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LHC Phenomenology of Supersymmetric Models

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Particle Physics is nowadays experiencing a time that is surely going to be remembered in the years to follow, whatever will be the final outcome of the currently running experiments. The first runs of the Large Hadron Collider (LHC) in Geneve have shown that the theoretical framework to explain Particle Physics constructed in the 60s and 70s, the Standard Model, enjoys good health since TeV scale Physics is showing nothing new. On December 13 2011 *tantalising hints* [1] of a signal possibly compatible with the Standard Model Higgs boson were claimed by the two larger collaborations at the LHC, CMS and ATLAS, and such a result later became an *observation* [2–4] on July 4 2012. On top of that the LHCb collaboration, heading for discoveries in flavour Physics, is slowly converging to the predictions of the Standard Model, ruling out large regions of the parameter space for New Physics models. Is this the dusk of Particle Physics as a Science on the front line? Have we already discovered what was to be discovered?

Aside on the fact that believing the Standard Model as the ultimate theory of Nature would be quite off the mindset of a scientist, whose aim should always be to test theories open-mindedly in order to disprove them, there are some issues that are still not explained in this framework, and some whose explanation is quite awkward. In order to construct a coherent theory to accomodate such issues, that we will address in greater details later in this chapter, one has to go beyond the Standard Model. In this chapter, after a short description of the Standard Model theory, necessary to set some notations, we will directly focus on the motivations for going beyond the latter framework. We will then introduce Supersymmetry as a possible explanation to some of the issues raised focusing then on some topics relevant for the rest of the thesis.

1.1 The Standard Model

The Standard Model (SM) [5–8] is the Quantum Field Theory (QFT) representing the theoretical framework currently used to describe the fundamental interactions of elementary particle physics[1]. It is a renormalizable Yang-Mills gauge theory based on the symmetry group $SU(3)_C \times SU(2)_L \times U(1)_Y$: the mediators of the interactions are spin 1 vector bosons belonging to the adjoint representation of the gauge group. The lagrangian density for the gauge fields can be written as

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}\text{Tr}(G_{\mu\nu}G^{\mu\nu}) - \frac{1}{4}\text{Tr}(W_{\mu\nu}W^{\mu\nu}) - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \tag{1.1}$$

where $G_{\mu\nu}$, $W_{\mu\nu}$ and $B_{\mu\nu}$ stand for the stress energy tensors of the gauge fields,

$$\begin{aligned} G^a_{\mu\nu} &= \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s\, f^{abc} G^b_\mu G^c_\nu \\ W^\alpha_{\mu\nu} &= \partial_\mu W^\alpha_\nu - \partial_\nu W^\alpha_\mu + g\, \varepsilon^{\alpha\beta\gamma} W^\beta_\mu W^\gamma_\nu \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \,, \end{aligned} \tag{1.2}$$

where f^{abc} and $\varepsilon^{\alpha\beta\gamma}$ are the structure constants of the groups $SU(3)_C$ and $SU(2)_L$ respectively.

Matter fields, known as quarks and leptons, belong to the spinorial representation of the Lorentz group and they are conveniently charged under the gauge interactions. The correct assignment of quantum numbers reproducing the physical characteristics of the elementary particles are as follows[2]:

$$\begin{aligned} q_i &= \begin{pmatrix} u_i \\ d_i \end{pmatrix} \to (\mathbf{3},\mathbf{2})_{+\frac{1}{6}}\,, \qquad {u^c}_i \to (\overline{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}} \quad {d^c}_i \to (\overline{\mathbf{3}},\mathbf{1})_{\frac{1}{3}} \\ l_i &= \begin{pmatrix} \nu_i \\ e_i \end{pmatrix} \to (\mathbf{1},\mathbf{2})_{-\frac{1}{2}}\,, \qquad\qquad\qquad\qquad {e^c}_i \to (\mathbf{1},\mathbf{1})_{\mathbf{1}}\,. \end{aligned} \tag{1.3}$$

Three different copies of the matter content just described exist in Nature, bringing about families of different flavours. The flavour index was denoted by i in equation 1.3.

The lagrangian density describing the interaction of matter fields, that are described by Weyl spinors, and gauge bosons is given by the usual Dirac term with

[1] Aside from gravity as we shall comment later.

[2] The notation is name of the field $\to (\mathbf{SU(3)_C}, \mathbf{SU(2)_L})_{\mathbf{U(1)_Y}}$.

minimal substitution:

$$\mathcal{L}_{\text{ferm}} = i\psi^\dagger \overline{\sigma}^\mu D_\mu \psi \quad \text{where} \quad D_\mu = \partial_\mu - i g_a A^a_\mu \tau^a \,, \tag{1.4}$$

with a running over those gauge groups under which the ψ field under analysis has no trivial behaviour and τ^a are the symmetry group generators.

Any field introduced until now happens to be massless. Experimental evidence tells us that three of the four vector bosons of the electroweak (EW) symmetry $SU(2)_L \times U(1)_Y$ are massive, along with the quarks and leptons. The issue of providing a mass to the latter fields without explicitly breaking gauge symmetry is addressed in the SM through the Higgs mechanism [9–12]. In order to break spontaneously the EW symmetry one introduces a complex scalar field with non trivial transformations under the gauge group. The easiest solution compatible with experimental data[3] envisages the introduction of the scalar field

$$h = \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} \to (\mathbf{1}, \mathbf{2})_{\frac{1}{2}} \,, \tag{1.5}$$

whose potential is the direct generalization to the non abelian group $SU(2)_L \times U(1)_Y$ of the mexican hat shaped one,

$$\mathcal{V}_h = \mu^2 h^\dagger h + \frac{\lambda}{2}(h^\dagger h)^2 \quad \text{with} \quad \mu^2 < 0 \,, \, \lambda > 0 \,. \tag{1.6}$$

Under the condition $\mu^2 < 0$ the vacuum state is not unique since there is a full circle of degenerate minima for the potential $\mathcal{V}_h$ and spontaneous symmetry breaking takes place. One then chooses a particular vacuum state, which obviously has non trivial transformations under the gauge group and in particular does not have the same symmetries of the potential. In the specific case of the SM, working in the unitarity gauge, one writes

$$\langle h \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v = \sqrt{-\frac{2\mu^2}{\lambda}} \end{pmatrix} \,. \tag{1.7}$$

Spontaneous symmetry breaking comes along with the presence of some massless modes, known as Goldstone bosons, corresponding to excitations in the vacua manifold. The latter become the longitudinal components of the gauge fields corresponding to broken symmetries, which get mass through the coupling hidden in the covariant derivative of the Higgs boson itself. From experiments one obtains $v \approx 246\,\text{GeV}$.

After EW symmetry breaking takes place only one of the four original EW bosons remains massless, the one associated to $U(1)_{\text{em}}$ gauge group, which is identified as the photon. The other EW gauge bosons, namely W^+, W^- and Z^0, acquire mass $\mathcal{O}(gv) \sim \mathcal{O}(100\text{GeV})$.

[3] That is exactly the SM solution.

As for the fermion fields, they get mass from the Yukawa interactions

$$\mathcal{L}_{\text{Yukawa}} = -\lambda_D^{ij} q_i h^\dagger d_j^c - \lambda_U^{ij} q_i h u_j^c - \lambda_E^{ij} l_i h^\dagger e_j^c + \text{h.c.}. \qquad (1.8)$$

We notice that in the original SM theory there is no mass term for neutrinos; a simple extension accounting for massive neutrinos would be obtained through the addition of a lagrangian term $\mathcal{L} = -\lambda_N^{ij} l_i h \nu_j^c$, similar to that of up quarks. Yukawa couplings are invariant under SM gauge symmetries and only after spontaneous EW symmetry breaking quarks and leptons get non vanishing masses, whose values can be obtained from the diagonalization of Yukawa matrices. The net effect of the latter procedure resides in a misalignment between the mass and the interaction fermionic eigenstates, giving rise to the phenomena of quark and lepton mixings. The analysis of the structures of these mixings and their effects on particle physics constitute the basis of flavour physics [13, 14].

1.2 Physics beyond the Standard Model: hints and motivations

High energy physics experiments have been teaching us that the SM[4] works remarkably well as it is virtually in good agreement with all the experimental data from accelerators. Still there are both direct and indirect hints that the SM will not turn out to be the *complete* theory of Nature. A short not exhaustive list of these hints should surely accounts that:

- The SM framework does not contain **gravitational interactions**, whose implementation in the context of Quantum Field Theories (QFT) is still an open issue;
- Cosmological observations require that 96% of the energy present in the universe is made of **Dark Matter** and **Dark Energy** [15, 16], which unfortunately do not find their explanation in the SM theory;
- Again on the cosmological side the SM does not provide for any mechanism accounting for **matter-antimatter asymmetry** [17] or **inflation** [15];
- The **strong CP problem** lacks of explanation [18];
- The $3.6\,\sigma$ discrepancy of the theoretical and experimental values of $(\mathbf{g-2})_\mu$ [19, 20];
- Generally speaking there are a lot of **flavour issues** to be addressed: the pattern of masses within the SM, the structure of the Yukawa couplings, ... [21] ;

[4]Eventually with the addition of Dirac right neutrino light fields in order to account for neutrino oscillations, as explained before.

- **Neutrino Physics**, that, despite being somehow connected to flavour physics, deserves a certain priority on its own because of the large effort needed to be fully understood [22].

Any of the issues just listed demands for New Physics to arise at higher energies, paving the way for the appearance of a **hierarchy problem** [23], that we will discuss in greater detail in the next section.

1.2.1 The hierarchy problem

The hierarchy (or naturalness) problem is usually stated as the fact that the mass of the Higgs boson of the SM is much lower than its natural value. In few words QFT tells us that being a spin zero particle its mass is not protected against quantum corrections by any symmetry. In order to explain this statement we spell out some of the underlying assumptions that are crucial for such a problem to arise.

After reading the list of hints for Physics Beyond the SM of the previous section we should be aware that the latter cannot be a Theory of Everything. This means that there will be a certain energy scale $\Lambda_{\rm NP}$, at which New Physics (new degrees of freedom, new interactions, ...) enters the game. $\Lambda_{\rm NP}$ has to be quite larger than v, the scale of EWSB, since experimental results have shown that New Physics should contribute very little to the SM framework at present collider energies. Moreover in the SM a single fundamental Higgs field is assumed to exist up to $\Lambda_{\rm NP}$.

Under the previous three considerations the problem of naturalness is easily seen to arise. As anticipated the SM is assumed to be valid with no modifications up to the scale $\Lambda_{\rm NP}$. We can then calculate the one loop corrections to the Higgs boson mass by cutting off the divergent unless integral $\Lambda_{\rm NP}$. Restricting for simplicity to the most relevant contribution, the Yukawa coupling with the top quark depicted in Figure 1.1a, after a straightforward calculation one obtains that

$$(m_h^2)_{1\,\rm loop} = (m_h^2)_{\rm bare} - 3\frac{|\lambda_t|^2}{8\pi^2}\Lambda_{\rm NP}^2 + \mathcal{O}\left(m_t^2 \ln\left(\frac{\Lambda_{\rm NP}}{m_t}\right)\right) , \tag{1.9}$$

where λ_t and m_t are respectively the top quark Yukawa coupling and its mass. The scale of New Physics enters quadratically the one loop corrections, and if it happens to be large that would imply large corrections to the bare mass value.

In particular, taking into account the onset of gravity at $M_{\rm Planck}$, one should consider that some New Physics is needed to protect the Higgs boson mass against radiative corrections without advocating finetuning. Two different approaches can thus be considered. The first one consists of relieving the tension by making the onset of gravity not so far from the EW scale, as in the context of Large Extra Dimensions (ED) [24] or of ED with a warp factor [25, 26]. Alternative possibilities are feasible if gravity lied at $M_{\rm Planck}$instead: there might be a strong sector that is the real responsible of the EWSB (Technicolor [27]) or the Higgs boson mass may be protected by some extra symmetry (Strong Interacting Light Higgs [28], Little Higgs [29], ...). Supersymmetry, that is going to be the main subject of this thesis, belongs to the latter category.

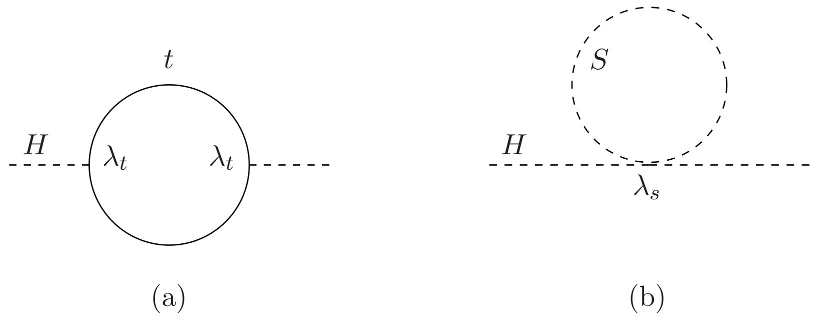

Figure 1.1: One loop contributions to Higgs boson two point function coming from: (a) SM like Yukawa coupling to a fermion, (b) four boson interaction with a scalar particle S.

1.3 Supersymmetry

Supersymmetry (SUSY) [30–32] challenges the hierarchy problem[5] through the introduction of new particles that cancel the leading quadratic contribution to the one loop correction to the Higgs boson mass. Suppose there was a complex scalar particle S of mass m_s coupling to the Higgs boson by means of a Lagrangian term $\mathcal{L}_{H-S} = -\lambda_s |S|^2 |H|^2$, then the diagram in figure 1.1b leads to a correction

$$\Delta m_h^2 = +\frac{\lambda_s^2}{16\pi^2}\Lambda_{\rm NP}^2 + \mathcal{O}\left(m_s^2 \ln\left(\frac{\Lambda_{\rm NP}}{m_s}\right)\right) . \tag{1.10}$$

If the coupling λ_s were equal to λ_t^2 it is evident that the different statistics of the particles running in the loops of Figures 1.1a and 1.1b would account for a cancelation. To be specific if any SM fermionic (bosonic) degree of freedom coupling to the Higgs boson had an associated bosonic (fermionic) one with appropriate couplings the cancelation of quadratic divergencies can be made exact[6].

Headed by this preliminary result one would be tempted by the possibility of associating to any particle of the SM a partner of different statistics. In facts such an approach would suggest the existence of a spacetime symmetry commuting with the group of internal symmetries so that its generic generator Q acted as

$$Q|\text{Boson}\rangle = |\text{Fermion}\rangle , \qquad Q|\text{Fermion}\rangle = |\text{Boson}\rangle . \tag{1.11}$$

What proves to be helpful in defining the algebra of such a symmetry are the Coleman Mandula [34] and Haag Lopuszanski Sohnius theorems [35]. We are not interested in giving a complete discussion of the SUSY algebra: we just remind that the operator Q commutes with the squared-mass operator P^2.

The property of commutation of Q with Poincaré invariants, along with that of commutation with internal symmetry groups, accounts for the possibility to cast

[5]There are actually also other hints and motivations for the introduction of SUSY, such as the prediction of a very nice gauge coupling unification, the presence of plausible Dark Matter candidates, a coherent framework for radiative EWSB.

[6]There still remain logarithmic divergences owed to SUSY breaking (implying $m_s \neq m_t$), as we shall see later. This feature accounts for a little hierarchy problem [33], but we shall not discuss it here.

the fermionic and bosonic states related by Q itself in a single multiplet, known as supermultiplet. In any supermultiplet in order to balance the number of fermionic and bosonic degrees of freedom off-shell it is necessary to place also some auxiliary fields. The latter are non dynamical fields, so they get integrated out through their equations of motion. In the following we will not focus on explaining the techniques of superfield formalism, even though they are going to be used often in this thesis. For the latter purpose the reader will find a more exhaustive discussion in [36]. Here we just set some notation and give some practical equations in terms of the component fields.

In the following we restrict to N = 1 globally supersymmetric theories, namely to the presence of just one generator Q (and its adjoint $Q^\dagger$). We consider a theory whose gauge group is G. Matter fields (fermions) ψ_i belong to chiral superfields along with their bosonic partners (sfermions) ϕ_i and auxiliary fields F_i

$$\Phi_i = (\phi_i, \psi_i, F_i)\,, \tag{1.12}$$

where the index i accounts for transformations under G. Obviously also Higgs fields belong to chiral supermultiplets, with their partners known as the higgsinos. The gauge bosons A^a_μ, instead, belong to vector superfields, along with their spin 1/2 partner (gauginos) λ^a and the related auxiliary fields D^a

$$\mathbf{A}^a = (A^a_\mu, \lambda^a, D^a)\,, \tag{1.13}$$

where a is an index of G in the adjoint representation.

The kinetic lagrangian density for gauge supermultiplets can be written in terms of the component fields as

$$\mathcal{L}_{\text{gauge kin}} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a + i\,\lambda^\dagger_a \overline{\sigma}^\mu D_\mu \lambda^a\,, \tag{1.14}$$

where $(D_\mu\lambda)^a = \partial_\mu\lambda^a + g f_{abc} A^b_\mu \lambda^c$ is the covariant derivative for gauginos. The kinetic lagrangian density for the components of the chiral superfields is

$$\mathcal{L}_{\text{chiral kin}} = D^\mu \phi^*_i D_\mu \phi^i + i\,\psi^\dagger_i \overline{\sigma}^\mu D_\mu \psi^i\,, \tag{1.15}$$

where $D_\mu\phi^i = \partial_\mu\phi^i - igA^a_\mu(T^a\phi)^i$ and $D_\mu\psi^i = \partial_\mu\psi^i - igA^a_\mu(T^a\psi)^i$ are the covariant derivatives of the scalar and fermion components respectively. SUSY also predicts the existence of couplings between the gauginos and the components of chiral superfields owed to the generalization in superfield formalism of covariant derivatives. In particular one has the term

$$\mathcal{L}_{\text{gaugino-fermion-sfermion}} = -\sqrt{2} g (\phi^* T^a \psi)\lambda_a\,. \tag{1.16}$$

For later convenience we introduce the superpotential W such that in terms of superfields it reads[7]

$$W = \frac{1}{2} M^{ij} \Phi_i \Phi_j + \frac{1}{3!} y^{ijk} \Phi_i \Phi_j \Phi_k + L^i \Phi_i\,, \tag{1.17}$$

[7]The linear term can be written only if Φ_i is a complete gauge singlet.

whose derivatives are named F-terms and are denoted by $\overline{F}_i = F_i^* = -W_i = -\frac{\partial W}{\partial \phi^i}$. Summing everything up one gets the interaction lagrangian density

$$\mathcal{L}_{\rm int} = -\frac{1}{2}\left(M_{ij}\psi^i\psi^j + y^{ijk}\phi_i\psi_j\psi_k + \text{h.c.}\right) - \sqrt{2}g(\phi^* T^a\psi)\lambda_a + \text{h.c.} - V(\phi^i, \phi^*)\,. \tag{1.18}$$

where $V(\phi^i) = F_k^* F^k + \frac{1}{2}D^a D^a$ is the scalar potential.

In this quick overview of SUSY formalism we now discuss the presence of SUSY breaking. The necessity for SUSY to be broken will be discussed in a later section; for now we just content ourselves with the notion that such a breaking has to be "soft" in order not to lose the nice cancelation of quadratic divergencies [37]. The soft breaking terms will make the different components of a supermultiplet to have different masses. The soft terms that can be considered are

$$\begin{aligned}\mathcal{L}_{\rm soft} &= -\left(\frac{1}{2}M_a\lambda^a\lambda^a + \frac{1}{3!}a^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}c^{ijk}\phi_i^*\phi_j\phi_k + \frac{1}{2}b^{ij}\phi_i\phi_j + t^i\phi_i\right) + \text{h.c.} \\ &\quad -(m^2)^{ij}\phi_i^*\phi_j\,,\end{aligned} \tag{1.19}$$

where just as argued before the terms proportional to t^i and c^{ijk} are possible only if ϕ_i belongs to a complete gauge singlet superfield, and will not play a role in the following. The terms above clearly break SUSY since they involve only the scalars and the gauginos and not their partners. Now that the ingredients for the lagrangian density of a spontaneously broken SUSY theory have been spelt out it is time to show the realization of the easiest SUSY extension of the SM, the Minimal Supersymmetric Standard Model (MSSM).

1.3.1 The lagrangian of the MSSM

The MSSM is the minimal supersymmetric extension of the SM: it requires nothing but the content of the SM extended to the SUSY partners. Actually, in order to construct a feasible SUSY theory, because of the constraint of analiticity of the superpotential, it is necessary to consider at least two Higgs doublets H_u and H_d, thus one has to supersymmetryze a Two Higgs Doublet Model (2HDM) [38]. Here we spell out some of the MSSM relevant features in order to set the notation for later convenience.

The gauge group of the MSSM is the usual $SU(3)_C \times SU(2)_L \times U(1)_Y$. The fermions of the SM are described by the fermionic components of the chiral superfields q, u^c, d^c, l, e^c, while the Higgses are the scalar components of the chiral superfields h_u and h_d. The gauge fields are the bosons of some vector superfields. All in all the field content of the MSSM can be resumed in Table 1.1.

The kinetic lagrangian density for gauge and chiral superfields are described by equations 1.14 and 1.15 respectively; interaction terms of the form gaugino-fermion-sfermion, instead, are modeled over equation 1.16. The superpotential of the theory is written in the superfield formalism as

$$W_{\rm MSSM} = \lambda_U u^c q h_u + \lambda_D d^c q h_d + \lambda_E e^c l h_d + \mu h_u h_d\,, \tag{1.20}$$

Superfield	fermion component	scalar component	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
q_i	q_i	$\tilde{q}_i$	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{6}$
u^c_i	u^c_i	$\tilde{u}^c_i$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$-\frac{2}{3}$
d^c_i	d^c_i	$\tilde{d}^c_i$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\frac{1}{3}$
l_i	l_i	$\tilde{l}_i$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$
e^c_i	e^c_i	$\tilde{e}^c_i$	$\mathbf{1}$	$\mathbf{1}$	1
h_u	$\tilde{h}_u$	h_u	$\mathbf{1}$	$\mathbf{2}$	$\frac{1}{2}$
h_d	$\tilde{h}_d$	h_d	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$
B	$\tilde{B}$ or λ_1	B_μ	$\mathbf{1}$	$\mathbf{1}$	0
W^α	$\tilde{W}^\alpha$ or λ_2^α	W^α_μ	$\mathbf{1}$	$\mathbf{3}$	0
G^a	$\tilde{g}^a$ or λ_3^a	g^a	$\mathbf{8}$	$\mathbf{1}$	0

Table 1.1: MSSM field content and notation. The reader will notice a slight abuse of notation, since the chiral superfields and some of their components are denoted by the same symbol. In the following it will be usually apparent if we will be referring to the superfield or the specific component. In situations in which confusion might arise the specific choice of notation will be spelt out.

where λ_i are 3x3 Yukawa matrices in flavour space and μ is responsible for a supersymmetric mass for the Higgs superfields.

The soft SUSY breaking terms that one has to consider because of SUSY breaking are contained in the lagrangian density

$$\begin{aligned} -\mathcal{L}_{\text{soft MSSM}} &= A_U \tilde{u}^c \tilde{q} h_u + A_D \tilde{d}^c \tilde{q} h_d + A_E \tilde{e}^c \tilde{l} h_d + B h_u h_d + \text{h.c.} \\ &\quad + m^2_{h_u} h_u^\dagger h_u + m^2_{h_d} h_d^\dagger h_d + \tilde{m}^2_q \tilde{q}^\dagger \tilde{q} + \tilde{m}^2_{u^c} \tilde{u}^{c\dagger} \tilde{u}^c \\ &\quad + \tilde{m}^2_l \tilde{l}^\dagger \tilde{l} + \tilde{m}^2_{d^c} \tilde{d}^{c\dagger} \tilde{d}^c + \tilde{m}^2_{e^c} \tilde{e}^{c\dagger} \tilde{e}^c \\ &\quad + \frac{1}{2} M_a \lambda_a \lambda_a + \text{h.c.}\,, \end{aligned} \tag{1.21}$$

where A_i and $\tilde{m}^2_{\tilde{f}}$ are 3x3 matrices in flavour space, B is the $B\mu$ term and M_a are the gaugino masses.

1.4 Supersymmetry breaking and its mediation

If SUSY were an exact symmetry of Nature then the fermion and sfermion mass matrices would be the same. Nonetheless experimental evidence shows that the mass of the sfermions has to be much larger than that of the corresponding fermions. Thus SUSY, as anticipated, has to be broken and such a breaking has to be "soft" to avoid the reappearance of quadratic divergencies in the theory.

The breaking of SUSY makes two relevant issues to pop up. The first one affects the mechanism through which SUSY is effectively broken; the second one, more phenomenological, concernes the effects of such a breaking on the low energy mass spectrum of the theory itself. In the following discussion we will be mainly focused on the second point.

The necessary and sufficient condition for SUSY to be broken is given by the fact that the vacuum energy of the ground state has to be larger than zero. Such a statement, since $V(\phi^i) = F_k^* F^k + \frac{1}{2} D^a D^a$, can be translated to the condition that SUSY is broken if the equations $\langle F_i \rangle = 0$ and $\langle D_a \rangle = 0$ cannot be simultaneously satisfied. The appearance of an F-term (or D-term) vev different from zero is a sufficient condition for SUSY breaking.

One could naively expect that in the case of SUSY breaking the fact that all the particles have a mass matrix equal with that of the corresponding antiparticle gets completely lost. Actually in case of spontaneuos SUSY breaking the masses of particles and antiparticles happen to be related by the supertrace formula [39]. The latter, valid at tree level for renormalizable theories for any set of conserved quantum numbers, states that the weighted sum of the squared masses of particle in the bosonic and fermionic sectors is

$$\mathrm{Str}\mathcal{M}^2 \equiv 3\mathrm{Tr}\mathcal{M}_1^2 - 2\mathrm{Tr}\mathcal{M}_{\frac{1}{2}}\mathcal{M}_{\frac{1}{2}}^\dagger + \mathrm{Tr}\mathcal{M}_0^2 = -2g\langle D^a \rangle \mathrm{Tr} T^a = 0\,, \tag{1.22}$$

where $\mathcal{M}_1$, $\mathcal{M}_{\frac{1}{2}}$, $\mathcal{M}_0$ stand respectively for the mass matrices of spin 1, 1/2 and 0 particles and the last equality holds for any non-anomalous gauge symmetry. The supertrace formula is crucial since it poses a strong phenomenological constraint: when applied to the SM quantum numbers it is apparent that while from theory one would get at tree level $\mathrm{Str}\mathcal{M}^2 = 0$, experimentally it is known that $\mathrm{Str}\mathcal{M}^2 > 0$. It is important for any model of SUSY breaking and of its mediation to circumvent this seeming threat and we will discuss about that later in more details.

As the vevs breaking SUSY cannot belong to the MSSM field content, SUSY breaking should take place in a hidden sector. The hidden sector in which SUSY breaking takes place has to share connections with the visible sector of observable fields. Such connections, organized in what is named messenger sector, thus mediate the communication of SUSY breaking and can give a rationale to the pattern of soft terms observed. Different mechanisms of mediation have been proposed in the literature, the most studied being the mediation through gauge interactions [40–42], the mediation through gravity interactions [43, 44] and mediation in Extra Dimensional (ED) scenarios [45–47]. We will focus on gauge mediation, that is going to be the main topic of this thesis, in the next section. Now we just give some remarks about gravity and anomaly mediation.

In gravity mediation [43,44] the hidden sector fields responsible for SUSY breaking couple to observable fields through gravitational interactions, giving rise to effects in the visible sector of order $\mathcal{O}\left(\frac{\langle F \rangle}{M_{\mathrm{Planck}}}\right)$. The constraint set by supertrace formula 1.22 is evaded through non renormalizability of gravitational interactions, and thus obtaining $\mathrm{Str}\mathcal{M}^2 > 0$ poses no further problems. Such a scenario is minimal, in the sense that it requires no extra messengers fields or interactions since gravity is already known to be an ingredient of Nature. However the main drawback of this mechanism lies in the fact that gravitational interactions, unless some specific mechanism of protection is assumed, have no reason not to generate possibly dangerous large contributions to flavour changing neutral currents (FCNC).

Mediation in the context of ED frameworks is usually based on the presence of a fifth compact dimension with two branes lying on its endpoints. The observable MSSM fields live on one brane, while the SUSY breaking ones live on the other: the separation of hidden and visible sectors becomes somehow geographical. In the gaugino mediation scenario [45, 46] gauge interactions are responsible of communicating SUSY breaking between the two branes, making the gauginos obtain a soft mass term from direct interaction, and the sfermion gaining one through interactions with gaugino themselves. Alternatively one can consider gauge interactions to be confined on the visible brane and gravitational ones to play the game of mediation, giving rise to the anomaly mediation framework [47]. The latter scenario is not viable in its minimal realization because of tachyonic masses for the sleptons, but the introduction of a universal scalar mass parameter can solve this issue. The two scenarios indicated here share some common phenomenology: as an example the sfermions tend to be lighter than the gauginos, in particular the lightest being the sleptons. Nonetheless an interesting point is given by the absence (or at least the suppression) of flavour violating sources giving rise to dangerous FCNC.

1.4.1 Gauge mediation

Gauge mediation [40–42] is a framework through which the effects of SUSY breaking are mediated to the observable sector by means of gauge interactions. The delicate issue of supertrace is circumvented by going to higher loop order: indeed in such a scenario the contributions to sfermion masses arise at the two loop level, making it possible to satisfy the phenomenological constraints.

In the following we will discuss in a simple model the main features of gauge mediation [48]. Suppose to have a supersymmetric gauge theory based on the simple group G. For what concernes SUSY breaking we just assume the presence of an unknown mechanism so that a superfield X, singlet under G, and belonging to the hidden sector takes a vev both in its scalar and F-term components:

$$\langle X \rangle = M + \theta^2 F\,. \tag{1.23}$$

The observable and the hidden sectors communicate through a messenger one that is composed of chiral superfields Φ and $\overline{\Phi}$ trasforming respectively under the fundamental and antifundamental representations of G. On top of that, such superfields couple to X by means of a superpotential term of the form

$$W = \lambda \overline{\Phi} X \Phi\,. \tag{1.24}$$

The presence of the F-term vev for the superfield X is responsible for the spectrum of the messenger superfields to be non supersymmetric, i.e. the fermion and scalar components will not share the same mass anymore, but they will experience a split $\mathcal{O}(\sqrt{F})$. The latter is the amount of SUSY breaking experienced by messenger fields. We notice that positivity of messengers' squared masses require $F < M^2$.

The chiral messengers couple to the observable fields by means of gauge interactions. In the visible sector SUSY breaking contributions are obtained by means of loop diagrams in which the chiral messenger components run: the effective soft terms that are induced are $\mathcal{O}(\frac{\alpha}{4\pi}F/M)$, that can be considered as the amount of SUSY breaking as felt by observable fields[8].

At one loop gaugino masses will arise by means of graph with a direct coupling to the messenger fields, as it is depicted in Figure 1.2 This contribution is given by

$$M_\lambda = \frac{\alpha}{4\pi}\frac{F}{M}\,, \tag{1.25}$$

where $\alpha = \frac{g^2}{4\pi}$, g being the coupling constant of the gauge group G.

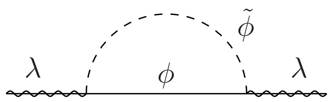

Figure 1.2: One loop contribution to gaugino mass in gauge mediation. For chiral superfields the tilde denotes the scalar component, for vector superfields the gaugino is denoted by λ.

Sfermion masses will arise at the two loop level: the interaction to messengers is mediated by gauge fields or by scalar exchange induced from D-terms. The different types of diagrams involved are shown in Figure 1.3. The total contribution, at the messenger scale, is given by

$$m^2_{\tilde{f}} = 2C^{\tilde{f}}\left(\frac{\alpha}{4\pi}\right)^2\left(\frac{F}{M}\right)^2\,, \tag{1.26}$$

where $C^{\tilde{f}}$ is the quadratic Casimir of the $\tilde{f}$ sfermion.

Trilinears in this theory do not arise through one loop messenger contribution and their principal contribution arise from the RG evolution proportional to gaugino masses. This in turn accounts for the trilinears to be relatively small in gauge mediation scenarios.

1.4.1.1 Pros and cons

The main advantage of the gauge mediation framework is provided by the safety of the resulting theory with respect to flavour violation problems. Because of the flavour diagonal structure of gauge interactions, indeed, the only sources to flavour violation are those coming from the Yukawa interactions, just as in SM. In such a context we happen to be in a Minimal Flavor Violation (MFV) scenario, and one could even generalize the usual GIM mechanism to a supersymmetric extension involving sparticles.

One of the drawbacks of gauge mediation is the μ and $\mu - B\mu$ problem, namely the generation of an EW scale μ term and that of a $B\mu$ term of comparable size [49]. In this framework, even if the solution of the μ problem can be quite easily obtained,

[8]It is usually assumed that $F/M^2 \ll 1$ in order to treat SUSY breaking effects as small perturbations of a SUSY preserving formalism. The latter assumptions will be of common use in the whole thesis.

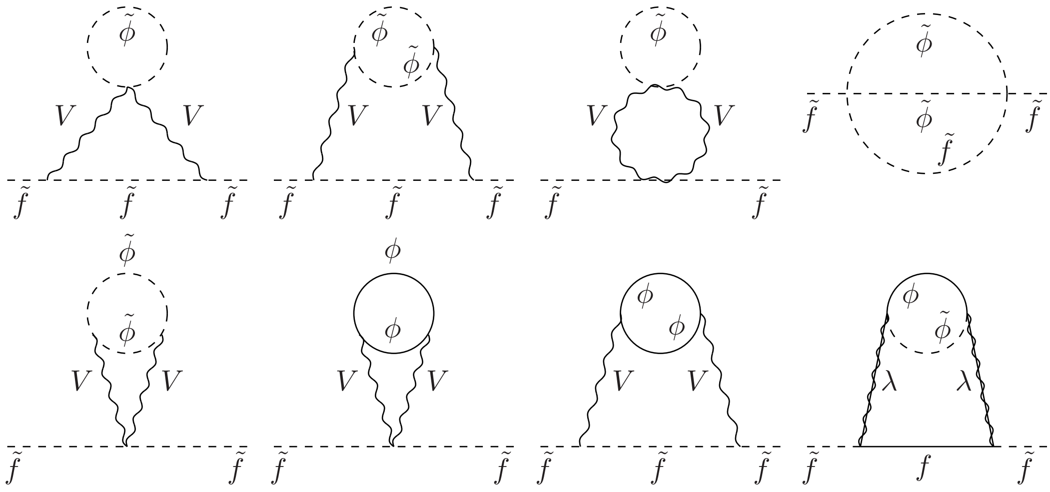

Figure 1.3: Two loop contributions to sfermion mass in gauge mediation. For chiral superfields the tilde denotes the scalar component, for vector superfields the gaugino is denoted by λ.

by neglecting the existence of a μ term at high scale and relating its generation to the SUSY breaking scale, still the $B\mu$ term is usually a loop factor larger than preferred. Such a problem is actually absent in theories of gravity mediation where generic Kähler interactions can solve the issue.

Finally concerning the most recent results at the LHC it is fair to say that gauge mediation in its easiest realization has some difficulties in generating an Higgs mass of order 125 GeV. Since trilinears are usually suppressed in the scenario the only possibility is that of raising the scale of SUSY breaking, but this accounts for an augmented little hierarchy problem. However there are some possible modifications slightly relaxing the issue as we shall see later on.

1.5 Outline

In this thesis we will study two models of gauge mediation and the phenomenology related to them, keeping the eyes wide open on the possible outcomes for such models at colliders. The first model is a model of Tree Level Gauge Mediation that was firstly proposed in 2009 [50,51] and then extended and more deeply studied recently. In chapter 2 we will discuss the model in its general theoretical realization, while we will devote chapter 3 to the phenomenological realization of its simplest $SO(10)$ realization at the LHC [52] and chapter 4 to the extension of the model to the exceptional symmetry groups case [53]. Then in chapter 5 we will discuss a brand new model of Yukawa-Gauge Mediation analyzing both the theoretical features of the framework and its phenomenology [54]. We will conclude this work through summary and conclusions in chapter 6

The original contribution of the thesis is contained in chapters 3, 4 and 5.

2

Tree Level Gauge Mediation

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In the previous chapter we introduced SUSY through both theoretical and experimental hints as a beyond the SM framework and identified the necessity for its breaking. In this chapter we will briefly review a particular mechanism of SUSY breaking mediation following the discussion given in [51]. This fairly recent mechanism, Tree Level Gauge Mediation (TGM), is quite interesting because it makes it possible to get sfermion masses at the tree level, while in usual Loop Gauge Mediation (LGM) scenarios, as the model proposed in section 1.4.1, they are obtained at the two loop level through the graphs of figure 1.3. In the next sections we will show how it is possible to circumvent the supertrace constraints, devising the general theory of the mechanism underlying TGM; then we will focus on the soft terms arising both at the tree level and at the one loop level, and mention some relevant issues about model building.

2.1 Tree Level Gauge Mediation: Why not?

For our purposes the net effect of SUSY breaking is the fact that some superfield takes an F-term vev in a hidden sector. One might wonder what kind of effective operator could transfer the information about such a breaking to the observable sector giving mass to the sfermions. The answer is of disarming easiness, the effective operator being

$$\int d^4\theta \, \frac{Z^\dagger Z Q^\dagger Q}{M^2} \,, \tag{2.1}$$

where Z is the superfield getting F-term vev, Q is an observable superfield and M is the mass scale of the mediating messengers: it is apparent that if $\langle Z \rangle = \theta^2 F$ then the scalar component of Q gets a mass. Directly from equation 2.1 one could try to guess how to draw a Feynman diagram responsible for the appearance of this operator. The answer is fairly simple: a tree level diagram in which a messenger field is exchanged, as in figure 2.1a.

Unfortunately the story is not over yet. In the previous chapter we introduced supertrace formula, equation 1.22, and argued that it holds at the tree level for renormalizable interactions: in usual LGM it can be circumvented since the effects of SUSY breaking are felt through diagrams with at least one loop. Tree level gauge mediation, instead, seems destined to a daunting failure because of the supertrace constraint. However in contexts inspired by supersymmetric Grand Unified Theories (GUTs) it can be revived quite easily.

If we restricted the supertrace formula to those superfields belonging to the MSSM it is apparent from experimental evidence that $\mathrm{Str}\mathcal{M}^2_{\mathrm{MSSM}} > 0$. But in GUTs there can easily arise many additional extra heavy fields, whose quantum numbers are the same as those of the MSSM fields, that can compensate the supertrace constraint, namely $\mathrm{Str}\mathcal{M}^2_{\mathrm{extra}} < 0$, so that $\mathrm{Str}\mathcal{M}^2_{\mathrm{MSSM}} + \mathrm{Str}\mathcal{M}^2_{\mathrm{extra}} = 0$. No phenomenological threat (i.e. tachyons) is posed by the extra fields since supertrace is just sensitive to the mass splitting between superpartners and not to their absolute mass values: indeed the negative SUSY breaking contribution adds up to a much larger supersymmetric mass[1].

Even though the presence of extra heavy fields can suggest us the path to a tree level realization of gauge mediation, another crucial point is still hidden in the supertrace formula. Since the latter holds for any set of conserved quantum numbers it can be easily reasoned that one has to consider an extension of the gauge group. Suppose we restricted ourselves to the fields with quantum numbers of the lightest down and up quark mass eigenstates. Then equation 1.22 would imply

$$m^2_{\mathrm{lightest}\,\tilde{d}} \leq m^2_d - \frac{1}{3} g' D_Y \qquad m^2_{\mathrm{lightest}\,\tilde{u}} \leq m^2_u + \frac{2}{3} g' D_Y \,, \tag{2.2}$$

[1] One could observe that if gaugino masses were very heavy, even if the latter would be a quite troublesome situation for naturalness, the direction of the previous inequalities would be reversed and no phenomenological threats were to appear. In facts what proves to be crucial in this case is the vanishing of the supertrace for any set of conserved quantum numbers, as explained in the next paragraph.

that cannot be solved simultaneously. Adding extra massive fields does not change the conclusions obtained: it is apparent that at least an extra $U(1)$ factor, contributing with a D-term of the same sign for both the equations 2.2, is needed.

It is quite evident that, to circumvent the troubles hidden behind the curtain of the supertrace formula and construct a TGM framework, an extension of both the gauge group and of the matter content of the theory is required. These, as we will see, can be easily obtained in the context of GUTs with a gauge group of Rank ≥ 5 by means of non standard matter embeddings. Anyway, before devoting to the phenomenological construction of a viable theory, we will at first build a well defined theoretical framework.

2.2 Tree Level soft terms: A formal approach

We consider a $N = 1$ globally supersymmetric gauge theory in four dimensions, whose gauge symmetry group is denoted by G. To be definite let g be the coupling constant of the gauge symmetry group and the orthonormalized generators be denoted as T_a's. The chiral content of the theory is composed of the superfields $\Phi \equiv (\Phi_1, \ldots, \Phi_n)$ belonging to suitable representations of the symmetry group G. The lagrangian density is made up of a canonical Kähler potential $K = \Phi^\dagger e^{2gV}\Phi$ for chiral superfields, the standard gauge kinetic function for the vector superfields associated to the generators of G and a superpotential $W(\Phi)$, analytic function of the various chiral superfields and obviously invariant under gauge transformations.

The gauge group G gets broken by the vev in the scalar component of some of the chiral superfields, $\phi_0 = \langle\phi\rangle$. Such a mechanism takes place at a scale much higher than the EW one, $g|\phi_0| \gg M_Z$, and leaves the subgroup $H \subset G$ unbroken. We obviously assume that $H \supset G_{\rm SM}$. After gauge symmetry breaking the generators of G will group into two sets, namely $T^l_a \in H : T^l_a\phi_0 = 0$ and $T^h_a \in G/H : T^h_a\phi_0 \neq 0$. The vector superfields associated to those generators belonging to the coset G/H become massive, their masses given by

$$(M^2_{V_0})_{ab} = g^2\phi_0^\dagger\{T^h_a, T^h_b\}\phi_0 = M^2_{V_a}\delta_{ab}\,, \tag{2.3}$$

where the last equality holds if one chooses a basis of the generators T^h_a in which the mass matrix is diagonal. For later convenience it proves useful to recast the chiral superfields as

$$\Phi = \phi_0 + \Phi' + \Phi^G, \quad \Phi^G = \sqrt{2}\,g\frac{\Phi^G_a}{M_{V_a}}T^h_a\phi_0, \quad \Phi' = \Phi'_i b_i\,, \tag{2.4}$$

where Φ^G are a collection of the Goldstone superfields, Φ' are the "physical" chiral superfields ($b_i^\dagger T_a\phi_0 = 0$, with $i = 1, \ldots, n - \dim(G/H)$), and ϕ_0 is the vev scalar component. Incidentally we notice that up to now we have just given some notation while discussing the supersymmetric Goldstone mechanism at play [55]: the theory is still perfectly supersymmetric, all the components of a specific heavy gauge vector superfield sharing the same mass. The same happens for the different components of the physical chiral superfields Φ', whose supersymmetric masses are given by

$$M^0_{ij} = \frac{\partial^2 W}{\partial \Phi'_i \partial \Phi'_j}(\phi_0) = M_i \delta_{ij} \quad M_i \geq 0\,, \tag{2.5}$$

where the second equality and the positiveness condition on the M_i might be obtained through a suitable choice of the b_i's.

