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Composite Higgs Models and Extra Dimensions

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Contents

Chapter 1

Introduction

The Standard Model (SM) of particle physics is so far our best understanding of nature at the fundamental level. This theory which describes the constituents of matter and their interactions has stood firmly after many years of experimental scrutiny. However, despite all its success and agreement with experimental data, the SM is still filled with problems and puzzles. To name few: i) among the particles of the SM there is no candidate for the observed dark matter of the universe, ii) the amount of CP violation in the SM is not enough to explain the asymmetry between matter and antimatter, iii) neutrino masses call for physics beyond the SM, although possibly at a very high energy scale, iv) the SM does not incorporate gravity, v) an unreasonable amount of fine tuning is required to account for the Higgs mass, known as the gauge hierarchy problem. These are all motivations to develop ideas and look for physics beyond the SM. In particular the gauge hierarchy problem will be the main motivation behind the work presented here and will be explained in more detail in the following.

The experimental status has finally come to a point to probe the weak scale. Recently both ATLAS and CMS collaborations at the Large Hadron Collider (LHC) have announced the discovery of a 125 GeV mass particle [1, 2] compatible with the long sought Higgs boson [3, 4, 5] , the last missing piece of the SM. Whether this is exactly the SM Higgs boson or not needs further investigation.

It has long been known that the mass of the Higgs boson in the SM suffers from quadratic divergences. specifically the main contribution to the Higgs mass comes from the top quark loop

$$m_h^2 = 2\mu^2 - \frac{3|Y_t|^2}{8\pi^2}\Lambda_{NP}^2$$

where Λ_{NP} is the cut-off of the theory, or where new physics shows up, and $\sqrt{2}\mu$ is the bare mass of the Higgs. On the other hand, apart from the recent discovery of a 125 GeV Higgs, there has been indirect constraints from precision electroweak measurements which put an upper bound on the mass of the Higgs roughly below 200 GeV. Although the bare parameter μ is absolutely free and one can adjust it to get the desired value for the Higgs mass, if new physics does not show up until very high energies, say the Planck energy, one must tune the bare mass to great accuracy which is highly unnatural. In other words the bare mass is extremely sensitive to the scale of new physics Λ_{NP}. This is the statement of the hierarchy problem. The general expectation is that there must be some new dynamics, not much above the weak scale, which is responsible for stabilizing the Higgs mass. This has also been the main motivation for the construction of the LHC.

There are two main classes of theories, namely weakly coupled and strongly coupled models, which aim to address this problem. Theories based on Supersymmetry, a symmetry which relates fermions and bosons, is an example of weakly coupled theories. In such theories the quadratic divergences cancel among fermion and boson loop contributions, solving in this way the hierarchy problem. A typical example of strongly coupled theories beyond the SM is Technicolor [6]. In Technicolor theories, inspired by chiral symmetry breaking in QCD, ElectroWeak Symmetry Breaking (EWSB) is triggered by strong dynamics. Also the smallness of the scale of strong dynamics which is essentially the weak scale is naturally explained, much like Λ_{QCD}, by renormalization group running. However these theories are highly constrained by precision electroweak data. Furthermore, incorporating flavour into these theories is quite challenging. A variant of these theories [7] which generally perform better under ElectroWeak Precision Tests (EWPT) is based on the idea that electroweak symmetry is not broken at the strong scale where bound states are formed, but rather strong dynamics will give rise, possibly among other bound states, to a composite Pseudo Goldstone Boson (PGB) with the quantum numbers of the Higgs which in turn breaks electroweak symmetry by acquiring a Vacuum Expectation Value (VEV). The PGB nature of this Higgs particle will guarantee its lightness, as its mass will be generated only radiatively. A common obstacle before model building in such theories is that due to strong coupling explicit computation of precision electroweak observables as well as the Higgs potential is not possible and one has to rely on arguments such as naive dimensional analysis to estimate these quantities.

