ISAS - INTERNATIONAL SCHOOL FOR ADVANCED STUDIES

DYNAMICAL SYMMETRY BREAKING IN QCD-LIKE GAUGE THEORIES

Stefania De Curtis

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Supervisor: Roberto Casalbuoni

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TRIESTE

1. INTRODUCTION

One outstanding problem in modern elementary particle physics is the problem of spontaneous symmetry breaking (SSB).(1) Some of the earliest applications of SSB in particle physics, dealt with the chiral symmetries of the strong interactions(2) and much of the progress of theoretical particle physics has occurred through exploration of the phenomenological consequences of spontaneous chiral symmetry breaking. However, the underlying mechanism of this phenomenon is still an open problem in the theory of strong interactions.

As it is well known, SSB can be realized by elementary or composite (dynamical symmetry breaking (DSB)) scalar fields. In both cases, the manifestation of symmetry breaking is the presence in the physical spectrum of a massless particle, the Goldstone boson,(3) which turns out to be elementary or composite in the respective two cases. Spin-0 hadrons are bound states of fermions (quarks and antiquarks),therefore they should not be described by fundamental spinless fields. Thus, the realization of the spontaneous breaking of chiral symmetry in hadron physics is closely connected with the realization of the dynamics of the tightly bound fermion-antifermion states. Here the Goldstone mechanism is not related to the fundamental Lagrangian but to an effective Lagrangian of hadron interaction at low energies when the hadronics structure is inessential.

The successes of the phenomenological description of spontaneous chiral symmetry breaking in hadron physics (see for example the numerous sum rules of current algebra, the low energy relations of pion physics or the analysis of the meson mass relations which also indicates that pseudoscalar mesons can be regarded as "almost" Goldstone bosons)[4], leaves no doubt about the reality of this phenomenon. This is the reason why the problem of the dynamical chiral symmetry breaking in QCD,which is the only candidate for a theory of strong interactions, has become of particular urgency.

Due to the previous motivations,R.Casalbuoni,D.Dominici,A.Barducci, R.Gatto and I, have started last year with a line of research intended to study DSB in QCD-like gauge theories from a dynamical point of view.[5] Our first priority, was to learn how to do a more quantitative computation of chiral symmetry breaking.

In order to make a systematic analysis ,we have used an effective potential for composite operators which is a convenient modification of the potential introduced by Cornwall,Jackiw and Tomboulis.[6]

We have assumed that the main contribution to the effective potential comes from short-distance effects as suggested by recent lattice calculations.[7] With such an approximation, it is sensible to perform a loop expansion of the effective potential

itself.Our strategy consists in introducing a parameter μ separating the infrared region from the ultraviolet one. We have assumed the self-energy of the fermions as a constant for energies lower than μ ,whereas, for greater energies we have taken the behaviour dictated by the operator product expansion.[15] This ansatz depends on a variational parameter wich is the fermion condensate renormalized at the point μ . Then, minimizing the potential with respect to the condensate, we have calculated the value of the condensate as a function of μ .
We review the basic features of spontaneous symmetry breaking in sect.2. In sect.3 we recall some fundamental aspects of the effective potential for composite operators as discussed by Cornwall, Jackiw and Tomboulis and, in sect.4 we introduce a modification of the CJT effective action which is more convenient from a computational point of view. In sect.5 we discuss some properties of chirally invariant QCD-like gauge theories and in sect.6 we deduce from the asymptotical equations for the Green's functions,the UV behaviour for the fermion self-energy. In sect.7 we show that the results obtained in sect.6 are consistent with the OPE analysis. Sect.8 represents the central part of this work. We perform the calculation of the effective potential as a function af a variational parameter χ whose physical meaning is explained in sect.9. In sect.10 we discuss the effective

potential using the case in which logarithmic corrections are neglected, for comparison. We find that dynamical symmetry breaking occurs provided that the coupling constant exceeds some critical value. Further, in our last section, the dimensional transmutation phenomenon is analyzed.

In appendix we collect the relevant diagrams we have obtained by numerical analysis.

2. SPONTANEOUS SYMMETRY BREAKING

The ground state of a theory can be quite generally classified through the study of the symmetries of the Lagrangian defining the theory itself. If the Lagrangian admits a continuos symmetry group G with generators S_i satisfying the Lie algebra:

$$[S_i, S_j] = i f_{ijk} S_k \qquad S_i \in \text{Lie } G \tag{2.1}$$

the classical Goldstone theorem[3] says that, if there are generators $R_\alpha \in$ Lie G, such that:

$$R_\alpha |0\rangle \neq 0 \tag{2.2}$$

then, together with any generator R_α, there exists a massless particle (Goldstone boson) having the same quantum numbers of R_α. If we denote by T_α the generators of Lie G annihilating the vacuum, i.e.

$$T_\alpha |0\rangle = 0 \tag{2.3}$$

one gets:

$$[T_\alpha, T_\beta] = i f_{\alpha\beta\gamma} T_\gamma \tag{2.4}$$

so they generate a subgroup of G (the stability group of the vacuum). By denoting such a group with H, one sees that the generators R_α lie in the quotient space:

$$R_\alpha \in \text{Lie } G / \text{Lie } H \tag{2.5}$$

When such a thing happens we say that the symmetry G is spontaneously broken down the symmetry H.[1]

The natural question is then how to study in general such a problem. A simple way is to consider operators A singlets with respect to H. In such a situation, let us look at the following matrix element:

$$\begin{aligned} \langle 0|A|0\rangle &= \langle 0|U^{-1}UAU^{-1}U|0\rangle \simeq \langle 0|(1-i\epsilon S)A'(1+i\epsilon S)|0\rangle \simeq \\ &\simeq \langle 0|A'|0\rangle - i\epsilon \langle 0|[S,A']|0\rangle \end{aligned} \tag{2.6}$$

with $U = e^{i\epsilon S} \simeq \mathbb{1} + i\epsilon S,\ S \in \mathrm{Lie}\,G$ and $A' = UAU^{-1}$

Now, let us consider the various possibilities:

i) $S \in$ Lie H; in this case $A' = A$ (because A is singlet under H) therefore $\langle 0|[S,A]|0\rangle = 0 \quad \Rightarrow \quad S|0\rangle = 0$

ii) $S \in$ Lie G/Lie H; in this case $A' \neq A$ and furthermore if $\langle 0|A|0\rangle \neq 0$ then $\langle 0|[S,A']|0\rangle \neq 0 \quad \Rightarrow \quad S|0\rangle \neq 0$.

We see that the breaking $G \rightarrow H$ can be tested by the vacuum matrix elements of an operator A singlet under H simply by studying if $\langle 0|A|0\rangle$ is equal or different from zero. Generally speaking we will have $\langle 0|A|0\rangle = f(g_i, m_i^2)$ where g_i and m_i^2 are the coupling constants and the mass parameters of the theory. We will call the space spanned by g_i and m_i^2 the "phase space" of the theory. Therefore, we will have to determine the "phase diagram" of a given theory, where each phase will be characterized by a convenient operator A such that $\langle 0|A|0\rangle \neq 0$.

We will call $\langle 0|A|0\rangle$ an <u>order parameter</u>.

When A is one of the elementary operators of the theory we say that the theory undergoes a <u>spontaneous symmetry breaking</u> whereas when A is a composite operator we speak of <u>dynamical symmetry breaking</u>.

3. THE EFFECTIVE POTENTIAL FORMALISM

It is clear that in order to study the problem of symmetry breaking one has to derive a systematic way to study the matrix elements of a given operator. In particular for the breakdown of chiral symmetry we need to know how to test wether the energy of the vacuum is lowered if some fermion bilinear acquires a non-zero vacuum expectation value. If the quantity acquiring a vacuum expectation value is a scalar field φ , we can use an object called the effective potential,(1) which is equal to the energy of the vacuum under the constraint that the vacuum expectation value of φ has some definite value φ_c . One needs only to compute this effective potential and minimize it with respect to φ_c to determine the vacuum values of φ , so the various phases of the theory are given by the extrema of this function. But, if one expects that the breaking of the theory is due to the formation of scalar bound states (condensates) playing the role of the previous elementary scalar field, and this is the case in strongly interacting fermionic theories, one needs an appropriate generalization of the effective potential for composite operators. The idea is to introduce inside the generating functional of the Green's functions of the theory, sources coupled to the operators we are interested in. For example, in the case of chiral symmetry breaking which we want to examine, to produce a vacuum expectation value of the operator $\bar{\psi}\psi$, we must, in principle, turn on some external field (analogous to a magnetic field orienting a potentially ferromagnetic system) coupled to this bilinear, construct the ordered vacuum in the presence of this field, and then see if the order in this vacuum survives when we turn this field off. We can realize this program by introducing a bilocal source coupled to a bilocal product of fermion fields.[8] The generating functional of the Green's functions will be:(writing down only the dependence on the fermion fields)

$$Z[K,\eta] = e^{iW[K,\eta]} = \frac{1}{N}\int \mathcal{D}\psi\mathcal{D}\bar{\psi}\, e^{i\left[I(\psi)+\bar{\eta}\psi+\bar{\psi}\eta+\bar{\psi}K\psi\right]} \qquad (\hbar=1) \tag{3.1}$$

where $I(\psi)$ is the classical action of the theory

N is a normalization factor

η and $\bar{\eta}$ are the usual sources and K is a bilocal source.

We have used the shorthand notation $\bar{\psi}K\psi$ to denote

$$\int d^4x\, d^4y\; \bar{\psi}_\alpha(x)\, K_{\alpha\beta}(x,y)\, \psi_\beta(y)$$

in which α, β are collective indices for spinor, flavor and color variables.

We will adjust $K(x,y)$ so that, in the Landau gauge (see later on):

$$\langle 0|\, T\psi_\alpha(x)\bar{\psi}_\beta(y)|0\rangle_K = S_{\alpha\beta}(x,y) = \int \frac{d^4p}{(2\pi)^4}\, e^{-ip(x-y)} \left(\frac{i}{\hat{p}-\Sigma(p^2)}\right)_{\alpha\beta} \tag{3.2}$$

for any generalized mass term $\Sigma(p^2)$.

Our goal is to determine for which function $\Sigma(p^2)$ this condition will be stable if we turn K off.

Let us define the variables:

$$\psi^c_\alpha(x) = \langle 0|\psi_\alpha(x)|0\rangle_{\eta,K} = \frac{\delta W}{\delta\bar\eta_\alpha(x)} \tag{3.3}$$

$$\bar\psi^c_\alpha(x) = \langle 0|\bar\psi_\alpha(x)|0\rangle_{\eta,K} = -\frac{\delta W}{\delta\eta_\alpha(x)} \tag{3.4}$$

In (3.2) we have introduced

$$S_{\alpha\beta}(x,y) = \langle 0|T\psi_\alpha(x)\bar\psi_\beta(y)|0\rangle_{\eta,K} = i\,\frac{\delta^2 W}{\delta\bar\eta_\alpha(x)\,\delta\eta_\beta(y)} \tag{3.5}$$

which for $\eta = \bar\eta = K = 0$ coincides with the exact propagator of the theory. It is easy to verify that:

$$\frac{\delta W}{\delta K_{\alpha\beta}(x,y)} = -S_{\beta\alpha}(y,x) - \psi^c_\beta(y)\,\bar\psi^c_\alpha(x) \tag{3.6}$$

which shows the relationship between the bilocal source K and our variable S.

It is useful to exchange S or Σ (see eq.(3.2)) for K as our dynamical variable. This can be done by permorming a double Legendre transformation of the generating functional of the connected Green's functions[6] (in complete analogy with Statistical Mechanics where we replace the Helmholtz by the Gibbs free energy). Let us define

$$\Gamma(\psi^c, S) = W[\eta,K] - \eta_\alpha\frac{\delta W}{\delta\eta_\alpha} - \bar\eta_\alpha\frac{\delta W}{\delta\bar\eta_\alpha} - K_{\alpha\beta}\frac{\delta W}{\delta K_{\alpha\beta}} \tag{3.7}$$

where now η, $\bar\eta$ and K are thought of as functions of ψ^c, $\bar\psi^c$ and S, obtained by inverting equations (3.3),(3.4) and (3.6).

Differentiating Γ with respect to ψ_c, $\bar\psi_c$ and K we obtain:

$$\frac{\delta\Gamma}{\delta\psi^c_\alpha} = \bar\eta_\alpha + \bar\psi^c_\beta K_{\beta\alpha} \qquad \frac{\delta\Gamma}{\delta\bar\psi^c_\alpha} = -\eta_\alpha - K_{\alpha\beta}\psi^c_\beta$$

$$\frac{\delta\Gamma}{\delta S_{\alpha\beta}} = K_{\beta\alpha} \tag{3.8}$$

In particular for vanishing sources, one gets that Γ must be stationary with respect to both the variables ψ^c and S:

$$\left.\frac{\delta\Gamma}{\delta\psi^c}\right|_{\eta=K=0}=0 \qquad \left.\frac{\delta\Gamma}{\delta\bar{\psi}^c}\right|_{\eta=K=0}=0 \qquad \left.\frac{\delta\Gamma}{\delta S}\right|_{\eta=K=0}=0 \tag{3.9}$$

The first two conditions give essentially the equations of motion of our system while, the third one, is nothing but the Schwinger-Dyson equation for the fermion propagator.

Γ is the generating functional in ψ^c and $\bar{\psi}^c$ of the two-particle irreducible Green's functions expressed in terms of the propagator S and it is called the <u>effective action for composite operators</u>. Γ reproduces the standard effective action when we set $K = 0$.

Let us start evaluating Γ in the free field case. One has:

$$Z[\eta,k] = e^{iW[\eta,k]} = \frac{\int \mathcal{D}\psi\mathcal{D}\bar{\psi}\; e^{i[\bar{\psi}iD^{-1}\psi+\bar{\eta}\psi+\bar{\psi}\eta+\bar{\psi}K\psi]}}{\int \mathcal{D}\psi\mathcal{D}\bar{\psi}\; e^{i[\bar{\psi}iD^{-1}\psi]}} \tag{3.10}$$

where D(x,y) is the free fermion propagator.

Here we have a gaussian integral, so we can calculate W:

$$W[\eta,k] = -i\,\mathrm{Tr}\ln(iD^{-1}+K) + i\,\mathrm{Tr}\ln(iD^{-1}) - \bar{\eta}(iD^{-1}+K)^{-1}\eta \tag{3.11}$$

from which:

$$\begin{aligned} \psi^c_\alpha &= -(iD^{-1}+K)^{-1}_{\alpha\beta}\eta_\beta \\ \bar{\psi}^c_\alpha &= -\bar{\eta}_\beta(iD^{-1}+K)^{-1}_{\beta\alpha} \\ S_{\alpha\beta} &= i(iD^{-1}+K)^{-1}_{\alpha\beta} \end{aligned} \tag{3.12}$$

that is

$$i S^{-1} = i D^{-1} + K \tag{3.13}$$

from which it is clear that S represents the fermionic propagator when the bilocal source K is on. We can invert the relations (3.12):

$$\begin{aligned} \eta_\alpha &= i S^{-1}_{\alpha\beta} \psi^c_\beta \\ \bar\eta_\alpha &= -i \bar\psi^c_\beta S^{-1}_{\beta\alpha} \\ K_{\alpha\beta} &= i S^{-1}_{\alpha\beta} - i D^{-1}_{\alpha\beta} \end{aligned} \tag{3.14}$$

and evalaate the double Legendre transform:

$$\Gamma(\psi_c, S) = I(\psi_c) + i \operatorname{Tr} \ln (D^{-1} S) - i \operatorname{Tr} (D^{-1} S) + \text{const.} \tag{3.15}$$

From this equation we get:

$$\frac{\delta \Gamma}{\delta S} = i S^{-1} - i D^{-1} \tag{3.16}$$

Then the stationary condition $\frac{\delta \Gamma}{\delta S} = 0$ is satisfied for $S = D$ that is the free fermion propagator. Evaluating Γ at the extremum one obtains the classical action:

$$\Gamma(\psi_c, S = D) = I(\psi_c) \tag{3.17}$$

We want now to derive a loop expansion for Γ when we consider interacting fields. First of all we notice that Γ can be obtained by performing the two Legendre transformations in a sequential way, that is taking first the Legendre transform for fixed K, (call it Γ^K), and after transforming with respect to K. Γ^K happens to be the ordinary effective action for a theory described by the classical action:

$$I^K(\psi) = I(\psi) + \bar\psi K \psi \tag{3.18}$$

Expressing $\Gamma^K(\psi_c)$ by the formal series given by Jackiw[11], and performing the Legendre transform with respect to K one gets the formal series for the Cornwall, Jackiw and Tomboulis effective action:

$$\Gamma(\psi_c, S) = I(\psi_c) + i\,\mathrm{Tr}\ln(D^{-1}S) - i\,\mathrm{Tr}\,\mathcal{D}^{-1}S + \Gamma_2 + \text{const.} \tag{3.19}$$

where $I(\psi_c)$ is the classical action

$$i D^{-1}_{\alpha\beta} = \left.\frac{\delta^2 I}{\delta\psi_\beta\,\delta\bar\psi_\alpha}\right|_{\psi=0}$$

$$i\mathcal{D}^{-1}_{\alpha\beta} = \left.\frac{\delta^2 I}{\delta\psi_\beta\,\delta\bar\psi_\alpha}\right|_{\psi=\psi_c} = iD^{-1}_{\alpha\beta} + \left.\frac{\delta^2 I_{int}}{\delta\psi_\beta\,\delta\bar\psi_\alpha}\right|_{\psi=\psi_c} \tag{3.20}$$

and Γ_2 generates all the two-particle irreducible (2PI) vacuum diagrams with respect to the action $\bar\psi\, iS^{-1}\psi + I_{int}(\psi,\psi_c)$

where $I_{int}(\psi,\psi_c) = I_{int}(\psi+\psi_c) - I_{int}(\psi_c) - \psi_\alpha \left.\frac{\delta I_{int}}{\delta\psi_\alpha}\right|_{\psi=\psi_c} +$

$$- \bar\psi_\alpha \left.\frac{\delta I_{int}}{\delta\bar\psi_\alpha}\right|_{\psi=\psi_c} - \bar\psi_\alpha\psi_\beta \left.\frac{\delta^2 I_{int}}{\delta\psi_\beta\,\delta\bar\psi_\alpha}\right|_{\psi=\psi_c}.$$

Recalling that $\delta\Gamma/\delta S = K$ and differentiating (3.19) with respect to S we obtain:

$$iS^{-1}_{\alpha\beta} = i\mathcal{D}^{-1}_{\alpha\beta} - \frac{\delta\Gamma_2}{\delta S_{\beta\alpha}} + K_{\alpha\beta} \tag{3.21}$$

For $K = 0$ this is just the Schwinger-Dyson equation for the propagator S. This in turn implies that $\frac{\delta\Gamma_2}{\delta S}$ is the fermion self-energy.

(Remark: we want to consider only renormalizable and chiral invariant actions in four dimensions, then $i\mathcal{D}^{-1}_{\alpha\beta} = iD^{-1}_{\alpha\beta} = i\hat\partial_{\alpha\beta}$

If we are interested only in solutions translationally invariant of $S_{\alpha\beta}$, which is our case (see (3.2)), we can choose the source K as a function of (x-y). In such a case we can factorize out the space-time volume and define the <u>effective potential V for composite operators</u>:

$$\Gamma(\psi_c, S) = -V(\psi_c, S)\int d^4x \tag{3.22}$$

The formal series for V can be easily obtained from the series for Γ.