2.2.1 Supersymmetry breaking

In order for SUSY to be broken some of the Φ' superfields should get an F-term vev, namely $\langle \Phi' \rangle = F_0 \theta^2$, where $|F_0| \ll M^2_{V_a}$. The latter assumption is true in most realistic cases, as already discussed in section 1.4.1, and makes it possible to calculate SUSY breaking effects using an explicitely supersymmetric formalism, treating them as small perturbations in the effective field theory for the observable superfields [48, 56]. In the end we can thus decompose the chiral superfields Φ' as

$$\Phi' = (Z, Q, \Phi^h)\,, \tag{2.6}$$

where Z represents the field taking F-term vev ($\langle Z \rangle = F_0 \theta^2$), Φ^h are a set of heavy superfields obtaining a large mass along with the breaking of G and Q are the observable light superfields. In equation 2.5 we respectively have $M_{\Phi^h} \gg \sqrt{|F_0|}$ and $M_Q \lesssim \sqrt{|F_0|}$.

Gauge invariance of the superpotential ($\delta_{\text{gauge}} W = F_i^\dagger T_a^{ij} \phi_j = 0$) along with the stationary condition for the scalar potential at the minimum ($\langle \partial V / \partial \phi_i \rangle = 0$) show that the F-term vevs induce D-term vevs for the heavy vector superfields. More in details we write

$$0 = \frac{\partial}{\partial \phi_k} F_i^\dagger T_a^{ij} \phi_j = \frac{\partial^2 W}{\partial \phi_k \partial \phi_i} (T_a \phi)^i + \frac{\partial W}{\partial \phi_i} T_a^{ik} \tag{2.7a}$$

$$0 = \langle F_j \rangle \left\langle \frac{\partial^2 W}{\partial \phi_j \partial \phi_i} \right\rangle \langle T_b \phi \rangle_i - g \langle D_a \rangle \left\langle \phi^\dagger T_a T_b \phi \right\rangle\,, \tag{2.7b}$$

from which at the minimum and by means of equation 2.3 we obtain

$$\left\langle D_a^h \right\rangle = -2g \frac{F_0^\dagger T_a^h F_0}{M^2_{V_a}} \tag{2.8a}$$

$$\left\langle D_a^l \right\rangle = 0\,. \tag{2.8b}$$

The induced D-terms determined so far give rise to tree level soft masses for the scalar components of the Φ' superfields. These contributions, along with the collection of all the other soft terms arising from the F-term vevs, can be formally derived through the effective theory obtained integrating out the heavy vector superfields and the Goldstone chiral superfields that have been eaten up.

2.2.2 Integrating out heavy superfields

The procedure of integrating out heavy superfields can be performed through the solution of the equations of motion. Here we shall briefly discuss the algorithm to perform such a calculation referring the interested reader to [51, 57] for greater detail.

In our framework it is important to preserve a manifestely supersymmetric formalism, thus it proves very useful to integrate heavy superfields directly at the superspace level. Unfortunately the usual truncation on the number of derivatives performed in non supersymmetric QFTs to obtain effective field theories would not preserve SUSY itself: indeed supersymmetric transformations would intertwine fields with different spin and different number of derivatives. The generalization of the common expansion order given by the number of derivatives n_∂ is given in supersymmetric theories by

$$n = n_\partial + \frac{1}{2}n_\psi + n_F + \frac{3}{2}n_\lambda + 2n_D\,, \tag{2.9}$$

where n_ψ is the number of fermions, n_F the number of chiral auxiliary fields, n_λ the number of gauginos and n_D the number of vector auxiliary fields. At superspace level chiral and vector superfields have $n = 0$, while any $d\theta$ integration or supercovariant derivative has $n = 1/2$. Putting the pieces together one has that $n = 2$ for the chiral lagrangian density and $n = 4$ for the gauge kinetic term. In general, when integrating out heavy superfields, contributions with different value of n arise; from the previous considerations, anyway, it is apparent that when seeking for threshold effects on the chiral lagrangian density (gauge kinetic function) we shall discard terms with $n \geq 3$ ($n \geq 5$).

The integration of heavy superfields out can be performed by calculating the equations of motion both for chiral and vector heavy superfields,

$$\frac{\partial W}{\partial \Phi}(\Phi_0^h) = 0 \quad \text{and} \quad \frac{\partial K}{\partial V}(V_0^h) = 0\,, \tag{2.10}$$

the latter two determining the values Φ_0^h and V_0^h at which heavy superfields are stabilized. Inverting equations 2.10 we get

$$\Phi^h = \Phi_0^h(\Phi^l) \quad \text{and} \quad V^h = V_0^h(\Phi^l, V^l)\,, \tag{2.11}$$

through which we algebraically determine the heavy superfields as a function of the light ones. The effective theory in terms of the light superfields only is then described by K_{eff} and W_{eff} given by

$$\begin{aligned} K_{\text{eff}}(\Phi^l, V^l) &= K(\Phi^l, \Phi_0^h(\Phi^l), V_0^h(\Phi^l)) \\ W_{\text{eff}}(\Phi^l) &= W(\Phi^l, \Phi_0^h(\Phi^l))\,. \end{aligned} \tag{2.12}$$

As already anticipated we discard terms with $n \geq 3$ for both $K_{\text{eff}}(\Phi^l, V^l)$ and $W_{\text{eff}}(\Phi^l)$.

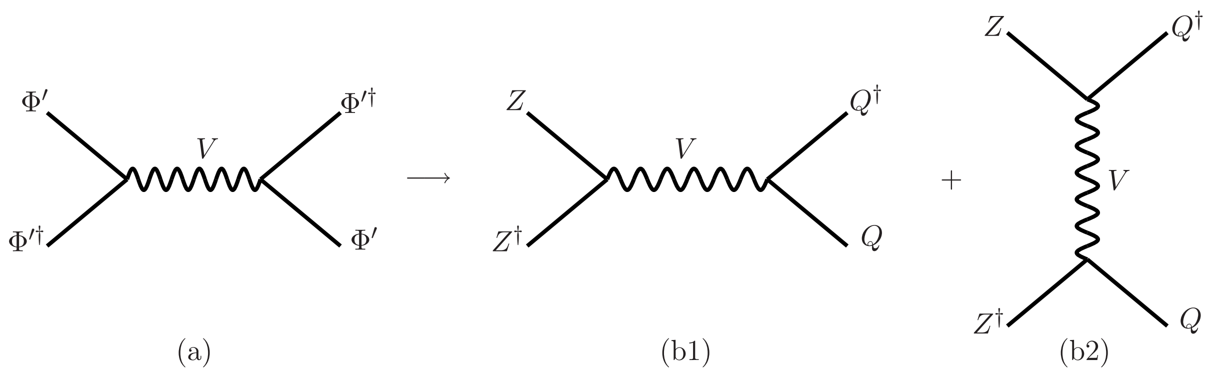

Figure 2.1: Soft supersymmetry breaking contributions to sfermion masses. (a) Tree Level Gauge Mediation supergraph generating the operator in equation 2.16 when integrating out the heavy vector superfield messengers. (b1) S-channel tree level contribution, as in first term of equation 2.19. (b2) T-channel tree level contribution as in second term of equation 2.19

2.2.3 Tree Level soft terms from an effective operator

After the short excursus on the procedure of integrating heavy superfields out we are ready to turn back to TGM theory. In the effective theory below the scale M_{V^h} we will be interested only in operators that are suppressed by at most two power of M_{V^h}, i.e. with dimension up to 6. We can thus use the equations of motion 2.10 to express V^h in terms of the light superfields

$$V_a^h\,(M_V^2)_{ab} = -\frac{1}{2}\frac{\partial}{\partial V_b^h}\Phi'^\dagger e^{2gV}\Phi'\,, \tag{2.13}$$

where

$$(M_V^2)_{ab} = \frac{1}{2}\frac{\partial^2}{\partial V_a^h \partial V_b^h}\left(\phi_0^\dagger e^{2gV}\phi_0\right)\bigg|_{V^h=0}. \tag{2.14}$$

The total effective contribution to the Kähler potential is then

$$K_{\rm eff} = -(M_V^2)_{ab}V_a^h V_b^h\,; \tag{2.15}$$

we are interested in the lowest order term given by the effective operator

$$\delta K_{\rm eff} = -\frac{g^2}{M_{V_a}^2}(\Phi'^\dagger T_a^h\Phi')(\Phi'^\dagger T_a^h\Phi')\,, \tag{2.16}$$

where we are considering a basis for the heavy generators T_a^h in which the mass matrix is diagonal.

The operator in equation 2.16 is sketched in figure 2.1a. The latter diagram shows that the extra contribution given by integrating out the heavy vector superfields can result in soft SUSY breaking terms arising at tree level. In particular when some of the chiral physical superfields Φ' take F-term vevs[2] the soft terms

[2]We incidentally note that the Φ''s taking F-term vev should belong to non trivial representation of the original symmetry group G in order for them to couple to the heavy vector messengers and thus give a contribution through TGM mechanism.

that are generated at tree level are

$$-\mathcal{L}_{\rm soft}^{\rm tree} = -\, F_{0i}\frac{\partial \hat{W}}{\partial \Phi_i} - 2g^2\frac{(F_0^\dagger T_a^h \psi')(\phi'^\dagger T_a^h \psi')}{M_{V_a}^2} + \text{h.c.}$$
$$+\, 2g^2\frac{(F_0^\dagger T_a^h F_0)(\phi'^\dagger T_a^h \phi')}{M_{V_a}^2} + 2g^2\frac{(\phi^\dagger T_a^h F_0)(F_0^\dagger T_a^h \phi')}{M_{V_a}^2} - F_0^\dagger F_0\,, \qquad (2.17)$$

where $\hat{W}$ is the superpotential in the effective theory,

$$\hat{W}(\Phi') = W(\phi_0 + \Phi')\,, \qquad (2.18)$$

choosing a gauge were the Goldstone superfields have been explicitely eaten up, so that $\Phi^G = 0$.

It is quite instructive to analyze the different terms in equation 2.17. The first term is connected to the presence of possible couplings among the SUSY breaking F-term vevs and the heavy chiral superfields Φ^h: such couplings will be crucial for the generation of gaugino masses at the one loop level. The possibility of couplings between the F-term vevs and the observable superfields is neglected, as we will comment in greater detail later.

The second term is of particular interest since it gives rise to Yukawa interactions whose size is particularly small, namely $\mathcal{O}(|F_0|/M_{V^h}^2)$. Such Yukawa couplings are of phenomenological relevance since they might be suitable for generating naturally tiny masses for neutrinos.

Finally the terms in the second line contribute to soft scalar mass terms. Summing up we have

$$\tilde{m}_{ij}^2 = 2g^2\left[(T_a^h)_{ij}\frac{F_0^\dagger T_a^h F_0}{M_{V_a}^2} + \frac{(T_a^h F_0)_i^*(T_a^h F_0)_j}{M_{V_a}^2}\right]\,, \qquad (2.19)$$

with the first and second terms respectively corresponding to the S-channel and T-channel contributions in figure 2.1b1 and 2.1b2 respectively. The T-channel contribution obviously happens to be relevant only for those superfields that are gauge partners of the Goldstino superfields Z, not disregarding the fact that the interaction vertex $ZQ^\dagger V$ has to be invariant under the unbroken gauge group $H \in G$.

2.2.3.1 Additional properties of the superpotential

In phenomenological applications the superfields taking F-term vev should be singlets under the SM gauge interactions. This consideration easily translates to the fact that there is no $G_{\rm SM}$ interaction through which the observable superfields and the SUSY breaking ones can feel each other. This can be easily extended to superpotential interactions[3] requiring that

$$\frac{\partial^2 \hat{W}}{\partial Z_j \partial Q_i}(Z, Q, \Phi^h = 0) = 0\,. \qquad (2.20)$$

[3]In the following we consider the case of renormalizable superpotentials and unbroken EW symmetry.

Fair enough we might say that the F-term vevs are hidden to the observable superfields.

Still we should care about the possible presence of a superpotential term of the form $Z_i Q_j \Phi^h_k$, which would be quite interesting, potentially being source for tree level generated A-terms. The chiral superfields Φ^h are heavy taking mass along with the breaking of the originary gauge symmetry group G. Thus in order to recover the effective field theory at lower energies we wish to integrate them out in a manifestely supersymmetric way by means of the chiral superfield equations of motion $\partial W/\partial\Phi^h = 0$. Suppose we write the superpotential $\hat{W}$ as

$$\hat{W} = \hat{W}_{\Phi^h=0} + \frac{M^h_{ij}}{2}\Phi^h_i\Phi^h_j + W_3(Z,Q,\Phi^h)\,, \tag{2.21}$$

then through the equations of motion we obtain

$$\Phi^h_i = -(M^h_{ij})^{-1}\frac{\partial W_3}{\partial \Phi^h_j}(Z,Q) + \mathcal{O}\left((M^h)^{-2}\right)\,. \tag{2.22}$$

The effective superpotential for Z and Q superfields is therefore

$$\hat{W}_{\text{eff}}(Z,Q) = \hat{W}_{\Phi^h=0} - \frac{(M^h_{ik})^{-1}}{2}\frac{\partial W_3}{\partial\Phi^h_k}\frac{\partial W_3}{\partial\Phi^h_i} + \mathcal{O}\left((M^h)^{-2}\right)\,, \tag{2.23}$$

and an effective extra contribution to the Kähler potential is also obtained

$$\delta K_{\text{eff}}(Z,Q) = \left[(M^h_{ij})^{-1}\frac{\partial W_3}{\partial\Phi^h_j}\right]\left[(M^h_{ik})^{-1}\frac{\partial W_3}{\partial\Phi^h_k}\right]^\dagger + \mathcal{O}\left((M^h)^{-3}\right)\,. \tag{2.24}$$

All in all integrating out the chiral heavy superfields accounts for tree level generated A-terms from $\hat{W}_{\text{eff}}$ and some extra negative contribution to the soft scalar masses from δK_{eff}. In particular the latter can be the origin of phenomenological troubles since they are of the same order of magnitude ($\widetilde{m} \sim F/M^h$) of the tree level ones, equation 2.19, and consequently they should be subleading. Such a feature can be easily obtained by imposing

$$\frac{\partial^3\hat{W}}{\partial Z\partial Q\partial\Phi^h} = 0\,. \tag{2.25}$$

The latter equation, that at first impact might seem to give a weird ad hoc solution, is actually automatically fulfilled in the minimal case we are going to discuss in chapter 3. Even in the next to minimal case of E_6, that will be the subject of chapter 4, equation 2.25 is satisfied with fairly minimal assumptions (see section 4.2). Incidentally we note that the condition in equation 2.25 forbids also the presence of tree level A-terms.

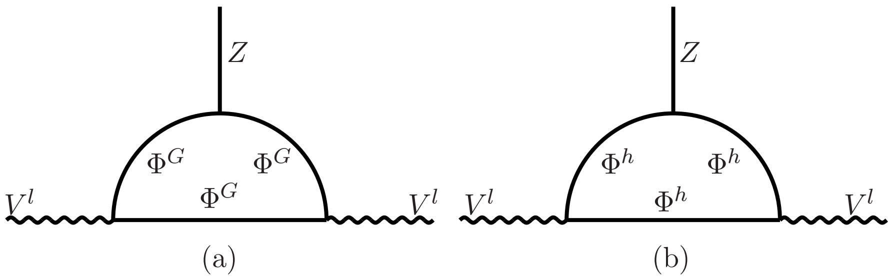

Figure 2.2: One loop contributions to light gaugino masses from the exchange of heavy vector (a) and chiral (b) degrees of freedom.

2.3 One Loop Soft terms

In the previous section we showed how tree level soft masses arise, but still we did not discuss about the generation of other soft terms, apart from the possible generation of trilinears at the tree level, that had to be dismissed because of phenomenological troubles. Other SUSY breaking contributions enter the game in TGM theories at the one loop level; obviously one should also not forget the possibly relevant contribution to soft scalar masses arising at the two loop level.

2.3.1 Gaugino masses

Gaugino masses arise from one loop corrections to the wave kinetic function. In particular they are generated through graphs in which heavy vectors or heavy chiral messengers do circulate as shown in figure 2.2: thus we easily see that

$$M^g_{ab} = (M^g_{ab})_V + (M^g_{ab})_\Phi\,. \tag{2.26}$$

One can notice the existence of a suppression for gaugino masses with respect to sfermion masses which is given by the extra loop factor in the diagram generating the former. The latter suppression could be the origin of some phenomenological issues, possibly pushing the sfermion masses to multi TeV values in order to satisfy the experimental bounds. Actually some numerical factors owed to the gauge structure of the theory will show of great help in reducing such a hierarchy.

In order to calculate the threshold corrections on gaugino masses coming from the exchange of heavy vector superfields we notice that after SUSY breaking the scalar ϕ^G_a and fermion ψ^G_a components of the Goldstone superfields are split by the mass term

$$-\frac{\partial^2 W}{\partial\Phi^G_a\partial\Phi^G_b}(\phi_0)\psi^G_a\psi^G_b = -m_{ab}\psi^G_a\psi^G_b\,. \tag{2.27}$$

The latter is non vanishing when a suitable F-term, non singlet under T^h_a, gets vev, since the gauge invariance condition for W applied to the broken generators,

$0 = \delta_{gauge} W = F_i^\dagger (T_a^h)^{ij} \phi_j$, yields

$$m_{ab} = g^2 \frac{F_0^\dagger \{T_a^h, T_b^h\} \phi_0}{M_{V_a} M_{V_b}} \,. \tag{2.28}$$

We can write this equation more fruitfully in a block diagonal form, recalling that the heavy vector representations are reducible under the unbroken group H to a set of irreducible components belonging to the representation r of mass $\hat{M}_{V_r}$. If T_a^h and T_b^h belong to the same representation we have

$$g^2 \phi_0^\dagger \{T_a^h, T_b^h\} F_0 = m_{ab}^* \hat{M}_{V_r} \equiv \frac{\partial \hat{M}_{V_r}}{\partial Z} |F_0| \delta_{ab} \,. \tag{2.29}$$

In the limit we are interested into, $F_0 \ll M_V^2$, the mass term m_{ab} can be treated as a perturbation in the loop calculation of gaugino masses, yielding a threshold contribution at the scale where the heavy vector superfields live given by

$$(M_{ab}^g)_V = -2 \frac{g^2}{(4\pi)^2} \sum_r S_{ab}(r) \frac{|F_0|}{\hat{M}_{V_r}^2} \frac{\partial \hat{M}_{V_r}^2}{\partial Z} , \tag{2.30}$$

where $S_{ab}(r) = \mathrm{Tr}\left(r(T_a^l) r(T_b^l) \right)$ is the Dynkin index of the representation $r : T \to r(T)$ of the generator T.

The chiral contribution to gaugino masses is usually dominant over the vector one and arises from one loop graphs analogous to those in LGM: the scalar and fermion components of the heavy chiral superfields couple to gauginos and, since they are split by SUSY breaking, they will induce a contribution $(M_{ab}^g)_\Phi$, as in figure 2.2b. The heavy chiral superfields have a supersymmetric mass term $M_{ij}^h \Phi_i^h \Phi_j^h$ due to the spontaneous breaking of the gauge symmetry group G and they can be casted into irreducible representations under H of mass $\hat{M}_r$. When SUSY is broken the scalar components acquire an extra contribution $F_{ij} \phi_i^h \phi_j^h$ given by

$$F_{ij} = -\frac{\partial^3 \hat{W}}{\partial \Phi_i^h \partial \Phi_j^h \partial Z} |F_0| \equiv \frac{\partial \hat{M}_r}{\partial Z} |F_0| \delta_{ij} \,, \tag{2.31}$$

that under our assumptions can be assumed to be subleading with respect to the supersymmetric term $\hat{M}_r$. Thus at leading order the total chiral contribution to gaugino masses is

$$(M_{ab}^g)_\Phi = \frac{g^2}{(4\pi)^2} \sum_r S_{ab}(r) \frac{|F_0|}{\hat{M}_r} \frac{\partial \hat{M}_r}{\partial Z} \,. \tag{2.32}$$

These contribution are obviously a sum of different terms arising at the scale at which the corresponding heavy chiral superfield gets integrated out.

2.3.2 Kähler contributions

We now turn to the other soft terms arising from the one loop exchange of heavy vector and chiral superfields: the Kähler potential gets a one loop correction given by [58]

$$\begin{aligned}\delta_{\text{1-loop}}K = & -\frac{1}{32\pi^2}\,\mathrm{Tr}\left[M_\Phi^\dagger M_\Phi\left(\log\frac{M_\Phi^\dagger M_\Phi}{\Lambda^2}-1\right)\right]+ \\ & +\frac{2}{32\pi^2}\,\mathrm{Tr}\left[M_V^2\left(\log\frac{M_V^2}{\Lambda^2}-1\right)\right], \end{aligned} \tag{2.33}$$

where

$$(M_\Phi)_{ij} = \frac{\partial^2 W}{\partial\Phi_i\partial\Phi_j}(\Phi)\,, \qquad (M_V^2)_{ab} = \frac{\partial^2 K}{\partial V_a \partial V_b}(\Phi, V=0)\,, \tag{2.34}$$

$K = \Phi^\dagger e^{2gV}\Phi$ is the canonical Kähler and the indices effectively run over the heavy superfields. Expanding the previous relations around ϕ_0 and spelling out the dependence on the Z superfield taking F-term vev we obtain the relevant contributions

$$\begin{aligned}\delta_{\text{1-loop}}K &= \left(\alpha_{ij}^{(1)} Z Q_i^\dagger Q_j + \frac{\beta_{ij}^{(1)}}{2} Z^\dagger Q_i Q_j + \text{h.c.}\right) + \\ &+ \alpha_{ij}^{(2)} Z^\dagger Z {Q_i}^\dagger Q_j + \left(\frac{\beta_{ij}^{(2)}}{2} Z^\dagger Z Q_i Q_j + \text{h.c.}\right) + \dots\,, \end{aligned} \tag{2.35}$$

where we remind that Q represent the observable superfields[4].

We now turn to the analysis of the different contributions, which is quite straightforward taking into account the number of θ's and $\bar{\theta}$'s present in each term. The first term $\alpha^{(1)}$ is responsible for the presence of the one-loop A-terms,

$$\mathcal{L}^A_{\text{1-loop}} = -A_{ij} q_i \frac{\partial \hat{W}}{\partial Q_j}(q) \quad \text{with} \quad A_{ij} = |F_0|\alpha^{(1)}\,, \tag{2.36}$$

where q_i is the scalar component of the Q_i superfield.

We wish to comment about this contribution, that is absent in LGM. In that case the trilinears are essentially generated by means of the interplay between the RGEs of gauginos and trilinear terms while running from the messengers' scale to the EW one. In TGM, instead, sizeable contribution can arise by means of one loop graphs in which the chiral heavy superfields get integrated out, as depicted in figure 2.3a. As one can easily spot from the graph the crucial term is the messenger matter mixing Yukawa interaction of the form $\lambda_{QQ\Phi^h} Q_i Q_j \Phi_k^h$ [49, 56].

[4]Terms with dependence $Z^\dagger Q_i$ have been omitted since they could destabilize the hierarchy [59]: their absence can be easily justified by stating that there are no light chiral superfields with the same quantum numbers of Z.

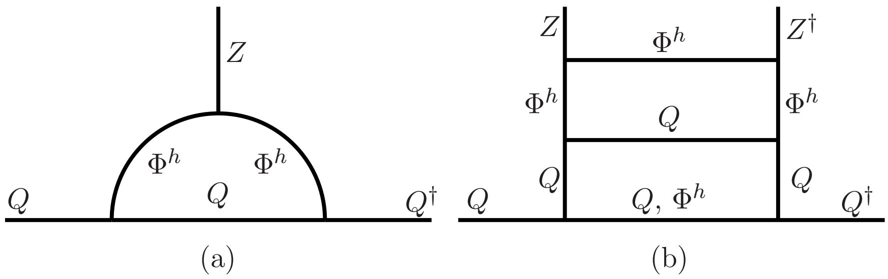

Figure 2.3: Additional contributions to SUSY breaking terms owed to messenger matter mixing: (a) one loop trilinear terms and (b) two loop scalar masses.

The second term $\beta^{(1)}$ evidently contributes to the μ-term in the superpotential

$$W^{\mu}_{\text{1-loop}} = \frac{M_{ij}}{2} Q_i Q_j, \quad \text{with} \quad \mu_{ij} = |F_0|\beta^{(1)}, \tag{2.37}$$

while $\beta^{(2)}$ accounts for a contribution to the term $q_i q_j$

$$\mathcal{L}^{B_\mu}_{\text{1-loop}} = -\frac{b_{ij}}{2} q_i q_j, \quad \text{with} \quad (B_\mu)_{ij} = -|F_0|^2 \beta^{(2)}\,, \tag{2.38}$$

thus being relevant for the B_μ term. Finally

$$\mathcal{L}^{\tilde{m}}_{\text{1-loop}} = \delta\tilde{m}^2_{ij} q_i^\dagger q_j, \quad \text{with} \quad \delta\tilde{m}^2_{ij} = -|F_0|^2 \alpha^{(2)}_{ij}\,, \tag{2.39}$$

is a one loop contribution to soft scalar masses that does not play an effective role since its F/M leading term vanishes because of an accidental cancelation [49, 60].

In this framework two loop contribution to scalar soft masses arises from two different sources. The first one is common to LGM and directly comes from gauge interactions: its importance depends on the size of gaugino masses, since they share a common origin. In particular in the case in which the latter are enhanced, so that the hierarchy between tree level soft masses and one loop gaugino ones is not severe, two loop soft terms will be sizeable and possibly even dominant over the peculiar contribution of TGM, driving the theory to a LGM scenario. On top of this contribution there is another one owed to the presence of the messenger matter mixing term $\lambda_{QQ\Phi^h} Q_i Q_j \Phi^h_k$ [49, 56]. Such a term is originated by two loop graphs like the one depicted in figure 2.3b and its importance is obviously dependent on the size of the Yukawa coupling $\lambda_{QQ\Phi^h}$.

2.4 Model Building: Guidelines

We wish to conclude this chapter pointing out some general considerations about the construction of a TGM phenomenologically viable theory. We will address then the explicit realization of the guidelines pointed out here in the next section, which will be dedicated to the analysis of the simplest realization of the TGM mechanism in GUT context.

Two obvious remarks are in order. First of all in a phenomenologically viable realization of the scenario the unbroken gauge group H should contain all the SM factors, namely $G_{\rm SM} \subseteq H$. Moreover the light matter content of the theory should contain the MSSM superfields, namely $(q_i, u_i^c, d_i^c, l_i, e_i^c) \subseteq Q$ for $i = 1, 2, 3$, and no tachyons are expected.

Focusing on the first point and assuming the gauge group G to be simple we notice that there are a few possibilities to be taken into account. As we already pointed out in section 2.1 we wish to consider a gauge group whose rank is larger than that of the Standard Model, so Rank$(G) \geq 5$. This consideration leads us to identify as plausible candidates in a GUT implementation of the framework the groups $SU(N)$, for $N > 5$, $SO(4n+2)$, for $n \geq 2$, and special groups, such as E_6. Still the assumption of G being a simple group is not compulsory: thus in principle one could consider any group $G' \subseteq G$ with $G' \equiv G_{\rm SM} \times U(1)_X$.

The effective prediction required to obtain a physically viable spectrum is that we expect the MSSM sfermions[5] to get a positive mass through the tree level diagram of figure 2.1. Such a positiveness condition divides the superfields with SM quantum numbers that arise from the decomposition of the GUT supermultiplets after the breaking $G \to H$ in two different categories. Those superfields whose scalar component gets a positive soft mass term can play the role of the MSSM sfermions, while the others should have a heavy supersymmetric mass exceeding the negative soft contribution and preventing them to become tachyonic.

Incidentally we will see that the latter constraint on the positiveness of soft terms will heavily drive model building: in particular in the simplest $SO(10)$ model it will make the usual matter embedding with a whole family in a single **16** representation unphysical, paving the way for an interesting non standard embedding that we are going to analyze in full details in the next chapter.

2.5 Invitation: $SU(5)$ invariant theory

The simplest possibility to embed the TGM framework in a GUT context, as we have just seen, requires the presence of the gauge group $SO(10)$[6]. Under the breaking chain $SO(10) \to SU(5) \times U(1)_X$ the two lightest representations of $SO(10)$ decompose as

$$\mathbf{16} \to \mathbf{10^{16}}_1 + \overline{\mathbf{5}}^{\mathbf{16}}{}_{-3} + \mathbf{1^{16}}_5 \qquad \mathbf{10} \to \overline{\mathbf{5}}^{\mathbf{10}}{}_2 + \mathbf{5^{10}}_{-2}\,, \tag{2.40}$$

where, for any $SU(5)$ representation on the right hand side, the subscript is the charge under the generator of $U(1)_X$ (X charge), while the superscript keeps track of the $SO(10)$ representation it originates from.

[5]We discard the possibility that the Z superfield taking F-term vev lies in the same supermultiplet with the light superfields Q.

[6]The other Rank 5 possibility, $SU(6)$, turns out not to be phenomenologically viable.

The breaking of the gauge group $SO(10) \to SU(5)$, restricting to representations of dimension d < 126, can be obtained through the vevs in the SM singlet direction of a $\mathbf{16}$ and a $\overline{\mathbf{16}}$, respectively N and $\overline{N}$, namely

$$\langle N \rangle = M \quad \langle \overline{N} \rangle = M \,. \tag{2.41}$$

SUSY breaking is achieved through the F-term vev of a $\mathbf{16}$ representation in its SM singlet direction, named Z. Gauge invariance, equations 2.7, and more specifically the fact that $F_i^\dagger T^h_{ij} \phi_j = 0$ tells us that it is not possible to embed both the $SO(10)$ breaking vev and the SUSY breaking one in the same superfield. It is then crucial to introduce a $\mathbf{16'}$ superfield such that

$$\langle Z \rangle = F\theta^2 \,. \tag{2.42}$$

We assume that matter is embedded in three copies of the smallest representations of $SO(10)$, thus $\mathbf{16}_i$ and $\mathbf{10}_i$. The rules to define such an embedding have been sketched in section 2.4. The F-term vev induces a D-term vev that can give rise to the tree level sfermion masses in equation 2.19. In particular, restricting to the case in which matter does not belong to the $\mathbf{16'}$ superfield, and thus the only S-channel in figure 2.1 is available, one obtains that

$$\tilde{m}_Q^2 = \frac{X_Q}{2X_Z} \left| \frac{F}{M} \right|^2 \,, \tag{2.43}$$

where X_Q and $X_Z = 5$ are respectively the X charges of the chiral superfield Q under analysis and of the F-term taking vev: depending on the relative sign of X_Q and X_Z the soft mass term can be either positive or negative. From equation 2.40 we notice that $\mathbf{10^{16}}$ and $\overline{\mathbf{5}}^{\mathbf{10}}$ get a positive soft term, while $\overline{\mathbf{5}}^{\mathbf{16}}$ and $\mathbf{5^{10}}$ get a negative one. Since light sfermions should have positive soft masses the only possible embedding for matter is[7]

$$(q_i, u_i^c, e_i^c) \subset (\mathbf{10^{16}})_i \quad \text{and} \quad (d_i^c, l_i) \subset (\overline{\mathbf{5}}^{\mathbf{10}})_i \,. \tag{2.44}$$

We notice that this embedding is different from the one usually considered in $SO(10)$ GUT, where a whole fermion family finds place in a single $\mathbf{16}_i$ representation [61]: the latter, indeed, would be non physical in TGM framework. On top of that we stress the very nice prediction on the ratio of sfermion masses

$$\frac{\tilde{m}^2_{q,u^c,e^c}}{\tilde{m}^2_{d^c,l}} = \frac{1}{2} \,. \tag{2.45}$$

Those fields getting a negative soft mass will get a heavy supersymmetric mass term by means of superpotential trilinear interactions in equation 2.47, solving possible phenomenological issues.

[7] We only consider *pure* embeddings, meaning that each SM fermion multiplet can be embedded into a single irreducible representation of the gauge group, and the representation is the same (or equivalent) for the three families. This assumption of pure embeddings is crucial to obtain flavour universal sfermion masses and can be easily achieved [51].

Before showing how this is effectively accomplished we firstly discuss the Higgs superfields embedding. If one restricts to the smallest $SO(10)$ representations, it turns out that $\mathbf{10}$, $\mathbf{16}$ and $\overline{\mathbf{16}}$ representations have the correct quantum numbers to accomodate h_u and h_d. In particular we shall call $\cos^2\theta_u$ ($\cos^2\theta_d$) the portion of the h_u (h_d) superfield in the $\mathbf{10}$ ($\mathbf{16}$) representation and correspondingly $\sin^2\theta_u$ ($\sin^2\theta_d$) the portion of h_u (h_d) superfield in $\overline{\mathbf{16}}$ ($\mathbf{16}$) representation. It is very tempting, in order to have the smallest number of superfields at play, to embed the Higgs superfields in the same superfields were already the $SO(10)$ and SUSY breaking vevs are. The Higgs superfields also get a soft mass term through the TGM mechanism: in particular one has

$$m_{h_u}^2 = \frac{-2\cos^2\theta_u + 3\sin^2\theta_u}{10}\left|\frac{F}{M}\right|^2 \quad m_{h_d}^2 = \frac{2\cos^2\theta_d - 3\sin^2\theta_d}{10}\left|\frac{F}{M}\right|^2 . \tag{2.46}$$

In this case the possibly negative soft term causes no phenomenological threats since one (or even both) the Higgs masses are expected to run negative in order to reach a successful EWSB.

The superpotential of the theory reads

$$W = h_{ij}\mathbf{16}_i\mathbf{10}_j\mathbf{16} + h'_{ij}\mathbf{16}_i\mathbf{10}_j\mathbf{16}' + y_{ij}\mathbf{16}_i\mathbf{16}_j\mathbf{10} + W_{\text{extra}} , \tag{2.47}$$

where $\mathbf{16}_i$ and $\mathbf{10}_i$ are the matter fields, $\mathbf{16}$ contains both the $SO(10)$ breaking vev and part of h_d, $\mathbf{16}'$ contains both the SUSY breaking F-term vev and another part of h_d, $\mathbf{10}$ contains portions of both h_u and h_d and finally W_{extra} takes care of the vevs and possibly other issues that for now we are not going to deal with. This superpotential accounts for flavour Yukawa interactions: the term proportional to y_{ij} is responsible for the top Yukawa coupling, while those proportional to h_{ij} and h'_{ij} are responsible for the bottom and the tau Yukawas.

After $SO(10)$ breaking, the superpotential 2.47 gives those superfields with negative soft mass terms ($\overline{\mathbf{5}}^{\mathbf{16}}$ and $\mathbf{5}^{\mathbf{10}}$) a large positive supersymmetric mass, i.e.

$$\overline{\mathbf{5}}_i^{\mathbf{16}}\mathbf{5}_j^{\mathbf{10}}\langle N\rangle = h_{ij}M\overline{\mathbf{5}}_i^{\mathbf{16}}\mathbf{5}_j^{\mathbf{10}} , \tag{2.48}$$

solving phenomenological issues and, along with the term $h'_{ij}\mathbf{16}_i\mathbf{10}_j\mathbf{16}'$, promoting them to the role of chiral messengers. The h' trilinear interactions, indeed, are responsible for chiral contributions to gaugino masses, as explained in section 2.3.1. The heavy chiral superfields $\overline{\mathbf{5}}^{\mathbf{16}}$ and $\mathbf{5}^{\mathbf{10}}$ are exchanged at scale hM and give rise to the term

$$M = \frac{\alpha}{4\pi}\mathrm{Tr}\left(h'h^{-1}\right)\frac{F}{M} , \tag{2.49}$$

where α is the $SU(5)$ coupling. The latter superpotential terms are also responsible for one loop trilinears, as explained in section 2.3.2. Moreover the presence of the superpotential mixing terms among matter, Higgs and chiral messenger superfields give rise to two loop contributions to soft scalar masses that adds up to the usual LGM contributions owed to gauge interactions.

3

TGM phenomenology at the LHC

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3.5.1 Analysis of the benchmark points 50

In this chapter we will analyze in full detail the easiest implementation in a simple group G of a phenomenologically viable TGM theory. The gauge group should have at least $\mathrm{Rank}(G) = 5$, as explained in section 2.4: considering the $SO(2n)$ family, this leads us to the case of $SO(10)$, whose analysis has been sketched in section 2.5. There we disregarded most of the subtleties connected to an effective physically viable implementation of the scenario, that we will deal with in this chapter, turning out to relate the gaugino mass ratios and more in general the soft terms to the flavour structure of the SM fermions. Then we will explore the phenomenology of the model

Field	$SO(10)$	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_X$	R_P
q_i	$\mathbf{16}_i$	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{6}$	1	-1
u_i^c	$\mathbf{16}_i$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$-\frac{2}{3}$	1	-1
d_i^c	$\mathbf{10}_i$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\frac{1}{3}$	2	-1
l_i	$\mathbf{10}_i$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	2	-1
e_i^c	$\mathbf{16}_i$	$\mathbf{1}$	$\mathbf{1}$	1	1	-1
S_i	$\mathbf{16}_i$	$\mathbf{1}$	$\mathbf{1}$	0	5	-1
D_i^c	$\mathbf{16}_i$	$\mathbf{3}$	$\mathbf{1}$	$-\frac{1}{3}$	-3	-1
$\overline{D_i^c}$	$\mathbf{10}_i$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\frac{1}{3}$	-2	-1
L_i	$\mathbf{16}_i$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	-3	-1
$\overline{L}_i$	$\mathbf{10}_i$	$\mathbf{1}$	$\mathbf{2}$	$\frac{1}{2}$	-2	-1
h_u	$\mathbf{10}$, $\overline{\mathbf{16}}$, $\overline{\mathbf{16}}'$	$\mathbf{1}$	$\mathbf{2}$	$\frac{1}{2}$	—	+1
h_d	$\mathbf{10}$, $\mathbf{16}$, $\mathbf{16}'$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	—	+1

Table 3.1: TGM field content. The various superfields are shown along with the SO(10) GUT representation they belong to. The last column shows the R-parity of the corresponding superfield.

and focus on the possible experimental hints that could be looked for at the LHC in order to discriminate the peculiarities of TGM models from other models of SUSY breaking mediation.

3.1 Fields and lagrangian

In order to study the TeV phenomenology of the model we only need to consider the lagrangian below the $SO(10)$ breaking scale. The matter ($R_P = -1$) field content consists of three $\mathbf{16}_i + \mathbf{10}_i$, whose SM decomposition is given in table 3.1. The lower case fields are (in first approximation) the light ones. The S_i are SM singlets, they may get mass at the non renormalizable level. The other capital letter fields get mass through $SO(10)$ breaking. We assume that only the light doublet components h_u, h_d of the Higgs fields ($R_P = 1$) survive below the GUT scale (see [51,62] for an example of how to achieve that). If the $SO(10)$ Higgs sector contains only representations with dimension $d < 120$ ($\mathbf{10}$, $\mathbf{16} + \overline{\mathbf{16}}$, $\mathbf{45}$, $\mathbf{54}$), the doublets can only belong to $\mathbf{10}$, $\mathbf{16}$, $\overline{\mathbf{16}}$ representations. To be general, we allow them to be superpositions of the doublets in those representations. That is why their X charge is not specified in table 3.1.

The goldstino superfield is also lighter than the $SO(10)$ breaking scale, but it contributes to the lagrangian at the TeV scale only through the soft terms induced by its SUSY breaking vev, we therefore do not include it.

Whatever is the dynamics above the $SO(10)$ breaking (GUT) scale, the lagrangian below that scale is described by the most general SM and R-parity invariant one. We first give a general parameterization of the latter, which is useful to incorporate radiative corrections through RGEs, then we show how that lagrangian is determined by the few relevant parameters of the model through the boundary conditions at the GUT scale.

The lagrangian below the GUT scale involves terms corresponding to the usual MSSM interactions and terms involving the extra heavy fields. Correspondingly, the superpotential is

$$W = W_{\rm MSSM} + W_{\rm TGM} + W_{\rm S} \, , \tag{3.1}$$

where $W_{\rm S}$ depends on the singlet fields S_i and is not relevant for our purposes (as long as R-parity is not spontaneously broken), and

$$\begin{aligned} W_{\rm MSSM} &= \lambda_U u^c q h_u + \lambda_D d^c q h_d + \lambda_E e^c l h_d + \mu h_u h_d \\ W_{\rm TGM} &= \hat\lambda_D D^c q h_d + \hat\lambda_E e^c L h_d + M_D \overline{D^c} D^c + M_{Dd} \overline{D^c} d^c + M_L \overline{L} L + M_{Ll} \overline{L} l \, . \end{aligned} \tag{3.2}$$

The terms $l\overline{L}$ or $d^c\overline{D^c}$ are supposed to be absent at the GUT scale but arise in the RGE running (see appendix A.2.4). The SUSY breaking lagrangian is

$$\mathcal{L}_{\rm SB} = \mathcal{L}^A_{\rm MSSM} + \mathcal{L}^A_{\rm TGM} + \mathcal{L}^m_{\rm MSSM} + \mathcal{L}^m_{\rm TGM} + \mathcal{L}^g_{\rm MSSM} \, , \tag{3.3}$$

with

$$\begin{aligned} -\mathcal{L}^A_{\rm MSSM} &= A_U \tilde u^c \tilde q h_u + A_D \tilde d^c \tilde q h_d + A_E \tilde e^c \tilde l h_d + B h_u h_d + \text{h.c.} \\ -\mathcal{L}^A_{\rm TGM} &= \hat A_D \tilde D^c \tilde q h_d + \hat A_E \tilde e^c \tilde L h_d + B_D \tilde{\overline{D^c}} \tilde D^c + B_{Dd} \tilde{\overline{D^c}} \tilde d^c + B_L \tilde{\overline{L}} \tilde L + B_{Ll} \tilde{\overline{L}} \tilde l + \text{h.c.} \\ -\mathcal{L}^m_{\rm MSSM} &= m^2_{h_u} h_u^\dagger h_u + m^2_{h_d} h_d^\dagger h_d + \tilde m^2_q \tilde q^\dagger \tilde q + \tilde m^2_{u^c} \tilde u^{c\dagger} \tilde u^c + \tilde m^2_l \tilde l^\dagger \tilde l + \tilde m^2_{d^c} \tilde d^{c\dagger} \tilde d^c + \tilde m^2_{e^c} \tilde e^{c\dagger} \tilde e^c \\ -\mathcal{L}^m_{\rm TGM} &= m^2_{D^c} \tilde D^{c\dagger} \tilde D^c + m^2_{\overline{D^c}} \tilde{\overline{D^c}}^\dagger \tilde{\overline{D^c}} + m^2_L \tilde L^\dagger \tilde L + m^2_{\overline{L}} \tilde{\overline{L}}^\dagger \tilde{\overline{L}} + (m^2_{Dd} \tilde D^{c\dagger} \tilde d^c + m^2_{Ll} \tilde L^\dagger \tilde l + \text{h.c.}) \\ -\mathcal{L}^g_{\rm MSSM} &= \frac{1}{2} M_a \lambda_a \lambda_a + \text{h.c.} \, . \end{aligned} \tag{3.4}$$

In the above equations we have suppressed the flavour indexes. The terms including the field breaking SUSY have also been omitted, but will discuss them in the next section.

