SISSA - SCUOLA
INTERNAZIONALE
SUPERIORE
STUDI AVANZATI
TRIESTE
Strada Costiera 11

ISAS - INTERNATIONAL SCHOOL FOR ADVANCED STUDIES

T E S I

DIPLOMA DI PERFEZIONAMENTO

DI

"DOCTOR PHILOSOPHIAE"

SUPERSYMMETRIC GRAND UNIFIED THEORIES

CANDIDATO:

Dott. Zhao Zhi Yong

RELATORE:

Prof. John Strathdee

Anno Accademico 1981/1982

TRIESTE

SUPERSYMMETRIC GRAND UNIFIED THEORIES

TEORIE GRANDI UNIFICATE SUPERSIMMETRICHE

超对称大统一理论

Zhao Zhi Yong

Department of Physics, Peking University

Peking, China

This thesis is submitted for the degree of Philosophiae Doctor of the International School for Advanced Studies in Trieste, Italy

September 1982

THIS THESIS IS DEDICATED

TO

MY LOVELY MOTHERLAND-CHINA,

WHERE I WAS BORN AND LIVE.

CONTERTS

PREFACE

The work presented in this thesis was carried out at the SCUOLA INTERNAZIONALE SUPERIORE DI STUDI AVANZATI (SISSA), Trieste, between November 1980 and July 1982, under the supervison of Prof. J.Strathdee, to whom I am deeply indebted for invaluable advice throughout this period.

It is a pleasure to thank C.S.Aulahk, N.Craigie, J.P.Derendinger, M.Dine, S.Ferrara, G.Furlan, L.Girardello, M.Grisaru, R.Jengo, A.Masiero , A.Namazie, J.C.Pati, N.Pavar, B.D.Peccei, S.Raby, S.Rapoot, Q.Shafi, J.Wess, A.Zee and Zhang Yuan-zhong for various discussions (or private correspondence) on the different subjects in this thesis.

In particular, I am deeply grateful to Prof.A.Salam for helpful advice and suggestions, and Dr.Y.Fujimoto for the many generous discussions concerning this thesis and other matters.

Finally, I would also like to thank Prof.P.Budinich and Prof.L.Fonda for hospitality at SISSA .

"Pure logical thinking cannot yield us any knowledge of the empirical world; all knowledge of reality starts from experiment and ends in it "

— A. Einstein

ABSTRACT

Supersymmetric grand unified theories such as SU(5) and SO(10) model are discussed in this thesis from the point of view of the gauge hierarchy and phenomenology.

INTRODUCTION

For several years, grand unified theories (GUTS) have been providing an appealing way of unifying strong, electromagnetic and weak interactions and dealing with some problems, such as charge quantization, $\mathrm{Sin}^2\theta_w$ and m_b/m_τ as well. Unfortunately, it is clear that there are still many open questions, for instance, the well-known gauge hierarchy puzzle, connected with GUTS.

On the other hand, as it is well-known, supersymmetric theories have a remarkable property, namely, "non-renormalization theorem", which enables us to solve the gauge hierarchy puzzle. Hence, one can imagine that it would be possible to make interesting physics by mixing both ingredients together. This is just a motivation of supersymmetric grand unified theories.

Supersymmetry is Fermi-Bose symmetry, whcih is of course one of the most remarkable ideas in particle physics. Unfortunately, so far we have not yet found where it would be in our understanding of nature. Recently, many physicists have tried to make a realistic model of particle physics through embedding the conventional grand unified theories into N = 1 global supersymmetry, even into local supersymmetry (supergravity). We shall be concerned only in this thesis, with global supersymmetry, namely, supersymmetric grand unified theories, in particular SU(5) and SO(10) SGUM's.

The outline of this article is as follows, First, in Chapter 1, we shall give a very brief introduction to supersymmetry which we shall need later on. Of course, it is not the purpose of this chapter to give sufficient material in order to enable the reader to master the subjects of supersymmetry. A basic knowlegde of both supersymmetric theory and grand unified theory is assumed.

In Chapter 2 we shall deal with supersymmetric grand unified models, such as SU(5) and SO(10) models; in particular, we shall concentrate our attention on the gauge hierarchy problem.Roughly speaking, we shall show how the gauge hierarchy problem is solved in the SU(5), SO(10) SGUM's and O'Raifaartaigh-Witten model, as well as the geometric hierarchy model.

In Chapter 3, we shall discuss supersymmetry breaking. It is obvious that if supersymmetry plays a role in nature it must be broken. As we know, supersymmetry breaking is perhaps the most difficult task. The point is how to break supersymmetry in order to make a realistic model. Several ways to break supersymmetry, such as soft explicit breaking, Fayet-Iliopoulos D-term, O'Raifeartaigh model, will be presented in this chapter. In addition, we shall point out the condition and the mass scale of supersymmetry breaking from the view point of phenomenology.

The problems that we shall tackle in Chapter 4 are then to discuss the phenomenological predictions of supersymmetric grand unified theories, which may involve the weak angle $\mathrm{Sin}^2\theta_w$, quark-lepton mass ratios m_b / m_τ, m_t / m_b, proton decay, CP problem and $N - \bar{N}$ oscillation. Those may yield us more or less a knowledge as to whether supersymmetric grand unified theories is relative to nature.

Finally, some conclusions and appendices will be given at the end of this thesis. My main contribution to this thesis is the supersymmetric SO(10) grand unified model (SO(10)SGUM), in which we have discussed the gauge hierarchy problem and the phenomenological predictions about $\mathrm{Sin}^2\theta_w$, quark-lepton mass ratios, proton decay, $N - \bar{N}$ oscillations.

Chapter 1 INTRODUCTION TO SUPERSYMMETRY

Before the discussion of supersymmetric grand unified models, let us recall some essentials of supersymmetry theories [1.1].

1.1 Notation

Our conventions are as follows:

The metric of space-time is taken to be

$$\eta_{\mu\nu} = \mathrm{diag}\,(+1,-1,-1,-1) \quad . \tag{1.1.1}$$

The Dirac matrices γ_μ satisfy

$$\{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu} \tag{1.1.2}$$

The frequently employed tensor and pseudoscalar combinations are defined by

$$\sigma_{\mu\nu} = \frac{i}{2}\,[\gamma_\mu, \gamma_\nu] \;, \qquad \gamma_5 = \gamma_0\gamma_1\gamma_2\gamma_3 \quad . \tag{1.1.3}$$

The charge conjugation matrix C is defined by

$$C^{-1}\gamma_\mu C = -\gamma_\mu^T \quad . \tag{1.1.4}$$

The Dirac spinor can be decomposed into the two-component spinors in terms of the chiral projections

$$\psi_\pm = \frac{1 \pm i\gamma_5}{2}\,\psi \quad . \tag{1.1.5}$$

The conjugate spinor ψ^c, is defined by

$$\psi^c = C\,\bar{\psi}^T \quad , \tag{1.1.6}$$

in which $\bar{\psi} = \psi^+\gamma_o$.

A Majorana spinor in one for which $\psi^c = \psi$ or, in terms of chiral projections,

$$\psi_\mp = C\,\bar{\psi}_\pm^T \quad . \tag{1.1.7}$$

1.2 Superspace and supertranslations

Superspace is defined by (x_μ, θ_α), in which θ_α are Majorana spinor, i.e.,

$$\{\theta_\alpha, \theta_\beta\} = 0 \quad , \tag{1.2.1}$$

where $\alpha, \beta = 1, 2, \ldots n$, for simple supersymmetry (N = 1 global supersymmetry), n = 4.

Supersymmetry can be understood as an extension of Poincaré symmetry with the following extended Poincaré transformations

$$\begin{aligned} \delta x_\mu &= b_\mu + \omega_{\mu\nu} x_\nu + \frac{i}{2} \bar{\varepsilon} \gamma_\mu \theta \\ \delta \theta &= \varepsilon - \frac{i}{4} \omega_{\mu\nu} \sigma_{\mu\nu} \theta \end{aligned} \quad , \tag{1.2.2}$$

where b_μ and $\omega_{\mu\nu} = - \omega_{\nu\mu}$ are infinitesimal real parameters. ε is an infinitesimal anticommuting Majorana spinor.

The infinitesimal transformation (1.2.2) yields the following algebra

$$\begin{aligned} &[P_\mu, P_\nu] = 0 \\ &[P_\mu, J_{\nu\lambda}] = i(\eta_{\mu\nu} P_\lambda - \eta_{\mu\lambda} P_\nu) \\ &[J_{\kappa\lambda}, J_{\mu\nu}] = i(\eta_{\lambda\mu} J_{\kappa\nu} - \eta_{\kappa\mu} J_{\lambda\mu} + \eta_{\kappa\nu} J_{\lambda\mu} - \eta_{\lambda\nu} J_{\kappa\mu}) \\ &[Q_\alpha, P_\mu] = 0 \\ &[Q_\alpha, J_{\mu\nu}] = \frac{1}{2} (\sigma_{\mu\nu} Q)_\alpha \\ &\{Q_\alpha, Q_\beta\} = -(\gamma_\mu C)_{\alpha\beta} P_\mu \end{aligned} \quad , \tag{1.2.3}$$

where the supertranslation generator Q_α tranforms as a Dirac spinor under Lorentz transformations.

The anticommutator in (1.2.3) can be rewritten as

$$\{ Q_\alpha \bar{Q}^\beta \} = (\gamma_\mu)_\alpha{}^\beta P_\mu \quad , \tag{1.2.4}$$

where $\bar{Q} = Q^+ \gamma_o$ (in unitary representations).

1.3 Superfields.

Superfield is defined to be the real scalar functions $\phi(x, \theta)$ in superspace (x, θ), which is local if it commutes with itself at spacelike separations,

(1.3.1)

This definition of locality is compatible with the usual requirement that Bose fields commute among themselves and with fermionic fields while fermionic fields anticommute among themselves.

Superfield $\Phi(x,\theta)$ may expand in power of θ as follows

$$\Phi(x,\theta) = A(x) + \bar{\theta}\psi(x) + \frac{1}{4}\bar{\theta}\theta F(x) + \frac{1}{4}\bar{\theta}\gamma_5\theta G(x) + \\ + \frac{1}{4}\bar{\theta} i\gamma_\nu\gamma_5\theta V_\nu(x) + \frac{1}{4}\bar{\theta}\theta\bar{\theta}\chi(x) + \frac{1}{32}(\bar{\theta}\theta)^2 D(x) \quad , \tag{1.3.2}$$

where A, F, G and D are scalars, V_ν is a vector, ψ and χ are spinors with respect to the Poincaré group.

Under supertranslation

$$\delta\Phi(x,\theta) = \bar{\varepsilon}\left(\frac{\partial}{\partial\bar{\theta}} + \frac{i}{2}\not{\partial}\theta\right)\Phi(x,\theta) \quad , \tag{1.3.3}$$

one obtains

$$\begin{aligned} \delta A &= \bar{\varepsilon}\psi \\ \delta\psi &= \frac{1}{2}(F + \gamma_5 G + i\gamma_\mu\gamma_5 V_\mu - i\not{\partial}A)\varepsilon \\ \delta F &= \frac{1}{2}\bar{\varepsilon}\chi - \frac{1}{2}\bar{\varepsilon}i\not{\partial}\psi \\ \delta G &= \frac{1}{2}\bar{\varepsilon}\gamma_5\chi + \frac{1}{2}\bar{\varepsilon}i\not{\partial}\gamma_5\psi \\ \delta V_\nu &= \frac{1}{2}\bar{\varepsilon}i\gamma_\nu\gamma_5\chi + \frac{1}{2}\bar{\varepsilon}i\not{\partial}i\gamma_\nu\gamma_5\psi \\ \delta\chi &= \frac{1}{2}(D - i\not{\partial}F + \gamma_5 i\not{\partial}G - i\gamma_\nu\gamma_5 i\not{\partial}V_\nu)\varepsilon \\ \delta D &= -\bar{\varepsilon}i\not{\partial}\chi \ . \end{aligned} \tag{1.3.4}$$

Since reality requires $\Phi^*(x,\theta) = \Phi(x,\theta)$. A, F, G, V and D components are real, ψ and χ are Majorana spinors.

The covariant derivative of Φ, which palys an important role in supersymmetry theories is defined by

$$D\,\Phi(x,\theta) = \left(\frac{\partial}{\partial\bar{\theta}} - \frac{i}{2}\not{\partial}\theta\right)\Phi(x,\theta) \quad . \tag{1.3.5}$$

D is covariant in the sense of transforming like a spinor with respect to homogeneous Lorentz transformations and is invariant with respect to translations and supertranslations, i.e.,

$$[D, J_{\mu\nu}] = \frac{1}{2}\sigma_{\mu\nu} D \quad ,$$
$$[D, P_\mu] = 0 \quad , \qquad (1.3.6)$$
$$\{D_\alpha, Q_\beta\} = 0 \quad .$$

From (1.3.5), it follows that the covariant derivatives generate the following algebra

$$\{D_\alpha, D_\beta\} = -(\gamma_\mu C)_{\alpha\beta}\, i\partial_\mu \quad . \qquad (1.3.7)$$

We can also define the chiral components $D_\pm$ and the conjugate $\bar{D}$,

$$D_\pm = \frac{1 \pm i\gamma_5}{2} D \quad , \qquad (1.3.8)$$

$$\bar{D}^\alpha = (C^{-1})^{\alpha\beta} D_\beta \quad \text{or} \quad \bar{D}_\pm = -D_\mp^T C^{-1} \quad . \qquad (1.3.9)$$

Two kinds of superfields, which are often employed, are the chiral superfield and the vector superfield.

Chiral superfields $\Phi_\pm$ are defined by

$$D_\mp \Phi_\pm = 0 \quad . \qquad (1.3.10)$$

In components $\Phi_\pm(x, \theta)$ can be expressed as

$$\Phi_\pm(x,\theta) = e^{\mp\frac{1}{4}\bar{\theta}\not{\partial}\gamma_5\theta}\left(A_\pm(x) + \bar{\theta}_\mp\psi_\pm(x) + \frac{1}{2}\bar{\theta}_\mp\theta_\pm F_\pm(x)\right) . \qquad (1.3.11)$$

Under supertranslations,

$$\delta A_\pm = \bar{\varepsilon}_\mp \psi_\pm \quad ,$$
$$\delta\psi_\pm = F_\pm \varepsilon_\pm - i\not{\partial} A_\pm \varepsilon_\mp \quad , \qquad (1.3.12)$$
$$\delta F_\pm = -\bar{\varepsilon}_\pm i\not{\partial}\psi_\pm \quad .$$

The vector superfield $V(x,\theta)$ can be defined in terms of the projector E_1 [1.1] acting on a general superfield $\Phi(x,\theta)$,

$$V(x,\theta) = E_1\, \Phi(x,\theta) \quad , \qquad (1.3.13)$$
$$E_1 = 1 + (\bar{D}D)^2/4\partial^2 \quad .$$

Its explicit form in wess-Zumino gauge [1.2] is

$$V(x,\theta) = \frac{1}{4}\bar{\theta}i\gamma_\nu\gamma_5\theta V_\nu(x) + \frac{1}{2\sqrt{2}}\bar{\theta}\theta\bar{\theta}\gamma_5\lambda(x) + \frac{1}{16}(\bar{\theta}\theta)^2 D(x) \quad . \qquad (1.3.14)$$

A crucial property of the chiral scalars in their closure with respect to multiplications

$$\Phi_\pm(x,\theta)\ \Phi'_\pm(x,\theta) = \Phi''_\pm(x,\theta) \quad , \tag{1.3.15}$$

in component form,

$$\begin{aligned} A''_\pm(x) &= A_\pm(x)\,A'_\pm(x) \ , \\ \psi''_\pm(x) &= A_\pm(x)\,\psi'_\pm(x) + \psi_\pm(x)\,A'_\pm(x) \ , \\ F''_\pm(x) &= A_\pm(x)\,F'_\pm(x) + \psi^T_\pm(x)\,C^{-1}\psi'_\pm(x) + F_\pm(x)\,A'_\pm(x) \ . \end{aligned} \tag{1.3.16}$$

In genral, for the anlytic function W(Φ_+) we have

$$W(\Phi_+) = e^{-\frac{1}{4}\bar\theta\gamma\gamma_5\theta}\left\{W(A_+) + \bar\theta_-\psi_+W'(A_+) + \frac{1}{2}\bar\theta_-\theta_+\left[F_+W'(A_+) + \frac{1}{2}\psi^T_+C^{-1}\psi_+W''(A_+)\right]\right\} \ , \tag{1.3.17}$$

where $\quad W'(A_+) = \dfrac{\partial W(A_+)}{\partial A_+}\ , \quad W''(A_+) = \dfrac{\partial^2 W(A_+)}{\partial A_+^2} \quad .$

The similar result may be derived for W(Φ_-).

1.4. Lagrangians

We only draw our attention on the supersymmatric Yang-Mills theories, which we need in constructing supersymmetric grand unified models. The action must be invariant with respect to both the supertranslations and the gauge tranform.

In order to obtain supersymmetric action, we must require $\mathcal{L}$ to transform as a D-component of vector superfield or as $F_\pm$ – component of chiral superfield,since, under supertanslation

$$\begin{aligned} \delta D &= -\partial_\mu(\bar\varepsilon\, i\gamma_\mu \chi) \ , \\ \delta F &= -\partial_\mu(\bar\varepsilon_+ i\gamma_\mu \psi_+) \ . \end{aligned} \tag{1.4.1}$$

One may define the gauge transformations which preserve local chirality

$$\begin{aligned} \Phi_+(x,\theta) &\to e^{i\Lambda_+(x,\theta)}\ \Phi_+(x,\theta) \ , \\ \Lambda_+(x,\theta) &= \Lambda^k_+(x,\theta)\ T^k \ , \end{aligned} \tag{1.4.2}$$

where T^k's are the generators of gauge group G on the Φ_+ bases.

For the sake of making kinetic terms gauge invariant, we introduce a new object $e^{2gV(x,\theta)}$ with the following gauge transformation

$$e^{2gV} \to e^{i\Lambda_+^+} \; e^{2gV} \; e^{-i\Lambda_+} \qquad (1.4.3)$$

It is easy to see that

$$\Phi_+^+ e^{2gV} \Phi_+ = \text{gauge inv.} , \qquad (1.4.4)$$

since under gauge transformations

$$\Phi_+ \to e^{i\Lambda_+} \Phi_+ ,$$
$$\Phi_+^+ \to \Phi_+^+ \; e^{-i\Lambda_+^+} . \qquad (1.4.5)$$

Therefore, the D-term of $\Phi_+^+ e^{2gV} \Phi_+$ is invariant with respect to both gauge transformation and supertranslation.

We will see later that $(\Phi_+^+ e^{2gV} \Phi_+)_D$ contains the kinetic terms of the scalr and the fermion in Φ_+ as well as their gauge couplings.

To construct super-gaugefield strengths, one observes that under gauge transformation

$$e^{-2gV} D_+ e^{2gV} \to \left(e^{i\Lambda_+} e^{-2gV} e^{-i\Lambda_+^+} \right) D_+ \left(e^{i\Lambda_+^+} e^{2gV} e^{-i\Lambda_+} \right) =$$
$$= e^{i\Lambda_+} \left(e^{-2gV} D_+ e^{2gV} \right) e^{-i\Lambda_+} + e^{i\Lambda_+} D_+ e^{-i\Lambda_+} ,$$

here we have used $D_+ \Phi_+^+ = 0$.

in order to eliminate inhomogeneous term, one may employ $\bar{D}_+ D_- = D_- D_-$,

$$D_- D_- \left(e^{i\Lambda_+} D_+ e^{-i\Lambda_+} \right) = e^{i\Lambda_+} D_- D_- D_+ e^{-i\Lambda_+} \sim$$
$$\sim e^{i\Lambda_+} D_- \not{\partial} e^{-i\Lambda_+} = 0 .$$

So the field strength may be defined as the following spinor superfield,

$$W_{++} = -\frac{i}{2\sqrt{2}g} \bar{D}_+ D_- \left(e^{-2gV} D_+ e^{2gV} \right) , \qquad (1.4.6)$$

Actually, under gauge transformation, it appears like a field strength

$$W_{++} \rightarrow e^{i\Lambda_+} W_{++} e^{-i\Lambda_+} \quad . \tag{1.4.7}$$

It is also easy to see that W_{++} is a chiral superfield, since

$$D_- W_{++} = 0 \quad . \tag{1.4.8}$$

Consequently, the F - term of $W^T_{++} C^{-1} W_{++}$ are both supersymmetry and gauge invariant. It follows that the Lagrangian for gauge fields can be written

$$\mathcal{L}_{gauge} = \frac{1}{4} \left(W^{kT}_{++} C^{-1} W^k_{++} \right)_F + h.c. \tag{1.4.9}$$

where W^k_{++}'s is defined by $W_{++} = W^k_{++} T^k$.

Substituting (1.3.14)into (1.4.6), we obtain that in Wess-Zumino gauge W_{++} can be expressed

$$W_{++} = e^{\frac{1}{2}\bar{\theta}_+ i\not\partial \theta_+} \left\{ \lambda_+ + \frac{i}{\sqrt{2}} \left(D + \frac{1}{2} \sigma_{\mu\nu} V_{\mu\nu} \right) + \frac{1}{2} \bar{\theta}_- \theta_+ \left(-i \not\nabla \lambda_- \right) \right\}, \tag{1.4.10}$$

where

$$V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu - ig \left[V_\mu, V_\nu \right] \quad , \tag{1.4.11}$$

$$\nabla_\mu \lambda_- = \partial_\mu \lambda_- - ig \left[V_\mu, \lambda_- \right] \quad . \tag{1.4.12}$$

Then the gauge field Lagrangian reduced to

$$\mathcal{L}_{gauge} = -\frac{1}{4} V^k_{\mu\nu} V^k_{\mu\nu} + \bar{\lambda}^k_- i\not\nabla \lambda^k_- + \frac{1}{2} D^k D^k \quad , \tag{1.4.13}$$

where λ^k_- are the gauge fermions, called gaugino.

The matter field Lagrangian is

$$\mathcal{L}_{matter} = \left\{ \Phi^+_+ e^{2gV} \Phi_+ + \Phi^+_- e^{-2gV} \Phi_- \right\}_F \quad , \tag{1.4.14}$$

which in Wess-Zumino gauge may be reexpressed

$$\begin{aligned} \mathcal{L}_{matter} = {} & \nabla_\mu A^+_+ \nabla_\mu A_+ + \bar{\psi}_+ i\not\nabla \psi_+ + F^+_+ F_+ + (i\sqrt{2} g_k A^+_+ \bar{\lambda}^k_- T^k \psi_+ + h.c.) + \\ & + g_k D^k A^+_+ T^k A_+ + \nabla_\mu A^+_- \nabla_\mu A_- + \bar{\psi}_- i\not\nabla \psi_- + F^+_- F_- + \\ & + (i\sqrt{2} g_k A^+_- \bar{\lambda}^k_+ T^k \psi_- + h.c.) + g_k D^k A^+_- T^k A_- \quad , \end{aligned} \tag{1.4.15}$$

where
$$\nabla_\mu A_\pm = \partial_\mu A_\pm - ig_k V_\mu^k T^k A_\pm \ ,$$
$$\nabla_\mu \psi_\pm = \partial_\mu \psi_\pm - ig_k V_\mu^k T^k \psi_\pm \ . \tag{1.4.16}$$

Finally, we also need to add a gauge-invariant self-interaction among the matter fields Φ_i, which may be easily generated from superpotential $W(\Phi_i)$ [13] being at most a cubic polynomial function of Φ_i for the sake of renormalization,

$$W(\Phi) = a + b_i \Phi_i + \frac{1}{2} m_{ij} \Phi_i \Phi_j + \frac{1}{3} g_{ijk} \Phi_i \Phi_j \Phi_k \ . \tag{1.4.17}$$

From (1.4.17), (1.3.17) and (1.4.15), we find that the part of the Lagrangian which describes tha scalar interactions is

$$\mathcal{L}^F_{scalar} = F_i^+ F_i + F_i \frac{\partial W(A)}{\partial A_i} + F_i^+ \frac{\partial W(A)^+}{\partial A_i} \ , \tag{1.4.18}$$

The equations of motion obtained by varying (1.4.18) with respect to F_i and F_i^+ are

$$F_i^+ = -\frac{\partial W}{\partial A_i} \ , \tag{1.4.19}$$

and their complex conjugates.

Substituting (1.4.19) into (1.4.18, and changing the sign, one obtains the part of the three approximation scalar potentials

$$V_1 = \sum_i \left| \frac{\partial W}{\partial A_i} \right|^2 \ . \tag{1.4.20}$$

From (1.4.13) and (1.4.15), we may pick up another part of the scalar interactions,

$$\mathcal{L}^D_{scalar} = \frac{1}{2}(D^k)^2 + g_k D^k A^+ T^k A \ . \tag{1.4.21}$$

Eliminating the auxiliary field D^k through its equations of motion, (1.4.21) gives a term of the scalar interactions,

$$V_2 = \frac{1}{2}(D^k)^2 = \frac{1}{2}\left(g_k A^+ T^k A\right)^2 \ . \tag{1.4.22}$$

If the gauge group contains U(1) factors, each U(1) factor may contribute to the sum a term of the form

$$\frac{1}{2} D^2 + g_1 D A^+ Y A + \xi D \quad , \tag{1.4.23}$$

where g_1 is the U(1) gauge coupling constant and Y is the U(1) charge of the scalar fields. The ξD term is the Fayet-Iliopoulos term [14]. (1.4.23) gives rise to

$$\frac{1}{2} (g_1 A^+ Y A + \xi)^2 \quad . \tag{1.4.24}$$

Therefore, the complete scalar potential is given by

$$V = \sum_i \left| \frac{\partial W}{\partial A_i} \right|^2 + \frac{1}{2} \sum_\alpha g_\alpha^2 (A^+ T^\alpha A)^2 + + \frac{1}{2} \sum_a (g_1^a A^+ Y^a A + \xi)^2 \quad . \tag{1.4.25}$$

1.5 Mass matrices:

After elimination of the auxiliary fields F_i and D in the Lagrangian (1.4.15), the tree level mass matrices for fermions, scalars and gauge bosons can be caculated as follows:

i) the mass matrix of the fermions ψ , λ is

$$M_{1/2} = \begin{pmatrix} W_{ab} & \sqrt{2} D_{\alpha b} \\ \sqrt{2} D_{\alpha a} & 0 \end{pmatrix} \quad ; \tag{1.5.1}$$

ii) the mass squared matrix of the scalars is

$$M_0^2 = \begin{pmatrix} \bar{W}^{ab} W_{bc} + D^a_\alpha D_{\alpha c} + D^a_{\alpha c} D_\alpha , & \bar{W}^{abc} W_b + D^a_\alpha D^c_\alpha \\ W_{abc} \bar{W}^b + D_{\alpha a} D_{\alpha c} , & W_{ab} \bar{W}^{bc} + D_{\alpha a} D^c_\alpha + D^c_{\alpha a} D_\alpha \end{pmatrix} ; \tag{1.5.2}$$

iii) the mass squared matrix of the gauge bosons is

$$M_1^2 = D_{\alpha a} D_\beta^a + D_\alpha^a D_{\beta a} = g_\alpha g_\beta \langle\phi\rangle^\dagger \{T_\alpha , T_\beta\} \langle\phi\rangle \quad ; \tag{1.5.3}$$

where

$$W_a = \frac{\partial W}{\partial \phi^a}, \quad W_{ab} = \frac{\partial^2 W}{\partial \phi^a \partial \phi^b},$$
$$W_{abc} = \frac{\partial^3 W}{\partial \phi^a \partial \phi^b \partial \phi^c}, \quad \overline{W}^{ab} = (W_{ab})^\dagger, \tag{1.5.4}$$
$$D_{\alpha a} = \frac{\partial D_\alpha}{\partial \phi^a}, \quad D_\alpha^a = \frac{\partial D_\alpha}{\partial \overline{\phi}_a}.$$

Chapter 2: GAUGE HIERARCHY IN SUPERSYMMETRIC GRAND UNIFIED THEORIES

2.1 Gauge hierachy puzzle

Conventional grand unified theories have many hierarchy puzzles [2.1.] that appear to be completely unnatural. For example,

a) the ratios of unification mass scale M_G to weak gauge boson mass M_W, $M_G/M_W \geq 10^{13}$, namely gauge hierarchy;

b) the ratio of M_W to matter fermion mass, such as $M_W/M_e \gtrsim 10^5$. Equivalently, the ratios of gauge coupling g to Yukawa coupling f is about $g/f \gtrsim 10^4$;

c) the ratio of Planck mass M_p to M_W, $M_p/M_W \geq 10^{17}$;

d) the ratios of different generation fermion masses, such as $m_\tau/m_\mu >> 1$, $m_\mu/m_e >> 1$;

e) QCD θ paramete, $\theta^{-1} >> 10^9$.

All above hierachy problems are difficult to solve within the framework of conventional grand unified models. Those require an artificial fine-tuning of parameters in potential. Such fine-tuning is unnatural since there is no-extra-symmetry to protect such fine-tuning from radiative corrections in conventional grand unified theory.

One's attention can now be concentrated on the gauge hierarchy puzzle [2.2]. There are three problems which should be solved in considering gauge-hierarchy. The first is whether it is possible to make an adjustment of the parameters in the Higgs potential so that the desired mass ratio M_X/M_W is generated. The second problem, even if such adjustment is possible, is whether it is natural. The technical meaning of the term natural is that the gauge hierarchy can exist for a finite range of the parameters in the model. Finally, the third problem is the following: can we get gauge hierarchy without the fine-tuning puzzle?

It has been shown [2.2] that a "fine-tuning" of the parameters of the Higgs potential is unstable under radiative corrections, because the smallness of the masses of scalar fields is not "natural". For instance, we consider a theory with the gauge symmetry SU(2)xU(1) that may be completely broken by two scalr Higgs fields, a doublet $\Phi = (\phi_0, \phi_-)$ and a singlet χ. The potential would appear:

$$V=-\frac{1}{2}\mu_1^2\varphi^\dagger\varphi+\frac{1}{4}\lambda_1(\varphi^\dagger\varphi)^2-\frac{1}{2}\mu_2^2\chi^*\chi+\frac{1}{4}\lambda_2(\chi^*\chi)^2+\frac{1}{2}\lambda_3(\varphi^\dagger\varphi)(\chi^*\chi) \quad . \tag{2.1.1}$$

The minimum of the above potential that introduces a gauge hierarchy is given by

$$\langle\varphi_-\rangle=0 \;, \quad \lambda_1 F^2+\lambda_3 f^2=\mu_1^2 \;, \quad \lambda_3 F^2+\lambda_2 f^2=\mu_2^2 \tag{2.1.2}$$

where $\langle\varphi^2\rangle = F^2$, $\langle\chi^2\rangle = f^2$.

