasoning as in section (4.4.1), for t_1 large $A_1(x_1) \mid 0 >$ will be mainly a superposition of P_1 states only:

$$A_1(x_1) \mid 0> = \sum_{\alpha_1} \int \frac{d^3p_1}{2(2\pi)^3 E(p_1)} F_1^{\alpha_1} \mid P_1, p_1, \alpha_1 > \exp(-i\vec{p}_1 \cdot \vec{x}_1 + iE(p_1)t_1)$$
$$+ \text{ exponentially small terms} \tag{9}$$

where we defined:

$$F_1^{\alpha_1} = < P_1, p_1, \alpha_1 \mid A_1(0) \mid 0 > \tag{10}$$

and for $-t_2$ large:

$$A_2(x_2) \mid 0> = \sum_{\alpha_2} \int \frac{d^3p_2}{2(2\pi)^3 E(p_2)} F_2^{\alpha_2} \mid P_2, p_2, \alpha_2 > \exp(-i\vec{p}_2 \cdot \vec{x}_2 + iE(p_2)t_2)$$
$$+ \text{ exponentially small terms} \tag{11}$$

where we defined:

$$F_2^{\alpha_2} = < P_2, p_2, \alpha_2 \mid A_2(0) \mid 0 > \tag{12}$$

α_1, α_2 denote collectively all the strong interaction quantum numbers of particles P_1, P_2 such as spin, parity, G-parity, strangeness, etc....

Substituting eq.(9) and eq.(11) in eq.(8), we get for both t_1 and $-t_2$ large:

$$F(x_1, x_2) = \sum_{\alpha_1, \alpha_2} \int \frac{d^3p_1}{2(2\pi)^3 E(p_1)} \frac{d^3p_2}{2(2\pi)^3 E(p_2)} F_1^{\alpha_1\,*} F_2^{\alpha_2} \times$$

$$\times < P_1, p_1, \alpha_1 \mid O(0) \mid P_2, p_2, \alpha_2 > \exp\{i\vec{p}_1 \cdot \vec{x}_1 - i\vec{p}_2 \cdot \vec{x}_2 - iE(p_1)t_1 + iE(p_2)t_2\} \tag{13}$$

Making the space Fourier-transform of $F(x_1, x_2) \rightarrow F(\vec{q}_1, \vec{q}_2, t_1, t_2)$ we can isolate in F the correlation between modes with given momentum $\vec{q}_1, \vec{q}_2$ only:

$$F(\vec{q}_1, \vec{q}_2, t_1, t_2) =$$
$$= \int d^3x_1 d^3x_2 \exp(-i\vec{q}_1 \cdot \vec{x}_1 + i\vec{q}_2 \cdot \vec{x}_2) F(x_1, x_2) =$$
$$= \sum\nolimits_{\alpha_1,\alpha_2} \frac{F_1^{\alpha_1} F_2^{\alpha_2\,*}}{4E(q_1)E(q_2)} \langle P_1, q_1, \alpha_1 \mid O(0) \mid P_2, q_2, \alpha_2 \rangle \exp(-iE(q_1)t_1 + iE(q_2)t_2) \tag{14}$$

Computing the 2-point correlation functions $F_{A_1 A_1^\dagger}$, $F_{A_2^\dagger A_2}$ we can determine (eq.4.4.3) $< 0 \mid A_1 \mid P_1 >$ and $< 0 \mid A_2 \mid P_2 >$ and then to extract from (eq.14) the required matrix element:

$$< P_1, \vec{q}_1, \alpha_1 \mid O(0) \mid P_2, \vec{q}_2, \alpha_2 > \tag{15}$$

We have thus proved the initial assertion.

The Feynman path-integral expression for F is:

$$F(x_1, x_2) = < A_1(x_1) O(0) A_2(x_2) > \tag{16}$$

Comparing eq.(14) with eq.(16) we get finally the path-integral expression for the matrix element (7):

$$\sum_{\alpha_1,\alpha_2} \frac{F_1^{\alpha_1\,*} F_2^{\alpha_2}}{4E(q_1)\ E(q_2)} < P_1, q_1, \alpha_1 \mid O(0) \mid P_2, q_2, \alpha_2 > \exp(-iE(q_1)t_1 + iE(q_2)t_2)$$

$$= \int d^3x_1 d^3x_2 \exp\{-i\vec{q}_1 \cdot \vec{x}_1 + i\vec{q}_2 \cdot \vec{x}_2\} < A_1(x_1) O(0) A_2(x_2) > \tag{17}$$

4.5 Symmetry relations

Symmetry properties of the static theory and the relativistic infinite mass theory can be derived doing manipulations on functional-integral expressions computed with the static QCD action (eq.4.2.2, 4.3.8). We will include the renormalization effects due to QCD in section (4.6).

4.5.1 Symmetry relations in the static theory

Consider the lowest-lying pseudoscalar P and vector mesons V composed of a heavy quark Q and a light antiquark $\overline{q}$. As interpolating fields (sources) we take respectively the time component of the axial current $A_4(x) = \overline{Q}(x)\gamma_4\gamma_k q(x)$ and one of the spatial components of the vector current $V_K(x) = \overline{Q}(x)\gamma_k q(x)$, $(k = 1, 2, 3)$.

Let us consider first the pseudoscalar case. Setting $A(x)^\dagger = B(x) = A_4(x)$ in eq.(4.4.6) gives:

$$\frac{1}{2M_P} |< 0 \mid A_4(0) \mid P >|^2 \, e^{-iM_P t} = \int d^3x < A_4(x) A_4(0) > \qquad (1)$$

Performing the (symbolical) integration over the quark fields ψ $\overline{\psi}$ and using Wick theorem the right hand side of eq.(1) becomes:

$$-\int d^3x < Tr(\gamma_4\gamma_5 S_Q(x \mid 0)\gamma_4\gamma_5 S_q(0 \mid x) >_A \qquad (2)$$

where we have introduced the notation $< ..... >_A$ to denote $1/Z$ times the functional integration over the gauge fields A_μ with the action $\tilde{S}[A_\mu]$.

$\tilde{S}[A_\mu] = S_{YM}[A_\mu] + \ln[det\Delta(A_\mu)]^{N_f}$, is an effective non local action, generating gluon field correlations only by integrating over gauge field, i.e. including all fermion loops. $\Delta(A_\mu)$ is the Dirac operator and N_f is the number of light quark flavors (according to the idea that heavy quark loops are unimportant). The trace is taken over spin and colour indices and minus sign comes from the fermionic loop.

Static theory enters at this point, in substituting static propagator for the heavy quark. Doing that, expression expression (2) reduces to:

$$< Tr((1-\gamma_4)/2P(A)S_q(0 \mid \vec{0},t) >_A e^{-iM_Q t} \tag{3}$$

where: $P(A) = P\exp(ig\int_0^t A_0(\vec{0},t')dt'$ and we used that

$$\gamma_4\gamma_5(1+\gamma_4)/2\gamma_4\gamma_5 = -(1-\gamma_4)/2 \tag{4}$$

Equating the first member of these relations with the last we have the result:

$$\frac{1}{2M_P} \mid< 0 \mid A_4(0) \mid P >\mid^2 e^{-i(M_P-M_Q)t} =< \; Tr[\frac{(1-\gamma_4)}{2}P(A)S_q(0 \mid \vec{0},t)] \; >_A \tag{5}$$

Since the right-hand side of eq.(5) doesn't contain M_Q, the left-hand side also is independent of M_Q, implying that:

a) The quantity $\epsilon = M_P - M_Q$ is independent on the heavy quark mass. This property can simply be derived noting that strong interactions in QCD are flavor independent and that in the static theory heavy quark masses $M_Q, M_{Q'} \ldots$ disappear from the lagrangian.