However, one has to interpret the results deriving from the study of the Cornwall, Jackiw and Tomboulis (CJT) potential with a certain care. For instance, in the free field case, it is easy to verify that V is not bounded from below. In fact, let us take massless Fermi fields in the Euclidean case and let us parametrize the propagator in momentum space by:

$$S(p) = \left(-i\hat{p} + M(p)\right)^{-1} \tag{3.23}$$

Recall that for Euclidean Fermi fields the effective action reads: $(D^{-1} = \hat{\partial})$

$$\Gamma(\psi_c, S) = I(\psi_c) + \mathrm{Tr}\ln(D^{-1}S) - \mathrm{Tr}(D^{-1}S) - \Gamma_2(S) + \mathrm{const.} \tag{3.24}$$

Since we are only interested in the dependence on M we have:

$$\Gamma(M) = -\mathrm{Tr}\ln\left(-i\hat{p} + M(p)\right) + \mathrm{Tr}\left(\frac{M(p)}{-i\hat{p} + M(p)}\right) + \mathrm{const.} \tag{3.25}$$

The first term gives:

$$-\mathrm{Tr}\ln\left(-i\hat{p}+M(p)\right) = -\int\frac{d^4p}{(2\pi)^4}\,\langle p|\ln\det\left(-i\hat{p}+M(p)\right)|p\rangle =$$

$$= \int\frac{d^4p}{(2\pi)^4}(2\pi)^4\delta^4(0)\,\langle p|\ln\det\left(-i\hat{p}+M(p)\right)|p\rangle =$$

$$= \int d^4p\,\langle p|\ln\det\left(-i\hat{p}+M(p)\right)|p\rangle\int d^4x$$

and, by using $(-i\hat{p}+M)\gamma_5(-i\hat{p}+M) = \gamma_5\,(p^2+M^2)$ we get

$$-\mathrm{Tr}\ln\left(-i\hat{p}+M(p)\right) = -2\int d^4p\,\ln\left(p^2+M^2\right)\int d^4x\,.$$

The second term gives:

$$4\int d^4p\,\frac{M^2(p)}{(p^2+M^2(p))}\int d^4x$$

so we obtain $\left[\,\Gamma(M) = V(M)\int d^4x\,\right]$

$$V(M) = \int d^4p\left(-2\ln\left(p^2+M^2\right) + 4\,\frac{M^2}{p^2+M^2}\right) \tag{3.26}$$

Let us perform

$$\frac{\delta V}{\delta M(p)} = \frac{-4M}{p^2+M^2} + \frac{8Mp^2}{(p^2+M^2)^2} = \frac{4M}{(p^2+M^2)^2}(p^2-M^2) \; . \qquad (3.27)$$

The correct value M(p)=0 is a solution to the condition $\delta V(M)/\delta M = 0$. However there is another solution at M(p)=p. Further, the shape of V(M) is such that, after attaining a local minimum at M(p) = 0, it rises until M(p) = p and then falls off, going to $(-\infty)$ as $M \to \infty$.

The stationary condition $\delta\Gamma/\delta S = 0$ does produce the correct solution for M but only if interpreted with a grain of salt; clearly we cannot use standard arguments about the absolute minimum. In fact the CJT effective action is not generally bounded from below and this problem arises already at the free case level.

4. A MODIFICATION OF THE CJT EFFECTIVE ACTION

We present an explicit derivation of an effective action for composite operators which has the same local extrema as the CJT one. Our effective action can be shown to reproduce in quadrilinear field theories, the effective action obtained with the method of collective variables at the one-loop level. It has the advantage of eliminating the problem of unboundness-below of the potential and, as we shall see, it is very convenient from a computational point of view.

Recall the formal series for Γ in euclidean space: (from now on we will denote ψ^c with ψ and Γ with Γ_{CJT})

$$\Gamma_{CJT} = I(\psi) - \mathrm{Tr}\,\ln S^{-1} - \mathrm{Tr}(D^{-1}S) - \Gamma_2 + \mathrm{const.} \tag{4.1}$$

From the very definition of Γ as a double Legendre transformation (see (3.7)), it is easy to show that the bilocal source is the derivative of Γ with respect to S:

$$K = \delta\Gamma/\delta S \tag{4.2}$$

From (4.1) one then gets:

$$K = S^{-1} - D^{-1} - \delta\Gamma_2/\delta S \tag{4.3}$$

By using this expression in (4.1) one obtains:

$$\begin{aligned}\Gamma_{CJT}(\psi,S) &= I(\psi) - \mathrm{Tr}\,\ln\left(K + D^{-1} + \frac{\delta\Gamma_2}{\delta S}\right) - \\ &- \mathrm{Tr}\left[\left(S^{-1} - \frac{\delta\Gamma_2}{\delta S} - K\right)S\right] - \Gamma_2 = I(\psi) + \\ &- \mathrm{Tr}\,\ln\left(D^{-1} + \frac{\delta\Gamma_2}{\delta S}\right) - \mathrm{Tr}\,\ln\left(1 + \left(D^{-1} + \frac{\delta\Gamma_2}{\delta S}\right)^{-1}K\right) + \\ &+ \mathrm{Tr}\left(\frac{\delta\Gamma_2}{\delta S}S\right) + \mathrm{Tr}(KS) - \Gamma_2 .\end{aligned} \tag{4.4}$$

and using again (4.3):

$$\mathrm{Tr}\ln\left[1+\left(D^{-1}+\frac{\delta\Gamma_2}{\delta S}\right)^{-1}K\right]=\mathrm{Tr}\ln\left(1+(S^{-1}+K)^{-1}K\right)=$$
$$=\mathrm{Tr}\ln\left(1-SK\right)^{-1} \tag{4.5}$$

one gets our final expression

$$\Gamma_{CJT}=\Gamma+\mathrm{Tr}\ln\left(1-SK\right)+\mathrm{Tr}(KS) \tag{4.6}$$

where

$$\Gamma=I(\psi)-\mathrm{Tr}\ln\left(D^{-1}+\frac{\delta\Gamma_2}{\delta S}\right)+\mathrm{Tr}\left(\frac{\delta\Gamma_2}{\delta S}S\right)-\Gamma_2+\mathrm{const.} \tag{4.7}$$

is the form of our effective action.

It is clear from eqs.(4.6) and (4.7) that the following identities hold:

$$\Gamma_{CJT}\big|_{K=0}=\Gamma\big|_{K=0} \tag{4.8}$$

$$\delta\Gamma_{CJT}/\delta S\big|_{K=0}=\delta\Gamma/\delta S\big|_{K=0}=0 \tag{4.9}$$

Eq. (4.9) means that Γ_{CJT} and Γ give both rise to the same Schwinger-Dyson equation

$$S^{-1}=D^{-1}+\delta\Gamma_2/\delta S \tag{4.10}$$

which is the extremum condition for the effective action.

We see that, in order to determine the extrema of the effective action, the choise between Γ_{CJT} and Γ is a pure matter of convenience. In particular, it will be shown that practical calculations are greatly simplified by the use of our Γ instead of Γ_{CJT}, and, in the case of gauge theories we will deal with, it turns out that, for any of the ansatz we shall use, Γ, at least at the two-loops level, is bounded from below. Furthermore our Γ does not have problems in the free case, simply because by turning the interaction off, it reduces to a constant functional.

We would like to mention another actractive property of the modified action (4.7).

Let us consider a field theory with quadrilinear interactions like the O(N) scalar model,[9] or the non-renormalizable Nambu Jona-Lasinio model[2] or the two dimensional Gross-Neveu model.[10]

All this theories can be reformulated in terms of collective variables by introducing into the generating functional composite fields bilinear in the elementary ones.

Defining the effective action for both elementary and composite fields, one can show, at least at the one-loop level, that it coincides with Γ (eq.(4.7)), after convenient identification between the composite fields and the variable S(x,x). For example let us show how our Γ reproduces the effective action of the Gross-Neveu model obtained with the collective variables method.

Such a model is described by the action

$$I(\psi) = \int d^2x \left[\bar{\psi} i \hat{\partial} \psi + \frac{1}{2} g^2 (\bar{\psi}\psi)^2 \right] \tag{4.11}$$

where ψ is a Fermi field with N components.

In two dimensions one defines a Clifford algebra with two generators

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu} \qquad g_{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{4.12}$$

As usual $\gamma_5 = \gamma_0\gamma_1$ such that

$$\begin{aligned} &\gamma_5^2 = 1 \\ &\{\gamma_\mu, \gamma_5\} = 0 \\ &\gamma_5^+ = \gamma_5 \qquad \gamma_0^+ = \gamma^0 \qquad \gamma_1^+ = -\gamma_1 \end{aligned} \tag{4.13}$$

The action I is invariant under the discrete chiral transformation

$\psi \to \gamma_5 \psi$ because

$$\begin{aligned} \bar\psi\gamma_\mu\psi &\to \bar\psi\gamma_\mu\psi \\ \bar\psi\psi &\to -\bar\psi\psi \end{aligned} \tag{4.14}$$

This invariance, which forbids a mass term for the fermion in the action, is dynamically broken and the fermions acquires a dynamical mass.

It is possible to reformulate the theory in terms of a collective variable.The idea is to introduce inside the generating functional a scalar field σ, having the same transformation properties of $\bar\psi\psi$: in such a way we substitute the original four-Fermi interaction with a Yukawian coupling of the fermions with the auxiliary scalar field σ.

This generating functional is

$$Z[\eta] = \frac{1}{N}\int \mathcal{D}\psi\mathcal{D}\bar\psi\mathcal{D}\sigma\, e^{i[\bar\psi i\hat\partial\psi - \frac{1}{2}\sigma^2 + g\sigma\bar\psi\psi + \bar\eta\psi + \bar\psi\eta]} \tag{4.15}$$

This is now a quadratic form in the fermionic fields, so it is possible to perform the functional integration over ψ and $\bar\psi$.

The result is

$$Z[\eta] = \frac{1}{N}\int \mathcal{D}\sigma\, e^{i(-\frac{1}{2}\sigma^2 - i\,\mathrm{Tr}\ln(i\hat\partial + g\sigma) + J\sigma)} \tag{4.16}$$

where we have turn the fermionic sources off because we are not interested in amplitudes with esternal fermions, while we have introduced a source J for σ.

For a constant $\sigma = \sigma_c$, one can read directly from (4.16) the expression for the effective potential for the Gross-Neveu model to the one-loop level:

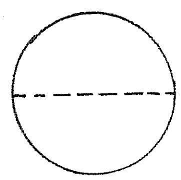
$$V(\sigma_c) = \frac{1}{2}\sigma_c^2 - N\int d^2p_E \, \ln\left(p_E^2 + g^2\sigma_c^2\right) \tag{4.17}$$

where we have calculated

$$i \,\mathrm{Tr}\ln\left(i\hat{\partial} + g\sigma_c\right) = i\int \frac{d^2p}{(2\pi)^2} \langle p| \ln\det\left(\hat{p} + g\sigma_c\right)|p\rangle =$$
$$= i\int d^2p \, \ln\left(-p^2 + g^2\sigma_c^2\right)\int d^2x \ ,$$

we have performed a Wick rotation and factorized out the space-time volume. (Remark: the integral in (4.17) is UV divergent and we have to regularize it with a cut-off procedure and then to introduce a counterterm to subtract the divergent contribution. We will not speak about these problems here).

Let us now perform the $N \to \infty$ limit with $g^2N = \lambda$ = const, N being the number of fermionic components. In this limit all the diagrams with two or more loops go to zero at least as 1/N. Let us consider for instance



———— ψ

------ σ

fig. 1

Remember that in the loop expansion for the effective potential, one has to calculate all the one-particle irreducible vacuum diagrams with respect to the action shifted by σ_c :(11)

$$I(\psi, \sigma_c, \sigma) = \bar{\psi}\left(i\hat{\partial} + g\sigma_c\right)\psi + g\sigma\bar{\psi}\psi - \frac{1}{2}\sigma^2 \tag{4.18}$$

The diagram in Fig.1 is the only one contributing to the two loops term in the effective potential. Let us calculate it in the $N \to \infty$ limit:

$$g^2 N \int \frac{d^2p}{(2\pi)^2} \frac{d^2k}{(2\pi)^2} \, tr\left(\frac{\hat{p} - g\sigma_c}{p^2+g^2\sigma_c^2} \cdot \frac{\hat{k} - g\sigma_c}{k^2+g^2\sigma_c^2} \right) =$$

$$= 2(g^2N)(g^2\sigma_c^2)\int \frac{d^2p \, d^2k}{(2\pi)^4} \frac{1}{p^2+g^2\sigma_c^2} \frac{1}{k^2+g^2\sigma_c^2} \underset{N\to\infty}{\sim} \frac{1}{N} .$$

It follows that $V(\sigma_c)$ as given in (4.17), that is at one loop level, is exact in this limit.

We want to show now that our effective action for composite fields Γ, reproduces (in the $N \to \infty$ limit), the effective action just calculated with the collective variables method.

We are only interested with $\langle\psi\rangle = \langle\bar{\psi}\rangle = 0$ so we put directly $\psi^c = \bar{\psi}^c = 0$ in the effective action. Then one has:

$$i\, \mathcal{D}^{-1}_{\alpha\beta} = \left.\frac{\delta^2 I}{\delta\psi_\beta \, \delta\bar{\psi}_\alpha}\right|_{\psi=0} = i\hat{\partial}_{\alpha\beta} \tag{4.18}$$

In the $N \to \infty$ limit, the main contribution to $\Gamma_2(0,S)$ is given by the diagram:

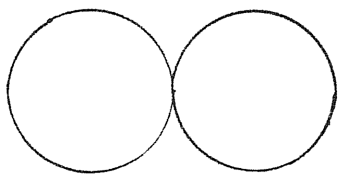


fig.2

that is

$$\Gamma_2 = \frac{1}{2} g^2 \int d^2x \left[(trS)^2 - tr\, S^2 \right] \tag{4.19}$$

To compare with the previous results, we assume S diagonal in the flavor and in the Lorentz space:

$$S^{ij}_{\alpha\beta}(x,x) = -\frac{1}{2Ng}\sigma_c(x)\,\delta_{\alpha\beta}\,\delta_{ij} \qquad i,j=1,..,n \tag{4.20}$$

Then, in the $N \to \infty$ limit, $(\mathrm{tr}\, S)^2$ dominates on $(\mathrm{tr}\, S^2)$ and we get:

$$\Gamma_2 = \frac{1}{2}\int d^2x\ \sigma_c^2(x) \tag{4.21}$$

We have also:

$$\frac{\delta \Gamma_2}{\delta S^{ii}_{\alpha\alpha}(x,x)} = g^2\, S^{jj}_{\beta\beta}(x,x) = -g\,\sigma_c(x) \tag{4.22}$$

and

$$\mathrm{Tr}\left[\frac{\delta\Gamma_2}{\delta S}\, S\right] = g^2\int d^2x\, S^{jj}_{\beta\beta}(x,x)\, S^{ii}_{\alpha\alpha}(x,x) = 2\,\Gamma_2 \tag{4.23}$$

By substituting in

$$\Gamma(0,S) = -i\,\mathrm{Tr}\ln\left(iD^{-1} - \frac{\delta\Gamma_2}{\delta S}\right) - \mathrm{Tr}\left(\frac{\delta\Gamma_2}{\delta S}S\right) + \Gamma_2 + \text{const.} \tag{4.24}$$

which is the form of our Γ in the Minkowski space, we have:

$$\Gamma(S) = -\frac{1}{2}\int d^2x\ \sigma_c^2(x) - i\ \mathrm{Tr}\ \ln\ (i\hat{\partial} + g\sigma_c) \tag{4.25}$$

from which one derives an effective potential coinciding with the expression (4.17).

5. CHIRAL SYMMETRY BREAKING IN QCD-LIKE GAUGE THEORIES

We present here an application of the method for calculating the effective potential within the functional formalism, to SU(N) color gauge theories with massless quarks. We will assume that the main contribution to the effective potential comes from the short-distance effects. With such an approximation, in virtue of the asymptotic freedom of the gauge theories, it is sensible to performe a loop expansion of the effective potential, and to consider only the lowest order contributions in the gauge coupling constant (two fermionic loops).

The calculations are for $\theta=0$ (θ is the parameter connected with axial anomaly). We will see that the phase diagram shows two phases: the chiral phase and the broken phase into the diagonal flavor subgroup. In particular, spontaneous chiral symmetry breaking occurs, when the coupling constant in the infrared regime exceedes some critical value (see later on).

The classical Lagrangian density of the SU(N) gauge field theory we are considering is:

$$\mathcal{L} = \bar{\psi}_{iA}\left(\gamma^\mu i\partial_\mu \delta^A_B - g\gamma^\mu A^a_\mu (T^a)^A_B\right)\psi^{iB} - \frac{1}{4} F^{\mu\nu a} F^a_{\mu\nu} + \text{+ ghost terms + gauge fixing.} \qquad (5.1)$$

where ψ_{iA} $i=1,..,n$ $A=1,..,N$ are n massless fermions each of them being assigned to the same representation r of the gauge group,

T^a $a=1,..,N^2-1$ are the hermitian generators of the gauge group in the r representation,

$$F_{\mu\nu}=F^a_{\mu\nu}t^a=\partial_\mu A_\nu-\partial_\nu A_\mu-g[A_\mu,A_\nu] \quad , \quad A_\mu=A^a_\mu t^a$$

and t^a are the generators of the gauge group in the adjoint representation.

This Lagrangian is invariant under the transformations of the flavor group $U(n)_L \times U(n)_R$. More precisely we have the invariance under the global chiral $SU(n)_L \times SU(n)_R$ group and the $U(1)_{L+R}$ group. (the divergence of the singlet axial-vector current $j_{5\mu}=\bar{\psi}_{iA}\gamma_\mu\gamma_5\psi^{iA}$ connected with the $U(1)_{L-R}$ group, is non zero even in the chiral limit due to the anomaly)

The chiral $SU(n)_L \times SU(n)_R$ symmetry implies the conservation of $2(n^2-1)$ currents (all of them are gauge group singlets):

$$\begin{aligned} j^r_{L\mu} &= \bar{\psi}_{iA}\gamma_\mu \frac{1-\gamma_5}{2}\frac{(\lambda_r)^i{}_j}{2}\psi^{jA} \\ j^r_{R\mu} &= \bar{\psi}_{iA}\gamma_\mu \frac{1+\gamma_5}{2}\frac{(\lambda_r)^i{}_j}{2}\psi^{jA} \end{aligned} \qquad r=1,..,n^2-1 \tag{5.2}$$

where $\lambda^r/2$ are the matrices of the fundamental representation of the SU(n) algebra. The $U(1)_{L+R}$ symmetry implies the conservation of the vector current $j_\mu=\bar{\psi}^{Ai}\gamma_\mu\psi_{Ai}$. The invariance of the Lagrangian density with respect to the transformations of the chiral group garantees that the mass term in the femion propagator will never appear in any order of thr perturbation theory. In fact, the structure of the propagator calculated in perturbation theory is a consequence of the invariance of the vacuum with respect to the following transformations:

$$U_r|0\rangle = 0 \qquad U_{r_5}|0\rangle = 0 \tag{5.3}$$

with

$$\begin{aligned}
U_r^{-1}\,\psi(x)\,U_r &= e^{i\alpha\frac{\lambda^r}{2}}\,\psi(x)\\
U_r^{-1}\,\bar\psi(x)\,U_r &= \bar\psi(x)\,e^{-i\alpha\frac{\lambda^r}{2}}\\
U_{r_5}^{-1}\,\psi(x)\,U_{r_5} &= e^{i\varphi\frac{\lambda^r}{2}\gamma_5}\,\psi(x)\\
U_{r_5}^{-1}\,\bar\psi(x)\,U_{r_5} &= \bar\psi(x)\,e^{i\varphi\frac{\lambda^r}{2}\gamma_5}
\end{aligned} \tag{5.4}$$

then

$$\begin{aligned}
S(x) = \langle 0|T\,\psi(x)\bar\psi(0)|0\rangle &=\\
&= \langle 0|T\,U_r^{-1}\,\psi(x)\,U_r U_r^{-1}\,\bar\psi(0)\,U_r|0\rangle =\\
&= e^{i\alpha\frac{\lambda^r}{2}}\,S(x)\,e^{-i\alpha\frac{\lambda^r}{2}}
\end{aligned} \tag{5.5}$$

and

$$S(x) = \exp\left(i\varphi\gamma_5\frac{\lambda^r}{2}\right) S(x)\,\exp\left(i\varphi\gamma_5\frac{\lambda^r}{2}\right)$$

Therefore $\quad [\lambda_r, S] = 0 \quad ; \quad \{\gamma_5\lambda^r, S\} = 0 \qquad (5.6)$

so S, perturbatively calculated, is diagonal in the global group indices and has the form:

$$S(p) = \frac{i}{Z(p^2)\,\hat p} \tag{5.6}$$

Under spontaneous symmetry breaking, $SU(n)_L \times SU(n)_R \rightarrow SU(n)_{L+R}$.