As we extend the space dimensions, other interesting possibilities arise. One interesting idea is to identify the internal component of the gauge field in a higher dimensional theory with compact extra dimensions as the Higgs field, the so called Gauge-Higgs Unification (GHU) scenarios [8]. In such models the gauge symmetry forbids a tree level potential for the Higgs, while the loop contribution, being a non-local effect, is finite. Another proposal introduced in [9] as a solution to the gauge hierarchy problem was to take a slice of AdS_5 and localize the matter fields on the IR brane, in this way the physical masses will be exponentially suppressed by the warp factor with respect to the mass parameters on the IR brane, while the Planck mass is insensitive to the compactification scale for small warp factors. It was noticed, though, that to provide a solution to the hierarchy problem it is only necessary for the Higgs to be localized close to the IR brane and embedding the matter fields in the bulk will give rise to a richer flavour structure.

Inspired by the AdS/CFT correspondence one can think of these extra dimensional theories in terms of their 4D dual descriptions which are strongly coupled conformal field theories [10]. In particular, GHU models in warped space can be interpreted as weakly coupled duals of 4D strongly coupled field theories [11] in which the Higgs arises as a PGB of the strong sector [12, 13]. In fact such a correspondence does not necessarily exist in the exact AdS/CFT sense, in which case the bulk of the 5D theory along with the IR boundary can be considered as the definition of the 4D strongly interacting sector.

In this work we will be interested in GHU models as they provide, due to their weak coupling nature, calculable examples of composite Higgs scenarios in which the Higgs potential and electroweak precision observables can be computed explicitly. Unfortunately model building in warped space is still a challenge and although a few 5D GHU models have been constructed so far [13, 14, 15], only in one model [16] (a modified version of a model introduced in [14] to accommodate a Dark Matter candidate) 1-loop corrections to S, T and δg_b (and the Higgs potential explicitly determined) have been analyzed and the EWPT successfully passed. However, as far as we are concerned with low energy phenomenology, we do not really need to consider the technically challenging warped models. Instead, we can rely on the much simpler flat space implementations of the GHU idea. The resulting models may still be reinterpreted as calculable 5D descriptions of 4D strongly coupled composite Higgs models. This is guaranteed by the holographic interpretation, which shows that the low-energy symmetries of the theory are independent of the specific form of the 5D metric.

The most constraining electroweak bounds on GHU models are given by the S and T parameters [17, 18] and by the deviation δg_b of the coupling between the left-handed (LH) bottom quark and the Z vector boson from its SM value. Couplings $g_{bt,R}$ between the right-handed (RH) top and bottom quarks with the $W^{\pm}$ vector bosons should also be taken into account, given the rather stringent experimental bounds on them of $\mathcal{O}(10^{-3})$ [19].

Unfortunately, the simplest constructions of GHU models in flat space (see [20] for an overview and for earlier references) turned out to be not fully satisfactory (see e.g. [21]). One of the reasons for this failure was the lack of some custodial protection mechanism for the electroweak precision parameters. If custodial symmetries are introduced, the situation improves but this is still not enough to build realistic theories, since one gets too low top and Higgs masses. Another key ingredient are the so called Boundary Kinetic Terms (BKT) [22]. When these are introduced and taken to be large, potentially realistic models can be constructed. In fact the BKT will be quantum mechanically generated any way [23] but small and large BKT are stable against radiative corrections.

This thesis is organized as follows. Chapter 2 gives an introduction to the subject starting with a discussion of composite Higgs models where the Higgs arises as a PGB. This is followed by introducing the Minimal Composite Higgs Model (MCHM) as a promising example. A simple fact regarding deconstructed models is then addressed as a link to theories with an extra dimension. Chapter 3 is devoted to certain 5D theories and their holographic interpretation. In the first part of this chapter the holographic method for gauge fields is discussed. This method is then applied to the gauge sector of the minimal composite Higgs model in 5D flat space with boundary kinetic terms. A comparison with the more standard KK approach is made afterwards. Introduction to the notion of holography for fermions will be the final part if this chapter. In chapter 4 we introduce three composite-Higgs/GHU models in flat space and study in detail their compatibility with EWPT. The gauge sector of these models is that of the MCHM described previously, and they differ in the way the SM fermions are embedded in complete multiplets of the bulk gauge group. In the first model, known as MCHM_5, fermions are embedded in four fundamental representations. In the second model fermions are embedded in two fundamentals, while in the third model they are embedded in one adjoint representation. Fermionic kinetic terms are also introduced on the UV boundary in the last two models. Details of the computation of the 1-loop corrections to the $Z\bar{b}_L b_L$ vertex are collected in Appendix A. In Appendix B after a short introduction to EWPT, 1-loop computation of electroweak precision observables as well as the χ^2 fit performed for our three models are explained in detail.