This would for $\lambda_1\lambda_2-\chi_3^2\neq 0$ yield

$$\frac{F^2}{f^2}=\frac{\lambda_2^2\,\mu_1^2-\lambda_3\,\mu_2^2}{\lambda_1\,\mu_2^2-\lambda_3\,\mu_1^2} \quad . \tag{2.1.3}$$

Due to symmetry breaking, three gauge bosons gains masses $O(F^2)$ and one picks up a mass $O(f^2)$ for $F^2 >> f^2$ as argued by Gildner, the gauge hierarchy depends critically upon the value of λ_3, which, due to one loop corrections can be given only with a presession of $O(g^4)$. Thus the radiative corrections would be important for gauge hierarchy. In fact, because of this, it has been shown [2.2] that there would be a limit to the hierarchy, $M_H^2/M_L^2 < O(\alpha^{-1})$ at one loop level, where M_H is high mass scale, at which SU(2)xU(1) breaks down to U(1), and M_L is low mass scale, at which U(1) is completely broken.

If one does the perturbation by using the physical parameters, masses of the various physical fields and couplings instead of the parameters of Higgs potential, μ_1, μ_2, λ_1, λ_2 and λ, one may also have large radiative corrections to the smaller masses that would spoil the light-heavy distinction. The low $(\text{mass})^2$ M_L^2 would get a correction of $O(g^2 M_H^2)$ at one loop level. The point is that the gauge hierarchy puzzle comes from the Higgs scalars. As 't Hooft has pointed out [2.3], in the case the parameter tends to zero, the theory acquires an extra symmetry. For instance, ordinary QED acquires a chiral symmetry when a fermion mass vanishes. Gauge symmetry can protect vector boson massless. Unfortunately, there is no such a extra symmetry in conventional theories to protect scalar boson massless.

So far we have found two possible solutions. The first is to assume that there are no elemenetary scalar fields and have their role taken over by billinear fermion condensates. This is namely the technicolour picture (TC) [2.4]. Unfortunately, TC is not sufficient to give masses to the fermions and it is necessary to enlarge the scheme by introducing Extended technicolour (ETC) [2.4] which runs into difficulties with flavour changing neutral currents and possibly with pseudo-Goldstone bosons. No viable phenomenological scheme has been proposed along this line in my knowledge.

The second solution is supersymmetry that is only extra symmetry so far known which can put a constraint on the mass of scalar particles .

The reason is that supersymmetric theories have a remarkable property: non-renormalization theorem[2.5],which guarantee that the light Higgs sector does not mix with the heavy one. This gives rise to some hope of soving the gauge hierarchy problem.

2.2 Non-renormalization theorem:

recently, many people have been interested in extending grand unified models SU(5), SO(10), SO(14) to supersymmetric ones, namely supersymmetric grand unified model. The main motivation is that supersymmetric grand unified models may provide a solution to the gauge hierarchy problem. It is well-known that supersymmetry iS Fermi-Bose symmetry. In supersymmetry theories, particles occur in supermultiplets, the number of Fermi degrees of freedom being always equal to the number of Bose degrees of freedom in the supermultiplet. For instance, in chiral supermultiplet, one has both a Majorana fermion and a complex scalar. In supersymmetry theories scalars occur naturally as members of supermultiplets and their masses can be naturally small if their fermionic partners have small masses protected by chiral invariance. In supersymmetric grand unified model, there are cancellations of infinities among Feynman graphs with fermion loops and graphs with boson loops so they are more convergent than conventional grand unified theories [2.5] . Furthermore, it has been prooved that[2.5]supersymmetry theory has fewer renormalization constants than non-supersymmetry theory, namely only a wave function renormalization for each chiral supermultiplet and for each vector supermultiplet, a renormalization constant for each gauge coupling. No separate mass, Yukawa or Higgs coupling renormalization constants are need.These results can be easily understood from the following well-known "non-renormalization therem"[7]:

To any finite order of perturbation theory, only D-term can receive radiative corrections, F-term can not.

This theorem can be easily prooved by means of supergraph technique (see Ref.2.5). Here we only give some explantion being concerned with the gauge hierarchy problem.

In a supersymmetric grand unified theory one can only construct two different supersymmetric invariant operators, one of them is D-term, which is obtained by integrating Lorentz invariant vector superfield $\Psi(x,\theta,\bar{\theta})$ over all superspace

$$O_1 = \int d^8z\ \Psi(x,\theta,\bar{\theta}) \equiv \int d^4x\, d^2\theta\, d^2\bar{\theta}\ \Psi(x,\theta,\bar{\theta}) \tag{2.2.1}$$

Another term is F-term, which is obtained by integrating Lorentz invariant chiral superfield $\Phi(x,\theta)$, which is a function of x and θ only, over x and θ

$$O_2 = \int d^6z\ \Phi(x,\theta) \equiv \int d^4x\, d^2\theta\ \Phi(x,\theta), \tag{2.2.2}$$

where θ and $\bar{\theta}$ are the anticommuting coordinates of negative positive chirality. It is obvious that any invariant which can be written in the form O_1 can also be written in the form O_2. In fact, if one defines

$$\Phi(x,\theta) = \int d^2\bar{\theta}\ \Psi(x,\theta,\bar{\theta}) \tag{2.2.3}$$

O_1 can be reexpressed

$$O_1 = \int d^6z\ \Phi(x,\theta) \quad . \tag{2.2.4}$$

this is nothing but just F-term. Indeed, there exist a type of supersymmetric invariants in susy theories, which can be written in the O_2 form but cannot be written in the O_1 form. For instance, all mass terms, all Yukawa couplings and all scalar couplings in renormalizable supersymmetric theories come from F-terms which cannot be written in O_1 form. In contrast , the kinetic energy terms, both for chiral superfields and for vector superfields, and gauge coupling terms, can be written in the O_1 form, i.e, D-term .

It follows that wave function renormalization occurs for both chiral and vector superfields, because the kinetic energy operators are D-terms. There is no separate mass, Yukawa and scalar couplings renormalization, because they come from F-terms. As a consequence there are no radiative corrections to the tree level vacuum expectation values of the scalar fields. This makes supersymmetric grand unified theories an obvious candidate for the solution of the gauge hierarchy problem.

So far two kinds of supersymmetric grand unified models have been presented :

a). Supersymmetry is broken explicitly but softly through dimension 2 or 3 term. In such a kind of theories gauge hierarchy occurs only by fine-tuning of parameters at tree level [2.6] . Due to non-renormalization theorem, such fine-tuning is natural at the technical sence that they will not spoilt by radiative corrections, since supersymmetry protects them from radiative corrections. So we only need fine-tuning once! This is different from fine-tuning made in non-supersymmetric grand unified theory. It is well-known that, in non-supersymmetric theory, such a fine-tuning will unfortunately be spoilt by radiative corrections [2.2] .

b). Supersymmetry is spontaneously broken at tree level. Gauge hierarchy occurs from radiative corrections: Witten's mechanism [2.3]. The key point of this mechanism is that the small mass scale M_W is assumed to be fundamental, the unification mass scale M_G being generated and based on radiative corrections. Along this line, recently Dimopoulos and Raby et al. have presented a geometric hierarchy model [2.8] in which supersymmetry is spontaneously broken at an intermediate mass scale, say $M_S \sim 10^{12}$ Gev, then both unification mass scale M_G and small mass scale M_W are generated from radiative corrections.

2.3 SU(5) supersymmetric grand unified model

Firstly, one briefly reviews the conventional SU(5) grand unified model, in which all matter fermions(quarks and leptons) of each generation have been signed in $\underline{5}^*$ and $\underline{10}$ representations as follows

$$\underset{\underline{5}^*}{\psi_i} = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ -\nu_e \end{pmatrix}_L , \qquad \underset{\underline{10}}{\psi^{[i,j]}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & u_3^c & -u_2^c & -u^1 & -d^1 \\ -u_3^c & 0 & u_1^c & -u^2 & -d^2 \\ u_2^c & -u_1^c & 0 & -u^3 & -d^3 \\ u^1 & u^2 & u^3 & 0 & -e^+ \\ d^1 & d^2 & d^3 & e^+ & 0 \end{pmatrix} . \qquad (2.3.1)$$

This model is anomaly free, since the anomalies of $\underline{5}^*$ and $\underline{10}$ representation exactly cancel each other.

Because SU(5) has 24 generators, there are 24 gauge bosons, which have been put in $\underline{24}$ adjoint representations. The covariant derivative D_μ is defined by

$$D_\mu = \partial_\mu - i g_\alpha T^\alpha A^\alpha_\mu \tag{2.3.2}$$

where T^α's are SU(5) generators, which obey SU(5) algebra

$$[T^\alpha, T^\beta] = i f^{\alpha\beta\gamma} T^\gamma , \quad \alpha,\beta,\gamma = 1,\cdots,24 , \tag{2.3.3}$$

in which $f^{\alpha\beta\gamma}$ are SU(5) structure constants.

g is SU(5) gauge coupling constant.

In order to break SU(5) down to $SU(3)_C xSU(2)_L xU(2)_Y$, further to $SU(3)_C xU(1)_{e.m}$ one needs a $\underline{24}$ and a $\underline{5}$ Higgs field.

$$\begin{aligned} &\phi^i_j \leftrightarrow \underline{24} , \quad \text{with } \phi^i_i = 0 , \quad i,j = 1 \text{ to } 5 \\ &\phi^i \leftrightarrow \underline{5} . \end{aligned} \tag{2.3.4}$$

ϕ^i_j may develop non vanishing expectation vacuum value as follows

$$\langle \phi^i_j \rangle = v \, \mathrm{diag}\,(1,1,1,-\tfrac{3}{2},-\tfrac{3}{2}) \tag{2.3.5}$$

which makes SU(5) break down to $SU(3)_C xSU(2)_L xU(1)_Y$ at mass scale M_G. M_G has been estimated at about 10^{15} Gev.

ϕ^i can receive non vanishing expectation vacuum value

$$\langle \phi^i \rangle = v' \delta^{i5} , \tag{2.3.6}$$

which breaks $SU(3)_C xSU(2)_L xU(1)_Y$ down to $SU(3)_C xU(1)_{e.m}$ at mass scale M_W, $M_W \sim 10^2$ Gev. The matter fermions may acquire mass through Yukawa coupling

$$\epsilon_{ijklm} \psi^{ij} \psi^{kl} \langle \phi^m \rangle \tag{2.3.7}$$

We now turn to the construction of the SU(5) supersymmetric grand unified model [2.6] , in which supersymmetry is explicitly but softly broken in order to protect the Higgs doublets from quadratic mass renormalization. This model requires a natural but incredibly accurate adjustment of parameters. The minimal SU(5) supersymmetric grand unified model should include the following supermultiplets :

Matter chiral supermultiplets:

$$
\begin{aligned}
&\underline{5}^{*}_{\alpha} \leftrightarrow M_{\alpha i} && i,j=1,\cdots,5 \\
&\underline{10}_{\alpha} \leftrightarrow M_{\alpha}^{[i,j]} = -M_{\alpha}^{[j,i]} && \alpha = e,\mu,\tau,\cdots
\end{aligned}
\tag{2.3.8}
$$

in which α is a family index, i, j are SU(5) indices, a vector supermultiplet which contains gauge bosons,

$$\underline{24} \leftrightarrow V^{i}_{j} \tag{2.3.9}$$

Higgs chiral supermultiplets :

$$\underline{24} \leftrightarrow \Sigma^{i}_{j}\ ,\quad \underline{5} \leftrightarrow H^{i} \quad \text{and} \quad \underline{5}^{*} \leftrightarrow H'_{i} \tag{2.3.10}$$

The $\underline{24}$ Higgs Σ^{i}_{j} will develop non vanishing VEV and break SU(5).

The scalar potential V is a sum of two terms. One term related by supersymmetry to the Yukawa couplings. this term may be easily obtained by introducing a new function, called the superpotential W. The superpotential W is a function of the scalar fields Φ's, but not of their complex conjugates Φ *'s. For a renormalizable theory W should be at the most cubic in the Φ 's. W is also restricted by gauge invariance. The corresponding contribution to the scalar potential is

$$V_1 = \sum_i \left| \frac{\partial W}{\partial \phi_i} \right|^2 , \tag{2.3.11}$$

where the sum is taken over all scalar fields.

The second term in the scalar potential comes from the gauge couplings. Let us define the functions usually called D-term [1.1]

$$D_\alpha = g_\alpha \, \phi^\dagger T^\alpha \phi \qquad \text{no sum on } \alpha . \qquad (2.3.12)$$

where T^α are the generators of the gauge group acting on the scalar representation. Then the corresponding term in the scalar potential coming from the gauge couplings is

$$V_2 = \frac{1}{2}\sum_\alpha D_\alpha^2 \qquad (2.3.13)$$

One therefore obtains the total scalar potential V

$$V = V_1 + V_2 = \sum_i \left| \frac{\partial W}{\partial \phi_i} \right|^2 + \frac{1}{2}\sum_\alpha D_\alpha^2 . \qquad (2.3.14)$$

If V vanishes when evaluated at the expectation value of the scalar fields, supersymmetry is unbroken. Because V_1 and V_2 are each non-negative, they must both be zero, as a consequence , the condition for supersymmetry to be unbroken at the tree level is that for each a and each i, we require

$$\frac{\partial W}{\partial \phi_i} = 0 , \qquad (2.3.15)$$

$$D_\alpha = 0 . \qquad (2.3.16)$$

If these equations have a simultaneous solution, supersymmetry is unbroken at the tree level. Otherwise, supersymmetry is spontaneously broken .

In the minimum SU(5) SGUM, the superpotential W is given by

$$W = \lambda_1 \left(\frac{1}{3} \Sigma^i_j \Sigma^j_k \Sigma^k_i + \frac{m}{2} \Sigma^i_j \Sigma^j_i \right) + \lambda_2 H'_i \left(\Sigma^i_j + 3m' \delta^i_j \right) H^j + $$
$$+ f_{\alpha\beta} \, \epsilon_{ijklm} H^i M^{jk}_\alpha M^{lm}_\beta + g_{\alpha\beta} H'_i M^{ij}_\alpha M_{\beta j} . \qquad (2.3.17)$$

Here we use same notation for both superfield and its scalar component . It follows that

$$\frac{\partial W}{\partial \Sigma^j_i} = \lambda_1 \left\{ \Sigma^i_k \Sigma^k_j - \frac{1}{5} \delta^i_j \operatorname{Tr} \Sigma^2 + m \Sigma^i_j \right\} + \lambda_2 \left(H'_j H^i - \frac{1}{5} \delta^i_j H'_k H^k \right) , \qquad (2.3.18)$$

$$\frac{\partial W}{\partial H^j} = \lambda_2 H'_i (\Sigma^i_j + 3m' \delta^i_j) + f_{\alpha\beta} \epsilon_{jiklm} M^{ik}_\alpha M^{lm}_\beta , \quad (2.3.19)$$

$$\frac{\partial W}{\partial H'_i} = \lambda_2 (\Sigma^i_j + 3m' \delta^i_j) H^j + g_{\alpha\beta} M^{ij}_\alpha M_{\beta j} , \quad (2.3.20)$$

$$\frac{\partial W}{\partial M^{jk}_\alpha} = 4 f_{\alpha\beta} \epsilon_{ijklm} H^i M^{lm}_\beta + g_{\alpha\beta} (H'_j M_{\beta k} - H'_k M_{\beta j}), \quad (2.3.21)$$

$$\frac{\partial W}{\partial M_{\beta j}} = g_{\alpha\beta} H'_i M^{ij}_\alpha \quad (2.3.22)$$

It is obvious that the potential V is minimized for

$$\langle H^i \rangle = \langle H'_i \rangle = \langle M^{ij}_\alpha \rangle = \langle M_{\alpha i} \rangle = 0 \quad (2.3.23)$$

and

$$\Sigma^i_k \Sigma^k_j - \frac{1}{5} \delta^i_j \, \mathrm{Tr}\, \Sigma^2 + m \Sigma^i_j = 0 \quad . \quad (2.3.24)$$

By SU(5) rotations we can choose the representation in which the <u>24</u> adjoint Higgs < Σ > is diagonal

$$\langle \Sigma^i_j \rangle = \mathrm{diag} (x_1, x_2, x_3, x_4, x_5) , \quad \sum_{i=1}^{5} x_i = 0 \quad . \quad (2.3.25)$$

Then one obtains the following equations

$$\left(x_i^2 - \frac{1}{5} \sum_{j=1}^{5} x_j^2 \right) + m x_i = 0 \quad . \quad (2.3.26)$$

Three solutions for VEV of Σ Higgs turn out to be

a) $$\langle \Sigma^i_j \rangle = 0 , \quad (2.3.27)$$

b) $$\langle \Sigma^i_j \rangle = \frac{m}{3} \mathrm{diag} (1, 1, 1, 1, -4) , \quad (2.3.28)$$

c) $$\langle \Sigma^i_j \rangle = 2m \, \mathrm{diag} (1, 1, 1, -\tfrac{3}{2}, -\tfrac{3}{2}) \quad . \quad (2.3.29)$$

Furthermore, it is easy to check that for the above VEV's all D_α vanish. So supersymmetry is not broken at tree level, but gauge group SU(5) may spontaneously be broken. Solution (a) does not break SU(5); solution (b) breaks SU(5) down to SU(4)xU(1). Finally, solution (c) breaks SU(5) down to $SU(3)_C \times SU(2)_L \times U(1)_Y$. These three physically inequivalent vacua are completely degenerate, as shown in fig. 1.

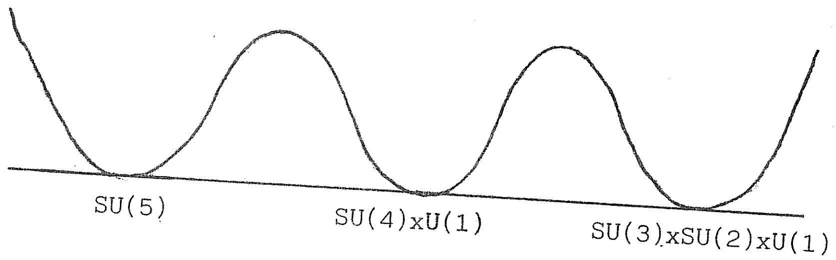


FIG. 1

This is a common situation in supersymmetric grand unified theories. The attractive solution is (c), which breaks SU(5) down to standard model $SU(3)_C \times SU(2)_L \times U(1)_Y$. The $SU(3)_C \times SU(2)_L \times U(1)_Y$ constants of $\underline{24}$ Higgs Σ are given by

$$\begin{array}{cccccccl} 24 = & (8,1) & + (1,3) & + (1,1) & + (3,2) & + (\bar{3},2) & & i,j = 1 \text{ to } 5, \\ \Sigma^i_j & & \Sigma^x_y & \Sigma^a_b & \delta^x_y \Sigma & \Sigma^x_a & \Sigma^a_x & x,y = 1,2,3, \\ & & & & & & & a,b = 4,5. \end{array} \tag{2.3.30}$$

We find that (3,2) and ($\bar{3}$,2) Higgs are massless Goldstone bosons and eaten by (3,2) and ($\bar{3}$,2) gauge bosons, which acquire masses of order m. On the other hand, (8,1), (1,3) and (1,1) Higgs bosons in $\underline{24}$ multiplet have mass squared = $6\lambda_1 m^2$.

For $\underline{5}$ and $\underline{5}^*$ Higgs bosons, the $SU(3)_C \times SU(2)_L \times U(1)_Y$ constants are

$$\begin{array}{llll} 5 = (3,1) & + (1,2) & & \\ H^i \quad H^x & H^a & & i=1 \text{ to } 5 \end{array} \tag{2.3.31}$$

$$\begin{array}{llll} 5^* = (\bar{3},1) & + (1,2) & & x=1,2,3 \\ H'_i \quad H'_x & H'_a & & a=4,5 \end{array} \tag{2.3.32}$$

As we can see from eq.(2.3.17) and (2.3.29), the SU(2) doublets Higgs H and H' have a mass squared which is proportional to $(m - m')^2$, whereas the colour triplets H^x and H'_x have mass of order $(2m+3m')$. The doublet should be very light, nearly massless, i.e. O(m), since their VEV's break $SU(3)_C \times SU(2)_L \times U(1)_Y$ down to $SU(3)_C \times U(1)_{em}$ at low mass scale M_W ($\sim$ 100 Gev). However, the colour triplets should be superheavy, otherwise they may cause the proton to decay too fast through Fig. 2

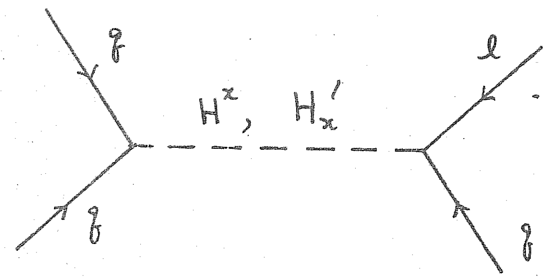


FIG. 2

Actually, if we set m' exactly equal to m, i.e.

$$m' - m = O(M_G) \tag{2.3.33}$$

then the $SU(2)_L$ doublets H^a, H'_a are massless. Whereas the colour triplets H^x, H'_x have mass of order m. According to non-renormalization theorem, such a fine-tuning is different from those we have made in the non supersymmetric case, because the fine-tuning in supersymmetric theory will not be spoilt by radiative correction. So we only need such a fine-tuning once at tree level. Precisely, the ratio m'/m is not renormalized at all, because

$$m'_R = \left(Z_{H'}^{\frac{1}{2}} Z_H^{\frac{1}{2}} Z_\Sigma^{\frac{1}{2}} / Z_{H'}^{\frac{1}{2}} Z_H^{\frac{1}{2}}\right) m'_0 = Z_\Sigma^{\frac{1}{2}} m'_0 ,$$

$$m_R = \frac{(Z_\Sigma^{\frac{1}{2}})^3}{(Z_\Sigma^{\frac{1}{2}})^2} m_0 = Z_\Sigma^{\frac{1}{2}} m_0 ,$$

therefore $$m'_R/m_R = m'_0/m_0 \quad ,$$

where $Z_{H'}^{-1/2}$, $Z_{H}^{-1/2}$, and $Z_{\Sigma}^{-1/2}$ denote the wave function renormalization of H', H and Σ multiplets respectively. This implies that the SU(2) doublet would exactly remain massless to all orders if supersymmetry is unbroken. However, supersymmetry should be broken spontaneously or explicitly, but softly at smaller mass scale, M_W , then the doublet may acquire a small mass of order M_W from radiative corrections, but cannot become superheavy through radiative correction.

2.4 SO(10) supersymmetric grand unified model

In this section we discuss SO(10) supersymmetric grand unified model [2.10]. SO(10) is a rank 5 group and contains SU(5) as a subgroup. As well-known, SO(10) has many advantages over the SU(5). for instance, all representations of the SO(10) group are safe, so we can put all light fermions (quarks and leptons) of each generation into a 16 spinor representation and Higgs fields into any irreducible representation as we wish, without worrying about Adler-Bell-Jahiw anomalies [2.11] . In addition, the SO(10) group has more attractive subgroups. Such as SU(5), SO(6)xSO(4) [$\sim$ SU(4)xSU(2)xSU(2)] , SU(3)xSU(2)xxU(1), SU(3)xSU(2)xU(1), thus SO(10) has more abundant patterns than SU(5) has. An attractive pattern is that SO(10) is broken down to $SU(3)_c \times U(1)_{e.m.}$ via $SU(4)_C \times SU(2)_L \times SU(2)_R$ (i.e. Pati-Salam model) [2.12] . SO(10) grand unified theory has left-right symmetry. So, in principle, we can have right neutrino, and we can discuss neutrino mass, neutrino-antineutrino as well as neutron-antineutron oscillations.

We now just follow the line of Dimopoulos and Geogi SU(5) SGUM; in order to construct a SO(10) supersymmetric grand unified model for solving the gauge hierarchy problem, the fine-tuning of parameters are needed.

2.4.1 SO(10) algebra [2.13]

SO(10) has 45 generators $M_{ij} = -M_{ji}$ $i \neq j = 1, \ldots, 10$, which satisfy

$$[M_{ij}, M_{k\ell}] = i(\delta_{ik}M_{j\ell} + \delta_{j\ell}M_{ik} - \delta_{i\ell}M_{jk} - \delta_{jk}M_{i\ell}) \qquad (2.4.1)$$

M_{ij}'s are antisymmetric. In terms of the generalized Dirac γ matrices of dimension $2^5 \times 2^5 = 32 \times 32$, M_{ij} can be expressed as follows

$$M_{ij} = \frac{1}{4i}(\Gamma_i\Gamma_j - \Gamma_j\Gamma_i), \qquad i,j = 1 \text{ to } 10, \qquad (2.4.2)$$

Γ_i satisfy the Clifford algebra

$$\{\Gamma_i, \Gamma_j\} = 2\delta_{ij} \qquad (2.4.3)$$

in the 10-dimensional Euclidean space.

In order to discuss SO(10) SGUM, it is convenient that we work in the following explicit representation of generalized Dirac matrices [2.13],[2.14]

$$\begin{aligned}
\Gamma_1 &= \sigma_1 \otimes \sigma_1 \otimes 1 \otimes 1 \otimes \sigma_2 \\
\Gamma_2 &= \sigma_1 \otimes \sigma_2 \otimes 1 \otimes \sigma_3 \otimes \sigma_2 \\
\Gamma_3 &= \sigma_1 \otimes \sigma_1 \otimes 1 \otimes \sigma_2 \otimes \sigma_3 \\
\Gamma_4 &= \sigma_1 \otimes \sigma_2 \otimes 1 \otimes \sigma_2 \otimes 1 \\
\Gamma_5 &= \sigma_1 \otimes \sigma_1 \otimes 1 \otimes \sigma_2 \otimes \sigma_1 \\
\Gamma_6 &= \sigma_1 \otimes \sigma_2 \otimes 1 \otimes \sigma_1 \otimes \sigma_2 \\
\Gamma_7 &= \sigma_1 \otimes \sigma_3 \otimes \sigma_1 \otimes 1 \otimes 1 \\
\Gamma_8 &= \sigma_1 \otimes \sigma_3 \otimes \sigma_2 \otimes 1 \otimes 1 \\
\Gamma_9 &= \sigma_1 \otimes \sigma_3 \otimes \sigma_3 \otimes 1 \otimes 1 \\
\Gamma_{10} &= \sigma_2 \otimes 1 \otimes 1 \otimes 1 \otimes 1 ,
\end{aligned} \qquad (2.4.4)$$

where σ_i (i = 1, 2, 3) are the usual Pauli matrices, i.e.,

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{2.4.5}$$

and I is the 2x2 identity matrix.

The chirality operator can be defined in terms of the Γ's (i = 1, .. 10) as

$$\Gamma_{11} = (-i)^5 \, \Gamma_1 \Gamma_2 \cdots \Gamma_{10} \quad . \tag{2.4.6}$$

Γ_{11} is a matrix of the block diagonal form, which can be written

$$\Gamma_{11} = \sigma_3 \otimes 1 \otimes 1 \otimes 1 \otimes 1 \tag{2.4.7}$$

by using the explicit Γ's matrices (2.4.4).

In SO(N) group there are two different types of representations, one of which is the spinor representation, the other is the vector representation. For SO(10) group, the fundamental spinor representation ψ is $2^5 = 32$ dimensional and decomposes into two inequivalent irreducible spinors (ψ_+, ψ_-) of dimension 16 by means of projection operators

$$\frac{1}{2} \left(1 \pm \Gamma_{11} \right) \tag{2.4.8}$$

i.e.,

$$\psi_\pm = \frac{1}{2} \left(1 \pm \Gamma_{11} \right) \psi \tag{2.4.9}$$

where I is 32x32 identity matrix.

This decomposition of Ψ into $\Psi_\pm$ is possible because Γ_{11} commutes with all the generators M_{ij} of SO(10).

SO(10) is rank five Lie group. The diagonal generators are given by the matrices σ_{12}, σ_{34}, σ_{56}, σ_{78} and σ_{910}, where $\sigma_{ij} = 2M_{ij}$.

Explicitly,

$$
\begin{aligned}
\sigma_{12} &= 1\otimes\sigma_3\otimes 1\otimes\sigma_3\otimes 1 \\
\sigma_{34} &= 1\otimes\sigma_3\times 1\otimes 1\otimes\sigma_3 \\
\sigma_{56} &= 1\otimes\sigma_3\otimes 1\otimes\sigma_3\otimes\sigma_3 \\
\sigma_{78} &= 1\otimes 1\otimes\sigma_3\otimes 1\otimes 1 \\
\sigma_{9\,10} &= \sigma_3\otimes\sigma_3\otimes\sigma_3\otimes 1\otimes 1 .
\end{aligned}
\tag{2.4.10}
$$

The electro-charge operator is

$$
\begin{aligned}
Q &= \frac{1}{6}(\sigma_{12}+\sigma_{34}-\sigma_{56})+\frac{1}{2}\sigma_{78} \\
&= \frac{1}{3}(M_{12}+M_{34}-M_{56})+M_{78} .
\end{aligned}
\tag{2.4.11}
$$

All quarks and leptons of each family are put in a <u>16</u> spinor representation as follows:

$$
\psi^T=(u_1\ u_2\ u_3\ \nu\ d_1\ d_2\ d_3\ e^-\ d_1^c\ d_2^c\ d_3^c\ e^+\ -u_1^c\ -u_2^c\ -u_3^c\ -\nu^c)_L . \tag{2.4.12}
$$

As we know, the above fermions (except ν^C) have been put in 5^*+ 10 representations in Georgi-Glashow SU(5) model.

Before discussing the gauge hierachy in SO(10) supersymmetric grand unified model, it is useful to recall some formulas about Kroenecker production and branching rule of SO(10) group.