When relation (5.3) is violated, a mass term in the fermion propagator can appear:

$$S(p) = \frac{i}{Z(p^2)\,\hat p + \Sigma(p^2)} \tag{5.7}$$

Thus, the dynamical realization of spontaneous chiral-symmetry breaking, requires necessarily to go beyond the framework of perturbation theory.

6. ULTRAVIOLET ASYMPTOTICS OF THE FERMION SELF-ENERGY.

As preliminarly considerations, we will obtain some restrictions on the mechanism of spontaneous chiral-symmetry breaking directly from the equations of the theory.[12] In particular we will consider some restrictions which follow from the asymptotical equations for Green's Functions.

Let us use for this purpose the Ward identities relating the unrenormalized proper axial-vector vertex functions $\Gamma^{(0)r}_{5\nu}$ with the fermion bare propagator $S^{(0)}$: (Λ is an ultraviolet cutoff)

$$P^{\nu}\,\Gamma^{(0)r}_{5\nu}(q_2,q_1) = i\gamma_5 \frac{\lambda^r}{2} S^{(0)-1}(q_1) + i\, S^{(0)-1}(q_2)\frac{\lambda^r}{2}\gamma_5 \tag{6.1}$$

(we are dealing with a chiral invariant Lagrangian so $\partial_\mu j^{r}_{5\mu} = 0$)

where $P = q_2 - q_1$

$S^{(0)-1}$ is the inverse bare propagator

$\Gamma^{(0)r}_{5\nu}$ are the bare amputated vertices of the colorless axial-vector currents $j^r_{5\nu} = \bar{\psi}\gamma_\nu\gamma_5 \frac{\lambda_r}{2}\psi$:

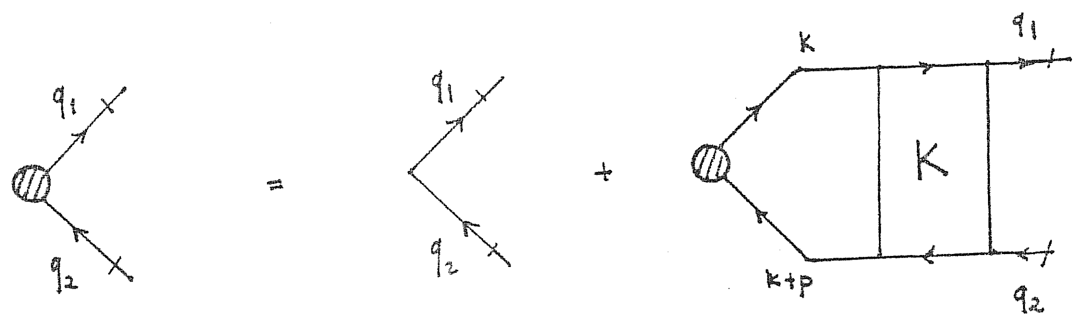
$$S^{(0)}(q_2)\,\Gamma^{(0)r}_{5\nu}(q_2,q_1)\,S^{(0)}(q_1) = \int d^4x_1\, d^4x_2\, e^{iq_2x_2 - iq_1x_1} \cdot$$

$$\cdot \langle 0|\, T\,\psi(x_2)\, j^r_{5\nu}(0)\,\bar{\psi}(x_1)\,|0\rangle$$

which satisfies to the equations of the Bethe-Salpeter type:

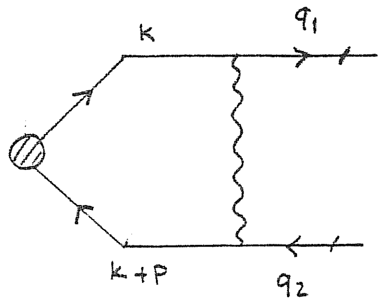
$$\left[\Gamma^{(0)r}_{5\nu}(q_2,q_1)\right]_{\alpha\beta} = \frac{\lambda^r}{2}(\gamma_\nu\gamma_5)_{\alpha\beta} + \int^{\Lambda}\frac{d^4k}{(2\pi)^4}\, K^{(0)}_{\alpha\beta;\alpha'\beta'}(q_2,q_1;k)\cdot \tag{6.2}$$

$$\cdot\left[S^{(0)}(k+p)\,\Gamma^{(0)r}_{5\nu}(k+p,k)\,S^{(0)}(k)\right]_{\alpha'\beta'}$$



where K is the fermion-antifermion scattering kernel.

In the lowest order approximation K consist of the single gluon-exchange graph in which lowest order expressions are used for both the gluon propagator and the gluon-fermion vertex function:



By substituting (6.2) into (6.1) we get:

$$\left(i\gamma_5 \frac{\lambda^r}{2} S^{(0)-1}(q_1) + i\, S^{(0)-1}(q_2) \frac{\lambda^r}{2}\gamma_5 \right)_{\alpha\beta} = \int^{\Lambda} \frac{d^4k}{(2\pi)^4}\, K^{(0)}_{\alpha\beta';\alpha'\beta'}(q_2,q_1;k)\, \cdot$$
$$\cdot \left(S^{(0)}(k+p) \left(i\gamma_5 \frac{\lambda^r}{2} S^{(0)-1}(k) + i\, S^{(0)-1}(k+p)\frac{\lambda^r}{2}\gamma_5 \right) S^{(0)}(k) \right)_{\alpha'\beta'} + \frac{\lambda^r}{2}(\hat{p}\gamma_5)_{\alpha\beta} \tag{6.3}$$

that is

$$\left(i\gamma_5 \frac{\lambda^r}{2} S^{(0)-1}(q_1) + i\, S^{(0)-1}(q_2) \frac{\lambda^r}{2}\gamma_5 \right)_{\alpha\beta} = \frac{\lambda^r}{2}(\hat{p}\gamma_5)_{\alpha\beta} + \tag{6.4}$$
$$+ \int^{\Lambda} \frac{d^4k}{(2\pi)^4}\, K^{(0)}_{\alpha\beta;\alpha'\beta'}(q_2,q_1;k) \left[S^{(0)}(k+p)\, i\gamma_5 \frac{\lambda^r}{2} + i\, \frac{\lambda^r}{2}\gamma_5 S^{(0)}(k) \right]_{\alpha'\beta'}$$

Now, by taking the limit $P \to 0$ and defining $q = \frac{q_1+q_2}{2}$ we get:

$$\left[\gamma_5 S^{(0)-1}(q) + S^{(0)-1}(q)\gamma_5\right]_{\alpha\beta} = \int^{\Lambda} \frac{d^4k}{(2\pi)^4} K^{(0)}_{\alpha\beta,\alpha'\beta'}(q,k)\left[S^{(0)}(k)\gamma_5 + \gamma_5 S^{(0)}(k)\right]_{\alpha'\beta'} \tag{6.5}$$

Recall that we have parametrized

$$\left[S^{(0)}(q)\right]^{-1} = -i\left(Z^{(0)}(q^2)\hat{q} - \Sigma^{(0)}(q^2)\right) \tag{6.6}$$

that is

$$S^{0}(q) = \frac{i Z^{(0)}(q^2)\hat{q} + i\Sigma^{(0)}(q^2)}{Z^{(0)2}(q^2)q^2 - \Sigma^{(0)2}(q^2)} \tag{6.7}$$

By using them into (6.4) we get:

$$(\gamma_5)_{\alpha\beta}\Sigma^{(0)}(q^2) = \int^{\Lambda}\frac{d^4k}{(2\pi)^4} K^{(0)}_{\alpha\beta;\alpha'\beta'}(q,k)(\gamma_5)_{\alpha'\beta'}\frac{\Sigma^{(0)}(k^2)}{\left(Z^{(0)2}(k^2)k^2 - \Sigma^{(0)2}(k^2)\right)} \tag{6.8}$$

or equivalently

$$(\gamma_5)_{\alpha\beta}\Sigma^{(0)}(q^2) = \int^{\Lambda}\frac{d^4k}{(2\pi)^4} K^{(0)}_{\alpha\beta;\alpha'\beta'}(q,k)\left(S^{(0)}(k)\gamma_5\Sigma^{(0)}(k^2)S^{(0)}(k)\right)_{\alpha'\beta'} \tag{6.9}$$

In the lowest order of perturbation theory

$$K^{(0)}_{\alpha\beta,\alpha'\beta'}(q,k) = -i\left(-ig^{(0)}(\Lambda)\right)^2 C_2 (\gamma^{\mu})_{\beta'\beta}(\gamma^{\nu})_{\alpha\alpha'} D_{\mu\nu}(q-k) \tag{6.10}$$

where $C_2 = \frac{N^2-1}{2N} = \sum_a T^a T^a$

$$D_{\mu\nu}(q) = \frac{1}{q^2}\left(g_{\mu\nu} - (1-\alpha)\frac{q_\mu q_\nu}{q^2}\right)$$

Our calculations will be performed in the Landau gauge, that is with $\alpha = 0$.

Going to renormalized quantities:

$$S(p) = Z_\psi^{-1} S^{(0)}(p) = \frac{i}{Z(p^2)\hat{p} - \Sigma(p^2)} \tag{6.11}$$

$$Z(p^2) = Z_\psi \, Z^{(0)}(p^2) \qquad ; \qquad \Sigma(p^2) = Z_\psi \, \Sigma^{(0)}(p^2) \tag{6.12}$$

and
$$K_{\alpha\beta,\alpha'\beta'}(q,k) = Z_\psi^2 \; K^{(0)}_{\alpha\beta,\alpha'\beta'}(q,k) \tag{6.13}$$

being K the proper four-fermion scattering amplitude,one obtains the equation:

$$(\gamma_5)_{\alpha\beta} \, \Sigma(q^2) = \int^\Lambda \frac{d^4k}{(2\pi)^4} \, K_{\alpha\beta,\alpha'\beta'}(q,k) \, \left(S(k)\gamma_5 \Sigma(k^2) S(k)\right)_{\alpha'\beta'} \tag{6.14}$$

where everything is expressed in terms of renormalized quantities and the cutoff dependence relies only in the upper limit of integration.

Let us go over to the deep euclidean region of momenta in eq.(6.14). Since the ultaviolet asymptotics of $Z(q^2)$ and $K_{\alpha\beta,\alpha'\beta'}(q,k)$ are insensitive to the mass term, they should not be changed when spontaneuos chiral symmetry breaking is taken into accont. Therefore,in the leading logarithmic approximation, one can take for them the expressions following from the renormalization group analysis.(13) The kernel K is a proper four-fermion scattering amplitude so it rescales in the following way:(17)

$$K(p,g,\mu) = \lambda^{-2} \, K(\bar{p}, \bar{g}(g,t)\mu) \, e^{-4\int_g^{\bar{g}(g,t)} dx \frac{\gamma_\psi(x)}{\beta(x)}} \tag{6.15}$$

where $\bar{p}$ is a reference scale of momentum

μ is the renormalization group parameter

$\lambda = \frac{p}{\bar{p}} \quad , \quad t = \ln\lambda$

$\bar{g}(g,t)$ is the running coupling constant which in the leading logarithmic approximation is given by:

$$\bar{g}^2\left(g, \ln\frac{p}{\bar{p}}\right) = \frac{g^2}{1 + \frac{g^2}{2b}\ln\frac{p^2}{\bar{p}^2}} \tag{6.16}$$

where $b = \dfrac{24\pi^2}{11N-2n}$, or equivalently by

$$g^2(p^2) = 2b \Big/ \ln\frac{p^2}{\Lambda^2_{QCD}} \tag{6.17}$$

where Λ_{QCD} is the renormalization group invariant mass: the free parameter of the theory (In QCD, $\Lambda_{QCD} \sim 250$ Mev),

and
$$\gamma_\psi(g) = \frac{1}{2}\,\frac{d\ln Z_\psi}{d\ln\mu}$$
$$\beta(g) = \frac{dg}{d\ln\mu} \tag{6.18}$$

In words, according to the renormalization group analysis, the behaviour of the kernel as a function of the asymptotic momentum p and the renormalized fermion-gluon coupling constant g, is governed by its behaviour as a function of a finite momentum $\bar{p}$ and of the running coupling constant $\bar{g}(g,t)$. In the $p \to \infty$ limit, thanks to the asymptotic freedom of the quark-gluon interaction, $g(p^2) \to 0$, so it is meaningfull to approximate the kernel K with the lowest perturbative order. Recalling that in the Landau gauge $\gamma_\psi = 0$ at this order, we get:

$$\lim_{p\to\infty} K(p, g, \mu) = \frac{\bar{p}^2}{p^2}\, iC_2\, g^2(p^2)(\gamma^\mu)_{\beta'\beta}(\gamma^\nu)_{\alpha\alpha'}\,\frac{1}{\bar{p}^2}\left(g_{\mu\nu} - \frac{\bar{p}_\mu \bar{p}_\nu}{\bar{p}^2}\right) = \tag{6.19}$$
$$= iC_2\, g^2(p^2)(\gamma^\mu)_{\beta'\beta}(\gamma^\nu)_{\alpha\alpha'}\,\frac{1}{p^2}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$$

Analogously for $Z(p^2)$, which represents the fermionic wave-function renormalization parameter, in the $p \to \infty$ limit, the renormalization group analysis gives (in the Landau gauge):

$$\lim_{p\to\infty} Z(p,g,\mu) = 1 \tag{6.20}$$

In this way $\Sigma(q^2)$ represents the dynamical mass function of the theory $m(q^2) = \frac{\Sigma(q^2)}{Z(q^2)} \simeq \Sigma(q^2)$. We observe that the expression (6.19) for the kernel K is not merely the ladder approximation, but, with the inser tion of the running coupling constant, it takes automatically into account the fermion-gluon vertex perturbative corrections at least in the leading logarithmic approximation; this "improved" ladder approximation faithfully represents the complete (relevant) kernel, and not merely the asymptotic limit of the ladder graph. Further,we assume the validity of the usual arguments reading that the region $k^2, (q-k)^2 \gg \Lambda^2_{QCD}$ gives the main contribution to the integral on the right-hand side of (6.14) in the limit $q^2 \to \infty$. In this region

$$S(k) \simeq \frac{i}{Z(k^2)\hat{k}} \simeq \frac{i}{\hat{k}} \tag{6.21}$$

substituting in eq. (6.14):

$$(\gamma_5)_{\alpha\beta}\,\Sigma(q^2) = iC_2 \int^{\Lambda} \frac{d^4k}{(2\pi)^4}\, g^2((q-k)^2)(\gamma^\mu)_{\beta'\beta}(\gamma^\nu)_{\alpha\alpha'} \left[g_{\mu\nu} - \frac{(q-k)_\mu (q-k)_\nu}{(q-k)^2} \right] \cdot \tag{6.22}$$

$$\cdot \left[\frac{i}{\hat{k}}\,\gamma_5\,\frac{i}{\hat{k}}\right] \frac{\Sigma(k^2)}{(q-k)^2} .$$

from which:

$$\Sigma(q^2) = -\frac{3iC_2}{(2\pi)^4} \int^{\Lambda} d^4k \,\frac{g^2((q-k)^2)}{k^2 (q-k)^2}\, \Sigma(k^2) \tag{6.23}$$

So, performing a Wick rotation, we find that eq. (6.14) in the Euclidean region at $q^2 \to \infty$ takes the form:

$$\Sigma(q^2) = \frac{3C_2}{16\pi^4} \int^{\Lambda} d^4k \,\frac{g^2((q-k)^2)}{k^2 (q-k)^2}\, \Sigma(k^2) \tag{6.24}$$

In the leading logarithmic approximation, in the region $k^2, (q-k)^2 \gg \Lambda^2_{QCD}$ the function $g^2((q-k)^2)$ can be substituted by the function $g^2(q^2)$ for $q^2 > k^2$ and by $g^2(k^2)$ for $q^2 < k^2$. In fact in the region $q^2 > k^2$, $k^2 \gg \Lambda^2_{QCD}$, $(q-k)^2 \gg \Lambda^2_{QCD}$ one has:

$$g^2((q-k)^2) = 2b \Big/ \ln \frac{q^2}{\Lambda^2_{QCD}} \left(1-\frac{k}{q}\right)^2 \sim 2b \Big/ \ln \frac{q^2}{\Lambda^2_{QCD}} = g^2(q^2)$$

and analogously for $k^2 > q^2$, $k^2 \gg \Lambda^2_{QCD}$, $(q-k)^2 \gg \Lambda^2_{QCD}$

$$g^2((q-k)^2) \simeq 2b \Big/ \ln k^2/\Lambda^2_{QCD} = g^2(k^2)$$

summing up

$$g^2((q-k)^2) = \theta(q^2-k^2)\, g^2(q^2) + \theta(k^2-q^2)\, g^2(k^2) \tag{6.25}$$

We substitute in (6.24):

$$\Sigma(q^2) = \frac{3C_2}{16\pi^4}\left[\int_{q^2}^{\Lambda^2} d^4k\, g^2(k^2)\, \frac{\Sigma(k^2)}{k^2}\frac{1}{(q-k)^2} + \right. \\ \left. + \int_0^{q^2} d^4k\, g^2(q^2)\, \frac{\Sigma(k^2)}{k^2}\frac{1}{(q-k)^2}\right]. \tag{6.26}$$

and then we integrate over the angles:

$$\int d^4k = \frac{1}{2}\int dk^2\, k^2\, d\Omega \\ \int d\Omega\, \frac{1}{(q-k)^2} = \frac{2\pi^2}{qk}\left(\theta(k-q)\frac{q}{k} + \theta(q-k)\frac{k}{q}\right) \tag{6.27}$$

The final result is:

$$\Sigma(q^2) = \frac{3C_2}{16\pi^2}\left[\int_{q^2}^{\Lambda^2} dk^2\, g^2(k^2)\, \frac{\Sigma(k^2)}{k^2} + \right. \\ \left. + \frac{g^2(q^2)}{q^2}\int_0^{q^2} dk^2\, \Sigma(k^2)\right] \tag{6.28}$$