More generally, in the static theory, all heavy-light mesons have identical properties since they simplify to a cloud of light $q\bar{q}$ pairs and gluons screening a static colour source.

b) The coefficient of the exponential on the left hand side of eq.(5) is independent on M_Q:

$$\frac{1}{2M_P} \mid< 0 \mid A_4(0) \mid P >\mid^2 = constant \text{ (independent of } M_Q) \tag{6}$$

Introducing the pseudoscalar meson decay constant f_P:

$$< P \mid A_4(0) \mid 0 >= i f_P M_P \tag{7}$$

we get the famous decay constant scaling law [10]:

$$f_P = \frac{const}{\sqrt{M_P}} \tag{8}$$

Repeating the previous computation with the vector meson source V_k instead of A_4 we arrive at the same right-hand side as eq.(5), since $\gamma_k(1+\gamma_4)/2\gamma_k = -(1-\gamma_4)/2$, like in eq.(4). Comparing eq.(5) with the vector case, we derive:

$$\begin{aligned} &\frac{1}{2M_V} \sum_{r=1}^{3} |< 0 \mid V_k(0) \mid V, r >|^2 \; e^{-i(M_V - M_Q)t} \\ = &\frac{1}{2M_P} \; |< 0 \mid A_4(0) \mid P >|^2 \; e^{-i(M_P - M_Q)t} \end{aligned} \tag{9}$$

a) Since the equality (9) holds for any t :

$$M_V = M_P \tag{10}$$

The equality of pseudoscalar and vector meson masses (10) can be understood in a simple way by considering the spin symmetries of the static theory [10]. The chromomagnetic moment $\vec{\mu}_Q$ associated with the heavy quark spin $\vec{S}_Q$ is a term of order $1/M_Q$. At the tree level:

$$\vec{\mu}_Q \;=\; \frac{g}{M_Q} \vec{S}_Q. \tag{11}$$

$\vec{\mu}_Q$ determines the spin interactions of the heavy quark with the meson cloud and has has the effect of removing the degeneracy (10). In the static theory $\vec{\mu}_Q = 0$. The pseudoscalar meson spin results from antiparallel orientation of the heavy quark and meson-cloud spin, both conserved in time. Rotating by 180^0 $\vec{S}_Q$ we transform the pseudoscalar meson into the vector one without any energy supply.

We expect the difference between pseudoscalar mass M_P and vector meson mass M_V to be inversely proportional to the average mass $M = (M_P + M_V)/2$. It holds then then the relation (see fig.1):

$$M_V^2 - M_S^2 = const \text{ (independent of } M) \tag{12}$$

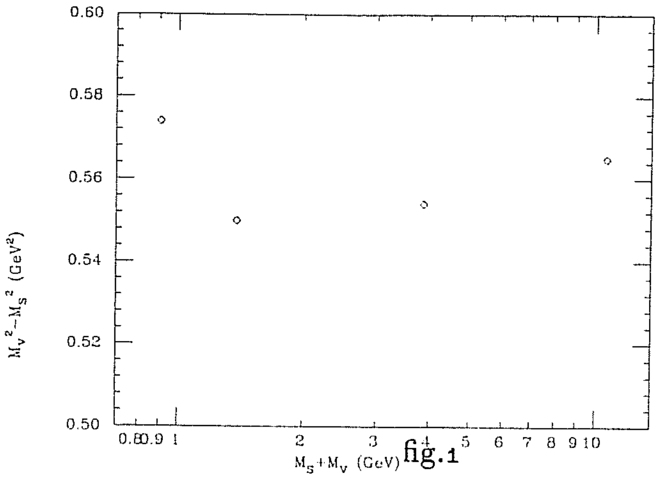


fig.1

It is surprising that the asymptotic law (12) is experimentally satisfied at the 5% already for light mesons (ρ, π) and strange mesons (K, K^*).

With an analogue computation we can prove the equality of the masses of the lowest lying scalar and axial mesons. The simplest choice of interpolating fields is the following:

$$\overline{Q}(x)q(x) \qquad \text{scalar meson}$$

$$\overline{Q}(x)\gamma_k\gamma_4 q(x) \qquad \text{axial meson}$$

b) Equating the coefficients of the exponentials on both side of eq.(9), we get:

$$\sum_{r=1}^{3} |< 0 \mid V_K(0) \mid V, r >|^2 = |< 0 \mid A_4(0) \mid P >|^2 = f_P^2 M_P^2 = const\ M_P \tag{13}$$

Let us introduce the vector meson annihilation constant f_V:

$$< 0 \mid V_K(0) \mid V, r > = \frac{M_V^2}{f_V} \epsilon_k^r \tag{14}$$

where ϵ^r is the polarization 3-vector of the state $| V, r >$. Putting eq.(14) into eq.(13) and using the completeness relation of polarization vectors ϵ^r, $\sum_{r=1}^{3} \epsilon_k^r \epsilon_l^r = \delta_{k,l}$ we get:

$$\frac{f_V \cdot f_P}{M} = 1 \tag{15}$$

4.5.2 Symmetry relations in the relativistic infinite mass theory

The normalization conditions and the relations among heavy meson form factors anticipated in section (4.3.1) can be derived computing 3-point correlation functions with the relativistic infinite mass theory lagrangian. The original derivation has been given by Isgur and Wise employing the spin-flavor symmetries of the infinite mass theory [11]. They constitute a formalisation of the qualitative considerations of section (4.3.1). The matrix element of the quark weak current

$$J_\mu^{b\to c}(x) = \bar{c}(x)\gamma_\mu(1-\gamma_5)b(x) = V_\mu(x) - A_\mu(x) \tag{16}$$

between an initial B meson state with velocity v_B and a final D meson state with velocity v_D

$$\langle D, v_D \mid J_\mu(0) \mid B, v_B \rangle = \langle D, v_D \mid V_\mu(0) \mid B, v_B \rangle - \langle D, v_D \mid A_\mu(0) \mid B, v_B \rangle \tag{17}$$

can be computed with the aid of the functional expression (4.4.17) (U. Aglietti and G. Martinelli) which yields:

$$\frac{F_D^* F_B}{4E(p_B)E(p_D)} < D, v_D \mid J_\mu(0) \mid B, v_B > \; exp\{-iE(p_D)t_D + iE(p_B)t_B\} =$$

$$= \int d^3x_B d^3x_D \; exp\{-i\vec{p}_D \cdot \vec{x}_D + i\vec{p}_B \cdot \vec{x}_B\} < A_D(x_D) J_\mu(0) A_B(x_B) > \tag{18}$$

where:

$$
\begin{aligned}
F_D &= \; < D, v_D \mid A_D(0) \mid 0 > \\
F_B &= \; < B, v_B \mid A_B(0) \mid 0 >
\end{aligned}
\tag{19}
$$

$A_B(x)$ and $A_D(y)$ are any two interpolating fields for the B and D mesons respectively. The simplest choice is:

$$
\begin{aligned}
A_B(x) &= \bar{b}(x)\gamma_5 q(x) \\
A_D(y) &= \bar{q}(x)\gamma_5 c(x)
\end{aligned}
\tag{20}
$$

where $q(x)$ is a light quark field: $q = u, d, s$. Of course, the result we derive are independent of the particular interpolating fields employed, since the propagation of a physical particle is independent of the generation mechanism. The matrix element of the interpolating field cancels in taking the ratio of the 3-point correlation function to the 2-point correlation functions.