1. Kroenecker products:

$16\times 16 = 10 + 120 + 126$

$16\times\overline{16} = 1 + 45 + 210$

$10\times 10 = 1 + 45 + 54$

$16\times 10 = 16 + 144$

$10\times 45 = 10 + 120 + 320$

$10\times 54 = 10 + 120' + 320$

$16\times 45 = 16 + 144 + 560$

$16\times 54 = 144 + 720$

$45\times 45 = 1 + 45 + 54 + 210 + 770 + 945$

$54\times 54 = 1 + 45 + 54 + 660 + 770 + 1386$

2. Branching rules :

SO(10) has many attractive subgroups, the most important of which are SU(5), $SU(4)_C \times SU(2)_L \times SU(2)_R$ and $SU(3)_C \times SU(2)_L \times U(2)_Y$,

a) $SO(10) \rightarrow SU(5)$:

$$\begin{aligned} 10 &= 5 + 5^* \\ 16 &= 1 + 5^* + 10 \\ 45 &= 24 + 10 + 10^* + 1 \\ 54 &= 24 + 15 + 15^* \\ 120 &= 45 + 45^* + 10 + 10^* + 5 + 5^* \\ 210 &= 75 + 40 + 40^* + 24 + 10 + 10^* + 5 + 5^* + 1 \end{aligned} \tag{2.4.13}$$

b) $SO(10) \rightarrow SU(4)_C \times SU(2)_L \times SU(2)_R$

$$\begin{aligned} 10 &= (6,1,1) + (1,2,2) \\ 16 &= (4,2,1) + (4^*,1,2) \\ 45 &= (15,1,1) + (1,3,1) + (1,1,3) + (6,2,2) \\ 54 &= (20,1,1) + (6,2,2) + (1,3,3) + (1,1,1) \\ 210 &= (15,1,1) + (20,2,2) + (15,3,1) + (15,1,3) + \\ &\quad + (6,2,2) + (1,1,1) \end{aligned} \tag{2.4.14}$$

c) $SO(10) \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y$

$$\begin{aligned} 10 &= (3,1)_{-\frac{2}{3}} + (3^*,1)_{\frac{2}{3}} + (1,2)_1 + (1,2)_{-1} \\ 16 &= (3,2)_{\frac{1}{3}} + (3^*,1)_{\frac{2}{3}} + (3^*,1)_{-\frac{4}{3}} + (1,2)_{-1} + (1,1)_2 + (1,1)_0 \\ 45 &= (8,1)_0 + (1,3)_0 + (1,1)_0 + (3^*,2)_{\frac{5}{3}} + (3,2)_{-\frac{5}{3}} + (3^*,2)_{-\frac{1}{3}} + (3,2)_{\frac{1}{3}} + \\ &\quad + (3^*,1)_{-\frac{4}{3}} + (3,1)_{\frac{4}{3}} + (1,1)_2 + (1,1)_0 + (1,1)_{-2} \\ 54 &= (8,1)_1 + (6,1)_{-\frac{1}{3}} + (3^*,1)_{-\frac{5}{3}} + (3^*,1)_{-\frac{1}{3}} + (3^*,2)_{\frac{5}{3}} + \\ &\quad + (3^*,2)_{-\frac{1}{3}} + (3,2)_{-\frac{5}{3}} + (3,2)_{\frac{1}{3}} + (1,3)_2 + (1,3)_0 + \\ &\quad + (1,3)_{-2} + (1,1)_0 \end{aligned} \tag{2.4.15}$$

where the subscript label denotes the quantum number Y.

2.4.2 Gauge hierarchy in SO(10) supersymmetric grand unified model.

Along the line of Dimopoulos-Georgi SU(5) model, one can make a SO(10) supersymmetric grans unified model in which the gauge hierarchy problem could be solved and the colour triplet Higgs is guaranted to be superheavy so that proton decay is not too fast, while the doublet Higgs, whcih may give masses to the light fermions at low energy, are light.

We have found that the possibility of solving the above problem on the basis of SO(10) is dependent on the breaking pattern. The one possible solution has been found [2.10] if SO(10) is broken down to $SU(3)_C xU(1)_{e.m.}$ via $SU(4)_C xSU(2)_L xSU(2)_R$.

The breaking chain is

$$SO(10) \xrightarrow[M_1]{54} SU(4)_C xSU(2)_L xSU(2)_R \xrightarrow[M_2]{16,16^*} SU(3)_C xSU(2)_L xU(1)_Y \xrightarrow[M_W]{10,10'} SU(3)_C xU(1)_{em}$$

As for the breaking of SO(10) down to $SU(4)_C xSU(2)_L xSU(2)_R$, we need, say, 54 Higgs, and for breaking $SU(4)_C xSU(2)_L xSU(2)_R$ down to $SU(3)_C xSU(2)_L xU(1)_Y$, one choses 16 and 16* Higgs. The need of both 16 and 16* Higgs is to guarantee the D-terms to be vanishing [2.15].

It has been proved [2.15] that if a group G breaks down to a little group g_z by the possible v.e.v.'s, Z, and the coset G/g_z does not contain any generator of G group transforming as a singlet under the little group g_z, then all D (z) = 0.

As well known, there are two generators Y and B-L in the coset $SU(4)_C xSU(2)_L xSU(2)_R$ / $Su(3)_C xSU(2)_L xU(1)_Y$ which are the singlets under the little group $SU(3)_C xSU(2)_L xU(1)_Y$. Therefore, the corresponding D-terms could probably be non-vanishing, if we only use a 16 Higgs for the breaking of $SU(4)_C xSU(2)_L xSU(2)_R$ down to $SU(3)_C xSU(2)_L xU(1)_Y$. However, we can take both a 16 and a 16* Higgs, the D-terms obviously vanish if the v.e.v.'s in 16 and 16* Higgs fields have the same little group and the same strength (in detail see eqs. (2.4.34), (2.4.35).

Furthermore, in order to give masses to quarks and leptons at least one of 10, 120 and 126 Higgs fields is required, since

$$16 \times 16 = 10 + 120 + 126 \quad . \tag{2.4.16}$$

It should be emphasized that for the sake of making a second hierarchy, i.e. the colour triplet Higgs superheavy but the doublet Higgs massless , we need two 10 Higgs fields. Consequently, the simple supersymmetric extension of the SO(10) grand unifed model must include the following superfields.

$$\begin{array}{ll} \underline{16} \text{ matter chiral superfiled} & \Phi_+ = (A_+, \psi_+, F_+) \\ \underline{45} \text{ gauge vector superfield} & \Psi_+ = (W_\mu, \lambda_+, D) \\ \underline{54} \text{ Higgs chiral superfield} & \Sigma_+ = (a_+, \zeta_+, f_+) \\ 16 \text{ Higgs chiral superfield} & N_+ = (b_+, \eta_+, h_+) \\ 16^* \text{Higgs chiral superfield} & N^*_+ = (b^*_+, \eta^*_+, h^*_+) \\ 10 \text{ Higgs chiral superfield} & \Omega_+ = (H_+, \xi_+, G_+) \\ 10 \text{ Higgs chiral superfield} & \Omega'_+ = (H'_+, \xi'_+, G'_+) \end{array} \tag{2.4.17}$$

The superspace potential W, which is at most cubic in a renormalization theory, is given by

$$W = f_1 AHA + f_2 (bH'b + b^* H' b^*) + mHH' + \\ + \lambda H a H' + \frac{1}{2} \mu a a + \frac{1}{3} k\, a a a \quad . \tag{2.4.18}$$

This superspace potential W is not only supesymmetric and SO(10) invariant, but also compatible with a global symmetry of

$$A \to e^{i\frac{\alpha}{2}} A, \quad b \to e^{-i\frac{\alpha}{2}} b, \quad b^* \to e^{-i\frac{\alpha}{2}} b^*, \quad H \to e^{-i\alpha} H, \quad H' \to e^{i\alpha} H' . \tag{2.4.19}$$

Such a global U(1) symmetry will be broken at M_2 mass scale, $M_2 \sim 10^{13}$ Gev. Thus, the strong CP problem can be avoided with invisible axion [2.16] . It is perhaps worth noticing that in supersymmetric theory we do not always need a symmetry reason to forbid an allowed term in the superpotential: if it is excluded by hand at tree level, the non-renormalization theorem guarantees that it will not be generated by loop diagrams.

The scalar potential V is

$$V = \sum_i \left| \frac{\partial W}{\partial \phi_i} \right|^2 + \frac{1}{2} D_{ij}^2 \quad , \tag{2.4.20}$$

in which φ_i stand for all scalar fields A, b, b^*, H, H' and a. The $D_{ij}=-D_{ji}$ (i, j = 1 to 10) is defined by

$$D_{ij} = g \varphi^{\dagger} M_{ij} \varphi \tag{2.4.21}$$

where M_{ij} denote the matrices of SO(10) generators on the φ fields. M_{ij} are anti-symmetric,i.e, $M_{ij} = - M_{ji}$, and obey the algebra (2.4.1). The vacuum expectation values of the Higgs fields are determined by minimizing V. If V=0 at some VEV's of the Higgs fields, supersymmetry of course is not broken. This will occur if and only if the following equations

$$\frac{\partial W}{\partial \varphi_i} = 0 \qquad \text{and} \qquad D_{ij} = 0 \tag{2.4.22}$$

have a simultaneous solution. Otherwise, supersymmetry is broken.

One requires that SO(10) gauge symmetry will be broken down to $SU(4)_C \times SU(2)_L \times SU(2)_R$, and futher to $SU(3)_C \times SU(2)_L \times U(1)_Y$ at high energy ,and finaly down to $SU(3)_C \times U(1)_{em}$ at the weak mass scale M_W ($\sim$100 Gev). We assume that supersymmetry is also broken at the weak mass scale M_W. For the first stage of SO(10) breaking, namely SO(10) down to $SU(4)_C \times SU(2)_L \times SU(2)_R$,all the D-terms vanish, since the breaking is achieved by the <u>54</u> Higgs. As we can see from Eq.(2.4.14) , <u>54</u> representation of SO(10) only contains one singlet of the little group $SU(4)_C \times SU(2)_L \times SU(2)_R$, so the coset SO(10)/ $SU(4)_C \times SU(2)_L \times SU(2)_R$ does not contain any generator being a singlet of the little group $SU(4)_C \times SU(2)_L \times SU(2)_R$.In addition, <u>16</u> and <u>10</u> representations of SO(10) do not contain any singlet of $SU(4)_C \times SU(2)_L \times SU(2)_R$. Their VEV's may arise at the second , or third stage of breaking,i.e, $SU(4)_C \times SU(2)_L \times SU(2)_R \rightarrow SU(3)_C \times SU(2)_L \times U(1)_Y$, $SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_{em}$ respectively.

From the superpotential (2.4.18) one may derive

$$\frac{\partial W}{\partial \bar{A}} = f_1 HA ,$$
$$\frac{\partial W}{\partial \bar{b}} = f_2 H' b ,$$
$$\frac{\partial W}{\partial \bar{b}^*} = f_2 H' b^* ,$$
$$\frac{\partial W}{\partial \bar{H}_i} = f_1 A \Gamma_{i+} A + m H_i' + \lambda a_{ij} H_j' , \tag{2.4.23}$$
$$\frac{\partial W}{\partial \bar{H}_i'} = f_2 (b \Gamma_{i+} b + b^* \Gamma_{i+} b^*) + m H_i + \lambda H_j a_{ji} ,$$
$$\frac{\partial W}{\partial a_{ij}} = \lambda (H_i H_j' + H_j H_i') + \mu a_{ij} + h \left(a_{ik} a_{kj} - \frac{1}{10} \delta_{ij} \, \mathrm{Tr}\, a^2 \right) ,$$

The VEV's of the Higgs fields are given by

$$\langle A \rangle = \langle H \rangle = \langle H' \rangle = 0 ,$$
$$b \Gamma_+ b = b^* \Gamma_+ b^* = 0 ,$$

and
$$\mu a_{ij} + h \left(a_{ik} a_{kj} - \frac{1}{10} \delta_{ij} a_{kl} a_{kl} \right) = 0 , \tag{2.4.24}$$
$$a_{ij} = a_{ji} , \quad \sum_i a_{ii} = 0 ,$$

where Γ_{i+} (i=1 to 10) are defined by

$$\Gamma_{i+} = \frac{1 + \Gamma_{11}}{2} \Gamma_i . \tag{2.4.25}$$

The five solutions of Eq.(2.4.24) turn out to be

1) $\langle a_{ij} \rangle = 0$ (2.4.26)

2) $\langle a_{ij} \rangle = \frac{1}{8} \frac{\mu}{h} \mathrm{diag} (1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ {-9})$ (2.4.27)

3) $\langle a_{ij} \rangle = \frac{1}{3} \frac{\mu}{h} \mathrm{diag} (1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ {-4}\ {-4})$ (2.4.28)

4) $\langle a_{ij} \rangle = \frac{3}{4} \frac{\mu}{h} \mathrm{diag} (1\ 1\ 1\ 1\ 1\ 1\ 1\ {-\frac{7}{3}}\ {-\frac{7}{3}}\ {-\frac{7}{3}})$ (2.4.29)

5) $\langle a_{ij} \rangle = \frac{2\mu}{h} \mathrm{diag} (1\ 1\ 1\ 1\ 1\ 1\ {-\frac{3}{2}}\ {-\frac{3}{2}}\ {-\frac{3}{2}}\ {-\frac{3}{2}})$ (2.4.30)

All the above states are degenerate at $E = 0$ as shown in Fig.3. It is obvious that in the case (1), SO(10) remains unbroken ; in the case (2), the residual unbroken symmetry is SO(9); in the case (3), it is $SO(8)\times SO(2)$; in the case (4), it is $SO(7)\times SO(3)$; finily, in the case (5), it is $SO(6)\times SO(4) \simeq SU(4)_C \times SU(2)_L \times SU(2)_R$.

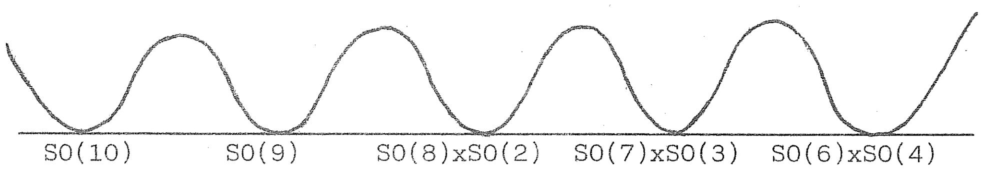


Fig.3 the five vacuum states in the SO(10) SGUM are degenerate at E=0 .

The case (5) is the most interesting one. However, we have no potential reason to choose the vacuum (5) rather than the others. Perhaps it would be possible to split the degenerate by means of embedding the SO(10)SGUM into supergravity, but we do not know whether the vacuum (5) is the physical one, by which means the lowest energy vacuum state .

We now draw attention to the interesting case (5), and show how to solve the gauge hierarchy problem by fine-tunig and how to make the colour triplet Higgs ,which may cause proton decay (see Fig.2),superheavy, but the SU(2) doublets light. From Eq.(2.4.15) one finds that all of $\underline{54}$,$\underline{16}$,$\underline{16}$, and $\underline{10}$ contain the colour triplets,however, the colour triplets in $\underline{54}$ Higgs do not cause too fast proton decay, since they are as heavy as the unification mass scale .It does not matter how heavy the masses of the colour triplets in $\underline{16}$,$\underline{16}^*$ are, as far as the proton decay concerned,beacuse there are no the trilinear $\underline{16}$x$\underline{16}$x$\underline{16}$ and $\underline{16}$x$\underline{16}^*$x$\underline{16}$ couplings due to the fact of $\underline{16}$ and $\underline{16}$* being spinor of SO(10). In concequence, the only dangerous colour triplets are those in the $\underline{10}$ Higgs multiplets.However, they can become superheavy by means of the fine- tuning of parameters in the potencil.

First of all, let us write down the contents of the $\underline{10}$ representation in $SU(4)_C\times SU(2)_L\times SU(2)_R$:

$$10 = (6,1,1) + (1,2,2) \qquad (2.4.31)$$

The colour triplets are in (6,1,1) , whereas the $SU(2)_L$ doublets in (1,2,2) . It is easy to see from Eq.(2.4.18) and (2.4.30) that the mass of the doublet is different from one of the colour triplet. We therefore can make an adjustment of the parameters in our model so that the doublet is massless , but the colour triplet is superheavy. Actually, for the case (5), the mass of the doublet Higgs is proportional to $m-3\mu\lambda/h$,so if one sets $m=3\mu\lambda/h$, they will be massless, whereas the colour triplet acquires a mass of the order of $5\mu\lambda/h$, which is just the mass scale of SO(10) down to $SU(4)_C xSU(2)_L xSU(2)_R$. This means that the colour triplet in 10 Higgs multiplet is also superheavy.

We now turn to consider the breaking of $SU(4)_C xSU(2)_L xSU(2)_R$ down to $SU(3)_C xSU(2)_L xU(1)_Y$ at a middle mass scale M_2 ($\sim 10^{12}$ Gev). This can be done by 16 and 16* Higgs fields, since the $SU(3)_C xSU(2)_L xU(1)_Y$ contents of 16 and 16* Higgs fields are given by

$$\begin{aligned} 16 &= (3,2)_{\frac{1}{3}} + (\bar{3},1)_{\frac{2}{3}} + (\bar{3},1)_{-\frac{4}{3}} + (1,2)_{-1} + (1,1)_{2} + (1,1)_{0} \\ 16^* &= (\bar{3},2)_{-\frac{1}{3}} + (3,1)_{-\frac{2}{3}} + (3,1)_{\frac{4}{3}} + (1,2)_{1} + (1,1)_{-2} + (1,1)_{0} \ , \end{aligned} \qquad (2.4.32)$$

where the subscript label denotes the weak-hypercharge Y. The neutral components $(1,1)_0$ (i.e, the $\nu^c (\nu)$ components) in 16 (16*) Higgs generate the breaking of $SU(4)_C xSU(2)_L xSU(2)_R$ to $SU(3)_C xSU(2)_L xU(1)_Y$.

It is easy to check that all F-terms vanish.However, we must say some thing about the D-terms: For $U(1)_Y$, it is obvious that

$$D_Y = 0 \ , \qquad (2.4.33)$$

because Y of $\nu^c (\nu)$ component in 16 (16*) is zero. For $U(1)_{B-L}$, the corresponding generator is $T_o = \frac{1}{2}\sqrt{\frac{3}{2}}(B-L)$, one then obtains

$$D_{B-L} = \left(\frac{1}{2}\sqrt{\frac{3}{2}}\right)^2 g^2 \left(|v|^2 - |v'|^2 \right) \qquad (2.4.34)$$

where v (v') is the VEV of the $(1,1)_0$ component in 16 (16*).

Consequently, we may get that

$$D_{B-L} = 0 \,, \qquad \text{if} \quad v = \pm v' . \tag{2.4.35}$$

All other D-terms are obviously vanishing. Therefore, supersymmetry still remains unbroken when SO(10) is broken down to $SU(3)_C xSU(2)_L xU(1)_Y$. We assume that supersymmetry would be explicitly or dynamicaly broken at the weak mass scale M_w, for instance, we can explicitly break supersymmetry in terms of the mass term of the <u>10</u> Higgs scalars. Such the breaking is soft as the dimension of the breaking term being two (see chapter 3). The last stage of breaking ,i.e, $SU(3)_C xSU(2)_L xU(1)_Y$ down to $SU(3)_C xU(1)_Y$, can be achieved by the VEV of the neutral component of the $SU(2)_L$ doublets in <u>10</u> Higgs multiplet.

Finally, it would be worthwhile to point out that the supersymmetric partners of matter fermions can not be a candidate for the Higgs fields responded to the breaking of supersymmetry or gauge symmtry , since they may lead to the " light fermion disaster"[2.17]due to the mixing of the light fermions and the gauginos ,and give rise to too far too much bayon number violation[2.18]. To see how this happen , as an example, let us discuss SU(5) supersymmetric model:

In SU(5)SGUM the supermultiplets containing the quark and lepton fields are as follows

$$M^{ij}_{\alpha} = - M^{ij}_{\alpha} \,, \qquad M'_{\alpha i} \,, \tag{2.4.36}$$

where i,j = 1 to 5, are the SU(5) indeces, α is the generation index ($\alpha = e, \mu, \tau$). M^{ij}_{α} set to <u>10</u> representation of SU(5), $M'_{\alpha i}$ set to <u>5*</u> representation. The $SU(3)_C xSU(2)_L xU(1)_Y$ symmetry is embedded such that the components of the <u>5</u> ($\bar{M}'^{i}_{e}$ say) correspond to quarks and leptons as follows :

$$\bar{M}'^{4}_{e} = d^{a} \,, \qquad \text{for } a = 1,2,3 \tag{2.4.37}$$

where a= 1 to 3 are the three colours, and

$$\bar{M}'^{4}_{e} = e^{+} \,, \qquad \bar{M}^{5}_{e} = \bar{\nu}_{e} \,. \tag{2.4.38}$$

If one makes the identification of Higgs scalars with the lepton partners, and the ν_μ component of $M'_{\mu i}$ gets a VEV, then the following SU(5) invariant term in the potencial

$$M_e^{ij} M'_{ei} M'_{\mu j} \tag{2.4.39}$$

may make a difficulty as concerns proton decay, because the couping of the triplet components of $M'_{\mu j}$, i.e, the SU(5) partners of the putative Higgs can madiate proton decay. Precisely, the Higgs triplet $M'_{\mu a}$ (a=1,2,3) can couple to (ignoring Cabibbo-KM mixing)

$$M_e^{ab} M'_{eb} + M_e^{a4} M'_{e4} + M_e^{a5} M'_{e5} = f^{abc} \bar{u}_b \bar{d}_c + u^a e^- + d^a \nu_e \quad . \tag{2.4.40}$$

This couping obviously produces proton decay.

<u>Conclution:</u>

It has been shown that the gauge hierarchy problem can be solved in supersymmetric grand unified models, in particular for SU(5)SGUM and , as we have demonstrated, for SO(10)SGUM. The second hierarchy problem, namely, the very small ratio of masses of the $SU(2)_L$ doublet Higgs and the associated colour triplet Higgs, can also be solved by means of an incradibly accurate, but "natural" adjustment of the parameters in the potencial. Such adjustments, usually calld fine-tunning, are quite different from those in non-supersymmetric GUM's in the sence that the fine-tunning one has made in supersymmetric GUM's will not disturbed by radiative corrections. This is because of the non-renormalization property of supersymmetry threoy. However, how to understand zeroth order fine-tuning in supersymmetric GUM's is still a problem. The question is : can we naturally solve the gauge hierarchy problem without any fine-tuning ?

2.5 Witten's mechanism

Along the lines of O'Raifeartaigh's model (see chapter 3), Witten has suggested a novel mechanism which may naturally yield a large ratio of gauge symmetry breaking parameters in a class of field-theories with spontaneous broken supersymmetry. The key idea is that in a theory characterized by a small mass scale $M \sim 1$ Tev, radiative corrections may produce large vacuum expectetion values (VEV's) of the order of $e^{1/\alpha}M$ in the fields whose VEV's can not determined at tree approximation. This means that the small mass scale is fundamental, the large one being generated dynamically.

Before going on to construct a realistic model, let us consider a simple example to explain Witten's mechanism:

The superpotential W of O'Raifeartaigh model is

$$W = \lambda X\,(A^2 - M^2) + g\,Y\,A^2 \quad , \tag{2.5.1}$$

which has a relevant global symmetry of $A \rightarrow -A$, $X \rightarrow e^{i} X$ and $Y \rightarrow e^{i} Y$.

From (2.5.1) we get the scalar potential

$$V = \lambda^2\,|A^2 - M^2|^2 + g^2\,|A|^2 + |2\lambda AX + gY|^2 \quad , \tag{2.5.2}$$

and

$$-F^*_A = \frac{\partial W}{\partial A} = 2(\lambda X + gY)A \tag{2.5.3a}$$

$$-F^*_X = \frac{\partial W}{\partial X} = \lambda(A^2 - M^2) \tag{2.5.3b}$$

$$-F^*_Y = \frac{\partial W}{\partial Y} = gA^2 \tag{2.5.3c}$$

The last two expressions (2.5.3b) and (2.5.3c) can not both vanish, so supersymmetry is spontaneously broken at tree level. Minimizing the potential V one finds that $A = 0$ or $A = (M^2 - g^2/2\lambda^2)^{1/2}$ denpending on whether $g/\lambda M$ is large or small. Besides, one also finds that both X and Y are undetermined at tree approximation, so long as $Y = -2\lambda A X/g$.

One of the attractive features of the Witten's model is that the A particles get large masses, $2\lambda X$, this theory therefore can have at tree level an arbitrarily large mass scale, totally unrelated to parameters in the Lagragian. Such A particles in any realistic model are not involved in the low energy symmetry breaking. However, it has been shown [2.20] that X and Y can be determined by the radiative corrections to the potencil. The one-loop corrections to the potential was carried out to be [2.21]

$$\Delta V_{1\text{-loop}} = \sum_i \frac{(-)^{2s}\hbar}{64\pi^2} M_i^4(\phi) \ln M_i^2(\phi)/\mu^2 \quad , \tag{2.5.4}$$

where the sum runs over all helicity states, M_i is the mass of the i-th such state, the s is the spin of the correponding state, the factor $(-1)^{2s}$ indicates that boson (scale and gauge boson) makes a positive contribution and fermion makes a negative contribution to the effective potencil, and μ is a renormalization mass.

The complex scalar A field is split by supersymmetry breaking into two real components, of which the mass squared are given by

$$m_B^2(A) = 4\lambda^2 |X|^2 + 2g^2 \pm 2M^2 \quad . \tag{2.5.5}$$

Their fermion partners have the mass squared

$$m_F^2(A) = 4\lambda^2 |X|^2 + 2g^2 \quad . \tag{2.5.6}$$

Having substituted the expressions (2.5.5) and (2.5.6) into (2.5.4), one gets

$$V = V_{tree} + V_{1\text{-loop}} = \lambda^2 M^4 \left[1 + \frac{\lambda^2}{8\pi^2} \ln \frac{|X|^2}{\mu^2} \right] \quad . \tag{2.5.7}$$

The above potential obviously dependents on X. As one can see from (2.5.7), the coefficient of the logarithm term is positive, which implies that X can not become arbitrarily large. However this is not the case in gauge theories. Witten observed that in gauge theories the logarithm term can have a negative coefficient. At one-loop level, actually, the potential in threories with gauge fields is found to be

$$V = a\lambda^4 M^4 \left\{ 1 + (b\lambda^2 - ce^2)\ln\frac{|X|^2}{\mu^2} \right\} , \qquad (2.5.8)$$

where λ is a scalar coupling, e is the gauge coupling constant, a,b and c are the calculative model-dependent positive constants. If $b\lambda^2 - ce^2 < 0$, X would seem become large without limit,and V becomes negative for large X. As we know, however, the energy in any globally supersymmetry threory can never become negative (see chapter 3). This suggest that the perturbation theory should break down at $e^2\ln X \sim 1$. If a stable minimum of V develops in the regime $e^2\ln X \gtrsim 1$, one may obtain a theory with an exponentially large mass scale characterized by $X \sim Me^{1/\alpha}$ and a mass scale M at which supersymmetry breaking takes place. The above mass scales can be interpreted as the scales of the grand unification and of weak interaction respectively. Indeed, by using the renormalization group analysis it has been found that a stable vacumm exist at a large X in some models such as SU(5) model. This is because of the asymptotic freedom which may force the effective e^2 to decrease at large X so that the effective coefficient $(b\lambda^2(t) - ce^2(t))$ can change sign when X becomes very large,however if λ is decreasing or not increasing too rapidly. This could stop the tendency for X from increasing and produce a stable vacuum.

Let us now see how the above idea is implemented in a simple SU(5) model. This model is constructed from two complex scalars A,Y in <u>24</u> representations

$$(A^i{}_j)^* = (A^+)^j{}_i , \quad (Y^i{}_j)^* = (Y^+)^j{}_i ,$$
$$\mathrm{Tr}\,A = \mathrm{Tr}\,Y = 0 , \qquad (2.5.9)$$

and a singlet X.

The superpotential W is given by

$$W(A,Y,X) = f\,\mathrm{Tr}\,A^2 Y + g X(\mathrm{Tr}\,A^2 - M^2) , \qquad (2.5.10)$$

which is invariant under $A \to -A$, $X \to e^{i\alpha}X$ and $Y \to e^{i\alpha}Y$. It follows that

$$-F_A^{\dagger} = f\{A,Y\} + 2gXA - \frac{2}{5} f \,\mathrm{Tr}\, AY \quad , \tag{2.5.11a}$$

$$-F_Y^{\dagger} = f A^2 - \frac{1}{5} f \,\mathrm{Tr}\, A^2 \quad , \tag{2.5.11b}$$

$$-F_X^{*} = g(\mathrm{Tr}\, A^2 - M^2) \quad . \tag{2.5.11c}$$

Since the equations $F_Y = 0$ and $F_X = 0$ are inconsistent for non-vanishing values of f and g , supersymmetry is spontaneously broken. The scalar potential is

$$V = \mathrm{Tr}\, F_A^{\dagger} F_A + \mathrm{Tr}\, F_Y^{\dagger} F_Y + |F_X|^2 + \\ + \frac{1}{4} e^2 \,\mathrm{Tr}\left([A,A^{\dagger}] + [Y,Y^{\dagger}]\right)^2 . \tag{5.12}$$

Minimizing the potential , we find

$$A = \frac{gM}{\sqrt{f^2+30g^2}} \,\mathrm{diag}\,(2\;2\;2\;{-3}\;{-3}) \quad , \tag{2.5.13}$$

but X is undetermined at this level and can be assumed to take any value so long as

$$Y = \frac{g}{f} X \,\mathrm{diag}\,(2\;2\;2\;{-3}\;{-3}) \quad . \tag{2.5.14}$$

The vacuum energy turns out to be

$$V_0 = \frac{f^2}{f^2+30g^2} g^2 M^4 \quad . \tag{2.5.15}$$

We see from the expressions (2.5.13) and (2.5.14) that the residual unbroken gauge symmetry is $SU(3)_C \times SU(2)_L \times U(1)_Y$.

Since X is undetermined at tree level , the quantum effects must be taken into account. The effective potential up to one-loop approximation is given by [2.22

$$V(X) = \frac{f^2 g^2 M^4}{f^2+30g^2}\left\{1+\frac{\hbar}{8\pi^2}\frac{3g^2}{f^2+30g^2}(29f^2-50e^2)\ln\frac{|X|^2}{M^2}\right\}. \quad (2.5.16)$$

Indeed, (2.5.16) implies a runaway behavior for $29f^2-50e^2<0$, namely, the X increases without limit so that vacuum energy becomes negative and arbitrary large,which is of course unwarrented. As mentioned before, this means that the perturbation theory breaks down when $|(29f^2-50e^2)\ \ln X^2/\mu^2\ | \gtrsim 1$. However, Since in SU(5) model the gauge coupling e is asymptotically free but the scalar coupling f is not[2.23], the effective coupling will make the coefficient $29\bar{f}^2(t)-50\bar{e}^2(t)$ to change sign as $X\to\infty$. So one then expects a stable vacuum to develop in the large X regime.

In this model, SU(5) is strontly broken by the expectation value of Y and the massive gauge bosons have masses of the order of X, whereas supersymmetry breaking is characterized by the smaller mass M as the vacuum energy being of the order of M . the heavy particles, for example, the $SU(3)_C \times SU(2)_L$ singlet components of A, Y and X may have the masses of the order of M and have Bose-Fermi degeneracy . Various particles which are massless to this order can only get masses smaller than M from higher-order corrections.