For the determination of the ultraviolet asymptotics of the dynamical mass function $\Sigma(q^2)$, one has to solve (6.28) with the upper limit of integration equal to Λ^2 and only after to perform the $\Lambda \to \infty$ limit. Only in this way we will have a criterion to choose between the distinct asymptotic behaviours of Σ.[14]

Eq. (6.28) can be transformed into a differential equation: let us differentiate with respect to q^2 in it:

$$\frac{d\,\Sigma(q^2)}{dq^2} = \frac{3C_2}{16\pi^2}\frac{d}{dq^2}\left(\frac{g^2(q^2)}{q^2}\right)\int_0^{q^2} dk^2\,\Sigma(k^2) \tag{6.29}$$

So

$$\frac{1}{\frac{d}{dq^2}\left(\frac{g^2(q^2)}{q^2}\right)}\;\frac{d\,\Sigma(q^2)}{dq^2} = \frac{3C_2}{16\pi^2}\int_0^{q^2} dk^2\;\Sigma(k^2) \tag{6.30}$$

By differentiating once again we find out that the solutions of (6.28) satisfy the second order differential equation:

$$\frac{16\pi^2}{3C_2}\,\frac{d}{dq^2}\left[\frac{1}{\frac{d}{dq^2}\left(\frac{g^2(q^2)}{q^2}\right)}\;\frac{d}{dq^2}\;\Sigma(q^2)\right] = \Sigma(q^2) \tag{6.31}$$

From (6.29) we get

$$q^2\,\frac{d\,\Sigma(q^2)}{dq^2}\bigg|_{q^2=\Lambda^2} = \frac{3C_2}{16\pi^2}\left[\frac{d}{dq^2}g^2(q^2) - \frac{g^2(q^2)}{q^2}\right]_{q^2=\Lambda^2}\int_0^{\Lambda^2} dk^2\,\Sigma(k^2) \tag{6.32}$$

and from (6.28)

$$\Sigma(q^2)\bigg|_{q^2=\Lambda^2} = \frac{3C_2}{16\pi^2}\left(\frac{g^2(q^2)}{q^2}\right)_{q^2=\Lambda^2}\int_0^{\Lambda^2} dk^2\,\Sigma(k^2) \tag{6.33}$$

so

$$\left[-\frac{d\ln g^2(q^2)}{d\ln q^2} + 1\right]\cdot\,\Sigma(q^2)\,\bigg|_{q^2=\Lambda^2} =$$

$$= \frac{3c_2}{16\pi^2}\left(\frac{g^2(q^2)}{q^2} - \frac{d\, g^2(q^2)}{dq^2}\right)_{q^2=\Lambda^2}\int_0^{\Lambda^2} dk^2\, B(k^2) \tag{6.34}$$

Hence the solutions of (6.28) must satisfy the boundary condition

$$\left[q^2 \frac{d\Sigma(q^2)}{dq^2} + \left(1 - \frac{d\ln g^2(q^2)}{d\ln q^2}\right)\Sigma(q^2)\right]_{q^2=\Lambda^2} = 0 \tag{6.35}$$

The general solution of (6.28) takes the form:

$$\Sigma(q^2) = c_1\Sigma_1(q^2) + c_2\Sigma_2(q^2) \tag{6.36}$$

and the functions $\Sigma_i(q^2)$ $i=1,2$ have the ultraviolet asymptotics of the form:

$$\Sigma_1(q^2) \sim \left(\ln\frac{q^2}{\Lambda^2_{QCD}}\right)^{-\frac{3C_2 b}{8\pi^2}} \tag{6.37}$$

$$\Sigma_2(q^2) \sim \frac{1}{q^2}\left(\ln\frac{q^2}{\Lambda^2_{QCD}}\right)^{\frac{3C_2 b}{8\pi^2}-1} \tag{6.38}$$

As it can be verified by substituting $\Sigma_1(q^2)$ and $\Sigma_2(q^2)$ respectively in (6.31) and by taking the asymptotic limit $q^2\to\infty$. So, calling $d=3C_2 b/8\pi^2$ and $\eta=\ln\frac{q^2}{\Lambda^2_{QCD}}$ we get:

$$\Sigma(q^2) \underset{q^2\to\infty}{\sim} c_1\eta^{-d} + \frac{1}{q^2}c_2\eta^{d-1} \tag{6.39}$$

By substituting into the boundary condition we obtain:

$$c_1\left[-d\eta^{d-1} + \eta^{-d} + \eta^{-d-1}\right]_{q^2=\Lambda^2} + c_2\left[-\frac{1}{\Lambda^2}\eta^{d-1} + \frac{1}{\Lambda^2}(d-1)\eta^{d-2} + \frac{1}{\Lambda^2}\left(1+\frac{1}{\eta}\right)\eta^{d-1}\right]_{q^2=\Lambda^2} = 0 \tag{6.40}$$

and retaining only the leading contribution for large values of Λ^2 we get: (remember that, for example, d=4/9 in QCD with three flavors)

$$c_1 = c_2\frac{1}{\Lambda^2}\frac{3C_2 b}{8\pi^2}\left(\ln\frac{\Lambda^2}{\Lambda^2_{QCD}}\right)^{\frac{3C_2 b}{4\pi^2}-2} \tag{6.41}$$

This is the relation between the coefficients c_1 and c_2 which must hold in order to satisfy the boundary conditon (6.35) for large but finite values of Λ^2.

Now we can remove the cut-off Λ finding that the constant c_1 goes to zero in the $\Lambda \to \infty$ limit. Therefore the ultraviolet asymptotics of the dynamical mass function is given only by Σ_2, that is:

$$\Sigma(q^2) \underset{q^2\to\infty}{\sim} \frac{1}{q^2}\left(\ln \frac{q^2}{\Lambda^2_{QCD}}\right)^{\frac{3C_2 b}{8\pi^2}-1} \tag{6.42}$$

This can be justified in an intuitive manner: the asymptotic behaviour of $\Sigma_1(q^2)$ in (6.37), exactly corresponds to what we find from a straightforward renormalization group analysis when we start with an explicit chiral symmetry breaking i.e. with a bare fermion mass different from zero. In fact the calculations of the Σ_1 solution from the improved ladder approximation, exactly correspond to the computation of the anomalous dimension γ_m. So $\Sigma_1(q^2)$ does not differ from the case in which quarks have non-zero bare mass and $\Sigma(q^2)$ does not have a purely dynamical origin. Hence, we expect that the solution which actually represents chiral symmetry realized in the Goldstone mode has the softer asymptotic behaviour of Σ_2.

7. OPERATOR PRODUCT EXPANSION ANALYSIS

We want to show that our results on the asymptotic behaviour of the dynamical mass function are consistent with the operator product expansion (OPE) analysis.[15]

We will apply the OPE to the quark propagator of our chiral symmetric quark-gluon model. The utility of this program rests on a theorem[16] which states that the Wilson's coefficient function in the OPE reflects the symmetries of the Lagrangian. In particular, if the Lagrangian is chiral invariant, we can compute the coefficient functions for large space-like momenta (so the running coupling constant is small) in perturbation theory, since perturbation theory does give the correct chiral transformation properties. All spontaneous symmetry breaking resides in the vacuum expectations of the local operators.

Remember that we have parametrized:

$$\mathrm{tr}\int d^4x\, e^{iqx}\langle 0|T\psi(x)\bar\psi(0)|0\rangle = \mathrm{tr}\left(\frac{i}{\hat q-\Sigma(q^2)}\right) = \mathrm{tr}\left(\frac{i\Sigma(q^2)}{q^2-\Sigma^2(q^2)}\right). \tag{7.1}$$

where tr is a trace on color and spinor indices, $\Sigma(q^2)$ is the quark self-energy and $Z(q^2)=1$ in the Landau gauge. For large Euclidean values of q^2 we can consider the operator product expansion of the left-hand side of (7.1):

$$\mathrm{tr}\int d^4x\, e^{iqx}\langle 0|T\psi(x)\bar\psi(0)|0\rangle = \sum_n c_n(q)\,\langle 0|\mathcal{O}^n|0\rangle = \tag{7.2}$$

$$= c_1(q)\,\langle 0|\mathbb{1}|0\rangle + c_2(q)\,\langle 0|\bar\psi\psi|0\rangle + \cdots$$

where $\bar{\psi}\psi = \bar{\psi}_\alpha^A(0)\ \psi_A^\alpha(0)$ i.e. is a Lorentz scalar and color singlet (for simplicity we are considering only one flavor). The ..... in (7.2) stand for terms with operators of higher dimension.

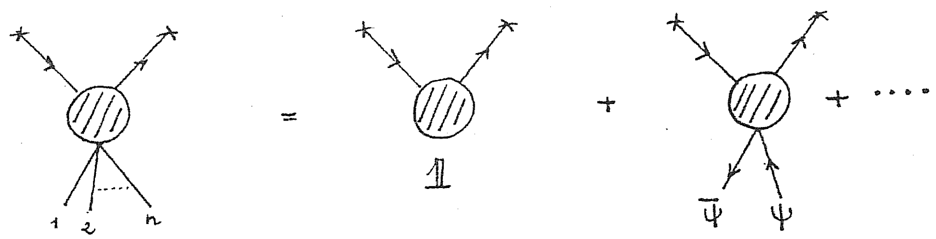


fig.3

Let the operators O_n, ψ and $\bar{\psi}$ be renormalized at μ. Standard renormalization group analysis of the Wilson coefficient functions c_n gives:(17)

$$c_n(q,g,\mu) = e^{t(2d_\psi - d_{O_n} - 4)}\ e^{\int_0^t dt'\,\gamma_n(t')}\ c_n(\bar{q},\bar{g}(g,t),\mu) \qquad (7.3)$$

or equivalently

$$c_n(q,g,\mu) = e^{\int_g^{\bar{g}(g,t)} dx\,\frac{\gamma_n(x)}{\beta(x)}}\ c_n(q,\bar{g}(g,t)\mu) \qquad (7.4)$$

where $\bar{q}$ is a reference scale of momentum (we will choose it equal to μ),

$t = \ln q/\bar{q}$

d_ψ, d_{O_n} are the dimensions of the fermionic operator and of O_n in momentum space,

$\bar{g}(g,t)$ is the running coupling constant $= g(q^2)$

$\gamma_n = 2\gamma_\psi + \gamma^{op}_{O_n}$ with γ_ψ and $\gamma^{op}_{O_n}$ the anomalous dimensions of the fermi field and the O_n:

$$\gamma_\psi = \frac{1}{2}\frac{\partial \ln Z_\psi}{\partial \ln \mu} \quad ; \quad \gamma^{op}_{O_m} = \mu \frac{\partial \ln Z^{op}_{O_m}}{\partial \mu}$$

and $$\beta(g) = \mu \frac{\partial g}{\partial \mu}$$

For q^2 large enough that $g(q^2)$ is small, the $c_n(q)$ are calculable in perturbation theory and behave as canonical powers of q times calculable logarithms. So the operators of higher dimensions in eq.(7.2) have coefficients that are down by negative powers of q. In fact, on a pure dymensional ground: $[C_1(q)] = q^{-1}$; $[C_2(q)] = q^{-4}$ and every other operator in the expansion is multiplied by a coefficient which goes to zero when $q^2 \to \infty$,faster than c_1 and c_2.

Let us calculate $c_1(q)$. The diagrams contributing are:

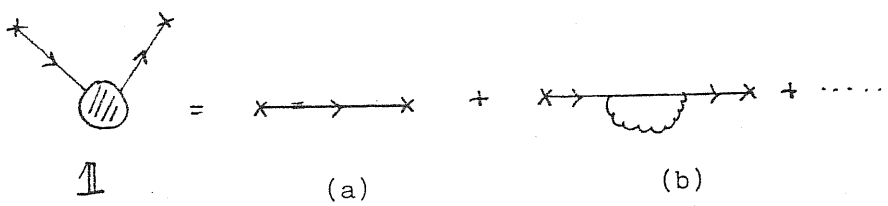


fig.4

To the zero order in g (graph (a)),we have:

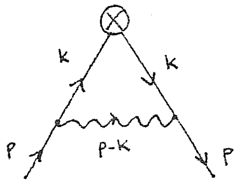
$$\int d^4x \, e^{iqx} \langle 0| T\psi^A_\alpha(x)\overline{\psi}^B_\beta(0)|0\rangle = \int d^4x \, e^{iqx} \delta_{AB} \int \frac{d^4k}{(2\pi)^4} \cdot \tag{7.5}$$

$$\cdot e^{-ikx}\left(\frac{i}{\hat{k}}\right)_{\alpha\beta} = \int d^4k \, \delta^4(k-q)\,\delta_{AB}\left(\frac{i}{\hat{k}}\right)_{\alpha\beta} = \delta_{AB}\left(\frac{i}{\hat{q}}\right)_{\alpha\beta}.$$

To the second order in g (graph (b)),we obtain zero in the Landau gauge. In this way,when we take the trace over the spinor indices, we obtain a vanishing contribution.

For the calculation of $c_2(q)$ we need $\gamma_{O_2} = \gamma_{\overline{\psi}\psi}$ to $O(g^2)$.

Let us compiute the graph:



which gives the lowest order contribution to $\gamma_{\bar\psi\psi}$.

$$(-ig)^2 C_2 \int \frac{d^4k}{(2\pi)^4} \gamma_\mu \frac{i}{\not k} \frac{i}{\not k} \gamma_\nu (-i) \left(g_{\mu\nu} - \frac{(p-k)_\mu (p-k)_\nu}{(p-k)^2} \right) \frac{1}{(p-k)^2} =$$

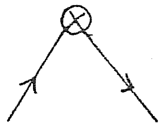
$$= -3ig^2 C_2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 (k-p)^2} = \frac{3g^2 C_2}{8\pi^2} \int dk \frac{k}{(p-k)^2} \qquad (7.6)$$

This integral is UV divergent.Regularizing it with a cutoff Λ we find the divergent part equal to:

$$\frac{3g^2 C_2}{16\pi^2} \ln \frac{\Lambda^2}{\mu^2} \qquad (7.7)$$

where μ is the renormalization parameter.

This divergent contribution will be compensated by the counterterm

$\delta Z_{\bar\psi\psi}$ 

So,to the g^2 order, we have:

$$Z_{\bar\psi\psi} = 1 - \frac{3g^2 C_2}{16\pi^2} \ln \frac{\Lambda^2}{\mu^2} \qquad (7.8)$$

Remember that:

$$Z^{op}_{\bar\psi\psi} = Z_{\bar\psi\psi} Z_\psi^{-1}$$

but,since in the Landau gauge the fermion wave function doesn't renormalize to the one loop order, we simply have:

$$Z^{op}_{\bar\psi\psi} = 1 - \frac{3g^2 C_2}{16\pi^2} \ln \frac{\Lambda^2}{\mu^2} \qquad (7.9)$$

$$\gamma^{op}_{\bar\psi\psi} = \frac{1}{\mu} \frac{\partial \ln Z^{op}_{\bar\psi\psi}}{\partial \mu} = \frac{3g^2 C_2}{8\pi^2} \qquad (7.10)$$

The diagrams contributing to c_2 are:

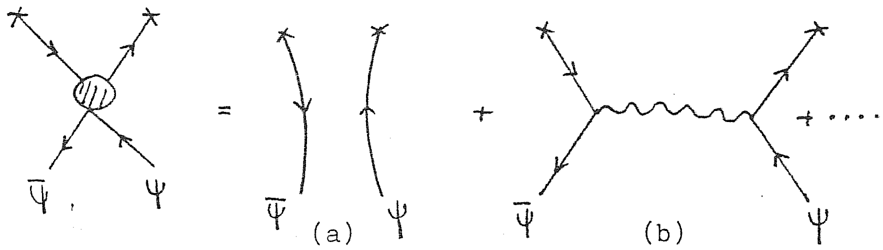


fig.5

To the zero order in g (graph (a)),we have in the short-distance limit:

$$\mathrm{tr}\int d^4x\, e^{iqx} \langle 0|T\psi(x)\,\overline{\psi}(0)|0\rangle \underset{x\to 0}{\sim} \int d^4x\, e^{iqx} \langle 0|\psi(0)\overline{\psi}(0)|0\rangle =$$

$$= -(2\pi)^4\,\delta^4(q) \quad \langle 0|\psi\overline{\psi}|0\rangle \tag{7.11}$$

To the second order in g (graph (b)),we have:

$$\frac{1}{2}\int d^4x\, e^{iqx} \langle 0|T\psi_\alpha^A(x)\,\overline{\psi}_\alpha^A(0)\left[-ig\,T^a\int d^4y\,\overline{\psi}(y)\gamma^\mu A_\mu^a(y)\psi(y)\right]\cdot$$

$$\cdot\left[-ig\,T^b\int d^4z\,\overline{\psi}(z)\gamma^\nu A_\nu^b(z)\psi(z)\right]|0\rangle .$$

By using the Wick theorem and by introducing the Fourier transformations for the propagators one gets:

$$\mathrm{tr}\int d^4x\, e^{iqx} \langle 0|T\psi(x)\overline{\psi}(0)|0\rangle\Big|_{O(g^2)} = ig^2\, C_2\,\delta_{A'B'}\cdot\int\frac{d^4k}{(2\pi)^4}\,d^4z\,e^{ikz}\cdot$$

$$\cdot\left(\frac{1}{\hat{q}}\gamma^\mu\right)_{\alpha\alpha'}\left[\frac{g_{\mu\nu}}{(k+q)^2}-\frac{(k+q)_\mu(k+q)_\nu}{(k+q)^4}\right]\left(\gamma^\nu\frac{1}{\hat{q}}\right)_{\beta'\beta}\cdot$$

$$\cdot\langle 0|T\,\overline{\psi}_{\beta'}^{B'}(z)\,\psi_{\alpha'}^{A'}(0)|0\rangle \xrightarrow[q\to\infty]{} 3ig^2C_2\,\frac{1}{q^4}\,\langle 0|\overline{\psi}\psi|0\rangle$$

Hence,to $O(g^2)$

$$c_2(q,\,g^2(q^2)) \sim -(2\pi)^4\delta^4(q) + 3iC_2\,\frac{g^2(q^2)}{q^4} \tag{7.12}$$

We are not interested in the δ -function term since it does not contribute to the connected quark propagator, so we will not consider it. By substituting in (7.4) :

$$c_2(q) = 3iC_2 \, \frac{g^2(q^2)}{q^4} \, e^{\int_g^{\bar{g}(q^2)} dx \, \frac{\gamma_m(x)}{\beta(x)}} \tag{7.13}$$

remembering that $\gamma_m(x) = \gamma^{op}_{\bar{\Psi}\Psi}(x) = \dfrac{3C_2 x^2}{8\pi^2}$ (in the Landau gauge)

and $\beta(x) = \dfrac{-x^3}{2b}$ $\qquad b = \dfrac{24\pi^2}{11N - 2n}$

we get:

$$c_2(q,\mu) \sim 3iC_2 \, \frac{g^2(q^2)}{q^4} \left(\frac{g^2(q^2)}{g^2(\mu^2)}\right)^{-d} \qquad d = \frac{3bC_2}{8\pi^2} \tag{7.14}$$

that is:

$$c_2(q,\mu) \sim 6ibC_2 \left(\ln \frac{\mu^2}{\Lambda^2_{QCD}}\right)^{-d} \frac{1}{q^4} \left(\ln \frac{q^2}{\Lambda^2_{QCD}}\right)^{d-1} \tag{7.15}$$

let us insert this asymptotic behaviour in (7.2):

$$t_2 \int d^4x \, e^{iqx} \langle 0|T\psi(x)\bar{\psi}(0)|0\rangle \underset{q^2\to\infty}{\sim} 6ibC_2 \left(\ln \frac{\mu^2}{\Lambda^2_{QCD}}\right)^{-d} \cdot \frac{1}{q^4} \left(\ln \frac{q^2}{\Lambda^2_{QCD}}\right)^{d-1} \langle 0|\bar{\Psi}\Psi|0\rangle \tag{7.16}$$

and by comparing with (7.1) we deduce the asymptotic behaviour of the quark self-energy:

$$\Sigma(q^2) \underset{q^2\to\infty}{\sim} \frac{6bC_2}{4N} \left(\ln \frac{\mu^2}{\Lambda^2_{QCD}}\right)^{-d} \langle 0|\bar{\Psi}\Psi|0\rangle \frac{1}{q^2} \left(\ln \frac{q^2}{\Lambda^2_{QCD}}\right)^{d-1} \tag{7.17}$$

(the 4N factor comes from the trace over spinor and color indices) which is just the asymptotic solution Σ_2 of eq.(6.38) which satisfies the boundary condition (6.35).