3.2 Generation of the soft terms

In the following sections we will describe how the SUSY breaking terms arise in the minimal framework under analysis and their relations to the unbroken $SO(10)$ superpotential discussed in appendix A.1, namely

$$W_2 = h_{ij} \mathbf{16}_i \mathbf{10}_j \mathbf{16} + h'_{ij} \mathbf{16}_i \mathbf{10}_j \mathbf{16'} + \frac{y_{ij}}{2} \mathbf{16}_i \mathbf{16}_j \mathbf{10} + W_2^{\rm NR} \, . \tag{3.5}$$

The discussion will follow the general structure presented in section 2.5.

3.2.1 Tree level generated boundary conditions

Tree level diagram of figure 2.1b1 induces soft mass terms for the sfermions when, as in section 2.5, an F-term vev in the SM singlet direction of a spinorial representation breaks SUSY. The parameter m_{10} is the common mass of the MSSM sfermions belonging to the **10** representation of $SU(5)$ ($\tilde{q}$, $\tilde{u}^c$, $\tilde{e}^c$) at the GUT scale. All sfermion masses are determined (at the tree level) by equation 2.43:

$$\tilde{m}_q{}^2 = \tilde{m}_{u^c}{}^2 = \tilde{m}_{e^c}{}^2 = \frac{1}{10}\left|\frac{F}{M}\right|^2 = m_{10}{}^2 \,, \quad \tilde{m}_l{}^2 = \tilde{m}_{d^c}{}^2 = 2\,m_{10}{}^2 \,. \tag{3.6}$$

The factor 2 is a prediction of the minimal unified realization of TGM, as already seen in section 2.5. It arises because the squared sfermion masses are proportional to their charges under the $U(1)_X$ mediating SUSY breaking (see table 3.1).

The D^c, $\overline{D^c}$, L and $\overline{L}$ soft masses are subdominant with respect to the much larger supersymmetric masses M_D, M_L in the superpotential[1] and, as the parameters m^2_{Dd}, m^2_{Ll}, are not relevant in our results. For completeness, they are given at the GUT scale by

$$m^2_{D_c} = m^2_L = -3m^2_{10}, \quad m^2_{\overline{D_c}} = m^2_{\overline{L}} = -2m^2_{10}\,, \quad m^2_{Dd} = m^2_{Ll} = 0. \tag{3.7}$$

The angles $0 \le \theta_u, \theta_d \le \pi/2$ account for the possibility that the light MSSM Higgs h_u and h_d are superpositions of doublets in different $SO(10)$ representations. Given the embedding of MSSM fields in table 3.1 (and up to non renormalizable contributions), the up and down Yukawa couplings $\lambda_U u^c q h_u$ and $\lambda_D d^c q h_d$ in equation 3.2 must come from $SO(10)$ interactions $\mathbf{16}\,\mathbf{16}\,\mathbf{10}_H$ and $\mathbf{10}\,\mathbf{16}\,\mathbf{16}_H$ respectively[2], as in equation 2.47. Therefore, h_u must have a component in $\mathbf{10}_H$ and h_d must have a component in $\mathbf{16}_H$. The simplest possibility is that this is it. On the other hand, to be general, we can consider the possibility that h_u has also a component in a $\overline{\mathbf{16}}$ and h_d in a **10** (there are no further possibilities as we only consider $SO(10)$ representations with dimension d < 120). In such a case, we use the angles θ_u and θ_d to measure the size of the Higgs components in the different representations:

$$\mathbf{10}_H \supset \cos\theta_u h_u + \ldots \qquad \mathbf{16}_H \supset \sin\theta_d h_d + \ldots. \tag{3.8}$$

In the *pure* case[3] in which the light Higgs are contained in the $\mathbf{10}_H$ and $\mathbf{16}_H$ only, their X charges are defined: $X_{h_u} = -(X_q + X_{u^c}) = -2$, $X_{h_d} = -(X_q + X_{d^c}) = -3$. The charges are negative because the MSSM Yukawas must be $U(1)_X$ invariant and the sfermions must have positive charges. Their soft masses are therefore negative at the tree level. In the general case, we have instead, reassessing equation 2.46,

$$m^2_{h_u} = (-2\cos^2\theta_u + 3\sin^2\theta_u)\, m^2_{10} \quad \text{and} \quad m^2_{h_d} = (2\cos^2\theta_d - 3\sin^2\theta_d)\, m^2_{10} \tag{3.9}$$

and the soft masses can both be positive or negative at the tree level.

[1] We will see in the next section how such masses arise in the framework.

[2] We can assume without loss of generality that $\mathbf{10}_H$ is the only **10** representation of $SO(10)$ containing h_u and $\mathbf{16}_H$ is the only **16** representation of $SO(10)$ containing h_d.

[3] We remind here that by *pure* embedding we mean that each SM fermion multiplet can be embedded into a single irreducible representation of the gauge group, and the representation is the same (or equivalent) for the three families

3.2.2 Heavy chiral messengers and their masses

The specification of the low energy lagrangian involves the determination of the detailed form of the masses of the heavy D^c, $\overline{D^c}$, L and $\overline{L}$ superfields in table 3.1 and of their couplings to SUSY breaking. Indeed, as briefly seen in section 2.5, they behave as chiral messengers being responsible for the generation at the one loop level of gaugino masses and trilinear terms and at the two loop level of contributions to soft masses. On top of that they also affect the low energy lagrangian through the RGE evolution from the GUT scale: it is therefore necessary to specify the masses M_D and M_L in equation 3.2 in order to determine the scale, tipically below the GUT scale, at which their contribution should be switched off.

Since the D^c, $\overline{D^c}$, L and $\overline{L}$ fields belong to different $SO(10)$ representations they acquire masses through $SO(10)$ breaking, specifically through the vev of the SM singlet components of a $\mathbf{16}+\overline{\mathbf{16}}$, denoted by $M > 0$[4], as already anticipated in section 2.5. It is therefore convenient to write the mass terms in equation 3.2 as

$$M_{Dij}\overline{D^c}_i D^c_j + M_{Lij}\overline{L}_i L_j = h_{Dij}M\overline{D^c}_i D^c_j + h_{Lij}M\overline{L}_i L_j\,. \tag{3.10}$$

Analyzing the superpotential of the theory as specified in equation 3.5 the mixing parameters M_{dD} and M_{lL} in equation 3.2 are predicted to vanish at the GUT scale at the renormalizable level, namely

$$M_{dD} = 0 \qquad M_{lL} = 0\,. \tag{3.11}$$

Non vanishing values are generated by the RGE running between the GUT and the messenger scales, as no unbroken quantum number distinguishes the d^c, l fields from the D^c, L ones.

SUSY breaking is provided by the F-term vev of the SM singlet component of spinorial representations of $SO(10)$, which are forced by gauge invariance not to coincide with $\mathbf{16}$, $\overline{\mathbf{16}}$ (see the discussion in section 2.5) and will therefore be denoted by $\mathbf{16}'$, $\overline{\mathbf{16}}'$. In order to obtain positive tree level sfermion masses, the F-term of the $\mathbf{16}'$ must be larger than the one of the $\overline{\mathbf{16}}'$ [51]. We will then assume for simplicity that only the SM singlet component of the $\mathbf{16}'$ field, Z, gets an F-term vev F. As $|F| \ll M^2$, the field Z should be included in the effective lagrangian below the GUT scale defined by equations 3.1, 3.2, 3.3, 3.4. The relevant terms are the superpotential couplings

$$W_Z = h'_{Dij}Z\overline{D^c}_i D^c_j + h'_{Lij}Z\overline{L}_i L_j\,. \tag{3.12}$$

As a further simplification, we will neglect the flavour structure of the above couplings h_D, h_L, h'_D, h'_L and consider only the diagonal elements, assuming that, as in the case of the SM Yukawa couplings, the deviation from the diagonal form, i.e. the breaking of the individual flavour numbers, is small. In such a case, the flavour

[4]The D-term condition for the $U(1)_X$ forces the two vevs to be equal in absolute value, up to negligible SUSY breaking effects. M can be taken positive without loss of generality.

structure we are neglecting does not significantly affect the collider observables we are interested in. Equations 3.10 and 3.12 involve six new parameters each. The latter are related to the Yukawa flavour structure, as detailedly discussed in appendix A.1.

To sum up, in this section we have specified the GUT scale boundary conditions for all the parameters in equation 3.2. The Yukawas λ_U, λ_D, λ_E are determined at low energy by the SM fermion masses, and $\hat{\lambda}_D$, $\hat{\lambda}_E$ from GUT scale relations. The messenger masses $M_{D,L}$ are specified by equations 3.10, while the parameters M_{dD}, M_{lL} are set to zero at the GUT scale. The μ parameter is determined by the EWSB condition and the specification of its sign.

3.2.3 One loop level generated boundary conditions: Gaugino masses

Gaugino masses are generated, as in LGM, at the one loop level because of the coupling of the field Z taking F-term vev to the heavy D^c, $\overline{D^c}$, L and $\overline{L}$, which play the role of chiral messengers of SUSY breaking. Nicely enough, in the $SO(10)$ model the latter coupling turns out to be automatically allowed by the gauge symmetry. Unlike LGM, in fact, the field Z (here the singlet component of the $\mathbf{16'}$) is charged under the gauge group and in particular under the $U(1)_X$ subgroup mediating SUSY breaking at the tree level, so that the possibility of a coupling to the chiral messengers is not a priori guaranteed. On the other hand, gauge invariance prevents the possibility of an explicit mass term for the messengers. The mass term must come from the vev of a field breaking the $U(1)_X$. Such a field must have the same quantum numbers as Z, but with independent couplings to the messenger fields, as discussed in section 2.5. In turn, this opens the possibility to enhance gaugino masses by means of the ratio of the coupling to SUSY breaking and the coupling to $U(1)_X$ breaking. In this section we show how such features are implemented in the $SO(10)$ model under consideration, and we point out a possible source of non minimality of gaugino masses.

Before proceeding further with the computation of gaugino masses we introduce some useful notation. The couplings to SUSY breaking h'_{D_i} and h'_{L_i} are conveniently traded for the parameters γ_{Di} and γ_{Li} defined by

$$\gamma_{Di} \equiv \left(\frac{h'_{Di}}{h_{Di}}\right)_{h_{Di}M} \qquad \gamma_{Li} \equiv \left(\frac{h'_{Li}}{h_{Li}}\right)_{h_{Li}M}, \tag{3.13}$$

where the couplings are supposed to be evaluated at the corresponding heavy field mass scale. Now we can introduce the averages $\gamma_D \equiv (\sum_{i=1}^3 \gamma_{Di})/3$, $\gamma_L \equiv (\sum_{i=1}^3 \gamma_{Li})/3$ and

$$r = \frac{\sum_{i=1}^3 \gamma_{Li}}{\sum_{i=1}^3 \gamma_{Di}}, \qquad \gamma = \frac{1}{6}\left(\sum_{i=1}^3 \gamma_{Di} + \sum_{i=1}^3 \gamma_{Li}\right). \tag{3.14}$$

Therefore, in order to specify the six parameters in equations 3.13 is equivalent to specify γ_D, γ_L and the four ratios

$$r_{D_i} = \gamma_{Di}/\gamma_D\,, \quad r_{L_i} = \gamma_{Li}/\gamma_L\,, \quad i = 1, 2\,. \tag{3.15}$$

Gaugino masses can be expressed in terms of the messenger masses and couplings to SUSY breaking in equations 3.10 and 3.12. The six vectorlike chiral messengers, $\overline{D^c}_i + D^c_i$ and $\overline{L}_i + L_i$, $i = 1, 2, 3$, have masses $h_{Di}M$ and $h_{Li}M$ respectively. Their scalar components get SUSY breaking mass terms given by $h'_{Di}F$ and $h'_{Li}F$. The contributions of the i-th family of messengers to the gaugino masses M_a, $a = 1, 2, 3$, are then

$$M_a^{D_i} = \frac{\alpha_a}{4\pi} b^D_a \gamma_{Di} \frac{F}{M} \quad (\text{scale } h_{Di}M) \qquad M_a^{L_i} = \frac{\alpha_a}{4\pi} b^L_a \gamma_{Li} \frac{F}{M} \quad (\text{scale } h_{Li}M), \tag{3.16}$$

where $b^L = (3/5, 1, 1)$, $b^D = (2/5, 1, 1)$ and the parameters γ_{Di}, γ_{Li} have just been defined in equation 3.13. Each of those contributions arise at the scale of the corresponding messenger and the gauge couplings in equations 3.16 are supposed to be evaluated at that scale, which is different for each contribution. The individual contributions in equations 3.16 can be formally obtained from the one loop running from the GUT scale of the hypothetical values

$$M_a^{D_i} = \frac{\alpha_a}{4\pi} b^D_a \gamma_{Di} \frac{F}{M} \qquad M_a^{L_i} = \frac{\alpha_a}{4\pi} b^L_a \gamma_{Li} \frac{F}{M} \quad (\text{GUT scale}), \tag{3.17}$$

where now the gauge couplings are supposed to be evaluated at the GUT scale with γ_D and γ_L evaluated as in 3.13. At the GUT scale, the individual contributions in equations 3.17 can be summed to give

$$M_a = 3\frac{\alpha_a}{4\pi}\left(2\frac{b^D_a + r b^L_a}{1+r}\right)\gamma\frac{F}{M} \quad (\text{GUT scale}), \tag{3.18}$$

where and r a ratio and γ is the average of the six parameters defined in equation 3.14. We can conveniently trade the parameter γ in terms of the more useful[5]

$$M_{1/2} \equiv 3\,\frac{\alpha_{\text{GUT}}}{4\pi}\gamma\frac{F}{M}\,, \tag{3.19}$$

and thus obtain the parameterization of gaugino masses in terms of $M_{1/2}$ and r in equations 3.27. As stressed above, those relations are valid at the GUT scale only in the sense that the gaugino masses at the scales at which they are actually generated and below can be obtained by running the formal GUT scale values with one loop RGEs.

Let us notice that $M_{1/2}$ can well be of the order of the tree level stop mass m_{10}, despite it is generated at the one loop level [50]. This is in part due to the fact that $F/M = \sqrt{10}\,m_{10}$, giving a factor $3\sqrt{10}$ enhancement of the loop suppressed value

$$M_{1/2} \equiv \frac{\alpha_{\text{GUT}}}{4\pi}(3\sqrt{10}\,\gamma)m_{10}\,. \tag{3.20}$$

[5]If gauge couplings do not unify one should use $(\alpha_2 + \alpha_3)/2$ instead of α_{GUT}.

And it is in part due to the fact that the unknown factor γ, being essentially a ratio of presumably hierarchical Yukawa couplings, can easily be larger (or smaller) than 1.

The gaugino masses obtained in this way are potentially non universal at the GUT scale, even in the case of precise gauge coupling unification, when the parameter r is different from 1. We note that small values of r can make the wino lighter than the bino. On top of that the measurement of non universal gaugino masses satisfying the sum rule 3.28 can be considered as another smoking gun of minimal unified TGM on top of the prediction on sfermion masses.

Let us close this section by discussing how concrete is such a possibility. The $SU(5)$ gauge symmetry, if unbroken, would force $\gamma_{Di} = \gamma_{Li}$ and $r = 1$. On the other hand, the $r \neq 1$ possibility is plausible because $SU(5)$ is broken and the same $SU(5)$ breaking corrections needed to make $\lambda_D \neq \lambda_E$ can as well make $h_D \neq h_L$ and $h'_D \neq h'_L$, so that $\gamma_{Di} \neq \gamma_{Li}$ and $r \neq 1$. Note that even in the limit in which the $SU(5)$ breaking effects are small and only affect significantly the small Yukawa couplings of the first families, the effect on r can be sizeable. In fact, the ratio of the small Yukawa couplings, potentially significantly different from 1, enters the r parameter with the same weight as the ratio of the third family Yukawas.

3.2.4 One loop level generated boundary conditions: Trilinear terms

The MSSM trilinear terms in equation 3.4 are generated through one loop graphs as the one sketched in figure 2.3a at the scale at which the heavy D^c, $\overline{D^c}$, L and $\overline{L}$ get integrated out. In particular, directly from equation 2.36, they have the form

$$\begin{aligned} A_U &= A_{u^c}\lambda_U + \lambda_U A_q + \lambda_U A_{h_u} \,, \\ A_D &= A_{d^c}\lambda_D + \lambda_D A_q + \lambda_D A_{h_d} \,, \\ A_E &= A_{e^c}\lambda_E + \lambda_E A_l + \lambda_E A_{h_d} \,. \end{aligned} \tag{3.21}$$

To be explicit the contributions induced by the coloured messengers D^c and $\overline{D^c}$ are

$$A_q(M_{Di}) = -\frac{1}{(4\pi)^2}\gamma_{Di}\hat{\lambda}_{Di}^2\frac{F}{M} \,, \tag{3.22a}$$

$$A_{h_d}(M_{Di}) = -\frac{3}{(4\pi)^2}\gamma_{Di}\hat{\lambda}_{Di}^2\frac{F}{M} \,, \tag{3.22b}$$

$$A_l(M_{Di}) = A_{d^c}(M_{Di}) = A_{u^c}(M_{Di}) = A_{e^c}(M_{Di}) = A_{h_u}(M_{Di}) = 0 \,, \tag{3.22c}$$

while the one induced by L and $\overline{L}$ are

$$A_{e^c}(M_{Li}) = -\frac{2}{(4\pi)^2}\gamma_{Li}\hat{\lambda}_{Ei}^2\frac{F}{M} \,, \tag{3.23a}$$

$$A_{h_d}(M_{Li}) = -\frac{1}{(4\pi)^2}\gamma_{Li}\hat{\lambda}_{Ei}^2\frac{F}{M} \,, \tag{3.23b}$$

$$A_l(M_{Li}) = A_{d^c}(M_{Li}) = A_{u^c}(M_{Li}) = A_q(M_{Li}) = A_{h_u}(M_{Li}) = 0 \,, \tag{3.23c}$$

Q	q_i	u_i^c	d_i^c	l_i	e_i^c	h_u	h_d
c_Q^1	1/60	4/15	1/15	3/20	3/5	3/20	3/20
c_Q^2	3/4	0	0	3/4	0	3/4	3/4
c_Q^3	4/3	4/3	4/3	0	0	0	0

Table 3.2: Quadratic Casimirs for the low energy superfields.

Note that only the third family A-terms are non negligible, as the first and second family ones are suppressed by powers of small Yukawa couplings.

3.2.5 Two loop level generated boundary conditions: soft terms

The coupling of the chiral messengers D^c, $\overline{D^c}$, L and $\overline{L}$ to SUSY breaking, equation 3.12, gives rise to the well known two loop contributions to sfermion masses. In this section we give their expressions in our model. Given that the chiral messengers have masses $h_i^D M$ and $h_i^L M$ and SUSY breaking mass terms given by $h'_{Di}F$ and $h'_{Li}F$, the contributions to sfermion masses, as the ones to gaugino masses, depend on the parameters γ_{Di} and γ_{Li}. We have in fact

$$
\begin{aligned}
(\widetilde{m}_Q^2)_{\text{LGM}} &= \sum_i (\widetilde{m}_Q^2)_{\text{LGM}}(M_{Di}) + (\widetilde{m}_Q^2)_{\text{LGM}}(M_{Li}) \\
&= 2\Bigg[\left(c_Q^3 \frac{\alpha_3^2(M_{Di})}{(4\pi)^2} + \frac{2}{5} c_Q^1 \frac{\alpha_1^2(M_{Di})}{(4\pi)^2}\right)\gamma_{Di}^2 \\
&\quad + \left(c_Q^2 \frac{\alpha_2^2(M_{Li})}{(4\pi)^2} + \frac{3}{5} c_Q^1 \frac{\alpha_1^2(M_{Li})}{(4\pi)^2}\right)\gamma_{Li}^2\Bigg]\left|\frac{F}{M}\right|^2 ,
\end{aligned}
\tag{3.24}
$$

where c_Q^a is the quadratic Casimir of the sfermion $\widetilde{Q}$ (or Higgs Q) relative to the gauge interaction a, as in table 3.2. The parameters $\gamma_{D_i,L_i}F/M$ are determined by the parameters $M_{1/2}$, r, r_{Di}, r_{Li}, $i = 1, 2$ through equations 3.15, 3.14 and 3.19.

On top of the usual LGM contributions, soft masses receive also two loop contributions because of messenger matter mixing, as discussed in section 2.3.2. Working in the third family approximation, that accounts for all the relevant contributions in our framework [63], one finds that extra sizeable corrections arise only for third family sfermions (and Higgses), while they are negligible for the first to families. All in all the corrections are

$$
\begin{aligned}
(4\pi)^4 \delta\tilde{m}_{q_3}^2 &= \left(\frac{7}{30}g_1^2 + \frac{3}{2}g_2^2 + \frac{8}{3}g_3^2 - 3\hat{\lambda}_{D_3}^2 - \frac{1}{2}(\lambda_{E_3}^2 + \hat{\lambda}_{E_3}^2)\right)\hat{\lambda}_{D_3}^2\gamma_{D_3}^2\left|\frac{F}{M}\right|^2 \\
&\quad + \frac{1}{2}\lambda_{D_3}^2\hat{\lambda}_{E_3}^2\gamma_{E_3}^2\left|\frac{F}{M}\right|^2
\end{aligned}
\tag{3.25a}
$$

$$
(4\pi)^4 \delta\tilde{m}_{l_3}^2 = \left(\frac{3}{2}\hat{\lambda}_{D_3}^2\lambda_{E_3}^2\gamma_{D_3}^2 + 2\lambda_{E_3}^2\hat{\lambda}_{E_3}^2\gamma_{E_3}^2\right)\left|\frac{F}{M}\right|^2 \tag{3.25b}
$$

$$(4\pi)^4\delta\tilde{m}^2_d = \left(6\hat{\lambda}^2_{D_3}\lambda^2_{D_3}\gamma^2_{D_3} + \hat{\lambda}^2_{E_3}\lambda^2_{D_3}\gamma^2_{E_3}\right)\left|\frac{F}{M}\right|^2 \tag{3.25c}$$

$$(4\pi)^4\delta\tilde{m}^2_{e_3} = \left(\frac{9}{5}g_1^2 + 3g_2^2 - 4\hat{\lambda}^2_{E_3} - 3(\lambda^2_{D_3} + \hat{\lambda}^2_{D_3})\right)\hat{\lambda}^2_{E_3}\gamma^2_{E_3}\left|\frac{F}{M}\right|^2 + 3\lambda^2_{E_3}\hat{\lambda}^2_{D_3}\gamma^2_{D_3}\left|\frac{F}{M}\right|^2 \tag{3.25d}$$

$$(4\pi)^4\delta\tilde{m}^2_{u_3} = \left(\lambda^2_{U_3}\hat{\lambda}^2_{D_3}\gamma^2_{D_3}\right)\left|\frac{F}{M}\right|^2 \tag{3.25e}$$

$$(4\pi)^4\delta\tilde{m}^2_{h_d} = \left(\frac{7}{10}g_1^2 + \frac{9}{2}g_2^2 + 8g_3^2 - 9\hat{\lambda}^2_{D_3} - \frac{3}{2}(\hat{\lambda}^2_{E_3} + \lambda^2_{U_3})\right)\hat{\lambda}^2_{D_3}\gamma^2_{D_3}\left|\frac{F}{M}\right|^2 + \left(\frac{9}{10}g_1^2 + \frac{3}{2}g_2^2 - 2\hat{\lambda}^2_{E_3} - \frac{3}{2}\hat{\lambda}^2_{D_3}\right)\hat{\lambda}^2_{E_3}\gamma^2_{E_3}\left|\frac{F}{M}\right|^2 \tag{3.25f}$$

$$(4\pi)^4\delta\tilde{m}^2_{h_u} = \frac{3}{2}\hat{\lambda}^2_{D_3}\lambda^2_{U_3}\gamma^2_{D_3}\left|\frac{F}{M}\right|^2 . \tag{3.25g}$$

3.3 Relevance of the various parameters

The specification of the boundary conditions for the soft terms given in the previous section accounts for all the parameters in our lagrangian terms 3.1 and 3.3 at the GUT or messenger scales. In order to obtain the sparticle spectrum and their couplings we use the RGE equations provided in appendix A.2.

Running down to the TeV scale we notice that not all the superpotential couplings have the same relevance on the final spectrum, but they do classify in two different sets: there will be parameters giving an $\mathcal{O}(1)$ impact on the theory, on which we will focus in the next section, and parameters having only a marginal (logarithmic) impact on it, that we will present in section 3.3.2.

3.3.1 Relevant parameters

The relevant parameters of the model are

$$m_{10}\,,\quad M_{1/2}\,,\quad r\,,\quad \tan\beta\,,\quad \mathrm{sign}(\mu)\,,\quad \theta_u\,,\quad \theta_d\,. \tag{3.26}$$

In the following subsections we will analyze them discussing the relevant boundary conditions.

The parameter m_{10} was introduced in section 3.2.1 and represents the tree level soft mass of the $\tilde{q}$, $\tilde{u}^c$ and $\tilde{e}^c$ sfermions. The h_u and h_d embedding, instead, defines θ_u and θ_d as already discussed in that section.

The parameters $M_{1/2}$ and r determine the gaugino masses M_1, M_2 and M_3 at the GUT scale. $M_{1/2}$ and r can also be traded for M_2 and M_3,

$$M_{1/2} = \frac{M_2 + M_3}{2}, \quad r = \frac{M_2}{M_3} \quad \text{(GUT scale)}, \tag{3.27}$$

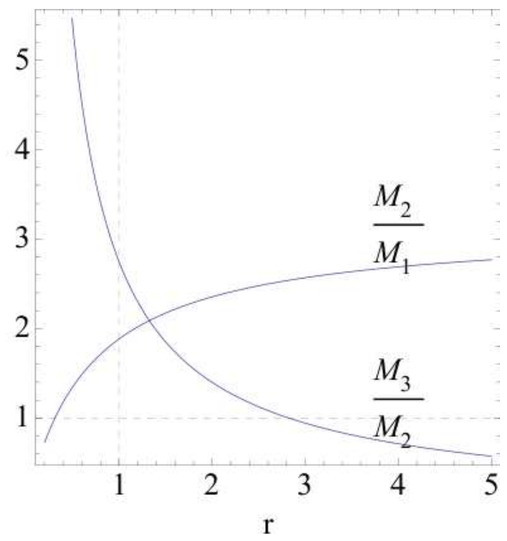

Figure 3.1: Dependence of the gaugino mass parameter ratios M_2/M_1 and M_3/M_2 evaluated at TeV scale on the parameter r. The wino mass term M_2 is lighter than the bino mass term M_1 for $r \lesssim 0.3$.

with M_1 given by the sum rule

$$M_1 = \frac{3}{5}M_2 + \frac{2}{5}M_3 \quad \text{(GUT scale)}. \tag{3.28}$$

The value $r = 1$ corresponds to universal gaugino masses. Largely non universal masses can arise for $r \neq 1$, despite $SO(10)$ unification, as already discussed in section 3.2.2 along with the subtleties connected to considering 3.27 and 3.28 to hold at the GUT scale. The dependence of the gaugino mass parameter ratios M_2/M_1 and M_3/M_2 on r at TeV scale is shown in figure 3.1.

As usual, $\tan\beta$ can be traded for the $B\mu$ parameter in equation 3.4 and sign(μ), together with the EWSB condition, determine the μ parameter.

3.3.2 Marginal parameters

Additional parameters are also needed in order to specify the detailed flavour structure of the lagrangian in equations 3.2, 3.3 and 3.4, but they have a marginal effect on the TeV spectrum.

We have checked that a $\mathcal{O}(1)$ variations of the four parameters r_{D_i} and r_{L_i}, defined in equation 3.15, have only a very mild (logarithmic) effect on our TeV scale predictions. This in turn happens because the gaugino masses are mostly affected by the total ratio $r = \gamma_L/\gamma_D$ and the most relevant trilinears are those belonging to the third family.

We have also checked that our TeV scale predictions have a very mild (logarithmic) dependence on $\mathcal{O}(1)$ variations of the parameters c_{D_i,L_i} and c'_{D_i,L_i}, defined in equation A.4.

3.4 Analysis of the parameter space

Let us now discuss the parameter space of the model. As pointed out in section 3.3, the relevant parameters to be specified are m_{10}, $M_{1/2}$, r, $\tan\beta$, sign(μ), θ_u and θ_d. Let us begin from a discussion of the allowed range for the angles θ_u and θ_d.

3.4.1 Allowed ranges of θ_u and θ_d

Two constraints have to be taken into account: reproducing the SM fermion masses and EWSB. Since the top Yukawa coupling is essentially given by $\lambda_t = y_3 \cos\theta_u$, see appendix A.1, we should have $\cos\theta_u = \mathcal{O}(1)$, if y_3 has to be kept perturbative and possibly of order one, as λ_t. Which means that the angle θ_u should be sizeable, with the maximal value θ_u also allowed. Similarly, as the bottom Yukawa coupling is $\lambda_b \lesssim \sin\theta_d \times \mathcal{O}(1)$ (see appendix A.1), we should have $\sin\theta_d \gtrsim \lambda_b = m_b/(\cos\beta v) \sim 10^{-2}\tan\beta$. Summarizing, we have

$$\begin{aligned} \theta_u &\sim \mathcal{O}(1) \\ \theta_d &\gtrsim 10^{-2}\tan\beta \end{aligned} \tag{3.29}$$

from the requirement of perturbativity of the couplings leading to the SM fermion masses.

The angles θ_u and θ_d also enter the EWSB conditions through the tree level expression for the Higgs soft masses. In order for EWSB to take place for a given value of $\tan\beta$ (and M_Z), the following two conditions have to be satisfied:

$$\begin{aligned} \frac{m^2_{h_d} - m^2_{h_u}\tan^2\beta}{\tan^2\beta - 1} &\geq M_Z^2/2 \\ (m^2_{h_d} - m^2_{h_u})\frac{\tan^2\beta + 1}{\tan^2\beta - 1} &+ M_Z^2 > 0\,. \end{aligned} \tag{3.30}$$

For moderately large values of $\tan\beta$ and in the typical fine tuned situation in which $|m^2_{h_u}| \gg M_Z^2$, the latter conditions become $m^2_{h_u} \lesssim 0$ and $m^2_{h_d} - m^2_{h_u} \gtrsim 0$. The corresponding constraints on θ_u and θ_d (taking into account the approximate analytical running of the Higgs mass parameters in appendix A.3) can be obtained in analytical form in the limit in which equations A.16 hold:

$$\cos^2\theta_d + \left(1 - \frac{\rho}{2}\right)\cos^2\theta_u \gtrsim \frac{6}{5} - \frac{\rho}{2} \qquad \cos^2\theta_u \gtrsim \frac{3/5 - \rho/2}{1 - \rho/2}\,. \tag{3.31}$$

For example, a typical value $\rho = 0.7$ gives

$$\cos^2\theta_d + 0.65\cos^2\theta_u \gtrsim 0.85 \qquad \cos^2\theta_u \gtrsim 0.4\,. \tag{3.32}$$

Again, the explicit form of the constraints above holds only in a typical fine tuned scenario with moderately large $\tan\beta$ and sfermions heavier than gauginos. Finally, some values of $\cos\theta_u$ and $\cos\theta_d$ may not be allowed because the constraints in equation 3.30 hold, but proper EWSB does not take place, for example because some particle becomes tachyonic.

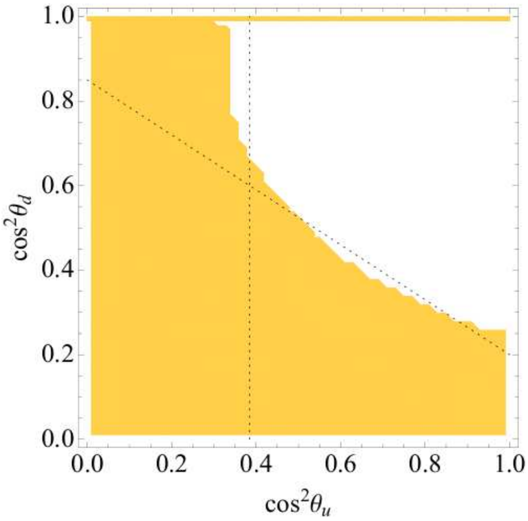

Figure 3.2: Constraints on θ_u and θ_d from proper breaking of the EW symmetry. The figure has been obtained for $\tan\beta = 10$, $m_{10} = 1.8\,\text{TeV}$, $M_{1/2} = 600\,\text{GeV}$, $r_{LD} = 1$. Also shown are the approximate constraints in equation 3.31 (dotted lines).

The constraints on θ_u and θ_d from proper EWSB should be merged with the ones from fermion masses (equations 3.29). The constraint $\theta_u = \mathcal{O}(1)$ is automatically satisfied once equations 3.30 hold, while the constraint on θ_d in equations 3.29 cuts an additional thin stripe of parameter space close to the $\cos^2\theta_d = 1$ axis. The overall constrain one gets in the $\cos^2\theta_u$–$\cos^2\theta_d$ plane is shown (for fixed values of the other parameters) in figure 3.2. The allowed points with $\cos^2\theta_u$ near the left vertical bound (where $m^2_{h_u}$ changes sign) correspond to smaller $|m^2_{h_u}|$ and therefore relatively smaller fine-tuning. We see from the figure that a *pure* embedding of the MSSM up Higgs in the $\mathbf{10}_H$ (with no component in $\overline{\mathbf{16}}_H$, $\cos\theta_u = 1$) is allowed, while the down Higgs must have a mixed embedding, with components in both the $\mathbf{16}_H$ and $\mathbf{10}_H$. A component in the $\mathbf{16}_H$ is needed to obtain non vanishing down quark masses (at the tree, renormalizable level), while a component in the $\overline{\mathbf{16}}_H$ is necessary for a correct EWSB.

3.4.2 A 125 GeV Higgs

In standard gauge mediation it is not easy to accommodate a rather heavy Higgs boson with a mass of about 125 GeV, as indicated by the recent discovery [3,4]. Such a mass needs in fact moderately large $\tan\beta$ and a rather heavy SUSY scale or large trilinear couplings, see, e.g. [66]. In standard gauge mediation it is usually assumed that the messengers have only gauge interactions with the SM fields and hence the trilinear couplings are strongly suppressed at the messenger scale. Sizeable trilinear terms can be generated by introducing superpotential messenger matter interactions. However, the latter potentially spoil the flavour universality of the soft terms, one of the main motivations for gauge mediation models [48] (see however [54, 67, 68]).

Things are different in our setup. Sizeable trilinears are generated because the

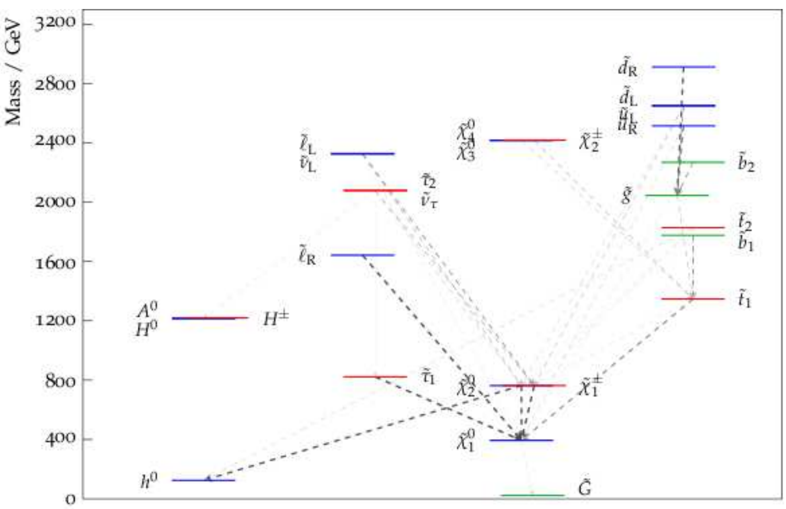

Figure 3.3: The SUSY spectrum of a point with a Higgs mass of 125 GeV calculated with our modified version of the `softSUSY` package [64]. The decays are depicted by the dashed grey arrows, which are scaled with the respective BR calculated via `SUSY-HIT` [65]: only BRs greater than 0.1 are shown. The parameters for the point under analysis are: $m_{10} = 1.5$ TeV, $\cos\theta_u = 0.95$, $\cos\theta_d = 0.9$, $\gamma = 16.7$, $r = 1$, $r_{D1,2} = r_{L1,2} = 0.3$, $\tan\beta = 10$, $\text{sign}(\mu) = +$

messengers unavoidably have Yukawa couplings to the MSSM fields, as we discussed in section 3.2.4. Such trilinears arise at the one loop level but they turn out to enjoy a potential enhancement by the same parameter γ enhancing gaugino masses. Moreover, because of the $SO(10)$ relations between them, the flavour structure of the messenger matter couplings is dictacted by the SM Yukawas. As a consequence, they do not spoil the solution of the supersymmetric flavour problem offered by our framework. The spectrum for a light Higgs of 125 GeV, thus compatible with the recent discovery, is shown in figure 3.7.

Alternatively the Higgs mass can be increased above the MSSM values in the presence of a mixing with a SM singlet chiral field S, as in the NMSSM [69]. In LGM, such a SM singlet would have vanishing soft mass at the messenger scale, as it does not couple to SM gauge interactions. This is not necessarily the case in TGM, as the soft masses are generated by $U(1)_X$ gauge interactions. Depending on the $SO(10)$ embedding of the Sh_uh_d interaction lifting the Higgs mass, such a singlet could acquire a positive, vanishing, or negative soft mass, with the latter case leading to a vev for the S field and therefore to a solution for the μ problem. For this reason, we can also take into account the possibility of a NMSSM like extra contribution to the Higgs mass. We will not enter the model building details associated to the possible presence of a NMSSM singlet in the TeV scale spectrum, leaving the latter to forthcoming studies.

3.4.3 NLSP

In TGM models, the Lightest Supersymmetric Particle (LSP) is the gravitino. The cosmology of the model is therefore determined first of all by the nature of the Next to LSP (NLSP) [50, 70]. The NLSP turns out to be a neutralino or the stau, depending on the region of the parameter space. Whether the lightest neutralino is bino like or wino like is essentially determined by the parameter r, as illustrated by figure 3.1. When $r \gtrsim 0.3$, the NLSP is either a bino like neutralino or a stau, while when $r \lesssim 0.3$ the NLSP is either a wino like neutralino or a stau. Figure 3.4 shows the part of the parameter space in which the NLSP is a neutralino (violet) or stau (light blue). On the left panel, $r = 1$ and the neutralino is bino like, while on the right panel $r = 0.2$ and the neutralino is wino like. The remaining parameters are $\tan\beta = 10$, $\cos\theta_u = 0.8$, $\cos\theta_d = 0.8$, $\text{sign}(\mu) = 1$. The figure shows that the NLSP is a neutralino in most of the parameter space. On the other hand, a stau stripe is present in both cases. In fact, the upper left boundary of the parameter space is due to the stau becoming tachyonic. A stau NLSP can therefore be obtained in a region close enough to that boundary. Also shown in figure are the regions (separated by a dotted line) in which the lightest coloured particle is the lightest stop or the gluino. Finally, the ratio of left and right handed squared selectron masses is also shown (dashed yellow lines). As a peculiar prediction of the minimal $SO(10)$ TGM scenario, that ratio is predicted to be 2 at the tree level. A deviation from two is induced by loop corrections due to SM gauge mediation effects. The figure shows that the deviation is small enough not to spoil the tree level prediction. The ratios of squark masses are affected by larger corrections.

In the light of the discussion above, we will consider three representative points in the parameter space in which the NLSP is a bino like neutralino, a wino like neutralino or a stau.

3.4.4 Three benchmark points

For the following phenomenological analysis we consider three benchmark points with different NLSP. Here we shall see their main features before devoting to collider searches for them.

3.4.4.1 Bino NLSP benchmark point

The case in which a bino like neutralino is the NLSP of the framework is probably the most promising from a phenomenological point of view. In turn it is a situation in which the TGM ratio can be easily obtained without many interferences from two loop contributions to soft masses and still get a reasonably light spectrum.

When the spectrum of figure 3.5 is considered one would expect that at the LHC collider the typical final states are characterized by a large presence of b-enriched final states accompanied by multileptonic signals. The produced b's come from

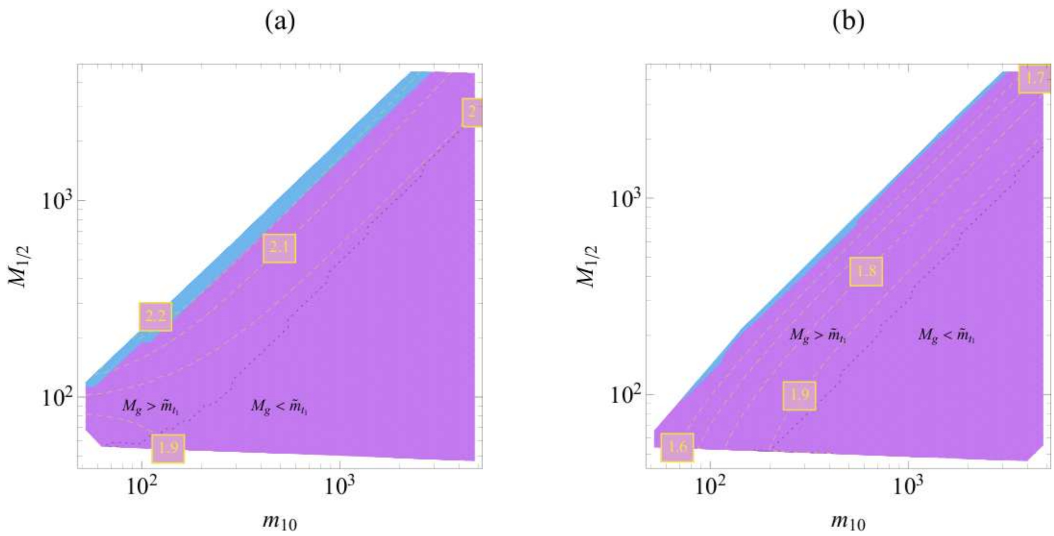

Figure 3.4: Nature of the NLSP in the allowed m_{10}–$M_{1/2}$ parameter space for $\tan\beta = 10$, $\cos\theta_u = 0.8$, $\cos\theta_d = 0.8$, $\text{sign}(\mu) = 1$, for $r = 1$ (a) and $r = 0.2$ (b). The NLSP is a neutralino in the violet region and a stau in the light blue region. The violet region corresponds to a bion like neutralino in the left panel (a) and to a wino like neutralino in the right panel (b). The regions in which the lightest coloured particle is a stop or a gluino are separated by a black dotted line. Also shown is the ratio $m^2_{\tilde{e}_L}/m^2_{\tilde{e}_R}$ of left and right handed squared selectron masses (dashed yellow lines).

electroweak decays. In particular it is very interesting the situation in which, as in our benchmark point, along with the usual W and Z channels, also the Higgs boson can be produced in the cascade[6], making the situation even more intriguing, because of the large branching fraction $H \to b\bar{b}$. Because of the large MET owed to the χ^0_1, the characteristic feature of such models would be the presence of both SUSY traces and the Higgs boson in the same event. The latter situation makes it profitable to consider such a scenario both with inclusive and exclusive dedicated searches.