Conclusion:

The main advantage of Witten's mechanism is that the gauge hierarchy can naturally occur without any fine-tuning problem. Supersymmetry is spontaneously broken by using an O'Raifeataigh -type model. The new idea è là Witten is that the small mass scale $M\sim 1$ Tev ,which coresponts the mass scale of supersymmetry breaking, is fundamental, then the radiative corrections may produce large VEV's (of the order $M\ e^{1/\alpha}$) in the X fields , whose VEV's can not determined at tree level. This large VEV, X , is reasonably interpreted as the unification mass scale M_G . In other words, the low -mass symmetry breaking drives the high-mass symmetry breaking, but the large VEV's can leave a larger unbroken gauge symmetry then the small VEV's do, thereby producing a gauge hierarchy .

In Witten's SU(5) model, the supersymmetry breaking mass scale M was assumed to be the same order of the weak interaction mass scale M_W. However, as one will point out in chapter 3 , supersymmetry may be broken at an intemediate mass scale , say $M_I \sim 10^{12}$ Gev [2.23] . In such a case, there exist at least three different mass scales , namely a unification mass scale M_G , a supersymmetry breaking mass scale M_I , and the weak mass scale M_W. In consequence, a natural question is that , along the line of the Witten's idea, can we make such a model so that only the supersymmetry breaking mass scale M_I is fundamental , whereas both the unification mass scale M_G and the weak interaction mass scale M_W are generated from radiative corrections.

2.6 Geometric hierarchy :

Very recently Dimopoulous and Raby have presented an attractive model, called Geometric hierarchy[2.8], in which they apply the Witten's mechanism and assume that supersymmetry is broken at an intermediate mass scale , $M_I \sim 10^{12}$ Gev, and both grand unification mass scale M_G and the weak mass scale M_w are generated from radiative corrections. Acturly they have found that $M_G \sim M_I\ e^{1/\alpha}$,and $M_w \sim f\ M_I^2/M_G$, where f is a function of both gauge and Yukawa couplings. Since M_I is approximately the geometric mean of M_G and M_w, they named it Germetric hierarchy.

This model looks very complicted since it contains too many supermultiplets, which may be needed for the breakings of supersymmetry, of SU(5) down to $SU(3)_C xSU(2)_L xU(1)_Y$,futher down to $SU(3)_C xU(1)_{em}$,for giving mass to both up quarks, down quarks and leptons,for avoiding the rapid proton decay,and so on. The superpotantial W in this model can be roughty divited into two parts: W_1 and W_2, in which, roughly speaking, W_1 only included the heavy sector and W_2 only the light sector.

$$W_1 = \lambda_1 \mathrm{Tr}(ZA^2) + \lambda_2 X(\mathrm{Tr}A^2 - M^2) + \\ + \lambda_3(\bar{H}_1 A H + \bar{H} A H_1) + \lambda_4(\bar{H}_1 H + \bar{H} H_1) B \quad , \tag{2.6.1}$$

where A and Z are $\underline{24}$ supermultiplets, X and B are SU(5) singlets, H, H_1 and $\bar{H}, \bar{H}_1$ are $\underline{5}$ and $\underline{5}^*$ supermultiplets, respectively. It is obvious that the first two terms in W_1 is exactly the O'Rafeartaigh-Witten model which drives supersymmetry spontaneouly breaking at tree level, the last two terms are necessary in order to guarantte no rapid proton decay and the right breaking pattern of SU(5), namely SU(5) to $SU(3)_C \times SU(2)_L \times U(1)_Y$.

W_2 is given by

$$W_2 = \lambda_5 Y (\bar{H}_1 H_1 + \eta C^2 + M' B - \tilde{M}^2) + \\ + g^u_{ij} H\, 10_i\, 10_j + g^d_{ij} \bar{H}\, 10_i\, \bar{5}_j \tag{2.6.2}$$

where all parameters $\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \eta$, g^u_{ij} and g^d_{ij} are dimensionless Yukawa couplings, M, M' and M are the mass parameters of the order M_I.

From W_1, we find

$$-F_{Z^{*i}_j} = \lambda_1 \left(\phi_A^2 - \tfrac{1}{5} 1 \left(\mathrm{Tr}\, \phi_A^2 \right) \right)^j_i \,, \tag{2.6.3a}$$

$$-F_{A^{*i}_j} = \{ [2(\lambda_1 \phi_Z + \lambda_2 \phi_X) \phi_A] - \\ - \tfrac{1}{5} \mathrm{Tr} [2(\lambda_1 \phi_Z + \lambda_2 \phi_X) \phi_A] 1 \}^j_i + \lambda_3 \phi^j_{\bar{H}} \phi_{H_j} \,, \tag{2.6.3b}$$

$$-F^*_X = \lambda_2 (\mathrm{Tr}\, \phi_A^2 - M^2), \tag{2.6.3c}$$

$$-F^*_B = \lambda_4 (\phi_{\bar{H}_1} \phi_H + \phi_{\bar{H}} \phi_{H_1}), \tag{2.6.3d}$$

$$-F^*_H = \phi_{\bar{H}_1} (\lambda_3 \phi_A + \lambda_4 \phi_B), \tag{2.6.3e}$$

$$-F^*_{\bar{H}} = (\lambda_3 \phi_A + \lambda_4 \phi_B) \phi_{H_1} \,, \tag{2.6.3f}$$

$$-F^*_{H_1} = \phi_{\bar{H}} (\lambda_3 \phi_A + \lambda_4 \phi_B), \tag{2.6.3g}$$

$$-F^*_{\bar{H}_1} = (\lambda_3 \phi_A + \lambda_4 \phi_B)\, \phi_H \qquad . \tag{2.6.3h}$$

It is always possible to set $F_H = F_{H_1} = F_{\bar{H}} = F_{\bar{H}_1} = F_A = F_B = 0$,however the equations $F_Z = 0$ and $F_X = 0$ are inconsistent, so supersymmetry is sponteneously broken at the scale M. The VEV's of ϕ_Z , ϕ_X are undetermined at tree level. Moreover, one can make the colour triplet Higgs superheavy but the $SU(2)_L$ doublets are massless by the expectation value $\langle \lambda_3\phi_A + \lambda_4\phi_B \rangle = M_G \mathrm{diag}(11100)$, which is just what we want.

The vacuum expectation valule for ϕ_A is found to be

$$\langle\phi\rangle_A = A_0 \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix} , \tag{2.6.4}$$

where

$$A_0 = \frac{\lambda_2 M}{\sqrt{\lambda_1^2 + 30\lambda_2^2}} \qquad , \tag{2.6.5}$$

and the VEV's of ϕ_Z , ϕ_X are

$$\langle \phi_X \rangle = X_0 \ ,$$

$$\langle \phi_Z \rangle = \frac{\lambda_2}{\lambda_1} X_0 \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix} \qquad . \tag{2.6.6}$$

Hence ϕ_A, ϕ_Z break SU(5) down to $SU(3)_C \times SU(2)_L \times U(1)_Y$ at M_G . The magnitude of X_0 is however undetermined at tree level. Consequently , the one-loop corrections to the potential must be taken into accunt for the sake of determing X_0. we find the poten il up to one loop level as follows

$$V = \frac{\lambda_1^2\lambda_2^2 M^4}{\lambda_1^2+30\lambda_2^2}\left\{1+\frac{\hbar}{8\pi^2}\frac{3\lambda_2^2}{\lambda_1^2+30\lambda_2^2}\times(29\lambda_1^2-50\lambda_2^2)\ln\frac{|X_0|^2}{\mu^2}\right\} \quad (2.6.7)$$

According to the argument of Sec.5, a stable vacuum can be developed with a large X_0 that determines the grand unification mass scale $M_G \sim M_I e^{1/\alpha}$, which is obviously much larger than the fundamental mass scale $M \sim M_I$ in the superpotencil W .

The λ_5 term with M' = 0 in the superpotencial W_2 is required for avording the massless Goldston boson. The last two terms in W_2 contain the standard Yukawa couplings of the scalar Higgs with quarks and leptons .

As mentioned above, supersymmetry is broken by $F_X \neq 0$ and $F_Z \neq 0$. Moreover, it is clear that F_X and F_Z appear in the Lagrangian only with the following expression

$$\delta\mathcal{L} = \mathrm{Tr}\left\{(\lambda_1 F_Z + \lambda_2 F_X)\,\phi_A^2\right\} \quad (2.6.8)$$

This indicates that the only states , which can directly feel supersymmetric breaking at tree level, are the A scalar states which have superheavy mass $\sim M_G$. In other words, supersymmetric breaking effects due to F_X and F_Z can only proceed via the superheavy states ϕ_A. Any state,which is massless at tree level, will remain massless until it knows about supersymmetry breaking[1.3] [2.5] [2.24]. Therefore, supersymmetry breaking effects into the light sector of the theory are expected to be an expansion in the small parameter M_I^2 / M_G^2 . For this reason, the effective supersymmetry breaking scale of the light sector is at most

$$\mu \sim \left(\frac{M_I^2}{M_G^2}\right) M_G \sim 10^5 - 10^3\ \mathrm{Gev} \quad (2.6.9)$$

for $M_G \sim 10^{19}$ Gev and $M_I \sim 10^{12} - 10^{11}$ Gev . It is this scale that will set the weak interaction mass scale M_W , namely $M_W \sim f\mu$, in which f is a function of both gauge and Yukawa couplings.

In such a model, any state being massless at tree level can only acquire a mass from the radiative corrections at most of the order M_I^2/M_G . For instance, a gauge fermion λ may receive a mass from Fig.6.1,which corresponds to the following operator

$$\frac{1}{M_G}\int d^2\theta \, X \; W^\alpha W_\alpha \qquad . \tag{2.6.10}$$

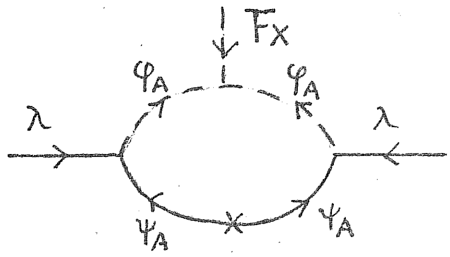


Fig.6.1 One-loop contribution to gaugino mass.

Having performed the loop integral in the momentum space, one finds

$$M_\lambda \approx \frac{\alpha}{2\pi}\,\frac{F_X}{M_G} \approx \frac{\alpha}{2\pi}\,\frac{M_I^2}{M_G} \quad , \tag{2.6.11}$$

where $\alpha = g^2/4\pi$, g is the relevant gauge coupling constant. The squarks, sleptons (the scalar partners of quarks,leptons)may obtain masses of the order M_I^2/M_G through Fig.6.2 corresponding to the operator

$$\frac{1}{M_G^2}\int d^4\theta \; X^* X^* \, \Phi \, \Phi \qquad , \tag{2.6.12}$$

where ϕ denotes the matter superfield.

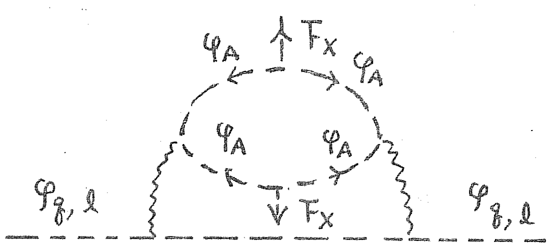


Fig.6.2 Two-loop contribution to masses of squarks, sleptons.

It is easy to get the masses of squarks and sleptons from Fig.6.2, which are of the order $\frac{\alpha}{2\pi} M_I^2/M_G$.

Conclusion:

The geometric heirarchy model is based on Witten's mechanism. The new point in the geometric hierarchy model is the assmption of a supersymmetry breaking scale $M_I \sim 10^{12}$Gev. As a concequence, both the unification mass scale M_G and the weak mass scale M_w can be naturally obtained through the radiative corrections. So called the fine-tuning problem is automatically avoided.

However, this model is complicated and contains many particles with masses of the order $M_I^2/M_G \sim 10^3 - 10^5$Gev. This makes trouble for the phenomenology, $\sin^2\theta_w$ in particular. If we input $M_I \sim 10^{12}$Gev, for example, then $\sin^2\theta_w \approx 0.25$ and the unification mass scale $M_G \approx 10^{21}$Gev. Obviously, both $\sin^2\theta_w$ and M_G are out of the range of the experimental values. If we in turn take $\sin^2\theta_w \approx 0.215$ as input, then $M_I \sim 10^4$Gev , $M_G \sim 10^{24}$Gev and $M_w \sim 10^{-18}$Gev , which in any case are unacceptable. An interesting question is whether we can make such a model so that both the unification mass scale M_G (and Plank mass scale M_p, if possible) and the weak mass scale M_w can be obtained from a single fundamental scale M_I where supersymmetry breaking occurs, without either the fine-tuning problem or the phenomenological problems.

Chapter 3 : SUPERSYMMETRY BREAKING

Supersymmetry is perhaps the most beautiful symmetry we know by now. If nature is really described by a supersymmetry theory, such a symmetry must be broken, since we have not observed in nature the degeneracies among fermion and bosons which would be predicated by supersymmetry. This is because of the following reason:

Supersymmetry charge Q_α ($\alpha = 1,\dots 4$) changes fermion and boson states each other. Namely,

$$Q_\alpha | s \rangle = \sqrt{\frac{E}{2}} \, | s \pm \frac{1}{2} \rangle , \qquad s - \text{spin} \tag{3.1}$$

and , as it is well known, the Hamiltonian H is given by

$$H = \frac{1}{2} \sum_\alpha Q_\alpha^2 . \tag{3.2}$$

Clearly Q_α do commute with the Hamiltonian H, so the fermion and boson on which Q_α acts have the same mass.

In this chapter we shall discuss various aspects related to supersymmetry breaking. We first discuss the conditions of supersymmetry breaking, and then discuss about the mass scale at which supersymmetry breaking occurs. Finally, a qustion is of course how to break supersymmetry.

3.1 Conditions of supersymmetry breaking

SUpersymmetry can be broken either explicit, if Lagrangian is not invariant under supersymmetric transformation, or spontaneously, if a vacuum is not supersymmetric but the Lagrangien is.

It is a simple matter to break supersymmetry explicitly. We only need to check whether Lagrangian is invariant under the supersymmetric transformation. For example, if we adds a mass term $m\phi^*\phi$ of a scale feild ϕ in Lagrangian , clearly the Lagrangian will not be supersymmetric any longer.

We now restrict ourselves to the spontaneous breaking.

THEOREM 1:

Supersymmetry is spontaneously broken if and only if the vacuum energy is positive [2.5] .

The proof is quate simple.

Note the fundamental relation of the algebra ,

$$\{ Q_\alpha , \bar{Q}_\beta \} = \gamma^\mu_{\alpha\beta} P_\mu \quad , \tag{3.1.1}$$

where Q_α is supersymmetry charge . $\bar{Q}_\beta = Q_\alpha \gamma^0_{\alpha\beta}$.

If we multiply Eq. (3.1.1) by $\gamma^0_{\beta\alpha}$, sum over β and α , and use the fact $(\gamma^0)^2 = 1$, $\mathrm{Tr}\,\gamma^0\gamma^\mu = 4\delta^{0\mu}$, we obtain

$$4P_0 = \sum_\alpha \{ Q_\alpha , Q_\alpha \} \quad . \tag{3.1.2}$$

Here P_0 is ,of couse, the Hamiltonian H , and $\{ Q_\alpha , Q_\alpha \} = \sum Q_\alpha^2$,

Therefore, it turns out to be

$$H = \frac{1}{2} \sum_\alpha Q_\alpha^2 \quad , \tag{3.1.3}$$

which is an impotant key to understanding supersymmetry breaking. Since Q_α is hermitian operator , Q_α^2 can never be negative. It follws immadiately that if supersymmetry is not spontaneously broken, then vacuum energy is zero. In fact, if supersymmetry is not spontaneously broken, by definition, we have

$$\delta |\phi\rangle = Q_\alpha |\phi\rangle = 0 \; , \tag{3.1.4}$$

where $|\phi\rangle$ denotes the vacuum state. δ means the infinite supersymmetry transformation.

Consequantly,

$$H |\phi\rangle = \frac{1}{2} \sum_\alpha Q_\alpha^2 |\phi\rangle = 0 \; . \tag{3.1.5}$$

Conversaly, if supersymmetry is spontaneously broken, i.e,

$$\delta |\phi\rangle = Q_\alpha |\phi\rangle \neq 0 \quad , \tag{3.1.6}$$

then

$$\langle\phi| H |\phi\rangle = \frac{1}{2} \sum_\alpha \langle\phi| Q_\alpha^2 |\phi\rangle = \frac{1}{2} \sum_\alpha \left| Q_\alpha |\phi\rangle \right|^2 > 0 , \tag{3.1.7}$$

in this case the vacuum energy is nonzero and positive.

THEOREM 2:

Supersymmetry is spontaneously broken at tree level if either F-term or D-term is non-zero.

The proof is again simple.

As we know, the scalar potential V at tree level can be written as

$$V = \sum_i |F_i|^2 + \frac{1}{2}\sum_\alpha D_\alpha^2 \tag{3.1.8}$$

where F_i and D_α are well defined

$$-F_i^* = \frac{\partial W}{\partial \phi_i}, \qquad D_\alpha = g_\alpha \varphi^+ T_\alpha \varphi, \tag{3.1.9}$$

in which W is superpotential, T_α's are the generators of the gauge group G, g_α is the corresponding gauge coupling constant.

Since anyterm in (3.1.8) is positive, so the vacuum energy is positive if one of F_i and D_α, evalueted at the vacuum state $|\phi\rangle$, is non-zero.

THEOREM 3:

If supersymmetry is not spontaneously broken at tree level, then supersymmetry will remain unbroken to all orders of perturbation.

Proof:

In order to break supersymmetry in finite orders of perturbation theory, we must give an expectation value to F or D . This means that we must obtain in the supersymmetric effective potential an operator linear in F or D, times fields with vacuum expectation values . As is well known, the only fields which can acquire non-zero vacuum expectation values are elementary scalar fields.

According to the non-renormalazation theorem, F-term , which is an integral of the type $\int d^2\theta \cdots$, can not be generated by loop corrections to any finite order . So a term linear in F does not appear and F does not get a non-VEV.

For the auxiliary D fields, it would seem can get an expectation value from the loop corrections, since it is an integral of type $\int d^4\theta \cdots$. However, This does not happen. The reason is following.

The D fields which could get vacuum expectation values are those associated with unbroken gauge symmetry . However,a D field associated with an unbroken gauge symmetry is always multiplied by charged scalar fields which of cousce have zero expectation values. Therefore, an expectation value of the D field is not induced in finate loop level.

Finaly, we come to an impotant criterion about supersymmetry spontaneous breaking.

First one defines an operator

$$(-)^F \equiv \exp(-2\pi i J_z) \quad , \quad J\text{-Spin} \qquad (3.1.10)$$

which distinguishes bosons from fermions, since for any boson state $|b\rangle$

$$(-)^F |b\rangle = |b\rangle \qquad (3.1.11a)$$

for any fermion state $|f\rangle$,

$$(-)^F |f\rangle = -|f\rangle . \qquad (3.1.11b)$$

<u>THEOREM 4:</u>

Supersymmetry is not spontaneously broken if

$$\Delta \equiv \mathrm{Tr}(-)^F \neq 0 \qquad (3.1.12)$$

where Δ is called Witten's index.

Proof:

It is easy to see that Δ may be identified with the difference $n_B^{E=0} - n_F^{E=0}$, in which $n_B^{E=0}(n_F^{E=0})$ is a number of the zero energy boson states(ferimon states).

As we know ,in supersymmetric theory the states of non-zero energy are in bose-fermi pairs, but the zero energy states are not paried in general since $H = \frac{1}{2}\sum_\alpha Q_\alpha^2$,any zero energy state (boson or fermion) can be annihilated by Q

$$Q|b, E=0\rangle = Q|f, E=0\rangle = 0 \quad . \qquad (3.1.13)$$

Hance the general feature of the spectrum of supersymmetric theory looks like as Fig.3.1

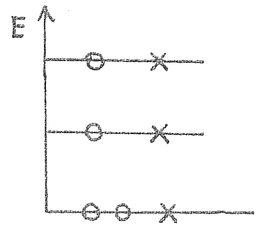


Fig.3.1 O denotes boson, X denotes fermion.

When we change the parameters of the theory, for example ,bare mass , coupling constants, the states of zero energy will move around in energy in boson-fermi paris as Fig.3.2

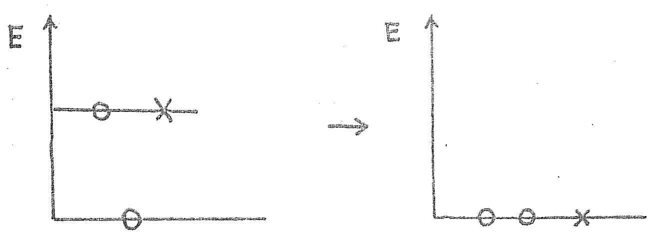


Fig.3.2 boson-fermion paris move around in energy when the parameters are changed. $n_B^{E=0} - n_F^{E=0} \neq 0$, supersymmetry is unbroken , since $E=0$.

In the Fig.3.2 it is shown that if $\Delta \neq 0$, all states can always be moved down to ground state, hance E = 0 and supersymmetry is not spontaneously broken.Hovewer, the inverse is not true. If $\Delta = 0$, supersymmetry may be spontaneously broken or may be not.

Witten's index Δ is caculable since it does not dependent on the parameters in the theory.

Conclusion:

the four theorems above are fundamantal and importent. As far as supersymmetry breaking is concerned, theorem 2 in fact is more useful and more practicable when we make a realistic supersymmetry theory. Several comments should be addressed. First, the vacuum energy may not be possitive in local supersymmetry (supergravity) theory even if supersymmetry spontaneously broking[3.1] . Second, in two dimensional theory, it is possible to break supersymmetry perturbatively, even if it is not spontaneously broken at tree level [3.2]. Finally, one would like to point out that global supersymmetry breaking is always accompanied with a massless Goldston fermion [3.14] .

3.2 The Mass Scale of Supersymmetry Breaking

From the point of view of phenomenology ,an interesting question is at what mass scale supersymmetry breaking takes place. In principle, supersymmetry can be broken either at very high energe ,say, Plank mass scale $M_p \sim 10^{19}$ Gev, or at low energy ,say, weak mass scale $M_W \sim 10^2$ Gev . Different mass scales may lead to completely different phenomenology. For instance, if supersymmetry is broken at M_p ,then influence of supersymmetry on the effective low energy physics may perhaps be negligible. However, if supersymmetry is spontaneously broken at M_W, then we can imagine that the influence of supersymmetry on taday's physics is very important .Obviously ,the phenomenology may in turn give some constraints on the mass scale of supersymmetry breaking .

We now give some constraints on supersymmetry breaking mass scale from several phenomenologies.

3.2.1 gauge hierarchy

Since weak mass scale M_W is about 100 Gev ,it is natural to assume that supersymmetry is broken at the same mass scale M_W in order to protect the gauge hierarchy from the radiative correction. However, if the SU(2) doublet Higgs ,which responds to the breaking of $SU(3)_C \times SU(2)_L \times U(1)_Y$ to the $SU(3)_C \times U(1)_{em}$,can not directly fell supersymmetry breaking , then the gauge hierarchy will not be spoilt by radiative correction even if the supersymmetry breaking mass scale M_I is much larger than M_W, say $M_I \sim 10^{12}$ Gev ,because the mass of the SU(2) Higgs doublet being set zero at tree level may acquired a mass at most of M_I^2 / M_G ,where M_G is unification mass scale(see Chapter 2: geometric hierarchy).

3.2.2 Cosmology [3.3]

It is well known that supergravity theories necessarily involve a massive spin $J = \frac{3}{2}$ particle ,the gravitino, whose mass m_g is related the scale F of spontaneous supersymmetry breaking by the formula [3.4]

$$m_g = \left(\frac{4\pi}{3}\right)^{1/2} \frac{F}{M_p} , \qquad (3.1.14)$$

where m_p is Plank mass $M_p \sim 1.2 \times 10^{19}$ Gev.

If we assume that the gravitino is stable enough to survive to the present, than from the observational bound on the cosmological mass density we find

$$m_g \lesssim 1 \text{ kev} \qquad (3.1.15)$$

Correspondingly, it follows that the upper bound on the scale of supersymmetry breaking is given by [3.3]

$$\sqrt{F} < 2 \times 10^6 \text{ Gev} \qquad (3.1.16)$$

However, if the gravitino is heavy enough so that almost all gravitinos would have decayed before the time of helium synthesis, then the lower bound on supersymmetry breaking scale is found to be[3.3]

$$\sqrt{F} > 10^{11} \text{ to } 10^{16} \text{ Gev} \tag{3.1.17}$$

3.2.3 $\sin\theta_W$

In the standard supersymmetric grand unified SU(5) model[2.6],it has been found that $\sin^2\theta_W = 0.236$ (for two light Higgs doublets) or $\sin^2\theta_W = 0.259$ (for four light Higgs doublets) which is too large to be compatible with the experiment limit . However,if we assume that supersymmetry is broken at an intermediate mass scale M_I ,and the breaking pattern of SU(5)SGUM is modified as follows

$$SU(5) \xrightarrow{M_G} SU(3)_C \times SU(2)_L \times U(1)_Y \xrightarrow{M_I} SU(3)_C \times SU(2)_L \times U(1)_Y \xrightarrow{M_W} SU(3)_C \times U(1)_{em}$$

In terms of the renormalization group equations and by using $\sin^2\theta_W = 0.215$ and $M_W \approx 85$ Gev as inputs, we find[3.5]

$$M_G \sim 4\times 10^{15}\text{Gev}$$
$$M_I \sim 1\times 10^{12}\text{Gev} \quad , \qquad \text{(for two light Higgs doublets)}$$

This may suggest that supersymmetry would be probably broken at very high mass scale, i.e, $M_I \sim 10^{12}$Gev . This conclusion is coincident with what we have obtained from cosmology.

3.2.4 Proton decay

As Weinberg has pointed out , there are the dimension five opertors in any supersymmetric grand unifield model with $SU(3)_C \times SU(2)_L \times U(1)_Y$ as a low energy theory. Such the operators may lead too fast a proton decay.However, for SU(5) SGUM, by considering renormalization effects the life-time of proton may probably be compatible with experiment limit even if there exist the dimension five operators in the theory [3.6] and supersymmetry breaking scale is much lower; about 1 Tev. However, this is not the case for SO(10) SUGM. We have found that [4.9] in the SO(10) SGUM supersymmetry would be broken at an intermediate mass scale $M_I \gtrsim 10^7$ Gev, if we want to get a suitable proton life time (See Chapter 4).

Conclusion:

Supersymmetry can possibly broken either at low mass scale, say $M_I \sim M_W$, or at high mass scale, $M_I \gtrsim 10^{11}$ Gev. In both cases, the gauge hierarchy puzzle can be removed. In addition, as we know, there is a well-known desert puzzle [2.1] in non-supersymmetric grand unified theory. This means that there is no new physies in the "desert" between $M_W \sim 100$ Gev and $M_G \sim 10^{15}$ Gev. If supersymmetry is realy broken at an intermediate mass scale, say $M_I \sim 10^6$ to 10^{12} Gev, this then would give interesting physics in the desert. The new physics we expect is just the supersymmetric phenomenology.

3.3 How to Break Supersymmetry

It has been emphasized that supersymmetry must be broken in order to construct realistic supersymmetry grand unified models. Several ways of breaking supersymmetry have been presented: one is an explicit but soft breaking of supersymmetry. The second is by the Fayet-Iliobolous D- term, The third way is by using O'Raifeataigh type models. Finally, supersymmetry may conceivably break dynamically, for example, through an instanton or condensate effects. It is also possible to break supersymmetry by M-term [3.10], if a global supersymmetry is embended in N=1 local supersymmetry.

3.3.1. Explicit but soft supersymmetry breaking

Definition:

i) Explicitly broken supersymmetry:

The Lagrangian $\mathcal{L}$ is not invarient under the supersymmetric transformation, i.e,

$$\delta\mathcal{L} \neq 0 \qquad (3.3.1)$$

ii) Soft broken supersymmetry:

Soft means that the quadratic divergences can not be generated from radiative correction in finite order perturbative theory. Otherwise, we call it hard broken supersymmetry.

In order to solve the gauge hierarchy problem in any supersymmetric grand unifield model,supersymmetry breaking must be soft. As we known, in theories with spontaneous broken supersymmetry the quadratic divergences are absent,so it is soft breaking. In theories with a explicit broken supersymmetry , the quadratic divergences can probably be present,therefore, an explicit broken supersymmetry can be either soft or hard. Consequently, the questions are :

(1) In global supersymmetric theories , what explicit breaking terms are soft ?

(2) For a given explicit but soft breaking term, what new logarithmically infinite terms can be generated in the effective action so as to require introduction of new counterterms and hance new parameters in the Lagrangian ?

The key observation for the answers of above questions is that supersymmetry is equivalant to translational invariance in superspace ($x, \theta, \bar\theta$). Therefore , supersymmetry is explicitly broken if one gives some superfields $\Phi(x,\theta,\bar\theta)$ a fixed, x-independent,$\theta,\bar\theta$-dependent values, since this destroys the translational invariance in superspace : $\Phi(x,\theta,\bar\theta) \rightarrow \Phi(x+\varepsilon\sigma\bar\theta+\theta\sigma\bar\varepsilon,\ \theta+\varepsilon,\ \bar\theta+\bar\varepsilon)$. Such a fixed superfield , for instance, can be choosed as $\Phi = \mu^2\theta^2\bar\theta^2,\ \mu\theta^2\bar\theta^2,\ \mu\theta^2,\ \mu^2\theta^2$ atc. Soft supersymmetry breaking then can be achieved by coupling in a manner consistent with the power counting criteria for renormalizability.

In the superspace picture , it has been shown that[2.5] the whole effective action can be written as an expession local in θ :

$$\Gamma(\phi,\bar{\phi},V)=\int d^4x_i\, d^4\theta\; G(\phi,\bar{\phi},D_\alpha\phi,V\cdots;x_i,\theta) \qquad (3.3.2)$$

In terms of power counting [3.7], the degree of divergence of any supergraph is given by

$$d=2-E-M_c \quad (\text{ -1 for a graph with only chiral or anti-chiral external lines }) \qquad (3.3.3)$$

where E_C is the number of external chiral and anti-chiral lines, and M_C is the number of internal (massive)$\langle\phi\,\phi\rangle$ and $\langle\bar{\phi}\,\bar{\phi}\rangle$ propagators. This result can be understood from the following dimension counting

$$[d^4x]=-4,\quad [d^2\theta]=[d^2\bar{\theta}]=1$$

$$[D_\alpha]=\frac{1}{2},\qquad [\phi]=1 \qquad (3.3.4)$$

$$[V]=0$$

where ϕ is a chiral superfield , V is a vector superfield. D is the superspace covariant derivatives.

We now give some examples of possible soft breaking terms of supersymmetry and the associated new logarithmic infinite terms they generate.