In section 8 we will use this asymptotic behaviour for our variational ansatz of the quark self-energy.

If we include explicit chiral symmetry breaking with a small mass term in the Lagrangian, we can deduce in a straightforward manner the asymptotic behaviour of the quark mass function. The prescription is just as before: the coefficient functions are computed in perturbation theory but now including an explicit mass in the Feynman rules.

We parametrize again: $S(q) = i / \hat{q} - \Sigma(q^2)$

and renormalize the theory in a way that the renormalized quark propagator in perturbation theory reads:

$$D^{-1}(q)\Big|_{q^2=-\mu^2} = \hat{q} - m(\mu) \qquad (7.18)$$

(Landau gauge)

Then the OPE analysis will give this asymptotic behaviour for $\Sigma(q^2)$:

$$\Sigma(q^2) \underset{q^2\to\infty}{\sim} m(\mu)\left(\frac{g^2(q^2)}{g^2(\mu^2)}\right)^{d} + 3C_2 \langle 0|\bar{\Psi}\Psi|0\rangle_{\mu} \cdot \frac{g^2(q^2)}{q^2}\left(\frac{g^2(q^2)}{g^2(\mu^2)}\right)^{-d} . \qquad (7.19)$$

where we are neglecting effects of $O(m^2/q^2)$.

As we have already observed, the first term, proportional to the explicit chiral symmetry breaking parameter m, corresponds to the $\Sigma_1(q^2)$ solution of the second order differential equation for the self-energy (6.31). This term dominate in the $q^2\to\infty$ limit showing that $\Sigma(q^2)$ has a softer asymptotic behaviuor when symmetry breaking is dynamical then when it is not.

8. EVALUATION OF THE EFFECTIVE POTENTIAL

With all these conclusions in mind, we are ready to study dynamical symmetry breaking in QCD-like gauge theories using an effective potential approach for composite operators.

Let us evaluate the effective action for QCD-like gauge theories within our modification of CJT functional formalism. To this end, it is enough to take into account fermion condensates. So we shall introduce only a bilocal source K(x,y) coupled to the fermion bilinear $\bar{\psi}(x)\psi(y)$ in the euclidean generating functional of the theory:

$$Z[\eta,K] = e^{-W[\eta,K]} = \frac{1}{N}\int \mathcal{D}\psi\,\mathcal{D}A_\mu\,\mathcal{D}c\; e^{-[I(\psi,A_\mu,c) + \bar{\eta}_\alpha\psi_\alpha + \bar{\psi}_\alpha\eta_\alpha + \bar{\psi}_\alpha K_{\alpha\beta}\psi_\beta]} \tag{8.1}$$

Remember that $I(\psi, A_\mu, c)$ is the gauge theory action; $\eta_\alpha, \bar{\eta}_\alpha$ are the usual local sources and the α index is a collective one for spinor, flavor and space-time variables (for the sake of simplicity we have not written down explicitly the local source terms for gauge bosons A_μ andghost fields c). Furthermore we will not be interested in amplitudes with external fermions. Therefore, we will put ψ_c and $\bar{\psi}_c$ equal to zero in the effective action. The expression to be evaluated is then:

$$\Gamma(S) = -\mathrm{Tr}\ln\left(\hat{\partial} + \frac{\delta\Gamma_2}{\delta S}\right) + \mathrm{Tr}\left(\frac{\delta\Gamma_2}{\delta S}S\right) - \Gamma_2 \tag{8.2}$$

with $\quad S_{\alpha\beta} = -\dfrac{\delta W}{\delta K_{\beta\alpha}}$

At the lowest order in the loop expansion (two loops), Γ_2 is given by the following diagram:

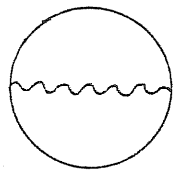


fig.6

In order to evaluate this diagram, one has to decide the form of the vertex and of the gluon propagator. As we have already seen in the discussion of the kernel of the Bethe-Salpeter equation for the proper axial-vector vertex, we can improve the lowest perturbative approximation by taking into account the renormalization group effects. We will use these arguments here too. In fact, as we have already underlined, we assume that the main contribution to the effective potential comes from the short-distance effects. For large momenta, the property of asymptotic freedom of QCD, gives the possibility to neglect multiloop contributions to the effective potential. For these reasons the renormalization group analysis allows to use the free vertex and gluon propagator in the calculation of the diagram in fig.6, and to improve this approximation with the running coupling constant. But, as far as the vertex is concerned, the situation is more subtle, because, in principle, one can run in some difficulties in order to satisfy the Ward identities.

Let us examine this point. Remember that we have parametrized:

$$S^{-1}(p) = -i\, Z(p^2)\,\hat{p} + \Sigma(p^2) \tag{8.3}$$

The Ward identity for the vertex function reads:

$$(q_1-q_2)_\mu \Gamma^\mu = S^{-1}(q_1) - S^{-1}(q_2) = -i\, Z(q_1^2)\hat{q}_1 + i\, Z(q_2^2)\hat{q}_2 + \Sigma(q_1) - \Sigma(q_2) \tag{8.4}$$

This equation can be satisfied by taking:

$$\Gamma_\mu = -i\gamma_\mu + \frac{(q_1-q_2)_\mu}{(q_1-q_2)^2}\Big[-i\,\big(Z(q_1^2)-1\big)\hat{q}_1 + i\big(Z(q_2^2)-1\big)\hat{q}_2 + \\ + \Sigma(q_1) - \Sigma(q_2)\Big] \tag{8.5}$$

However, in the evaluation of the previous vacuum contribution, (fig.6), Γ_μ is always saturated with the gluon propagator. Therefore, if we adopt the Landau gauge (transverse gluon propagator), we can safely use the free expression for the vertex.

Then, we get the following form for Γ_2 :

$$\Gamma_2 = -\frac{1}{2}\int \frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\,\big(-ig(p-q)\big)^2\, \mathrm{Tr}\,\big[S(p)\,T^a\gamma_\mu \cdot \\ \cdot\, S(q)\,T^a\gamma_\nu\big]\cdot D^{\mu\nu}(p-q)\int d^4x \tag{8.6}$$

$$\Big(\,(\Gamma_2)_{EUCL.} = (i\,\Gamma_2)_{MINK.}\Big)$$

Here, we have assumed a fermion propagator function only of the space-time difference, S(x,y)=S(x-y). From this, translational invariance of the effective action follows, and as a consequence, the space-time volume $\Omega = \int d^4x$ factorizes out. In eq.(8.6)

$$D^{\mu\nu}(k) = \left(g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right)\frac{1}{k^2} \tag{8.7}$$

T^a are the generators of the gauge group in the fermion representation and $g(p-q)$ in the leading log approximation is given by: (see (6.25))

$$g(p-q) = \theta(p-q)\, g(p) + \theta(q-p)\, g(q) \tag{8.8}$$

where $g(p)$ is the running coupling constant. However the running coupling constant becomes singular for $p^2 = \Lambda^2_{QCD}$ where Λ_{QCD} is the renormalization group invariant mass. The singularity is of course due to the use of perturbation theory in a region where the coupling becomes strong. Unfortunately, in eq. (8.6) one has to integrate upon all the range of momenta and consequently one has to make some ansatz for the coupling constant in the infrared region. Under the assumptions made, the theory should not depend too much from the infrared behaviour of the various quantities, therefore we will assume that $g(p)$ is a constant for energies lower than the scale μ we introduce to separate the large distance and small distance effects. That is, we will write:[18]

$$g^2(p) = g^2(\mu) \left[\theta(\mu-p) + \theta(p-\mu) \frac{\ln \mu^2/\Lambda^2_{QCD}}{\ln p^2/\Lambda^2_{QCD}} \right] \tag{8.9}$$

where $g^2(\mu)$ is the coupling constant renormalized at μ :

$$g^2(\mu) = \frac{2b}{\ln \mu^2/\Lambda^2_{QCD}} \quad ; \quad b = \frac{24\pi^2}{(11N-2n)} \tag{8.10}$$

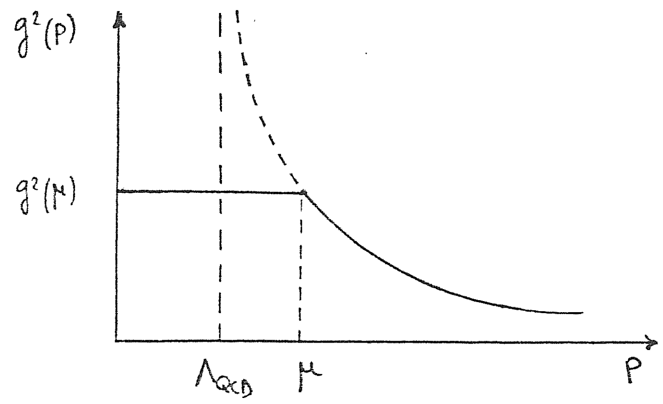


fig.7

In order to evaluate Γ_2 it is convenient to parametrize the fermion propagator in the following way:

$$\left[S(p)\right]^{A}_{B}{}^{i}_{j} = \delta^{A}_{B}\left(i\, A^{i}_{j}(p^2)\hat{p} + B^{i}_{j}(p^2)\right) \tag{8.11}$$

Our method will consist in making a convenient ansatz for $B(p^2)$ in terms of a set of parameters related to the fermionic condensates and then, in minimizing the effective potential with respect to these parameters. In this way we will able to explore the possibility of dynamical symmetry breaking.

Substituting in eq. (8.6) and taking the trace over the γ - matrices, one finds:

$$\Gamma_2 = 6N\Omega C_2 \int \frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\Big\{ tr\left[B(p^2)B(q^2)\right] D(p-q) - 2tr\left[A(p^2)A(q^2)\right] E(p,q)\Big\} g^2(p-q) \tag{8.12}$$

where C_2 is the Casimir of the fermion representation, the trace is only on the flavor indices $B = B_i^{\,j}$; $A = A_i^{\,j}$

$i,j = 1,\dots,n$

$$D(p) = \frac{1}{p^2}$$

and

$$E(p,q) = 1 - \frac{1}{2}\left[(p^2+q^2) + \frac{(p^2-q^2)^2}{(p-q)^2}\right]\frac{1}{(p-q)^2} \tag{8.13}$$

In eq. (8.12), $g^2(p-q)$ does not depend on the angle between P and q (see eq. (8.8)), therefore one can perform the angular integration by the help of the following formulae:

$$\int d\Omega \quad \frac{1}{(p-q)^2} = \frac{2\pi^2}{pq}\, e^{-|\log q/p|} \tag{8.14}$$

$$\int d\Omega \quad \frac{1}{(p-q)^4} = \frac{2\pi^2}{pq}\, \frac{e^{-|\log q/p|}}{|p^2-q^2|} \tag{8.15}$$

Then, it follows:

$$\int d\Omega \quad E(p,q) = 0 \tag{8.16}$$

We see that Γ_2 does not depend on $A(p^2)$; this is obviously related to the fact that there is no wave function renormalization in the Landau gauge. In fact, from equation (8.2), one gets the Schwinger-Dyson equation (see (4.10))

$$S^{-1} = \hat{\partial} + \delta\Gamma_2/\delta S \tag{8.17}$$

which is the extremum condition for the effective action. By using (8.6) for Γ_2 we obtain, in the momentum space:

$$S^{-1}(p) = -i\hat{p} + \int \frac{d^4q}{(2\pi)^4}\, T^a \gamma_\mu S(q) T^a \gamma_\nu D^{\mu\nu}(p-q)\, g^2(p-q) \tag{8.18}$$

By using the expressions (8.3) and (8.11) for S^{-1} and S respectively and evaluating $\mathrm{Tr}\, S^{-1}$ and $\mathrm{Tr}(\gamma_\mu S^{-1})$,one finds:

$$\Sigma(p^2) = 3C_2 \int \frac{d^4q}{(2\pi)^4}\, B(q^2)\, D(p-q)\, g^2(p-q) \tag{8.19}$$

$$Z(p^2) = 1 - \frac{C_2}{p^2}\int \frac{d^4q}{(2\pi)^4} A(q^2)\, E(p,q)\, g^2(p-q) \qquad (8.20)$$

Using again eq. (8.16), we see that in the Landau gauge, and at this order of approximation , $Z(p^2)=1$.

We are now left with the following expression for Γ_2 :

$$\Gamma_2 = 6N\Omega C_2 \int \frac{d^4p}{(2\pi)^4}\frac{d^4q}{(2\pi)^4}\, \mathrm{tr}\left[B(p^2)\, B(q^2)\right] D(p-q)\, g^2(p-q) \qquad (8.21)$$

It turns out that the more convenient variable for our variational problem is not $B(p)$, but rather $\frac{\delta\Gamma_2}{\delta S}(p)$. This last quantity coincides with the fermion self-energy when the Schwinger-Dyson equation (see (8.17)) is satisfied. (that is on the extremum of the action). Furthermore, the expression (8.21) for Γ_2 can be simplified a lot by reexpressing $B(p)$ in terms of $\frac{\delta\Gamma_2}{\delta S}$. By defining

$$\tilde{\Sigma}(p) = \frac{\delta\Gamma_2}{\delta S}(p) \qquad (8.22)$$

($\tilde{\Sigma}(p) \equiv \Sigma(p)$ when we turn off the external source, that is at the extremum of Γ), we get from eq. (8.21):

$$\tilde{\Sigma}(p) = 3C_2 \int \frac{d^4q}{(2\pi)^4} \frac{B(q)}{(p-q)^2}\, g^2(p-q) \qquad (8.23)$$

Performing the angular integration by means of eq. (8.14) we get:

$$\tilde{\Sigma}(p) = \frac{3C_2}{8\pi^2}\frac{1}{p}\int_0^\infty q^2 dq\, B(q)\left[\frac{q}{p} g^2(p)\,\theta(p-q) + \right.$$
$$\left. + \frac{p}{q} g^2(q)\,\theta(q-p)\right] \qquad (8.24)$$

This integral relation can be easily inverted by applying an appropriate differential operator to both sides of this expression. Let us do it in two steps: first of all we apply $\frac{d}{dp}$:

$$\frac{d}{dp}\tilde{\Sigma}(p) = \frac{d}{dp}\left[\frac{3C_2}{8\pi^2}\frac{g^2(p)}{p^2}\int_0^p dq\, q^3 B(q) + \right.$$

$$\left. + \frac{3C_2}{8\pi^2}\int_p^\infty dq\; g^2(q)\, B(q)\, q\right] = \tag{8.25}$$

$$= \frac{3C_2}{8\pi^2}\frac{d}{dp}\left(\frac{g^2(p)}{p^2}\right)\int_0^p dq\; q^3 B(q)$$

Dividing this expression by $\frac{d}{dp}\left(\frac{g^2(p)}{p^2}\right)$ and differentiating once again with respect to p, we get finally:

$$B(p) = \frac{8\pi^2}{3C_2}\frac{1}{p^3}\frac{d}{dp}\left[\frac{1}{\frac{d}{dp}\left(\frac{g^2(p)}{p^2}\right)}\frac{d\tilde{\Sigma}(p)}{dp}\right] \tag{8.26}$$

The big advantage we get from this invertion is that we can reexpress Γ_2 by using the Euler theorem for homogeneous functionals:

$$2\Gamma_2 = \mathrm{Tr}\left(\frac{\delta\Gamma_2}{\delta S}S\right) = \mathrm{Tr}\left(\tilde{\Sigma}S\right) \tag{8.27}$$

that is

$$\Gamma_2 = \frac{2N}{3C_2}\Omega\int_0^\infty dp\; \mathrm{tr}\left[\tilde{\Sigma}(p)\frac{d}{dp}\frac{1}{\frac{d}{dp}\left(\frac{g^2(p)}{p^2}\right)}\frac{d\tilde{\Sigma}}{dp}\right] \tag{8.28}$$

(as usual tr is the trace on the flavor indices). Integrating by parts, we get:

$$\Gamma_2 = \frac{2N}{3C_2}\Omega\left[\frac{1}{\frac{d}{dp}\left(\frac{g^2(p)}{p^2}\right)}\mathrm{tr}\left(\tilde{\Sigma}(p)\frac{d\tilde{\Sigma}(p)}{dp}\right)\right]_0^\infty +$$

$$- \frac{2N}{3C_2} \Omega \int_0^\infty dp \frac{1}{\frac{d}{dp}\left(\frac{g^2(p)}{p^2}\right)} \mathrm{tr} \left[\left(\frac{d\tilde{\Sigma}(p)}{dp} \right)^2 \right] \tag{8.29}$$

This expression is completely general, however if we assume that $\tilde{\Sigma}(p)$ has the same momentum behaviour as the self-energy (eq. (6.42)), we get that the finite term goes as $\frac{1}{p^2}$ times $\log S$ for $p \to \infty$. As far as the behaviour for $p \to 0$ is concerned, let us consider the relation:

$$S = iA\hat{p} + B = \left(-i\hat{p} + \Sigma\right)^{-1} \tag{8.30}$$

(remember $Z=1$ in the Landau gauge), from which we get, after diagonalization in flavor space:

$$A = \frac{1}{p^2 + \Sigma^2} \quad , \qquad B = \frac{\Sigma}{p^2 + \Sigma^2} \tag{8.31}$$

Taking eq. (8.24) at the extremum, where $\tilde{\Sigma} = \Sigma$ and using the relation (8.31) we obtain the Schwinger-Dyson equation for Σ, which for $p \to 0$ gives:

$$\lim_{p \to 0} \Sigma(p) = \frac{3C_2}{8\pi^2} \int_0^\infty \frac{dq}{q} \frac{q^2\, \Sigma(q)}{q^2 + \Sigma^2(q)}\, g^2(q) \tag{8.32}$$

This integral is convergent for $q^2 \to \infty$, because $\Sigma \xrightarrow[q^2 \to \infty]{} 1/q^2$.
Now suppose that $\Sigma(q) \to q^{-\alpha}$ for $q \to 0$ with $\alpha > 0$. Then at the lower limit of integration, the integrand goes like $q^{1+\alpha}$ and therefore the limit in (8.32) is finite, contrarily to the assumption. It follows that for $p \to 0$, $\Sigma(p)$ must go to some finite constant. We will assume the same kind of behaviour for $p \to 0$ also for $\tilde{\Sigma}$.