3.4.4.2 Wino NLSP benchmark point

The case in which a wino like neutralino is the NLSP of the framework is not as promising as the one previously discussed, but it still yields some interesting phenomenology. It is a perfect benchmark for an inclusive analysis, and it is again characterized by a quite clear evidence of the soft mass ratios since the two loop contributions do not spoil too much the tree level prediction. In this case the inversion of the hierarchy between the two lightest gauginos comes along with the presence of a compression of the mass spectrum of the lightest neutralinos, thus making the decay $\chi^0_2 \to \chi^0_1 H$ kinematically forbidden. In this respect it is then comparatively more profitable to look at this situation with semi- and full-leptonic

[6]This is possible if the decay $\chi^0_2 \to \chi^0_1 H$ is kinematically allowed.

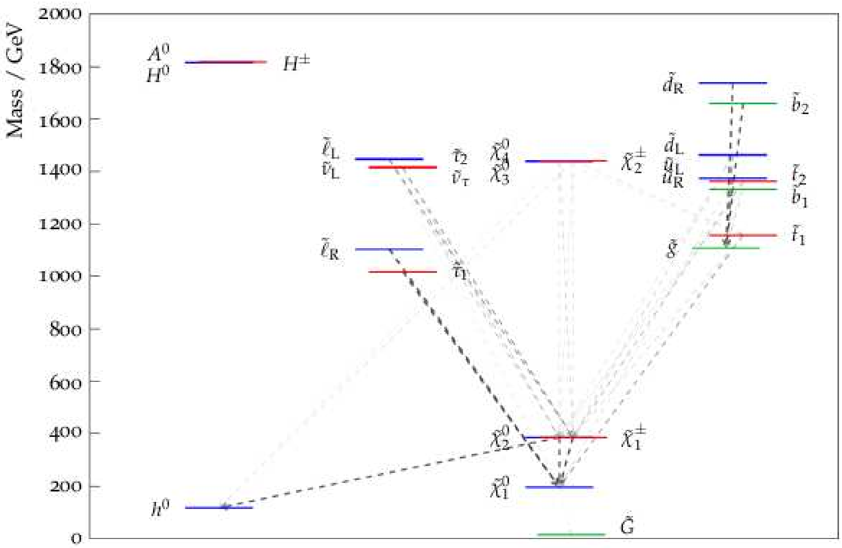

Figure 3.5: The SUSY spectrum of the benchmark point with bino NLSP calculated with our modified version of the `softSUSY` package [64]. The decays are depicted by the dashed grey arrows, which are scaled with the respective BR calculated via `SUSY-HIT` [65]: only BRs greater than 0.1 are shown. The parameters for the point under analysis are: $m_{10} = 1.0$ TeV, $\cos\theta_u = 0.9$, $\cos\theta_d = 0.9$, $\gamma = 15$, $r = 1$, $r_{D1,2} = r_{L1,2} = 1$, $\tan\beta = 10$, sign$(\mu) = +$

channels.

3.4.4.3 Stau NLSP benchmark point

The case in which the stau is the NLSP of the theory is probably the less interesting from the TGM point of view. In those regions of parameter space characterized by a stau NLSP the tree level and the two loop contributions to coloured sfermion masses are comparable. In this situation one can use searches for heavy charged stable particles, on top of inclusive ones. The stau indeed is not expected to decay to the gravitino within the collider as such a decay is very slow since mediated by gravitational interactions.

3.5 Razor and TGM smoking gun

The study of unbalanced events allows to probe the production of SUSY events at the LHC. The ATLAS and CMS experiments have collected so far $\sim$ 5 fb^{-1} at 7 TeV and are expected to collect $\sim$ 20 fb^{-1} at 8 TeV. The current limits are pushing the masses of the coloured superpartners above the 1 TeV threshold for generic MFV models [71, 72], while lower masses are allowed for stop and sbottom in the case of models with large mass splitting among the third family and the others [73, 74]. So far, the possibility of light charginos and neutralinos has been tested only through

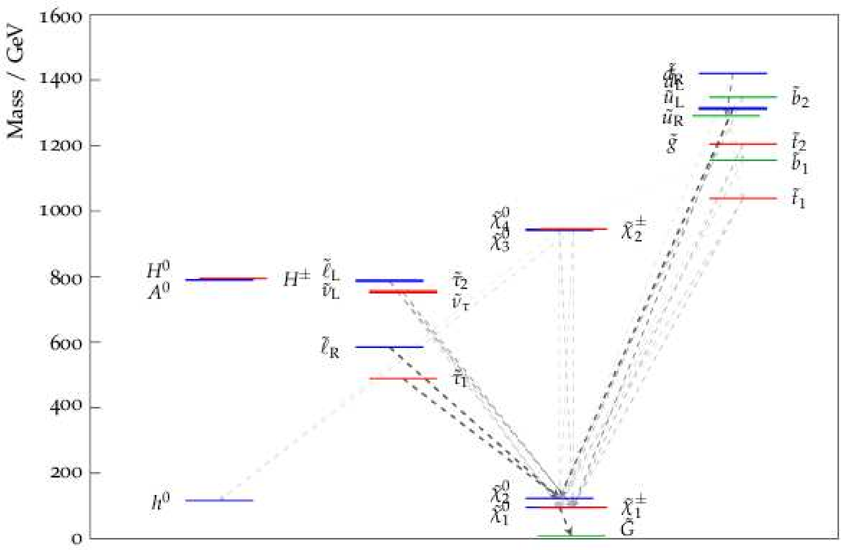

Figure 3.6: The SUSY spectrum of the benchmark point with wino NLSP calculated with our modified version of the `softSUSY` package [64]. The decays are depicted by the dashed grey arrows, which are scaled with the respective BR calculated via `SUSY-HIT` [65]: only BRs greater than 0.1 are shown. The parameters for the point under analysis are: $m_{10} = 550$ GeV, $\cos\theta_u = 0.9$, $\cos\theta_d = 0.9$, $\gamma = 20$, $r = 0.2$, $r_{D1,2} = r_{L1,2} = 1$, $\tan\beta = 10$, sign$(\mu) = +$

multi lepton final states [75, 76], which suffer from the suppression coming from $Z \to \ell\ell$ and $W \to \ell\nu$ branching ratios. The increase in the center of mass energy will be beneficial to push the mass limits on squarks and gluino above the TeV scale, while the search for light EW gauginos will be pushed by the larger collected luminosity.

In this scenario, a possible hint of new physics could emerge by the end of 2012, but even in this situation the mission would be far from being accomplished. The search for SUSY would be completed by the characterization of a possible excess in terms of a specific SUSY model, to possibly underline the nature of the SUSY breaking mechanism and of its mediation. Accomplishing this goal, sometimes referred to as the *inverse LHC problem* [77], would imply the use of kinematic variables sensitive to the mass of the produced particles in as many final states as possible.

The TGM class of models offers a very reach phenomenology at the LHC, challenging the experiments on several fronts at the same time (e.g. high mass searches, compressed gaugino spectra, . . .) and producing many interesting scenarios, such as Higgs production in SUSY cascades. In this respect, TGM is an interesting playground on which the performances of different searches (e.g. hadronic vs. leptonic searches) could be compared, and, on top of that, it comes with a specific prediction on the ratio of sfermion masses, which should be tested by experiments in case an excess is found.

Keeping in mind the latter consideration we think that the razor inclusive anal-

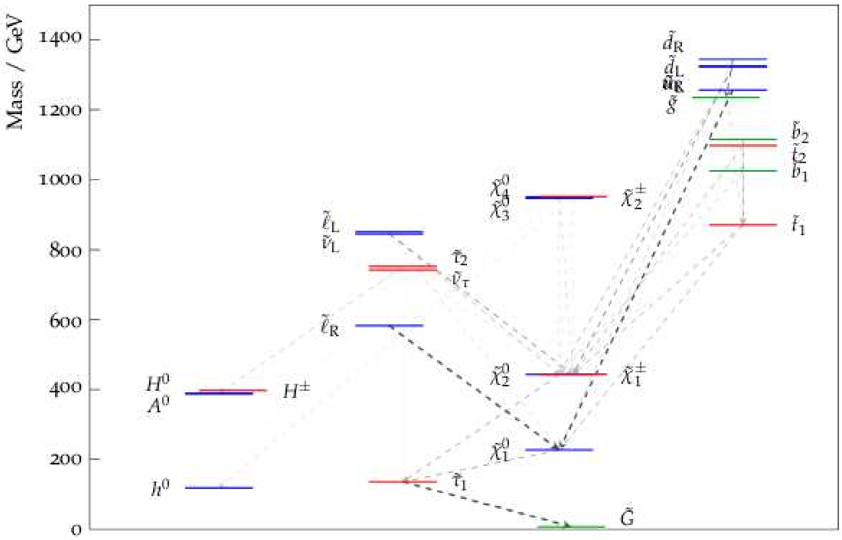

Figure 3.7: The SUSY spectrum of the benchmark point with stau NLSP calculated with our modified version of the softSUSY package [64]. The decays are depicted by the dashed grey arrows, which are scaled with the respective BR calculated via SUSY-HIT [65]: only BRs greater than 0.1 are shown. The parameters for the point under analysis are: $m_{10} = 800$ GeV, $\cos\theta_u = 0.9$, $\cos\theta_d = 0.9$, $\gamma = 35$, $r = 1$, $r_{D1,2} = r_{L1,2} = 1$, $\tan\beta = 35$, $\text{sign}(\mu) = +$

ysis by CMS [74,78] would be an ideal tool to highlight the smoking gun of minimal TGM. From the general discussion in appendix A.4 we see that as far as our spectrum is characterized by two well defined mass scales, namely corresponding to $\tilde{q}, \tilde{u}^c, \tilde{e}^c$ and $\tilde{d}^c, \tilde{l}$ sfermions, the distribution of the M_R variable will identify the latter as two different peaks of definite mass. More specifically such peaks will occur for those values of M_Δ, see equation A.19, corresponding to the decays of the squarks towards the NLSP. The peculiar phenomenological prediction of TGM, the ratio in equation 3.6, would then be translated to a ratio between the position of the two peaks in the distribution of M_R given by

$$\frac{M_\Delta^{d^c,l}}{M_\Delta^{q,u^c,e^c}} = \sqrt{2}\left(1 + \frac{m_{\text{NLSP}}^2}{2\tilde{m}_{10}^2} + \dots\right) . \qquad (3.33)$$

Unfortunately the situation just depicted is too simplistic as many different effects tend to broaden the M_R distribution, causing a partial or total overlap of the different peaks. Anyway, with high luminosity and sufficient separation ($\gtrsim 30\%$ of the peak position) one could distinguish the peaks even in presence of detector effects.

The main issue connected with the use of razor, just as any other inclusive analysis tool, is the lack of probes for light gauginos directly hitting the detector, since the trigger is based on the detection of jets (or jet + one lepton) with broad cuts. Since in our scenario, along with many others, there is a large presence of such superpartners coming from direct production channels, this represents a possible phenomenological issue.

In this respect, the comparison of the leptonic vs. hadronic razor *boxes* could be used to disentangle the EW part of the SUSY spectrum from the tipically heavier coloured one. To this purpose one could improve the framework adapting the razor strategy to the current dilepton and multi lepton analyses. For instance, the low M_R region of the razor plane could be investigated using the dilepton triggers, applying cuts on M_R at the level of roughly 100 GeV, and thus making it possible to spot the peaks owed to the decays of light gauginos. The obvious drawback of the latter procedure, represented by large SM backgrounds, could be easily circumvented by means of tight cuts on R. It is quite obvious then that a specific dilepton razor analysis performed for large integrated luminosity could complement the search for coloured particles in events with jets.

To sum up the razor analysis, eventually upgraded with dileptonic trigger, would certainly be a perfect tool for analyzing TGM scenarios. While a detailed analysis of the TGM parameter space would imply a CPU need that goes beyond our possibilities, we show here what our TGM benchmark points, introduced in section 3.4.4, would look like in a razor search. We also associate to each benchmark point an upper limit on the SUSY cross section, derived following the instructions provided by the CMS collaboration [79].

3.5.1 Analysis of the benchmark points

We start by computing the SUSY spectrum evolving the parameters of equations 3.2 and 3.3 with the RGEs described in appendix A.2 down to low energies using a modified version of `softSUSY` package [64]; knowing the spectrum we calculate the branching ratios via `SUSY-HIT` [65]. Then we generate a sample of SUSY events at the center of mass energy of 7 TeV using `PYTHIA8` [80]. We cluster jets from the stable particles in the event, ignoring neutrinos and the NLSP, with the *anti-Kt* jet algorithm [81] as implemented in `FASTJET` [82,83]. The energy of the generator level jets is then modified in order to take into account the detector resolution of the CMS detector [84]. The resolution is modeled according to a Gaussian response function both for the jet transverse momenta and the missing transverse energy (MET).

The CMS collaboration provides the information to reinterpret the razor analysis, limited to the hadronic *box*: our simulation of the jet and MET reconstruction would then be sufficient to the scope. Nevertheless we decided to show the distribution of M_R and R^2 also in other boxes, obtained considering muons and electrons at the generator level. This simplification is good enough to reconstruct the kinematic properties of the event, but it overestimates the signal yield, since no inefficiency is associated to the lepton reconstruction. These plots should then be interpreted as a qualitative illustration of the sensitivity to TGM in the leptonic boxes, waiting for more information being released by the CMS collaboration.

We show in figure 3.8 the M_R and R^2 projections for the hadronic, leptonic, and semileptonic boxes in the benchmark points under analysis. One could notice that the different decay chains produce different distributions, even within one model.

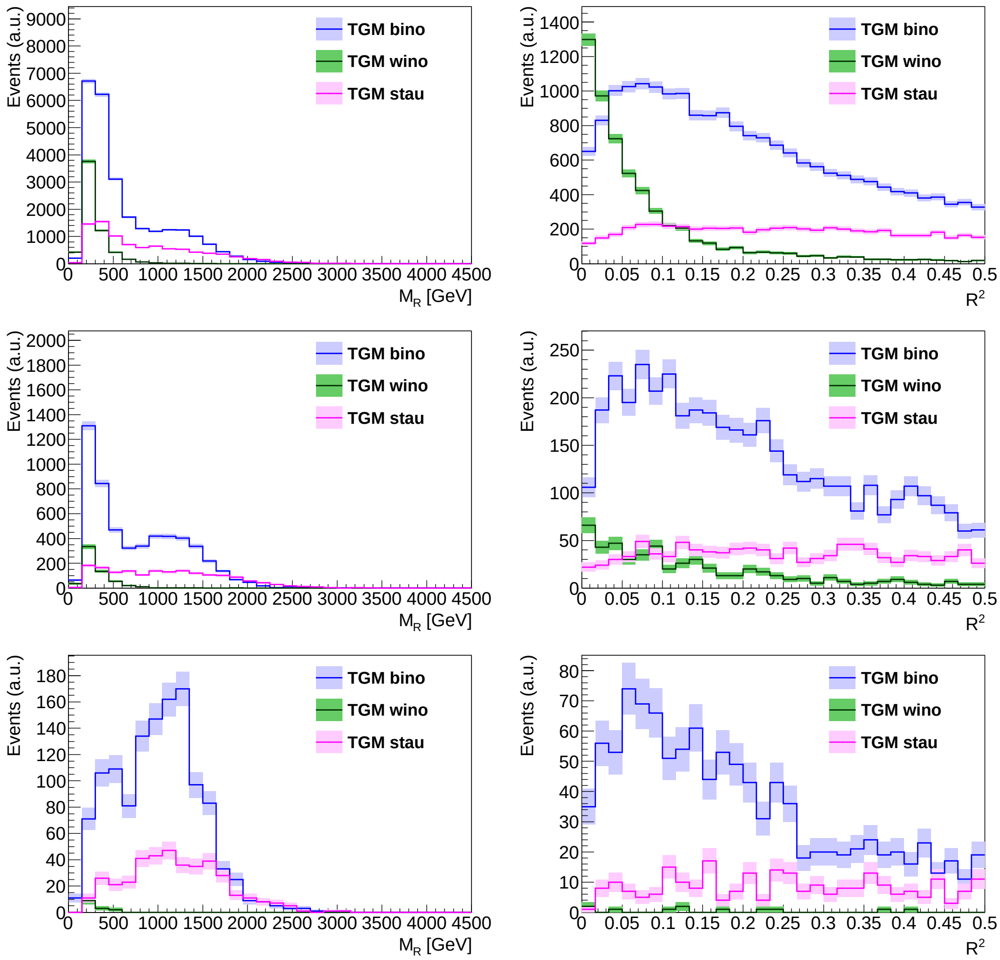

Figure 3.8: M_R (left) and R^2 (right) distributions for a set of TGM benchmark points, as obtained for the CMS razor hadronic (top), leptonic (center), and dileptonic (bottom) boxes.

The presence of two competitive decay chains in one model generates a multimodal distribution, each local maximum corresponding to a different mass split between the produced sparticle and the NLSP. One should notice that we further assume the stable staus to be too slow to be detected with the ordinary event reconstruction[7].

In the M_R distribution for two of the benchmark models two peaks arise, which are associated to the decays of the squarks and of the Next to NLSP to the NLSP itself. On top of that the broad peak one observes for the squarks extends beyond the width expected from the experimental resolution, showing how such a peak is best fitted by two overlapping bells. It is also interesting to stress the fact that

[7]Recently, it was also pointed out that these particles could receive a boost if produced in the cascade decay of heavier particles. In this case they should be detected as ordinary muons, with no missing energy in the event. In this sense, any conclusion we obtain neglecting this effect overestimates the sensitivity of the razor analysis to these models, since a misidentification of the stau as a muon would reduce the value of R^2 and consequently the efficiency of the analysis.

the relative composition changes in different boxes. This illustrates how one could obtain extra handles for model discrimination in this analysis.

The majority of the squark events happen to be at $M_R \sim 1$ TeV, meaning that the benchmark models considered are characterized by an energy scale higher than what was tested with 7 TeV collisions. In this respect, the update of the razor search to the 8 TeV data could give interesting results. Following the instructions given by CMS [79] we compute the excluded cross section for each benchmark model and compare it to the next to leading order (NLO) value, obtained from `PROSPINO` [85]. Such a limit is obtained only from the hadronic box; a more stringent one could be obtained with the information related to the leptonic boxes that still have to be released by the CMS collaboration. In the case of the stau benchmark point one would need a more detailed detector simulation to correctly take into account the fraction of events in which the two staus actually contribute to the missing transverse energy in the event. If this fraction is small, the limit would be much weaker than what quoted in table 3.3.

Model	NLO SUSY cross section [pb]	excluded cross section [pb]
TGM bino	0.008	0.082
TGM wino	0.006	0.053
TGM stau	0.021	0.21

Table 3.3: Theoretical NLO SUSY cross section for the three benchmark points obtained from `PROSPINO` [85] compared to the excluded cross section calculated following the indications of the CMS collaboration [79].

Keeping these warnings in mind, one could then compare the predicted and excluded cross sections, showing that the benchmark poits are still not ruled out by the experiments. A much higher luminosity (and more inclusive triggers) would be needed to test this kind of scenarios, which can be obtained from considering also the collisions at 8 TeV in the center of mass that are currently run.

Finally, in the case in which the NLSP is the stau some bounds on its mass can be set from the searches on the heavy charged stable particles, as anticipated in section 3.4.4.3: the stau, indeed, decays to the gravitino outside the detector. Such limits in the TGM framework are in general less restrictive than those in LGM since the additive tree level contribution to stau soft mass term accounts for a comparably smaller production cross section. As shown in figure 3.9 the recent experimental results accounts for a TGM stau mass larger than $220 \div 250$ GeV.

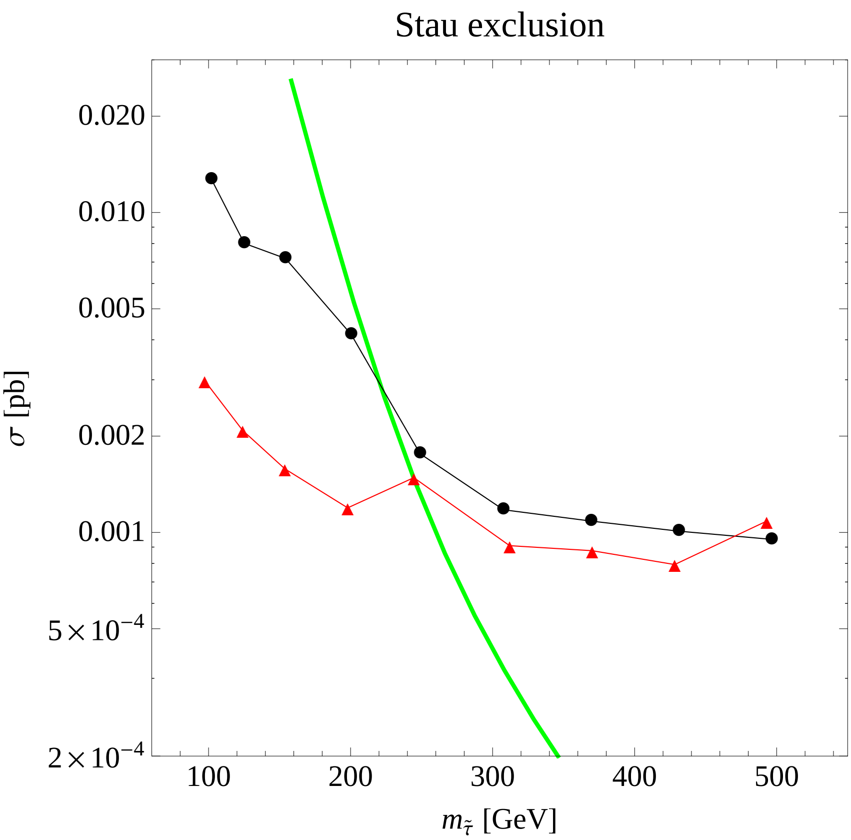

Figure 3.9: In order to set some limits on the NLSP stau mass we calculated the predicted theoretical cross section and then compared the latter with the observed 95% CL upper limit [86]. The black line represent the experimental bound on the cross section taking into account only the selection based on the tracker, while the red line is based also on the time of flight (TOF). The green line the theoretical direct production cross sections for stau on which we added the subleading contribution of the indirect stau production owed to the squark and gluino channels, all of these contribution computed through `PROSPINO`. All in all we can give a mass bound for the stau of 220 ÷ 250 GeV.

4

TGM beyond $SO(10)$

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In this chapter we extend the TGM framework to the case of E_6, which, besides being independently well motivated and widely studied in the literature [87–90], is strongly motivated by TGM. As we have seen in chapter 3, in the context of $SO(10)$ the chiral superfield spectrum needed for TGM to work contains three families of $\mathbf{16}+\mathbf{10}$ representations, which, together with an $SO(10)$ singlet, form precisely the fundamental of E_6: $\mathbf{27} = \mathbf{16}+\mathbf{10}+\mathbf{1}$. The E_6 vector spectrum contains four new SM singlets and thus allows for a variety of possibilities for combining the corresponding D-terms. To be general, we will consider the possibility that part of E_6 is broken by boundary conditions in the context of Extra Dimensions [91, 92], so that we will deal with an effective theory below the compactification scale with a gauge group that is a Rank 5 or Rank 6 subgroup G of E_6. Despite the large number of possible E_6 subgroups containing G_{SM}, we will see that there are only three different cases corresponding to the number of vector messengers. As far as the tree level sfermion mass prediction is concerned, what matter are the structure and the breaking of the SM singlet generators by scalar and F-term vevs, inducing D-terms for the corresponding vector superfields. Their number is either one, two or four and we will study the three cases in sections 4.2, 4.3 and 4.4 respectively. As a robust prediction we will find that sfermion masses are $SU(5)$ invariant at the GUT scale, even if the gauge group does not contain $SU(5)$ itself. We notice that some of the notations of this chapter will differ from those presented up to now since we believe that in this way the discussion is clearer in some crucial points. With the help of the tables in appendix B.2, anyway, there will be no troubles for the reader.

4.1 General Framework

Let us determine the possible TGM SUSY breaking messengers. The gauge group we want to consider is a subgroup G of E_6 (including E_6) containing the SM group $G_{\rm SM}$. The messengers are a subset of the SM singlet E_6 vectors. In order to identify the latter, let us decompose the E_6 adjoint with respect to $G_{\rm SM}$ and consider the embedding of the SM group $G_{\rm SM}$ in E_6 through the maximal subgroup $SO(10) \times U(1)_{10}$. The relevant subgroup chain is

$$E_6 \to SO(10) \times U(1)_{10} \to \mathrm{SU}(5) \times U(1)_5 \times U(1)_{10} \to G_{\rm SM} \times U(1)_5 \times U(1)_{10}\,, \tag{4.1}$$

and the corresponding decomposition of the E_6 adjoint $\mathbf{78}$ is (we illustrate the decompositions of the fundamental and adjoint representation of E_6 in tables B.2 and B.3 at the end of appendix B.2)

$$\mathbf{78} \to \mathbf{45}_0 + \mathbf{16}_{-3} + \overline{\mathbf{16}}_3 + \mathbf{1}_0 \tag{4.2a}$$

$$\begin{aligned} &\mathbf{45}_0 \to \mathbf{24}_{0,0} + \mathbf{10}_{-4,0} + \overline{\mathbf{10}}_{4,0} + \mathbf{1}_{0,0} \qquad \mathbf{1}_0 \to \mathbf{1'}_{0,0} \\ &\mathbf{16}_{-3} \to \overline{\mathbf{5}}_{-3,-3} + \mathbf{10}_{1,-3} + \mathbf{1}_{5,-3} \qquad \overline{\mathbf{16}}_3 \to \mathbf{5}_{3,3} + \overline{\mathbf{10}}_{-1,3} + \mathbf{1}_{-5,3}\,. \end{aligned} \tag{4.2b}$$

Therefore the new four SM singlets contained in E_6 are the fields $\mathbf{1}_{0,0}, \mathbf{1'}_{0,0}, \mathbf{1}_{5,-3}, \mathbf{1}_{-5,3}$, the first two corresponding to the $U(1)$ factors $U(1)_5$ and $U(1)_{10}$. Since all these generators commute with $SU(5)$ the sfermion masses from TGM will be $SU(5)$ invariant (provided the embedding of MSSM fields is in full $SU(5)$ multiplets[1]). This constitutes one of the main phenomenological predictions of TGM.

As for the SM matter, we will consider an embedding in irreps of G that arises from the fundamental representation $\mathbf{27}$ of E_6. Under the subgroup chain in equation 4.1 the $\mathbf{27}$ decomposes as

$$\mathbf{27} \to \mathbf{16}_1 + \mathbf{10}_{-2} + \mathbf{1}_4 \tag{4.2c}$$

$$\mathbf{16}_1 \to \overline{\mathbf{5}}_{-3,1} + \mathbf{10}_{1,1} + \mathbf{1}_{5,1} \quad \mathbf{10}_{-2} \to \overline{\mathbf{5}}_{2,-2} + \mathbf{5}_{-2,-2} \quad \mathbf{1}_4 \to \mathbf{1}_{0,4}\,. \tag{4.2d}$$

We therefore have some freedom to embed the SM fields, namely we can choose whether to embed d^c, l into $\overline{\mathbf{5}}_{-3,1}$ or $\overline{\mathbf{5}}_{2,-2}$ (or a linear superposition of both). The choice will be dictated by the requirement that the sfermion masses from TGM are positive. Moreover we only want to consider *pure* embeddings of the MSSM matter fields in the $\mathbf{27}$ of E_6. By *pure* embedding we mean that each SM fermion multiplet can be embedded into a single irreducible representation of the gauge group, and the representation is the same (or equivalent) for the three families. This assumption of

[1] We will see that this is a well motivated assumption, even if G does not contain $SU(5)$.

pure embeddings is crucial to obtain flavour universal sfermion masses (since there is no flavour problem with Higgs soft masses we allow mixed embeddings for the MSSM Higgs multiplets).

In order to break gauge symmetry and SUSY we will need scalar and F-term vevs of SM singlet fields. For this purpose we introduce a certain number of Higgs fields, which are distinguished from the matter fields by means of a symmetry like matter or R-parity. The SM components of the matter will be denoted by small letters and the components of higgs fields by capital letters. We will consider only singlets contained in the $\mathbf{27}, \overline{\mathbf{27}}$ and $\mathbf{78}$ of E_6, which are $\mathbf{1}_{0,4}, \mathbf{1}_{5,1}, \mathbf{1}_{-5,3}$ + conjugated $= N^{c\prime}, N^c, S'_+, \overline{N}^{c\prime}, \overline{N}^c, S'_-$.

As a gauge group we will consider not only E_6 but also a generic Rank 5 or Rank 6 subgroup G of E_6 which contains G_{SM}. This is because we want to consider the possibility that part of E_6 is broken (to G) through boundary conditions in the context of Extra Dimensional GUT models [91, 92]. The model is in this case supposed to describe the effective theory below the compactification scale. Without loss of generality we can assume that $G \supseteq G_{\text{SM}} \times U(1)_X \equiv G_{\text{min}}$, where the $U(1)_X$ is a generic linear combination of the two $U(1)$'s that appear in the subgroup chain in equation 4.1. The SM singlet generators contained in G can be either just the linear combination $U(1)_X$, both $U(1)_5$ and $U(1)_{10}$ or all four singlets $\mathbf{1}_{0,0}, \mathbf{1'}_{0,0}, \mathbf{1}_{5,-3}, \mathbf{1}_{-5,3}$ (which form a $U(1)' \times SU(2)'$ subgroup of E_6). We now analyze the three possibilities in this order.

4.2 One Messenger Case: $G \supset U(1)_X$

We will start with the simplest case in which there is only one SM singlet generator in G, corresponding to a $U(1)_X$ subgroup. We assume that one can choose suitable boundary conditions such that this generator is given as general linear combination of the normalized generators $\hat{t}_5$ and $\hat{t}_{10}$

$$\hat{t}_X \equiv \sin\theta_X \hat{t}_5 + \cos\theta_X \hat{t}_{10} \qquad \theta_X \in [0, \pi] \ . \tag{4.3}$$

Sfermion masses arise from the breaking of this generator by scalar and F-term vevs according to the first term of equation 2.19. The dependence on these vevs can be parametrized by a single real parameter m_X^2, whose expression in terms of the vevs can be found in appendix B.1. We obtain for the sfermion mass of the sfermion f with X-charge X_f

$$m_f^2 = X_f m_X^2 \, , \tag{4.4}$$

so that the sfermion masses of the candidate matter fields in the $\mathbf{10}_{1,1}, \overline{\mathbf{5}}_{-3,1}, \overline{\mathbf{5}}_{2,-2}$ are given by

$$m^2(\overline{\mathbf{5}}_{-3,1}) = (-3\hat{s}_X + \hat{c}_X) m_X^2 \tag{4.4a}$$

$$m^2(\mathbf{10}_{1,1}) = (\hat{s}_X + \hat{c}_X) m_X^2 \tag{4.4b}$$

$$m^2(\overline{\mathbf{5}}_{2,-2}) = 2(\hat{s}_X - \hat{c}_X) m_X^2 \, , \tag{4.4c}$$

where $\hat{s}_X \equiv 1/\sqrt{40}\sin\theta_X$ and $\hat{c}_X \equiv 1/\sqrt{24}\cos\theta_X$. These masses satisfy the useful tree level identity

$$m^2(\overline{\mathbf{5}}_{-3,1}) + m^2(\overline{\mathbf{5}}_{2,-2}) + m^2(\mathbf{10}_{1,1}) = 0\,. \tag{4.5}$$

We now show that if we assume *pure* embeddings of MSSM matter and require sfermion masses to be positive, then the embeddings and therefore sfermion masses are also $SU(5)$ invariant. First note that the embedding of $u^c_{\rm SM}$, $q_{\rm SM}$, $e^c_{\rm SM}$ in the $\mathbf{27}$ is unique and $SU(5)$ invariant, namely all fields must reside in the $\mathbf{10}_{1,1}$. As for $d^c_{\rm SM}$ and $l_{\rm SM}$, in principle we have two possibilities for each of them: $d^c_{\rm SM} = d^c \subset \overline{\mathbf{5}}_{-3,1}$ or $d^c_{\rm SM} = d'^c \subset \overline{\mathbf{5}}_{2,-2}$ and $l_{\rm SM} = l \subset \overline{\mathbf{5}}_{-3,1}$ or $l_{\rm SM} = l' \subset \overline{\mathbf{5}}_{2,-2}$. But the relation 4.5 implies that at least one of the soft terms $m^2(\mathbf{10}_{1,1}), m^2(\overline{\mathbf{5}}_{-3,1}), m^2(\overline{\mathbf{5}}_{2,-2})$ must be negative. Since we require that $m^2(\mathbf{10}_{1,1})$ is positive, either $m^2(\overline{\mathbf{5}}_{-3,1})$ or $m^2(\overline{\mathbf{5}}_{2,-2})$ can be positive. This means that $d^c_{\rm SM}$ and $l_{\rm SM}$ must be embedded in the same $\overline{\mathbf{5}}$, which is $\overline{\mathbf{5}}_{-3,1}$ if $m^2(\overline{\mathbf{5}}_{-3,1}) > 0$ and $\overline{\mathbf{5}}_{2,-2}$ if $m^2(\overline{\mathbf{5}}_{2,-2}) > 0$. Therefore we have the tree level prediction that sfermion soft masses are $SU(5)$ invariant and flavour universal:

$$(\tilde{m}^2_{d^c})_{ij} = (\tilde{m}^2_l)_{ij} = \tilde{m}^2_{\bar{5}}\delta_{ij} \qquad (\tilde{m}^2_{u^c})_{ij} = (\tilde{m}^2_q)_{ij} = (\tilde{m}^2_{e^c})_{ij} = \tilde{m}^2_{10}\delta_{ij}\,, \tag{4.6}$$

with generic $\tilde{m}^2_{\bar{5}}$ and $\tilde{m}^2_{10}$ depending only on θ_X and m^2_X.

We can consider the two simplifying cases in which either $U(1)_X = U(1)_5$ or $U(1)_X = U(1)_{10}$. In the first case we have $\hat{s}_X = 1/\sqrt{40}, \hat{c}_X = 0$, which implies that we need $m^2_X > 0$ and the light sfermions are $d^{c\prime}, l'$ in $\overline{\mathbf{5}}_{2,-2}$. The ratio $\tilde{m}^2_{10}/\tilde{m}^2_{\bar{5}_{2,-2}}$ is fixed to be 1/2 and we merely reproduced the $SO(10)$ model already described in full detail in chapter 3.

In the second case $\hat{c}_X = 1/\sqrt{24}, \hat{s}_X = 0$ we need again $m^2_X > 0$, but now the light sfermions are d^c, l in $\overline{\mathbf{5}}_{-3,1}$. We have $\tilde{m}^2_{10} = \tilde{m}^2_{\bar{5}_{-3,1}}$ and therefore obtain $SO(10)$ invariant sfermion masses, which follows immediately from the fact that $U(1)_{10}$ commutes with $SO(10)$ (and the SM fermions are embedded in a single $SO(10)$ representation). Note that in this way we can reproduce the popular boundary conditions of Constrained MSSM (CMSSM) for sfermion masses at the scale where $U(1)_X$ is broken (except for the Higgs masses). In this scenario they are naturally flavour-universal since they arise from (extra) gauge interactions which are universal for *pure* embeddings.

What regards the MSSM higgs soft masses we can have in principle a mixed embedding of h_u and h_d in the $\mathbf{27}, \overline{\mathbf{27}}$ and $\mathbf{78}$ higgs fields. That is, h_d and h_u can in general be a linear combination of the fields $L^{27}, L'^{27}, L^{\overline{27}}, L^{78}$ and $\overline{L}^{27}, \overline{L'}^{27}, \overline{L}^{\overline{27}}, \overline{L}^{78}$, respectively. The only requirement is that the coefficient of that field that actually couples to the light MSSM matter fields is sizable, i.e. $\overline{L}^{27}$ for h_u and L^{27} (L'^{27}) for h_d if the light fields d^c_{SM}, l_{SM} are in $\overline{\mathbf{5}}_{2,-2}$ ($\overline{\mathbf{5}}_{-3,1}$). The Higgs soft masses depend on the precise embedding but can range only in certain intervals that are set by the soft masses of $L^{27}, L'^{27}, L^{\overline{27}}, L^{78}$ and $\overline{L}^{\overline{27}}, \overline{L'}^{\overline{27}}, \overline{L}^{27}, \overline{L}^{78}$. We find that

$$m^2_{h_d} \in \left[\min\{-3\tilde{m}^2_{10}, -\tilde{m}^2_{\overline{5}} - \tilde{m}^2_{10}\}, \max\{2\tilde{m}^2_{10}, \tilde{m}^2_{\overline{5}}\}\right] \quad (4.7)$$

$$m^2_{h_u} \in \left[\min\{-2\tilde{m}^2_{10}, -\tilde{m}^2_{\overline{5}}\}, \max\{3\tilde{m}^2_{10}, \tilde{m}^2_{\overline{5}} + \tilde{m}^2_{10}\}\right] \ , \quad (4.8)$$

In order to discuss gaugino masses we have to specify, at least in part, the superpotential. Let us start from identifying the relevant fields. We first have the chiral matter fields (defined by an appropriate assignment of a negative matter or R parity) associated to subrepresentations of three E_6 fundamentals, $\mathbf{27}_i$, $i = 1, 2, 3$, and grouped of course in a set of full G representations. Besides the fields of a whole SM family and two singlets, the $\mathbf{27}$ of E_6 contains additional 10 degrees of freedom. We have in fact two copies of the down quark and lepton fields, d^c, $d^{c\prime}$, l, l' and one copy of fields with conjugate quantum numbers, $\overline{d^c}$, $\overline{l}$. This is welcome, as such extra degrees of freedom need to be (and can be easily made) heavy and, as such, they can play the role of the chiral messenger responsible of gaugino masses, as in ordinary LGM[2]. Let us see how they get heavy.

As the candidate chiral messengers have different charges under $U(1)_X$, a mass term for them can only come from the vev of a SM singlet breaking $U(1)_X$. In particular, the only possibility is to use the N^c, $N^{c\prime}$ and $\overline{N^c}$, $\overline{N^{c\prime}}$ contained in Higgs $\mathbf{27}$ and $\overline{\mathbf{27}}$. Without loss of generality, we can choose a basis in the flavour space of each of such singlets in which only one of them, say N^c_M, $N^{c\prime}_M$, $\overline{N^c}_M$, or $\overline{N^{c\prime}}_M$, gets a vev. Mass terms for the chiral messengers then arise from the following superpotential interactions

$$(h^l_M)_{ij}\bar{l}_i l_j N^c_M + (h^d_M)_{ij}\overline{d^c}_i d^c_j N^c_M + (h^{l\prime}_M)_{ij} l'_i \bar{l}_j N^{c\prime}_M + (h^{d\prime}_M)_{ij} d'^c_i \overline{d^c}_j N^{c\prime}_M \,. \quad (4.9)$$

The couplings in the superpotential terms above can be related to each other and to other superpotential couplings by gauge invariance, depending on the choice of G.

Assuming that all the couplings are non vanishing, we need a scalar vev either for N^c_M or $N^{c\prime}_M$, but not for both, in order to avoid mixed embeddings. In order to generate gaugino masses, the fields that get a heavy mass term must also couple to SUSY breaking (but not the light ones, in order to avoid negative contributions to sfermion masses). This can again be achieved only by coupling them to N^c, $N^{c\prime}$, $\overline{N^c}$, $\overline{N^{c\prime}}$ singlets getting an F-term vev. The relevant superpotential interactions have the same form as above,

$$(h^l_F)_{ij}\bar{l}_i l_j N^c_F + (h^d_F)_{ij}\overline{d^c}_i d^c_j N^c_F + (h^{l\prime}_F)_{ij} l'_i \bar{l}_j N^{c\prime}_F + (h^{d\prime}_F)_{ij} d'^c_i \overline{d^c}_j N^{c\prime}_F \,. \quad (4.10)$$

Gauge invariance (see section 2.2.1) is automatically satisfied if the field getting F-term vev is different from the field getting scalar vev.

In summary we can distinguish two cases depending on the embedding of the light fields d^c_{SM}, l_{SM}

[2] If the gauge group is not E_6, or it does not contain $SU(2)'$ (see below), those extra components could actually be absent. We are obviously not interested in such a case.

A) $N^{c\prime}_M = 0$, $N^{c\prime}_F = 0$, $N^c_M = M$, $N^c_F = F\theta^2$ $\quad (d^c_{SM}, l_{SM}$ are $d^{c\prime}, l'$ in $\overline{\mathbf{5}}_{2,-2})$

B) $N^c_M = 0$, $N^c_F = 0$, $N^{c\prime}_M = M$, $N^{c\prime}_F = F\theta^2$ $\quad (d^c_{SM}, l_{SM}$ are d^c, l in $\overline{\mathbf{5}}_{-3,1})$.

This gives rise to one loop gaugino masses M_i given by

$$M_3 = \frac{g_3^2}{16\pi^2}\frac{F}{M}\,\mathrm{Tr}\left[h^d_F\left(h^d_M\right)^{-1}\right] \tag{4.11a}$$

$$M_2 = \frac{g_2^2}{16\pi^2}\frac{F}{M}\,\mathrm{Tr}\left[h^l_F\left(h^l_M\right)^{-1}\right] \tag{4.11b}$$

$$M_1 = \frac{g_1^2}{16\pi^2}\frac{F}{M}\,\mathrm{Tr}\left[\frac{3}{5}h^l_F\left(h^l_M\right)^{-1} + \frac{2}{5}h^d_F\left(h^d_M\right)^{-1}\right] \tag{4.11c}$$

for case A, and for case B with the replacements $h^d_{F,M} \to h^{d\prime}_{F,M}$ and $h^l_{F,M} \to h^{l\prime}_{F,M}$.

Note that the d^c and l contributions to gaugino masses can be split into three contributions each, corresponding to the three messenger mass eigenstates that are related to the three eigenvalues of $(h^d_M)_{ij}M$ and $(h^l_M)_{ij}M$. Each of these contributions should be evaluated at the corresponding mass scale. If $G \supset SU(5)$, one gets universal gaugino masses, up to corrections from non renormalizable operators, as discussed in chapter 3.

4.3 Two Messengers Case: $G \supset U(1)_5 \times U(1)_{10}$

We now consider the case with two SM singlet generators corresponding to the $U(1)_5 \times U(1)_{10}$ subgroup. Since the discussion of gaugino masses and Higgs soft masses is exactly same as before we will not repeat it again and restrict to tree level sfermion masses.