The axial part of the matrix element (17) vanishes due to parity conservation of strong interactions:

$$
< D, v_D \mid A_\mu(0) \mid B, v_B >= 0 \tag{21}
$$

We then proceed by considering only the vector current matrix element.

Performing the functional integration over the quark fields $\overline{\psi}$, ψ, and employing Wick theorem, we get for the right hand side of eq.(18):

$$
\begin{aligned}
&- \int d^3x_B d^3x_D \; \exp(-i\vec{p}_D \cdot \vec{x}_D + i\vec{p}_B \cdot \vec{x}_B) \\
&< Tr[\; \gamma_5 \; S_q(x_B \mid x_D) \; \gamma_5 \; S_c(x_D \mid 0) \; \gamma_\mu \; S_b(0 \mid x_B) \;] \; >_A
\end{aligned}
\tag{22}
$$

Inserting in expression (22) eq.(4.3.23) for both b and c quarks, and taking into account that $t_B < 0$, $t_D > 0$, we get:

$$\frac{F_D^* F_B}{4E(p_B)E(p_D)} < D, v_D \mid J_\mu(0) \mid B, v_B > \ \exp\{-iE(p_D)t_D + iE(p_B)t_B\} =$$

$$= \frac{-1}{\gamma_B\ \gamma_D} \exp\{-i\vec{q}_D \cdot \vec{u}_D t_D + i\vec{q}_B \cdot \vec{u}_B t_B - iM_c t_D/\gamma_D + iM_b t_B/\gamma_B\} \times$$

$$\times < Tr\ [\ P_b\ \frac{1+\hat{v}_B}{2}\ \gamma_5\ S_q(t_B, \vec{u}_B t_B \mid t_D, \vec{u}_D t)\ \gamma_5\ \frac{1+\hat{v}_D}{2}\ P_c\ \gamma_\mu\ ]\ >_A \tag{23}$$

where $\vec{u}_B$, $\vec{u}_D$ are the 3-velocities and γ_B, γ_D the relativistic time dilation factors of B, D mesons respectively. P_b, P_c are the P-line factors of the propagators of the b and c quarks:

$$\begin{aligned} P_b &= P\exp(ig\int_{t_B}^{0} A(t, \vec{u}_B t) \cdot v_B \frac{dt}{\gamma_B}) \\ P_c &= P\exp(ig\int_{0}^{t_D}\ _D A(t, \vec{u}_D t) \cdot v_D \frac{dt}{\gamma_D}) \end{aligned} \tag{24}$$

We have identified the velocity of the b/c quark with that of the B/D meson; otherwise the bound state would decompose as time goes on into the cloud and the heavy quark.

Solving with respect to the weak current matrix element we arrive at:

$$< D, v_D \mid J_\mu(0) \mid B, v_B >= K \cdot Tr\ [\frac{1+\hat{v}_B}{2}\gamma_5 L \gamma_5 \frac{1+\hat{v}_D}{2}\gamma_\mu] \tag{25}$$

where we have defined:

$$K = \frac{4M_B\ M_D}{F_D^* F_B} \tag{26}$$

and:

$$L = -\exp\{\epsilon(\frac{t_D}{\gamma_D} - \frac{t_B}{\gamma_B})\}\ <\ P_b\ S_q(t_B, \vec{u}_B t_B \mid t_D, \vec{u}_D t_D)\ P_c\ >_A \tag{27}$$

$\epsilon = M_B - M_b = M_D - M_c$ can be interpreted after renormalization as the heavy-light meson (universal) binding energy.

Since L is integrated over all gauge field configurations and is independent of times t_B, t_D, as it stems from eq.(25), it may depend only on the 4-velocities v_B, v_D. According to Lorentz symmetry, it can be expanded into:

$$L = M_1 + M_2\hat{v}_B + M_3\hat{v}_D + M_4\hat{v}_B\hat{v}_D + M_5\hat{v}_D\hat{v}_B \tag{28}$$

where M_i, $i = 1,2\ldots5$ are matrices in colour indices only, depending on the scalar $v_B \cdot v_D$.

Higher powers of $\hat{v}_B$, $\hat{v}_D$ are not linearly independent with respect to the terms in eq.(28) and γ_5-terms cannot appear due to parity conservation of the QCD action.

It is more convenient to express L in terms of projection operators:

$$\begin{aligned} L = C_1 + C_2\frac{1+\hat{v}_B}{2}\,\frac{1+\hat{v}_D}{2} + C_3\frac{1+\hat{v}_B}{2}\,\frac{1-\hat{v}_D}{2} \\ + C_4\frac{1-\hat{v}_B}{2}\,\frac{1+\hat{v}_D}{2} + C_5\frac{1-\hat{v}_B}{2}\,\frac{1-\hat{v}_D}{2} \end{aligned} \tag{29}$$

where C_i are linear combinations of the matrices M_i.

Substituting eq.(29) into eq.(25) we arrive at:

$$< D, v_D \mid J_\mu(0) \mid B, v_B >= (v_B + v_D)_\mu \cdot \sqrt{M_D M_B}\;\xi(v_B \cdot v_D) \tag{30}$$

where ξ, the Isgur-Wise function [11], is defined by:

$$\begin{aligned} \xi &= (K/\sqrt{M_D\, M_B}) \cdot Tr[C_5] \\ &= \frac{4\sqrt{M_B\, M_D}}{F_D^* F_B}\; Tr(C_5) \end{aligned} \tag{31}$$

The factor $\sqrt{M_D\, M_B}$ is introduced for convenience (see later).