Therefore, using our preferred renormalization point μ, we will make the following ansatz:

$$\tilde{\Sigma}^i_j(p) = \chi^i_j \mu \left[\Theta(\mu-p)+\Theta(p-\mu)\left(\frac{\mu}{p}\right)^2 \frac{g^2(p)}{g^2(\mu)}\left(\frac{\ln p^2/\Lambda^2_{QCD}}{\ln \mu^2/\Lambda^2_{QCD}}\right)^d\right] \tag{8.33}$$

i,j=1,...,n

with $d=3C_2 b/8\pi^2 = 9C_2/(11N-2n)$.

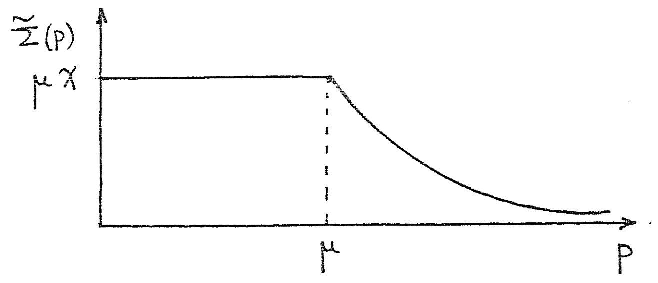


fig.8

The constants χ^i_j will be our variational parameters. In other words, we assume a constant behaviour of $\tilde{\Sigma}$ in the infrared region and the OPE prediction for momenta greater than μ. With this kind of behaviour the finite term in eq. (8.29) can be completely neglected. In fact $\frac{d\tilde{\Sigma}}{dp} = 0$ at p=0. However, notice that also for a smoother behaviour of $\tilde{\Sigma}$ for $p \to 0$, there is always the term

$$\left(\frac{d}{dp}\frac{g^2(p)}{p^2}\right)^{-1} \xrightarrow[p\to 0]{} \left(\frac{1}{p^3}\cdot\text{logs}\right)^{-1}$$

responsible for the vanishing of the finite term for $p \to 0$. With these considerations in mind we get:

$$\Gamma_2 = -\frac{2N}{3C_2}\Omega\int_0^\infty dp\, \frac{1}{\frac{d}{dp}\left(\frac{g^2(p)}{p^2}\right)}\, \mathrm{tr}\left[\frac{d\tilde{\Sigma}(p)}{dp}\right]^2 \tag{8.34}$$

This expression can be further simplified by evaluating the derivative of $\tilde{\Sigma}$, from eq. (8.33):

$$\frac{d\tilde{\Sigma}(p)}{dp} = \mu\, \chi^i_j\, \theta(p-\mu)\ \frac{\mu^2}{g^2(\mu^2)} \left\{ \frac{d}{dp}\left(\frac{g^2(p)}{p^2}\right) F(p) + \right. \qquad (8.35)$$

$$\left. + \frac{g^2(p)}{p^2}\ \frac{dF(p)}{dp} \right\}$$

where we have defined

$$F(p) = \left(\frac{\ln p^2/\Lambda^2_{QCD}}{\ln \mu^2/\Lambda^2_{QCD}} \right)^d \qquad (8.36)$$

Inserting (8.35) into eq. (8.34) one gets:

$$\Gamma_2 = -\frac{2N}{3C_2}\,\Omega\ \mu^2 \left(\frac{\mu^2}{g^2(\mu^2)}\right)^2 \int_\mu^\infty dp \left\{ F^2 \frac{d}{dp}\left(\frac{g^2(p)}{dp^2}\right) + \right. \qquad (8.37)$$

$$\left. + 2F\ \frac{g^2(p)}{p^2}\frac{dF}{dp} + \frac{1}{\frac{d}{dp}\left(\frac{g^2(p)}{p^2}\right)}\ \frac{g^4(p)}{p^4}\left(\frac{dF}{dp}\right)^2 \right\} \mathrm{tr}\, \chi^2$$

Integrating by parts the first term one finds:

$$\Gamma_2 = \frac{2N\Omega}{3C_2}\,\frac{\mu^4}{g^2(\mu)} \left\{ 1 - \frac{\mu^2}{g^2(\mu)} \int_\mu^\infty dp\ \frac{1}{\frac{d}{dp}\,\frac{g^2(p)}{p^2}} \cdot \right. \qquad (8.38)$$

$$\left. \cdot \left[\frac{g^2(p)}{p^2}\ \frac{dF}{dp} \right]^2 \right\} \mathrm{tr}\, \chi^2$$

Notice that Γ_2 is definite positive because $\frac{d}{dp}\,\frac{g^2(p)}{p^2} < 0$, and also that, with our ansatz for $\tilde{\Sigma}$, the infrared region ($p < \mu$) does not contribute.

Using again the Euler theorem, eq. (8.2) can be rewritten as:

$$\Gamma = \Gamma_2 - \Omega \int \frac{d^4p}{(2\pi)^4}\ 2N \ln\left(p^2 + \tilde{\Sigma}^2\right) \qquad (8.39)$$

where $\tilde{\Sigma}$ has been diagonalized in the flavor indices by a unitary transformation. We get in this way our final result:

$$\Gamma = \frac{N}{4\pi^2}\mu^4 \Omega \sum_{i=1}^{n} V(\chi_i) \tag{8.40}$$

where χ_i are the eigenvalues of the matrix χ_j^i , and

$$V(\chi) = \frac{8\pi^2}{3C_2 g^2(\mu)}\chi^2 \left\{1 - \frac{\mu^2}{g^2(\mu)}\int_\mu^\infty dp \frac{1}{\frac{d}{dp}\left(\frac{g^2(p)}{p^2}\right)}\left[\frac{g^2(p)}{p^2}\frac{dF}{dp}\right]^2\right\} +$$
$$- \frac{1}{\mu^4}\int_0^\mu dp\, p^3 \ln\left(p^2+\mu^2\chi^2\right) + \tag{8.41}$$
$$- \frac{1}{\mu^4}\int_\mu^\infty dp\, p^3 \ln\left(p^2+\chi^2\left(\frac{\mu}{p}\right)^4\mu^2\,\frac{g^4(p)}{g^4(\mu)}\,F^2(p)\right).$$

Apart from an infinite additive constant which can be removed adding $\frac{1}{\mu^4}\int_0^\infty dp\, p^3 \ln p^2$, $V(\chi)$ is a completely finite quantity, both in the ultraviolet and in the infrared regime. The theory is regularized in the infrared by the assumed constant behaviour of the self-energy for $p \to 0$, whereas the convergence in the ultraviolet follows from the physical meaning of χ which will be explained in the next section.

9. THE PHISICAL MEANING OF χ

Our variational parameter χ can be related to the fermionic condensate when evaluated at the extremum of the action. This can be seen directly by comparison of our ansatz, (eq.(8.33), with the OPE expansion given in eq. (7.17). It is however instructive to derive this relation by direct calculation. If we evaluate the fermion propagator at equal space-time points, in terms of a cut-off Λ, we have:

$$\langle 0|\bar{\psi}\psi|0\rangle_{\Lambda} = -\frac{1}{(2\pi)^4}\int^{\Lambda} d^4p \, \mathrm{tr}\left(\frac{-i\not{p}+\Sigma}{p^2+\Sigma^2}\right) \qquad (9.1)$$

where the trace is over spinor and color indices. This integral is ultraviolet divergent, and its leading divergence can be evaluated by using the asymptotic behaviour for Σ, (eq. (8.33)):

$$\langle 0|\bar{\psi}\psi|0\rangle_{\Lambda} = \frac{N}{2\pi^2}\frac{\mu^3}{g^2(\mu)}\int^{\Lambda}\frac{dp}{p}g^2(p)\left(\frac{\ln p/\Lambda_{QCD}}{\ln \mu/\Lambda_{QCD}}\right)^d \chi \approx$$

$$\approx \frac{N}{2\pi^2}\frac{\mu^2}{g^2(\mu)}\frac{b}{d}\left(\frac{\ln \Lambda/\Lambda_{QCD}}{\ln \mu/\Lambda_{QCD}}\right)^d \chi$$

Recalling that $b/d = 8\pi^2/3C_2$ we get:

$$\langle 0|\bar{\psi}\psi|0\rangle_{\Lambda} \simeq \frac{4N\mu^3}{3C_2 g^2(\mu)}\left(\frac{\ln \Lambda/\Lambda_{QCD}}{\ln \mu/\Lambda_{QCD}}\right)^d \chi \qquad (9.2)$$

The composite operator $\bar{\psi}\psi$ must be renormalized. From the renormalization group analysis we can derive the relation between $\langle 0|\bar{\psi}\psi|0\rangle_{\Lambda}$ and the condensate renormalized at the point $\bar{\mu}$:[19]

$$\langle 0|\bar{\Psi}\Psi|0\rangle_{\bar{\mu}} = \left(\frac{\ln \bar{\mu}/\Lambda_{QCD}}{\ln \Lambda/\Lambda_{QCD}}\right)^{d} \langle 0|\bar{\Psi}\Psi|0\rangle_{\Lambda} \tag{9.3}$$

Therefore, our variational parameter χ is proportional to the condensate:

$$\langle 0|\bar{\Psi}\Psi|0\rangle_{\bar{\mu}} = \frac{4N\mu^3}{3C_2\, g^2(\mu)} \left(\frac{\ln \bar{\mu}/\Lambda_{QCD}}{\ln \mu/\Lambda_{QCD}}\right)^{d} \chi \tag{9.4}$$

The relation is, of course, particularly simple by choosing μ as renormalization point:

$$\langle 0|\bar{\Psi}\Psi|0\rangle_{\mu} = \frac{4N\mu^3}{3C_2\, g^2(\mu)} \chi \tag{9.5}$$

It follows that χ has operator dimension equal to three. However, due to the chiral invariance, linear terms in χ are forbidden so that the potential V must start at least with χ^2 in an expansion in the composite field. Therefore, due to the absence of operators with dimensions lower or equal to 4, no UV divergences are expected in V. This explains the result founded at the end of the previous section. Equation (9.5) shows that χ is nothing but the order parameter of the theory relatively to the chiral symmetry. We will study V as a function of χ, and the phase in which V is a minimum for nonvanishing values of χ will correspond to the chiral symmetry breaking phase.

10. DISCUSSION OF THE EFFECTIVE POTENTIAL

Let us start evaluating the effective potential in a situation in which one neglects the logarithmic corrections. It corresponds to a fixed coupling constant $g^2(p) = g^2(\mu)$, and to $F(p) = \left(\frac{\ln p/\Lambda_{QCD}}{\ln \mu/\Lambda_{QCD}}\right)^d$ equal to 1 (this is equivalent to take equal to zero the anomalous dimension of $\bar{\psi}\psi$ which is proportional to d).

This approximation is equivalent to neglect all the corrections coming from the renormalization group equation. The reason why we are interested in this simplified case is that, as we shall verify later on, the qualitative conclusions are not different from the complete case, and also quantitatively the differences are not very large. However, in this simple case one can evaluate V analitically and this will turn out to be very useful also to understand the asymptotic behaviour of the complete case. Then, by calling $V^{(0)}$ the potential without the logarithmic corrections, we get:

$$V^{(0)}(\chi) = c\chi^2 - \int_0^1 dy\, y^3 \ln\left(y^2 + \chi^2\right) - \int_1^\infty dy\, y^3 \ln\left(y^2 + \frac{\chi^2}{y^4}\right) \tag{10.1}$$

where we have introduced the quantity:

$$c = \frac{8\pi^2}{3 C_2 g^2(\mu)} \tag{10.2}$$

It is interesting to notice that the phase structure of the theory depends now from the single combination $C_2 g^2(\mu)$.

The integrals in eq.(10.1) can be evaluated esplicitily and the result is:

$$V^{(0)}(\chi) = \left(c-\frac{1}{4}\right)\chi^2 + \frac{1}{4}\chi^4 \ln\frac{1+\chi^2}{\chi^2} + \frac{1}{8}\chi^{4/3}\ln\frac{1-\chi^{2/3}+\chi^{4/3}}{(1+\chi^{2/3})^2} + \tag{10.3}$$

$$-\frac{\sqrt{3}}{4}\chi^{4/3}\left(\frac{\pi}{2} - \operatorname{arctg}\frac{2-\chi^{2/3}}{\sqrt{3}\,\chi^{2/3}}\right).$$

From this equation we get easily the behaviour of $V^{(0)}$ for small fields:

$$V^{(0)} \underset{\chi\to 0}{\longrightarrow} (c-1)\chi^2 + \frac{3}{16}\chi^4 - \frac{1}{4}\chi^4 \ln\chi^2 \tag{10.4}$$

and for large fields:

$$V^{(0)} \underset{\chi\to\infty}{\longrightarrow} c\chi^2 - \frac{\pi}{2\sqrt{3}}\chi^{4/3} \tag{10.5}$$

From (10.4) we see that, at the point c=1 an instability occurs, and in fact, by a careful numerical analysis of eq. (10.3), one finds that the theory breaks spontaneously the chiral symmetry for $c < 1$ that is for $\alpha_s = \frac{g^2(\mu)}{4\pi} > \frac{2\pi}{3C_2}$ .In QCD with triplet fremions the critical point is for $\alpha_s = \frac{\pi}{2}$.

Furthermore, from eq.(10.5), it follows that, along the direction choosen in the functional space in which $\tilde{\Sigma}(p)$ lives, the effective potential is bounded from below.

In the appendix we have some diagrams showing the behaviour of $V^{(0)}(\chi)$. In FIG.1 $V^{(0)}$ is plotted as a function of χ for four values of c and in FIG.2 the phase transition is showed by plotting $V^{(0)}(c,\chi)$.

Furthermore, if we want to consider also the pseudoscalar bound-states, we have to parametrize the fermion propagator in this way:

$$S(p) = \left(+iA\hat{p} + B + i\gamma_5 C\right) = \left(-i\, Z\hat{p} + \Sigma + i\gamma_5 \Sigma_5\right)^{-1} \tag{10.6}$$

The calculation are exatly the same as before. In fact, making for $\tilde{\Sigma}_5(p^2)$ an ansatz of the form: (recall that we are neglecting logs)

$$\tilde{\Sigma}_5(p^2) = \Pi\, \mu \left\{ \theta(\mu - p) + \theta(p-\mu)\left(\frac{\mu}{p}\right)^2 \right\} \tag{10.7}$$

the final expression of $V^{(0)}$, due to the explicit chiral invariance, will follow simply by substituting $\chi^2 \to \Pi^2 + \chi^2$ into eq.(10.5). In FIG.3 $V^{(0)}(\chi, \Pi)$ is plotted. For c=1 it has a minimum for $\chi = \Pi = 0$ while in FIG.4 we see that for c=0.6 we have degenerate minima lying on a circle.

In the particular case in which we neglect the logarithmic corrections, one can study the contribution of the infrared behaviour to the effective potential.

First of all, we notice that, with our ansatz, such a contribution is not present in Γ_2, and therefore all the infrared effect comes from the second term in eq. (10.1).

In this way we get:

$$V^{(0)}_{UV} = c\chi^2 + \frac{1}{4}\ln\left(1+\chi^2\right) + \frac{1}{8}\chi^{4/3}\ln\frac{1-\chi^{2/3}+\chi^{4/3}}{\left(1+\chi^{2/3}\right)^2} + \tag{10.8}$$
$$- \frac{\sqrt{3}}{4}\chi^{4/3}\left(\frac{\pi}{2} - \operatorname{arctg}\frac{2-\chi^{2/3}}{\sqrt{3}\,\chi^{2/3}}\right)$$

$$V_{I.R} = -\frac{1}{4}\chi^2 - \frac{1}{4}\ln\left(1+\chi^2\right) + \frac{1}{4}\chi^4 \ln\frac{1+\chi^2}{\chi^2} \tag{10.9}$$

The behaviour for small fields is given by:

$$V_{UV}^{(o)} \underset{\chi\to 0}{\approx} \left(c-\frac{1}{2}\right)\chi^2 \tag{10.10}$$

$$V_{IR}^{(o)} \underset{\chi\to 0}{\approx} -\frac{1}{2}\chi^2 \tag{10.11}$$

whereas for large fields we get:

$$V_{UV}^{(o)} \underset{\chi\to\infty}{\approx} c\chi^2 - \frac{\pi}{2\sqrt{3}}\chi^{4/3} + \frac{1}{4}\ln\chi^2 \tag{10.12}$$

$$V_{IR}^{(o)} \underset{\chi\to\infty}{\approx} -\frac{1}{4}\ln\chi^2 \tag{10.13}$$

We see that the critical point is very sensitive to the infrared behaviour; however,if the symmetry is badly broken, that is if $c \ll 1$, the minimum of the potential is very large and one can reproduce it by using the asymptotic behaviuor (10.5). In fact from (10.5) one has:

$$\frac{dV^{(o)}}{d\chi} \underset{\chi\to\infty}{\approx} 2c\chi - \frac{2\pi}{3\sqrt{3}}\chi^{1/3} \qquad \chi_{exstr.}^{2/3} = \frac{\pi}{3\sqrt{3}}\frac{1}{c} \tag{10.14}$$

that is ,the smaller is c the larger is the value of the extremum of the potential. In such a situation, the influence of the infrared part is less and less effective decreasing c. In practice it turns out that for $c \lesssim 0.3$, the minimum evaluated with (10.12) differs of about 20% from the one evaluated with (10.5).

From this considerations, it appears plausible that,at least for $c \lesssim 0.3$ our ansatz is not very sensitive to the paricular form that we give to the infrared part and, furthermore, the ultraviolet appears to dominate in agreement of our initial hypothesis. From the phenomenological study in the massive case, it turns out that the relevant value for c is $\simeq 0.32$

(see sect.11). Also, eq. (10.5), is very accurate in order to get the minimum of the potential, in fact, the error at c=0.5 is only 5%.

One can study numerically the minima of the potential of eq.(10.1) and determine the behaviour of the minima itself with c (remember $c= 8\pi^2/3C_2\, g^2(\mu)$). Due to the connection between χ and the condensate renormalized at μ (see (9.5)),we get the behaviour of $\langle\bar\psi\psi\rangle_\mu$ with the coupling constant. This is illustrated in FIG.5 (see appendix), where the ratio

$$\left[\frac{1}{N}\,\frac{\langle\bar\psi\psi\rangle_\mu}{\mu^3}\right]^{1/3} = \left[\frac{c\;\chi(c)}{2\pi^2}\right]^{1/3} \tag{10.15}$$

is plotted as a function of c , showing explicitily the phase transition happening at c=1.

Let us go back now to the complete case with logarithmic corrections taken into account. First of all we will show that also in this case the effective potential $V(\chi)$ of eq. (8.41) is bounded from below.To this end let us consider:

$$V(\chi) - V^{(0)}(\chi) = -\frac{c\,\mu^2\chi^2}{g^2(\mu)}\int_\mu^\infty dp\, \frac{1}{\frac{d}{dp}\left(\frac{g^2(p)}{p^2}\right)}\left[\frac{g^2(p)}{p^2}\,\frac{dF}{dp}\right]^2 + -\frac{1}{\mu^4}\int_\mu^\infty dp\, p^3 \ln\left(\frac{1+\chi^2\left(\frac{\mu}{p}\right)^6 \frac{g^4(p)}{g^4(\mu)}\,F^2(p)}{1+\chi^2\left(\frac{\mu}{p}\right)^6}\right) \tag{10.16}$$

with $V^{(0)}$ given by eq.(10.1). In this expression, the first term is positive definite because $\frac{d}{dp}\left(\frac{g^2(p)}{p^2}\right) < 0$; as far as the second termis concerned, one has to study the quantity

$$A = \frac{g^2(p)}{g^2(\mu)} F(p) = \left(\frac{\ln p/\Lambda_{QCD}}{\ln \mu/\Lambda_{QCD}} \right)^{d-1} \tag{10.17}$$

The expression (10.16) is positive definite for $A \leq 1$.