The sfermion masses of the candidate matter fields in the $\mathbf{10}_{1,1}, \overline{\mathbf{5}}_{-3,1}, \overline{\mathbf{5}}_{2,-2}$ depend only on their charges under $U(1)_5 \times U(1)_{10}$ and the two parameters m_5^2 and m_{10}^2 that are calculated in appendix B.1. We get

$$m^2(\overline{\mathbf{5}}_{-3,1}) = -3m_5^2 + m_{10}^2 \tag{4.11d}$$

$$m^2(\mathbf{10}_{1,1}) = m_5^2 + m_{10}^2 \tag{4.11e}$$

$$m^2(\overline{\mathbf{5}}_{2,-2}) = 2m_5^2 - 2m_{10}^2\,, \tag{4.11f}$$

with the tree level identity

$$m^2(\overline{\mathbf{5}}_{-3,1}) + m^2(\overline{\mathbf{5}}_{2,-2}) + m^2(\mathbf{10}_{1,1}) = 0\,. \tag{4.12}$$

As in the previous section we can use this identity to show that for *pure* embeddings of the matter fields and positive sfermion masses we get $SU(5)$ invariant sfermion masses. Therefore we have the tree level prediction that sfermion soft masses are $SU(5)$ invariant and flavour universal:

$$(\tilde{m}^2_{d^c})_{ij} = (\tilde{m}^2_l)_{ij} = \tilde{m}^2_5\delta_{ij} \qquad (\tilde{m}^2_{u^c})_{ij} = (\tilde{m}^2_q)_{ij} = (\tilde{m}^2_{e^c})_{ij} = \tilde{m}^2_{10}\delta_{ij}\,, \tag{4.13}$$

with generic $\tilde{m}^2_{\bar{5}}$ and $\tilde{m}^2_{10}$ that depend on the scalar and F-term vevs according to the formulae given in appendix B.1. We did not find simplifying limits with definite predictions for sfermion mass ratios other than $\tilde{m}^2_{\bar{5}}/\tilde{m}^2_{10} = 1/2$, which was considered already in [50]. The ranges for the Higgs masses are the same as in section 4.2.

4.4 Four Messenger Case: $G \supset U(1)' \times SU(2)'$

Let us now consider the case in which all the four E_6 candidate SUSY breaking messengers belong to G. The four messengers correspond to the E_6 subgroup $U(1)' \times SU(2)'$. The $SU(2)'$ is the one appearing in the E_6 maximal subgroup $E_6 \supset SU(6) \times SU(2)'$ and the $U(1)'$ is the subgroup of $SU(6)$ that commutes with $SU(5)$, as shown in appendix B.2. We denote the corresponding generators as t' and t'_a, $a = 1, 2, 3$. The two additional generators, with respect to the previous section, are t'_1 and t'_2, which can be combined into two complex generators $t'_\pm = (t'_1 \pm i t'_2)/\sqrt{2}$, while t'_3 and t' are linear combinations of t_5 and t_{10} given by $t'_3 = (t_{10} - t_5)/8$ and $t' = (3t_5 + 5t_{10})/4$.

The role of the $SU(2)'$ symmetry is to make the two $\mathbf{\bar{5}}$ and the two singlets of $SU(5)$ in the $\mathbf{27}$ of E_6 equivalent, i.e. belonging to the same $SU(2)'$ doublet. Denoting by $(a, b)_q$ the representation which transforms as (a, b) under $SU(5) \times SU(2)'$ and has $t' = q$, we have in fact

$$\mathbf{\bar{5}}_{-3,1} + \mathbf{\bar{5}}_{2,-2} = (\mathbf{\bar{5}}, \mathbf{2})_{-1} \qquad \mathbf{1}_{0,4} + \mathbf{1}_{5,1} = (\mathbf{1}, \mathbf{2})_5\,, \tag{4.14}$$

while the $\mathbf{10}$ and $\mathbf{5}$ of $SU(5)$ in the $\mathbf{27}$ are $SU(2)'$ singlets and have charge $t' = 2, -4$ respectively. This makes a qualitative difference in the way sfermion masses are generated but will not alter the conclusion in equation 4.17.

The masses of the SUSY breaking messengers and the breaking of $U(1)' \times SU(2)'$ are due to the vevs of the singlets $N^{c\prime}, N^c, \overline{N^{c\prime}}, \overline{N^c}$, S'_+, S'_-, as before, which are now grouped into doublets and triplets of $SU(2)' \times U(1)'$. As shown in appendix B.1, in the presence of an arbitrary number of such representations, the masses for the sfermions in the case of the $SU(2)'$ singlets in the $\mathbf{27}$ are given by

$$m^2((\mathbf{10}, \mathbf{1})_2) = 2m_1^2 \tag{4.15a}$$
$$m^2((\mathbf{5}, \mathbf{1})_{-4}) = -4m_1^2\,. \tag{4.15b}$$

Note that the need for non negative tree level soft terms for the sfermions embedded in the $\mathbf{10}$ of $SU(5)$ requires $m_1^2 \geq 0$. The $SU(2)'$ doublets in the $\mathbf{27}$ can mix, and their mass matrices are given by

$$m^2((\mathbf{\bar{5}}, \mathbf{2})_{-1}) = \begin{pmatrix} \dfrac{m_3^2}{2} - m_1^2 & \dfrac{m_+^2}{\sqrt{2}} \\ \dfrac{m_-^2}{\sqrt{2}} & -\dfrac{m_3^2}{2} - m_1^2 \end{pmatrix} \tag{4.16a}$$

$$m^2((\mathbf{1},\mathbf{2})_5) = \begin{pmatrix} \frac{m_3^2}{2} + 5m_1^2 & \frac{m_+^2}{\sqrt{2}} \\ \frac{m_-^2}{\sqrt{2}} & -\frac{m_3^2}{2} + 5m_1^2 \end{pmatrix} . \tag{4.16b}$$

The four parameters m_3^2, m_1^2, $m_\pm^2$ correspond to the four messengers. The first two are real, while $m_+^2 = (m_-^2)^*$.

The MSSM masses of the sfermions that can be embedded in a $\mathbf{10}$ of $SU(5)$ are universal and given by $2m_1^2$ at the tree level. In order to identify the masses of the MSSM sfermions that can be embedded in a $\overline{\mathbf{5}}$ of $SU(5)$, we have to identify the light d^c_{SM} and l_{SM} in the multiplets $(\overline{\mathbf{5}},\mathbf{2})_{-1}$. In principle, the three light leptons l^i_{SM} could be superpositions of the three $t'_3 = 1/2$ lepton doublets l_i and of the three $t'_3 = -1/2$ lepton doublets l'_i contained in three $(\overline{\mathbf{5}},\mathbf{2})_{-1}$. On the other hand, it can be shown that the natural solution of the flavour problem requires that it must be possible to identify the three light leptons with, for example, the $t'_3 = 1/2$ lepton doublets l_i: $l^i_{\text{SM}} = l_i$, up to an $SU(2)'$ rotation. This is indeed what is obtained in simple models, as shown below. The three leptons turn then out to have universal soft terms proportional to $m^2_{\overline{5}} = m_3^2/2 - m_1^2$.

If the gauge group contains $SU(5)$, the same results hold in the d^c sector. If not, the three light d^c_{SM} are also aligned in $SU(2)$ space, but they could in principle be oriented in a different direction. We will see that, under plausible hypotheses, this is not the case, so that we get again the prediction that the sfermion soft masses are $SU(5)$ invariant and flavour universal at the tree level:

$$(\tilde{m}^2_{d^c})_{ij} = (\tilde{m}^2_l)_{ij} = \tilde{m}^2_{\overline{5}}\delta_{ij} \qquad (\tilde{m}^2_{u^c})_{ij} = (\tilde{m}^2_q)_{ij} = (\tilde{m}^2_{e^c})_{ij} = \tilde{m}^2_{10}\delta_{ij} \, , \tag{4.17}$$

with generic $\tilde{m}^2_{\overline{5}}$ and $\tilde{m}^2_{10}$.

The discussion of MSSM Higgs soft masses is similar as before. Now h_d and h_u can in general be a linear combination of the doublets in $(\overline{\mathbf{5}},\mathbf{2})_{-1}$,$(\overline{\mathbf{5}},\mathbf{1})_{-6}$, $(\overline{\mathbf{5}},\mathbf{1})_4$ and $(\mathbf{5},\mathbf{2})_1$, $(\mathbf{5},\mathbf{1})_6$, $(\mathbf{5},\mathbf{1})_{-4}$ respectively. The range for the Higgs masses (for simplicity we consider the case $m_+^2 = 0$) is

$$\begin{aligned} m^2_{h_d} &\in \left[\min\{-6m_1^2, \frac{m_3^2}{2} - m_1^2, -\frac{m_3^2}{2} - m_1^2\}, \max\{\frac{m_3^2}{2} - m_1^2, -\frac{m_3^2}{2} - m_1^2, 4m_1^2\}\right] \\ m^2_{h_u} &\in \left[\min\{-\frac{m_3^2}{2} + m_1^2, \frac{m_3^2}{2} + m_1^2, -4m_1^2\}, \max\{6m_1^2, -\frac{m_3^2}{2} + m_1^2, \frac{m_3^2}{2} + m_1^2\}\right] . \end{aligned}$$

The presence of the $SU(2)'$ guarantees that the MSSM $l_{\text{SM}} = l_i$ and $d^c_{\text{SM}} = d^c_i$ (and the singlets N^c needed to generate masses) come together with $SU(2)'$ partners l'_i and $d^{c\prime}_i$ (and $N^{c\prime}$), which need to be heavy and, as such, can play the role of the chiral SUSY breaking messengers responsible for one loop gaugino masses through ordinary LGM mechanism. Since they must get heavy with their conjugates, the presence of the $\overline{l}_i$, $\overline{d^c_i}$ from the $\mathbf{27}_i$ is also guaranteed.

Let us see how they get heavy. First, let us denote the three $SU(2)'$ doublets containing the light fields as $\mathbf{l}_i = (l_i, l'_i)^T$, $\mathbf{d^c}_i = (d^c_i, d'^c_i)^T$. Mass terms for the extra charged matter fields can only come from superpotential interactions

$$(h^l_M)_{ij}\bar{l}_i\mathbf{l}_j\mathbf{N}^c_M + (h^d_M)_{ij}\overline{d^c}_i\mathbf{d^c}_j\mathbf{N}^c_M\,, \tag{4.18}$$

where we have assumed for simplicity that only one doublet $\mathbf{N}^c_M = (N^{c\prime}, N^c)^T$ gets a vev in the scalar component. If $G \subset SU(5)$, $h^l_M = (h^d_M)^T$, up to corrections from non renormalizable operators, as discussed in chapter 3. We can rotate without loss of generality the vev in the N^c component: $\langle \mathbf{N}^c_M \rangle = (0, M)^T$. Then, the l_i and d^c_i fields automatically end up being also massless, and the flavour problem is naturally solved. Note also that this represents an improvement with respect to the $SO(10)$ theory studied in [50, 51] and with respect to the one and two messenger cases studied in the previous sections. In those cases, in fact, the possible presence of a bare mass term $\mu_{ij}\bar{l}_i l_j$ could give rise to a non *pure* embedding and to flavour non universal soft masses. In this case, such a bare mass term is forbidden by the $SU(2)'$ symmetry.

If more than one $\mathbf{N^c}$ gets a vev coupled to the light fields, the flavour problem is automatically solved if, in an appropriate $SU(2)'$ basis, all those vevs lie in the N^c component only. If that is the case, we can use a basis in the $\mathbf{N^c}$ flavour space such that only one of them gets a vev, and we can still use equation 4.18. In order to avoid negative, tree level contributions to sfermion masses from chiral superfield exchange, we need the $N^{c\prime}$ components not to get an F-term either. In order to generate gaugino masses, one of the N^c must however take an F-term vev. Gauge invariance (see section 2.2.1) is automatically satisfied if the field getting the F-term vev, $\mathbf{N}^c_F$, with $\langle \mathbf{N}^c_F \rangle = (0, F\theta^2)^T$, is different from the one getting the scalar vev, $\mathbf{N}^c_M$. Let

$$(h^l_F)_{ij}\bar{l}_i\mathbf{l}_j\mathbf{N}^c_F + (h^d_F)_{ij}\overline{d^c}_i\mathbf{d^c}_j\mathbf{N}^c_F \tag{4.19}$$

be its coupling to the chiral messengers. The gaugino masses are then given by

$$M_3 = \frac{g_3^2}{16\pi^2}\frac{F}{M}\,\mathrm{Tr}\left[h^d_F(h^d_M)^{-1}\right] \tag{4.20a}$$

$$M_2 = \frac{g_2^2}{16\pi^2}\frac{F}{M}\,\mathrm{Tr}\left[h^l_F(h^l_M)^{-1}\right] \tag{4.20b}$$

$$M_1 = \frac{g_1^2}{16\pi^2}\frac{F}{M}\,\mathrm{Tr}\left[\frac{3}{5}h^d_F(h^d_M)^{-1} + \frac{2}{5}h^d_F(h^d_M)^{-1}\right]\,. \tag{4.20c}$$

The d^c and l contributions to gaugino masses can be split into three contributions each, corresponding to the three messenger mass eigenstates, namely to the three eigenvalues of M_{d^c} and M_l. Each of the three contributions should be evaluated at the corresponding mass scale.

4.5 Phenomenology

In this section we briefly comment on some general aspects of TGM phenomenology in the setup we considered. In TGM models sfermion masses arise at tree level, while gaugino masses arise at one loop. As mentioned, the hierarchy between gaugino and sfermion masses that one might naively expect, potentially leading to sfermions outside the reach of the LHC and to a serious fine tuning problem, turns out to be reduced by various effects down to a mild hierarchy. The hierarchy could actually easily be fully eliminated, but a mild hierarchy is actually welcome, as it makes the ordinary two loop gauge mediated pollution of tree level sfermion masses subleading, and will be assumed in the following.

The Higgs sector parameters are not tightly related to sfermion and gaugino masses. The μ and $B\mu$ parameters are highly model dependent[3], and the Higgs soft masses depend on the Higgs embedding, which is allowed to be mixed in different representation of the gauge group, as discussed above equations 4.7. The coefficients X_{eff} will be conveniently taken in their ranges, while μ and $B\mu$ will be treated as free parameters and as usual traded for M_Z and $\tan\beta$.

Trilinear A-terms arise typically at one loop as they are generated by the exchange of heavy chiral messengers that couple directly to MSSM fields in the superpotential. Their value is model dependent, as it is controlled by unknown superpotential parameters, but it can safely neglected in a sizeable part of the parameter space [50–52].

In the following we present two representative low energy spectra that can be obtained in the present framework. As a result of the previous sections we found that the main phenomenological prediction of extended TGM is that the tree level contribution to the sfermion masses is $SU(5)$ invariant and flavour universal, and thus parametrized by two parameters $\tilde{m}^2_{\bar{5}}, \tilde{m}^2_{10}$ which are independent in the general case. These tree level predictions for sfermion masses hold at the messenger scale where the soft terms are generated. In order to recover the low energy spectra we have to keep into account both the finite two loop contributions from LGM and the RG effects. Since sfermion masses are in our example heavier than gaugino masses, the predictions for the sfermion mass patterns are approximately preserved at low energy. One therefore expects two separated sets of sfermions grouped according to their $SU(5)$ representation. In figure 4.1 we show two illustrative spectra, one in the case $\tilde{m}^2_{\bar{5}} > \tilde{m}^2_{10}$ and the other in the case $\tilde{m}^2_{\bar{5}} < \tilde{m}^2_{10}$. In the specific case where $\tilde{m}^2_{\bar{5}} = \tilde{m}^2_{10}$ as in section 4.2 the spectrum we obtain is analogue to the CMSSM case with non universal Higgs masses [93–98]. The remarkable point is that, in contrast to the CMSSM case, in which universality of sfermion masses is an *ad hoc* phenomenological assumption, in our extended TGM setup it follows from the fact that SUSY breaking is mediated by a heavy $U(1)$ gauge field which universally couples to the MSSM fields.

Finally, we comment about the gravitino. A general feature of TGM is the fact that the gravitino is the LSP, just as in ordinary gauge mediation. Its mass is given by

$$m_{3/2} = \frac{F_0}{\sqrt{3}M_P}\,, \tag{4.21}$$

[3]For some possible implementations see [51].

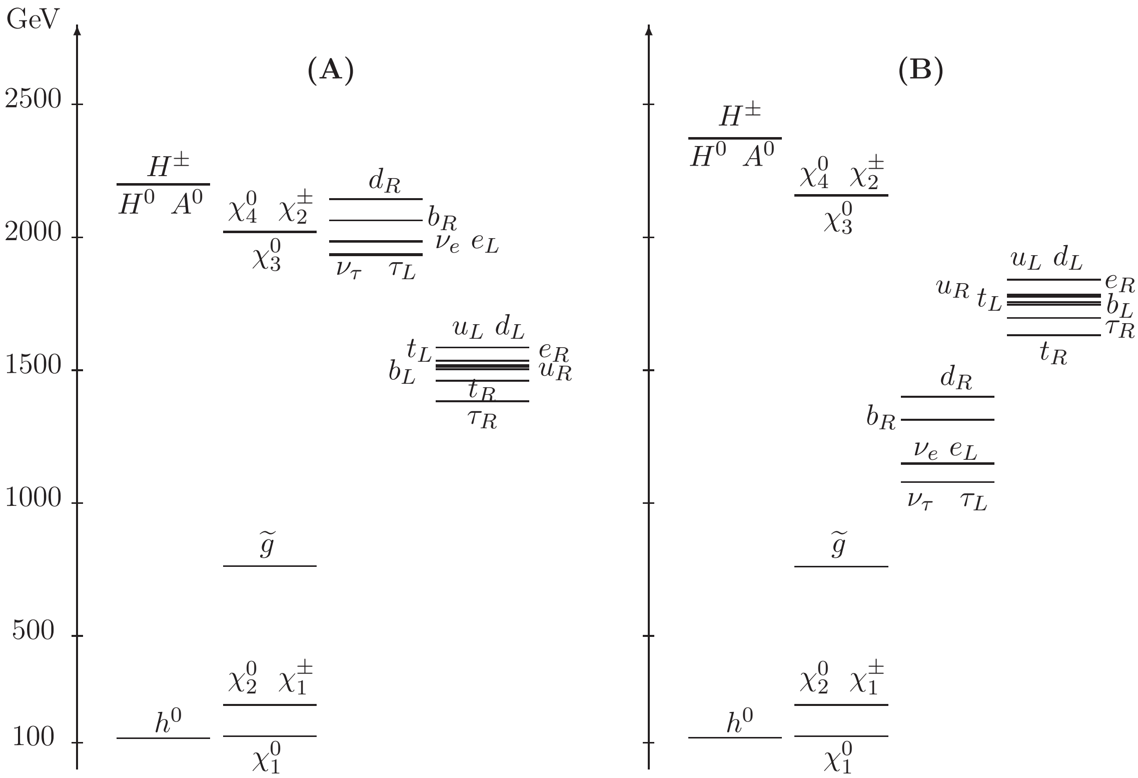

Figure 4.1: Overall parameters: $m = F/M = 4.5\,\text{TeV}$, $m^2_{h_u} = -1/5\,m^2$, $m^2_{h_d} = 3/40\,m^2$, $\tan\beta = 30$. Case A: $m^2_{\bar{5}} = 1/5\,m^2$, $m^2_{\bar{5}} = 2\,m^2_{10}$; Case B: $m^2_{\bar{5}} = 1/14\,m^2$, $m^2_{\bar{5}} = 1/2\,m^2_{10}$.

where $M_P = (8\pi G_N)^{-1/2} = 2.4 \times 10^{18}$ GeV is the reduced Planck mass and $F_0^2 = \sum_i |F_i|^2$, where the sum runs over all the fields taking F-term vevs.

We note that, contrary to the minimal case, the ratios of gravitino mass and other superpartners masses are not fixed, since they depend on the specific pattern of F-term vevs. This happens because the F-terms vevs of different fields enter the gravitino mass through $F_0^2 = \sum_i |F_i|^2$, while they enter the expression for sfermion masses weighted by their charges. A lower bound for the ratio is obtained when just one F-term vev is switched on. For example, in the one messenger case discussed in section 4.2 one obtains

$$\frac{m^2_{3/2}}{m^2_{\widetilde{f}}} \gtrsim \frac{X_F}{X_f\sqrt{3}}\frac{M^2}{M_P^2} = 4 \times 10^{-5}\frac{X_F}{X_f}\left(\frac{M}{2 \times 10^{16}\,\text{GeV}}\right)^2, \tag{4.22}$$

where X_f (X_F) is the charge of the sfermion (singlet breaking SUSY), and M is the scalar vev responsible for $U(1)$ breaking. On the other hand, the gravitino mass cannot be made arbitrarily large. While gauge contributions to sfermion masses are flavour universal, gravitational ones are expected not to be. Their typical size is set by the gravitino mass, thus one has to require that (for $m^2_{\tilde{f}}$ around TeV scale)

$$\frac{m^2_{3/2}}{m^2_{\tilde{f}}} \sim \frac{\left(m^2_{\tilde{f}}\right)_{i\neq j}}{\left(m^2_{\tilde{f}}\right)_{i=j}} \lesssim 10^{-4} \tag{4.23}$$

in order to avoid flavour problems [99].

5

Minimal Yukawa-Gauge Mediation

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In the last months LHC SUSY searches have pushed the lower bounds on coloured sparticles up to the TeV scale. ATLAS [71] and CMS [72] most recent results have shown that both squarks and gluinos must be relevantly heavier than what expected in the pre LHC era. Even if such analyses have been performed in specific frameworks, such as the Constrained MSSM (CMSSM), and under specific hypothesis, doubtless at present we do not expect coloured superpartners much lighter than 1 TeV. What SUSY searches have not specifically excluded so far is the possible presence of low energy neutralinos or gravitino, for which the strictest bounds arise from direct and indirect detection of Dark Matter (DM) [100–103].

The most recent analysis on LHC data have shown that the preferred hypothetical SUSY spectrum is quite peculiar and points in the direction of a scenario close to split SUSY [104, 105] or high scale SUSY [106]. Indeed these frameworks may accomodate heavy coloured sparticles, relatively light neutralinos and a quite large Higgs boson mass, as it has been already noticed in some recent papers [107, 108].

What at the moment seems quite clear is that if SUSY exists the low scale masses are obtained by a quite large tuning of the parameters. This means that the fine tuning principle has to be reassessed if taken as a guideline in model building.

In this chapter we present a new SUSY breaking framework in which the spectrum sketched so far is easily achieved. In our scenario SUSY breaking is communicated via the combined effect of Yukawa and Gauge Mediation: in particular we revisit in a minimal version the old idea of Yukawa-Gauge Mediation (Y-GM) [109–111]. In section 5.1 we will describe the mechanism of SUSY breaking mediation firstly with a toy model and then in the MSSM context. Then in section 5.2 we will show how the soft terms arise and we will focus on the phenomenological predictions of the model. We will discuss in detail the sparticle spectrum, the Higgs boson mass and EW Precision Tests (EWPT). We will also show that the model is safe with respect to FCNC processes and then conclude the chapter giving some cosmological considerations.

5.1 The model

As anticipated the model we are going to propose is a simpler version of Y-GM scenarios already present in literature [109–111]. As we will see it is simpler because we do not ask for a GUT completion and we use the MSSM Higgs doublets to act as SUSY breaking messengers. In this way we avoid the use of large representations and the number of new superfields added to the MSSM ones is really basical.

5.1.1 General implant: a toy model

In this section we use a toy model to introduce our SUSY breaking mechanism. Let us consider a SUSY gauge theory based on the simple group G. Matter superfields are charged under G and denoted by Q_i. Here we do not address the origin of SUSY breaking, assuming it happens because of an unknown mechanism in a secluded sector. For our purposes we just consider that the net effect of such a breaking can be parametrized by a gauge singlet chiral superfield[1] $X = X + \psi_X\theta + F_X\theta^2$ developing vev in its scalar $\langle X\rangle = M/k$ and auxiliary $\langle F_X\rangle = F/k$ components; we define $B_\phi = F/M$. The coupling constant k, whose meaning will soon be apparent, is introduced in the vev definition for later convenience. Such a chiral superfield cannot mediate SUSY breaking, thus we have to couple it to a charged superfield sector that effectively communicates SUSY breaking to matter. At this level the scenario is similar to LGM [48]: the field that develops a SUSY breaking vev does not couple directly to MSSM fields. However, contrary to LGM, X couples only to an additional gauge singlet Φ, and the latter interacts with charged superfields, thus our mechanism works through two messenger sectors and therefore in two different steps.

[1] As anticipated in caption to table 1.1 we will make use of a slight abuse of notation by referring with the same letter to the whole superfield and to its scalar component (fermion component in the case of SM matter fields). In cases in which confusion might arise the precise choice of notation will be spelt out explicitely.

The superfield Φ is identified as first messenger: it couples to charged superfields, H_i (and their partners with opposite charge $\overline{H}_i$), that effectively perform the SUSY breaking mediation. The superpotential that implements the two step mechanism just sketched has the form

$$W_{\text{messengers}} = kX\Phi\Phi + \lambda_{ij}\Phi H_i \overline{H}_j . \tag{5.1}$$

The toy model superpotential contains also the mass term for the second messengers

$$W_{\text{mass}} = \mu_{ij} H_i \overline{H}_j \, . \tag{5.2}$$

While the first messenger, Φ, is typically thought quite heavy and decouples from the low energy spectra, the H_i superfields could be decoupled or not according to the structure of μ_{ij}. In practice we will see that in our realization a subset of the H_i fields becomes part of the low energy spectrum.

The H_i superfields are the effective mediators of SUSY breaking to the MSSM superfields. Gaugino masses are given by two loop graphs in which a gaugino couples through gauge interactions to H_i and then the latter couples to the Φ superfield loop, as shown in figure 5.1. The effects of SUSY breaking to the matter sector are mediated by means of superpotential trilinear interactions

$$W_{\text{matter}} = h_{ijk} Q_i Q_j H_k + \overline{h}_{lmn} Q_l Q_m \overline{H}_n \, , \tag{5.3}$$

where the indices (ijk) and (lmn) are contracted to give gauge invariants. In this scenario the SUSY breaking trilinears are induced at one loop level, figure 5.1b, while sfermion masses arise at two loops as shown in figure 5.1c.

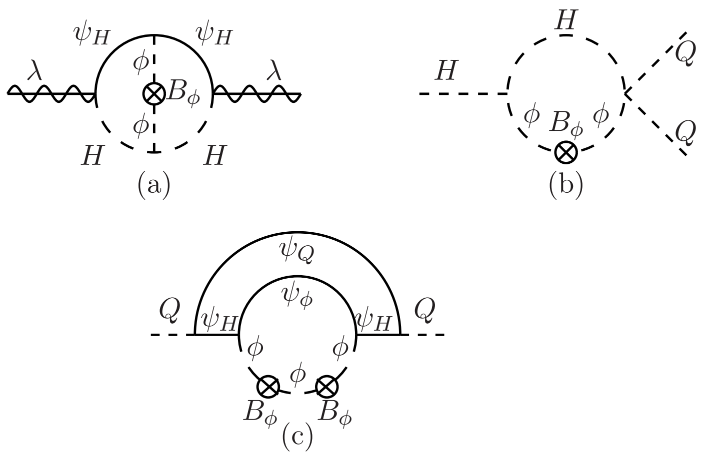


Figure 5.1: Loop graphs giving rise to the different soft terms: (a) gaugino masses, (b) trilinear terms, (c) sfermion masses. The dashed (full) lines stand for scalar (fermionic) fields propagating.

Superfield	fermion component	scalar component	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
q_i	q_i	$\tilde{q}_i$	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{6}$
u_i^c	u_i^c	$\tilde{u}_i^c$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$-\frac{2}{3}$
d_i^c	d_i^c	$\tilde{d}_i^c$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\frac{1}{3}$
l_i	l_i	$\tilde{l}_i$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$
e_i^c	e_i^c	$\tilde{e}_i^c$	$\mathbf{1}$	$\mathbf{1}$	1
h_u	$\tilde{h}_u$	h_u	$\mathbf{1}$	$\mathbf{2}$	$\frac{1}{2}$
h_d	$\tilde{h}_d$	h_d	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$
T	ψ_t	t	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\frac{1}{3}$
$\overline{T}$	$\psi_{\overline{t}}$	$\overline{t}$	$\mathbf{3}$	$\mathbf{1}$	$-\frac{1}{3}$
Φ	ψ_ϕ	ϕ	$\mathbf{1}$	$\mathbf{1}$	0

Table 5.1: Superfield matter content of the theory.

5.1.2 General implant: explicit construction of the model

We now show how the scenario proposed can be implemented in a physically viable theory. The ingredients are essentially those anticipated in the toy model.

The effect of the secluded sector where SUSY breaking effectively takes place is parametrized by the presence of the gauge singlet chiral superfield $X = X + \psi_X \theta + F_X \theta^2$ that develops a vev both in its scalar and auxiliary components, M/k and F/k respectively. X couples to a gauge chiral singlet Φ through a superpotential term $kX\Phi\Phi$. The gauge group G is the SM gauge group and we identify the second messenger sector $(H_i, \overline{H}_i)$ with the MSSM Higgs superfields. In order to prevent too heavily suppressed gluino masses we should add an extra heavy coloured triplet with the quantum numbers of the down quark superfield, that does not couple dangerously to MSSM fields by means of a Z_2 discrete symmetry. Thus the second messengers of the framework happen to be $(H_i, \overline{H}_i) = (h_u \oplus T, h_d \oplus \overline{T})$. In this way all the three gauginos receive mass at the same loop level. Clearly the matter fields Q_i are the MSSM matter superfields. The model field content is summarized in table 5.1.

The extra triplets must be heavy, but not necessarily of order the GUT scale. Dangerous operators mediating the proton decay are actually forbidden by the unbroken Z_2 symmetry. In order to correctly mediate SUSY breaking the triplets have not to be decoupled at the scale where X gets vev. We can set a lower bound on M_T asking for gauge coupling unification to be achievable in the scenario.

At the GUT scale the superpotential is given by two contributions

$$W = W_{\mathrm{MSSM}} + W_\Phi \,, \tag{5.4}$$

with

$$\begin{aligned} W_{\mathrm{MSSM}} &= \lambda_U \, q \, h_u \, u^c - \lambda_D \, q \, h_d \, d^c - \lambda_E \, l \, h_d \, e^c + \mu \, h_u \, h_d \,, \\ W_\Phi &= h_0 \, h_u \, h_d \, \Phi + \frac{1}{3} \eta \, \Phi \, \Phi \, \Phi + kX \, \Phi \, \Phi + M_T \, T \, \overline{T} + h_t \, T \, \overline{T} \, \Phi . \end{aligned} \tag{5.5}$$

As anticipated the extra Z_2 discrete symmetry prevents the triplets to couple with matter fields. When X develops its vev and breaks SUSY, Φ communicates such a breaking to the MSSM fields giving rise to the standard MSSM soft potential

$$\begin{aligned} -\mathcal{L}_{\text{soft MSSM}} &= A_U\tilde{u}^c\tilde{q}h_u + A_D\tilde{d}^c\tilde{q}h_d + A_E\tilde{e}^c\tilde{l}h_d + Bh_uh_d + \text{h.c.} \\ &+m^2_{h_u}h_u^\dagger h_u + m^2_{h_d}h_d^\dagger h_d + \tilde{m}^2_q\tilde{q}^\dagger\tilde{q} + \tilde{m}^2_{u^c}\tilde{u}^{c\dagger}\tilde{u}^c \\ &+\tilde{m}^2_l\tilde{l}^\dagger\tilde{l} + \tilde{m}^2_{d^c}\tilde{d}^{c\dagger}\tilde{d}^c + \tilde{m}^2_{e^c}\tilde{e}^{c\dagger}\tilde{e}^c \\ &+\frac{1}{2}M_a\lambda_a\lambda_a + \text{h.c.}\,, \end{aligned} \tag{5.6}$$

that is summed to the SUSY invariant scalar potential

$$V_{\text{SUSY}} = \left|\left(\left.\frac{\partial W_{\text{MSSM}}}{\partial\Omega_j}\right|_{\Omega_j=\tilde{\Omega}_j}\right)\right|^2 + \sum_a\left|\tilde{\Omega}_j^\dagger T_a\tilde{\Omega}_j\right|^2\,, \tag{5.7}$$

where Ω_j are the various superfields of the theory. Below the scale M (M_T) Φ ($T,\overline{T}$) decouple and we are left with the standard MSSM. For this reason we have not included terms involving these fields in equation 5.6. We neglect threshold effects arising from the decoupling of these heavy fields. The soft terms structure is strongly correlated to all the superpotential parameters, because of the Y-GM mechanism. This in turn gives rise to a novel spectrum, that is the subject of next sections.

5.2 Phenomenological predictions

This section is devoted to the phenomenological predictions of our model. The mass spectrum is peculiar to this singlet Yukawa-gauge mediation realization and quite different from that obtained in LGM frameworks.

5.2.1 Spectrum

We have already anticipated that the spectrum of the theory is quite uncommon: indeed the third family sfermions are in general heavier than those of the first two families, a feature owed to the role played by the Yukawa interactions in the SUSY breaking mediation mechanism. A similar hierarchy has been recently considered in [112]. The structure of the low energy spectrum is determined by the RG evolution of the boundary contributions generated when integrating out the first messenger, Φ, at its scale M. In the following sections we will show that in order to be phenomenologically acceptable our model requires $M \lesssim 10^{14}$. Thus for simplicity in the following we assume that $M \sim M_T < M_{GUT}$ and leave the possibility of a low energy SUSY breaking realization to further studies. In particular it could be interesting to connect the superfield Φ to the generation of neutrino masses, see section 5.3. In addition we will assume that the democratic contribution to sfermion mass matrices arising because of the gravitino is negligible. We will comment on this assumption in section 5.2.6. Moreover we consider anomaly mediation contributions [47] to be subleading.

To deeper analyze the implant of the theory and the differences with respect to standard scenarios we should remember that the MSSM fields couple in a quite peculiar way to the SUSY breaking vev. In particular at one loop level no soft mass terms are generated. The first contributions appear at two loops, where both gauginos and sfermions get a mass term. The structure of the two terms (see section C.3) is

$$M_{\rm gaugino} = {\rm Lp}^2 A^{i,j} g_i^2 h_j^2 B_\phi \,, \qquad\qquad m^2_{\rm sfermion} = {\rm Lp}^2 B^r \mathcal{Y}_r h_0^2 B_\phi^2 \,, \tag{5.8}$$

where $B_\phi = F/M \ll M$, ${\rm Lp} = (4\pi)^{-2}$ is a loop factor, $\mathcal{Y}_{u(d)} = Y^\dagger_{u(d)} Y_{u(d)}, Y_{u(d)} Y^\dagger_{u(d)}$ for right and left up (down) quark respectively and $h_j = h_0,\ h_t$. With respect to LGM there is an effective extra loop factor in gaugino masses, keeping them smaller than third family sfermions. Indeed sfermion masses are proportional to the Yukawa couplings and thus they result heavily suppressed in the case of the first two families. Anyway the contribution of LGM coming from three loop graphs become competing or even more important of the Yukawa mediated two loop one in this case. Such a contribution yields terms of the form

$$m^2_{\rm sfermion} = {\rm Lp}^3 \Big(C_1^{i,j,k} r_i^2 r_j^2 r_k^2 + C_2^{i,j,k} r_i^2 r_j^2 \mathcal{Y}_k + C_3^{i,j,k} r_i^2 \mathcal{Y}_j \mathcal{Y}_k \Big) B_\phi^2 \,, \tag{5.9}$$

where $r_i = g_{j=1,2,3},\ h_0,\ h_t,\ \eta$. The three loop contributions happen to be competing with the two loop one only in the case of third family down and lepton sfermions, that are characterized by small Yukawa couplings compared to the top one. On the contrary they are dominant in the case of the first two families for all the flavours, since in that case the Yukawa couplings give rise to negligible terms. Consequently the first two families are essentially degenerate in mass, a feature preserved even after the evolution to low energies. For what concerns the first two families the hierarchy with respect to gaugino masses is milded because of the extra loop factor, but still present.

Below the scale M we are left with the particle content of the MSSM, thus the evolution of soft terms can be simply obtained using the beta functions reported in [113].

Since our theory predicts the presence of a split spectrum in which sfermions and higgsinos are much heavier than gauginos, we improved the calculation of gaugino masses by integrating out sfermions and higgsinos at their mass scales and then determining new evolution equations. Such a procedure is explained in details in next subsection to show that gauge coupling unification predictions are not affected by this spectrum. In figure 5.2 we report an example of low energy spectrum obtained within our framework.

5.2.2 Gauge coupling unification

In this section we briefly discuss how gauge coupling unification is realized in our scenario. Notice that in our model unification is not mandatory.

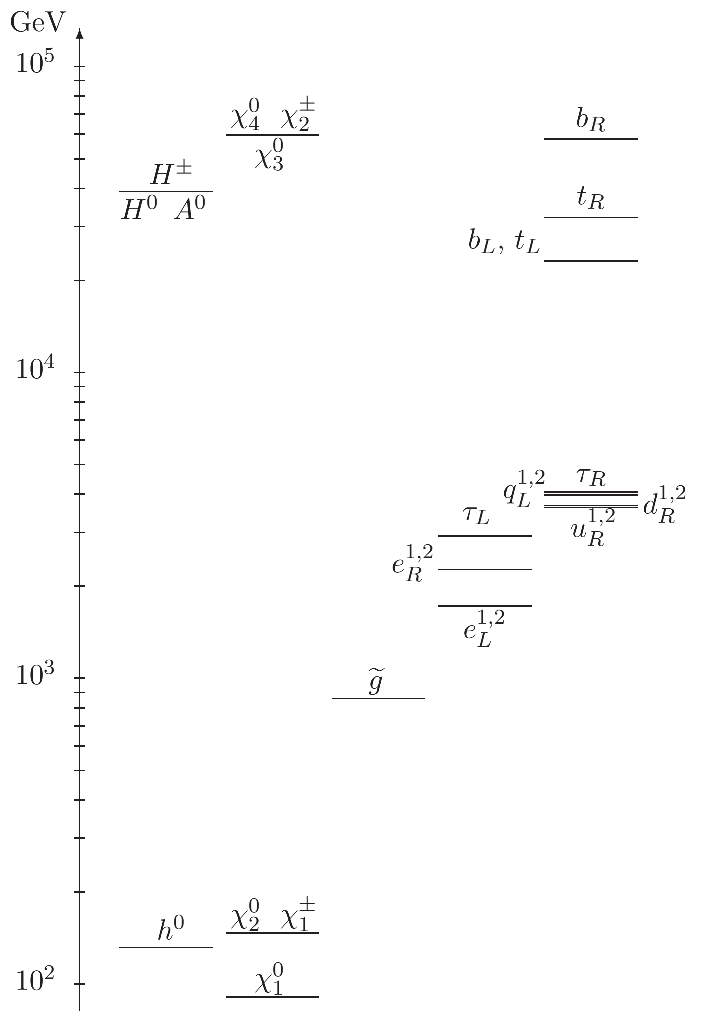
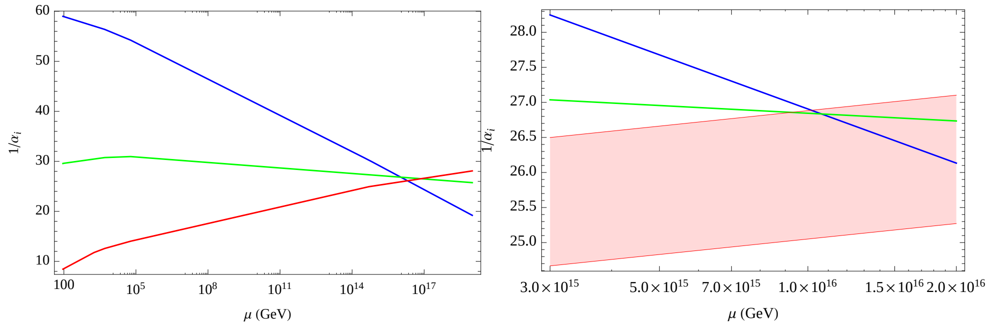


Figure 5.2: Typical spectrum arising in the framework. The superpotential parameters are $h_0 = 0.80$, $h_t = 1.25$, $B_\phi = 6.6 \times 10^6$ GeV, $\eta = 0.1$.



(a) Evolution of gauge couplings (b) Unification scale under a magnifying glass

Figure 5.3: Gauge coupling evolution.

We take into account the one loop RGEs with a series of intermediate scales dictated by the spectrum shown in figure 5.2. Here we just report the main results while all the detailed calculations can be found in appendix C.1.

Neglecting the EW scale, characterized by the SM degrees of freedom, in the spectra of figure 5.2 we identify four SUSY scales: the lower one, $M^{(4)}_{\rm SUSY} \sim$ hundreds of GeV, is that of $SU(2)_L \times U(1)_Y$ gauginos. Then follow the gluino scale, $M^{(3)}_{\rm SUSY}$, that LHC constraints fix around the TeV, and the light sfermions one $M^{(2)}_{\rm SUSY}$. The first scale, $M^{(1)}_{\rm SUSY}$, corresponds to the the heavy third family sfermions, higgsinos and heavy scalars. Finally we have to consider the scale of the heavy triplets M_T. The evolution is then computed taking as inputs the low energy gauge coupling values and the result is shown in figure 5.3a. In order to test gauge coupling unification we show in figure 5.3b a plot at energies around unification scale under the magnifying glass. The red strip represents the region for the strong coupling within three sigmas of the experimental value: thus we can easily see that it is compatible with unification.

5.2.3 EWSB and the Higgs boson

We already commented that the spectrum is essentially divided in two different sets. While the gauginos are expected to lie at lower energy ($M^{(4,3)}_{\rm SUSY} \gtrsim 100\,{\rm GeV} \div {\rm TeV}$), all the other sparticles (namely the sfermions and the higgsinos) are pushed up to multi-TeV energies, $M^{(2,1)}_{\rm SUSY}$. Such a spectrum might be easily confused with a split SUSY one, but it actually differs from that because in our picture the higgsinos are heavy and the lightest neutralino is essentially a bino or wino.