An analogue computation for the hadronic matrix element of the semileptonic decay $B \to D^*$ gives:

$$< D^*, v_D, \epsilon \mid J_\mu(0) \mid B, v_B > = \; K \cdot Tr[\;\frac{1+\hat{v}_B}{2}\gamma_5 L\hat{\epsilon}\frac{1+\hat{v}_D}{2}\gamma_\mu(1-\gamma_5)\;] \tag{32}$$

where we used the relation between the vector and pseudoscalar annihilation constants (15), the mass degeneracy relation (10), and the completeness relation of the polarization 4-vectors:

$$\sum_{r=1}^{3} \epsilon_\mu^r \epsilon_\nu^r = g_{\mu\nu} - \frac{k_\mu k_\nu}{M^2}. \tag{33}$$

Computing the γ-matrix algebra, we get for the vector part:

$$Tr[\frac{1+\hat{v}_B}{2}\gamma_5 \; L \; \hat{\epsilon}\frac{1+\hat{v}_D}{2}\gamma_\mu] = Tr(C_5)\cdot \epsilon_{\mu\alpha\rho\lambda}\epsilon^\alpha v_D^\rho v_B^\lambda \tag{34}$$

and for the axial part:

$$Tr[\frac{1+\hat{v}_B}{2}\gamma_5 L\hat{\epsilon}\frac{1+\hat{v}_D}{2}\gamma_\mu\gamma_5] = i \; Tr(C_5)[\epsilon_\mu(1+v_D\cdot v_B) - v_B\cdot\epsilon \; v_{D\mu}] \tag{35}$$

and then:

$$< D^*, v_D, \epsilon \mid V_\mu(0) \mid B, v_B > \;=\; \epsilon_{\mu\alpha\rho\lambda}\epsilon^\alpha v_D^\rho v_B^\lambda \; \sqrt{M_D \; M_B} \; \xi(v_B\cdot v_D) \tag{36}$$

$$< D^*, v_D, \epsilon \mid A_\mu(0) \mid B, v_B > = i[\epsilon_\mu(1+v_D v_B) - v_B\epsilon(v_D)_\mu]\sqrt{M_D \; M_B} \; \xi(v_B v_D) \tag{37}$$

Equations (30) (36) and (37) imply that the six form factors characterizing $B \to D, D^*$ semileptonic decays can all be expressed in terms of a single function $\xi \;=\; \xi(v_B\cdot v_D)$ (Isgur and Wise [11]).

For determining the absolute normalization of hadronic matrix elements (17), i.e. the value of $\xi(v_B\cdot v_D = 1)$, let us limit ourselves to the temporal component of the vector current $V_{\mu=0}$ and take in eq.(23):

$$v_B \;=\; v_D \;=\; (1,\vec{0}) \tag{38}$$

We have:

$$\begin{aligned}< D, rest \mid V_0(0) \mid B, rest > &= -K < Tr[\frac{1-\gamma_0}{2} C\, S_q(t_B, \vec{0} \mid t_d, \vec{0})] >_A \\ &= \frac{K}{\sqrt{4 M_D\, M_B}} F_D^* F_B \\ &= 2\sqrt{M_B\, M_D}\end{aligned} \tag{39}$$

where we used equation (5) in the form:

$$< Tr[\frac{1-\gamma_0}{2} C\, S_q(0, \vec{0} \mid t, \vec{0})] >_A = \frac{1}{\sqrt{4 M_D\, M_B}} F_D^* F_B \tag{40}$$

On the other hand:

$$< D, rest \mid V_0(0) \mid B, rest > = 2\sqrt{M_D\, M_B}\, \xi(v_B v_D = 1) \tag{41}$$

Comparing eq.(39) with eq.(41) we get the wanted normalization condition (Isgur and Wise [11]):

$$\xi(\, v_B v_D = 1\,) = 1 \tag{42}$$

This relation can also be immediately derived by using the conservation of the vector current, which holds in the infinite mass limit.

4.6 Renormalisation

The static theory and its generalization to an arbitrary velocity discussed in section (4.2 ,4.3) are effective theories and the methods and considerations developed in chapter (3) apply to them as particular cases. In the following we will consider the renormalization properties of the operators discussed in sec.(4.5).

4.6.1 Renormalization of the static theory

Let us discuss in detail the renormalization of the operators in the static theory. A comparison of the Feynman rules of the static theory (eq.4.2.2) with that of the high energy one (eq.4.2.1), allows us to recognize that tree level amplitudes in the two theories, for equal external states, differ only by terms of order $1/M_Q$.

Loop corrections to the above amplitudes can be computed by introducing an ultraviolet cut-off Λ_{UV} in both theories. Differences of the form

$$\ln(\Lambda_{UV}/M_Q) \tag{1}$$

will appear which do not vanish even at small external momenta. These differences originate from the fact that, in intermediate virtual states, particles with momenta up to Λ_{UV} are excited, which propagate and interact in a quite different way in the two theories. The ultraviolet behaviour of the two theories is different because in the full theory $M_Q < \Lambda_{UV}$ while in the effective theory M_Q is larger than any scale, i.e. $M_Q > \Lambda_{UV}$. As it is has been explained in chapter (3), effective theories can be refined in such a way to include radiative corrections. Terms of the form (1) can be reabsorbed introducing suitable renormalization constants Z_i for the fields, coupling, masses and Green functions of the effective theory. The constants Z_i are determined by a consistency requirement: the low energy and the high energy version of the same physical theory must generate the same transition amplitudes for small external momenta. In the case of the static theory, amplitudes have to coincide at the lowest order in $1/M_Q$.

Let us start with a specific example. For the weak interaction phenomenology a relevant quantity is the axial current $\tilde{A}_\nu$:

$$\tilde{A}_\nu(x) = \bar{b}\gamma_\nu\gamma_5 q(x) \tag{2}$$

where the b field is treated in the static approximation and q is a light quark field: $q = u,\ d,\ s$. We will consider now the renormalization of this operator.

In the full theory the axial current is partially conserved and then undergoes a finite renormalization. In the static theory this property doesn't hold due to the different ultraviolet behaviour of the theory, and the axial current acquires anomalous dimensions, i.e. a renormalization point μ dependence:

$$\tilde{A}_\nu = \tilde{A}_\nu(\mu) \tag{3}$$

There is not any contradiction with the Ademollo-Gatto theorem because we cannot consider any more the heavy quark mass M_Q as a small,infrared, axial-symmetry breaking term. Let us give an intuitive explanation. We may think that the mass of the heavy quark M_Q acts in the full theory as a cut-off for the radiative corrections to the axial vertex. Modes with momenta $p \gg M_Q$ give a negligible contribution to the amplitude, because they 'see' essentially a conserved current. We can then let $\Lambda_{UV} \to \infty$ in the full theory without generating any divergence. In the effective theory it is $\Lambda_{UV} \ll M_Q$, because $M_Q \to \infty$. Going to the effective theory, then, we loose a physical cut-off, the heavy quark mass. M_Q specifies the (finite) amount of axial symmetry violations in the real world. To make amplitudes finite, we have to introduce in the effective theory a new arbitrary cut-off or renormalization point μ.

The differences in the full and in the effective theories originate from the fact that the limits

$$\begin{aligned} &\Lambda_{UV} \to \infty \\ &M_Q \to \infty \end{aligned} \tag{4}$$

do not commute with each other.

The effective axial current $\tilde{A}_\nu(\mu)$ is multiplicatively renormalized and is related to the physical one A_ν by a renormalization constant $C_A(\mu)$:

$$A_\nu = C_A(\mu)\tilde{A}_\nu(\mu) \tag{5}$$

$C_A(\mu)$ has been computed in LLA by many authors [12,13]. The most convenient renormalization prescription is the $\overline{MS}$ scheme. We only need to compute the divergent parts of the 1-loop corrections to the axial vertex of the effective theory (see fig.1).