Because $p \geq \mu$ it follows that $A \leq 1$ according to $(d-1) \leq 0$.

We have $d=3bC_2/8\pi^2$ that is $d=9C_2/(11N-2n)$ then, in QCD with color triplet fermions ($N=3$, $C_2=4/9$), one has $d=12/(33-2n)$ and $d<1$ only for $n<21/2$. So the expression (10.16) is positive definite if we have less than six families.

In such a situation we are guaranteed that V is buonded from below, because $V(\chi) \geq V^{(0)}(\chi)$ for any value of χ.

As far as the critical behaviour is concerned, one has to study $v(\chi)$ numerically. Now, $V(\chi)$ is a function only of the ratio $\frac{\mu}{\Lambda_{QCD}}$; one can show that there is dynamical symmetry breaking for $\mu/\Lambda_{QCD} < 1.355$ which corresponds to $\alpha_s = g^2(\mu)/4\pi > 0.33\pi$ in QCD with color triplet fermions. In FIG.6 (see appendix) we have plotted the potential as a function of χ in the case of QCD with $n=3$, for three values of μ/Λ_{QCD}.

Remember that, in the previous discussion we have found a broken phase occurring for $\alpha_s > \pi/2$.

We conclude that the overall qualitative picture remains unaltered when we take the logarithmic corrections into account and , also quantitatively, the observable quantities do not change very much (13% for μ/Λ_{QCD}).

11. DETERMINATION OF $g^2(\mu)$

We have shown that the effective potential is UV finite due to the non-appearance of operators of dimension $\lesssim 3$.

However, this situation is peculiar for the massless case, because if a mass term is allowed, a linear term in the condensate can appear in the potential. Correspondingly one expects a divergence to come out. We can illustrate this situation in the simplified case in which the logarithms are neglected. Furthermore, just as an illustration, is convenient to consider the case of a single flavor. The effective potential is gotten exactly performing the same calculations as before, except that now $\hat{\partial} \to \hat{\partial} + m$. The ansatz that we use is the same as in the massless case and the final result is:

$$\Gamma = \frac{N\mu^4}{4\pi^2}\Omega\left[c\chi^2 - \int_0^{\Lambda/\mu} dy\, y^3 \ln\left[y^2 + \left(\frac{m}{\mu} + \chi f(y)\right)^2\right]\right] \tag{11.1}$$

where we have introduced a cut-off Λ and:

$$f(y) = \theta(1-y) + \frac{1}{y^2}\theta(y-1) \tag{11.2}$$

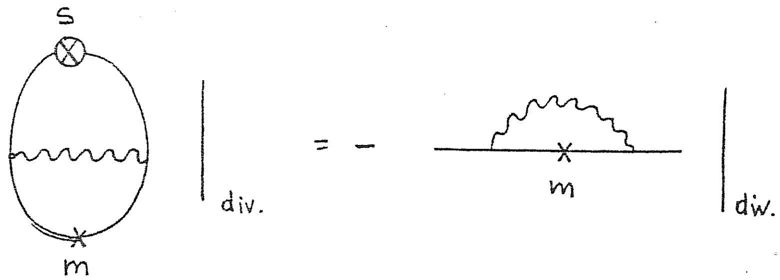
Expanding the log for large y (we have again subtracted an infinite constant), we get:

$$\ln\left[1 + \frac{1}{y^2}\left(\frac{m}{\mu} + \frac{\chi}{y^2}\right)^2\right] \approx \frac{1}{y^2}\frac{m^2}{\mu^2} + \frac{2m}{\mu}\frac{\chi}{y^4} + \cdots \tag{11.3}$$

The first term is quadratically divergent, but it does not depend on the field χ and it can be removed. The second term, linear in the condensate, gives the expected logarithmic divergence. By calling $s = \langle 0|\bar{\psi}\psi|0\rangle_\mu$, one can write the divergent piece in the following way:

$$-\delta m\, s = -\left(\frac{3g^2(\mu)C_2}{16\pi^2}\ln\frac{\Lambda^2}{\mu^2}\right) m\, s \qquad (11.4)$$

which is nothing but the divergence corresponding to the following diagram:



The effective action can then be renormalized by adding $\delta m\, s\, \Omega$ to eq. (11.1). However, there is a condition that Γ must satisfy in the $m \to 0$ limit. In fact, let us consider the generating functional of the connected Green's functions W in the presence of a small mass term. Because we can think to add the mass by the replacement $K(x,y) \to K(x,y) + m\,\delta^4(x-y)$, we can write:

$$W(m,K) \underset{m\to 0}{\Longrightarrow} W(0,K) + \int d^4x\; m\, \frac{\delta W(K)}{\delta K(x,x)} \qquad (11.5)$$

where a trace over spinor and color indices is understood. Then, we can evaluate the effective action at its extremum (K=0), obtaining:

$$\Gamma(m)\Big|_{extr.} = W(m,0) \underset{m\to 0}{\Longrightarrow}$$

$$\underset{m\to 0}{\Longrightarrow} \quad W(0,0) - m\int d^4x\, S(x,x) = \Gamma(0)\Big|_{extr.} + m s\,\Omega \tag{11.6}$$

because $S(x,x)$ is nothing but minus the condensate. Therefore we have to require:

$$\lim_{m\to 0} \frac{1}{\Omega}\frac{1}{m}\frac{\partial\Gamma}{\partial s}\Big|_{extr.} = 1 \tag{11.7}$$

This is a not trivial condition, and we shall see that it determines indeed the coupling constant at the point μ. The situation is very much similar to what happens in other massless models, and it is nothing but dimensional transmutation working for us. In fact, one has to appreciate that μ is not really introduced in the theory as a parameter, but rather as a privileged renormalization point, and eq. (11.7) tells us that we can fix g at the point μ and parametrize the theory in terms of μ itself.

Now, in order to solve for $g^2(\mu)$, we need to expand (11.1) for small m. We get:

$$\begin{aligned}\Gamma &= \Gamma\Big|_{m=0} - \frac{N\mu^4\Omega}{4\pi^2}\int_0^{\Lambda/\mu} dy\; y^3\, \frac{2m\chi}{\mu}\, \frac{f(y)}{y^2+\chi^2 f^4(y)} + \delta m s\Omega = \\ &= \Gamma\Big|_{m=0} + \frac{N(\chi)}{2c}\, m s\,\Omega\end{aligned} \tag{11.8}$$

where we have defined:

$$\begin{aligned}N(\chi) &= -2\left[\int_0^1 dy\, \frac{y^3}{y^2+\chi^2} + \int_1^\infty dy\left(\frac{y^5}{y^6+\chi^2} - \frac{1}{y}\right)\right] \equiv \\ &\equiv N_{IR}(\chi) + N_{UV}(\chi)\end{aligned} \tag{11.9}$$

and we have separated the infrared part from the ultraviolet one. We get:

$$N_{IR}(\chi) = -1 + \chi^2 \ln \frac{1+\chi^2}{\chi^2} \tag{11.10}$$

$$N_{UV}(\chi) = \frac{1}{3} \ln \left(1+\chi^2\right) \tag{11.11}$$

Our boundary condition (11.7), requires that:

$$N(\chi(c)) = 2c \tag{11.12}$$

where χ (c) is obtained extremizing Γ at m=0. Having determined χ (c), one can plot N(χ (c)) and solve eq. (11.12), (see fig.6 in appendix). The result is $c \simeq 0.32$. Notice that also in this case $N_{UV}(\chi)$ is dominating over $N_{IR}(\chi)$ for large values of χ ; in fact:

$$N_{IR}(\chi) \xrightarrow[\chi\to\infty]{} -\frac{1}{2\chi^2} \tag{11.13}$$

$$N_{UV}(\chi) \xrightarrow[\chi\to\infty]{} 2/3 \ln \chi \tag{11.14}$$

Because $\chi(0.32) \approx 2.5$, again the influence of the infrared part in fixing c is only of the order of 10%.

As we have seen, the condition (11.7) plays a crucial role because it allows us to eliminate $g^2(\mu)$ and to remain with a theory which depends only from a mass scale.

CONCLUSIONS

We have shown that in massless QCD-like gauge theories chiral symmetry breaking can occur provided that the coupling constant exceeds some critical value. In such a situation the $SU(n)_L \times SU(n)_R$ flavor group breaks down to the diagonal subgroup $SU(n)_{L+R}$.

The effective potential we have found is UV and IR finite. However, it acquires a logarithmic divergence as soon as we turn on a mass term . This fact gives rise to a boundary condition (equivalent to the Adler and Dashen mass relation),which allowes us to eliminate the coupling constant in favour of our scale parameter μ.

In principle the parameter μ can be determined by evaluating some physical quantities like the pion decay constant f_π . Such a calculation is feasible in our formalism simply by evaluating the effective action for non constant configurations of the pion field. The value one finds in this way is $\mu \approx 300$ Mev to each corresponds a value of $\Lambda_{QCD} \approx 250$ Mev.[20]

APPENDIX

CAPTION TO FIGURES

FIG.1 Behaviour of $V^{(0)}(\chi)$ near the critical point c=1

FIG.2 Phase transition by plotting $V^{(0)}(c, \chi)$

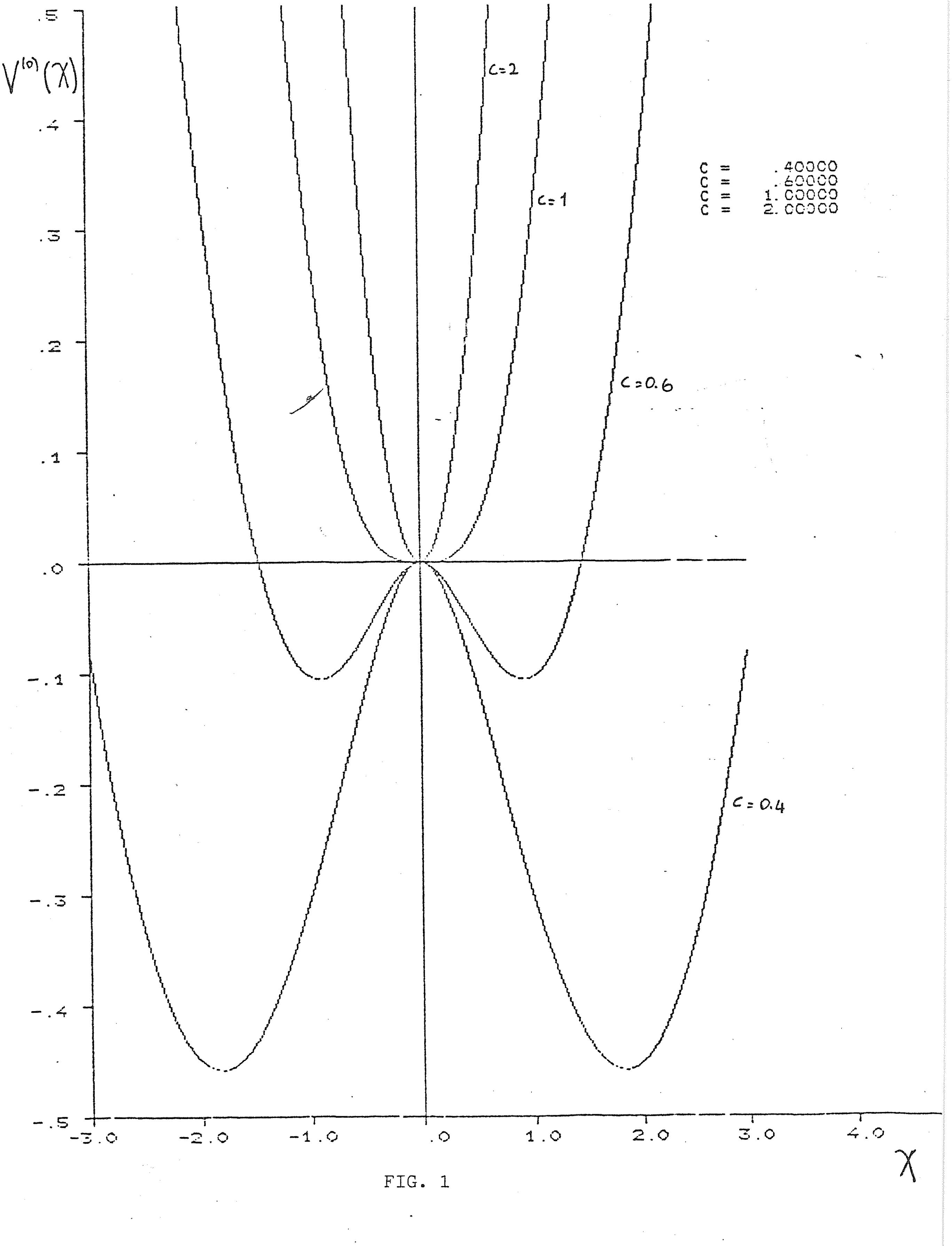
FIG.3 Behaviour of $V^{(0)}(\pi, \chi)$ at the critical point c=1

FIG.4 Behaviour of $V^{(0)}(\pi, \chi)$ in the broken phase (c=0.6)

FIG.5 The condensate is exhibited as a function of c

FIG.6 Behaviour of $V(\chi)$ near the critical point $\mu/\Lambda_{QCD}=1.355$

FIG.7 Graphical solution of the eq.(11.12)



$V^{(0)}(\chi)$
C = .40000
C = .60000
C = 1.00000
C = 2.00000
c=2
c=1
c=0.6
c=0.4
.5
.4
.3
.2
.1
.0
-.1
-.2
-.3
-.4
-.5
-3.0
-2.0
-1.0
.0
1.0
2.0
3.0
4.0
χ

FIG. 1

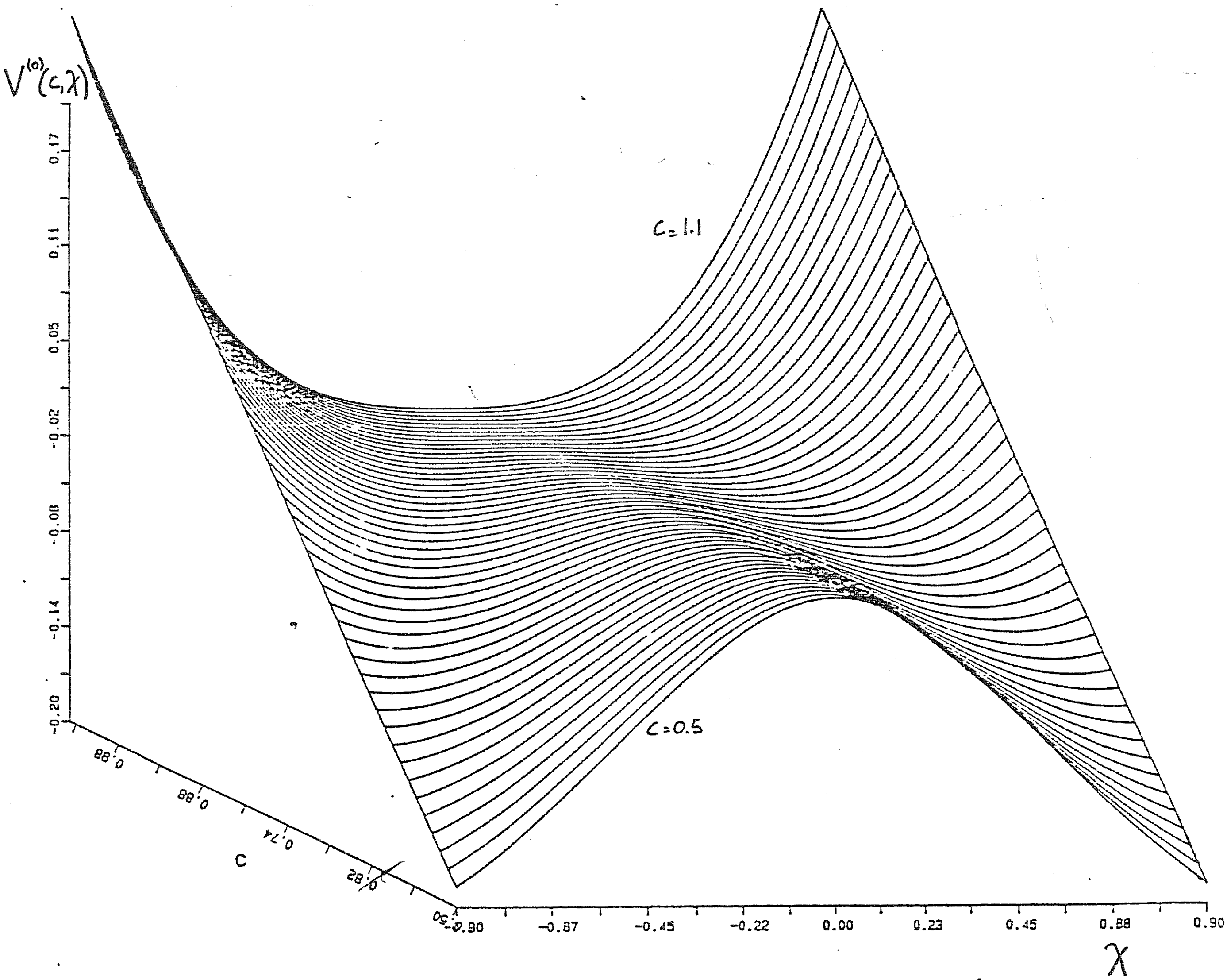

$V^{(0)}(c,\chi)$
0.17
0.11
0.05
-0.02
-0.08
-0.14
-0.20
$c=1.1$
$c=0.5$
c
χ
-0.90
-0.87
-0.45
-0.22
0.00
0.23
0.45
0.88
0.90