The light Higgs boson mass is obtained by the standard procedure used in SUSY scenarios, by decoupling heavy particles in turn and computing their threshold effects as the energy decreases. Discussing gauge coupling unification we identified four scales at which sequentially heavy particles decouple. At the highest scale below GUT scale, $M^{(1)}_{\rm SUSY}$, we decouple third family sfermions, higgsinos and heavy

Higgs scalars. Indeed only one linear combination of h_u and h_d

$$h = h_d \cos\alpha + i\sigma_2 h_u^* \sin\alpha, \tag{5.10}$$

remains part of the theory, corresponding to the light Higgs boson. The orthogonal combination

$$H = -h_d \sin\alpha + i\sigma_2 h_u^* \cos\alpha, \tag{5.11}$$

corresponds to an $SU(2)_L$ heavy doublet, whose mass is roughly fixed by the soft terms. The two masses are given at $M^{(1)}_{\rm SUSY}$ by

$$m^2_{h,H}(M^{(1)}_{\rm SUSY}) \;=\; \frac{1}{2}\left[m^2_{h_u} + m^2_{h_d} + 2|\mu|^2 \pm (m^2_{h_d} - m^2_{h_u})/\cos 2\alpha\right] . \tag{5.12}$$

It is clear that m_h^2 is the quadratic term of the Higgs scalar potential that has to run to negative values at the EW scale to induce EWSB. On the contrary m_H^2 has to be greater than zero being the mass of the heavy doublet. The parameter α in equations 5.10, 5.11 and 5.12 is meant to be the mixing angle between the light and heavy states before EWSB, defined by

$$\tan 2\alpha = -\frac{2B}{(m^2_{h_u} - m^2_{h_d})} . \tag{5.13}$$

The light state coincides roughly with the SM Higgs boson and it is responsible of EWSB. Its mixing with H induces a negligible vev v_H, thus the mixing angle α and the ratio $\beta = v_u/v_d$ do essentially coincide. In our framework $B\mu$ is tightly connected to μ and to the soft $m^2_{H_{u,d}}$, but this can lead to some inconsistencies in obtaining a successful EWSB in the model. The difficulties can be reasoned by means of a contemporary solution to the $\mu - B\mu$ problem: an example can be given by eliminating in the superpotential the explicit μ term and adding an extra part inspired by [114]. One can easily see that, by means of a suitable choice of the parameters, it is possible to make the framework viable.

The effective theory below $M^{(1,2)}_{\rm SUSY}$ contains the SM matter content and the three gauginos. The doublet h is nothing but the usual SM Higgs field whose scalar potential, $V(h)$, is characterized by the quartic coupling λ. As usual λ is given by the SUSY tree level contributions

$$\lambda_{\rm SUSY} = \frac{1}{4}\Big(g_2^2(M^{(1)}_{\rm SUSY}) + \frac{3}{5}g_1^2(M^{(1)}_{\rm SUSY})\Big)\cos^2 2\beta(M^{(1)}_{\rm SUSY}), \tag{5.14}$$

and the one-loop threshold contribution obtained integrating out heavy sfermions via box and triangle one-loop diagrams [115]. The dominant contribution arises from diagrams involving the stop, even if in our case the stop is the heaviest sfermion, and it is given by

$$\delta\lambda = \frac{3\lambda_t^4(M^{(1)}_{\rm SUSY})}{16\pi^2}\left(2\frac{X_t^2}{{M^{(1)}_{\rm SUSY}}^2} - \frac{X_t^4}{6{M^{(1)}_{\rm SUSY}}^4}\right) , \tag{5.15}$$

where $X_t = A_t(M_{\rm SUSY}^{(1)}) + \mu(M_{\rm SUSY}^{(1)})\cot\beta(M_{\rm SUSY}^{(1)})$.

The quartic coupling λ is then evolved through the RGEs given in appendix C.2 up to the EW scale, where the one loop effective Coleman Weinberg potential [116] is computed

$$V(h) = m_h^2|h|^2 + \frac{1}{2}\lambda|h|^4 + \frac{1}{16\pi^2}\sum_{k=1}^{5} a_k\mathcal{M}_k^4\left(\ln\frac{\mathcal{M}_k^2}{\mu^2} + b_k\right)$$

where

$$\mathcal{M}_1^2 = m_h^2 + \frac{\lambda}{2}h^2\,, \qquad \mathcal{M}_2^2 = m_h^2 + \frac{\lambda}{6}h^2\,, \qquad \mathcal{M}_3^2 = \frac{1}{4}g_2^2h^2\,,$$
$$\mathcal{M}_4^2 = \frac{1}{4}\Big(g_2^2 + \frac{3}{5}g_1^2\Big)h^2\,, \qquad \mathcal{M}_5^2 = m_h^2 + \frac{\lambda_t^2}{2}h^2\,,$$

and

$$a_1 = \frac{1}{4}\,, \qquad a_2 = \frac{3}{4}\,, \qquad a_3 = -3\,, \qquad a_4 = \frac{3}{2}\,, \qquad a_5 = \frac{3}{4}\,,$$
$$b_1 = b_2 = b_3 = -\frac{3}{2}\,, \qquad b_4 = b_5 = -\frac{5}{6}\,.$$

The procedure used to find the minimum of the Higgs potential is the one depicted in [117]. The running mass of the Higgs boson is defined as the second derivative of the potential evaluated at the minimum, namely

$$\hat{m}_h^2 = \left.\frac{\partial^2 V(h)}{\partial^2 h}\right|_{\langle h\rangle = v_W}\,, \tag{5.16}$$

where v_W is the EW scale. Finally the physical Higgs boson mass is obtained by computing the pole mass. The relation between the Higgs running mass and the pole one can be evaluated as follows. We can write

$$M_h^2 = \hat{m}_h^2 + \Delta\Pi\,, \tag{5.17}$$

where M_h^2 is the pole propagator mass, $\hat{m}_h^2$ is the running Higgs mass defined in 5.16 and $\Delta\Pi$ is the difference of the renormalized self energy calculated at the pole mass and at zero momentum: $\Delta\Pi \equiv \Pi(p^2 = M_h^2) - \Pi(p^2 = 0)$. $\Delta\Pi$ receives contribution from many sources, the top contribution being the most relevant.

In the parameter point corresponding to figure 5.2 the pole mass of the light Higgs boson is

$$M_h^2 = 132\,\text{GeV}\,, \tag{5.18}$$

that is too large taking into account the recent Higgs boson discovery [3,4]. Generally speaking, the minimal model presented here faces the problem of a Higgs boson mass heavier than what expected, lying in the range $129 \div 135\,\text{GeV}$ in most of the parameter space. A tempting solution to such phenomenological issue is the extension of the framework by means of an extra light singlet, going to a NMSSN like scenario. In this case the prediction on the Higgs boson mass can be clearly slightly lowered, thus satisfying the recent experimental results.

5.2.4 EWPT

As in any theory providing a SM extension we have to check the consistence of our model through the oblique corrections classified [118–122] by means of the three parameters T, S, U, written in terms of the physical gauge boson vacuum polarizations as [123]

$$\begin{aligned} T &= \frac{4\pi}{e^2 c_W^2 m_Z^2}\left[A_{WW}(0) - c_W^2 A_{ZZ}(0)\right] , \\ S &= 16\pi \frac{s_W^2 c_W^2}{e^2}\left[\frac{A_{ZZ}(m_Z^2) - A_{ZZ}(0)}{m_Z^2} - A'_{\gamma\gamma}(0) - \frac{(c_W^2 - s_W^2)}{c_W s_W} A'_{\gamma Z}(0)\right] , \\ U &= -16\pi \frac{s_W^2}{e^2}\left[\frac{A_{WW}(m_W^2) - A_{WW}(0)}{m_W^2} - c_W^2 \frac{A_{ZZ}(m_Z^2) - A_{ZZ}(0)}{m_Z^2} + \right. \\ & \left. -s_W^2 A'_{\gamma\gamma}(0) - 2 s_W c_W A'_{\gamma Z}(0)\right] , \end{aligned} \tag{5.19}$$

where s_W, c_W are sine and cosine of θ_W and e is the electric charge. Roughly speaking for any $SU(2)_L$ doublet T, S, U are sensitive to the mass splitting of the doublet components and thus vanish in the limit of degenerate masses [124]. As it can be easily checked by looking at the spectra shown in figure 5.2 the SUSY breaking scale is so high that the components of the $SU(2)_L$ doublets happen to be still almost degenerate after the EWSB, and right and left sfermion mixing is thus completely negligible. Contemporaneously at the EW scale the wino and the light chargino form a degenerate doublet, so the EW parameters do not receive any new contribution arising from new particles, the unique contribution being that of the SM like Higgs, which, given the new experimental evidence, is in perfect agreement with the data [125].

5.2.5 Flavour constraints

A detailed analysis of flavour processes is beyond the purposes of this work. Intuitively such processes should not further constrain the model because of the very heavy sparticle spectrum, but we should take care of them because of the Yukawa structure of the SUSY breaking mediation mechanism. Being the third family the heaviest and the first two lighter and almost degenerate, we are in presence of a hierarchical squark spectrum inverted with respect to the discussion developed in [126]. However we may use their formalism to estimate the contribution to $\Delta F = 1, 2$ processes.

As an example we may consider the gluino loop contributions in the down sector. The latter in the case of $d_i^L \to d_j^L$ ($\Delta F = 1$) and $d_i^L \leftrightarrow d_j^L$ ($\Delta F = 2$) may be parametrized as

$$
\begin{aligned}
A(\Delta F=1) &= \mathcal{W}_{d_i^L \tilde{d}_I}\, f\left(\frac{\tilde{m}^2_{d_I}}{M_3^2}\right) \mathcal{W}^*_{d_j^L \tilde{d}_I} \\
A(\Delta F=2) &= \mathcal{W}_{d_i^L \tilde{d}_I}\, \mathcal{W}_{d_i^L \tilde{d}_J}\, g\left(\frac{\tilde{m}^2_{d_I}}{M_3^2}, \frac{\tilde{m}^2_{d_J}}{M_3^2}\right) \mathcal{W}^*_{d_j^L \tilde{d}_I}\, \mathcal{W}^*_{d_j^L \tilde{d}_J} \,,
\end{aligned}
\tag{5.20}
$$

where f and g are loop functions, in particular $g(x,y) = g(x) - g(y)/(x-y)$. $\mathcal{W}$ diagonalizes the full 6×6 down squark mass matrix in the basis in which the down quarks are diagonal, namely $\mathcal{M}_D^2$. In our specific framework $\mathcal{W}$ is very simple and block diagonal because the soft terms are so heavy that we can neglect left-right squark mixing[2]. In particular this means that gluino loops cannot mediate $\Delta F = 1$ processes, like $b \to s\gamma$, and in the following we will concentrate only on $\Delta F = 2$ processes. If we assume that λ_U and λ_D have a Froggatt Nielsen symmetric structure [127] – thus implying that $V_L^u \sim V_L^d \sim V_R^u \sim V_R^d \sim \mathcal{O}(V_{CKM})$ – the structure of $\mathcal{M}^2_{D_{L,R}}$ at M in our specific framework is roughly given by

$$
\mathcal{M}^2_{D_{L,R}} = \left[\mathrm{Lp}^2 B^r \mathcal{Y}_r h_0^2 + \mathrm{Lp}^3 \Big(C_1^{i,j,k} r_i^2 r_j^2 r_k^2 + C_2^{i,j,k} r_i^2 r_j^2 \mathcal{Y}_k + C_3^{i,j,k} r_i^2 \mathcal{Y}_j \mathcal{Y}_k\Big)\right] B_\phi^2
\tag{5.21}
$$

where the notation is the same of equations 5.8 and 5.9. Equation 5.21 may be rewritten as

$$
\mathcal{M}^2_{D_{L,R}} = \frac{M_3^2}{\mathrm{Lp}^2}\left[\mathcal{Y}\frac{h_0^2}{g_3^4 h_t^4} + \mathrm{Lp}\left(C_1^{i,j,k}\frac{r_i^2 r_j^2 r_k^2}{g_3^4 h_t^4} + C_2^{i,j,k}\frac{r_i^2 r_j^2}{g_3^4 h_t^4}\mathcal{Y}_k + C_3^{i,j,k}\frac{r_i^2}{g_3^4 h_t^4}\mathcal{Y}_j\mathcal{Y}_k\right)\right],
$$

where $\mathcal{Y} = B^r \mathcal{Y}_r$. Among the terms that arise at three loop level the dominant one is that proportional to C_1. The diagonal two loop entries dominate over the three loop ones if

$$
(\mathcal{Y})_{ii} > \mathrm{Lp}\, C_1^{i,j,k}\frac{r_i^2 r_j^2 r_k^2}{h_0^2} \sim \mathrm{Lp}\, c_g\, g^4 \sim 10^{-1}\,,
\tag{5.22}
$$

where $c_g \sim \mathcal{O}(10)$, that clearly is realized only for the third family. Thus the two loop term controls the heavy third familiy squark masses and the off diagonal entries, while the first three loop term dominates the degenerate two lightest families. For

[2]The order of magnitude of the various terms in the squark mass matrix $\mathcal{M}_D^2$ is $\mathcal{M}^2_{D_L} \sim \mathcal{M}^2_{D_R} \sim \widetilde{m}^2$ while $\mathcal{M}^2_{D_{LR}} \sim v_W \widetilde{m}$, where $\widetilde{m} = \mathrm{Lp}\, B_\phi$. Thus the mixing between left and right down squarks is roughly given by $v_W/B_\phi \mathrm{Lp} \ll 1$ and may be neglected.

the following discussion we are interested in the ratios

$$
\begin{aligned}
\Delta_{12}^{L,R} &= \frac{(\mathcal{M}^2_{D_{L,R}})_{12}}{(\mathcal{M}^2_{D_{L,R}})_{22}} \sim \frac{1}{c_g \mathrm{Lp}} \frac{(\mathcal{Y})_{12}}{g^4} \sim 10\lambda_C^5 \frac{m_b^2}{m_t^2}\frac{\tan\beta^2}{g^4} \sim 10^{-3} \div 10^{-2}\,, \\
\Delta_{23}^{L,R} &= \frac{(\mathcal{M}^2_{D_{L,R}})_{23}}{(\mathcal{M}^2_{D_{L,R}})_{33}} \sim \frac{(\mathcal{Y})_{23}}{(\mathcal{Y})_{33}} \sim \lambda_C^2 \sim 10^{-2}\,, \\
\Delta_{13}^{L,R} &= \frac{(\mathcal{M}^2_{D_{L,R}})_{13}}{(\mathcal{M}^2_{D_{L,R}})_{33}} \sim \frac{(\mathcal{Y})_{13}}{(\mathcal{Y})_{33}} \sim \lambda_C^3 \sim 10^{-3}\,.
\end{aligned} \tag{5.23}
$$

In equations 5.23 m_t (m_b) are the top (bottom) quark mass, $\lambda_C \approx 0.2$ the Cabibbo angle and as usual $\tan\beta = v_u/v_d$. According to our considerations $\mathcal{W}$ has the structure

$$
\mathcal{W} = \begin{pmatrix} \mathcal{W}^L & 0 \\ 0 & \mathcal{W}^R \end{pmatrix}. \tag{5.24}
$$

with

$$
\mathcal{W}_{L,R} \simeq \begin{pmatrix} \cos\theta^L_{12} & \sin\theta^L_{12} & \Delta^L_{13} \\ -\sin^L\theta_{12} & \cos\theta^L_{12} & \Delta^L_{23} \\ \Delta^L_{23}\sin\theta^L_{12} - \Delta^L_{13}\cos\theta^L_{12} & -\Delta^L_{23}\cos\theta^L_{12} - \Delta^L_{13}\sin\theta^L_{12} & 1 \end{pmatrix} \tag{5,25}
$$

where

$$
\tan 2\theta_{12} = 2\frac{(\mathcal{M}_{D_L})_{22}\Delta^L_{12}}{(\mathcal{M}_{D_L})_{22} - (\mathcal{M}_{D_L})_{11}} \sim \frac{(\mathcal{Y})_{12}}{(\mathcal{Y})_{22}}\,. \tag{5.26}
$$

The generalization to up squarks and charged sleptons is trivial.

Following [126] in our model we have

$$
A(\Delta F = 2) \sim g^{(1)}(x)(\hat{\delta}^{LL}_{ij})^2 + \frac{x^2}{3!}g^{(3)}(x)(\delta^{LL}_{ij})^2\,, \tag{5.27}
$$

with $x = \tilde{m}^2_L/M_3^2$ and we can set $\tilde{m}^2_{d_{L,R}} \sim \tilde{m}^2_{s_{L,R}} \sim \tilde{m}^2_{L,R}$ because of their approximate degeneracy. The δ parameters are defined as

$$
\begin{aligned}
\hat{\delta}^{LL}_{ij} &= \mathcal{W}^L_{d_i\tilde{d}}\mathcal{W}^{L*}_{d_j\tilde{d}} + \mathcal{W}^L_{d_i\tilde{s}}\mathcal{W}^{L*}_{d_j\tilde{s}} = \delta_{ij} - \mathcal{W}^L_{d_i\tilde{b}}\mathcal{W}^{L*}_{d_j\tilde{b}} \\
&\simeq \delta_{ij} - \delta_{33} - \left[(\delta_{ik}\delta_{j3} + \delta_{jk}\delta_{i3})\Delta^L_{k3}\right]_{k=1,2}\,, \\
\delta^{LL}_{ij} &= (\delta_{i2}\delta_{j3} + \delta_{j2}\delta_{i3})\Delta^L_{12}\,.
\end{aligned} \tag{5.28}
$$

Clearly the term that depends on $\hat{\delta}^{LL}_{ij}$ controls $B_{d,s} - \overline{B}_{d,s}$ oscillation, while that on δ^{LL}_{ij} controls $K - \overline{K}$. In the limit $x \gg 1$ it turns out that $g^{(1)}(x), g^{(3)}(x) \sim 1/x, 1/x^3$ respectively [128], and we get

$$
A(\Delta F = 2) = \mathcal{F}^{LL}_{ij} \sim \alpha_s C_q \left(\frac{M_3^2}{\tilde{m}^2}\right)\left[(\hat{\delta}^{LL}_{ij})^2 + \frac{1}{6}(\delta^{LL}_{ij})^2\right]\,, \tag{5.29}
$$

where C_q is a colour factor. From [129] we have

$$
\Delta M_F = M_F f_F^2 B_F \frac{8}{3M_3^2}\mathcal{F}^{LL}_{ij}\,, \tag{5.30}
$$

B_F	M_F (GeV)	f_F (GeV)	B_F
B_d	5.2795	0.1928 ± 0.0099	1.26 ± 0.11
B_s	5.3664	0.2388 ± 0.0095	1.33 ± 0.06
K	0.497614	0.1558 ± 0.0017	0.725 ± 0.026

Table 5.2: *Properties of neutral mesons [130]: in the table the values for parameters in equation 5.30 are given.*

where B_F is a parameter of order 1, f_F is the decay constant of the meson $F = K, B_d, B_s$ and ij the transition responsible of its oscillations. By combining equations 5.28, 5.29 and 5.30 we get

$$\begin{aligned}
\Delta M_K &= C_q \alpha_s^2 M_K f_K^2 B_K \frac{4}{9\tilde{m}^2}(\Delta_{12}^L)^2 \sim M_K \left(\frac{f_K^2}{\text{GeV}^2}\right) B_K (10^{-14} \div 10^{-16})\,, \\
\Delta M_{B_d} &= C_q \alpha_s^2 M_{B_d} f_{B_d}^2 B_{B_d} \frac{8}{3\tilde{m}^2}(\Delta_{13}^L)^2 \sim M_{B_d} \left(\frac{f_{B_d}^2}{\text{GeV}^2}\right) B_{B_d} 10^{-15}\,. \\
\Delta M_{B_s} &= C_q \alpha_s^2 M_{B_s} f_{B_s}^2 B_{B_s} \frac{8}{3\tilde{m}^2}(\Delta_{23}^L)^2 \sim M_{B_s} \left(\frac{f_{B_s}^2}{\text{GeV}^2}\right) B_{B_s} 10^{-13}\,.
\end{aligned}$$

By comparing our results and the experimental bounds as reported in table 5.2 we see that the flavour processes mediated by the sfermions are a few orders of magnitude below the experimental bounds. Similar results are expected in the case of loops in which circulate other superpartners.

We also briefly comment about the possibility of gravity mediated flavour changing processes. The latter are generated by the complete democracy of gravity mediated interactions in flavour space and thus can be neglected only if they are subleading with respect to the other contributions. Such a constraint imposes bounds on the mass of the gravitino that will be taken into account in Section 5.2.6.

B_F	$\lvert\Delta M_F^{exp}\rvert$ (GeV)	$\lvert\Delta M_F^{res}\rvert$ (GeV)
B_d	$(3.337 \pm 0.006) \times 10^{-13}$	$< 10^{-16}$
B_s	$(1.170 \pm 0.008) \times 10^{-11}$	$< 10^{-14}$
K	$(3.500 \pm 0.006) \times 10^{-11}$	$< 10^{-16}$

Table 5.3: *Properties of neutral mesons [130] and the model predictions. The last column report our rough estimation of the meson oscillation mass splitting ΔM_F^{res} taking as reference values the spectrum given in figure 5.2.*

5.2.6 Cosmology

In LGM the LSP is usually the gravitino [48]. Here we discuss if our framework behaves as the minimal case or not, allowing the bino or the wino to be the LSP.

The gravitino takes mass through gravitational interactions: their coupling strenght is given by the inverse reduced Planck mass and, assuming a vanishing cosmological constant, the mass $m_{3/2}$ is

$$m_{3/2} = \frac{F_0}{\sqrt{3}M_P}\,, \tag{5.31}$$

where $M_P = (8\pi G_N)^{-1/2}$ is the reduced Planck mass, F_0 is the total contribution of the F-term SUSY breaking vev to the vacuum energy, thus $V = F_0^2$ in the minimum. Actually the effective F-term vev felt by the messenger Φ is related to F_0 by means of the superpotential interaction $kX\Phi\Phi$, thus being $F_0 = F/k$. The ratio among the two quantities directly reflects the way in which SUSY breaking is mediated, and in our case it is simply given by the coupling constant $k \lesssim 1$ to preserve perturbativity at high energy scale. The gravitino mass can thus be rewritten as

$$m_{3/2} = \frac{B_\phi M}{k\sqrt{3}M_P}\,. \tag{5.32}$$

To point out the nature of the LSP it turns useful rewriting the gravitino mass in terms of the wino one. Reminding that

$$M_2 = \mathrm{Lp}^2 g_2^2 h_0^2 B_\phi\,, \tag{5.33}$$

we get

$$m_{3/2} = \Big(\frac{M_2(M)}{250\,\mathrm{GeV}}\Big)\Big(\frac{0.7}{g_2(M)}\Big)^2\frac{1}{h_0^2(M)}\frac{0.9}{k(M)}3.4\times 10^{-12}M, \tag{5.34}$$

where M is the Φ superfield mass scale and thus gives rise to the SUSY breaking boundary conditions. As one can easily see from the formula above the gravitino is the LSP if a relatively low energy SUSY breaking takes place. If the scale of SUSY breaking is of order $M \gtrsim 10^{14}\,\mathrm{GeV}$ than the relative weight of the adimensional parameter entering into equation 5.34 establishes whether the LSP is the gravitino or neutralino, while for larger values of M, such as the GUT scale, the gravitino is surely not the LSP of the framework. At first let us briefly comment on this possibility.

lf R-parity is conserved and the neutralino (wino or bino) is the LSP of the model the decay of the gravitino must happen before Big Bang Nucleosynthesis (BBN) in order not to destroy the successful predictions of BBN itself [131]. In particular if gravitino decays have to be completed before BBN at $t \sim 1s$ we must have

$$m_{3/2} \gtrsim 10\,\mathrm{TeV} \Longrightarrow M \gtrsim 10^{15} \div 10^{16}\,\mathrm{GeV}\,, \quad B_\phi \lesssim 10^7 \div 10^8\,\mathrm{GeV}\,. \tag{5.35}$$

Such a large gravitino mass would give rise to a very large contribution to sfermion mass matrices that would induce dangerous FCNC and our hypothesis to neglect gravitino contribution to sfermion masses would fail.

Consequently in our scenario the gravitino has to be the LSP, condition that can be rephrased as $M \lesssim 10^{14}$ GeV. On the other side bounds on gluino mass impose $B_\phi > 10^6$ GeV and then the perturbativity of the ratio F/M asks at least for $M > 10^7$ GeV implying $m_{3/2} \gtrsim 1\,\text{keV}$. Light gravitino DM scenarios are therefore not realizable in our framework, while the possibility of a superWIMP DM is left open [16]. The analysis of the gravitino as superWIMP candidate is left for further studies.

5.3 R-parity breaking and neutrino masses

In section 5.2.6 we deduced that the allowed range for M is $10^7\,\text{GeV} < M \lesssim 10^{14}\,\text{GeV}$ in order not to affect FCNC processes and gluino mass bounds. One very interesting possibility is promoting Φ to be both the SUSY breaking messenger and the source of light neutrino masses, through its fermionic component. This picture shares features both with the νGMSB model [132] and the bilinear R-parity breaking scenarios [133–140], since Φ is a singlet superfield. As in [132] light neutrinos receive type I seesaw like mass contribution arising from the explicit R-parity breaking term – the coupling $y_i l_i h_u \Phi$ – yielding

$$m_\nu^I \sim \frac{v_u^2}{M} y_i \cdot y_i^T \,, \tag{5.36}$$

where the y_i are thought as three dimensional vector columns. When sneutrinos and the scalar component of Φ acquire a tiny vev a further contribution to left handed neutrino masses is generated. This contribution is nothing but that presents in models with explicit linear R-parity breaking term and discussed in details in [133]. Using their notation and assuming a spectrum similar to that given in figure 5.2 the second contribution to neutrino masses is given by

$$m_\nu^{Rbr} \sim \frac{g_2^2 + 3/5 g_1^2}{4\mu^2 M_0} \Lambda_i \cdot \Lambda_i^T \,, \tag{5.37}$$

where we have approximated $M_1 \sim M_2 \sim M_0 \ll \mu$ and Λ_i is defined as

$$\Lambda_i = \mu v_i + y_i v_d v_\phi \,, \tag{5.38}$$

with v_i (v_ϕ) is the sneutrino (Φ) vev. As long as v_i and y_i are disaligned[3], neglecting the one loop contributions, the effective light neutrino mass matrix has two non vanishing eigenvalues and lepton mixing is completely determined.

This scenario is quite appealing for its predictivity in neutrino sector and we leave a detailed analysis to a a forthcoming analysis. Notice that in this case a late decaying gravitino should be the DM candidate [141].

[3]This may be realized because of the sneutrino soft mass terms that are not aligned to $y_i y_i^T$. Indeed the soft sneutrino masses receive a contribution both proportional to y_i from $y_i l_i h_u \Phi$ and to $\lambda_E^\dagger \lambda_E$ from $(\lambda_E)_{ij} l_i h_d {e^c}_j$.

6

Conclusions

In this thesis we analyzed two models of gauge mediation and the related phenomenology, focusing on the possibilities of detecting their predictions at colliders.

Chapters 2, 3 and 4 were devoted to the analysis of the Tree Level Gauge Mediation (TGM) framework. In chapter 2 we reviewed the general mechanism, that generates sfermion masses at the tree level which are naturally flavour universal since they arise from gauge interactions, and in chapter 3 we focused on its implementation in the simplest situation, based on the $SO(10)$ gauge group. We discussed the embedding of the various MSSM superfields and the generated soft terms, identifying a minimal set of relevant parameters for the phenomenology. The most striking phenomenological features of the full analysis happen to be a nice prediction on the ratio of the sfermion masses, namely $\tilde{m}^2_{10}/\tilde{m}^2_5 = 1/2$, and the possibility of non universal gaugino masses, which anyway satisfy the GUT scale sum rule $5M_1 = 3M_2 + 2M_3$. The most important outcome of the non universality of gaugino masses lies in the possibility of the NLSP of the framework to be a bino like neutralino, a wino like neutralino or the stau, leading to quite different phenomenologies. We then detailedly described the possible searches to be set at the LHC in order to spot the relevant features of the TGM framework and distinguish it from other models of gauge mediation. In particular we do believe that inclusive searches conducted with the CMS razor analysis, eventually upgraded with dilepton searches to fully consider EW gauginos, are a perfect tool for spotting both the prediction on the ratio of sfermion masses and the low energy consequences of the GUT scale sum rule for gauginos. In the case of the stau NLSP, on top of that, one can also combine the razor analysis with the search for heavy charged particles because of the slow decay of the stau to the gravitino. We ended chapter 3 by showing the application of such tools to a set of benchmark points with different type of NLSP.

In chapter 4 we have extended the framework of TGM to the case of extensions of the SM gauge group derived from E_6, a unified group that, besides its interest for other reasons, is strongly motivated by TGM. To be general, we have allowed for the possibility that part of E_6 is broken by boundary conditions in extra dimensions, so that we performed our analysis for an effective theory with a gauge group that is a Rank 5 or Rank 6 subgroup of E_6. Despite the large number of possible gauge groups, we needed to study only three cases, depending on the number of vector messengers that could be one, two or four. As a result we have found that for pure embeddings of MSSM fields we obtain $SU(5)$ invariant (and flavour universal) sfermion masses provided that they are positive: this feature is a pretty robust prediction of TGM. In the case of a Rank 6 subgroup the ratio $\tilde{m}_{10}^2/\tilde{m}_5^2$ remains undetermined in the general case but can be fixed by considering special limits in the parameter space of scalar and F-term vevs to be $1/2$, which is the same prediction obtained in the minimal $SO(10)$ case. In the case of Rank 5 subgroup the ratio is fixed and depends only on the specific form of the $U(1)$ factor orthogonal to the $SU(5)$ embedding the SM. If this is the $U(1)$ subgroup of E_6 that commutes with $SO(10)$ we can obtain $SO(10)$ invariant sfermion masses. Therefore TGM offers an interesting possibility to reproduce the popular CMSSM boundary conditions for sfermion masses in a novel scenario.

Finally in chapter 5 we focused on the idea of Yukawa gauge mediation in what we may define its minimal version. The minimality resides in the small number of extra degrees of freedom with respect to those of the MSSM. A chiral superfield $\hat{\Phi}$, coupling to a F-term vev, is the first messenger of SUSY breaking and due to its singlet nature SUSY breaking happen to be communicated to the MSSM fields in two sequential steps, after the subsequent coupling to a charged superfield sector. Such second messengers are identified with the MSSM Higgs fields with the addition of extra colour triplets that ensure a large enough gluino mass. This framework gives rise to a novel split spectrum that is consistent with the indication arising from the most recent LHC data. The peculiarities of such a spectrum are an inverted hierarchy among the sfermions of the third family and those of the first two and an almost complete pureness of the gaugino components in the lightest neutralinos and chargino. On top of that we have shown that a successful EWSB takes place and that the model poses no threats for both EWPT and flavour Physics constraints. The mass scale of the scalar superpartners is fairly heavy, above few TeVs, and only the gauginos will be soon in the range of LHC searches. We did not analyze the specific signatures of the model presented, but from the features of the predicted spectrum we may easily deduce that few processes could be testable at the LHC, such as excesses in the monojet channel owed to initial state radiation combined with the production of two gluinos.

A

Appendix for Chapter 3.

In this appendix we will present some relevant formulae connected to the discussion tackled in chapter 3 about the minimal model of TGM in GUTs.

A.1 Flavour structure of the superpotential

As we have seen in section 3.2.2, the breaking of $SO(10)$ and SUSY must involve spinorial representations. In particular, the $\mathbf{16}$, $\overline{\mathbf{16}}$ fields acquire a vev M in the scalar, SM singlet component and $\mathbf{16'}$, $\overline{\mathbf{16}}'$ acquire an a vev in the F-term SM singlet component. As in section 3.2.2, we will actually assume for simplicity that only $\mathbf{16'}$ gets an F-term and we further assume that they are the only spinorial representations coupling to matter bilinears. The most general R-parity invariant superpotential bilinear in the matter fields $\mathbf{16}_i + \mathbf{10}_i$ and involving the above fields is then

$$W_2 = h_{ij}\mathbf{16}_i\mathbf{10}_j\mathbf{16} + h'_{ij}\mathbf{16}_i\mathbf{10}_j\mathbf{16'} + \frac{y_{ij}}{2}\mathbf{16}_i\mathbf{16}_j\mathbf{10} + W_2^{\mathrm{NR}}, \tag{A.1}$$

up to a mass term $\mu_{ij}\mathbf{10}_i\mathbf{10}_j$, assumed to be absent to obtain a *pure* embedding of the SM fields in $SO(10)$ representations and to avoid reintroducing the flavour problem [51]. The non renormalizable part is not specified and takes care of fixing the fermion mass ratios to the phenomenologically correct values.

In order to identify the light Yukawa couplings we need to specify better the embedding of the light Higgs fields, deepening the discussion in section 3.2.1. Under the hypothesis of minimal Higgs embedding, $\mathbf{16}_H = \mathbf{16}$, the light h_d can thus be contained in the doublet component of the $\mathbf{16}$, h_d^{16}, in the doublet of the $\mathbf{16'}$, $h_d^{16'}$ or in a $\mathbf{10}$, with the size of the total component in spinorial representations given by $\sin\theta_d$. The field h_d could be in principle also be embedded in a spinorial representation different from $\mathbf{16}$ and $\mathbf{16'}$ and not coupling to the matter bilinears, but we assume that this is not the case. We can use an angle α to measure how h_d is shared by the two spinorial representations:

$$h_d^{16} = \sin\theta_d\cos\alpha\, h_d\,, \quad h_d^{16'} = \sin\theta_d\sin\alpha\, h_d\,. \tag{A.2}$$

From equations A.1 and A.2 we can recover the SM Yukawa couplings $\lambda_{U,D,E}$ and $\hat{\lambda}_{D,E}$ in equation 3.2 as follows:

$$\begin{aligned}
&\lambda_U = \cos\theta_u\, y + \lambda_U^{\rm NR} \\
&\lambda_E = \sin\theta_d(\cos\alpha h + \sin\alpha h') + \lambda_E^{\rm NR} \qquad \lambda_D = \sin\theta_d(\cos\alpha h + \sin\alpha h') + \lambda_D^{\rm NR} \\
&\hat{\lambda}_E = \cos\theta_d\, y + \hat{\lambda}_E^{\rm NR} \qquad\qquad\qquad\qquad\;\; \hat{\lambda}_D = \cos\theta_d\, y + \hat{\lambda}_D^{\rm NR}
\end{aligned} \tag{A.3}$$

where the superscript "NR" denotes a correction vanishing in the limit $W_2^{\rm NR} \to 0$.

Following the previous considerations the simplest possible prediction for the messenger mass parameters $h_{D,L}$ is that they are proportional to the corresponding SM Yukawa couplings: in order to be general we introduce the new parameters c_{Di}, c_{Li}, $i = 1, 2, 3$

$$\begin{aligned}
h_{Di} &= c_{Di}\lambda_{Di}/\sin\theta_d \qquad & h_{Li} &= c_{Li}\lambda_{Li}/\sin\theta_d \\
h'_{Di} &= c'_{Di}\lambda_{Di}/\sin\theta_d \qquad & h'_{Li} &= c'_{Li}\lambda_{Li}/\sin\theta_d\,.
\end{aligned} \tag{A.4}$$

The c_{D_i,L_i} and c'_{D_i,L_i} coefficients can be written as functions of the parameters in equations A.3:

$$\begin{aligned}
c_{L_i} &= \frac{1}{\cos\alpha + \sin\alpha\,\gamma_{Li}} + (c_{L_i})_{\rm NR}, \qquad & c_{D_i} &= \frac{1}{\cos\alpha + \sin\alpha\,\gamma_{Di}} + (c_{D_i})_{\rm NR} \\
c'_{L_i} &= \frac{\gamma_{Li}}{\cos\alpha + \sin\alpha\,\gamma_{Li}} + (c_{L_i})_{\rm NR}, \qquad & c'_{D_i} &= \frac{\gamma_{Di}}{\cos\alpha + \sin\alpha\,\gamma_{Di}} + (c_{D_i})_{\rm NR}\,.
\end{aligned} \tag{A.5}$$

The equations above allow to set an appropriate range for these coefficients. In the limit in which h_d lies in the **16** only ($\alpha = 0$), $c_{D_i,L_i} = 1$ at the renormalizable level. In the limit in which h_d lies in the $\mathbf{16'}$ only ($\alpha = \pi/2$), on the other hand, the parameters c_{D_i,L_i} can be smaller, especially if the parameters $\gamma_{D,L}$ in 3.13 enhance gaugino masses.

A.2 One-loop RGE equations

In this section we shall present the RGEs for the full theory below the GUT scale [113]. In all of the following equations we will use the common definition $t \equiv \ln\mu$ where μ is the renormalization scale.

A.2.1 Gauge couplings

The RGEs for the gauge couplings are

$$(4\pi)^2\frac{{\rm d}g_a}{{\rm d}t} = \beta^{(1)}g_a\,, \tag{A.6}$$

where

$$\beta^{(1)}g_a = g_a^3\sum_R B_a(R) \tag{A.7}$$

and

$$B_3 = \sum_R B_3(R) = -3 + \frac{N_{D^c} + N_D}{2}\,, \tag{A.8a}$$

$$B_2 = \sum_R B_2(R) = 1 + \frac{N_L + N_{L^c}}{2}\,, \tag{A.8b}$$

$$B_1 = \sum_R B_1(R) = \frac{33}{5} + \frac{3}{5}\Big(\frac{1}{3}N_{D^c} + \frac{1}{3}N_D + \frac{1}{2}N_L + \frac{1}{2}N_{L^c}\Big)\,. \tag{A.8c}$$

We notice that integrating out the chiral messengers accounts for an obvious modification for the running of the gauge couplings.