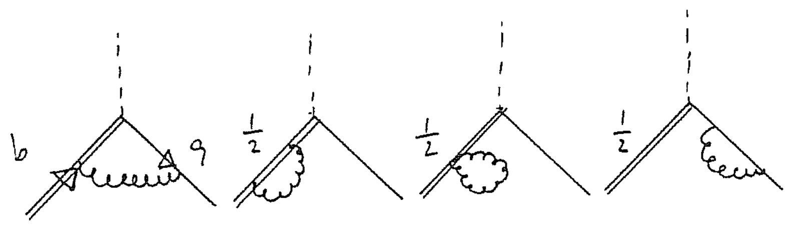


fig.1

The whole series of leading logs is summed up with the usual technique: we integrate over a finite scale transformation the R.G. equation that express the μ independence of the right hand side of eq.(5):

$$(\mu\frac{d}{d\mu} - \gamma_A)C_A(\mu) \;=\; 0 \tag{6}$$

where γ_A is the anomalous dimension of the effective axial current $\tilde{A}_\nu$.

The matching condition in LLA is easily derived by noting that for $\mu = M_Q$ there are not large logs in the difference of matrix elements of A_ν and $\tilde{A}_\nu(\mu)$. We then have:

$$C_A(\mu = M_Q) \;=\; 1 \tag{7}$$

Note that the static theory "doesn't know" the heavy quark mass, M_Q. A dependence of M_Q of its matrix elements is generated only through renormalization, with matching conditions. The computation yields for the matching constant $C_A(\mu)$:

$$C_A(\mu) \;=\; [\frac{\alpha_S(M_Q)}{\alpha_S(\mu)}]^{-6/(33-2N_f)} \tag{8}$$

where N_f is the number of active flavors.

The renormalization of the axial current $\tilde{A}_\nu(\mu)$ generates logarithmic corrections to the asymptotic scaling law of the pseudoscalar meson decay constant f_P. By definition, for a meson P at rest, we have:

$$< 0 \mid A_4 \mid P > = \; i f_P^{phys} M_P, \tag{9}$$

and by eq.(4.5.6)

$$\frac{1}{2M_P} \mid < 0 \mid \tilde{A}_4(\mu) \mid P > \mid^2 = \; const \tag{10}$$

Using the relation (5)

$$< 0 \mid A_\nu \mid P > = \; C_A(\mu) < 0 \mid \tilde{A}_\nu(\mu) \mid P > \tag{11}$$

we derive for the physical decay constant the following scaling law:

$$f_P^{phys} = \frac{const}{\sqrt{M_P}} [\frac{\alpha_S(M_Q)}{\alpha_S(\mu)}]^{-6/(33-2N_f)} \tag{12}$$

Dividing by the same expression with the replacement $M_P \to M_{P'}$, $\; Q \to Q'$, we derive finally the equation:

$$\frac{f_P^{phys}}{f_{P'}^{phys}} = (\frac{M_P}{M_{P'}})^{-1/2} [\frac{\alpha_S(M_Q)}{\alpha_S(M_{Q'})}]^{-6/(33-2N_f)} \tag{13}$$

which tells us how the decay constant should vary as a function of the heavy quark (meson) mass. At this order we can identify the mass of the heavy quark with that of the meson, since differences arise at order $1/M_Q$. The additional dependence on the heavy quark mass produced by the last factor in eq. (13) is mild.

4.6.2 Renormalization of the relativistic infinite mass theory

The renormalization of the relativistic infinite mass theory follows the same lines of that of the static theory. The mass renormalization δm_v and the field renormalization constants Z_v are independent of the heavy quark velocity v in a regularization of the theory (4.3.8) that preserves Lorentz symmetry.

In numerical simulations of the relativistic infinite mass theory it is required to adopt lattice regularization, which breaks Lorentz symmetry. In this case δm_v and Z_v become complicated functions of the velocity v of the heavy quark and it is necessary to introduce velocity-dependent factors in the matching of the relativistic infinite mass theory with the complete one. There are differences between a continuum and a lattice space-time only in the modes with wavelength $\lambda \lesssim a$. Since high-energy modes are weakly coupled in QCD, if the ultraviolet cut-off $\Lambda_{UV} = 1/a$ in numerical simulations is large enough, continuum-lattice matching can be done in perturbation theory. At 1-loop level the dependence on v of δm_v and Z_v is generated by hard gluons radiative corrections ($\lambda \sim a$, where a=lattice spacing). For simplicity, however, we assume in the following dimensional regularization.

The renormalization of heavy meson form factors is essential for computing the hadronic matrix elements of the weak Hamiltonian between heavy quark bound states. It has also a methodological interest because it involves a sequence of effective theories and matching conditions.

Relations between semileptonic $B \to D, D^*$ decay form factors (eq.4.5.30, 4.5.36, 4.5.37) are true at leading order in the effective theory for both $M_c, M_b \to \infty$. We introduce matching constants Z_A, Z_V relating the weak currents $\tilde{V}_\mu$, $\tilde{A}_\mu$

in the effective theory to that in the complete one V_μ, A_μ:

$$\begin{aligned} V_\mu &= Z_V \tilde{V}_\mu \\ A_\mu &= Z_A \tilde{A}_\mu \end{aligned} \tag{14}$$

The computation has been performed in LLA assuming $M_b \gg M_c \gg \Lambda_{QCD}$ (A. Falk et al. [14,15]). Let us consider the renormalization of a generic heavy quark current

$$G(x) = \bar{c}(x)\Gamma b(x) \tag{15}$$

where Γ is a matrix in the Clifford algebra.

The running of the current G in the mass region $M_c - M_b$ is easily computed with $\overline{MS}$ renormalization scheme of the Feynman diagrams of fig.(2), involving a heavy b quark and a light c quark:

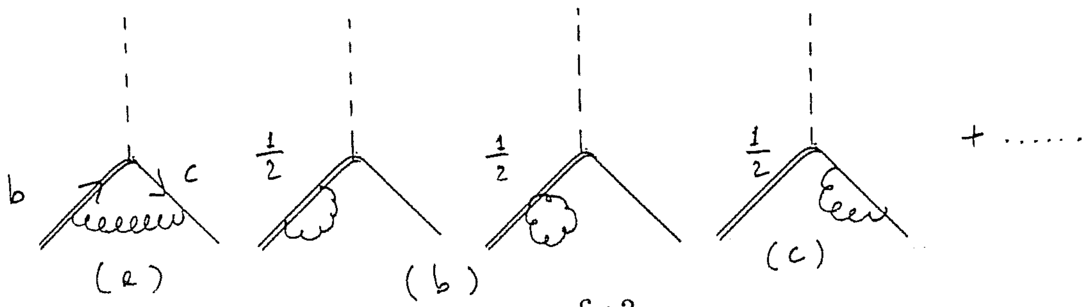


fig.2

Diagrams (b) and (c) are standard heavy and light quark selfenergy corrections. A straightforward application of Feynman rules of ordinary QCD and relativistic infinite mass theory gives for the vertex correction (a):

$$D_2 = -i\frac{4}{3}g^2 \int \frac{d^4k}{(2\pi)^4} \frac{\hat{v}_b(\hat{k} + M_c\hat{v}_c + M_c)\Gamma}{(k^2 + 2M_c v_c k)k^2 v_b \cdot k} \tag{16}$$

Computing the anomalous dimensions of the current (the ultraviolet log-divergent part of $(a) + 1/2(b) + 1/2(c)$) and summing leading logs with standard R.G. technique, we get the current scaling factor:

$$\left(\frac{\alpha_S(M_b)}{\alpha_S(M_c)}\right)^{a_I} \tag{17}$$

where $a_I = -6/(33-2N_f)$ and $N_f = 4$ is the number of active flavors in the mass interval $M_b \to M_c$ that has been integrated. The factor (17) is the equal to the renormalization constant of the axial current in the static theory.