(1) $$\mathcal{L}_{break}=\int d^4\theta\; U\bar{\phi}\phi=\frac{\mu^2}{2}(A^2+B^2) \qquad (3.3.5)$$

where $U=\mu^2\theta^2\bar{\theta}^2$,

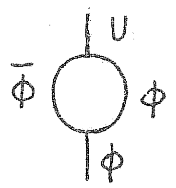
and $$\phi=\frac{1}{\sqrt{2}}(A+iB)+\theta\psi+\frac{1}{\sqrt{2}}\theta^2(F-iG) \qquad (3.3.7)$$

It is obvious that the dimension of $\mathcal{L}_{break}$ is 2 , and (3.3.5) is a mass term of scalar components. The induced new logarithmic divergent terms are

$$\Delta\mathcal{L}=\int d^4\theta\; U\phi+h.c\sim\mu^2 m A \qquad (3.3.8)$$

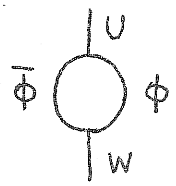
and $$\Delta\mathcal{L} = \int d^4\theta\, U D^\alpha W_\alpha \sim \mu^2 D$$ (if U(1) gauge field is present) (3.3.9)

which are shown in Fig. 3.3 (a),(b).



$$\sim \int d^4\theta\, U\phi \sim \mu^2 m A$$

(a)



$$\sim \int d^4\theta\, U D^\alpha W_\alpha \sim \mu^2 D$$

(b)

Fig.3.3 (a),(b). The logarithmic divergence terms generated from $\mathcal{L}_{break} = \int d^4\theta\, U \bar{\Phi}\phi$.

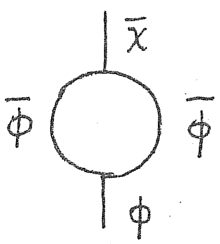
(2) $$\mathcal{L}_{break} = \int d^2\theta\, \chi\phi^2 + h.c. = \mu^2(A^2 - B^2)$$ (3.3.10)

where $\chi = \mu^2\theta^2$, $[\mathcal{L}_{break}] = 2$.

Its corresponding new logarithmic div. term is

$$\Delta\mathcal{L} = \int d^4\theta\, \bar{\chi}\phi + h.c \sim F$$ (3.3.11)

as shown in Fig.3.4



$$\sim \int d^4\theta\, \bar{\chi}\phi + h.c \sim F$$

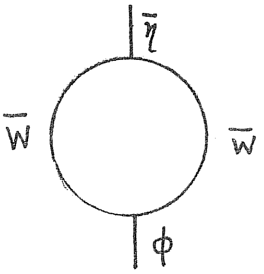
Fig.3.4 The new logarithmic div. term generated by $\mathcal{L}_{break} = \int d^4\theta\, \chi\phi^2 + h.c.$

(3) $$\mathcal{L}_{break} = \int d^2\theta\, \eta W^\alpha W_\alpha + h.c. \sim \mu \bar{\lambda} \lambda \tag{3.3.12}$$

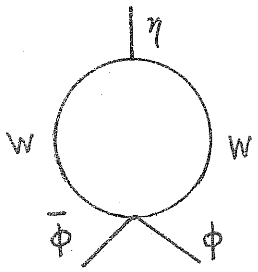
where $\eta = \mu \theta^2$

$[\mathcal{L}_{break}] = 3$. (3.3.12) is a mass term of gaugino λ.

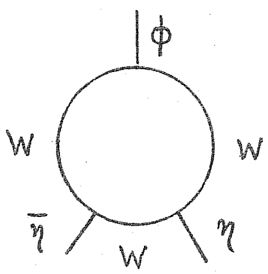
The following logarithmic divergent terms can be generated



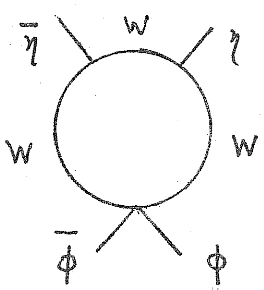
$$\sim \int d^4\theta\, \bar{\eta} \phi + h.c \sim F \tag{3.3.13}$$



$$\sim \int d^4\theta\, \eta \bar{\Phi} \phi + h.c. \sim FA + GB \tag{3.3.14}$$



$$\sim \int d^4\theta\, \eta \bar{\eta}\, \phi + h.c. \sim A \tag{3.3.15}$$



$$\sim \int d^4\theta\, \bar{\eta} \eta \bar{\Phi} \phi \sim A^2 + B^2 \tag{3.3.16}$$

The operator (3.3.12) ,which has dimension three, is soft. However, an operator with dimension three, in general, is not soft. For example, the operator

$$\mathcal{L}_{break} = \int d^4\theta \; U D^\alpha \phi D_\alpha \phi + h.c. \sim \mu \bar{\Psi} \Psi ,$$

where $U = \mu \theta^2 \bar{\theta}^2$,may lead to hard supersymmetry breaking , since this operator can generate the quardratic divergences.

Any operator, whose dimension D $\geq$ 4,is not soft.

Conclusion:

Dimension-two operators(of course,and dimension-one,too,) are soft, but the opertors with dimension more then three are not soft, and in general, the dimension three operators are not soft.

3.3.2 Supersymmetry breaking produced by Fayet-Iliopolous D-term

It has been pointed out in chapter 1 that supersymmetry can be spontaneously broken when F or D-term is no-vanishing. Following this idea, Fayet and Iliopolous have presented a mechanism to break supersymmetry: In a class of supersymmetric gauge theories which contain U(1) gauge symmetry, supersymmetry can be spontaneously broken by adding an D-term to the Lagarangien[3.8].namely,

$$\mathcal{L} = \mathcal{L}_0 + \xi (V)_D \quad , \qquad (3.3.17)$$

where $\mathcal{L}_0$ is an original Lagaragien ,$\xi(V)_D$ is the Fayet-Iliopolous supersymmetry breaking term, called Fayet-Iliopolous D-term. V is an U(1) vector superfield, D means the D-component,i.e, $(V)_D = \int d^4\theta \, V$, ξ is a real parameter.

Noticing that

$$D_\alpha = g_\alpha \varphi^{+} T_\alpha \varphi$$

and restricting to the generator T of the isolated U(1) factor of the group G, we then get

$$D = g \varphi T \varphi + \xi \qquad (3.3.18)$$

As an illustration of Fayet-Iliopolous mechanism, let us consider a toy model:

Toy model:

In this toy model there is only an U(1) gauge group , and two chiral superfields A, B, in which A carries a U(1) charge g, B carries -g, as well as an U(1) vector superfield V. Then the Lagaragian is given by

$$\mathcal{L} = \int d^4\theta\,(A^* e^{2gV} A) + \int d^4\theta\, B^* e^{-2gV} B + \frac{1}{4}\int d^2\theta\, W^\alpha W_\alpha + \\ + \xi \int d^4\theta\, V + \left|\frac{\partial W}{\partial A}\right|^2 + \left|\frac{\partial W}{\partial B}\right|^2, \qquad (3.3.19)$$

where the superpoten ial W is chosen as follows

$$W = h\,AB \qquad (3.3.20)$$

The scalar potential V then turn out to be

$$V = h^2(|A|^2 + |B|^2) + \frac{g^2}{2}(|A|^2 - |B|^2 + \xi/g)^2 . \qquad (3.3.21)$$

It is easy to see that in this toy model supersymmetry is spontaneously broken only by Fayet-Iliopolous D-term. Indeed, if we set $\xi = 0$, supersymmetry remains unbroken.

Minimizing the potential V, one get

$$\frac{\partial V}{\partial A^*} = h^2 A + g^2(|A|^2 - |B|^2 + \xi/g)\,A = 0 \qquad (3.3.22)$$

$$\frac{\partial V}{\partial B^*} = h^2 B - g^2(|A|^2 - |B|^2 + \xi/g)\,B = 0 \qquad (3.3.23)$$

From (3.3.22) and (3.3.23) we obtain

$$(|A|^2 - |B|^2 + \xi/g)\,AB = 0 \qquad (3.3.24)$$

The vacuum is found to be

i) $\langle A \rangle = \langle B \rangle = 0$, if $g\xi < h^2$, (3.3.25)

ii) $\langle A\rangle = 0$, $\langle B\rangle = \sqrt{g\xi - h^2}/g$ if $g\xi > h^2$ (3.3.26a)

or $\langle B\rangle = 0$, $\langle A\rangle = \sqrt{-(g\xi + h^2)}/g$ if $g\xi < -h^2$ (3.3.26b)

First, let us assme $g\xi < h^2$, then the vacuum is given by $\langle A\rangle = \langle B\rangle = 0$.

we now proceed to calculate the masses of all particles in this toy model. The fermion mass metrix is given by

$$M_{1/2} = \begin{array}{c c} & \begin{array}{ccc} \psi_A & \psi_B & \lambda \end{array} \\ \begin{array}{c} \psi_A \\ \psi_B \\ \lambda \end{array} & \begin{pmatrix} 0 & h & 0 \\ h & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{array} \tag{3.3.27}$$

It is easy to get the eigen states and eigen values,

$$\psi_{1,2} = \frac{1}{\sqrt{2}}(\psi_A \pm \psi_B), \qquad M_{1,2} = \pm h \tag{3.3.28}$$

$$\psi_3 \equiv \psi_G = \lambda \qquad M_G = 0 \tag{3.3.29}$$

ψ_G is the Goldston fermion, which is exactly massless, namely, it can not acquire any mass from radiative corrections. This is a general phenomenon in a spontaneous broken supersymmetry theory ,just like the case of a spontaneous broken gauge symmetric theory, in which there exist the Goldston bosons.

The scalar mass squared matrix is as follows

$$M_0^2 = \begin{array}{c c} & \begin{array}{cccc} \varphi_A & \varphi_B & \varphi_A^* & \varphi_B^* \end{array} \\ \begin{array}{c} \varphi_A^* \\ \varphi_B^* \\ \varphi_A \\ \varphi_B \end{array} & \begin{pmatrix} h^2+g\xi & 0 & 0 & 0 \\ 0 & h^2-g\xi & 0 & 0 \\ 0 & 0 & h^2+g\xi & 0 \\ 0 & 0 & 0 & h^2-g\xi \end{pmatrix} \end{array} \tag{3.3.30}$$

The eigen states and eigen values are found to be

$$\begin{aligned} &\varphi_1 = \mathrm{Re}\,\varphi_A && M_1^2 = h^2 + g\xi \\ &\varphi_2 = \mathrm{Im}\,\varphi_A && M_2^2 = h^2 + g\xi \\ &\varphi_3 = \mathrm{Re}\,\varphi_A && M_3^2 = h^2 - g\xi \\ &\varphi_4 = \mathrm{Im}\,\varphi_B && M_4^2 = h^2 - g\xi \end{aligned} \qquad (3.3.31)$$

In this case, no massless scalar exists, since U(1) gauge group remains unbroken.

The gauge boson A has no mass, i.e,

$$M_1^2 \equiv M^2(A_\mu) = 0 \qquad (3.3.32)$$

From (3.3.27) to (3.3.30) one gets a mass sum rule [3.9]:

$$\sum_J (-)^{2J}(2J+1)\, M_J^2 = 0 \qquad (3.3.33)$$

In theories with spontaneous broken supersymmetry this mass sum rule generally valied at tree level, but not at loop level.

For the solusion ii) $\langle A\rangle = 0$, $\langle B\rangle = \sqrt{\xi g - h^2}/g$, $g\xi > h^2$,
The fermion mass matrix is given by

$$M_{1/2} = \begin{array}{c} \\ \psi_A \\ \psi_B \\ \lambda \end{array} \begin{array}{c} \begin{array}{ccc} \psi_A & \psi_B & \lambda \end{array} \\ \begin{pmatrix} 0 & h & 0 \\ h & 0 & -\sqrt{2}\,g\langle B\rangle \\ 0 & -\sqrt{2}\,g\langle B\rangle & 0 \end{pmatrix} \end{array} \qquad (3.3.34)$$

The eigen states and eigen values turn out to be

$$\psi_{1,2} = \left(h\psi_A \mp \sqrt{2g\xi - h^2}\;\psi_B + \sqrt{2(g\xi - h^2)}\,\lambda\right) \Big/ \sqrt{2(2g\xi - h^2)} \qquad (3.3.35)$$

$$\psi_3 \equiv \psi_G = \left(\sqrt{2(g\xi - h^2)}\;\psi_A + h\lambda\right) \Big/ \sqrt{2g\xi - h^2} \qquad (3.3.36)$$

and
$$M_{1,2} = \pm\sqrt{2g\xi - h^2} \tag{3.3.37}$$

$$M_3 = M_G = 0 \tag{3.3.38}$$

ψ_G is the Goldston fermion which is the company of a broken global supersymmetry. In order to get the scalar masses, it is convenient to use the shifited fields

$$A = \frac{A_1 \pm i A_2}{\sqrt{2}}$$

$$B = \langle B \rangle + \frac{B_1 \pm i B_2}{\sqrt{2}} \quad , \tag{3.3.39}$$

when a scalar poten ial V is not complicated.

Inserting (3.3.39) into the potential V (3.3.21), we get

$$V = h^2\left[\frac{A_1^2}{2} + \frac{A_2^2}{2} + \left(\langle B \rangle + \frac{B_1}{\sqrt{2}}\right)^2 + \frac{B_2^2}{2}\right] + + \frac{g^2}{2}\left[\frac{A_1^2}{2} + \frac{A_2^2}{2} - \left(\langle B \rangle + \frac{B_1}{\sqrt{2}}\right)^2 - \frac{B_2^2}{2} + \frac{\xi}{g}\right]^2 , \tag{3.3.40}$$

One then picks out all the quadratic terms,

$$V_2 = \left(\frac{h^2}{2} + \frac{g^2}{2}\cdot\frac{\xi}{g} - \frac{g^2}{2}\langle B \rangle^2\right)(A_1^2 + A_2^2) + \left(\frac{h^2}{2} + g^2\langle B \rangle^2 + \frac{g^2}{2}\left(\langle B \rangle^2 - \frac{\xi}{g}\right)\right)B_1^2 + + \left[\frac{h^2}{2} + \frac{g^2}{2}\left(\langle B \rangle^2 - \frac{\xi}{g}\right)\right]B_2^2 = h^2(A_1^2 + A_2^2) + + (g\xi - h^2)B_1^2 . \tag{3.3.41}$$

From (3.3.41) we obtain the following eigen states and eigen values

$$\phi_{1,2} = \frac{\phi_A \pm \phi_A^*}{\sqrt{2}} , \qquad M_{1,2}^2 = h^2$$

$$\phi_3 = \frac{\phi_B + \phi_B^*}{\sqrt{2}} , \qquad M_3^2 = g\xi - h^2 \tag{3.3.42}$$

$$\phi_4 \equiv \phi_G = \frac{\phi_B - \phi_B^*}{\sqrt{2}} , \qquad M_G^2 = 0 .$$

ϕ_G is the massless Goldston boson associated with the breaking of the U(1) gauge group.

The U(1) gauge boson has a mass as follows

$$M_1^2 = g^2 \langle B \rangle^2 = g\xi - h^2 . \tag{3.3.43}$$

The mass sum rule becomes

$$\sum (-)^{2J} (2J+1) M_J^2 = 2(h^2 - 2g\xi) \neq 0 \tag{3.3.44}$$

The above mass sum rule is somewhat different from (3.3.33), since B field here which carries a U(1) charge develops a non-vanishing vacuum expectation value. In general, the mass sum rule is [3.7]

$$\sum (-)^{2J} (2J+1) M_J^2 = 2g\, D_\alpha \,\mathrm{Tr}\, T_\alpha \tag{3.3.45}$$

where D_α is defined by

$$D_\alpha = g \langle \phi \rangle^+ T_\alpha \langle \phi \rangle + \xi \tag{3.3.46}$$

Conclusion:

Supersymmetry can spontaneously broken by Fayet-Iliopoulos D-term, if gauge group contains a U(1) factor. Since the low-energy symmetry is $SU(3)_C \times SU(2)_L \times U(1)_Y$, so the Fayet-Iliopoulos D-term mechanism is available, however, one should be careful to avoid the breaking of $U(1)_Y$ sub-group. If the gauge group does not contain any U(1) factor, we can add an extra $\tilde{U}(1)$, and we then go futher to break both supersymmetry and the extra $\tilde{U}(1)$ simultaneously. Obviously, this will have no effect on the effctive low-energy physies if we assme that all physical particles (quarks, leptons, gauge bosons and so on) do not couple with the $\tilde{U}(1)$ sector.

3.3.3 O'Raifeartaigh model

We are going to discuss a class model with supersymmetry spontaneously broken by F-term at tree level— the well-known O'Raifeartaigh model[2.19]. As we have seen in chapter 2 , O'Raifeartaigh model has plaid an important rȯle in Witten's type of supersymmetric grand unified models.

A simplest O'Raifeartaigh model may contain three complex scalar feilds A, X and Y. The superpotencial W is given by hand as follows

$$W = g A Y + \lambda X (A^2 - M^2) , \qquad (3.3.47)$$

which is compatible with the discreat symmetry $A \to -A$, $Y \to -Y$, and $X \to X$.

It follows that

$$-F_A^* = \frac{\partial W}{\partial A} = gY + 2\lambda X A \qquad (3.3.48)$$

$$-F_Y^* = \frac{\partial W}{\partial Y} = gA \qquad (3.3.49)$$

$$-F_X^* = \frac{\partial W}{\partial X} = \lambda (A^2 - M^2) . \qquad (3.3.50)$$

It is easy to see that supersymmetry is spontaneously broken at tree level, since F_Y and F_X could not vanish simultaneously .So the key point of O'Raifeartaigh model is to break supersymmetry spontaneously at tree level in terms of F-term.

From (3.3.48) to (3.3.50) we derive the scalar potential V

$$V = |gY + 2\lambda X A|^2 + g^2 |A|^2 + \lambda^2 |A^2 - M^2|^2 , \qquad (3.3.51)$$

The VEV of A scalar can be determined by minimizing the last two terms in the potencial V (3.3.51)

$$\frac{\partial V}{\partial A^*} = g^2 A + 2\lambda^2 (A^2 - M^2) A^* = 0 \qquad (3.3.52a)$$

$$\frac{\partial V}{\partial A} = g^2 A^* + 2\lambda^2 (A^{*2} - M^2) A = 0 \qquad (3.3.52b)$$

One finds two solutions:

i) $A = 0$ for $(g/\lambda M)^2 > 2$ (3.3.53a)

ii) $A = \sqrt{M^2 - g^2/4\lambda^2}$ for $(g/\lambda M)^2 < 2$ (3.3.53b)

It it obvious that supersymmetry is spontaneously broken in either case. However, X and Y can not be determined uniquely, although the minimization of the potencial determines A. Since the only term in the potential ,which depends on X and Y, is the first term, i.e, $|gY + 2\lambda XA|^2$, we can choose any X, so long as

$$Y = -2\lambda AX/g \qquad (3.3.54)$$

The energy is minimized so long as (3.3.54) is satisfied. X may be arbitrarily large. such degeneracies are a common feature of O'Raifeartaigh type models.

It would be worthwhile to point out that although the energy is independent of X, the masses of particles are not. It is this fact that provides the possibility of generating the gauge hierarchy dynamically - Witten's mechanism. In fact, for case i), if $X \gg M$, then the fermion mass matrix is

$$M_{1/2} = \begin{array}{c c} & \begin{array}{ccc} \psi_A & \psi_Y & \psi_X \end{array} \\ \begin{array}{c} \psi_A \\ \psi_Y \\ \psi_X \end{array} & \begin{pmatrix} 2\lambda X & g & 0 \\ g & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{array} \qquad (3.3.55)$$

The eigen states and eigen values of $M_{1/2}$ turn out to be

$$\psi_1 = (2\lambda X\, \psi_A + g\psi_Y)/\sqrt{4\lambda^2X^2+g^2}\,, \qquad M_1 = 2\lambda X + \frac{g^2}{2\lambda X} + O(\tfrac{1}{X^2})\,, \tag{3.3.56a}$$

$$\psi_2 = (g\psi_A - 2\lambda X\, \psi_Y)/\sqrt{4\lambda^2X^2+g^2}\,, \qquad M_2 = -\frac{g^2}{2\lambda X} + O(\tfrac{1}{X^2})\,, \tag{3.3.56b}$$

$$\psi_3 = \psi_G = \psi_X\,, \qquad M_G = 0\,. \tag{3.3.56c}$$

where ψ_G is the Goldston fermion.

The scalar mass squared matrix is

$$M_0^2 = \begin{array}{c} \\ \varphi_A^* \\ \varphi_Y^* \\ \varphi_A \\ \varphi_Y \end{array} \begin{array}{c} \begin{array}{cccc} \quad\varphi_A \quad & \quad\varphi_Y\quad & \quad\varphi_A^*\quad & \quad\varphi_Y^* \end{array} \\ \begin{pmatrix} 4\lambda^2X^2+g^2 & 2\lambda gX & -2\lambda^2M^2 & 0 \\ 2\lambda gX & g^2 & 0 & 0 \\ -2\lambda^2M^2 & 0 & 4\lambda^2X^2+g^2 & 2\lambda gX \\ 0 & 0 & 2\lambda gX & g^2 \end{pmatrix} \end{array} . \tag{3.3.57}$$

The eigen states and eigen values are following

$$\varphi_{1,2} \approx \varphi_A - \left(1 + \frac{g^2(1 \pm g^2/2\lambda^2M^2)}{4\lambda^2X^2}\right)\varphi_A^* + \frac{g^2}{2\lambda X}\left(1 - \frac{g^2 \pm 2\lambda^2M^2}{4\lambda^2X^2}\right)\varphi_Y -$$

$$- \frac{g}{2\lambda X}\left(1 \pm \frac{g^4 - 4\lambda^4M^4}{8\lambda^4M^2X^2}\right)\varphi_Y^*\,, \tag{3.3.58a}$$

$$M_{1,2}^2 = 4\lambda^2X^2 + 2g^2 \pm 2\lambda^2M^2\,, \tag{3.3.58b}$$

$$\varphi_{3,4} \approx \varphi_A - \frac{8\lambda^2X^2}{g^2 \mp 2\lambda^2M^2}\varphi_A^* - \frac{2\lambda gX}{g^2 \mp 2\lambda^2M^2}\varphi_Y + \frac{16g\lambda^3X^3}{(g^2 \mp 2\lambda^2M^2)^2}\varphi_Y^*\,, \tag{3.3.59a}$$

$$M_{3,4}^2 = \pm 2\lambda^2M^2 + O(\tfrac{1}{X^2})\,, \tag{3,3.59b}$$

$$\text{For } \varphi_X\,, \qquad M_X^2 = 0\,. \tag{3.3.60}$$

To construct a realistic supersymmetric grand unified model one has to embed a gauge group, say SU(N), into the supersymmetric theory. The O'Raifeartaigh type model in such case can usually be made from two complex fields A, Y in the adjoint representation and a singlet X, for instance, in SU(5) model, O'Raifeartaigh sector is

$$W = \lambda_1 \,\mathrm{Tr}\, A^2 Y + \lambda_2 X \left(\mathrm{Tr}\, A^2 - M^2\right) , \tag{3.3.61}$$

where A, Y belon to <u>24</u> representations of SU(5), X is a singlet.

However, we have found that O'Raifeartaigh model could not work in SU(2N) case,beacuse in this case we can find such a solution in the O'Raifeartaigh sector so that supersymmetry remains unboken. Indeed,from (3.3.61) one may obatain that

$$\frac{\partial W}{\partial A^i_j} = 2\lambda_1 \left[(AY)^j_i - \frac{1}{2N}\delta^j_i \,\mathrm{Tr}\, AY \right] + 2\lambda_2 X A^j_i , \tag{3.3.62a}$$

$$\frac{\partial W}{\partial Y^i_j} = \lambda_1 \left[(A^2)^j_i - \frac{1}{2N}\delta^j_i \,\mathrm{Tr}\, A^2 \right] , \tag{3.3.62b}$$

$$\frac{\partial W}{\partial X} = \lambda_2 \left(\mathrm{Tr}\, A^2 - M^2\right) . \tag{3.3.62c}$$

We observe from (3.3.62b)that the matrix $A^2 \propto I$, in which I is the identical matrix of SU(2N). Therefore, one can find the following vacuum

$$\begin{aligned} A &= \frac{M}{\sqrt{2N}}\, \mathrm{diag}\,(\underbrace{1\ 1 \cdots 1}_{N}\ \underbrace{-1\ -1 \cdots -1}_{N}) \\ Y &= A , \quad \text{and} \quad X = 0 \end{aligned} \tag{3.3.63}$$

so that all Eqs. $\frac{\partial W}{\partial A} = 0, \frac{\partial W}{\partial Y} = 0$ and $\frac{\partial W}{\partial X} = 0$ are consistant. Hance supersymmetry remains unbroken.In contrast to SU(2N) case, it is impossible to find such a solution like (3.3.63) when the gauge group is SU(2N+1). Therefore, in SU(2N+1) case, O'Raifeartaigh model is available.

<u>Conclusion:</u>

In contrast to D-breaking of Fayet-Iliopoulos type, supersymmetry breaking is achived in O'Raifeartaigh model by F-term.Since in O'Raifeartaigh model the X feild can not determined at tree level and the masses of the particles depende on X, this makes the possiblity to generate the gauge hierarchy from radiative corrections, as what Witten has done. For SU(2N) gauge symmetry O'Raifeartaigh mechanism may not be avaliable. Even if for U(1) case, we also have found that supersymmetry can be broken by O'Raifeartaigh model , but the masses of the particles may be independent on X field, in addtion, the U(1) gauge coupling is not asymptotic free , this may **lead** difficulty to generating the gauge hierarchy in terms of Witten's mechanism.

3.3.4 Ovrut-Wess mechanism[3.10]

Ovrut and Wess have presented a new mechanism to break a global supersymmetry by the M-term if a supersymmetric gauge theory is embedded in N=1 supergravity. Such the breaking of supersymmetry is manifested by the explicit operators with dimension two or less, so it is soft.

As we know, the action for supersymmetric gauge theories before the coupling tosupergravity is

$$S=\int d^8z\, \varphi^+ e^{gV}\varphi + \frac{1}{2g^2C_2(A)} \mathrm{Tr}\int d^6s\, W^\alpha W_\alpha - \\ -\left(\int d^6s \left(f\varphi + \frac{m}{2}\varphi^2 + \frac{\lambda}{3}\varphi^3 \right) + h.c \right) , \qquad (3.3.64)$$

where $$W_\alpha = -\frac{1}{4}\bar{D}^2\left(e^{-gV} D_\alpha e^{gV} \right) \qquad (3.3.65)$$

and φ(V) is a chiral (vector) superfield. $C_2(A)$ is the second-order Casimir invariant. It has been shown[3.11] that when S is coupled to supergravity it becomes

$$S=\int d^6s\,\mathcal{Z}\,\Xi\,\varphi+\frac{1}{2g^2C_2(A)}\,\mathrm{Tr}\int d^6s\,\mathcal{Z}\,\mathcal{M}^\alpha\mathcal{M}_\alpha-$$
$$-\left[c^+\int d^6s\,\mathcal{Z}\left(-\tfrac{1}{4}\bar{\mathcal{D}}^2-8R\right)\varphi^++h.c.\right]-\left[\int d^6s\,\mathcal{Z}\left(f\varphi+\tfrac{m}{2}\varphi^2+\tfrac{\lambda}{3}\varphi^3\right)+h.c.\right] \tag{3.3.66}$$

where
$$\mathcal{M}_\alpha=-\tfrac{1}{4}\left(\bar{\mathcal{D}}^2-8R\right)\left(e^{-gV}\mathcal{D}_\alpha e^{gV}\right) \tag{3.3.67}$$

$\mathcal{D}_\alpha$ is the covariant derivative in curved superspace and R is the scalar curvature superfield . The chiral density supermultiplet, $\mathcal{Z}$, is given by

$$\mathcal{Z}=a+\theta\rho+\theta^2\hat{f} \tag{3.3.68}$$

where

$$\begin{aligned}a&=e\\ \rho&=ie\,\sigma^a\bar{\psi}_a\\ \hat{f}&=-eM^+-\tfrac{e}{4}\bar{\psi}_m\left(\bar{\sigma}^m\sigma^n-\bar{\sigma}^n\sigma^m\right)\bar{\psi}_n\end{aligned} \tag{3.3.69}$$

in which $e=\sqrt{-\det g_{ab}}$, ψ_a is the gravitino, and M is scalar auxilary field in the supergravity multiplet. The Ξ is defined by

$$\Xi=-\tfrac{1}{4}\left(\bar{\mathcal{D}}^2-8R\right)\varphi^+e^{gV} \tag{3.3.70}$$

We must add to (3.3.66) the pure supergravity action, $\kappa^{-2}S_{S.G}$,whose precise form need not concern us. One then add the following term

$$S_c=-\frac{\Delta^+}{\kappa^2}\int d^6s\,\mathcal{Z}\;+h.c \tag{3.3.71}$$

to the action. This term is called the M-term.

We can henceforth ignore the terms of $O(\kappa)$ and take the flat space limit $g_{\mu\nu}\to\eta_{\mu\nu}$, if the graviton and gravitino interactions are turned off and the cosmological constant cancelled. It follows that e=1 and ψ_a =0 , Evaluating above the terms in this limit one finds

1) $$\int d^6s\,\mathcal{Z}\,\Xi\,\varphi=\int d^8z\;\tilde{\varphi}^+e^{gV}\tilde{\varphi} \tag{3.3.72}$$

where $\tilde{\varphi}$ is φ with the complex auxiliary field F replaced by

$$\mathcal{F} = F - \frac{1}{3}M^{+}A \quad , \tag{3.3.73}$$

2) $$\int d^6 s\, \mathcal{Z}\, \mathcal{M}^{\alpha}\mathcal{M}_{\alpha} = \int d^6 s\, W^{\alpha}W_{\alpha} \quad , \tag{3.3.74}$$

3) $$\kappa^{-2} S_{S.G.} = -\kappa^{-2} M^{+}M \quad , \tag{3.3.75}$$

4) $$\int d^6 s\, \mathcal{Z}\left[-\frac{1}{4}\left(\bar{\mathcal{D}}^2 - 8R\right)\varphi^{+}\right] = \int d^8 z\, \mathcal{Z}\, \tilde{\varphi}^{+} \quad . \tag{3.3.76}$$

Writing F in terms of $\mathcal{F}$, we find that all $M^{+}A^{3}$ terms (and their hermition conjugates) cancell. This cancellation insures that the dimension 3 opertors, which may break supersymmetry and destroy renormalizability, do not appear in the action. the equation of motion for M is found to be

$$M = \Delta^{+} + \kappa^{2}\left(\frac{2}{3}fA + \frac{m}{6}A^{2} + C^{+}\mathcal{F}^{+}\right) \tag{3.3.77}$$

Eliminating M using (3.3.77) and dropping terms of $O(\kappa)$, the total action becomes

$$\begin{aligned} S = &\int d^8 z\, \tilde{\varphi}^{+} e^{gV} \tilde{\varphi} + \frac{1}{2g^2 C_2(A)} \mathrm{Tr}\int d^6 s\, W^{\alpha}W_{\alpha} - \\ &- \left[\int d^6 s\left(f'\tilde{\varphi} + \frac{m}{2}\tilde{\varphi}^{2} + \frac{\lambda}{3}\tilde{\varphi}^{3}\right) + h.c\right] + \\ &+ \int d^4 x \left\{\frac{\Delta\Delta^{+}}{\kappa^{2}} + \left[\left(\frac{2}{3}\Delta f A + \frac{\Delta m}{6}A^{2}\right) + h.c.\right]\right\} \quad , \end{aligned} \tag{3.3.78}$$

where $f' = f - \Delta^{+}C$. Supersymmetry is obviously broken by the last term in Eq.(3.3.78).