A.2.2 Gaugino masses

The running of gaugino masses is straightforward. In terms of the results obtained for the gauge couplings one has

$$(4\pi)^2 \frac{\mathrm{d}M_a}{\mathrm{d}t} = 2g_a^2 B_a M_a\,. \tag{A.9}$$

A.2.3 Yukawa couplings

When some of the flavours of the heavy chiral messengers get integrated out at their mass scale we simply consider that the corresponding entries of the Yukawa matrices become zero. We note that the part proportional to the gauge coupling does not depend on the number of flavours that are switched on since it is directly related to the specific λ parameter under study. Incidentally we note that if some of the flavours are frozen out this will also act on the meaning of the various traces appearing in the equations.

$$(4\pi)^2 \frac{\mathrm{d}\lambda_U}{\mathrm{d}t} = \lambda_U\Big[\mathrm{Tr}(3\lambda_U^\dagger\lambda_U) + 3\lambda_U^\dagger\lambda_U + \lambda_D^\dagger\lambda_D + \hat{\lambda}_D{}^\dagger\hat{\lambda}_D - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\Big] \tag{A.10a}$$

$$\begin{aligned}(4\pi)^2 \frac{\mathrm{d}\lambda_D}{\mathrm{d}t} &= \lambda_D\Big[\mathrm{Tr}(3\lambda_D^\dagger\lambda_D + 3\hat{\lambda}_D{}^\dagger\hat{\lambda}_D + \lambda_E^\dagger\lambda_E + \hat{\lambda}_E{}^\dagger\hat{\lambda}_E) + 3\lambda_D^\dagger\lambda_D + 3\hat{\lambda}_D{}^\dagger\hat{\lambda}_D + \lambda_U^\dagger\lambda_U\Big]\\ &\quad - \lambda_D\Big[\frac{16}{3}g_3^2 + 3g_2^2 + \frac{7}{15}g_1^2\Big]\end{aligned} \tag{A.10b}$$

$$\begin{aligned}(4\pi^2) \frac{\mathrm{d}\lambda_E}{\mathrm{d}t} &= \Big[\mathrm{Tr}(3\lambda_D^\dagger\lambda_D + 3\hat{\lambda}_D{}^\dagger\hat{\lambda}_D + \lambda_E^\dagger\lambda_E + \hat{\lambda}_E{}^\dagger\hat{\lambda}_E) + 3\lambda_E\lambda_E^\dagger + 3\hat{\lambda}_E\hat{\lambda}_E^\dagger\Big]\lambda_E\\ &\quad - \Big[3g_2^2 + \frac{9}{5}g_1^2\Big]\lambda_E\end{aligned} \tag{A.10c}$$

$$\begin{aligned}(4\pi^2) \frac{\mathrm{d}\hat{\lambda}_D}{\mathrm{d}t} &= \hat{\lambda}_D\Big[\mathrm{Tr}(3\lambda_D^\dagger\lambda_D + 3\hat{\lambda}_D{}^\dagger\hat{\lambda}_D + \lambda_E^\dagger\lambda_E + \hat{\lambda}_E{}^\dagger\hat{\lambda}_E) + 3\hat{\lambda}_D{}^\dagger\hat{\lambda}_D + 3\lambda_D^\dagger\lambda_D + \lambda_U^\dagger\lambda_U\Big]\\ &\quad - \hat{\lambda}_D\Big[\frac{16}{3}g_3^2 + 3g_2^2 + \frac{7}{15}g_1^2\Big]\end{aligned} \tag{A.10d}$$

$$\begin{aligned}(4\pi^2) \frac{\mathrm{d}\hat{\lambda}_E}{\mathrm{d}t} &= \Big[\mathrm{Tr}(3\lambda_D^\dagger\lambda_D + 3\hat{\lambda}_D{}^\dagger\hat{\lambda}_D + \lambda_E^\dagger\lambda_E + \hat{\lambda}_E{}^\dagger\hat{\lambda}_E) + 3\lambda_E\lambda_E^\dagger + 3\hat{\lambda}_E\hat{\lambda}_E^\dagger\\ &\quad - 3g_2^2 - \frac{9}{5}g_1^2\Big]\hat{\lambda}_E\end{aligned} \tag{A.10e}$$

A.2.4 The μ parameter and other bilinear terms in the superpotential

In this section we consider the running of the bilinear parameters. Again if a field has been integrated out it should not enter the running anymore, which can be implemented by setting to zero the corresponding term (essentially its matrix elements in flavour space).

$$(4\pi)^2\frac{\mathrm{d}\mu}{\mathrm{d}t} = \mu\Big[\mathrm{Tr}(3\lambda_D^\dagger\lambda_D + 3\hat{\lambda}_D{}^\dagger\hat{\lambda}_D + 3\lambda_U^\dagger\lambda_U + \lambda_E^\dagger\lambda_E + \hat{\lambda}_E{}^\dagger\hat{\lambda}_E) - 3g_2^2 - \frac{3}{5}g_1^2\Big] \tag{A.11a}$$

$$(4\pi)^2\frac{\mathrm{d}M_D}{\mathrm{d}t} = 2\hat{\lambda}_D\Big(\hat{\lambda}_D{}^\dagger M_D + \lambda_D{}^\dagger M_{dD}\Big) - \Big(\frac{16}{3}g_3^2 + \frac{4}{15}g_1^2\Big)M_D \tag{A.11b}$$

$$(4\pi)^2\frac{\mathrm{d}M_{dD}}{\mathrm{d}t} = 2\lambda_D\Big(\hat{\lambda}_D{}^\dagger M_D + \lambda_D{}^\dagger M_{dD}\Big) - \Big(\frac{16}{3}g_3^2 + \frac{4}{15}g_1^2\Big)M_{dD} \tag{A.11c}$$

$$(4\pi)^2\frac{\mathrm{d}M_L}{\mathrm{d}t} = \hat{\lambda}_E^T\Big(\lambda_E^* M_{lL} + \hat{\lambda}_E^* M_L\Big) - \Big(3g_2^2 + \frac{3}{5}g_1^2\Big)M_L \tag{A.11d}$$

$$(4\pi)^2\frac{\mathrm{d}M_{lL}}{\mathrm{d}t} = \lambda_E^T\Big(\lambda_E^* M_{lL} + \hat{\lambda}_E^* M_L\Big) - \Big(3g_2^2 + \frac{3}{5}g_1^2\Big)M_{lL} \tag{A.11e}$$

A.2.5 Trilinear SUSY breaking interactions

Now we turn to the study of the SUSY breaking interaction terms of the Lagrangian.

$$\begin{aligned}(4\pi)^2\frac{\mathrm{d}A_U}{\mathrm{d}t} &= A_U\Big[\mathrm{Tr}(3\lambda_U^\dagger\lambda_U) + 5\lambda_U^\dagger\lambda_U + \lambda_D^\dagger\lambda_D + \hat{\lambda}_D{}^\dagger\hat{\lambda}_D - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\Big]\\ &+ 2\lambda_U\Big[\mathrm{Tr}(3\lambda_U^\dagger A_U) + 2\lambda_U^\dagger A_U + \lambda_D^\dagger A_D + \hat{\lambda}_D{}^\dagger\hat{A}_D\\ &+ \frac{16}{3}M_3 g_3^2 + 3M_2 g_2^2 + \frac{13}{15}M_1 g_1^2\Big]\end{aligned} \tag{A.12a}$$

$$\begin{aligned}(4\pi)^2\frac{\mathrm{d}A_D}{\mathrm{d}t} &= A_D\Big[\mathrm{Tr}(3\lambda_D^\dagger\lambda_D + 3\hat{\lambda}_D{}^\dagger\hat{\lambda}_D + \lambda_E^\dagger\lambda_E + \hat{\lambda}_E{}^\dagger\hat{\lambda}_E) + 5\lambda_D^\dagger\lambda_D + 5\hat{\lambda}_D{}^\dagger\hat{\lambda}_D\\ &+ \lambda_U^\dagger\lambda_U - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2\Big]\\ &+ 2\lambda_D\Big[\mathrm{Tr}(3\lambda_D^\dagger A_D + 3\hat{\lambda}_D{}^\dagger\hat{A}_D + \lambda_E^\dagger A_E + \hat{\lambda}_E{}^\dagger\hat{A}_E) + 2\lambda_D^\dagger A_D + 2\hat{\lambda}_D{}^\dagger\hat{A}_D\\ &+ \lambda_U^\dagger A_U + \frac{16}{3}M_3 g_3^2 + 3M_2 g_2^2 + \frac{7}{15}M_1 g_1^2\Big]\end{aligned} \tag{A.12b}$$

$$\begin{aligned}(4\pi)^2\frac{\mathrm{d}A_E}{\mathrm{d}t} &= A_E\Big[\mathrm{Tr}(3\lambda_D^\dagger\lambda_D + 3\hat{\lambda}_D{}^\dagger\hat{\lambda}_D + \lambda_E^\dagger\lambda_E + \hat{\lambda}_E{}^\dagger\hat{\lambda}_E) + 5\lambda_E^\dagger\lambda_E - 3g_2^2 - \frac{9}{5}g_1^2\Big]\\ &+ 2\lambda_E\Big[\mathrm{Tr}(3\lambda_D^\dagger A_D + 3\hat{\lambda}_D{}^\dagger\hat{A}_D + \lambda_E^\dagger A_E + \hat{\lambda}_E{}^\dagger\hat{A}_E) + 2\lambda_E^\dagger A_E\\ &+ 3M_2 g_2^2 + \frac{9}{5}M_1 g_1^2\Big] + 5\hat{A}_E\hat{\lambda}_E{}^\dagger\lambda_E + 4\hat{\lambda}_E\hat{\lambda}_E{}^\dagger A_E\end{aligned} \tag{A.12c}$$

$$\begin{aligned}(4\pi)^2\frac{\mathrm{d}\hat{A}_D}{\mathrm{d}t} &= \hat{A}_D\Big[\mathrm{Tr}(3\lambda_D^\dagger\lambda_D + 3\hat{\lambda}_D{}^\dagger\hat{\lambda}_D + \lambda_E^\dagger\lambda_E + \hat{\lambda}_E{}^\dagger\hat{\lambda}_E) + 5\lambda_D^\dagger\lambda_D + 5\hat{\lambda}_D{}^\dagger\hat{\lambda}_D\\ &+ \lambda_U^\dagger\lambda_U - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2\Big]\end{aligned}$$

$$+ 2\hat{\lambda}_D\Big[\text{Tr}(3\lambda_D^\dagger A_D + 3\hat{\lambda}_D{}^\dagger\hat{A}_D + \lambda_E^\dagger A_E + \hat{\lambda}_E{}^\dagger\hat{A}_E) + 2\lambda_D^\dagger A_D + 2\hat{\lambda}_D{}^\dagger\hat{A}_D + \lambda_U^\dagger A_U + \frac{16}{3}M_3 g_3^2 + 3M_2 g_2^2 + \frac{7}{15}M_1 g_1^2\Big] \quad \text{(A.12d)}$$

$$(4\pi)^2\frac{\mathrm{d}\hat{A}_E}{\mathrm{d}t} = \hat{A}_E\Big[\text{Tr}(3\lambda_D^\dagger\lambda_D + 3\hat{\lambda}_D{}^\dagger\hat{\lambda}_D + \lambda_E^\dagger\lambda_E + \hat{\lambda}_E{}^\dagger\hat{\lambda}_E) + 5\hat{\lambda}_E{}^\dagger\hat{\lambda}_E - 3g_2^2 - \frac{9}{5}g_1^2\Big] + 2\hat{\lambda}_E\Big[\text{Tr}(3\lambda_D^\dagger A_D + 3\hat{\lambda}_D{}^\dagger\hat{A}_D + \lambda_E^\dagger A_E + \hat{\lambda}_E{}^\dagger\hat{A}_E) + 2\hat{\lambda}_E{}^\dagger\hat{A}_E + 3M_2 g_2^2 + \frac{9}{5}M_1 g_1^2\Big] + 5A_E\lambda_E{}^\dagger\hat{\lambda}_E + 4\lambda_E\lambda_E{}^\dagger\hat{A}_E \quad \text{(A.12e)}$$

A.2.6 The $B\mu$ term and other bilinear SUSY breaking parameters

For what concernes trilinear SUSY breaking interactions we get

$$(4\pi)^2\frac{\mathrm{d}B}{\mathrm{d}t} = B\Big[\text{Tr}(3\lambda_U^\dagger\lambda_U + 3\lambda_D^\dagger\lambda_D + 3\hat{\lambda}_D{}^\dagger\hat{\lambda}_D + \lambda_E^\dagger\lambda_E + \hat{\lambda}_E{}^\dagger\hat{\lambda}_E) - 3g_2^2 - \frac{3}{5}g_1^2\Big] + 2\mu\Big[\text{Tr}(3\lambda_U^\dagger A_U + 3\lambda_D^\dagger A_D + 3\hat{\lambda}_D{}^\dagger\hat{A}_D + \lambda_E^\dagger A_E + \hat{\lambda}_E{}^\dagger\hat{A}_E) + 3M_2 g_2^2 + \frac{3}{5}M_1 g_1^2\Big] \quad \text{(A.13a)}$$

$$(4\pi)^2\frac{\mathrm{d}B_D}{\mathrm{d}t} = 2\hat{\lambda}_D\Big(\hat{\lambda}_D{}^\dagger B_D + \lambda_D^\dagger B_{dD}\Big) + 4\hat{A}_D\Big(\hat{\lambda}_D{}^\dagger M_D + \lambda_D^\dagger M_{dD}\Big) - B_D\Big(\frac{16}{3}g_3^2 + \frac{4}{15}g_1^2\Big) + M_D\Big(\frac{32}{3}M_3 g_3^2 + \frac{8}{15}M_1 g_1^2\Big) \quad \text{(A.13b)}$$

$$(4\pi)^2\frac{\mathrm{d}B_{dD}}{\mathrm{d}t} = 2\lambda_D\Big(\hat{\lambda}_D{}^\dagger B_D + \lambda_D^\dagger B_{dD}\Big) + 4A_D\Big(\hat{\lambda}_D{}^\dagger M_D + \lambda_D^\dagger M_{dD}\Big) - B_{dD}\Big(\frac{16}{3}g_3^2 + \frac{4}{15}g_1^2\Big) + M_{dD}\Big(\frac{32}{3}M_3 g_3^2 + \frac{8}{15}M_1 g_1^2\Big) \quad \text{(A.13c)}$$

$$(4\pi)^2\frac{\mathrm{d}B_L}{\mathrm{d}t} = \hat{\lambda}_E^T\Big(\lambda_E^* B_{lL} + \hat{\lambda}_E^* B_L\Big) + 2\hat{A}_E^T\Big(\lambda_E^* M_{lL} + \hat{\lambda}_E^* M_L\Big) - B_L\Big(3g_2^2 + \frac{3}{5}g_1^2\Big) + M_L\Big(6M_2 g_2^2 + \frac{6}{5}M_1 g_1^2\Big) \quad \text{(A.13d)}$$

$$(4\pi)^2\frac{\mathrm{d}B_{lL}}{\mathrm{d}t} = \lambda_E^T\Big(\lambda_E^* B_{lL} + \hat{\lambda}_E^* B_L\Big) + 2A_E^T\Big(\lambda_E^* M_{lL} + \hat{\lambda}_E^* M_L\Big) - B_{lL}\Big(3g_2^2 + \frac{3}{5}g_1^2\Big) + M_{lL}\Big(6M_2 g_2^2 + \frac{6}{5}M_1 g_1^2\Big)\,. \quad \text{(A.13e)}$$

A.2.7 Soft scalar masses

Finally we study the running of the masses parameters for sfermions and higgsinos. It is convenient to define the quantity

$$\mathcal{S} = m_{h_u}^2 - m_{h_d}^2 + \text{Tr}(m_q^2 - 2m_{u^c}^2 + m_{d^c}^2 - m_l^2 + m_{e^c}^2 + m_{D^c}^2 - m_{\overline{D^c}}^2 - m_L^2 + m_{\overline{L}}^2)\,. \quad \text{(A.14)}$$

As usual, if some of the degrees of freedom happen to be integrated out the corresponding m^2 parameter in $\mathcal{S}$ will vanish. The RGE equations are then

$$\begin{aligned}
(4\pi)^2\frac{\mathrm{d}m^2_{h_u}}{\mathrm{d}t} &= 6\,\mathrm{Tr}\Big((m^2_{h_u}+m^2_q)\lambda_U^\dagger\lambda_U + \lambda_U^\dagger m^2_{u^c}\lambda_U + A_U^\dagger A_U\Big) \\
&\quad - 6|M_2|^2 g_2^2 - \frac{6}{5}|M_1|^2 g_1^2 + \frac{3}{5}g_1^2\mathcal{S} \qquad \text{(A.15a)}\\
(4\pi)^2\frac{\mathrm{d}m^2_{h_d}}{\mathrm{d}t} &= \mathrm{Tr}\Big(6(m^2_{h_d}+m^2_q)\lambda_D^\dagger\lambda_D + 6(m^2_{h_d}+m^2_q)\hat\lambda_D{}^\dagger\hat\lambda_D + 2(m^2_{h_d}+m^2_l)\lambda_E{}^\dagger\lambda_E \\
&\quad + 2(m^2_{h_d}+m^2_L)\hat\lambda_E{}^\dagger\hat\lambda_E + 2\lambda_E^\dagger\hat\lambda_E m^2_{lL} + 2\hat\lambda_E{}^\dagger\lambda_E m^2_{lL}{}^\dagger + 6\lambda_D^\dagger m^2_{dD}\hat\lambda_D \\
&\quad + 6\hat\lambda_D{}^\dagger m^2_{dD}{}^\dagger\lambda_D + 6\lambda_D^\dagger m^2_{d^c}\lambda_D + 6\hat\lambda_D{}^\dagger m^2_{D^c}\hat\lambda_D + 2\lambda_E^\dagger m^2_{e^c}\lambda_E + 2\hat\lambda_E{}^\dagger m^2_{e^c}\hat\lambda_E\Big) \\
&\quad + 2\,\mathrm{Tr}\Big(3A_D^\dagger A_D + 3\hat A_D{}^\dagger\hat A_D + A_E^\dagger A_E + \hat A_E{}^\dagger\hat A_E\Big) \\
&\quad - 6|M_2|^2 g_2^2 - \frac{6}{5}|M_1|^2 g_1^2 - \frac{3}{5}g_1^2\mathcal{S} \qquad \text{(A.15b)}\\
(4\pi)^2\frac{\mathrm{d}m^2_q}{\mathrm{d}t} &= (m^2_q+2m^2_{h_u})\lambda_U^\dagger\lambda_U + (m^2_q+2m^2_{h_d})(\lambda_D^\dagger\lambda_D + \hat\lambda_D{}^\dagger\hat\lambda_D) \\
&\quad + (\lambda_U^\dagger\lambda_U + \lambda_D^\dagger\lambda_D + \hat\lambda_D{}^\dagger\hat\lambda_D)m^2_q + 2(A_U^\dagger A_U + A_D^\dagger A_D + \hat A_D{}^\dagger\hat A_D) \\
&\quad + 2(\lambda_U^\dagger m^2_{u^c}\lambda_U + \lambda_D^\dagger m^2_{d^c}\lambda_D + \hat\lambda_D{}^\dagger m^2_{D^c}\hat\lambda_D + \hat\lambda_D{}^\dagger m^2_{dD}{}^\dagger\lambda_D + \lambda_D{}^\dagger m^2_{dD}\hat\lambda_D) \\
&\quad - \frac{32}{3}|M_3|^2 g_3^2 - 6|M_2|^2 g_2^2 - \frac{2}{15}|M_1|^2 g_1^2 + \frac{1}{5}g_1^2\mathcal{S} \qquad \text{(A.15c)}\\
(4\pi)^2\frac{\mathrm{d}m^2_l}{\mathrm{d}t} &= (m^2_l+2m^2_{h_d})\lambda_E^\dagger\lambda_E + m^2_{lL}{}^\dagger\hat\lambda_E{}^\dagger\lambda_E + \lambda_E^\dagger\lambda_E m^2_l + \lambda_E^\dagger\hat\lambda_E m^2_{lL} \\
&\quad + 2\lambda_E^\dagger m^2_{e^c}\lambda_E + 2A_E^\dagger A_E - 6|M_2|^2 g_2^2 - \frac{6}{5}|M_1|^2 g_1^2 - \frac{3}{5}g_1^2\mathcal{S} \qquad \text{(A.15d)}\\
(4\pi)^2\frac{\mathrm{d}m^2_{u^c}}{\mathrm{d}t} &= 2(m^2_{u^c}+2m^2_{h_u})\lambda_U\lambda_U^\dagger + 2\lambda_U\lambda_U^\dagger m^2_{u^c} + 4\lambda_U m^2_q\lambda_U^\dagger + 4A_U A_U^\dagger \\
&\quad - \frac{32}{3}|M_3|^2 g_3^2 - \frac{32}{15}|M_1|^2 g_1^2 - \frac{4}{5}g_1^2\mathcal{S} \qquad \text{(A.15e)}\\
(4\pi)^2\frac{\mathrm{d}m^2_{d^c}}{\mathrm{d}t} &= 2(m^2_{d^c}+2m^2_{h_d})\lambda_D\lambda_D^\dagger + 2m^2_{dD}\hat\lambda_D\lambda_D^\dagger + 2\lambda_D\lambda_D^\dagger m^2_{d^c} + 2\lambda_D\hat\lambda_D{}^\dagger m^2_{dD}{}^\dagger \\
&\quad + 4\lambda_D m^2_q\lambda_D^\dagger + 4A_D A_D^\dagger - \frac{32}{3}|M_3|^2 g_3^2 - \frac{8}{15}|M_1|^2 g_1^2 + \frac{2}{5}g_1^2\mathcal{S} \qquad \text{(A.15f)}\\
(4\pi)^2\frac{\mathrm{d}m^2_{e^c}}{\mathrm{d}t} &= 2(m^2_{e^c}+2m^2_{h_d})(\lambda_E\lambda_E^\dagger + \hat\lambda_E\hat\lambda_E{}^\dagger) + 2(\lambda_E\lambda_E^\dagger + \hat\lambda_E\hat\lambda_E{}^\dagger)m^2_{e^c} \\
&\quad + 4(\lambda_E m^2_l\lambda_E^\dagger + \hat\lambda_E m^2_L\hat\lambda_E{}^\dagger + \lambda_E m^2_{lL}{}^\dagger\hat\lambda_E{}^\dagger + \hat\lambda_E m^2_{lL}\lambda_E^\dagger) + 4(A_E A_E^\dagger + \hat A_E\hat A_E{}^\dagger) \\
&\quad - \frac{24}{5}|M_1|^2 g_1^2 + \frac{6}{5}g_1^2\mathcal{S} \qquad \text{(A.15g)}\\
(4\pi)^2\frac{\mathrm{d}m^2_{D^c}}{\mathrm{d}t} &= 2(m^2_{D^c}+2m^2_{h_d})\hat\lambda_D\hat\lambda_D{}^\dagger + 2m^2_{dD}{}^\dagger\lambda_D\hat\lambda_D{}^\dagger + 2\hat\lambda_D\hat\lambda_D{}^\dagger m^2_{D^c} + 2\hat\lambda_D\lambda_D^\dagger m^2_{dD} \\
&\quad + 4\hat\lambda_D m^2_q\hat\lambda_D{}^\dagger + 4\hat A_D\hat A_D{}^\dagger - \frac{32}{3}|M_3|^2 g_3^2 - \frac{8}{15}|M_1|^2 g_1^2 + \frac{2}{5}g_1^2\mathcal{S} \\
&\qquad \text{(A.15h)}
\end{aligned}$$

$$(4\pi)^2\frac{\mathrm{d}m^2_{D^c}}{\mathrm{d}t} = -\frac{32}{3}|M_3|^2g_3^2 - \frac{8}{15}|M_1|^2g_1^2 - \frac{2}{5}g_1^2\mathcal{S} \tag{A.15i}$$

$$(4\pi)^2\frac{\mathrm{d}m^2_{dD}}{\mathrm{d}t} = 2(m^2_{d^c} + 2m^2_{h_d})\lambda_D\hat{\lambda}_D^\dagger + 2\lambda_D\lambda_D^\dagger m^2_{dD} + 2\lambda_D\hat{\lambda}_D^\dagger m^2_{D^c} + 2m^2_{dD}\hat{\lambda}_D\hat{\lambda}_D{}^\dagger + 4\lambda_D m_q^2\hat{\lambda_D}^\dagger + 4A_D\hat{A}_D^\dagger \tag{A.15j}$$

$$(4\pi)^2\frac{\mathrm{d}m^2_{\tilde{L}}}{\mathrm{d}t} = -6|M_2|^2g_2^2 - \frac{6}{5}|M_1|^2g_1^2 + \frac{3}{5}g_1^2\mathcal{S} \tag{A.15k}$$

$$(4\pi)^2\frac{\mathrm{d}m^2_L}{\mathrm{d}t} = (m^2_L + 2m^2_{h_d})\hat{\lambda}_E{}^\dagger\hat{\lambda}_E + m^2_{lL}\lambda_E^\dagger\hat{\lambda}_E + \hat{\lambda}_E{}^\dagger\hat{\lambda}_E m^2_L + \hat{\lambda}_E{}^\dagger\lambda_E m^2_{lL}{}^\dagger + 2\hat{\lambda}_E{}^\dagger m^2_{e^c}\hat{\lambda}_E + 2\hat{A}_E{}^\dagger\hat{A}_E - 6|M_2|^2g_2^2 - \frac{6}{5}|M_1|^2g_1^2 - \frac{3}{5}g_1^2\mathcal{S} \tag{A.15l}$$

$$(4\pi)^2\frac{\mathrm{d}m^2_{lL}}{\mathrm{d}t} = (m^2_L + 2m^2_{h_d})\hat{\lambda}_E^\dagger\lambda_E + m^2_{lL}\lambda_E^\dagger\lambda_E + \hat{\lambda}_E^\dagger\lambda_E m_l^2 + \hat{\lambda}_E^\dagger\hat{\lambda}_E m^2_{lL} + 2\hat{\lambda}_E^\dagger m^2_{e^c}\lambda_E + 2\hat{A}_E^\dagger A_E\,. \tag{A.15m}$$

A.3 Approximate analytical running of Higgs mass parameters

A sometimes useful simple approximation for the solutions of the RGEs for the soft mass terms is obtained in the limit in which $\tan\beta$ is moderate, so that only the top Yukawa coupling is relevant in the equations above, and the squared gaugino masses and A-terms are negligible compared to m^2_{10}. In such a case, the only soft terms that run significantly are $m^2_{h_u}$ and the stop squared mass parameters $\tilde{m}^2_{q3}$ and $\tilde{m}^2_{u_3^c}$, for which we have

$$\begin{aligned} m^2_{h_u}(M_Z^2) &= m^2_{h_u}(M_{\rm GUT}) - \frac{1}{2}m^2_U\,\rho = -\frac{1}{2}m^2_{10}\,(4 + 5(-2+\rho)\sin^2\theta_u) \\ \tilde{m}^2_{q3}(M_Z^2) &= \tilde{m}^2_{q3}(M_{\rm GUT}) - \frac{1}{6}m^2_U\,\rho = m^2_{10}\Big(1 - \frac{5}{6}\rho\,\sin^2\theta_u\Big) \\ \tilde{m}^2_{u_3^c}(M_Z^2) &= \tilde{m}^2_{u_3^c}(M_{\rm GUT}) - \frac{1}{3}m^2_U\,\rho, = m^2_{10}\Big(1 - \frac{5}{3}\rho\,\sin^2\theta_u\Big)\,, \end{aligned} \tag{A.16}$$

where $m^2_U = (m^2_{h_u} + \tilde{m}^2_{q3} + \tilde{m}^2_{u_3^c})_{M_{\rm GUT}} = 5\sin^2\theta_u\,m^2_{10}$, $m^2_{h_u}(M_{\rm GUT}) = (-2\cos^2\theta_u + 3\sin^2\theta_u)m^2_{10}$, $\tilde{m}^2_{q3}(M_{\rm GUT}) = \tilde{m}^2_{u_3^c}(M_{\rm GUT}) = m^2_{10}$ and

$$\rho = 1 - e^{12\int\frac{dt}{(4\pi)^2}\lambda_t^2(t)}\,, \quad 0<\rho<1\,. \tag{A.17}$$

A typical value of ρ is $\rho\sim 0.7$.

A.4 Razor

The razor analysis [78] is a fairly recent approach that has been introduced by CMS collaboration to discriminate New Physics signals over SM backgrounds in situations in which there is a large presence of $E_T^{\rm miss}$. The framework is designed to perfectly fit to a situation in which from parton collision two heavy particles (G_1, G_2), whose mass is significantly larger than those of SM particles, are produced. The decades of the G_i's are then forced to be described in a dijet topology, in which any of the G_i decades to a massive unseen particle χ_i, contributing to $E_T^{\rm miss}$, and a massless seen particle Q_i, being detected as a jet. In SUSY theories the benchmark scenario for such this approach would thus be the case in which two heavy squarks are produced and then decay to a quark and a neutralino:

$$pp \to G_1 G_2 \to Q_1 \chi_1 + Q_2 \chi_2 \Longrightarrow pp \to \widetilde{q}\widetilde{q} \to 2j + \text{MET}\,. \tag{A.18}$$

For any of the decay chains $G_i \to Q_i + \chi_i$ one can define the variable

$$M_{\Delta_i} = \frac{M_{G_i}^2 - M_{\chi_i}^2}{M_{G_i}}\,, \tag{A.19}$$

which, in the approximation where the heavy G_i's are produced at threshold and the Q_i's are massless, corresponds to twice the energy of the Q_i's in the center of mass (CM) frame.

The reconstruction of the CM frame in events with two undetected particles is not conceivable, but still it is possible to perform an event by event reconstruction of the specific reference frame in which the three-momenta of the observed jets coincide. This reference frame, named R-frame, is an estimator of the CM frame itself: working in it one can construct a transverse mass M_T^R,

$$M_T^R \equiv \sqrt{\frac{E_T^{miss}(p_T^{j_1} + p_T^{j_1}) - \overrightarrow{E}_T^{miss}(\overrightarrow{p}_T^{j_1} + \overrightarrow{p}_T^{j_1})}{2}}\,, \tag{A.20}$$

whose distribution would have an edge at M_Δ corresponding to the case in which CM and R frame coincide, and

$$M_R \equiv \sqrt{(E_{j_1} + E_{j_2})^2 - (p_z^{j_1} + p_z^{j_1})^2}\,, \tag{A.21}$$

which peaks at M_Δ for signal events.

Given the tools described one could easily discriminate between background and signal events by means of the razor variable, defined as

$$R \equiv \frac{M_T^R}{M_R}\,. \tag{A.22}$$

For signal events the distribution of R peaks around 1/2, while for any SM background it is quite lower: this allows to discriminate between the two by means of smart cuts on the value of R.

$\mathcal{B}$

Appendix for Chapter 4.

In this appendix we will present some relevant formulae connected to the discussion tackled in chapter 4 about the E_6 inspired models of TGM.

B.1 Expressions of Sfermion Masses

In the presence of n SUSY breaking vector messengers associated to broken generators T^h_a, $a = 1 \dots n$, sfermion masses are given by the general expression

$$\tilde{m}^2_{ij} = 2g^2 (T^h_a)_{ij} (M^2_V)^{-1}_{ab} F^\dagger_{0k} (T^h_b)_{kl} F_{0l}\,, \tag{B.1}$$

where the indices $ijkl$ denote the chiral superfields, F_{0i} are the corresponding F-term vevs, and M^2_V is the $n \times n$ messenger vector mass matrix. No matter how complicated is the Higgs mechanism giving rise to M^2_V and F_{0i}, the sfermion masses effectively depend only on the n real parameters $m^2_a \equiv 2g^2 (M^2_V)^{-1}_{ab} F^\dagger_{0k} (T^h_b)_{kl} F_{0l}$:

$$\tilde{m}^2_{ij} = (T^h_a)_{ij} m^2_a\,. \tag{B.2}$$

The real parameters can be of course combined in complex parameters corresponding to complex generators, if needed. In each of the three cases considered in this paper, the parameters m^2_a can be recovered as functions of the parameters of the model.

In the case of one and two messengers sfermion masses arise from scalar and F-term vevs of the SM singlets

$$N^{c\prime}, N^c, S'_+, \overline{N}^{c\prime}, \overline{N}^c, S'_- \tag{B.3}$$

which are understood as vectors in flavour space. We denote $(x, y) = \sum_i x^*_i y_i$, $|x|^2 = (x, x)$, where i runs over the flavour indices and introduce the shorthand notation

$$\begin{aligned} x &\equiv |N^c|^2 + |\overline{N^c}|^2 & y &\equiv |N^{c\prime}|^2 + |\overline{N^{c\prime}}|^2 & z &\equiv |S'_+|^2 + |S'_-|^2 \\ f_x &= |F_{N^c}|^2 - |F_{\overline{N^c}}|^2 & f_y &= |F_{N^{c\prime}}|^2 - |F_{\overline{N^{c\prime}}}|^2 & f_z &= |F_{S'_+}|^2 - |F_{S'_-}|^2\,, \end{aligned} \tag{B.4}$$

where we have denoted the vevs by the same symbol used for the fields and called $F_{N^{c\prime}}, F_{N^c}, F_{\overline{N^{c\prime}}}, F_{\overline{N^c}}, F_{S'_+}, F_{S'_-}$ the F-term vevs of $N^{c\prime}, N^c, \overline{N^{c\prime}}, \overline{N^c}$, S'_+, S'_- , respectively.

In the one messenger case sfermion masses depend on these parameters only through a single parameter m_X^2 given by

$$m_X^2 \equiv \frac{(5\hat{s}_X + \hat{c}_X)f_x + 4\hat{c}_X f_y + (-5\hat{s}_X + 3\hat{c}_X)f_z}{(5\hat{s}_X + \hat{c}_X)^2 x + 16\hat{c}_X{}^2 y + (-5\hat{s}_X + 3\hat{c}_X)^2 z}, \tag{B.5}$$

where $\hat{s}_X \equiv 1/\sqrt{40}\sin\theta_X$ and $\hat{c}_X \equiv 1/\sqrt{24}\cos\theta_X$.

In the two messenger case we have two parameters m_5^2 and m_{10}^2 for which we get

$$\begin{pmatrix} m_5^2 \\ m_{10}^2 \end{pmatrix} = \frac{1}{20(xy + xz + yz)} \begin{pmatrix} f_x(4y + 3z) + f_y(3z - x) - f_z(x + 4y) \\ 5f_x z + 5f_y(x + z) + 5f_z x \end{pmatrix}. \tag{B.6}$$

Note that at least two among x, y, z must be non vanishing in order to completely break $U(1)_5 \times U(1)_{10}$, since a single vev would leave a linear combination of the two $U(1)$ factors unbroken.

In the four messenger case sfermion masses are generated by the scalar and F-term vevs of n flavours of doublets and antidoublets and m flavours of triplets

$$(\mathbf{1}, \mathbf{2})_5 = \begin{pmatrix} N^{c\prime} \\ N^c \end{pmatrix} \quad (\mathbf{1}, \mathbf{2})_{-5} = \begin{pmatrix} \overline{N}^{c\prime} \\ \overline{N}^c \end{pmatrix} \quad (\mathbf{1}, \mathbf{3})_0 = \begin{pmatrix} S'_+ \\ S'_0 \\ S'_- \end{pmatrix}. \tag{B.7}$$

In addition to equations B.4 we define

$$w \equiv |S'_0|^2, \quad \alpha \equiv (N^{c\prime}, N^c) + (\overline{N}^c, \overline{N}^{c\prime}), \quad \beta \equiv (S'_+, S'_0) + (S'_0, S'_-), \quad \gamma \equiv (S'_+, S'_-),$$

where $\alpha, \beta, \gamma \in \mathbb{C}$, and $|\alpha| \leq \sqrt{xy}$, $|\beta| \leq \sqrt{2zw}$, $|\gamma| \leq z/2$. We use the same notation as before for the F-term vevs and further denote by $F_{S'_0}$ the F-term of S'_0.

The sfermion masses depend on the above vevs through four parameters $m_+^2, m_-^2, m_3^2, m_1^2$ given by

$$\begin{pmatrix} m_+^2 \\ m_-^2 \\ m_3^2 \\ m_1^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1/\sqrt{60} \end{pmatrix} 2g^2(\hat{M}_V^2)^{-1} \begin{pmatrix} F_+^2 \\ F_-^2 \\ F_3^2 \\ F_1^2 \end{pmatrix}, \tag{B.8}$$

where

$$\begin{aligned} F_+^2 &\equiv F_0^\dagger \hat{T}'_+ F_0 = \frac{f_\alpha}{\sqrt{2}} - f_\beta & F_-^2 &\equiv F_0^\dagger \hat{T}'_- F_0 = \left(F_+^2\right)^* \\ F_3^2 &\equiv F_0^\dagger \hat{T}'_3 F_0 = f_z + \frac{f_y - f_x}{2} & F_1^2 &\equiv F_0^\dagger \hat{T}' F_0 = \frac{5}{\sqrt{60}}(f_x + f_y) \end{aligned} \tag{B.9}$$

$$\hat{M}_V^2 = g^2 \begin{pmatrix} \frac{x+y+2z+4w}{2} & -2\gamma^* & -\beta^* & \sqrt{\frac{5}{6}}\alpha^* \\ -2\gamma & \frac{x+y+2z+4w}{2} & -\beta & \sqrt{\frac{5}{6}}\alpha \\ -\beta & -\beta^* & \frac{x+y+4z}{2} & \frac{1}{2}\sqrt{\frac{5}{3}}(y-x) \\ \sqrt{\frac{5}{6}}\alpha & \sqrt{\frac{5}{6}}\alpha^* & \frac{1}{2}\sqrt{\frac{5}{3}}(y-x) & \frac{5}{6}(x+y) \end{pmatrix}. \tag{B.10}$$

B.2 Rank 6 Subgroups of E_6 containing G_{SM}

We now provide a complete list of the Rank 6 subalgebras $\mathfrak{g}$ of the E_6 Lie algeba containing the SM algebra. We distinguish the two ($t'_\pm \notin \mathfrak{g}$) and four ($t'_\pm \in \mathfrak{g}$) messenger cases. We will write the subalgebras as direct sums of the decomposition of the E_6 Lie algebra with respect to $G_{\text{min}} = G_{\text{SM}} \times U(1)_{10} \times U(1)_5$ and $G'_{\text{min}} = G_{\text{SM}} \times U(1)' \times SU(2)'$ respectively.

The **78** decomposes as in equation 4.1. The G_{min} irreducible subalgebras (besides the ones in $\mathfrak{g}_{\text{min}}$) can be labelled as follows:

$$\begin{gathered} \mathbf{24}_{0,0} + \mathbf{1}_{0,0} + \mathbf{1'}_{0,0} = \mathfrak{g}_{\text{min}} + V_{0,0} + \overline{V}_{0,0} \\ \mathbf{10}_{-4,0} = q_{-3,1/2} + u^c_{-3,1/2} + e^c_{-3,1/2} \qquad \overline{\mathbf{10}}_{4,0} = \overline{q}_{3,-1/2} + \overline{u^c}_{3,-1/2} + \overline{e^c}_{3,-1/2} \\ \mathbf{10}_{1,-3} = q_{-3,-1/2} + u^c_{-3,-1/2} + e^c_{-3,-1/2} \qquad \overline{\mathbf{10}}_{-1,3} = \overline{q}_{3,1/2} + \overline{u^c}_{3,1/2} + \overline{e^c}_{3,1/2} \\ \overline{\mathbf{5}}_{-3,-3} = l_{-6,0} + d^c_{-6,0} \qquad \mathbf{5}_{3,3} = \overline{l}_{6,0} + \overline{d^c}_{6,0} \\ \mathbf{1}_{5,-3} = s'_{0,-1} \qquad \mathbf{1}_{-5,3} = s'_{0,1}\,. \end{gathered} \tag{B.11}$$

where $r_{a,b}$ denotes the subalgebra with the quantum numbers of the SM representation r and with $t' = a$, $t'_3 = b$ (we use t' and t'_3 instead of t_5, t_{10} here because it makes easier to compute commutators). V denotes the $(\mathbf{3}, \mathbf{2}, -5/6)$ SM representation that describes the heavy $SU(5)/G_{\text{SM}}$ vectors.

The Rank 6 subalgebras $\mathfrak{g}$ of the E_6 Lie algeba containing the SM algebra, but not $t'_\pm$, are then

$$\mathfrak{su}(5) + \mathfrak{u}(1)_5 + \mathfrak{u}(1)_{10} = \mathfrak{g}_{\text{min}} + V_{0,0} + \overline{V}_{0,0} \tag{B.12a}$$

$$\mathfrak{su}(5)_f + \mathfrak{u}(1)_{5f} + \mathfrak{u}(1)_{10} = \mathfrak{g}_{\text{min}} + q_{-3,1/2} + \overline{q}_{3,-1/2} \tag{B.12b}$$

$$\mathfrak{su}(4)_c + \mathfrak{su}(2)_L + \mathfrak{u}(1)_{3R} + \mathfrak{u}(1)_{10} = \mathfrak{g}_{\text{min}} + u^c_{-3,1/2} + \overline{u^c}_{3,-1/2} \tag{B.12c}$$

$$\mathfrak{su}(3)_c + \mathfrak{su}(2)_L + \mathfrak{su}(2)_R + \mathfrak{u}(1)_{B-L} + \mathfrak{u}(1)_{10} = \mathfrak{g}_{\text{min}} + e^c_{-3,1/2} + \overline{e^c}_{3,-1/2} \tag{B.12d}$$

$$\mathfrak{su}(3)_c + \mathfrak{su}(3)_L + \mathfrak{u}(1)'_8 + \mathfrak{u}(1)'_3 = \mathfrak{g}_{\text{min}} + l_{-6,0} + \overline{l}_{6,0} \tag{B.12e}$$

$$\mathfrak{su}(4)_{cf} + \mathfrak{su}(2)_L + \mathfrak{u}(1)'_3 + \mathfrak{u}(1)_{10f} = \mathfrak{g}_{\text{min}} + d^c_{-6,0} + \overline{d^c}_{6,0} \tag{B.12f}$$

$$\mathfrak{so}(10) + \mathfrak{u}(1)_{10} = \mathfrak{g}_{\text{min}} + (V_{0,0} + q_{-3,1/2} + u^c_{-3,1/2} + e^c_{-3,1/2} + \text{conj}) \tag{B.12g}$$

$$\mathfrak{su}(6) + \mathfrak{u}(1)'_3 = \mathfrak{g}_{\text{min}} + (V_{0,0} + l_{-6,0} + d^c_{-6,0} + \text{conj}) \tag{B.12h}$$

$$\mathfrak{su}(6)_f + \mathfrak{u}(1)'_{3R} = \mathfrak{g}_{\text{min}} + (q_{-3,1/2} + u^c_{-3,-1/2} + l_{-6,0} + \text{conj}) \tag{B.12i}$$

$$\mathfrak{su}(5)_f + \mathfrak{su}(2)'_R + \mathfrak{u}(1)'_f = \mathfrak{g}_{\rm min} + (q_{-3,1/2} + e^c_{-3,-1/2} + \text{conj}) \tag{B.12j}$$

$$\mathfrak{su}(6)_f + \mathfrak{su}(2)'_R = \mathfrak{g}_{\rm min} + (q_{-3,1/2} + u^c_{-3,-1/2} + l_{-6,0} + e^c_{-3,-1/2} + \text{conj}) \tag{B.12k}$$

$$\mathfrak{su}(4)_c + \mathfrak{su}(2)_L + \mathfrak{su}(2)_R + \mathfrak{u}(1)_{10} = \mathfrak{g}_{\rm min} + (u^c_{-3,1/2} + e^c_{-3,1/2} + \text{conj}) \tag{B.12l}$$

$$\mathfrak{su}(5)'_{fR} + \mathfrak{su}(2)_L + \mathfrak{u}(1)'_{fR} = \mathfrak{g}_{\rm min} + (u^c_{-3,1/2} + e^c_{-3,-1/2} + d^c_{-6,0} + \text{conj}) \tag{B.12m}$$

$$\mathfrak{su}(3)_c + \mathfrak{su}(3)_L + \mathfrak{su}(2)_R + \mathfrak{u}(1)_R = \mathfrak{g}_{\rm min} + (e^c_{-3,1/2} + l_{-6,0} + \text{conj})\,, \tag{B.12n}$$

besides of course $\mathfrak{su}(3)_c + \mathfrak{su}(2)_L + \mathfrak{u}(1)_Y + \mathfrak{u}(1)_5 + \mathfrak{u}(1)_{10} = \mathfrak{g}_{\rm min}$. For the definition of the $U(1)$ factors see table B.1.

	Generator	Definition
$\mathrm{U}(1)_{5f}$	t_{5f}	$(t_5 + 24y)/5$
$\mathrm{U}(1)_{3R}$	t_{3R}	$(t_5 - 6y)/10$
$\mathrm{U}(1)_{B-L}$	t_{B-L}	$(t_5 + 4y)/5$
$\mathrm{U}(1)'_8$	y'	$(-3t_5 + 48y - 5t_{10})/60$
$\mathrm{U}(1)'_3$	t'_3	$(t_{10} - t_5)/8$
$\mathrm{U}(1)_{10f}$	t_{10f}	$(3t_5 + 5t_{10} + 72y)/20$
$\mathrm{U}(1)'_{3R}$	t'_{3R}	$(t_5 - 5t_{10} + 24y)/40$
$\mathrm{U}(1)'_f$	t'_f	$(3t_5 + 25t_{10} + 72y)/20$
$\mathrm{U}(1)'_{fR}$	t'_{fR}	$(-3t_5 + 5t_{10} + 18y)/5$
$\mathrm{U}(1)_R$	t_R	$(-3t_5 - 12y + 5t_{10})/30$
$\mathrm{U}(1)'$	t'	$(5t_{10} + 3t_5)/4$
$\mathrm{U}(1)'_{10f}$	t'_{10f}	$(3t_5 + 5t_{10} + 72y)/20$
$\mathrm{U}(1)'_c$	t'_c	$(-3t_5 - 5t_{10} + 18y)/5$
$\mathrm{U}(1)_{8L}$	y_L	$(-3t_5 - 5t_{10} - 12y)/30$

Table B.1: Definition of U(1) factors

Some comments are in order. All the subgroup factors in equations B.12 are orthogonal. Adding a subalgebra with opposite values of t'_3 leads to an equivalent embedding that can be obtained from the original one by means of a $SU(2)'$ rotation flipping the sign of t'_3. The subalgebra $\mathfrak{su}(5)_f$ gives the flipped embedding of $SU(5)$ in $SO(10) \subset E_6$ with the flipped $U(1)$ generator t_{5f}. The "flipped $SU(4)_c$" subalgebra $\mathfrak{su}(4)_{cf}$ can be seen as the $SU(4)$ subgroup of $SU(6)$ generated by $\mathfrak{su}(3)_c + d^c_{-6,0} + \overline{d^c}_{6,0}$ and the "flipped $B-L$" generator $t^f_{B-L} \equiv (t' - 2y)/5$. The flipped $\mathfrak{su}(6)_f$ subalgebra is spanned by $\mathfrak{su}(5)_f + \overline{\mathbf{5}}_{-3,-3} + \mathbf{5}_{3,3} + \mathfrak{u}(1)'_f$. The $\mathrm{SU}(5)'_{fR}$ subgroup is the one obtained from the unification of $SU(3)_c$ and $SU(2)'_R$ instead of $SU(2)_L$.

In the case in which the gauge group contains $SU(2)'$ (i.e. $t'_\pm \in \mathfrak{g}$), it is convenient to decompose the E_6 adjoint with respect to $G'_{\rm min}$. One has

$$\begin{gathered}
\mathbf{78} \to (\mathbf{24},\mathbf{1})_0 + (\mathbf{5},\mathbf{1})_6 + (\overline{\mathbf{5}},\mathbf{1})_{-6} + (\mathbf{1},\mathbf{1})_0 + (\mathbf{1},\mathbf{3}) + (\mathbf{10},\mathbf{2})_{-3} + (\overline{\mathbf{10}},\mathbf{2})_3 \\
(\mathbf{24},\mathbf{1})_0 + (\mathbf{1},\mathbf{1})_0 + (\mathbf{1},\mathbf{3})_0 = \mathfrak{g}'_{\rm min} + (V,1)_0 + (\overline{V},1)_0 \\
(\mathbf{10},\mathbf{2})_{-3} = (q,2)_{-3} + (u^c,2)_{-3} + (e^c,2)_{-3} \qquad (\overline{\mathbf{10}},\mathbf{2})_3 = (\overline{q},2)_3 + (\overline{u^c},2)_3 + (\overline{e^c},2)_3 \\
(\overline{\mathbf{5}},\mathbf{1})_{-6} = (l,1)_{-6} + (d^c,1)_{-6} \qquad (\mathbf{5},\mathbf{1})_6 = (\overline{l},1)_6 + (\overline{d^c},1)_6\,,
\end{gathered} \tag{B.13}$$

where $(a,b)_q$ denotes a subalgebra with quantum numbers a under $SU(5)$ (first line) of $G_{\rm SM}$ (other lines), b under $SU(2)'$, and $t' = q$.