The running of the current G in the interval $\mu - M_c$, where μ is a given renormalization point for the b, c effective theory, is computed by the one loop Feynman diagrams of fig.(3) with heavy b and c quarks:

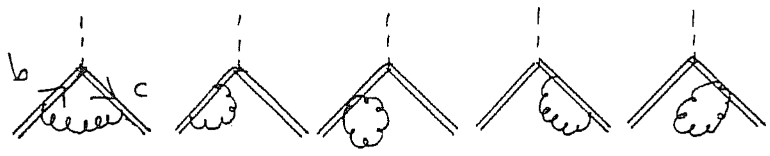


fig.3

The vertex correction reads:

$$D_1 = -i\frac{4}{3}g^2 \int \frac{d^4k}{(2\pi)^4} \frac{v_b v_c \Gamma}{(v_b \cdot k)\ k^2\ (v_c \cdot k)} \tag{18}$$

Then, the running of the current in the interval $M_c \to \mu$ introduces the further scaling factor:

$$\left(\frac{\alpha_S(M_c)}{\alpha_S(\mu)}\right)^{a_L} \tag{19}$$

where:

$$a_L = \frac{8(v_b \cdot v_c r(v_b \cdot v_c) - 1)}{33 - 2N_f} \tag{20}$$

$N_f = 3$ is the number of active flavour for $\mu < M_c$ and $r(x) = (1/\sqrt{x^2-1})\ln(x + \sqrt{x^2-1})$. The anomalous dimension of G in this mass region is velocity dependent, because in the relativistic infinite mass theory fields that create particles with different velocities are different fields (velocity superselection rule of eq.4.3.9).

Multiplying factors (17) and (19) we get the required matching constants in LLA, connecting heavy b, c theory with the 'true' one:

$$Z_A = Z_V = \left(\frac{\alpha_S(M_c)}{\alpha_S(\mu)}\right)^{a_L} \left(\frac{\alpha_S(M_b)}{\alpha_S(M_c)}\right)^{a_I} \tag{21}$$

Inserting eq.(21) in the relations between hadronic matrix elements (4.5.30), (4.5.36) and (4.5.37) we get the same rescaling for the Isgur-Wise function $\xi_0(v_b v_c)$ of the effective theory:

$$\xi(v_b v_c) = \Xi(v_b v_c, M_b, M_c, \mu)\ \xi_0(v_b v_c, \mu) \tag{22}$$

where:

$$\Xi(v_b v_c, M_b, M_c, \mu) = \left(\frac{\alpha_S(M_c)}{\alpha_S(\mu)}\right)^{a_L} \left(\frac{\alpha_S(M_b)}{\alpha_S(M_c)}\right)^{a_I} \tag{23}$$

and $\xi(v_b v_c)$ is the Isgur-Wise function of the complete theory. It is μ independent, then the μ dependence of Ξ compensates the one of ξ_0. Since a_L is velocity dependent a change in μ amounts to transferring part of the (perturbative) $v_b v_c$ dependence between Ξ and ξ_0, just like in the evolution of structure functions with Altarelli-Parisi equation. ξ_0 is the true non perturbative contribution of strong interactions. If it is computed with Montecarlo simulations of QCD, it is necessary to introduce a further matching constant relating continuum and lattice regularization.

Let us discuss an interesting subleading contribution to full-effective matching. Spin interactions in the effective theory differ from that of the complete theory: the effective theory propagator $S_F^0(v, k)$ is obtained setting $p = mv + k$ in the complete one $S_F(p)$ and dropping the $\hat{k}$ term in the numerator. As a consequence, radiative corrections to the Γ vertex generate different Lorentz structures in the full and in the effective theory. Let us consider the matching at the M_c threshold, between the effective theory for b quark only and the one for both b and c quarks. Dropping the terms that simply renormalize the lowest order result, we get for the spin dependent matching contribution [14]:

$$
\begin{aligned}
D_2 - D_1 &= - i\frac{4}{3}g^2 \int \frac{d^4k}{(2\pi)^4} \frac{\hat{v}_b \hat{k} \Gamma}{(k^2 + 2M_c vk)k^2 v_b k} + \cdots \\
&= - \frac{g(M_c)^2}{6\pi^2} r(v_b v_c) \hat{v}_b \Gamma + \cdots
\end{aligned}
\tag{24}
$$

Thus, the matching at the M_c scale is realised adding to Γ the term (24), incorporating as a new interaction for the effective b, c theory the correct effect of charm spin interactions:

$$
\Gamma \to \Gamma - \frac{g(M_c)^2}{6\pi^2} r(v_b v_c) \hat{v}_b \Gamma + \cdots \tag{25}
$$

In quite analogous way, at the M_b threshold, due to different spin interactions of the b quark in the complete theory and the effective theory for the b quark, we have for the spin dependent matching [14]:

$$
\Gamma \to \Gamma - \frac{g(M_b)^2}{24\pi^2} \gamma_\mu \hat{v}_b \Gamma \hat{v}_b \gamma_\mu + \cdots \tag{26}
$$

Combining (25) and (26) we get the spin dependent matching term relating heavy b, c theory current with the complete one. Up to order α_S we have:

$$
\Gamma \to \Gamma - \frac{g(M_b)^2}{24\pi^2} \gamma_\mu \hat{v}_b \Gamma \hat{v}_b \gamma_\mu - \frac{g(M_c)^2}{6\pi^2} r(v_b v_c) \hat{v}_b \Gamma + \cdots \tag{27}
$$

The matrix on the right-hand side up to order α_S generates the same matrix elements in the effective theory as Γ in the complete one.

For the case of the vector and axial vector currents $\Gamma = \gamma_\mu, \gamma_\mu \gamma_5$ we have respectively:

$$
\gamma_\mu \to (1 + k)\gamma_\mu + (\lambda_b - \lambda_c(v_b v_c)) \hat{v}_b \gamma_\mu \tag{28}
$$

and

$$\gamma_\mu\gamma_5 \rightarrow (1+k)\gamma_\mu\gamma_5 - (\lambda_b + \lambda_c(v_b v_c))\hat{v}_b\gamma_\mu\gamma_5 \quad (29)$$

where:

$$\lambda_b = \frac{\alpha_S(M_b)}{3\pi}, \qquad \lambda_c = \frac{2\alpha_S(M_c)}{3\pi} r(v_b v_c) \quad (30)$$

and k is a renormalization constant of order α_S that depends on the renormalization prescription which is adopted.