Conclusion:

Ovrut-Wess mechanism is essentially a type of a F-term breaking, but different from O'Raifeartaigh model. O'Raifeartaigh model is rather ad hoc ,since the breaking terms are put by hand. The breaking term in Ovrut-Wess mechanism is induced. So it would be interesting to apply this new mechanism to the construction of a realistic supersymmetric grand unified model.

Conclution:

We have discussed how supersymmetry can be broken either explicitly but softly with the dimension $d \leqslant 3$ operators, or spontaneously by Fayet-Iliopoulos D-term,or O'Raifeartaigh model, or Ovrut-Wess mechanism. In conclusion, one would like to make the following two remarks:

1) Supersymmetry may also be broken dynamically [1.3] ,for instance, in terms of instantons, or condensates, but unfortunately, so far no realistic (even toy) model has been constructed along this line . On the contrary, it seems that condensates, for example, $\lambda\lambda$ condensate in which λ is a gaugino, could not break supersymmetry but only chiral symmetry [3.13] .

2) The explicit and soft breaking terms in section 3.3.a. can be derived from the spontaneous supersymmetry breaking by the aid of Slavnov mechanism [3.12] . Thus in this view, all the soft breakings are spontaneous rather then explicit.

Chapter 4 : PHENOMENOLOGICAL PREDICTIONS OF SUPERSYMMETRIC GRAND UNIFIED MODELS

It has been shown in chapter 2 that supersymmetric grand unified theories are found to be succesful as a candidate of avoiding the gauge hierarchy puzzle . We shall procced to discuss in this chapter some other phenomenological predictions,for instance, the unification mass scale M_G, $\sin^2\theta_W$, m_b/m_τ, proton decay, CP-violation and N-$\bar{N}$ oscillation , in supersymmetric grand unifield theories, particularly, SU(5) and SO(10) SGUM's .

4.1 Unification mass scale M_G and the weak angle $\sin^2\theta_W$:

As is well known, the conventional grand unified models(such as the Geogi-Glashow SU(5) model) have scored one very impressive sucsses. They predict

$$\sin^2\theta_W(M_W) = 0.216 + 0.004 (N_H - 1) - 0.006\ln(\Lambda_{\overline{MS}} / 0.1 \text{ Gev}) \qquad (4.1.1)$$

where N_H is the number of relatively light Higgs doublets and $\Lambda_{\overline{MS}}$ is the QCD scale parameter evaluated in the modified minimal subtraction . For the minimal case $N_H=1$, and using $\Lambda_{\overline{MS}}=0.1$Gev obtained from Upsilon decay, Eq.(4.1.1) gives $\sin^2\theta_W(M_W) = 0.216$. This prediction is in remarkably good agreement with the average experimental value obtained from deep-inelastic -hadron scattering (including radiative corrections)

$$\sin^2\theta_W(M_W) = 0.215 \pm 0.010 \pm 0.004 \qquad (4.1.2)$$

where $\pm$ 0.004 being the approximate theoretical uncertainty.

The powerful tool to estimate M_G and $\sin^2\theta_W$ is the renormalization group analysis, which is the same as the non-supersymmetric case except of the β- function being modified to

$$b_N = 3N - \sum_i T(\phi_i) \tag{4.1.3}$$

where $T(\phi)$ is the index of the representation, ϕ, and is defined by

$$\mathrm{tr}\, T_i T_j = T(\phi)\,\delta_{ij} \tag{4.1.4}$$

The formule (4.1.3) is easily proved by noting that [4.6]

$$b_N = \frac{11}{3}N - \frac{4}{3}T(f) - \frac{1}{6}T(s) \tag{4.1.5}$$

in which the first term is the gauge-boson contribution and the second and third terms denote the fermion and scalar-boson contribution, respectively. Since in the supersymmetric case a vector supermultiplet contains the gauge boson and their Majorana fermion partners (gauginos), and a Chiral supermultiplet contains the Majorana fermions and their complex scalar partners, it follows that

$$b_N = \left(\frac{11}{3}N - \frac{4}{3}\times\frac{1}{2}N\right) - \sum_i\left(\frac{4}{3}\times\frac{1}{2}T(\phi_i) - \frac{1}{6}\times 2\,T(\phi_i)\right) = 3N - \sum_i T(\phi_i) \, . \tag{4.1.6}$$

4.1.1 SU(5) SGUM.

As we know, the standard SU(5) SGUM contains quark and lepton supermultiplets Q_{10} and $Q_{\bar{5}}$, as well as Higgs supermultiplets in the adjoint Σ_{24} and the fundamental representations (H_5, $H_{\bar{5}}$), except for the gauge supermultiplet.

We assume that supersymmetry is broken at the M_W mass scale. Then by integrating the renormalization group equations up to two loop approximation for the gauge couplings of $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$ we obtain [4.7]

$$\alpha_3^{-1}(M_W) = \alpha_3^{-1}(M_G) - \frac{1}{2\pi}(9-2n_g)\ln\frac{M_G}{M_W} + \frac{1}{4\pi}\left\{ \frac{54-\frac{68}{3}n_g}{9-2n_g}\ln\frac{\alpha_3(M_G)}{\alpha_3(M_W)} + \right.$$

$$\left. + \frac{-3n_g}{6-2n_g-\frac{1}{2}N_H}\ln\frac{\alpha_2(M_G)}{\alpha_2(M_W)} + \frac{-\frac{11}{15}n_g}{-2n_g-\frac{3}{10}N_H}\ln\frac{\alpha_1(M_G)}{\alpha_1(M_W)}\right\},$$

$$\alpha_2^{-1}(M_W) = \alpha_2^{-1}(M_G) - \frac{1}{2\pi}\left(6-2n_g-\frac{1}{2}N_H\right)\ln\frac{M_G}{M_W} + \frac{1}{4\pi}\left\{ \frac{-8n_g}{9-2n_g}\ln\frac{\alpha_3(M_G)}{\alpha_3(M_W)} + \right.$$

$$\left. + \frac{24-14n_g-\frac{7}{2}N_H}{6-2n_g-\frac{1}{2}N_H}\ln\frac{\alpha_2(M_G)}{\alpha_2(M_W)} + \frac{-\frac{2}{5}n_g-\frac{3}{10}N_H}{-2n_g-\frac{3}{10}N_H}\ln\frac{\alpha_1(M_G)}{\alpha_1(M_W)}\right\},$$

$$\alpha_1^{-1}(M_W) = \alpha_1^{-1}(M_G) - \frac{1}{2\pi}\left(-2n_g-\frac{3}{10}N_H\right)\ln\frac{M_G}{M_W} + \frac{1}{4\pi}\left\{ \frac{-\frac{88}{15}n_g}{9-5n_g}\ln\frac{\alpha_3(M_G)}{\alpha_3(M_W)} + \right.$$

$$\left. + \frac{-\frac{6}{5}n_g-\frac{9}{10}N_H}{6-2n_g-\frac{1}{2}N_H}\ln\frac{\alpha_2(M_G)}{\alpha_2(M_W)} + \frac{-\frac{38}{15}n_g-\frac{9}{50}N_H}{-2n_g-\frac{3}{10}N_H}\ln\frac{\alpha_1(M_G)}{\alpha_1(M_W)}\right\},$$

$$(4.1.7)$$

with the relationship

$$\alpha^{-1}(M_W) = \alpha_2^{-1}(M_W) + \frac{5}{3}\alpha_3^{-1}(M_W) \quad , \qquad (4.1.8)$$

which comes from

$$Q = T_3 + \frac{Y}{2} \quad \text{and} \quad \frac{Y}{2} = \sqrt{\frac{5}{3}}\, T_B \quad ,$$

where T_B is the normalized $U(1)_Y$ generator, and also by using the boundary conditions [4.8]

$$\alpha_1^{-1}(M_G) = \alpha_2^{-1}(M_G) - \frac{1}{6\pi} = \alpha_3^{-1}(M_G) - \frac{1}{4\pi} \quad . \qquad (4.1.9)$$

n_g and N_H in Eq.(4.1.7) are the number of fermion generations and Higgs $SU(2)_L$ doublets, respectively.

It follows from (4.1.7), (4.1.8)and (4.1.9) that

$$\frac{\alpha(M_W)}{\alpha_3(M_W)} = \frac{3}{8}\Big\{1 - \frac{\alpha(M_W)}{2\pi}(18+N_H)\ln\frac{M_G}{M_W} + \frac{\alpha(M_W)}{2\pi} + \frac{\alpha(M_W)}{4\pi}\Big(\frac{144-\frac{384}{9}n_g}{9-2n_g}\ln\frac{\alpha_3(M_G)}{\alpha_3(M_W)} +$$

$$+\frac{-24+8n_g+5N_H}{6-2n_g-\frac{1}{2}N_H}\ln\frac{\alpha_2(M_G)}{\alpha_2(M_W)} + \frac{\frac{8}{3}n_g+\frac{3}{5}N_H}{-2n_g-\frac{3}{10}N_H}\ln\frac{\alpha_1(M_G)}{\alpha_1(M_W)}\Big\} \tag{4.1.10}$$

and

$$\sin^2\theta_W(M_W) = \frac{3}{8}\Big\{1 - \frac{\alpha(M_W)}{2\pi}\left(10-\frac{1}{3}N_H\right)\ln\frac{M_G}{M_W} + \frac{5\alpha(M_W)}{18\pi} +$$

$$+\frac{\alpha(M_W)}{4\pi}\Big(\frac{-\frac{32}{9}n_g}{9-2n_g}\ln\frac{\alpha_3(M_G)}{\alpha_3(M_W)} + \frac{40-\frac{64}{3}n_g-\frac{13}{3}N_H}{6-2n_g-\frac{1}{2}N_H}\ln\frac{\alpha_2(M_G)}{\alpha_2(M_W)} +$$

$$\frac{\frac{32}{9}n_g-\frac{1}{5}N_H}{-2n_g-\frac{3}{10}N_H}\ln\frac{\alpha_1(M_G)}{\alpha_1(M_W)}\Big)\Big\} \tag{4.1.11}$$

Carrying out the analysis described above for $n_g = 3, N_H = 2$ or 4 and a range of $\Lambda_{\overline{MS}}$ values, we find M_G and $\sin^2\theta_W$ given in table I and II.

$\Lambda_{\overline{MS}}$ (Gev)	M_W (Gev)	$\sin^2\theta_W(M_W)$	M_G (Gev)	$\alpha_3(M_W)$	$\alpha^{-1}(M_W)$
0.05	78.2	0.248	2.1×10^{15}	0.093	127.68
0.10	78.8	0.239	4.8×10^{15}	0.102	127.65
0.20	79.4	0.235	1.1×10^{16}	0.113	127.63
0.30	79.7	0.233	1.7×10^{16}	0.122	127.59
0.40	80.0	0.232	2.4×10^{16}	0.128	127.58

TableI. SU(5) SGUM predictions for $N = 2$.

$\Lambda_{\overline{MS}}$ (Gev)	M_W (Gev)	$\sin^2\theta_W(M_W)$	$M_G(10^{14}$ Gev)	$\alpha_3(M_W)$	$\alpha^{-1}(M_W)$
0.05	75.0	0.263	1.3	0.093	127.75
0.10	75.5	0.260	2.6	0.103	127.72
0.20	75.8	0.258	5.5	0.114	127.70
0.30	76.1	0.256	8.5	0.122	127.67
0.4.	76.3	0.255	12	0.129	127.66

Table II. SU(5) SUGM predications fOr $N = 4$.

From the above numerical analysis we learn that the predictions of supersymmdtry grand unified models are very sensitive to the Higgs content of the theory. For $\Lambda_{\overline{MS}} \approx 0.1$ Gev, $\sin\theta_W(M_W)$ and M_G are found to be

$$\text{i)} \quad M_G \approx 4.8 \times 10^{15} \text{ Gev}, \quad \sin^2\theta_W \approx 0.239 \quad \text{for } N_H = 2,$$

$$\text{ii)} \quad M_G \approx 2.6 \times 10^{14} \text{ Gev}, \quad \sin^2\theta_W \approx 0.260 \quad \text{for } N_H = 4 \quad . \qquad (4.1.12)$$

Obviously, $\sin^2\theta_W$ is uncomfortably large when compared to the experiment value (4.1.2). This a well-known problem in SU(5) SGUM.

4.1.2 SO(10)SGUM.

All the coupings in the SO(10) SGUM are α_{10}, α_4, α_{2L}, α_{2R}, α_3, α_2, α_1 and α corresponded to SO(10), $SU(4)_c$, $SU(2)_L$, $SU(2)_R$, $SU(3)_c$, $SU(2)_L$, $U(1)_Y$ and $U(1)_{em}$, respectively.

Having integrated the one-loop renormalization group equations by using standard procedures, one may obtain the solutions for the running coupling constant as follows

$$\begin{aligned}
\alpha_4^{-1}(M_2) &= \alpha_4^{-1}(M_1) - \frac{b_4}{2\pi} \ln \frac{M_1}{M_2}, \\
\alpha_{2L}^{-1}(M_2) &= \alpha_{2L}^{-1}(M_1) - \frac{b_{2L}}{2\pi} \ln \frac{M_1}{M_2}, \\
\alpha_{2R}^{-1}(M_2) &= \alpha_{2R}^{-1}(M_1) - \frac{b_{2R}}{2\pi} \ln \frac{M_1}{M_2}, \\
\alpha_3^{-1}(M_W) &= \alpha_3^{-1}(M_2) - \frac{b_3}{2\pi} \ln \frac{M_2}{M_W}, \\
\alpha_2^{-1}(M_W) &= \alpha_2^{-1}(M_2) - \frac{b_2}{2\pi} \ln \frac{M_2}{M_W}, \\
\alpha_1^{-1}(M_W) &= \alpha_1^{-1}(M_2) - \frac{b_1}{2\pi} \ln \frac{M_2}{M_W},
\end{aligned} \qquad (4.1.13)$$

with the boundary conditions

$$\alpha_4^{-1}(M_1) = \alpha_{2L}^{-1}(M_1) = \alpha_{2R}^{-1}(M_1) = \alpha_{10}^{-1}(M_1) \equiv \alpha_{SGUM}^{-1} ,$$

$$\alpha_3^{-1}(M_2) = \alpha_4^{-1}(M_2) ,$$

$$\alpha_2^{-1}(M_2) = \alpha_{2L}^{-1}(M_2) , \qquad (4.1.14)$$

and the relationships

$$\frac{5}{3}\alpha_1^{-1}(M_2) = \alpha_{2R}^{-1}(M_2) + \frac{2}{3}\alpha_4^{-1}(M_2) ,$$

$$\alpha^{-1}(M_W) = \alpha_2^{-1}(M_W) + \frac{5}{3}\alpha_1^{-1}(M_W) . \qquad (4.1.15)$$

Since in the SO(10) model,

$$\sqrt{\frac{5}{3}}\, T_0 = T_{3R} + \sqrt{\frac{2}{3}}\, T_C ,$$

$$Q = T_{3L} + \sqrt{\frac{5}{3}}\, T_B ,$$

$$\sqrt{\frac{5}{3}}\, T_B = \frac{Y}{2} . \qquad (4.1.16)$$

where Q is an electrocharge operator, T_{3L}, T_{3R} are generators of the $SU(2)_L$ $SU(2)_R$ subgroups, T_c is a generator of the $SU(4)_c$ subgroup, and T_B is the generator of $U(1)_Y$, Y is the weak-hypercharge.

From Eqs.(4.1.13)—(4.1.16), one obtains

$$\frac{\alpha(M_W)}{\alpha_3(M_W)} = \frac{3}{8}\left\{1 + \frac{\alpha}{2\pi}\left[(b_{2L}+b_{2R}-2b_4)\ln\frac{M_1}{M_2} + \left(\frac{5}{3}b_1+b_2-\frac{8}{3}b_3\right)\ln\frac{M_2}{M_W}\right]\right\} \qquad (4.1.17)$$

$$\sin^2\theta_W(M_W) = \frac{3}{8}\left\{1+\frac{\alpha}{2\pi}\left[\left(\frac{2}{3}b_4+b_{2R}-\frac{5}{3}b_{2L}\right)\ln\frac{M_1}{M_2} + \frac{5}{3}(b_1-b_2)\ln\frac{M_2}{M_W}\right]\right\} \qquad (4.1.18)$$

It should be born in mind that in the SO(10) SGUM there exist a $\underline{45}$ vector supermultiplet, a $\underline{16}$ matter chiral supermultiplet, a $\underline{54}$ Higgs chiral supermultiplet, and $\underline{16}^*$ Higgs chiral supermultiplets, as well as two $\underline{10}$

Higgs chiral supermultiplets.

As we know, the 54 Higgs and the colour triplets in two 10 Higgs are superheavy, they give no contribution to the β-functions. For the 16 and 16*Higgs, one may assume that the (4,2,1) component of 16 and $(\bar{4},2,1)$ compont of 16*Higgs have masses of the order M_1, while the $(\bar{4},1,2)$ componet of 16 and (4,1,2) compont of 16* have masses of the order M_W.

Therefore, when $M_2 < \mu < M_1$, the surviving particles are:

$(\bar{4},1,2)$ component of 16 Higgs supermultiplet;

(4,1,2) component of 16*Higgs supermultiplet;

16 matter supermultiplet;

the four doublets in two 10 Higgs supermultiplets;

(15,1,1),(1,3,1) and (1,1,3) gauge bosons of $SU(4)_C$,$SU(2)_L$ and $SU(2)_R$ and their fermion partners.

When $M_W < \mu < M_2$, the only surviving parlticles are:

16 matter supermultiplet; $(\bar{3},1)_{2/3}$, $(\bar{3},1)_{-4/3}$, $(1,1)_2$ and $(1,1)_0$ components of 16 Higgs supermultiplet; $(3,1)_{-2/3}$, $(3,1)_{4/3}$, $(1,1)_{-2}$ and $(1,1)_0$ components of 16* Higgs multiplet;

the four doublets in two 10 Higgs supermultiplets,

the $(8,1)_0$, $(1,3)_0$, and B gauge bosons of $SU(3)_C$,$SU(2)_L$ and $U(1)_Y$ and their fermion partners.

In table III, we display b_N at one-loop approximation for three generations.#

b_4	b_{2L}	b_{2R}	b_3	b_2	b_1
4	-2	-6	1	-2	-52/5

Table III: b_N values in the SO(10)SGUM at one-loop level for $n_g = 3$.

The b_N values here are different from those in Ref. 4.9 since 16* Higgs supermultiplet has been introduced here while it is not in Ref. 4.9 .

Substituting the above b_N values into Eqs. (4.1.17), (4.1.18), we get

$$\frac{\alpha(M_W)}{\alpha_3(M_W)} = \frac{3}{8}\left\{1 - \frac{\alpha}{\pi}\left(8 \ln \frac{M_1}{M_2} + 11 \ln \frac{M_2}{M_W}\right)\right\}, \tag{4.1.19}$$

$$\sin^2\theta_W(M_W) = \frac{3}{8}\left(1 - \frac{7\alpha}{\pi} \ln \frac{M_2}{M_W}\right) . \tag{4.1.20}$$

INorder to determine M_1, M_2 and $\sin^2\theta_W$, one chooses as imputs the electromagnetic coupling constant $\alpha(M_W)$ and the strong coupling $\alpha_3(M_W)$ determined from low energy physics

$$\alpha(M_W) = 1/128.5 \quad , [4.10] \tag{4.1.21}$$

$$\alpha_3(M_W) = 0.11 \quad , [4.11] \tag{4.1.22}$$

where the values of $\alpha_3(M_W)$ used here are based on the experimental values $\alpha_3(10\text{Gev}) = 0.1476 \pm 0.9994$ with $\Lambda_{\overline{MS}} = 157^{+55}_{-40}$ Mev, from the analysis of experimental data in e^+e^- deep inelastic structure functions and purely hadronic reactions.

We also need to introduce as imput the ratio of two mass scales M_1, M_2. Let us define

$$R = \ln \frac{M_1}{M_2} . \tag{4.1.23}$$

In table IV, we display the mass scales M_1, M_2 and the weak angle $\sin^2\theta_W(M_W)$ as functions of R for both the SO(10) SGUM and SO(10) GUM. Notice that M_1 and $\sin^2\theta_W(M_W)$ monotonically increase with R, whereas M_2 decreases.

Model	SO(10) SGUM				SO(10) GUM			
R	0	2	4	7	0	2	4	10
$M_1(10^{15}\text{Gev})$	0.73	1.3	2.2	5.0	0.73	1.4	2.7	20
$M_2(10^{15}\text{Gev})$	0.73	0.I7	0.04	4.1×10^{-3}	0.73	0.19	0.05	9×10^{-3}
$\sin^2\theta_W(M_W)$	0.181	0.190	0.200	0.215	0.206	0.244	0.282	0.396

Table IV: M_1, M_2 and $\sin^2\theta_W(M_W)$ as functions of R for both the SO(10)SGUM and SO(10)GUM.

It can be seen from table IV that if choosing R = 7 , then $\sin^2\theta_W(M_W) = 0.215$ which is just its experionental value (4.1.2). This unification mass scale $M_1 \approx 5\times10^{15}$ Gev. Thus unification mass scale is quite close to that in SU(5)GUM with two doublet Higgs. (See (4.1.12)). However, in contrast to SU(5) GUM, the SO(10)SGUM has no problem with $\sin^2\theta_W$.

Conclusion

Unification mass scale M_G and the weak angle $\sin^2\theta_W$ are model-dependent, particulary dependent on the Higgs content of the theory. When the number of SU(2) Higgs doublets increases, then M_G will decrease whereas $\sin^2\theta_W$ will increase . The SU(5)SGUM has a difficulty with $\sin^2\theta_W$, which is too large to compare with the experimental value, but the SO(10) SGUM has no such a problem.

4.2 Quark-lepton Mass Ratios m_b/m_τ, m_t/m_b and m_τ/m_{ν_τ} .

Another succesful prediction of the conventional grand onified models is the ratio m_b/m_τ .

In supersymmetric grand unified theories both γ_m (mass anomalous dimension) and b_N (β - function) will be changed. The general formula for the anomalous dimension γ_m is changed from

$$\gamma_m = 6 \sum_{a,j} (T_L^a)_{ij} (T_R^a)_{ji} \qquad (4.2.1)$$

in the non -supersummetric case to [4.7, 4.9]

$$\gamma_m = 4 \sum_{a,j} (T_L^a)_{ij} (T_R^a)_{ji} \qquad (4.2.2)$$

in the supersymmetric model , where $T^a_{L(R)}$ denotes the generator T^a in the representation of the left (right) matter fermions (quarks and leptons). The point is that in addition to the usual self - energy graph (Fig.3.2 - 1a) contributing to the anomalous dimension of the mass operator γ_m , there is also the graph (Fig. 3.2 - 1b) with the gauge fermion and complex scalar partner of the fermion.

(a) (b)

Fig (4 .2-1): Graph contributing to the one-loop anomalous dimension. ψ — chiral fermion, φ —scalar partner of the fermion ψ, A —gauge boson, λ — the gauge fermion. (a) gauge boson contribution (b) gauge fermion contribution.

One now may recall the mass renormalization group equation:

$$\frac{\partial m(\mu)}{\partial \ln \mu} = m(\mu)\,\gamma_m(\mu) \quad . \tag{4.2.3}$$

Since
$$\frac{dg}{d\ln\mu} = \beta(g) \quad ,$$

Eq.(4.2.3) can be rewritten as follows

$$\ln \frac{m(\mu)}{m(\mu_0)} = \int_{g(\mu_0)}^{g(\mu)} \gamma_m(g)\,\frac{dg}{\beta(g)} \quad . \tag{4.2.4}$$

It is well known that at one -loop approximation,

$$\beta(g) = -16\pi^2 b_N g^3 , \quad \gamma_m(g) = -16\pi^2 \gamma_m g^2 , \tag{4.2.5}$$

where b_N is defined in (4.1.3), γ_m is one in (4.2.2) for supersymmetric case. From (4.2.4)and (4.2.5) one finds

$$\frac{m(\mu)}{m(\mu_0)} = \left(\frac{\alpha(\mu)}{\alpha(\mu_0)}\right)^{\gamma_m/2b_N} \tag{4.2.6}$$

If we have semi-simple group $G \simeq \bigcup_i G_i$, then

$$\frac{m(\mu)}{m(\mu_0)} = \prod_i \left(\frac{\alpha_i(\mu)}{\alpha_i(\mu_0)}\right)^{\gamma_m^i/2b_i} \tag{4.2.6}$$

where $\prod_i$ runs over all subgroup G_i.

4.2.1 SU(5) SGUM.

Assuming that supersymmetry breaking occurs at M_W , we find for the SU(5) supersymmetric model with 3 grnerations

$$\frac{m_b}{m_\tau} = \left(\frac{\alpha_3(m_b)}{\alpha_3(m_t)}\right)^{\frac{12}{13}} \left(\frac{\alpha_3(m_t)}{\alpha_3(M_W)}\right)^{\frac{4}{7}} \left(\frac{\alpha_3(M_W)}{\alpha_{SGUM}}\right)^{\frac{8}{9}} \tag{4.2.8}$$

While in the conventional SU(5) model

$$\frac{m_b}{m_\tau} = \left(\frac{\alpha_3(m_b)}{\alpha_3(m_t)}\right)^{\frac{12}{23}} \left(\frac{\alpha_3(m_t)}{\alpha_{GUM}}\right)^{\frac{4}{7}} \quad . \tag{4.2.9}$$

Here m_b , m_t present the masses of b, t-quarks. We have neglected the contribution from the $U(1)_Y$ subgroup, which in both cases gives a correction of O(10 %). For $\alpha_{SGUM} \simeq \frac{1}{24}$, $\alpha_{GUM} \simeq \frac{1}{41}$, at m_t = 20 Gev. we obtain

$$\left.\frac{m_b}{m_\tau}\right|_{SGUM} \approx 3.7 , \qquad \left.\frac{m_b}{m_\tau}\right|_{GUM} \approx 3.4 \quad . \tag{4.2.10}$$

Thus the prediction of the SU(5) SGUM does not differ greatly from that of the SU(5) GUM.

4.2.2 SO(10) SGUM

We assume that $m_b(M_1) = m_\tau(M_1)$ and $m_t(M_1) = m_{\nu_\tau}(M_1)$ at the unification mass scale M_1, and also assume that supersymmetry breaking occurs at M_W. Then the quark and lepton masses at lower energies are renormalized by the radiative corrections, with the following results, at present energies in the SO(10) SGUM,

$$\left.\frac{m_b}{m_\tau}\right|_{SGUM} = \left(\frac{\alpha_3(m_b)}{\alpha_3(m_t)}\right)^{\frac{12}{23}} \left(\frac{\alpha_3(m_t)}{\alpha_3(M_W)}\right)^{\frac{4}{7}} \left(\frac{\alpha_3(M_W)}{\alpha_3(M_2)}\right)^{\frac{8}{3}} \left(\frac{\alpha_1(M_W)}{\alpha_1(M_2)}\right)^{\frac{5}{78}} \left(\frac{\alpha(m_\tau)}{\alpha(m_b)}\right)^{-\frac{27}{76}} \left(\frac{\alpha(m_b)}{\alpha(m_t)}\right)^{-\frac{3}{10}} \left(\frac{\alpha(m_t)}{\alpha(m_W)}\right)^{-\frac{1}{4}} \tag{4.2.11}$$

$$\left.\frac{m_t}{m_b}\right|_{SGUM} = \left(\frac{\alpha_1(M_W)}{\alpha_1(M_2)}\right)^{-\frac{9}{208}} \left(\frac{\alpha(m_b)}{\alpha(m_t)}\right)^{-\frac{3}{80}} \left(\frac{\alpha(m_t)}{\alpha(M_W)}\right)^{\frac{3}{32}} , \tag{4.2.12}$$

$$\left.\frac{m_\tau}{m_{\nu_\tau}}\right|_{SGUM} = \left(\frac{\alpha_1(M_W)}{\alpha_1(M_2)}\right)^{-\frac{27}{208}} \left(\frac{\alpha(m_\tau)}{\alpha(m_b)}\right)^{\frac{27}{76}} \left(\frac{\alpha(m_b)}{\alpha(m_t)}\right)^{\frac{27}{80}} \left(\frac{\alpha(m_t)}{\alpha(M_W)}\right)^{\frac{9}{32}} \quad . \tag{4.2.13}$$

From Eqs.(4.1.13), (4.1.14) and (4.1.15) and $M_1 \approx 5 \times 10^{15}$ Gev, $M \approx 4.1 \times 10^{12}$ Gev we find

$$\alpha_3^{-1}(M_2) \approx 13, \quad \alpha_1^{-1}(M_2) \approx 20, \quad \alpha_2^{-1}(M_2) \approx 20,$$

and

$$\alpha_4^{-1}(M_1) = \alpha_{2L}^{-1}(M_1) = \alpha_{2R}^{-1}(M_1) = \alpha_{SGUM} \approx 24 \quad . \tag{4.2.14}$$

Emplaying the following imputs

$$\alpha_3^{-1}(m_b) \approx 5.7, \quad \alpha_3^{-1}(m_t) \approx 7.0, \quad \alpha_3(M_W) \approx 0.11,$$

$$\alpha_1^{-1}(M_W) \approx 61, \quad \alpha(m_\tau) \approx \alpha(m_b) \approx \alpha(m_t) \approx \alpha(M_W), \tag{4.2.15}$$

those mass ratios in SO(10) SGUM tnrn out to be

$$\left.\frac{m_b}{m_\tau}\right|_{SGUM} \approx 3.2,$$

$$\left.\frac{m_t}{m_b}\right|_{SGUM} \approx 1.1,$$

$$\left.\frac{m_\tau}{m_{\nu_\tau}}\right|_{SGUM} \approx 1.2 \quad . \tag{4.2.16}$$

On the other hand, the above mass ratios in the conventional SO(10) GUM (ignoring Higgs contribution) are given by

$$\left.\frac{m_b}{m_\tau}\right|_{GUM} = \left(\frac{\alpha_3(m_b)}{\alpha_3(m_t)}\right)^{\frac{12}{23}} \left(\frac{\alpha_3(m_t)}{\alpha_3(M_2)}\right)^{\frac{4}{7}} \left(\frac{\alpha_1(M_W)}{\alpha_1(M_2)}\right)^{\frac{1}{4}} \times$$
$$\left(\frac{\alpha(m_\tau)}{\alpha(m_b)}\right)^{-\frac{27}{76}} \left(\frac{\alpha(m_b)}{\alpha(m_t)}\right)^{-\frac{3}{10}} \left(\frac{\alpha(m_t)}{\alpha(M_W)}\right)^{-\frac{1}{4}}, \tag{4.2.17}$$

$$\left.\frac{m_t}{m_b}\right|_{GUM} = \left(\frac{\alpha_1(M_W)}{\alpha_1(M_X)}\right)^{-\frac{3}{40}} \left(\frac{\alpha(m_b)}{\alpha(m_t)}\right)^{-\frac{3}{80}} \left(\frac{\alpha(m_t)}{\alpha(M_W)}\right)^{\frac{3}{32}} , \qquad (4.2.18)$$

$$\left.\frac{m_\tau}{m_{\nu_\tau}}\right|_{GUM} = \left(\frac{\alpha_1(M_W)}{\alpha_1(M_X)}\right)^{-\frac{9}{40}} \left(\frac{\alpha(m_\tau)}{\alpha(m_b)}\right)^{\frac{27}{76}} \left(\frac{\alpha(m_b)}{\alpha(m_t)}\right)^{\frac{27}{80}} \left(\frac{\alpha(m_t)}{\alpha(M_W)}\right)^{\frac{9}{32}} . \qquad (4.2.19)$$

Using the imputs

$$\alpha_3^{-1}(M_X)_{GUM} \approx \alpha_1^{-1}(M_X)_{GUM} \approx 47 , \qquad (4.2.20)$$

one obtains

$$\left.\frac{m_b}{m_\tau}\right|_{GUM} \approx 3.1 ,$$

$$\left.\frac{m_t}{m_b}\right|_{GUM} \approx 1.0 ,$$

$$\left.\frac{m_\tau}{m_{\nu_\tau}}\right|_{GUM} \approx 1.1 . \qquad (4.2.21)$$

By a comparison between Eqs.(4.2.8)—(4.2.10), (4.2.16) and (4.2.21), we find that the predictions of quark-lepton mass ratios are mostly model-independent. However, we also find that the predictions of m_t/m_b and m_τ/m_{ν_τ} are still out of the range of experiment, just like those in the non-supersymmetric SO(10) model. This problem is caused by the wrong assmption of $m_{\nu_\tau}(M_1) = m_t(M_1)$. If one employs both 10 and 126 Higgs for instance to give matter fermion masses, the relation $m_{\nu_\tau}(M_1) = m_t(M_1)$ will be washed away.