Chapter 2

Composite Higgs Models

2.1 Introduction

In a general model of composite Higgs, apart from the elementary sector, one postulates the existence of a strongly interacting sector which incorporates the Higgs as a bound state and mixes with the elementary sector. A more promising scenario which naturally gives rise to a light Higgs is when the strong sector has a global symmetry G, which is broken by strong dynamics at some scale f, the analogue of the pion decay constant, to a subgroup $H_1 \in G$ and results in a number of Goldstone Bosons, including one with the quantum numbers of the Higgs. This global symmetry is not exact, hence leading to a PGB which has a mass generated only at 1-loop order. In fact this symmetry is explicitly broken by the mixing terms with the elementary sector. Another source of explicit breaking of G is that a subgroup H_0 of G, which includes the electroweak gauge group $G_{SM} = SU(2)_L \times U(1)_{em}$, is gauged. So G_{SM} is in fact included in the intersection of H_0 and H_1. This situation is described schematically in fig.(2.1). The number of Goldstone bosons resulting from the spontaneous symmetry breaking $G \to H$ is dim(G) - dim(H_1), among which dim(H_0) - dim($H_0 \cap H_1$) are eaten to give mass to the gauge bosons, so we are left with a number of dim(G) - dim($H_0 \cup H_1$) PGBs in the theory. As we mentioned above, G_{SM} must be included in $H_0 \cap H_1$, so the minimal choice would be to take $H_0 = H_1 = G_{SM}$. To have a Higgs doublet among the PGBs we need to have dim(G) - dim(H_1) ≥ 4, the simplest possibility is then to take $G = SU(3)$ [1]. However this model will give large corrections to the Peskin-Takeuchi T parameter due to the absence of a custodial symmetry[2]. To have a custodial symmetry, the subgroup H_1 has to be extended

[1] This choice will lead to an incorrect prediction for the mixing angle $\sin^2\theta_W = 3/4$, which can be avoided by introducing BKT [12, 24] , which also helps overcome the problem of too low top and Higgs masses [24].

[2] See section B.1.1

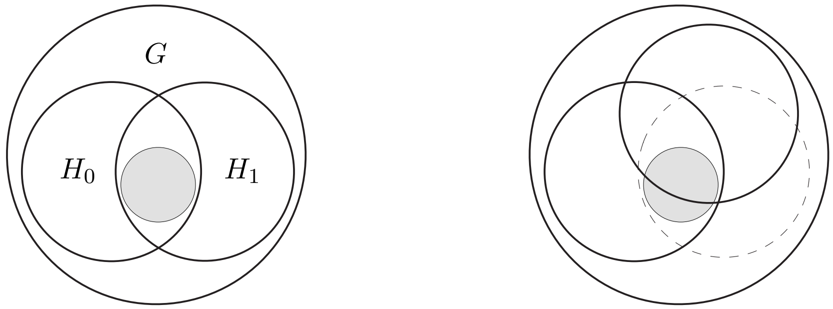


Figure 2.1: The left diagram shows the pattern of symmetry breaking at the strong scale G/H_1 and the gauged subgroup H_0. The gray blob represents G_{SM}. After EWSB, the vacuum misaligns with the G_{SM} preserving direction, this is depicted by the diagram on the right.

$H_1 = SU(2)_L \times SU(2)_R \simeq SO(4)$. In this case the minimal choice for the global symmetry is $G = SO(5)$ which gives rise to the Higgs as the only PGB. In the following we will restrict ourselves to this more promising scenario based on the $SO(5)/SO(4)$ symmetry breaking pattern. Before moving to the discussion of this model in more detail, we will briefly discuss, as a preliminary step, the tool for parametrizing the Goldstone bosons arising from a general symmetry breaking pattern G/H.