4.7 Large energy effective theory

Consider the exclusive hadronic B decay

$$B \rightarrow D + \pi \quad (1)$$

Process (1) is generated by the action of an effective hamiltonian H_{eff} consisting at tree level of the product of two colour singlet $V - A$ currents:

$$H_{eff} = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* \, O_1(x) \quad (2)$$

where $O_1(x) = J_\mu^{(b\rightarrow c)}(x)\; J^{\mu(u\rightarrow d)}(x)$. One loop gluonic corrections of (2) require renormalization and introduce, due to colour exchange between quark bilinears, the octet-octet operator $O_2(x)$ in the effective Hamiltonian:

$$O_2(x) = \bar{c}(x)\gamma_\mu(1-\gamma_5)t^a b(x)\; \bar{d}(x)\gamma_\mu(1-\gamma_5)t^a u(x) \quad (3)$$

We assume that typical momentum transfers q_μ between quarks and gluons occurring in the generation of final bound states (D and π mesons) are of order Λ_{QCD}:

$$q^2 \sim \Lambda_{QCD}^2 \quad (4)$$

At partonic level, process (1) is described by spectator b decay:

$$b \rightarrow c + \overline{u} + d \tag{5}$$

The energy E transferred to light $\overline{u}$ and d quarks, assuming b quark at rest, is given by:

$$E \geq \frac{M_b^2 - M_c^2}{2M_b} \simeq 2.2\ GeV \gg \Lambda_{QCD} \tag{6}$$

To form a low mass particle, with a large energy release, the $\overline{u}$ and d quarks have to be emitted collinearly, with momenta that differ at most by order Λ_{QCD}:

$$p_{\overline{u}} = xp + k \tag{7}$$

$$p_d = (1-x)p - k \tag{8}$$

where k is residual momentum of order $\sim \Lambda_{QCD}$, $p_0 = E$ and $p^2 \sim 0$.
$x(1-x)$ represents the fraction of momentum transferred in partonic decay to $\overline{u}(d)$ quark. We naturally assume that x distribution is not peaked at $x = 0, 1$.

Spectator quark q_s ends up in the D meson, since this channel implies a momentum transfer to q_s of order Λ_{QCD} instead of E. In intuitive terms, spectator quark starts to move screening the colour of the c quark, that has a Lorentz factor $\gamma_L \simeq 1.8$ and then transfer a momentum $\sim \gamma_L \Lambda_{QCD} \sim \Lambda_{QCD}$ (the dynamic is the same as in $B \rightarrow D$ semileptonic decay at $q^2 \simeq 0$).

Decay of the type (1) can be computed with an effective theory for energetic light quarks, organised as a $1/E$ expansion (large enrgy effectiev theory, Dugan and Grinstein [16]). The expected contribution of terms of order N is $(\Lambda_{QCD}/E)^N$. Setting $p = En$, where n is a null vector $n^2 = 0$ with $n_0 = 1$, we get for the energetic collinear light quark propagators at lowest order in $1/E$:

$$S_{\overline{u}}(k,n):\ \frac{x\hat{p} + \hat{k}}{(xp+k)^2 + i\epsilon} \rightarrow \frac{\hat{n}}{2}\frac{1}{n \cdot k + i\epsilon} + O(\frac{k}{E}) \tag{9}$$

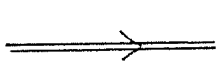
and:

$$S_d(k,n) \;=\; S_{\overline{u}}(k,n), \tag{10}$$

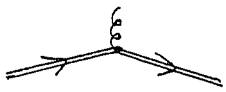
since at lowest order there is not x dependence. Note that the expansion is singular at $x = 0, 1$, when the leading momentum of $\overline{u}/d$ vanishes.
Light quark/antiquark in the large energy effective theory is treated as an infinite momentum particle with zero rest mass. Propagators (9) and (10) can be derived from the lagrangian:

$$L_{LEET} \;=\; \psi^{\dagger} i \; n_{\mu} D^{\mu} \psi \tag{11}$$

where the field ψ satisfies the condition $\hat{n}\psi = 0$. The simplest version of Feynman rules for the large energy effective theory is:



$$= \frac{i}{n \cdot k + i\epsilon} \tag{12}$$



$$= -igt^{a} n_{\mu} \tag{13}$$

At the end we have to multiply by $\hat{n}/2$ for every light quark in the process.

Weak interactions, with respect to hadron dynamic, act in decay (1) as an external agency, transferring the energy E to the light degrees of freedom. With $1/E$ expansion we extract analytically the dependence of amplitudes on the 'high energy' E. With the additional application of the heavy quark effective theory for b and c quarks, Λ_{QCD} is the only mass scale left in the process.

The large energy effective theory can be made covariant by summing $L_{LEET}(n,x)$ over all possible null vectors n_{μ} with a Lorentz invariant measure, just as in the case of the relativistic infinite mass theory (involving timelike vectors v_{μ}, with $v^2 = 1$). Also in the case of the large energy effective theory there is a superselection rule, related to the spatial direction of light quark motion, that is not changed by finite momentum transfer.

4.8 Factorization

The large energy effective theory discussed in the previous section exhibits factorization of decay amplitude (1) at leading order in E and at every order in the loop expansion (Dugan and Grinstein [16]):

$$<\pi D \mid O_1(0) \mid B> = <\pi \mid J_\mu(0)^{(u\to d)} \mid 0 >< D \mid J^{\mu(b\to c)}(0) \mid B> \qquad (1)$$

Amplitude factorization is equivalent to say that the process consists of independent subprocesses.

Indeed, in the light cone gauge:

$$n_\mu \cdot A^\mu = 0 \qquad (2)$$

the interaction vertex vanishes on contraction with gluon propagator. The only non zero radiative corrections involve the $b \to c$ current, like in semileptonic B decays.

Factorization allows a prediction of non leptonic decay rate of (1) on the basis of the knowledge of pion decay constant f_π and B semileptonic decay form factors $f^+(q^2)$, $f^-(q^2)$.

$O_2(x)$ doesn't contribute to leading order in $1/E$ to decay (1) since physical states B, D, π are colour singlets:

$$\begin{aligned} &<\pi D \mid O_1(0) \mid B> = \\ &\quad = < D \mid \bar{c}(0)\gamma_\mu(1-\gamma_5)t^a b(0) \mid B > < \pi \mid \bar{d}(0)\gamma_\mu(1-\gamma_5)t^a u(0) \mid 0 > \qquad (3) \\ &\quad = 0 \end{aligned}$$

Matching constants of the effective theory for light quarks with the complete theory have been computed in LLA, assuming b(and c) as heavy and employing the heavy quark effective theory [16]. The corrections however are not realistic since the mass scales M_b, M_c, E are not well separated and $1/E$ corrections are expected to dominate. More investigation will clarify this important point. An experimental analysis by the CLEO collaboration has shown that factorization works reasonably well for the B decays observed so far, within still sizable experimental errors.

In quite analogous way, we can prove factorization in Λ_b hadronic decay:

$$\Lambda_b \rightarrow \Lambda_c + \pi \tag{4}$$

Let us end this section with some qualitative observations on other decays than (1).
We expect $b \rightarrow u$ transitions ending into two light mesons, such as

$$B \rightarrow \pi + \rho \tag{5}$$

to be further suppressed in addition to the small factor $\mid V_{ub} \mid^2 / \mid V_{cb} \mid^2$. We assume that a quark antiquark pair in b decay has to be highly collinear to combine into a $\pi(\rho)$ meson. The third light quark coming from b decay has an energy $\sim E$. To form the second $\rho(\pi)$ meson, it has to tranfer to the spectator quark a momentum of order E. The amplitude is then reduced by a factor $\sim \Lambda_{QCD}/E$.