Conclusion

The predictions of quark-lepton mass ratios are almostly model-independent. Thus supersymmetric grand unified models share those succesful predictions with the conventional grand unified models.

4.3 Proton Decay.

The most exciting consequence of grand unification is the existence of heavy gauge bosons and Higgs particles which violates baryon number B and lepton number L conservation laws. Such new interaction may cause proton decay. In this section we study proton decay in supersymmetric cases.

4.3.1 Proton life time τ_p.

The proton life time τ_p is determined by the total decay width Γ of the proton, which is given by

$$\Gamma = \lambda A^2 \sigma |\psi(0)|^2 , \qquad (4.3.1)$$

where λ is the SU(6) weight factor. A is the enhancement factor due to the renormalization effect of the effective interactions which are responsible for proton decay. σ denotes the basic cross section for the process $qq \to \bar{q}\bar{l}$,in which q, ℓ denote quark and lepton respectively,

$$\sigma = \frac{g^4}{48\pi M_G^4} \langle m_{qq}^2 \rangle , \qquad (4.3.2)$$

if only dimension six operators are taken into account, M_G is the unification mass. $\langle m_{qq}^2 \rangle$ is the average of the total center-of-momentum energy squared for the process $qq \to \bar{q}\bar{\ell}$.

$$\langle m_{qq}^2 \rangle \approx 0.7 \times \left(\frac{2}{3}\right)^2 m_p^2 \approx 0.274 \text{ Gev}^2 . \qquad (4.3.3)$$

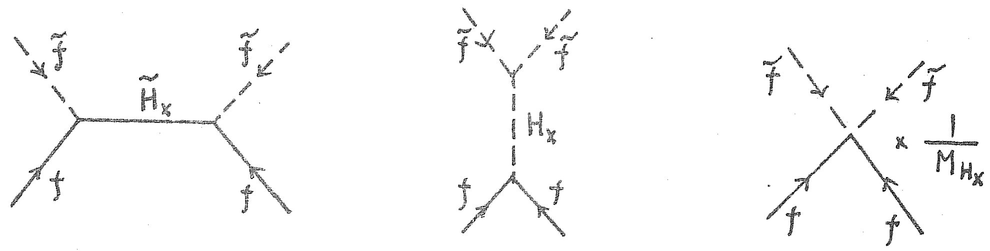
$|\psi(0)|^2$ in Eq.(4.3.1) is the square of the quark wave function inside the proton evaluated at the origin, which has been estimated from an analyses of weak hyperon decay.[4.12] The values of A and σ in supersymmetric grand unified models will be different from those in conventional grand unified models, but one can imagine that λ and $|\psi(0)|^2$ may remain unchanged.

It is well known that a remarkable feature about proton decay in supersymmetric grand unified theories is that apart from the dimension six operators, there are some dimension five operators which may cause faster proton decay, say $\tau_p \sim 10^{24-28}$ years. Weinberg has suggested that such new operators can be ruled out by introducing an extre U(1) symmetry. However, recently J.Ellis et al re-analyed the proton decay of SU(5)SGUM by considering renomalization effects and found that the proton lifetime is compatible with the experiment limit without the need of such U(1) symmtry to kill the dimension five operators.

Here we only pay attention to the dimension five operators. First we discuss the minimal supersymmetric SU(5) model, then SO(10) model.

A. SU(5) SGUM.

The diamension five operators which are dangerous for proton decay in SGUM's are shown in Fig.4.1



(a) (b) (c)

Fig.4.1 Diagrams generating the dimension five operators being responsible for proton decay in SGUM's, a) Higgs-fermion exchange, b) Higgs exchange, c) local two scalar - two fermion operators $\tilde{f}\tilde{f}ff$ which are reduced from both (a) and (b) at low energies $\mu \ll m_{H_X}$.

The corresponding effective Lagrangian would be of order

$$\mathcal{L}_{eff} \approx \frac{g_H^2}{M_{\tilde{H}_X}} \left(\tilde{q}\tilde{q}ql \ \text{ or } \ qqq\tilde{l} \right) , \qquad (4.3.4)$$

where g_H is a typical Higgs-matter Yukawa couping, $\tilde{H}_X$ is the fermion partner of a colour triplet Higgs H_X. $\tilde{q}$ and $\tilde{l}$ are the squark and slepton.

It has been poind out that there exist two possible dimension five operator, which could break proton down [4.14]

$$Q_L = \varepsilon^{abc} \int d^2\theta \; Q_L^a Q_L^b Q_L^c L_L , \qquad (4.3.5)$$

$$Q_R = \varepsilon^{abc} \int d^2\theta \; \mathcal{U}_R^{*a} \mathcal{U}_R^{*b} D_R^{*c} E_R^{*} , \qquad (4.3.6)$$

where Q_L, L_L denote respectively $SU(2)_L$ doublet chiral supermultiplet of the quark and lepton. U_R, D_R and E_R are $SU(2)$ singlet u, d-quark and the right hand electron chiral supermultiplets. The a,b,c are the $SU(3)_c$ colour indexes. In order to get the four-fermions operators for proton decay one must consider the one-loopdiagrams of Fig.4.2, with an exchange of the gaugino $\tilde{W}$, $\tilde{G}$ and $\tilde{B}$.

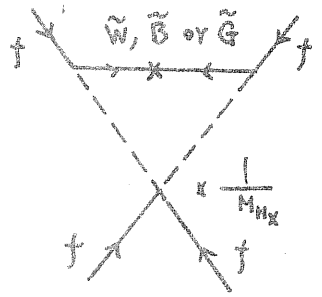


Fig.4.2 A four-fermion operator (ffff) for proton decay generated from Fig.4.1(c) by the exchange of a $\tilde{W}$, $\tilde{G}$ or $\tilde{B}$.

If $M_{\tilde{W}}$ or $M_{\tilde{B}} \gg m_{\tilde{f}}$, the Fig.4.2 can be pinch into the form of Fig.4.3.

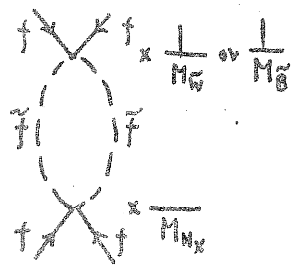


Fig. 4.3 The simple one-loop diagram pinched from Fig.4.2, when $M_{\tilde{W}}$ or $M_{\tilde{B}} \gg m_{\tilde{f}}$.

Thereby we obtain the effective Lagrangian

$$\mathcal{L}_{eff}^{Susy} \approx \frac{g_H^2}{M_{H_X}}\left(\frac{g_2^2}{M_{\tilde{W}}} \text{ or } \frac{g_1^2}{M_{\tilde{B}}}\right)(qqql) \qquad (4.3.7)$$

The loop of Fig.4.3 gives a factor

$$\frac{O(1)}{16\pi^2}\,\frac{g_2^2}{2}\cdot\frac{g_H^2}{M_{\tilde{W}} M_{H_X}} \qquad \text{for } \tilde{W} \text{ exchange} ,$$

$$\frac{1}{16\pi^2}\,\frac{8g_3^2}{3}\cdot\frac{g_H^2 M_{\tilde{G}}}{M_{\tilde{q}}^2 M_{H_X}} \qquad \text{for } \tilde{G} \text{ exchange}$$

$$\frac{1}{16\pi^2}\,\frac{8g_1^2}{3}\cdot g_H^2 M_{\tilde{B}}/M_{\tilde{q}}^2 M_{H_X} \qquad \text{for } \tilde{B} \text{ exchange} \qquad (4.3.8)$$

g_H^2 in (4.3.7) is given by

$$g_H^2 \approx \sqrt{2}\, G_F m_s m_u 2K \qquad (K \geq 1) , \qquad (4.3.9)$$

in which K is an extra factor depending on the ratio of the VEV's of 5 and $\overline{5}$ Higgs.

We now recall the formula of the renormalization enhancement factor A of an operator O for a gauge group $G = \bigotimes_i G_i$,

$$A = \prod_{i,a} \left(\frac{\alpha_i(\mu_{a+1})}{\alpha_i(\mu_a)} \right)^{\gamma_0^i / 2b_i(a)} \qquad (4.3.10)$$

,

where γ_0^i is the anomalous dimension of the operator in G_i group, $b_i(a)$ is the β-function of G_i in the energy range between μ_a to μ_{a+1}. Both γ_0^i and b_i depend on whether supersymmetry is broken. In non-supersymmetric cases, the renormalizations of both wave function and vertex can contribute to the operator anomalous dimension γ_0, but in supersymmetric cases, only wave function renormalizations (no separate vertex renormalizations as those vertices being F-terms) should take into account as conccerns γ_0 . Consequently, it would be better to divite the scales of interest into the range between M_{proton} ($\approx$ 1 Gev) and $M_{\tilde{W}}$ and the range between $M_{\tilde{W}}$ and $M_{\tilde{H}_X}$. In the former supersymmetry is broken, whereas in the latter supersymmetry remains unbroken.

Between 1 Gev and $M_{\tilde{W}} \approx M_W$, the Higgs vertices (4.3.8) get renormalizad downwards, while the operator (4.3.7) gets renormalized upwards by the strong interactions, resulting enhancement A_I is

$$A_I = \left(\frac{\alpha_3(1\mathrm{Gev})}{\alpha_3(m_c)} \right)^{-\frac{6}{9}} \left(\frac{\alpha_3(m_c)}{\alpha_3(m_b)} \right)^{-\frac{18}{25}} \left(\frac{\alpha_3(m_b)}{\alpha_3(m_t)} \right)^{-\frac{18}{23}} \left(\frac{\alpha_3(m_t)}{\alpha_3(M_W)} \right)^{-\frac{6}{7}} . \qquad (4.3.11)$$

Between M_W and $M_{\tilde{H}_X} \approx M_G$, both g_H and the dimension five operators (4.3.5) , (4.3.6) only get renormalized by external wave functions as supersymmetry being unbroken. so the coresponding enhancemet A_{II} is given by

$$A_{II} = \left(\frac{\alpha_3(M_W)}{\alpha_{SGUM}} \right)^{-\frac{4}{9}} \times \begin{cases} \left(\frac{\alpha_2(M_W)}{\alpha_{SGUM}} \right)^{-\frac{3}{2}} \left(\frac{\alpha_1(M_W)}{\alpha_{SGUM}} \right)^{\frac{5}{396}} & \text{for } O_L \\ \left(\frac{\alpha_2(M_W)}{\alpha_{SGUM}} \right)^{\frac{3}{2}} \left(\frac{\alpha_1(M_W)}{\alpha_{SGUM}} \right)^{-\frac{25}{396}} & \text{for } O_R \end{cases} \qquad (4.3.12)$$

To compare with the cnventional SU(5) GUM, we recall the coresponding enhancement A [4.15]

$$A = \left(\frac{\alpha_3(1\,\mathrm{Gev})}{\alpha_3(m_c)}\right)^{\frac{2}{9}} \left(\frac{\alpha_3(m_c)}{\alpha_3(m_b)}\right)^{\frac{6}{25}} \left(\frac{\alpha_3(m_b)}{\alpha_3(m_t)}\right)^{\frac{6}{23}} \left(\frac{\alpha_3(m_t)}{\alpha_{GUM}}\right)^{\frac{2}{7}} \times \left(\frac{\alpha_2(M_W)}{\alpha_{GUM}}\right)^{\frac{27}{38}} \left(\frac{\alpha_1(M_W)}{\alpha_{GUM}}\right)^{-\frac{23}{82}} . \tag{4.3.13}$$

Putting together all the factors (4.3.7), (4.3.8), (4.3.9), (4.3.11),(4.3.12) and (4.3.13), and using the imputs

$$m_s(1\mathrm{Gev}) \approx (150 \text{ to } 500)\ \mathrm{Mev}\ , \quad m_u \approx 0.03\ m_s\ ,$$

$$\alpha_3(M_W) \approx 0.12\ , \quad \alpha_2^{-1}(M_W) \approx 31\ , \quad \alpha_1^{-1}(M_W) \approx 50\ ,$$

$$\alpha_{GUM}^{-1} \approx 41\ , \quad \alpha_{SGUM}^{-1} \approx 24\ , \quad M_{GUM} \approx 4 \times 10^{14}\ \mathrm{Gev}\ , \tag{4.3.14}$$

we obtain

$$A_I \approx 0.91\ , \quad A_{II} \approx 0.22\ , \quad A \approx 3.9$$

and

$$S \equiv \frac{\mathcal{L}_{eff}^{susy}}{\mathcal{L}_{eff}^{non-susy}} \approx (0.37 \text{ to } 4.1) \times 10^{-11} \times (1.6 \times 10^{29}\ \mathrm{Gev}^2) \left\{ \frac{2}{M_{\tilde{W}} M_{\tilde{H}_X}} \text{ or } \frac{20 M_{\tilde{G}}}{M_{\tilde{g}}^2 M_{\tilde{H}_X}} \right\} K . \tag{4.3.15}$$

Assuming $S^2 \lesssim 10$, and using $M_{\tilde{H}_X} \approx M_{SGUM} \approx (0.4 \text{ to } 1.6) \times 10^{16}$ Gev, we find from (4.3.15) that

$$M_{\tilde{W}} \geq (10 \text{ to } 600)\ \mathrm{Gev}\ . \tag{4.3.16}$$

The limit (4.3.16) suggest that the proton life time in SU(5) SGUM is still close to the present experimental lower limit.

B. SO(10) SGUM

In SO(10) model, if SO(10) is broken down to $SU(3)_c \times SU(2)_L \times U(1)_Y$ via

$SU(4)_C \times SU(2)_L \times SU(2)_R$, the effective Lagrangian related to proton decay (dimension six operator) is [4.16]

$$\mathcal{L}_{eff} = G\,\varepsilon_{ijk}\Big\{(\bar{u}^c_{kL}\gamma_\mu u_{jL})(\bar{e}^+_R\gamma_\mu d_{iR} + 2\,\bar{e}^+_L\gamma_\mu d_{iL}) - (\bar{u}^c_{kL}\gamma_\mu d_{jL})(\bar{\nu}^c_{eR}\gamma_\mu d_{iR}) - (\bar{d}^c_{kL}\gamma_\mu u_{jL})(\bar{e}^+_R\gamma_\mu u_{iR}) + \cdots\Big\} \quad (4.3.19)$$

In deriving (4.3.19) the Higgs contributions and the mixing angles between quarks of different generations have been ignored. If we include the renormalization effects, the effectve Lagrangian (4.3.19) becomes

$$\mathcal{L}_{eff} = A\,G\,\varepsilon_{ijk}\Big\{2(\bar{u}^c_{kL}\gamma_\mu u_{jL})\big[b(\bar{e}^+_L\gamma_\mu d_{iL}) + (\bar{e}^+_R\gamma_\mu d_{iR})\big] - 2(\bar{u}^c_{kL}\gamma_\mu d_{jL})(\bar{\nu}^c_{eR}\gamma_\mu d_{iR}) + \cdots\Big\} . \quad (4.3.20)$$

For the conventional SO(10) GUM one obtains

$$A = \left(\frac{\alpha_3(1\,\mathrm{GeV})}{\alpha_3(m_c)}\right)^{\frac{2}{9}}\left(\frac{\alpha_3(m_c)}{\alpha_3(m_b)}\right)^{\frac{6}{25}}\left(\frac{\alpha_3(m_b)}{\alpha_3(m_t)}\right)^{\frac{6}{23}}\left(\frac{\alpha_3(m_b)}{\alpha_3(M_2)}\right)^{\frac{2}{7}} \times \left(\frac{\alpha_2(M_W)}{\alpha_2(M_2)}\right)^{\frac{27}{40}}\left(\frac{\alpha_1(M_W)}{\alpha_1(M_2)}\right)^{-\frac{11}{80}} , \quad (4.3.21)$$

$$b = \left(\frac{\alpha_1(M_W)}{\alpha_1(M_2)}\right)^{-\frac{3}{20}} , \quad (4.3.22)$$

in which the smmall renormalization effects due to gauge bosons of $SU(4)_C$, $SU(2)_L$ and $SU(2)_R$ between M_1 and M_2 have been neglected.

Employing (4.2.15) ,(4.2.20) and

$$\alpha_3^{-1}(m_c) \approx 4.2 , \qquad \alpha_2^{-1}(M_W) \approx 28 , \quad (4.3.23)$$

one finds

$$A \approx 2.9 , \qquad b \approx 1.04 . \quad (4.3.24)$$

For the supersymmetric SO(10) model, the dimmension five operators will give rise to a new effective Lagrangian

$$\mathcal{L}'_{eff} \approx \frac{g_H^2}{m_{\tilde{H}}}\left(\frac{g_2^2}{m_{\tilde{W}}} \text{ or } \frac{g_1^2}{m_{\tilde{B}}}\right)(q q q l) \quad . \tag{4.3.25}$$

In order to estimate the enhancement A, we divide the scales of interest into two ranges: one between 1 Gev and M_W, another between M_W and M_1. Hence the enhancement factor will be changed to

$$A \rightarrow A' = A'_I \, A'_{II} \quad , \qquad b \rightarrow b' \quad , \tag{4.3.26}$$

where A'_I is the enhancement between 1 Gev and M_W , which being the same as (4.3.11) , A'_{II} is one between M_W and M_1 , being different from (4.3.12) due to the different β-functions. Using the b_N-values of the SO(10)SGUM in the table III , we find

$$\begin{aligned} A'_{II} &= \left(\frac{\alpha_3(M_W)}{\alpha_3(M_2)}\right)^{-\frac{4}{3}} \left(\frac{\alpha_2(M_W)}{\alpha_2(M_2)}\right)^{\frac{3}{4}} \left(\frac{\alpha_1(M_W)}{\alpha_1(M_2)}\right)^{-\frac{25}{624}} , \\ b' &= \left(\frac{\alpha_2(M_W)}{\alpha_2(M_2)}\right)^{-\frac{3}{2}} \left(\frac{\alpha_1(M_W)}{\alpha_1(M_2)}\right)^{\frac{30}{624}} , \end{aligned} \tag{4.3.27}$$

in which the small renormalization effects between M_2 and M_1 have been neglected. Using

$$\alpha_3(M_W) \approx 0.11 , \quad \alpha_2^{-1}(M_W) \approx 28 , \quad \alpha_1^{-1}(M_W) \approx 61 ,$$

and

$$\alpha_3^{-1}(M\ ) \approx 13 , \quad \alpha_1^{-1}(M_2) \approx \alpha_2^{-1}(M_2) \approx 20 , \qquad \text{(see(4.2.14))},$$

it turns out to be

$$A'_{II} \approx 0.50 , \quad b' \approx 1.57 \quad . \tag{4.3.28}$$

Employing $\alpha_3^{-1}(1\,\text{Gev}) \approx 3.9$, $\alpha_3^{-1}(m_c) \approx 4.2$, $\alpha_3^{-1}(m_b) \approx 5.7$, $\alpha_3^{-1}(m_t) \approx 7.0$, we find

$$A'_I \approx 0.51 \quad , \tag{4.3.29}$$

therefore, the enhancement A' is

$$A' = A' A' \approx 0.26 \tag{4.2.30}$$

Employing (4.3.24), (4.3.25), (4.3.30) and

$$g_H^2 \approx G m_u m_s \approx 1.3 \times 10^{-8} \quad [4.12] \quad ,$$

one obtains

$$\hat{R} \equiv \frac{(\tau_p)_{SGUM}}{(\tau_p)_{GUM}} \approx \frac{128\pi^4}{g_H^2} \left(\frac{\alpha_{GUM} \ A \ M_{\tilde{W}}}{\alpha_2(M_W) \ A' \ M_1} \right)^2 , \tag{4.3.31}$$

where α_{GUM} is the coupling of the non-supersymmetric SO(10) model at the unification mass scale. We take

$$\alpha_{GUM}^{-1} \approx 47 \quad . \tag{4.3.32}$$

Finally we find

$$\hat{R} \simeq 1.7 \times 10^{-18} \ M_{\tilde{W}}^2 \quad . \tag{4.3.33}$$

If one imposes $\hat{R} \geq 10^{-2}$, the gaugino mass $M_{\tilde{W}}$ must be much large than the weak mass scale M_W,

$$M_{\tilde{W}} \gtrsim 8 \times 10^7 \ \text{Gev}. \tag{4.3.34}$$

This is unacceptable if supersymmetry is broken at the weak mass scale M_W. So this simply imply that supersummetry could be broken at an intermediate mass scale which is larger than 10^7 Gev.

Conclusion:

If the renormalization enhahcement of the dimension five operators, which lead proton decay , is taken into accaunt, the proton lifetime in the SU(5) SGUM with a supersymmetry breaking scale $\sim$100 Gev is compatible with the experiment limit. However, in contract with SU(5) SGUM, we find that the SO(10) SGUM may still has problem with τ_p, if supersymmetry breaking mass scale is about 100 Gev. This suggest that supersymmetry would be broken at an intermediate scale of order $\gtrsim 10^7$ Gev.

4.3.2 Proton Decay Modes.

The new dimension five operators in Fig.4.1 may generate a Yukawa coupling among the Higgs superfields and the matter superfields, for example, in the minimal SU(5) SGUM, the Yukawa Lagrngian is [3.6] [4.17]

$$\mathcal{L}_Y^{Susy} = \int d^2\theta \left\{ -\frac{1}{4} \varepsilon^{\alpha\beta\gamma\delta\eta} \chi_{\alpha\beta} H_{1\gamma} \mathcal{M}_2 \chi_{\delta\eta} - \frac{1}{\sqrt{2}} \chi_{\alpha\beta} \mathcal{M}_1 (H_2^\alpha \psi^\beta - H_2^\beta \psi^\alpha) \right\} , \quad (4.3.35)$$

where $H_1 (H_2)$ is 5 ($\bar{5}$) Higgs, $\chi (\psi)$ is 10 ($\bar{5}$) matter field, and $\mathcal{M}_1, \mathcal{M}_2$ are matrices in generation space. The diagonal forms of $\mathcal{M}_{1,2}$ give fermion masses

$$\mathcal{M}_1 = \frac{1}{v_1} \mathrm{diag} (m_d, m_s, m_b) ,$$

$$\mathcal{M}_2 = \frac{1}{v_2} \mathrm{diag} (m_u, m_c, m_t) . \quad (4.3.36)$$

The one -loop diagrams of Fig 4.2 with an exchange of the gauginos $\tilde{W}$, $\tilde{G}$ and $\tilde{B}$ yield the following effective interactions for proton decay,

$$\mathcal{L}_{eff}^{Susy} \approx k_1 O_{\tilde{W}} + k_2 O_{\tilde{G}} + k_3 O_{\tilde{B}} \quad (4.3.37)$$

where the operators $O_{\tilde{W}}$, $O_{\tilde{G}}$ and $O_{\tilde{B}}$ correspond to an exchange of $\tilde{W}$, $\tilde{G}$ and $\tilde{B}$.

It has been found for the first two generations that

$$
\begin{aligned}
O_{\tilde{G}} = \frac{1}{2}\Big\{ & m_c m_s \sin\theta_c \big[-2\sin^2\theta_c (\bar{u}^c d_L)(\bar{d}^c \nu_{\mu L}) + 2\sin\theta_c\cos\theta_c (\bar{u}^c d_L)(\bar{s}^c \nu_{\mu L}) + \\
& + 2\sin\theta_c\cos\theta_c (\bar{u}^c s_L)(\bar{d}^c \nu_{\mu L}) \big] + m_c m_d \cos\theta_c \,(\text{analogous terms with} \\
& \nu_\mu \to \nu_e) + m_u m_s \cos\theta_c \big[\sin\theta_c (\bar{u}^c d_L)(\bar{u}^c \mu_L) - \cos\theta_c (\bar{u}^c s_L)(\bar{u}^c \mu_L) + \\
& + 2(\bar{d}^c s_L)(\bar{u}^c \nu_{\mu L}) + (2\sin^2\theta_c + \sin\theta_c\cos\theta_c)(\bar{u}^c d_L)(\bar{d}^c \nu_{\mu L}) + \\
& + (\sin^2\theta_c - 2\sin\theta_c\cos\theta_c)(\bar{u}^c d_L)(\bar{s}^c \nu_{\mu L}) - (\cos^2\theta_c + 2\sin\theta_c\cos\theta_c) \times \\
& \times (\bar{u}^c s_L)(\bar{d}^c \nu_{\mu L}) \big] - m_u m_d \sin\theta_c \,(\text{analogous terms} \\
& \text{with } \nu_\mu \to \nu_e \text{ and } \mu \to e) \Big\}
\end{aligned}
\qquad (4.3.38)
$$

$$
\begin{aligned}
O_{\tilde{W}} = \frac{1}{2}\Big\{ & m_c m_s \sin\theta_c \big[2\sin^2\theta_c (\bar{u}^c d_L)(\bar{d}^c \nu_{\mu L}) - 2\sin\theta_c\cos\theta_c (\bar{u}^c d_L)(\bar{s}^c \nu_{\mu L}) - \\
& - 2\sin\theta_c\cos\theta_c (\bar{u}^c s_L)(\bar{d}^c \nu_{\mu L}) \big] + m_c m_d \cos\theta_c \,(\text{analogous terms with} \\
& \nu_\mu \to \nu_e) + m_u m_s \cos\theta_c \big[\sin\theta_c (\bar{u}^c d_L)(\bar{u}^c \mu_L) - \cos\theta_c (\bar{u}^c s_L)(\bar{u}^c \mu_L) + \\
& + 2(\bar{d}^c s_L)(\bar{u}^c \nu_{\mu L}) + (\sin\theta_c\cos\theta_c - 6\sin^2\theta_c)(\bar{u}^c d_L)(\bar{d}^c \nu_{\mu L}) + \\
& + (\sin^2\theta_c + 6\sin\theta_c\cos\theta_c)(\bar{u}^c d_L)(\bar{s}^c \nu_{\mu L}) + (6\sin\theta_c\cos\theta_c - \cos^2\theta_c) \times \\
& \times (\bar{u}^c s_L)(\bar{d}^c \nu_{\mu L}) \big] - m_u m_d \sin\theta_c \,(\text{analogous terms} \\
& \text{with } \nu_\mu \to \nu_e, \text{ and } \mu \to e) \Big\} ,
\end{aligned}
\qquad (4.3.39)
$$

$$
O_{\tilde{B}} = -\frac{1}{18}\, O_{\tilde{G}} \quad . \qquad (4.3.40)
$$

From (4.3.38) to (4.3.40), the following hierarchy for proton decay rates have been derived

$$
\begin{aligned}
& \Gamma(p \to \bar{\nu}_\mu K^+) > \Gamma(p \to \bar{\nu}_\mu \rho^+) > \Gamma(p \to \bar{\nu}_\mu K^{*+}, \nu_\mu \pi^+) > \\
& > \Gamma(p \to \bar{\nu}_e K^+) > \Gamma(p \to \bar{\nu}_e \rho^+, \bar{\nu}_e K^{*+}) > \Gamma(p \to \bar{\nu}_e \pi^+) > \\
& > \Gamma(p \to \mu^+ K^0) \gg \Gamma(p \to \mu^+ \eta^0, \mu^+ \rho^0, \mu^+ \pi^0, \mu^+ \omega) \gg \\
& \gg \Gamma(p \to e^+ K^0) \gg \Gamma(p \to e^+ \eta^0, e^+ \rho^0, e^+ \pi^0, e^+ \omega, e^+ K^{*0}) \; .
\end{aligned}
\qquad (4.3.41)
$$

So it is evident from (4.3.41) that the dominant proton decay modes should be $\bar{\nu}_\mu$ +strange particles. This is completely defferent from the conventional GUT in which the dominant proton decay modes are e^+ + non-strange.

Conclusion:

The dominant proton decay modes in the minimal SU(5) supersymmetric grand unified model are $p \rightarrow \bar{\nu}_\mu$ + strange particles, in contrast with the conventional GUT's, in which the dominant one is $p \rightarrow e^+$ + non- strange. This is a signal to distinguish supersymmetric grand unified theories from conventional grand unified theories. We have also found that the dominant proton decay modes in SO(10) SGUM are also $p \rightarrow \bar{\nu}_\mu$ + strange particls; because the SO(10) involves the same dimension five operators. Therefore, the similar calculation for the SO(10) SGUM is eliminated in this article.