2.2 The CCWZ prescription

In this section we review briefly the prescription introduced in [25, 26] for writing down low energy effective Lagrangians for theories with spontaneous symmetry breaking. Consider a global symmetry group G acting on a field configuration, and suppose that the fields pick up a VEV and break G down to a subgroup H. The authors of [25] have classified all possible nonlinear transformation laws of fields in a neighbourhood of the VEV which can be summarized in the following way. As we know every element of G can be written in a unique way as a product of the form

$$e^{\pi^{\hat{a}} T^{\hat{a}}} h' \tag{2.1}$$

where $T^{\hat{a}}$ are the broken generators of G, and h' is an element of the unbroken subgroup H. As a result, for any element $g \in G$ one can write

$$g\, e^{\pi^{\hat{a}} T^{\hat{a}}} = e^{\pi'^{\hat{a}} T^{\hat{a}}} h \tag{2.2}$$

in which $\pi'^{\hat{a}}$ and $h \in H$, obviously, depend on $\pi^{\hat{a}}$ and g. It was shown that with a suitable redefinition of the fields, one can always find a "standard" parametrization, which we call $(\pi^{\hat{a}}, \psi^i)$ [3] here, with the following transformation laws under g

$$\pi^{\hat{a}} \to \pi'^{\hat{a}}, \qquad \psi^i \to D(h)^i_j \psi^j \tag{2.3}$$

where $\pi'^{\hat{a}}$ and h are given by eq.(2.2), and D is a linear representation of H. Note that one can equivalently work with $U \equiv \exp(\pi^{\hat{a}} T^{\hat{a}})$ rather than $\pi^{\hat{a}}$ in which case, according to (2.2), the transformation under $g \in G$ is

$$U \to g U h^{-1}(U, g). \tag{2.4}$$

Notice also that $\pi'^{\hat{a}}$ will transform linearly under the subgroup H. Clearly any transformation of the form (2.3) will also give a nonlinear realization of G. So all the pairs $(\pi^{\hat{a}}, \psi^i)$ with the transformation law (2.3) span all possible nonlinear realizations of the group G.

In a subsequent work [26], Callan, Coleman, Wess and Zumino (CCWZ) classified all the G invariant Lagrangians constructed out of $\pi^{\hat{a}}$ (or equivalently U), ψ^i and their derivatives. the fields $\pi^{\hat{a}}$, ψ^i are already in the standard form described above, but their derivatives are not. It turns out the standard form of their derivatives can be given in term of the two fields $d^{\hat{a}}_\mu$ and E^a_μ defined by

$$U^{-1} \partial_\mu U = d^{\hat{a}}_\mu T^{\hat{a}} + E^a_\mu T^a \tag{2.5}$$

where T^a are the unbroken generators of G. In fact $d^{\hat{a}}_\mu$ is the derivative of $\pi^{\hat{a}}$ in the standard form, and the standard form of the derivative of ψ^i is given by

$$\nabla_\mu \psi^i \equiv \partial_\mu \psi^i + E^a_\mu\, \rho(T^a)^i_j \psi^j \tag{2.6}$$

where ρ is the representation of the algebra of G associated with the Lie group representation D. from eq.(2.5) one can easily read off the transformations of $d_\mu \equiv d^{\hat{a}}_\mu T^{\hat{a}}$ and $E_\mu \equiv E^a_\mu T^a$ under $g \in G$

$$d_\mu \to h d_\mu h^{-1}, \qquad E_\mu \to h E_\mu h^{-1} + h \partial_\mu h^{-1} \tag{2.7}$$

[3] we have used here the same symbol $\pi^{\hat{a}}$ as the fields parametrizing the the group elements generated by $T^{\hat{a}}$

from which the transformation of $\nabla_\mu \psi$ also follows

$$\nabla_\mu \psi^i \to D(h)^i_j \nabla_\mu \psi^j \tag{2.8}$$

A G invariant Lagrangian constructed out of $\pi^{\hat{a}}$, ψ^i and their derivatives $\partial_\mu \pi^{\hat{a}}$ and $\partial_\mu \psi^i$, is equivalent to one which is constructed out of U, ψ^i, d_μ, and $\nabla_\mu \psi^i$. But in writing the Lagrangian in terms of these new fields, the field U cannot appear explicitly, because it can be eliminated by a U^{-1} transformation, while under such a transformation, ψ^i, d_μ, and $\nabla_\mu \psi^i$ will not change because of the fact that $h(U, U^{-1}) = 1$.