FIG. 2

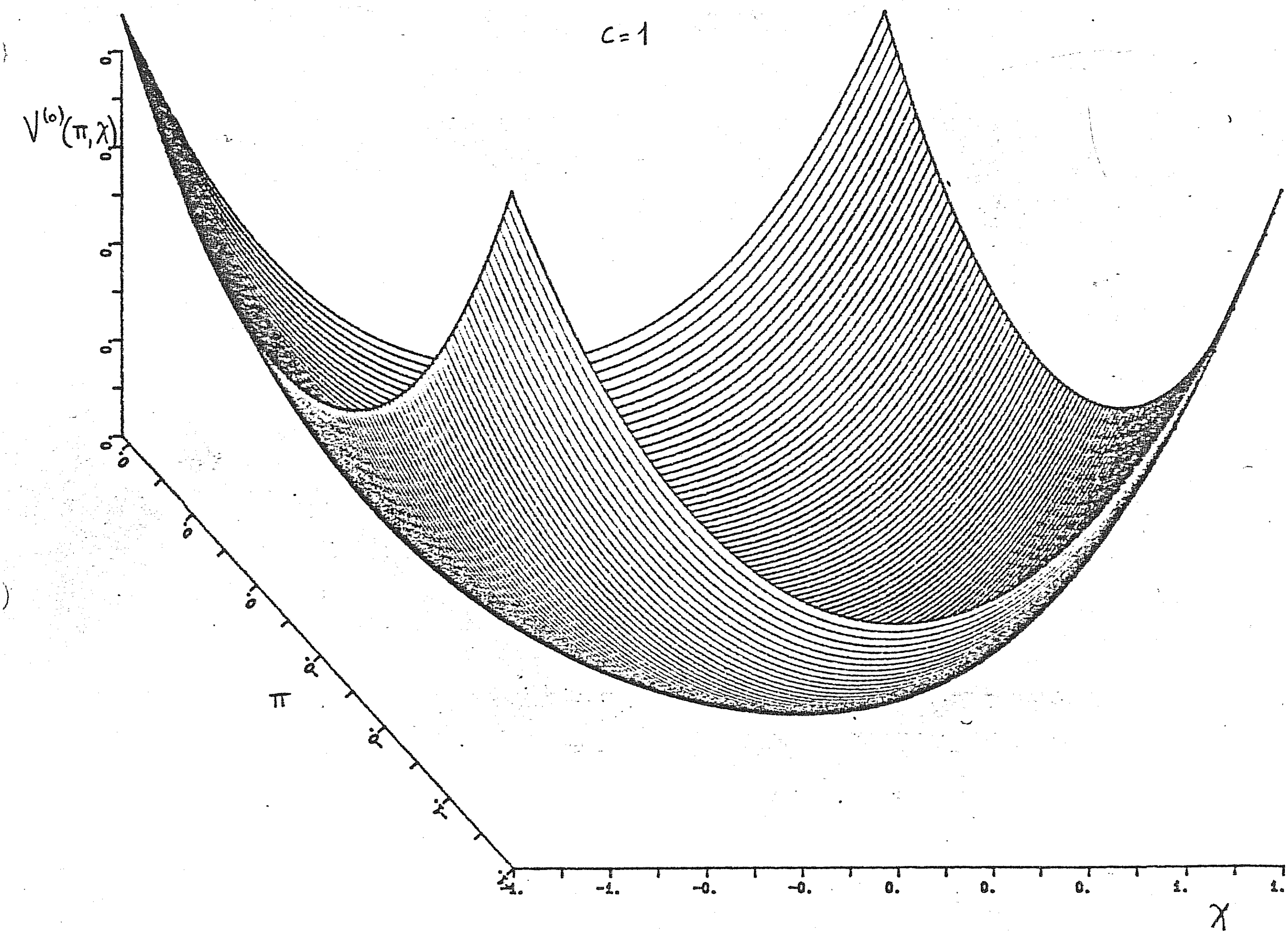
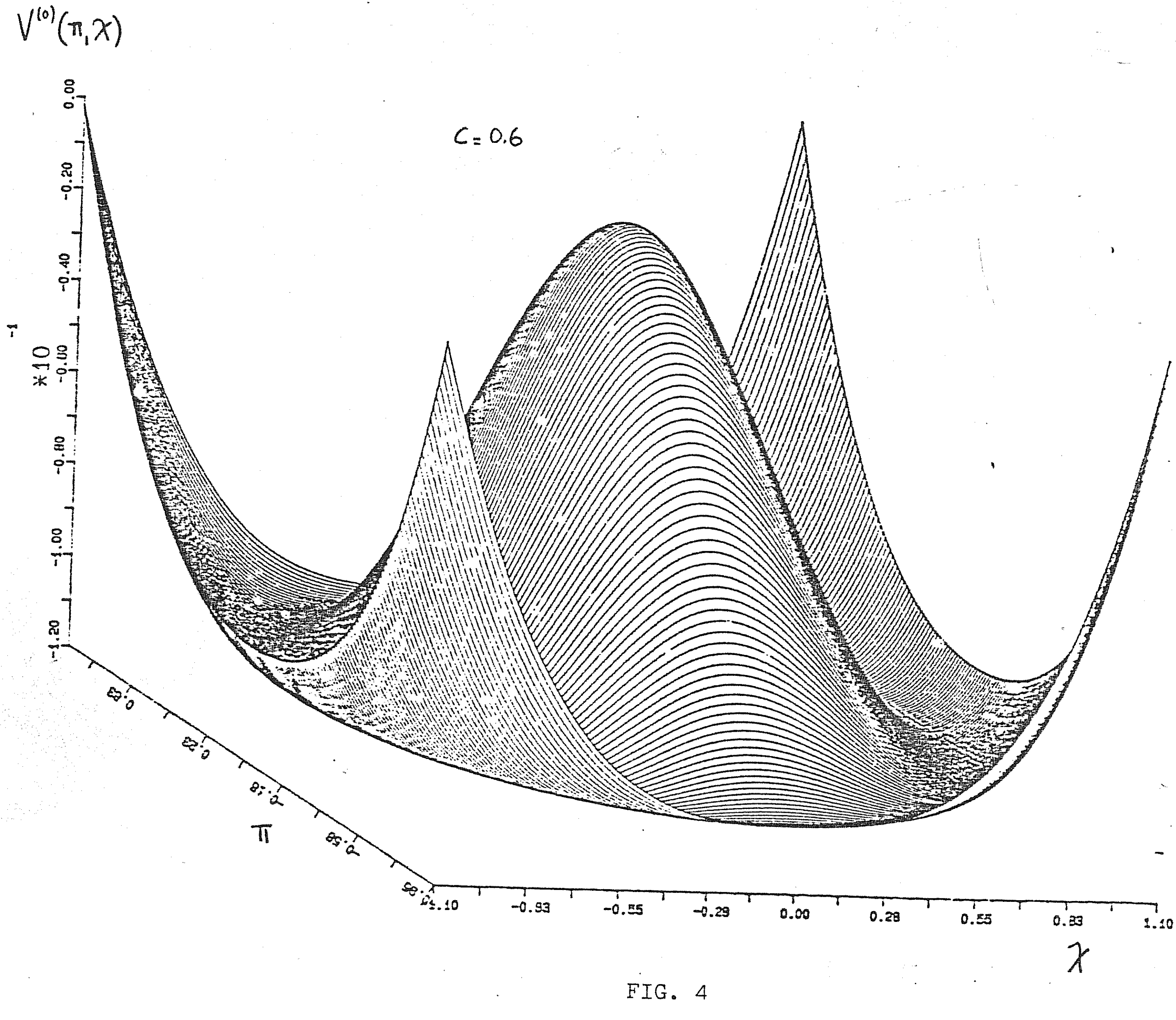
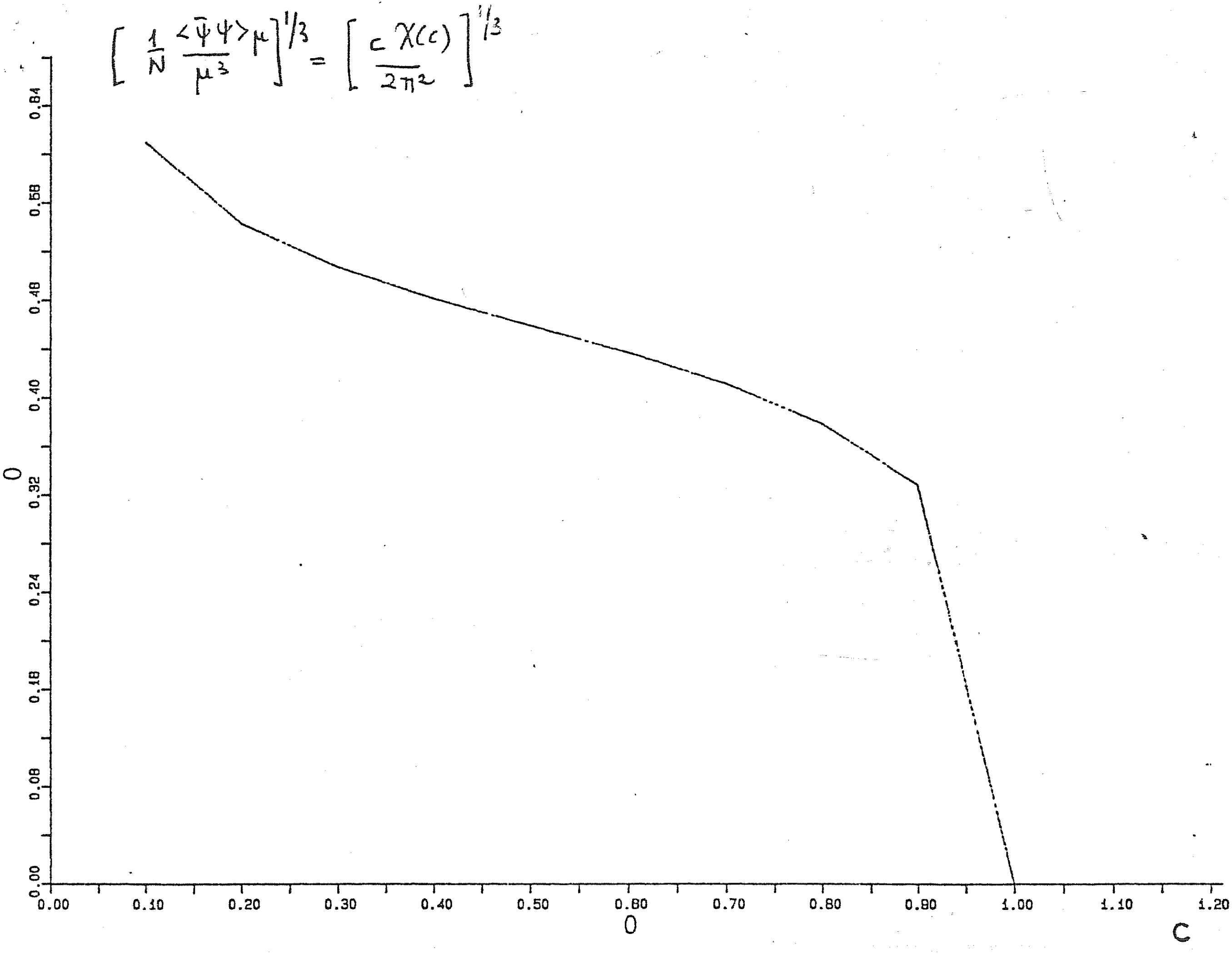


FIG. 3



$V^{(0)}(\pi,\chi)$
$c = 0.6$
$*10^{-1}$
0.00
-0.20
-0.40
-0.60
-0.80
-1.00
-1.20
π
-1.10
-0.83
-0.55
-0.28
0.00
0.28
0.55
0.83
1.10
χ

FIG. 4



$\left[\frac{1}{N} \frac{\langle \bar{\Psi}\Psi \rangle_\mu}{\mu^3} \right]^{1/3} = \left[\frac{c\,\chi(c)}{2\pi^2} \right]^{1/3}$
0.64
0.56
0.48
0.40
0.32
0.24
0.16
0.08
0.00
0
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00
1.10
1.20
0
c

FIG. 5

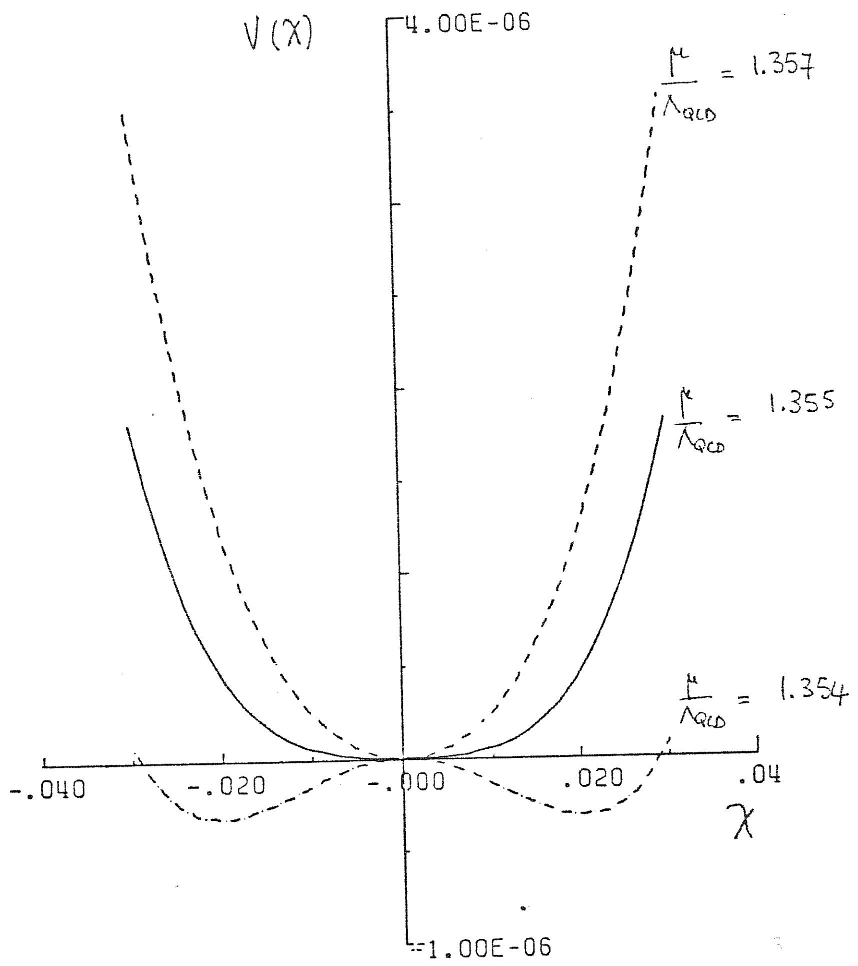
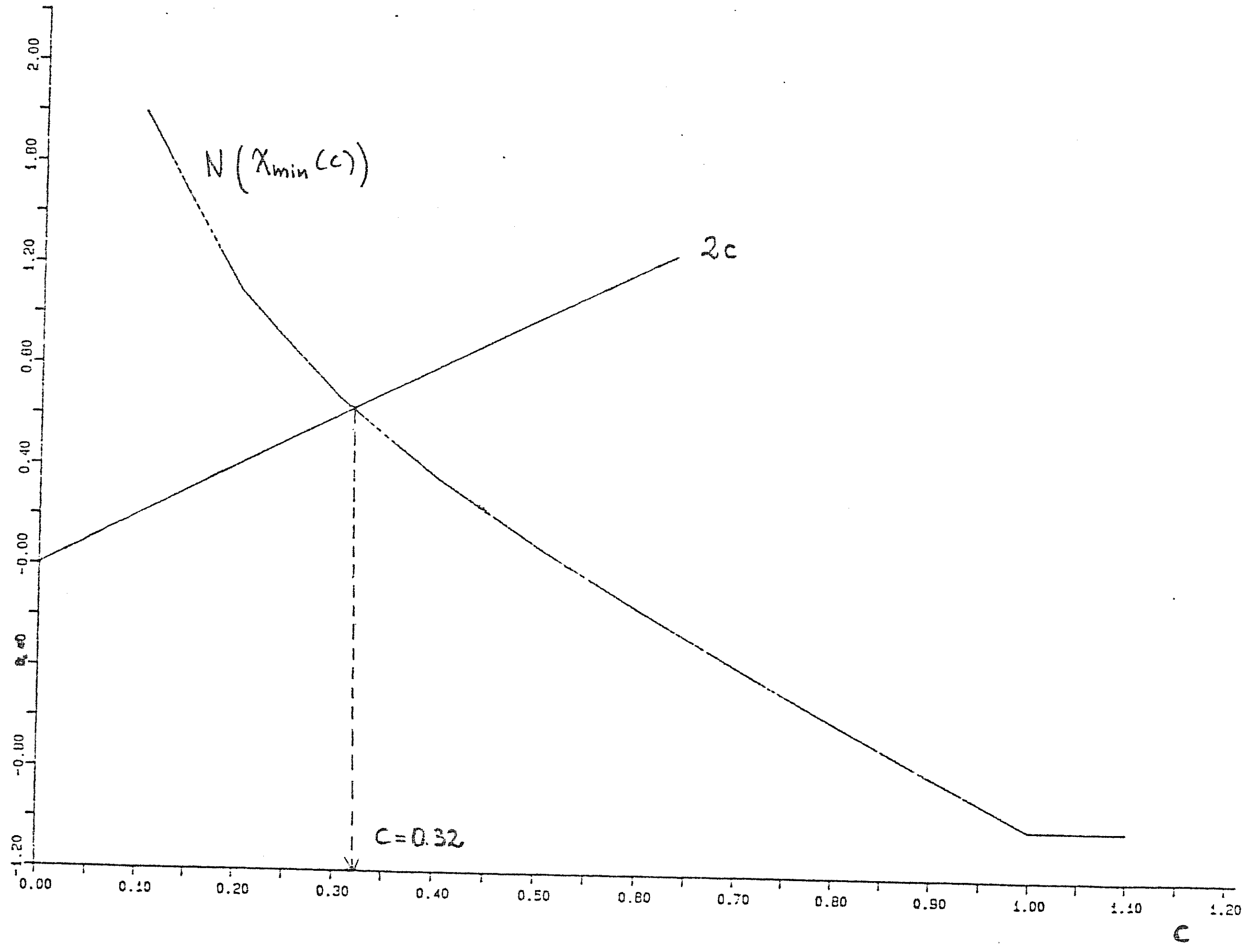

V(χ)
4.00E-06
$\frac{\mu}{\Lambda_{QCD}} = 1.357$
$\frac{\mu}{\Lambda_{QCD}} = 1.355$
$\frac{\mu}{\Lambda_{QCD}} = 1.354$
-.040
-.020
-.000
.020
.04
χ
-1.00E-06

FIG. 6



$N(X_{min}(c))$
$2c$
$C=0.32$
2.00
1.60
1.20
0.80
0.40
-0.00
-0.80
-1.20
0.00
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00
1.10
1.20
c

FIG. 7

REFERENCES

(1) S.Coleman:"Secret Symmetries" in Laws of Hadronic Matter, edited by A.Zichichi (N.Y. 1975)

(2) see for example M.Gell Mann and M.Levy:Nuovo Cimento 16,705(1960) and Y.Nambu and G.Jona-Lasinio:Phys.Rev. 122,345(1961)

(3) J.Goldstone:Nuovo Cimento 19,154(1961)

(4) S.Adler and R.Dashen:"Current Algebra" (N.Y.1968) and V.De Alfaro,S.Fubini,G.Furlan and C.Rossetti:"Currents in Hadron Physics (Amsterdam and London,1973)

(5) R.Casalbuoni,S.De Curtis,D.Dominici and R.Gatto:Phys.Lett. 140B, 228,ib.357(1984) and Phys.Lett.150B,295(1985)

(6) J.M.Cornwall,R.Jackiw and E.Tomboulis:Phys.Rev.D10,2428(1974) for a review see M.Peskin:Lectures at the Les Huoches Summer School (1982)

(7) J.B.Kogut:University of Illinois preprint,ILL-(TH)-85-75(1985)

(8) G.Domokos and P.Suranyi:Sov.J.Nucl.Phys.2,361(1966)

(9) R.Casalbuoni,E.Castellani and S.De Curtis:Phys.Lett.131B,95(1983)

(10) D.J.Gross and A.Neveu:Phys.Rev.D10,3235(1974)

(11) R.Jackiw:Phys.Rew.D9,1686(1974)

(12) P.I.Fomin,V.P.Gusynin,V.A.Miranski and Yu.Sitenko:Rivista del Nuovo Cimento 6,1(1984)

(13) K.Lane:Phys.Rev.D10,2605(1974)

(14) V.A.Miranski:Sov.J.Nucl.Phys.38,280(1983)

(15) H.D.Politzer:Nucl.Phys.B117,397(1976)

(16) C.Bernard,A.Duncan,J.Lo Secco and S.Weinberg:Phys.Rev. D12,792(1985)

(17) D.J.Gross:Lectures at the Les Houches Summer School (1975)

(18) K.Higashijima:Phys.Lett.124B,257(1983)

(19) M.A.Shifman,A.I.Vainstein and V.I.Zakharov:Nucl.Phys.B147, 385(1978)

(20) R.Casalbuoni:Università di Firenze preprint DDF85-21(1985)