Finally, factorization is expected to hold in hadronic decays with many energetic high collinear pions.

CONCLUSIONS

The study of weak processes involving quarks is still a wide and open research field. It involves crucial tests of the Standard Model because there is a high sensitivity to short distance effects. On the either hand, these phenomena can reveal new physics beyond the Standard Model itself. As we have stressed in various circumstances, a detailed understanding of strong dynamic is essential. In our opinion the most promising technique for this task is lattice QCD. We aim to perform a first principle computation of the form factors for $B \to D/D^*$, $B \to \pi/\rho$ semileptonic decays. With present computer facilities it is not possible to study b physics on the lattice and it is necessary to implement the effective theory discussed in chapter (4). We have then discretized the continuum infinite mass theory with a proper definition of particle velocities on the lattice. To convert the measures on the lattice of the form factors in physical unit it is necessary the renormalization of the lattice infinite mass theory. That is a quite long and involved work. At present we are finishing the computation of the relevant one loop Feynman diagrams (we do not reported in the thesis the lattice action, the lattice Feynman rules and the lattice loop integrals because they are not essential for the physical understanding of the problem). This will be followed by numerical study.

ACKNOWLEDGEMENTS

I am extremely grateful to my supervisor Prof. Guido Martinelli who has taught me many things during this last year. He also answered all the questions in every topic of physics I asked to him.

I am very glad to thank Prof. Giorgio Salvini for his very kind hospitality in his room at the Rome University, for frequent discussions on physics and for encouragement.

I would like to express particular thanks to Marco Crisafulli; we compared and discussed together every result of the computations performed.

I thank also Laura Reina for useful suggestions and Arianna Borrelli, Vittorio Lubicz, Marco Masetti and Carlotta Pittori for their kind hospitality and for discussions.

I thank my old friends Stefano and Serena Colonna for their help in making the figures and Pasquale Pavone and Francesca Sartogo for their help in "computer science".

REFERENCES

Chapter 1

[1] S. Weinberg, Phys. Rev. Lett. 19: 1264(1967); A. Salam in "Elementary Particle theory", ed. N. Svartholm, p.637, Stockholm: Almquist & Wiksells (1968).

[2] H. Albrecht et all. (ARGUS collaboration), Desy report DESY-87-029(1987).

[3] S. Glashow, J. Iliopulos and L. Maiani, Phys. Rev. D2(1970) 1285.

[4] T.P. Cheng and L.F. Li, "Gauge theory of elementary particles", Clarendon Press, Oxford(1984).

[5] T. Kobayashi and M. Maskawa, Progr. Theor. Phys. 49(1973) 652.

[6] L. Maiani, Proceedings of the Int. Symposium on lepton and Photon Interactions at High Energies, Hamburg(1977).

[7] L. Wolfenstein, Phys. Rev. Lett. 51(1983) 1945.

[8] W.J. Marciano, BNL preprint BNL-45999.

[9] D.G. Hitlin, Caltech preprint CALT-68-1722.

[10] M. Ademollo and R. Gatto, Phys. Rev. Lett. 13, 264(1964).

[11] A.J. Buras, Rev. Mod. Phys. 52 n.1(1980).

[12] G. Martinelli, Nucl. Phys. B(Proc. Suppl.) 10A(1989), 146-198.

[13] G. Altarelli et al., Nucl. Phys. B208(1982), 365-380.

[14] B. Grinstein et al., Phys. Rev Lett. 56 n.4(1986).

[15] E. Eichten et al., Phys. Rev. D17 3090(1979); 21, 203 (1980).

Chapter 2

[1] L.B. Okun, "Leptoni e quark", Editori Riuniti(1986).

[2] T.D. Lee, "Particle physics and introduction to field theory", Harwood academic publishers(1982).

[3] P.J. Franzini, Cern preprint CERN-TH-5083.

[4] L.D. Landau and E.M. Lifsits, "Meccanica Quantistica (teoria non relativistica)", vol.3, Editori Riuniti.

[5] T. Inami and L.S. Lim, Progr. Theor. Phys. 65(1981) 297.

[6] L. Reina, "CP violation and the present of heavy quarks: the case of $K^0 - \overline{K}^0$ mixing", Sissa master thesis (1990).

[7] F.J. Gilman and M.B. Wise Phys. Rev. D 27(1985) 1128.

[8] M.I. Vysotskiĭ, Sov. J. Nucl. Phys. 31(1980) 797.

[9] A. Datta et al., Particles and fields 46(1990) 63-70.

[10] C.R. Allton et al., SHEP 89/90-11.

[11] M. Lusignoli et al., Rome preprint n.792(1991).

Chapter 3

[1] E. Witten, Nucl. Phys. B122(1977) 109-143.

[2] A. Falk et al. HUTP-90/A044.

[3] W.E. Caswell and G.P. Lepage, Phys. Lett. B167(1986) 437.

[4] C. Itzykson and J.B. Zuber, "Quantum field theory", Mc Graw-Hill(1980).

[5] G. Altarelli and L. Maiani, Phys. Lett. B52(1974) 351.

[6] M.K. Gaillard and B.W. Lee, Phys. Rev. Lett. 33(1974) 108.

[7] F.J. Gilman and M.B. Wise, Phys. Lett. B83 (1979) 83.

[8] F.J. Gilman and M.B. Wise, Phys. Rev. D20(1979) 2392.

Chapter 4

[1] G. Martinelli, Nucl. Phys. B(Proc. Suppl.) 10A(1989) 146-198.

[2] E. Eichten and F. Feinberg, Phys. Rev. D23(1981) 2724-2744.

[3] G.P. Lepage and B.A. Tacker, Nucl. Phys B(Proc. Suppl.) 4(1988), 199-203.

[4] J.D.Bjorken and S.D. Drell, "Relativistic Quantum Mechanics",

Mc Graw-Hill Book Company(1964).

[5] L. Maiani, Rome preprint n.786(1991).

[6] E. Eichten and B.Hill, Phys. Lett. B234(1990) 511.

[7] E. Eichten and B. Hill, Phys. Lett. B240(1990) 1930.

[8] W. Wirbel, Dortmund preprint, DO-TH 89/4.

[9] H. Georgi, Phys. Lett. B240(1990) 447.

[10] N. Isgur and M.B. Wise, Phys. Lett. B232 n.1 (1989) 113.

[11] N. Isgur and M.B. Wise, Caltech preprint CALT-68-1608.

[12] M.B. Voloshin and M.A. Shifman, Sov. J. Nucl. Phys. 45(1987) 292.

[13] H.D. Politzer and M.B. Wise, Phys. Lett. B206 n.4(1988) 681.

[14] F.Falk et al. Harward preprint HUTP-90/A011.

[15] H. Georgi, B. Grinstein and M.B. Wise Harward preprint HUTP-90/A052.

[16] M.J. Dugan and B. Grinstein, Phys. Lett. B255 n.4(1991) 583.