4.4 $N - \bar{N}$ Oscillation in SO(10) SGUM. [4.19]

Apart from proton decay, another interest consequence of B and L non - conservation interactions is neutron -antineutron oscillation. It has been noted that in the context of left - right symmetric models (for example SO(10) model), where the generator of the U(1) part of the eletroweak gauae symmetry is identified with B - L quantum number, the breakdown of left - right symmetry leads naturally to $N - \bar{N}$ oscillation (B = 2) or Májorana neutrinoes oscillation (L = 2).

A crucial question in planning experiments to detect $N - \bar{N}$ oscillation is the theoretical lower bound on $\tau_{N-\bar{N}}$ that is consistent with the known upper limits on nuclesr instability. One can parametrize N-$\bar{N}$ oscillation by a neutron Majorana mass term

$$\mathcal{L}_{eff} = \delta m\, N^T C N + h.c\,, \tag{4.4.1}$$

which implies $N - \overline{N}$ oscillations in empty space with a time

$$\tau_{N-\overline{N}} \approx \frac{1}{\delta m}, \qquad \delta m \approx G_{N-\overline{N}} |\psi(0)|^4 , \tag{4.4.2}$$

where $|\psi(0)|^2 \approx \frac{1}{\pi R^3}$ (R being the nucleon radius) is the probability of finding two quarks at the same point in a nucleon. The effective coupling constant $G_{N-\overline{N}}$ depends on the specific model.

In the conventional SO(10) GUM, the operators relevant to $N - \overline{N}$ oscilation have dimension 9, namely

$$\mathcal{L}_{eff} \approx qqqqqq \quad .$$

The Feynman graph leading to $N - \overline{N}$ oscillation is shown in fig.4.4.1

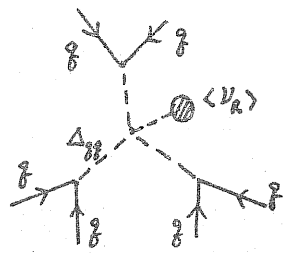


Fig. 4.4.1 Feyman graph contributing to $N - \overline{N}$ oscillation.

In fig.4.4.1 Δ_{qq} is diquark Higgs, whose mass is $M_{\Delta_{qq}}$. From Fig.4.4.1 one gets

$$\delta m \approx \lambda h^3 \langle \nu_R \rangle \Big/ M^6_{\Delta_{qq}} R^6 \quad , \tag{4.4.3}$$

where R is the radius of the nucleon and the factor R^{-6} is induced for taking into account the effect of the 3 - quark wave function inside the nucleon.

In order to estimate δm, it is important to know the mass of Δ_{qq} diquark Higgs. In the conventional SO(10) GUM, $M_{\Delta_{qq}} \sim M_c \sim 10^{10}$ GeV (M_c is the mass scale of $SU(4)_c \times SU(2)_L \times SU(2)_R \rightarrow SU(3)_c \times U(1) \times SU(2)_L \times SU(2)_R$). So δm is too small to be observable, $N - \overline{N}$ oscillation in SO(10) GUM is suppressed.

We now turn to discuss the $N-\overline{N}$ oscillation in SO(10) SGUM. As is the case of proton decay , SGUM provides a new kind of operators with dimension 7 and 8 (shown in Fig.4.4.2), which lead $n-\overline{n}$ oscillations.

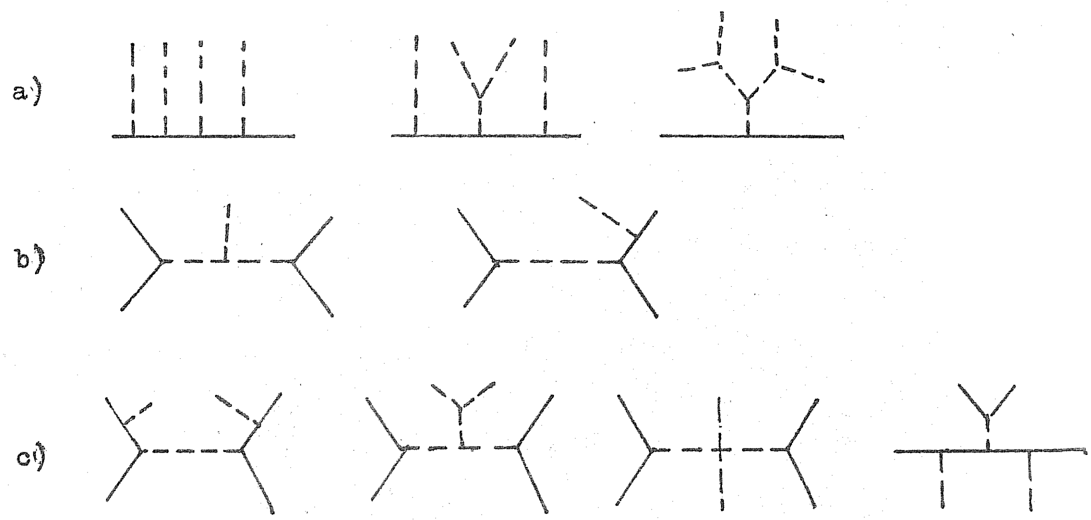


Fig4.4.2 The dimension 7and 8 operators responsible to $N-\overline{N}$ oscillations: a) dimension 7 opertor $(\Phi\Phi\bar{\Phi}\Phi\Phi\Phi)_F$, b) dimension 7 operator $(\Phi^*\Phi\Phi\Phi\Phi)_D$, c) dimension 8 operator $(\Phi^*\Phi^*\Phi\Phi\Phi\Phi)_D$.

Out of those dimension 7 and dimension 8 operators, one can form $N-\overline{N}$ oscilletion diagrams as Fig. 4.4.3

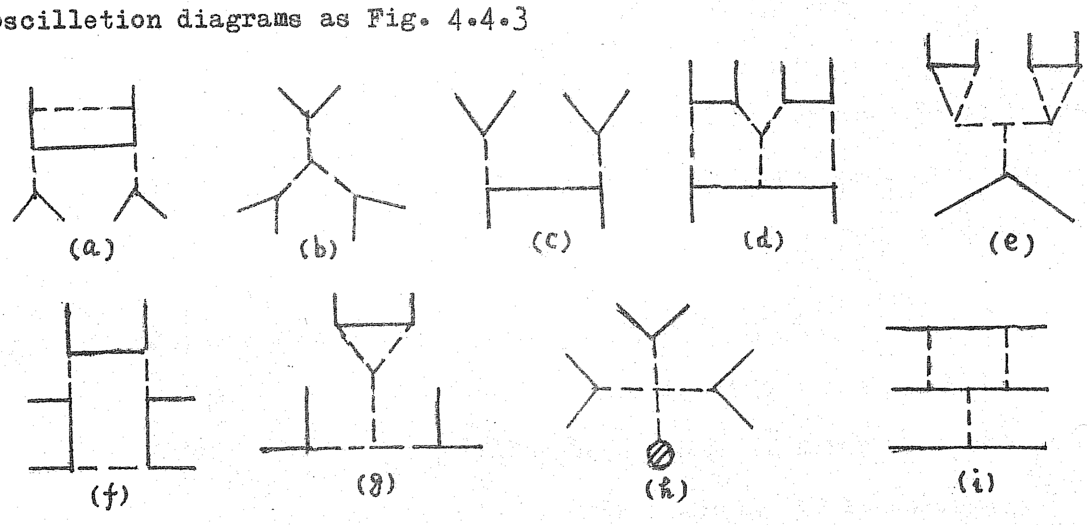


Fig.4.4.3 $N-\overline{N}$ oscillation diagrams formed from the dimension 7 and 8 operators.

The effective coupling $G_{N-\bar{N}}$ for $N-\bar{N}$ oscillation is roughly estimated as follows :

$$
\begin{aligned}
&\text{for Fig.4.4.3 (a)} \sim f^6 m_s^{-5}, && (b) \sim \lambda f^3 m_s^{-6}, \\
&(c) \sim f^4 m_s^{-4} m_f^{-1}, && (d) \sim \lambda f^6 m_s^{-6}, \\
&(e) \sim \lambda^3 f^5 m_s^{-8}, && (f) \sim f^6 m_s^{-5}, \\
&(g) \sim \lambda^2 f^4 m_s^{-7}, && (h) \sim \lambda' f^3 m_s^{-6} \langle v \rangle, \\
&(i) \sim f^6 m_s^{-5}.
\end{aligned}
\tag{4.4.4}
$$

where λ, λ', f ; m_s and m_f denote scalar and Yukawa couplings; scalar and fermion masses, respectively. In general we would expect the presence of the diagrams (a) $\sim$ (i). However which diagrams appear has to be checked for each model. For instance SU(5)SGUM without R- symmetry has (c) which contributes most among (a)$\sim$(i) . If any of those diagrams appears and if $m_s \sim 1$ TeV (= supersymmetry breaking scale) $N-\bar{N}$ oscillation is too large. Besides proton decay will be too large. Therefore such diagrams have to be eliminated from the model by, for example , imposing an extra symmetry. In SO(10) SGUM we don't find diagram (c) due to the difference in the manner Yukawa couplings are formed.

In the SO(10) SGUM, the dominant $N-\bar{N}$ oscillation diagram is shown in Fig.4.4.4

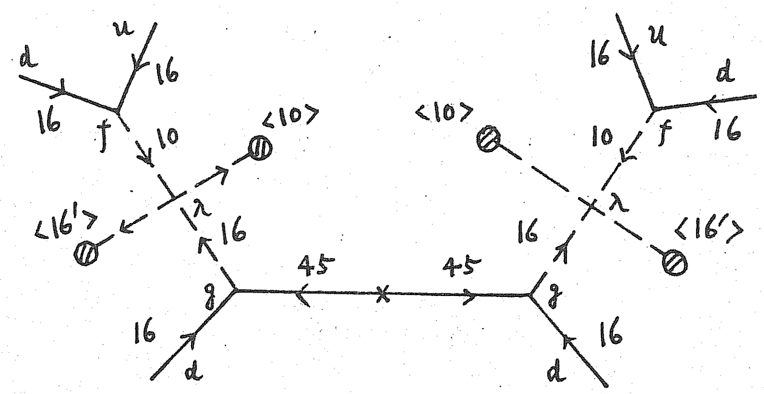


Fig.4.4.4 the dominant $N-\bar{N}$ oscillation diagram in the SO(10)SGUM.

Fig.4.4.4. gives

$$G_{N-\bar{N}} \approx \lambda^2 f^2 g^2 \langle 10 \rangle^2 \langle 16' \rangle^2 / m_{16}^4 \, m_{10}^4 \, M_W \quad , \qquad (4.4.5)$$

where λ, f and g are scalar, Yukawa and gauge couplings, respectively. m_{16}, m_{10} are masses of the diquark in 16 and 10 representations. The 16 scalar is the partener of the quark, so $m_{16} \approx M_W$, and the 10 diquark is colour triplet,which should be superheavy, thus $m_{10} \approx M_1 \approx 10^{15}$ Gev . Since the VEV of 16 Higgs breaks $SU(4)_C xSU(2)_L xSU(2)_R$ down to $SU(3)_C xSU(2)_L xU(1)_Y$, and the VEV of 10 Higgs breaks $SU(3)_C xSU(2)_L xU(1)_Y$ down to $SU(3)_C xU(1)_{em}$, so $\langle 16' \rangle \approx M_2$ and $\langle 10 \rangle \approx M_W$. Consequently, (4.4.5) becomes

$$G_{N-\bar{N}} \approx \lambda^2 f^2 g^2 M_2^2 / M_1^4 \, M_W^3 \quad . \qquad (4.4.6)$$

Taking $\lambda \approx 1$, $f \lesssim 10^{-3}$, $g^2/4\pi \approx 10^{-1}$, $M_2 \approx 10^{12}$ Gev ,we get

$$G_{N-\bar{N}} \approx 10^{-48} (Gev)^{-5} \quad . \qquad (4.4.7)$$

This implies that the N-$\bar{N}$ oscillation in SO(10) SGUM will be highly suppressed.

Conclusion :

In supersymmetric grand unified theories, in addition to the original dimmension 9 operator, there exist the dimmension 7 and 8 operators which may also lead N-$\bar{N}$ oscillation. However, the N-$\bar{N}$ oscillation in SO(10)SGUM is still highly suppresed.

4.5 CP-violation: [4.22]

As we know, CP-violation can appear not only by the explicit weak interaction but also can be associated with non-perturbative effcts in the stron interaction sector. Actually, although the normal QCD Lagrangian ($= -\frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu}$) is CP-invariant , one can add an additional term

$$\mathcal{L}_\theta = \frac{\theta}{32\pi^2} F^a_{\mu\nu} \tilde{F}_a^{\mu\nu} \quad , \qquad (4.4.8)$$

where $\tilde{F}_a^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\sigma\rho} F_{\sigma\rho,a}$ is the dual tensor of $F^{\mu\nu}$. It is clear that such a $\mathcal{L}_\theta$ term will induce CP-violations in the strong interaction Lagrangian.

CPviolation effects in strong interaction have to be very small as required by the electric dipole moment of the neutron (d_N):

$$d_N = c\,\theta / m_N \quad . \tag{4.4.9}$$

From the limit on d_N one obtains a very low upper bound on θ

$$\theta \leq 10^{-8} - 10^{-10} \quad . \tag{4.4.10}$$

The effective θ in (4.4.8) can be expressed

$$\theta = \theta_{QCD} + \theta_{QFD} \tag{4.4.11}$$

where θ_{QCD} is the bare value from the QCD Lagrangian and

$$\theta_{QFD} = \arg \det M \quad , \tag{4.4.12}$$

where det M is the determinant of the quark mass matrix. Hence the renormalization of θ would be induced by chiral rotations on guark fields necessitated by the renormalization of quark mass matrix.

A problem is : can we make θ so small or zero in a natural way? It is clear that the renormalization of θ parameter in supersymmetry theories vanishes, since θ renormalization is related to mass renormalizations which vanish in supersymmetry theories, according to non- renormalization theorem. If supersymmetry is broken, θ may get a renormalization, the magnitude of which depends on the scale of supersymmetry breaking.

More precisely, let us see how θ renormalization is related to mass renormalization.[4.23] At zeroth order in the non- strong interactions the inverse quark propagator is

$$S_0^{-1}(p) = \not{p} - M \tag{4.4.13}$$

with M a real and diagonal matrix . We denote by $\Sigma(p)$ the sum of irreducible non -strong diagram contributions to the quark propagator. $\Sigma(p)$ can be decomposited as follows

$$\Sigma(p) = A\not{p}L + B\not{p}R + MCL + DMR \quad , \tag{4.4.14}$$

where hermiticity imposes the constraints

$$A = A^{+}, \quad B = B^{+}, \quad C = D^{+} \quad . \tag{4.4.15}$$

Combining (4.4.13) and (4.4.14) to get the full inverse quark propagator we see that the left and right and right- handed quark fields must undergo wave - function trnormalizations

$$\psi_L = \frac{1}{\sqrt{1-A}}\psi_L^{Ren}, \quad \psi_R = \frac{1}{\sqrt{1-B}}\psi_R^{Ren} \tag{4.4.16}$$

and that the quark mass matrix becomes

$$M^{Ren} = \frac{1}{\sqrt{1-B}} M(1+C)\frac{1}{\sqrt{1-A}} \quad . \tag{4.4.17}$$

When one makes this mass matrix real and diagonal by a U(1)rotation through an angle

$$\delta\theta = \arg \det M^{Ren} \quad , \tag{4.4.18}$$

it is apparent from (4.4.17) that

$$\delta\theta = \arg \det(1+C) = \mathrm{Im}\,\mathrm{Tr}\,C + \cdots \quad , \tag{4.4.19}$$

because the hermiticity of A and B and the real and diagonal nature of M. In supersymmetry theories C = 0 since C is related to the supersymmetric mass counterterm. Therefore , it follows from (4.4.19) that

$$\delta\theta = 0 \quad . \tag{4.4.20}$$

For this reason it is technically "natural" to set $\theta=0$ in supersymmtry theoris. However, a finite corrections to θ parameter may arise when supersymmetry is broken. For the supersymmetric extention of the Kobayashi- Mashawa model , it is estimated that

$$\delta\theta = \sum_{q} O\left(\frac{\alpha_s}{\pi} \text{ or } \frac{\alpha}{\pi}\right) \mathrm{Im}(UV^{+})_{qq} \frac{\Delta m^2_{q\tilde{q}}}{m^2_{\tilde{q}} \text{ or } m^2_{\tilde{G}}} \cdot \frac{m_{\tilde{G}}}{m_q} , \tag{4.4.21}$$

where $m_{\tilde{q}}$, $m_{\tilde{G}}$ and m_q are squark, gaugino and quark masses respectively, U and V are the gaugino cauplings to left - and right - handed quark-squark combinations.

Conclusion:

Since θ renormalization is related to mass renormalization, it is technically "natural" to set $\theta = 0$ in supersymmetry theories. A finite corrections to θ parameter may arise when supersymmetry is broken. In this sense, supersymmetric grand unified theories may also be available for solving the strong CP problem.

CONCLUSION

We have discussed most aspects of supersymmetic grand unified theories: the gauge hierarely problem, supersymmetry breaking and phenomenological predictions. In particular, I myself have presented an SO(10) super-symmetric ground unified model, in which the gauge hierarely problem and most phenomenological predictions have been examined. We find that, by compareson with the minimum SU(5) supersymmetric grand unified model, the SO(10) SGUM has some attrative advantages, as concerns the phenomenology.

The key point of supersymmetry theory is its remarkable property — the wellknown "non - renormalization" theorem. This enable us to deal with many difficult problems which emerged in the non - supersymmetric GUT's, where those problem are caused by renormalizations ,for example, the gauge hierarely puzzle. In addition, another exciting feature of supersymmetric GUT's may perhaps be that supersymmetry theories may provide a hopful solusion of the "desert" problem in GUT's, if super-symmetry is broken at an intermediate mass scale between M_W and M_G.

However , it is still too early to say that supersymmetry theory is suitable to describle nature, because we have paid a heavy price for solving the gauge hierarely problem, and perhaps the desert problem as well. The cost may be summarized as follows:

1). The proton decay is still an ambiguous problem. As we demonstrated, the proton lifetime in SU(5) supersymmetric model can be compatible with the lowest experimental limit, but not in the SO(10) SGUM, if the supersymmetry breaking scale is $M_W \sim 100$ Gev .

2). $\sin^2\theta_W$ in SU(5) SGUM and the geometric model is unacceptably large compared with the experiment. Besides, we find that it is unstable against the number of Higgs multiplets.

3). In any supersymmetric grand unified model there are too many new particles: squarks, sleptons, Higgs fermions, whose masses are assumed to be very great, since none has been observed so far. In my opinion, this is not a satisfactory feature. Even in W – S model, the Higgs pasticles are not yet understood very well. Hence, many people have attempted to improve the W – S model without Higgs particles. At the begining, we expected to identify the scalar partners of the matter fermions with the Higgs scalars so that the Higgs particles would be naturly introduced in the supersymmetry theory, Unfortunately, we have finally found that this attempt fails. Therefore, in order to break the gauge group, and supersymmetry, and to give the matter fermions masses one has to ask for extra Higgs supermultiplets to come to the rescue. On the one hand, we have the scalar partners of quarks, leptons, which so far useless; on the other hand, we have the fermion partners of Higgs scalar, which are also unwanted.

4). In any global supersymmetry theory, there are always vacuum degeneracies, for example, in SU(5) SGUM, there are three vacuums, which are respectively SU(5), SU(4) × U(1) and SU(3) × SU(2) × U(1) invariant, and are degenerate. Which vacuum do we believe ? There is no potential reason to choose SU(3) × SU(2) × U(1) but not SU(4) × U(1) .

5). In supersymmtric grand unified theories, in particular, the geometric hierarely model, the unification mass scale is boosted as high as Plank mass scale M_p , even higher. In this case, gravity should be included in supersymmetry theory.* In addition, many old problems, such as family puzzle, could not yet be answered by supersymmetry theories.

In view of the above comments, I suggest the following directions to myself :

a). From 4) and 5), it would be better to embed global supersymmetry grand unified model into supergravity theory. In a supergravity theory the degeneracy of vacuum can be split. Perhaps we can find a theory in which $SU(3)_C \times SU(2)_L \times U(1)_Y$ correspones to the lowest vacuum. In such a theory the supersymmtry breaking can be naturelly derived by M — term. Of cause, one may imagine that some new problem will probably come in. As we know, supergravity theory is not renormalizable (except perhaps N = 8 supergravity theory in which renormalizations may be finite although it is also unrenormalizable.)

b). From 3), also 2), three directions may be of interest:

(i) To extend the supersymmetric grand unified theory into high dimension. Then it would be possible that the Higgs scales come from the dimension reduction. However, the problem we may have in such a theory would be that it is difficult to get a suitable scalar potential so that the breaking direction is just what we would wish, since the potential is also deduced from the dimension reduction, therfore, it is almost fixed.

(ii) To explore the new mechanism of both supersymmetry and gauge symmetry breakings without the introduction of any Higgs particle, i.e., dynamical breaking.

---

*This is now being considered. (Arnowitt, Rhamseddnio & Nath)

(iii) To construct a supersymmetric sub - quark model, in which, we may only have a minimal number of sub -particles, from which all quarks, leptons, gauge bosons and Higgs scales are made.[4.25]

c). From 2). it would perhaps be interest to construct a SO(10) geometric hierarchy model, since we have found that both unification mass scale M_G and $\sin^2\theta_W$ in SO(10) SGUM are lower than those in SU(5) SGUM. Therefore we can expect that $\sin^2\theta_W$ in SO(10) geometric hierarchy model may be compatibal with the experiments.[4.26]

REFERENCES

1.1 A.Salam and J. Strathdee, Fortschrilte der Physik, 26 57 (1978), A.Salam and J. Strathdee, Phys. Rev. D11 1521 (1975).

1.2 J.Wess, B. Zumino, Nucl. Phys. B 78 , 1 (1974).

1.3 E.Witten, Nucl. Phys. B 188 513 (1981).
B.Zumino, LBL - 13691 (1981).

1.4 P. Fayet, J. Iliopoulos, Phys. Lett. 51B 461 (1974).

2.1 Paul Langacker, SLAC - PUB -2544 (1980).

2.2 E. Gildener, Phys. Rev. D14 1667 (1976).
S. Weinberg, Phys. Lett. 82B 387 (1979).

2.3 G 't Hooft, Cargese Lectures in Recent Developments in Gauge Theories (1979).

2.4 S. Weinberg, Phys. Rev. D13 974 (1976).
S. Weinberg, Phys. Rev. D19 1277 (1979).
L. Susskind, Phys. Rev. D20 2619 (1979).
S. Dimopolous, Nucl.Phys. B168 69 (1980).

2.5 J. Wess and B. Zumino, Phys. Lett. 49B 52 (1974).
J. Iliopoulos and B. Zumino, Nucl. Phys. B76 (1974) 310;
S. Ferrare, J. Iliopoulos and B. Zumino, Nucl. Phys. B77 (1974) 413;
M. T. Grisaru, W. Siegel and M. Rocek, Nucl. Phys. B159 (1979) 429;

2.6 S. Dimopoulos and H. Georgi, Nucl. Phys. B193 (1981) 150
N. Sakai, Tohoku Univ. preprint TU/ 81/ 225
R. K. Kaul, Nucl. Phys. B199 36 (1982).

2.7 E. Witten, Phys. Lett. 105B 267 (1981).

2.8 S. Dimopoulos and S. Raby, LA - UR - 82 - 1282
J. Polchinski and J. Susskind, Stanford Univ. Preprint (1982)

2.9 H. Geprgi and S. L. Glashow, Phys. Rev. Lett. 32 (1974) 438.

2.10 Zhao Zhiyong , J. Phys. G : Nucl. Phys. 8 (1982) 1019;

2.11 S. L. Adler, Lectures on Elementary Particles and Quantum Field Theory. (1970)
R. Jakiw, Lectures on Current Algebra and Itis Applications. (1970)

2.12 J.C. Pati and A. Salam, Nucl. Phys. B150 (1979) 76.

2.13 S. Rajpoot, Phys. Rev. D22 (1980) 2244.

2.14 N. P. Chang and J. Perez - Mercader , Phys. Rev. D18 4721 (1978).

2.15 F. Buccella, J. P. Derendinger, C.A. Savoy, TH 3212 - CERN, 1981.

2.16 R.D. Peccei and H.K.Quinn, Phys. Rev. Lett. 38 1440 (1978),
Phys. Rev. D16, 1791 (1977)
S.Weinberg, Phys. Rev. Lett. 40 223 (1978),
F.Wilczek, Phys. Rev. Lett. 40 279 (1978),
W.A. Bardeen and S.-H. H. Tye, Phys. Lett. 74B 229 (1981).

2.17 T. Sherry, ICTP Preprint IC/79/105

2.18 H. Georgi, HUTP - 81/A039

2.19 L. O'Raifeartaigh, Nucl.Phys. B96 (1975) 331.

2.20 M. Hug, Phys. Rev. D14 (1976) 3548.

2.21 S. Caleman and E. Weinberg, Phys. Rev. D7 (1973) 1888.

2.22 P. H. Frampton, H. Georgi and J. E. Kim, HUTP -82/A025 IFP- 177 -UNC.

2.23 H. Yamagishi, Princeton Preprint. (1982).

2.24 K. Fujikawa and W. Lang, Nucl. Phys. B88 61 (1975).
S. Weinberg, Phys. Lett. 62B 111 (1976).

3.1 E. Cremmer, S. Ferrara, L. Girardello, A.V. Proeyen, TH 3312-CERN (1982)

3.2 L.A. Gaume', HUTMP 81/B111.

3.3 S. Weinberg, Taxas preprint 78712 (1982)
H. Pagels and J.R.Primack, Phys. Rev. Lett. 48 223 (1982)
M. Fukugita, N. Sakai, KEK - TH 42, (1982)

3.4 S. Deser and B. Zumino, Phys. Rev. Lett. 38 1433 (1977)
D. Z. Freedman and A. Das, Nucl. Phys. B120 221 (1977)
P.K. Townsend, Phys. Rev. D15 2802 (1977)

3.5 Zhao Zhiyong, private communication with S. Rajpoot, (1982)
S. Rajpoot and J.G. Taylor, King's College Preprint, (1982)

3.6 J. Ellis , D.V. nanopoulos, S. Rudaz, Nucl. Phys. B202 43 (1982)
S. Dimopoulos, S. Raby and F. Wilczek, Phys. Lett. 112B 133 (1982)

3.7 L. Girardello and M.T. Grisaru, Nucl. Phys. B194 65 (1982)

3.8 P. Fayet and J. Iliopoulos , Phys. Lett. 51B 461 (1974)
P. Fayet, ibid 69B 489 (1977)

3.9 S. Ferrara, L. Girardello and F.Palumbo, Phys. Rev. D20 403 (1979)

3.10 B.T. Ovrut and J. Wess, Karlsruke,Univ. Preprint, Oct. (1981)

3.11 J. Wess and B. Zumino, Phys.Lett. 74B 51 (1978)

3.12 A.A. Slavnor, Nucl. Phys. B124 301 (1977);
N. O hta Phys. Lett. B112 215 (1982)

3.13 H.P. Nilles, CERN Preprint TH 3238 -CERN (1982)

3.14 A. Salam and J. Strathdee, ICTP preprint IC/75/38

4.1 H. Geogi and S.Glashow, Phys. Rev. Lett. 32 438 (1974)

4.2 H. Geogi, H. Quinn and S.Weinberg, Phys. Rev. Lett. 33 451 (1974)

4.3 W.A. Bardeen, A.J.Buras, D.W. Duke and T. Muta, Phys. Rev. D18 3998 (1978)

4.4 P. Mackenzie and G.P. Lepage, Phys. Rev. Lett. 47 1244 (1981).

4.5 W.T. Mariano and A. Sirlin, Phys. Rev. Lett. 46 163 (1981),
Nucl. Phys. B189 442 (1981), Reckefeller (00 —-2232B - 206).

4.6 L-F. Lee, Phys. Rev. D9 1723 (1974).

4.7 M.B.Einhorn and D.R.T. Jones, Nucl. Phys. B196, 475 (1982).
W.J. Marciano and G. Senjanovic, Brookhaven national Laboratory Preprint.

4.8 W. Marciano and A. Sirlin, Preceedings of the Second Workshop on Grand Unification, Ann. Arbor. 1981.

4.9 Zhao Zhiyong, SISSA Preprint 26/82 E.P. to appear in Nucl.Phys. B.

4.10 P. Langacker Pennsylvania Univ. Preprint UPR - 0186 T (1982).

4.11 M. Chaichian, Y.N. Kolmahov and N.F. Nelipa, Research Institute for Theoretical Physicas, Univ. of Helsinki Preprint, HU - TFI 81 - 31 (1981).

4.12 J. Fingord and M.K. Gaillard, Phys. Rev. D22 778 (1980)

4.13 S. Weinbery, Harvard Univ. Preprint HUTP - 81/A047 (1980)

4.14 N. Sakai, Z.Phys. C11 153 (1982)
N. Sakai, T. Yaneyida, Nucl. Phy. B197 533 (1982)
T.M. Aliev, M.I. Vysitsky, ITEP - 77 (1982)

4.15 F.A. Wilizek and A. Zee, Phys. Rev. Lett. 43 157 (1979)
H.A.Welden and A. Zee, Nucl. Phys. B173 269 (1980).

4.16 Yasuo Hara, T.Matsuki and N.Yamamoto, UTHEP - 89 (1982)

4.17 P. Salati and J.C. Wallet, LAPP - TH - 55 (1982)

4.18 M.B. Gavela, A. Le Yaouan , L. Oliver, O. Pena and J. C. Rynal,
Phys. Rev. D23 1580 (1980)
J. Ellis, M.K. Gaillard and D.V..Nanopoulous, Phys. Lett. 88B320 (1979)

4.19 Y. Fujimoto and Zhao Zhiyong. ICTP Preprint IC/82/60

4.20 V.A. Kuzmin, JETP Leters 12 228 (1970)
S.L. Glashow, Harvard Preprint HUTP - 79 /A040 (1979)
R.E. Marshak and R.N. Mohapatra, Phys. Lett. 94B 183 (1980)

4.21 R.N. Mohapatra, CCNY - TEP - 82/1 (1982)

4.22 G. Girardi, LAPP - TH - 58 (1982)
J. Ellis, S. Ferera and D.V. Nanopoulos, CERN Prepint TH3272
- CERN (1982)

4.23 J. Ellis and M.K. Gaillard, Nucl. Phys. B150 (1979) 141.

4.24 M. Kobayashi and T. Mashawa, Prog. Theor. Phys. 49 (1973) 652.

4.25 A.Salam,J.C.Pati and Zhao Zhiyong , (in preparation)

4.26 Y.Fujimoto, S. Rapoot and Zhao Zhiyong (in preparation)