The Rank 6 subalgebras $\mathfrak{g}$ of the E_6 Lie algebra containing the SM algebra and $t'_\pm$, are then

$$\mathfrak{su}(5) + \mathfrak{u}(1)' + \mathfrak{su}(2)' = \mathfrak{g}'_{\rm min} + [(V,1)_0 + \text{conj}] \tag{B.14a}$$
$$\mathfrak{su}(6) + \mathfrak{su}(2)' = \mathfrak{g}'_{\rm min} + [(V,1)_0 + (l,1)_{-6} + (d^c,1)_{-6} + \text{conj}] \tag{B.14b}$$
$$\mathfrak{so}(10)'_f + \mathfrak{u}(1)'_{10f} = \mathfrak{g}'_{\rm min} + [(q,2)_{-3} + (d^c,1)_{-6} + \text{conj}] \tag{B.14c}$$
$$\mathfrak{su}(5)_c + \mathfrak{su}(2)_L + \mathfrak{u}(1)'_c = \mathfrak{g}'_{\rm min} + [(u^c,2)_{-3} + \text{conj}] \tag{B.14d}$$
$$\mathfrak{su}(6)_c + \mathfrak{su}(2)_L = \mathfrak{g}'_{\rm min} + [(u^c,2)_{-3} + (e^c,2)_{-3} + (d^c,1)_{-6} + \text{conj}] \tag{B.14e}$$
$$\mathfrak{su}(3)_c + \mathfrak{su}(2)_L + \mathfrak{su}(3)' + \mathfrak{u}(1)_{8L} = \mathfrak{g}'_{\rm min} + [(e^c,2)_{-3} + \text{conj}] \tag{B.14f}$$
$$\mathfrak{su}(3)_c + \mathfrak{su}(3)_L + \mathfrak{su}(3)' = \mathfrak{g}'_{\rm min} + [(e^c,2)_{-3} + (l,1)_{-6} + \text{conj}] \tag{B.14g}$$
$$\mathfrak{su}(3)_c + \mathfrak{su}(3)_L + \mathfrak{su}(2)' + \mathfrak{u}(1)'_8 = \mathfrak{g}'_{\rm min} + [l_{-6,0} + \text{conj}] \tag{B.14h}$$
$$\mathfrak{su}(4)_{cf} + \mathfrak{su}(2)_L + \mathfrak{su}(2)' + \mathfrak{u}(1)_{10f} = \mathfrak{g}'_{\rm min} + [d^c_{-6,0} + \text{conj}]\,, \tag{B.14i}$$

besides of course $\mathfrak{e}_6$ itself.

Table B.2: Decomposition of **27**

$SU(6)\times SU(2)'$	$SU(5)\times SU(2)'\times U(1)'$	$SU(5)\times U(1)_5\times U(1)_{10}$	SM	$SO(10)\times U(1)_{10}$
$(\mathbf{15},\mathbf{1})$	$(\mathbf{10},\mathbf{1})_2$	$\mathbf{10}_{1,1}$	q,u^c,e^c	$\mathbf{16}_1$
	$(\mathbf{5},\mathbf{1})_{-4}$	$\mathbf{5}_{-2,-2}$	$\overline{d^c},\overline{l}$	$\mathbf{10}_{-2}$
$(\overline{\mathbf{6}},\mathbf{2})$	$\overline{\mathbf{5}}_{-3,1}$	$(\overline{\mathbf{5}},\mathbf{2})_{-1}$	d^c,l	$\mathbf{16}_1$
		$\overline{\mathbf{5}}_{2,-2}$	d'^c,l'	$\mathbf{10}_{-2}$
	$(\mathbf{1},\mathbf{2})_5$	$\mathbf{1}_{0,4}$	$\nu^{c\prime}$	$\mathbf{1}_4$
		$\mathbf{1}_{5,1}$	ν^c	$\mathbf{16}_1$

Table B.3: Decomposition of **78**

$SU(6)\times SU(2)'$	$SU(5)\times SU(2)'\times U(1)'$	$SU(5)\times U(1)_5\times U(1)_{10}$	SM	$SO(10)\times U(1)_{10}$
$(\mathbf{35},\mathbf{1})$	$(\mathbf{24},\mathbf{1})_0$	$\mathbf{24}_{0,0}$		$\mathbf{45}_0$
	$(\mathbf{5},\mathbf{1})_6$	$\mathbf{5}_{3,3}$		$\overline{\mathbf{16}}_3$
	$(\overline{\mathbf{5}},\mathbf{1})_{-6}$	$\overline{\mathbf{5}}_{-3,-3}$		$\mathbf{16}_{-3}$
	$(\mathbf{1},\mathbf{1})_0$	$\mathbf{1}_{0,0}$	s'	$(\mathbf{1}_0,\mathbf{45}_0)$
$(\mathbf{20},\mathbf{2})$	$(\mathbf{10},\mathbf{2})_{-3}$	$\mathbf{10}_{-4,0}$		$\mathbf{45}_0$
		$\mathbf{10}_{1,-3}$		$\mathbf{16}_{-3}$
	$(\overline{\mathbf{10}},\mathbf{2})_3$	$\overline{\mathbf{10}}_{-1,3}$		$\overline{\mathbf{16}}_3$
		$\overline{\mathbf{10}}_{4,0}$		$\mathbf{45}_0$
$(\mathbf{1},\mathbf{3})$	$(\mathbf{1},\mathbf{3})_0$	$\mathbf{1}_{-5,3}$	s'_+	$\overline{\mathbf{16}}_3$
		$\mathbf{1}_{0,0}$	s'_0	$(\mathbf{1}_0,\mathbf{45}_0)$
		$\mathbf{1}_{5,-3}$	s'_-	$\mathbf{16}_{-3}$

C

Appendix for Chapter 5

In this appendix we will present some relevant formulae connected to the discussion tackled in chapter 5 about the Minimal Yukawa-Gauge Mediation model.

C.1 Gauge coupling unification

In this appendix we discuss the details of gauge coupling evolution, whose results are reported in section 5.2.2; in particular we show that unification is easily realized in our framework.

The one loop evolution of the gauge couplings is given by

$$(4\pi)^2 \frac{\mathrm{d}g_i}{\mathrm{d}t} = g_i^3 b_i \,, \tag{C.1}$$

where the b_i's have to be calculated considering all the fields that are charged under the i-th interaction. For a generic theory the different contributions owed to scalars and fermions may be obtained as in [142–145]. Thus we calculate the various b_i's considering only those fields present in the effective field theory at a given scale: indeed at scale μ all the fields heavier than μ decouple and do not contribute to the running.

Our framework is characterized by the presence of five scales:

- winos and bino, $M^{(4)}_{\mathrm{SUSY}} \sim 100 \div 300\,\mathrm{GeV}$;
- gluinos, $M^{(3)}_{\mathrm{SUSY}} \sim 1 \div 1.5\,\mathrm{TeV}$;
- light sfermions, $M^{(2)}_{\mathrm{SUSY}} \sim 4 \div 7\,\mathrm{TeV}$;
- heavy sfermions, heavy Higgs and higgsinos, $M^{(1)}_{\mathrm{SUSY}} \sim 30 \div 50\,\mathrm{TeV}$;
- extra triplets, $M_T \sim 10^{14} \div 10^{15}$ GeV.

The different b_i's at the energy scale μ are:

- $\mu < M^{(4)}_{\mathrm{SUSY}}$: the theory coincides with the SM thus $b_i = \left\{\frac{41}{10}, -\frac{19}{6}, -7\right\}$;

- $M_{\rm SUSY}^{(3)} < \mu < M_{\rm SUSY}^{(4)}$: the theory is SM + winos + bino thus $b_i = \left\{\frac{41}{10}, -\frac{11}{6}, -7\right\}$;
- $M_{\rm SUSY}^{(2)} < \mu < M_{\rm SUSY}^{(3)}$: the theory is SM + winos + bino + gluino thus $b_i = \left\{\frac{41}{10}, -\frac{11}{6}, -5\right\}$;
- $M_{\rm SUSY}^{(2)} < \mu < M_{\rm SUSY}^{(1)}$: the theory is SM + winos + bino + gluino + first two family sfermions thus $b_i = \left\{\frac{163}{30}, -\frac{1}{2}, -\frac{11}{3}\right\}$;
- $M_{\rm SUSY}^{(1)} < \mu < M_T$, the theory is MSSM thus $b_i = \left\{\frac{33}{5}, 1, -3\right\}$;
- $\mu > M_T$, the theory is MSSM + heavy triplets thus $b_i = \{7, 1, -2\}$.

We can check the compatibility of gauge unification with the observables at the EW scale as follows. The low energy values at M_Z, the Z boson mass scale, of α, $\sin^2\theta_W$ and α_s – in the $\overline{MS}$ renormalization scheme – are given by [125]

$$\begin{aligned} M_Z &= 91.1876 \pm 0.0021\,\text{GeV}\,, \\ \alpha(M_Z)^{-1} &= 127.916 \pm 0.015\,, \\ \sin^2\theta_W(M_Z) &= 0.23150 \pm 0.00016\,, \\ \alpha_s(M_Z) &= 0.1184 \pm 0.0012\,. \end{aligned}$$

We notice that the observable with the biggest uncertainties is α_s. We evolve g_1 and g_2 to high energies and we determine a tentative unification scale. Then we run back the strong coupling and check if it is compatible with the experimental value. We find that for the typical scales corresponding to the spectrum shown in figure 5.2 the strong coupling calculated with this procedure falls always within three sigmas of the experimental value.

C.2 RG evolution of the effective theory for calculation of Higgs mass

In section 5.2.3 we have seen that below $M_{\rm SUSY}^{(1)}$ the heavy fields start to decouple from the theory. In the following we write down the evolution equations for the parameters involved to get the Higgs running mass at the minimum of the potential. As already anticipated in appendix C.1 we spotted the presence of different intermediate scales to consider. Not to be redundant we do not re-write the gauge coupling RGEs, that can be found in appendix C.1.

Between $M_{\rm SUSY}^{(1)}$ and $M_{\rm SUSY}^{(2)}$

In this region the heavy third family sfermions, the higgsinos and the heavy Higgs doublet have decoupled. The relevant equations needed for the evolution of the various parameters are the following. The couplings evolve through

$$(4\pi)^2 \frac{\mathrm{d}\xi}{\mathrm{d}t} = \beta_\xi. \tag{C.2}$$

We considered the various β_ξ and γ_h, the anomalous dimensions, in the third family approximation, where only the top Yukawa coupling is relevant:

$$
\begin{aligned}
\beta_{M_1} &= 8g_1^2 M_1\,, && \text{(C.3)}\\
\beta_{M_2} &= -4g_2^2 M_2\,, && \text{(C.4)}\\
\beta_{M_3} &= -10g_3^2 M_3\,, && \text{(C.5)}\\
\beta_{\lambda_t} &= \lambda_t\left(\frac{9}{2}\lambda_t^2 - \frac{17}{20}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2\right)\,, && \text{(C.6)}\\
\beta_\lambda &= 12\lambda^2 + \lambda\left(12\lambda_t^2 - \frac{9}{5}g_1^2 - 9g_2^2\right) + \frac{9}{2}\left(\frac{3}{50}g_1^4 + \frac{g_2^4}{2} + \frac{g_1^2 g_2^2}{5}\right) - 12\lambda_t^4\,, && \text{(C.7)}\\
\beta_{m_h^2} &= m_h^2(6\lambda + 6\lambda_t^2 - \frac{9}{2}g_2^2 - \frac{9}{10}g_1^2)\,, && \text{(C.8)}\\
\gamma_h &= 3\lambda_t^2 - \frac{9}{4}g_2^2 - \frac{9}{20}g_1^2. && \text{(C.9)}
\end{aligned}
$$

Between $M_{\mathbf{SUSY}}^{(2)}$ and $M_{\mathbf{SUSY}}^{(3)}$

At scale $M_{\rm SUSY}^{(3)}$ the light sfermions (namely those of the first two families) decouple. The various β_ξ and γ_h in the third family approximation are now:

$$
\begin{aligned}
\beta_{M_1} &= 0\,, && \text{(C.10)}\\
\beta_{M_2} &= -12g_2^2 M_2\,, && \text{(C.11)}\\
\beta_{M_3} &= -18g_3^2 M_3\,, && \text{(C.12)}\\
\beta_{\lambda_t} &= \lambda_t\left(\frac{9}{2}\lambda_t^2 - \frac{17}{20}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2\right)\,, && \text{(C.13)}\\
\beta_\lambda &= 12\lambda^2 + \lambda\left(12\lambda_t^2 - \frac{9}{5}g_1^2 - 9g_2^2\right) + \frac{9}{2}\left(\frac{3}{50}g_1^4 + \frac{g_2^4}{2} + \frac{g_1^2 g_2^2}{5}\right) - 12\lambda_t^4\,, && \text{(C.14)}\\
\beta_{m_h^2} &= m^2(6\lambda + 6\lambda_t^2 - \frac{9}{2}g_2^2 - \frac{9}{10}g_1^2)\,, && \text{(C.15)}\\
\gamma_h &= 3\lambda_t^2 - \frac{9}{4}g_2^2 - \frac{9}{20}g_1^2. && \text{(C.16)}
\end{aligned}
$$

Between $M_{\mathbf{SUSY}}^{(3)}$ and $M_{\mathbf{SUSY}}^{(4)}$

Below $M_{\rm SUSY}^{(3)}$ also the gluinos cease to be part of the theory. β_ξ and γ_h are now given by

$$
\begin{aligned}
\beta_{M_1} &= 0\,, && \text{(C.17)}\\
\beta_{M_2} &= -12g_2^2 M_2\,, && \text{(C.18)}\\
\beta_{\lambda_t} &= \lambda_t\left(\frac{9}{2}\lambda_t^2 - \frac{17}{20}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2\right)\,, && \text{(C.19)}\\
\beta_\lambda &= 12\lambda^2 + \lambda\left(12\lambda_t^2 - \frac{9}{5}g_1^2 - 9g_2^2\right) + \frac{9}{2}\left(\frac{3}{50}g_1^4 + \frac{g_2^4}{2} + \frac{g_1^2 g_2^2}{5}\right) - 12\lambda_t^4\,, && \text{(C.20)}\\
\beta_{m_h^2} &= m_h^2(6\lambda + 6\lambda_t^2 - \frac{9}{2}g_2^2 - \frac{9}{10}g_1^2)\,, && \text{(C.21)}
\end{aligned}
$$

$$\gamma_h = 3\lambda_t^2 - \frac{9}{4}g_2^2 - \frac{9}{20}g_1^2. \qquad (C.22)$$

Below $M^{(4)}_{\mathbf{SUSY}}$

Below $M^{(4)}_{\text{SUSY}}$ the theory is nothing but the SM. The evolution can be obtained as in [146]. For the sake of completeness we report the relevant β_ξ's:

$$\beta_{\lambda_t} = \lambda_t \left(\frac{9}{2}\lambda_t^2 - \frac{17}{20}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2 \right), \qquad (C.23)$$

$$\beta_\lambda = 12\lambda^2 + \lambda \left(12\lambda_t^2 - \frac{9}{5}g_1^2 - 9g_2^2 \right) + \frac{9}{2} \left(\frac{3}{50}g_1^4 + \frac{g_2^4}{2} + \frac{g_1^2 g_2^2}{5} \right) - 12\lambda_t^4, \qquad (C.24)$$

$$\beta_{m_h^2} = m_h^2 (6\lambda + 6\lambda_t^2 - \frac{9}{2}g_2^2 - \frac{9}{10}g_1^2), \qquad (C.25)$$

$$\gamma_h = 3\lambda_t^2 - \frac{9}{4}g_2^2 - \frac{9}{20}g_1^2. \qquad (C.26)$$

C.3 Calculation of the SUSY breaking terms

In this appendix we derive the SUSY breaking terms in our theory. In general such terms can be derived by means of diagrammatic loop calculation in which the SUSY breaking F-term vev enters. In our scenario such a calculation is quite involved, as it would require to consider graphs up to three loops. It is far more convenient to use an approach based on the properties of SUSY theories renormalization. Following the seminal paper [56, 147] we review this method and its realization in our framework.

General theory

As already discussed in section 5.1 we parametrize the breaking of SUSY through the presence of a single chiral superfield X taking vev both in its scalar and auxiliary components, $\langle X \rangle = M/k + \theta^2 F/k$, where k is a coupling constant that has been reabsorbed in the vevs for later convenience. The only source for the appearance of soft terms is the vev F. Thus the SUSY breaking contributions can be casted in an expansion in terms of powers of F, or to be more precise, in terms of the dimensionless parameter F/M^2. If interested in the regime $F \ll M^2$, one can keep track of the SUSY breaking effects in a manifestly supersymmetric framework, considering soft terms just as small modifications of the latter. Being a bit sloppy we can explain the procedure as follows. Let us consider a SUSY gauge theory based on the gauge group G. It is well known that in SUSY theories the renormalization effects are owed to the renormalization of the kinetic terms of gauge and matter, while the superpotential does not renormalize. Thus one can calculate the evolution of the matter wave functions and of the gauge couplings in the SUSY limit from the high cutoff scale Λ_{UV} down to low energies across the scale M using the RGEs and then substitute $M \to \sqrt{XX^\dagger}$ into the gauge real couplings $\mathcal{R}(M,\mu)$[1] and in the

[1] The real coupling is the gauge coupling defined for canonically normalized fields and it is different from the holomorphic coupling. The relation among the two is given in equation C.37.

wave function renormalizations $Z_r(M,\mu)$. Since the X superfield takes both scalar and F-term vevs, such a procedure implies the appearance of SUSY breaking terms in the lagrangian of the theory. Extracting them is then simply a matter of knowing the dependence on F of the various $\mathcal{R}$ and Z_r's.

To be definite we start by considering the high energy theory defined by the lagrangian

$$\begin{aligned}\mathcal{L} \;&=\; \int \mathrm{d}^4\theta \left[Z_M^> \Phi^\dagger e^{V(\Phi)} \Phi + \sum_r Z_r^> Q_r^\dagger e^{V(Q_r)} Q_r \right] \\ &\quad + \int \mathrm{d}^2\theta \frac{1}{2} S^> \,\mathrm{Tr}\Big(W^\alpha W_\alpha \Big) + \text{h.c.} \\ &\quad + \int \mathrm{d}^2\theta (\lambda X \Phi\Phi + \mu_{rs} Q_r Q_s + \lambda_{rst} Q_r Q_s Q_t) + \text{h.c.} \end{aligned} \tag{C.27}$$

where Φ is a chiral messenger (to be general we assume it charged under G) and the Q_r are the matter superfields, X is the chiral superfield taking vevs and μ_{rs}, λ_{rst} are the SUSY superpotential mass terms and Yukawa interactions respectively. Finally $Z_M^>, Z_r^>, S^>$ are the renormalization wave functions above the scale M. At this scale the messenger superfield Φ takes mass and decouples, thus at lower energies one gets

$$\begin{aligned}\mathcal{L} \;&=\; \int \mathrm{d}^4\theta \sum_r Z_r^< Q_r^\dagger e^{V(Q_r)} Q_r + \int \mathrm{d}^2\theta \frac{1}{2} S^< \,\mathrm{Tr}\Big(W^\alpha W_\alpha \Big) + \text{h.c.} \\ &\quad + \int \mathrm{d}^2\theta (\mu_{rs} Q_r Q_s + \lambda_{rst} Q_r Q_s Q_t) + \text{h.c.}\,. \end{aligned} \tag{C.28}$$

The above equations C.27 and C.28 define the theory in which the RG evolution takes place. Once one knows the γ's of the superfields and the β functions of the couplings involved it is possible to determine the $\mathcal{R}(M,\mu)$'s and the $Z_r(M,\mu)$'s (Z_r is defined as $Z_r^>$ above M and $Z_r^<$ below it). By substituting $M \to \sqrt{XX^\dagger}$ and redefining

$$Q_r \to Z_r^{1/2} \left(1 + \frac{1}{2} \frac{\partial \ln Z_r(X, X^\dagger, \mu)}{\partial \ln |X|} \frac{F}{M} \theta^2 \right) Q_r' \tag{C.29}$$

the soft SUSY breaking terms, defined by

$$\mathcal{L}_{soft} = -\frac{1}{2}(M_\lambda \lambda\lambda + \text{h.c.}) - \tilde{m}^2_{Q_r} \tilde{Q}_r^\dagger \tilde{Q}_r - \Big(\sum_r A_r \tilde{Q}_r \partial_{\tilde{Q}_r} W(\tilde{Q}) + \text{h.c.} \Big) \tag{C.30}$$

can be easily extracted:

$$M_\lambda(\mu) = -\frac{1}{2} \frac{\partial \ln \mathcal{R}(X,\mu)}{\partial \ln |X|} \bigg|_{X=M} \frac{F}{M}\,, \tag{C.31}$$

$$\tilde{m}^2_{Q_r}(\mu) = -\frac{1}{4} \frac{\partial^2 \ln Z_r(X, X^\dagger, \mu)}{(\partial \ln |X|)^2} \bigg|_{X=M} \frac{FF^\dagger}{MM^\dagger}\,, \tag{C.32}$$

The reason for the presence of two different couplings is well explained in [148, 149]. In particular while the holomorphic coupling has the nice property of renormalizing just at one loop, the real coupling is the canonically normalized one that actually couples to matter, thus giving the strenght of the interaction.

$$A_r(\mu) = \frac{\partial \ln Z_r(X, X^\dagger, \mu)}{\partial \ln |X|}\bigg|_{X=M} \frac{F}{M}\,. \tag{C.33}$$

In the following sections we compute explicitly the wave function RG evolutions and extract the soft terms.

Coupling evolutions

As seen in equation C.31 the fundamental ingredient to compute gaugino masses M is the knowledge of the X dependence of the real coupling $\mathcal{R}$. The latter can be easily obtained once the evolution of the holomorphic coupling S is known.

Since we wish to calculate the X dependence of the holomorphic coupling at low energies we have to calculate its RG evolution from the high energy scale $\Lambda_{\rm UV}$ down to the scale μ accross the threshold μ_X of the physical messenger scale [149, 150],

$$\mu_X^2 = \frac{X X^\dagger}{Z_M^2(\mu_X)}\,. \tag{C.34}$$

The evolution can be split in two different regions, namely above (where quantities are denoted with the superscript $>$) and below (where quantities are denoted with the superscript $<$) the scale μ_X. The coupling at μ_X is related to the high energy one by the simple formula

$$S^>(\mu_X) = S^>(\Lambda) + \frac{b^>}{16\pi^2}\ln\frac{\mu_X}{\Lambda}\,, \tag{C.35}$$

where $b = 3T_G - \sum_\phi T_\phi$ and ϕ runs over all the matter fields present in the theory at a certain scale; T_ϕ is the Dynkin index of the representation ϕ (the subscript G refers as usual to the adjoint representation). Writing the equivalent formula below μ_X one obtains the low energy value of the holomorphic coupling after applying matching conditions at μ_X scale,

$$S(\mu) = S^>(\Lambda) + \frac{b^>}{16\pi^2}\ln\frac{\mu_X}{\Lambda} + \frac{b^<}{16\pi^2}\ln\frac{\mu}{\mu_X}\,. \tag{C.36}$$

The relation between the holomorphic coupling and the interaction one is given by [148, 149]

$$\mathcal{R}(\mu) = \mathrm{Re}\Big(S(\mu)\Big) + \frac{T_G}{8\pi^2}\ln \mathrm{Re}\Big(S(\mu)\Big) - \sum_r \frac{T_r}{8\pi^2}\ln Z_r(\mu)\,, \tag{C.37}$$

where by $\mathrm{Re}(S)$ we mean the real part of S and the Z_r's are the wave functions of the matter fields. From the previous considerations on S we can easily obtain the matching condition

$$\begin{aligned}\mathcal{R}^>(\mu_X) \;&=\; \mathcal{R}^>(\Lambda) + \frac{b^>}{16\pi^2}\ln\frac{\mu_X^2}{\Lambda^2} + \frac{T_G}{8\pi^2}\ln\frac{\mathrm{Re}\Big(S^>(\mu_X)\Big)}{\mathrm{Re}\Big(S^>(\Lambda)\Big)} \\ &\quad - \sum_r \frac{T_r}{8\pi^2}\ln\frac{Z_r^>(\mu_X)}{Z_r^>(\Lambda)} - \frac{T_M}{8\pi^2}\ln\frac{Z_M(\mu_X)}{Z_M(\Lambda)}\,,\end{aligned} \tag{C.38}$$

that yields for the low energy real coupling

$$\mathcal{R}(\mu) = \mathcal{R}^{>}(\mu_X) + \frac{b_i^<}{16\pi^2}\ln\frac{\mu^2}{\mu_X^2} + \frac{T_G}{8\pi^2}\ln\frac{\mathrm{Re}\Big(S^<(\mu)\Big)}{\mathrm{Re}(S^<\Big(\mu_X)\Big)} - \sum_r \frac{T_r}{8\pi^2}\ln\frac{Z_r^<(\mu)}{Z_r^<(\mu_X)}\,. \tag{C.39}$$

Z_r evolution

The calculation of soft terms is just a step away: we still have to calculate the wave function renormalization dependence on X. Once we will have done that the whole determination of the SUSY breaking parameters will be straightforward.

The RG evolution of the matter fields is strictly connected to the knowledge of the γ functions of the fields under consideration. In particular it is governed by the well known differential equation

$$\frac{\mathrm{d}\ \ln Z_r}{\mathrm{d}t} = \gamma_r\,. \tag{C.40}$$

The integration of such a differential equation across the scale μ_X is the simple task to obtain the wave function Z of the field r: indeed one gets

$$\ln Z_r(\mu) = \int_\Lambda^{\mu_X} \mathrm{d}t\, \gamma_r^>(t) + \int_{\mu_X}^{\mu} dt\, \gamma_r^<(t,\mu_X)\,. \tag{C.41}$$

Soft terms

All the ingredients for the determination of the soft SUSY breaking terms are now at our disposal, thus it is easy to perform the needed calculations.

Gaugino masses

As we saw in equation C.31 gaugino masses are given by

$$M_\lambda(\mu) = -\frac{1}{2}\frac{\partial\ \ln\mathcal{R}(\mu)}{\partial\ \ln|X|}\frac{F}{M}\,.$$

It is easy to recast the derivation with respect to $\ln|X|$ in the following way:

$$\frac{\partial}{\partial\ \ln|X|} = \frac{\partial\ \ln\mu_X}{\partial\ \ln|X|}\frac{\partial}{\partial\ \ln\mu_X}\,.$$

As we saw in equation (C.34) the scale μ_X is given by

$$\mu_X^2 = \frac{XX^\dagger}{Z_M^2(\mu_X)}\,, \tag{C.42}$$

so that

$$\frac{\partial\ \ln\mu_X}{\partial\ \ln|X|} = 1 - \frac{\partial\ \ln Z_M(\mu_X)}{\partial\ \ln|X|} \sim 1 - \frac{\partial\ \ln Z_M(\mu_X)}{\partial\ \ln\mu_X} = 1 - \gamma_M^>\,.$$

Calculating the gaugino mass is now just a matter of bookkeeping:

$$
\begin{aligned}
M_\lambda = & -\frac{1}{2}\frac{\partial\,\ln\mu_X}{\partial\,\ln|X|}\frac{\partial\,\ln\mathcal{R}(\mu)}{\partial\,\ln\mu_X}\frac{F}{M} = -\frac{1}{2\mathcal{R}(\mu_X)}\Big(1-\gamma_M^>\Big)\Bigg[\frac{b^> - b^<}{8\pi^2} \\
& + \frac{T_G}{8\pi^2}\frac{1}{\mathrm{Re}\Big(S(\mu_X)\Big)}\left(\frac{b^> - b^<}{8\pi^2}\right) - \sum_r \frac{T_r}{8\pi^2}\Big(\gamma_r^> - \gamma_r^<\Big) - \frac{T_M}{8\pi^2}\gamma_M^>\Bigg]. \qquad \text{(C.43)}
\end{aligned}
$$

Sfermion masses

The soft masses for the low energy fields of the theory are given by equation C.32

$$
\tilde{m}^2_{Q_r}(\mu) = -\frac{1}{4}\frac{\partial^2\,\ln Z_r(\mu)}{\partial\,\ln|X|^2}\frac{FF^\dagger}{MM^\dagger},
$$

thus we just have to deal with the derivatives of Z_r as calculated in equation C.41. The first derivative with respect to μ_X is

$$
\begin{aligned}
\frac{\partial\,\ln Z_r(\mu)}{\partial\,\ln\mu_X} &= \frac{\partial}{\partial\,\ln\mu_X}\left[\int_\Lambda^{\mu_X}\mathrm{d}t\,\gamma_r^>(t) + \int_{\mu_X}^{\mu}\mathrm{d}t\,\gamma_r^<(t,\mu_X)\right] \\
&= \gamma_r^>(\mu_X) - \gamma_r^<(\mu_X) + \int_{\mu_X}^{\mu}\mathrm{d}t\,\frac{\partial\,\gamma_r^<(t,\mu_X)}{\partial\,\ln\mu_X}, \qquad \text{(C.44)}
\end{aligned}
$$

and the second one is

$$
\frac{\partial^2\,\ln Z_r(\mu)}{\partial\,\ln\mu_X^2} = \frac{\partial}{\partial\,\ln\mu_X}\Big(\gamma_r^>(\mu_X) - \gamma_r^<(\mu_X)\Big) - \left.\frac{\partial\,\gamma_r^<(t,\mu_X)}{\partial\,\ln\mu_X}\right|_{\mu_X}. \qquad \text{(C.45)}
$$

Suppose we now consider the generic coupling λ. Its RG evolution is controlled by the β_λ function defined through

$$
\frac{\mathrm{d}\,\lambda}{\mathrm{d}t} = \beta_\lambda\,. \qquad \text{(C.46)}
$$

The formal solution of equation C.46 from the high scale Λ_{UV} down to low energies across the scale μ_X is

$$
\lambda(\mu) = \lambda(\Lambda_{\mathrm{UV}}) + \int_\Lambda^{\mu_X}\mathrm{d}t\,\beta_\lambda^>(t) + \int_{\mu_X}^{\mu}\mathrm{d}t\,\beta_\lambda^<(t,\mu_X)\,. \qquad \text{(C.47)}
$$

By means of the above equations we can easily translate the derivation with respect to the scale μ_X to the one with respect to the running couplings of the theory using the known β functions. In particular we obtain

$$
\tilde{m}^2_{Q_r} = -\frac{1}{4}\Big(1-\gamma_M^>\Big)^2\left(\frac{\partial\Delta\gamma_r}{\partial\lambda}\beta_\lambda^> - \frac{\partial\gamma_r^<}{\partial\lambda}\Delta\beta_\lambda\right)\frac{FF^\dagger}{MM^\dagger}, \qquad \text{(C.48)}
$$

where $\Delta\gamma_r = \gamma_r^> - \gamma_r^<$ and $\Delta\beta_\lambda = \beta_\lambda^> - \beta_\lambda^<$ are defined to be the difference of the shown quantities above and below the μ_X scale.

Trilinears

In order to complete the computation of the soft SUSY breaking terms we have to focus on the trilinears terms A_{rst} of equation C.28. In general the computation of the latter can be obtained through the summation over the vertex corrections owed to the different fields involved in the interaction. In particular one obtains that for any of the field of the interaction it is possible to write, as shown in C.33,

$$A_i(\mu) = \frac{\partial \ln Z_i(\mu)}{\partial \ln |X|} \frac{F}{M}\,, \tag{C.49}$$

that following the same procedure of the previos subsection yields

$$A_i(\mu) = \left(1 - \gamma_M^{>}\right) \Delta\gamma_i \frac{F}{M}\,. \tag{C.50}$$

The trilinear soft term entering the lagrangian can now be easily obtained by summing the contribution coming from any of the vertices entering the diagram, thus the resulting SUSY breaking lagrangian term will be

$$A_{rst}\tilde{Q}_r\tilde{Q}_s\tilde{Q}_t = \left(1 - \gamma_M^{>}\right) \left(\sum_{i=r,s,t} \Delta\gamma_i \, \tilde{Q}_i \partial_{\tilde{Q}_i} W(\tilde{Q}) \right) \frac{F}{M}\,. \tag{C.51}$$

SUSY breaking contributions in our model

In the following we write all the SUSY breaking soft terms in our framework calculated using the formulae just calculated. To easily spot the numbers of loop factors at which any of the following terms arise we use $\mathrm{Lp} = (4\pi)^{-2}$.

Trilinears

$$A_U = \mathrm{Lp}\, h_0^2 \, \lambda_U \, B_\phi \tag{C.52a}$$
$$A_D = \mathrm{Lp}\, h_0^2 \, \lambda_D \, B_\phi \tag{C.52b}$$
$$A_E = \mathrm{Lp}\, h_0^2 \, \lambda_E \, B_\phi \tag{C.52c}$$

Bilinear terms

$$B = \mathrm{Lp}\, h_0^2 \, \mu \, B_\phi \tag{C.53}$$

Gaugino masses

$$M_1 = \mathrm{Lp}^2 \, g_1^2 \left(\frac{3}{5} h_0^2 + \frac{2}{5} h_t^2 \right) B_\phi \tag{C.54a}$$
$$M_2 = \mathrm{Lp}^2 \, g_2^2 \, h_0^2 \, B_\phi \tag{C.54b}$$
$$M_3 = \mathrm{Lp}^2 \, g_3^2 \, h_t^2 \, B_\phi \tag{C.54c}$$

Soft squared masses: two loop contributions

$$m_q^{2\,(2)} = \frac{\mathrm{Lp}^2}{2}\, h_0^2 \left(\lambda_D^\dagger \lambda_D + \lambda_U^\dagger \lambda_U\right) B_\phi^2 \tag{C.55a}$$

$$m_l^{2\,(2)} = \frac{\mathrm{Lp}^2}{2}\, h_0^2\, \lambda_E^\dagger \lambda_E\, B_\phi^2 \tag{C.55b}$$

$$m_{u^c}^{2\;(2)} = \mathrm{Lp}^2\, h_0^2\, \lambda_U \lambda_U^\dagger\, B_\phi^2 \tag{C.55c}$$

$$m_{d^c}^{2\;(2)} = \mathrm{Lp}^2\, h_0^2\, \lambda_D \lambda_D^\dagger\, B_\phi^2 \tag{C.55d}$$

$$m_{e^c}^{2\;(2)} = \mathrm{Lp}^2\, h_0^2\, \lambda_E \lambda_E^\dagger\, B_\phi^2 \tag{C.55e}$$

$$m_{H_u}^{2\;(2)} = \frac{\mathrm{Lp}^2}{2}\, h_0^2 \left[\frac{3g_1^2}{5} + 3g_2^2 - 4h_0^2 - 3h_t^2 - \mathrm{Tr}\left(\lambda_E^\dagger \lambda_E + 3\lambda_D^\dagger \lambda_D\right) - 2\eta^2\right] B_\phi^2 \tag{C.55f}$$

$$m_{H_d}^{2\;(2)} = \frac{\mathrm{Lp}^2}{2}\, h_0^2 \left[\frac{3g_1^2}{5} + 3g_2^2 - 4h_0^2 - 3h_t^2 - 3\,\mathrm{Tr}\left(\lambda_U^\dagger \lambda_U\right) - 2\eta^2\right] B_\phi^2 \tag{C.55g}$$

Soft squared masses: three loop contributions

$$\begin{aligned} m_q^{2\,(3)} = \mathrm{Lp}^3 \Bigg\{ & h_0^2 \Bigg[\frac{1}{50} g_1^4 + \frac{3}{2} g_2^4 - 2\left(\lambda_D^\dagger \lambda_D \lambda_D^\dagger \lambda_D + \lambda_U^\dagger \lambda_U \lambda_U^\dagger \lambda_U\right) \\ & - \left(\frac{1}{3} g_1^2 + 3g_2^2 + \frac{8}{3} g_3^2 - 3h_0^2 - 3h_t^2 - 2\eta^2\right)\left(\lambda_U^\dagger \lambda_U + \lambda_D^\dagger \lambda_D\right)\Bigg] \\ & + \left(\frac{1}{75} g_1^4 + \frac{8}{3} g_3^4\right) h_t^2 \Bigg\} B_\phi^2 \end{aligned} \tag{C.56a}$$

$$\begin{aligned} m_l^{2\,(3)} = \mathrm{Lp}^3 \Bigg\{ & h_0^2 \Bigg[\frac{9}{50} g_1^4 + \frac{3}{2} g_2^4 - 2\lambda_E^\dagger \lambda_E \lambda_E^\dagger \lambda_E \\ & - \left(\frac{3}{5} g_1^2 + 3g_2^2 - 3h_0^2 - 3h_t^2 - 2\eta^2\right)\lambda_E^\dagger \lambda_E\Bigg] + \frac{3}{25} g_1^4 h_t^2 \Bigg\} B_\phi^2 \end{aligned} \tag{C.56b}$$

$$\begin{aligned} m_{u^c}^{2\;(3)} = \mathrm{Lp}^3 \Bigg\{ & h_0^2 \Bigg[\frac{8}{25} g_1^4 + 2\left(\lambda_D \lambda_D^\dagger + \lambda_U \lambda_U^\dagger\right)\lambda_U \lambda_U^\dagger \\ & - \left(\frac{5}{3} g_1^2 + 3g_2^2 + \frac{16}{3} g_3^2 - 6h_0^2 - 6h_t^2 - 4\eta^2\right)\lambda_U \lambda_U^\dagger\Bigg] \\ & + h_t^2 \left(\frac{16}{75} g_1^4 + \frac{8}{3} g_3^4\right) \Bigg\} B_\phi^2 \end{aligned} \tag{C.56c}$$

$$m_{d^c}^{2\;(3)} = \mathrm{Lp}^3 \Bigg\{ h_0^2 \Bigg[\frac{2}{25} g_1^4 - 2\left(\lambda_U \lambda_U^\dagger + \lambda_D \lambda_D^\dagger\right)\lambda_D \lambda_D^\dagger$$

$$
\begin{aligned}
&- \Big(\frac{13}{15}g_1^2 + 3g_2^2 + \frac{16}{3}g_3^2 - 6h_0^2 - 6h_t^2 - 4\eta^2\Big)\lambda_D\lambda_D^\dagger\Bigg] \\
&+ h_t^2\Big(\frac{4}{75}g_1^4 + \frac{8}{3}g_3^4\Big)\Bigg\}B_\phi^2 \qquad \text{(C.56d)}
\end{aligned}
$$

$$
\begin{aligned}
m_{e^c}^{2\,(3)} =& \,\mathrm{Lp}^3\Bigg\{h_0^2\Bigg[\frac{18}{25}g_1^4 - 2h_0^2\lambda_E\lambda_E^\dagger\lambda_E\lambda_E^\dagger \\
&- \Big(3g_1^2 + 3g_2^2 - 6h_0^2 - 6h_t^2 - 4\eta^2\Big)\lambda_E\lambda_E^\dagger\Bigg] + \frac{12}{25}g_1^4h_t^2\Bigg\}B_\phi^2 \qquad \text{(C.56e)}
\end{aligned}
$$

$$
\begin{aligned}
m_{H_u}^{2\,(3)} =& \,\mathrm{Lp}^3\Bigg\{h_0^2\Bigg[-\frac{201}{100}g_1^4 - \frac{9}{10}g_1^2g_2^2 - \frac{9}{4}g_2^4 - \frac{6}{5}g_1^2h_0^2 - 6g_2^2h_0^2 + 9h_0^4 \\
&- \Big(\frac{7}{5}g_1^2 + 9g_2^2 + 6g_3^2 - 15h_0^2 - 9\,\mathrm{Tr}(\lambda_D^\dagger\lambda_D) - 3\,\mathrm{Tr}(\lambda_E^\dagger\lambda_E)\Big)\,\mathrm{Tr}(\lambda_D^\dagger\lambda_D) \\
&- \Big(\frac{9}{5}g_1^2 + 3g_2^2 - 5h_0^2 + h_0^2\,\mathrm{Tr}(\lambda_E^\dagger\lambda_E)\Big)\,\mathrm{Tr}(\lambda_E^\dagger\lambda_E) + 9h_0^2\,\mathrm{Tr}(\lambda_U^\dagger\lambda_U) \\
&- 3\,\mathrm{Tr}(\lambda_E^\dagger\lambda_E\lambda_E^\dagger\lambda_E) + 3\,\mathrm{Tr}(\lambda_E^\dagger\lambda_E\lambda_D^\dagger\lambda_D) + 9\,\mathrm{Tr}(\lambda_D^\dagger\lambda_D\lambda_D^\dagger\lambda_D) \\
&- 9\,\mathrm{Tr}(\lambda_U^\dagger\lambda_U\lambda_U^\dagger\lambda_U) + h_t^2\Big(\frac{1}{10}g_1^2 + \frac{9}{2}g_2^2 - 16g_3^2 + 3h_0^2 + 6h_t^2 + 12\eta^2\Big) \\
&+ \eta^2\Big(\frac{3}{5}g_1^2 + 3g_2^2 + 10h_0^2 + 8\eta^2\Big)\Bigg] + \frac{3}{25}g_1^4h_t^2\Bigg\} \qquad \text{(C.56f)}
\end{aligned}
$$

$$
\begin{aligned}
m_{H_d}^{2\,(3)} =& \,\mathrm{Lp}^3\Bigg\{h_0^2\Bigg[-\frac{201}{100}g_1^4 - \frac{9}{10}g_1^2g_2^2 - \frac{9}{4}g_2^4 - \frac{6}{5}g_1^2h_0^2 - 6g_2^2h_0^2 + 9h_0^4 \\
&- \Big(\frac{13}{5}g_1^2 + 9g_2^2 + 16g_3^2 - 15h_0^2 - \mathrm{Tr}(\lambda_U^\dagger\lambda_U)\Big)\,\mathrm{Tr}(\lambda_U^\dagger\lambda_U) \\
&+ 9\,\mathrm{Tr}(\lambda_U^\dagger\lambda_U\lambda_U^\dagger\lambda_U) - 9\,\mathrm{Tr}(\lambda_D^\dagger\lambda_D\lambda_D^\dagger\lambda_D) - 3\,\mathrm{Tr}(\lambda_E^\dagger\lambda_E\lambda_E^\dagger\lambda_E) \\
&+ 9h_0^2\,\mathrm{Tr}(\lambda_D^\dagger\lambda_D) + 3h_0^2\,\mathrm{Tr}(\lambda_E^\dagger\lambda_E) + h_t^2\Big(\frac{1}{10}g_1^2 + \frac{9}{2}g_2^2 - 16g_3^2 + 3h_0^2 \\
&+ 6h_t^2\Big) + \frac{3}{5}g_1^2\eta^2 + 3g_2^2\eta^2 + 10h_0^2\eta^2 + 12h_t^2\eta^2 + 8\eta^4\Bigg] + \frac{3}{25}g_1^4ht^2\Bigg\} \\
&\qquad \text{(C.56g)}
\end{aligned}
$$

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