At higher derivatives the quantity $E_{\mu\nu} = \partial_\mu E_\nu - \partial_\nu E_\mu - [E_\mu, E_\nu]$ also appears (second derivative) which transforms covariantly $E_{\mu\nu} \to h E_{\mu\nu} h^{-1}$, as well as derivatives of ψ, d_μ and $E_{\mu\nu}$ of arbitrary order, using $\nabla_\mu \equiv \partial_\mu + E^a_\mu T^a$, in which T^a must be understood in the appropriate representation. This gives the general form of a G invariant Lagrangian. From the way that these fields transform it is clear that constructing G invariants is equivalent to constructing invariants under local H transformations.

One can generalize to the case of continuous G transformations by introducing gauge fields $A_\mu = A^{\hat{a}}_\mu T^{\hat{a}} + A^a_\mu T^a$ with the usual transformation law $A_\mu \to g A_\mu g^{-1} - g \partial_\mu g^{-1}$ under the group G, and modify eq.(2.5) to

$$U^{-1}(\partial_\mu - A_\mu)U = d^{\hat{a}}_\mu T^{\hat{a}} + E^a_\mu T^a. \tag{2.9}$$

It is easy to see that for d_μ and E_μ defined in the above way, the transformations (2.7) are still valid. Note that E_μ is now of first order in the derivatives. In this gauged case two other ingredients are added to the Lagrangian, namely $F^+_{\mu\nu}$ and $F^-_{\mu\nu}$ which are the projections of $U^{-1} F_{\mu\nu} U$ along the unbroken and broken generators respectively, with $F_{\mu\nu}$ being the field strength of A_μ. These fields clearly transform as $F^\pm_{\mu\nu} \to h F^\pm_{\mu\nu} h^{-1}$ under $g \in G$.

2.3 The Minimal Composite Higgs Model

We will introduce in this section the model based on the $SO(5)/SO(4)$ symmetry breaking pattern which was originally introduced in the context of 5D theories [13]. According to the discussion of section 2.1 this symmetry breaking pattern will give rise to 4 PGBs which form a fundamental representation of $SO(4)$ or equivalently a bidoublet of $SU(2)_L \times SU(2)_R$. However, in order to reproduce the correct hypercharges for the fermions we need to introduce a $U(1)_X$ factor under which the fermions are charged with appropriate X charges. So we will consider a strong sector with global symmetry $SO(5) \times U(1)_X$ which is broken by strong interactions to $SO(4) \times U(1)_X$. The subgroup G_{SM} is gauged with the hypercharge generator defined by $Y = T^3_R + X$.

Using the CCWZ formalism introduced in the previous section, one can immediately write down the effective Lagrangian for the SM gauge fields and the Goldstone bosons.

Since we are finally interested in the Higgs potential we will ignore the derivatives of the Higgs field and treat it as a constant field. To find the effective Lagrangian we promote the SM gauge fields to complete representations of $SO(5)$ by adding spurious fields which will be finally set to zero, and write the most general Lagrangian in terms of these fields which are invariant under local G transformations.

Before proceeding we define the basis for the algebra of $SO(5)$ that we will use throughout this thesis. These are chosen such that T_L^a and T_R^a, respectively the generators of $SU(2)_L$ and $SU(2)_R$ are given by

$$\begin{aligned}(T_L^a)_{ij} &= \frac{i}{2}(\epsilon^{abc}(\delta_j^b\delta_i^c - \delta_i^b\delta_j^c) + (\delta_j^4\delta_i^a - \delta_j^a\delta_i^4)) \\ (T_R^a)_{ij} &= \frac{i}{2}(\epsilon^{abc}(\delta_j^b\delta_i^c - \delta_i^b\delta_j^c) - (\delta_j^4\delta_i^a - \delta_j^a\delta_i^4))\end{aligned} \tag{2.10}$$

with $a = 1, 2, 3$, while the broken generators are

$$(T_B^k)_{ij} = \frac{i}{\sqrt{2}}(\delta_i^k\delta_j^5 - \delta_j^k\delta_i^5) \tag{2.11}$$

with $k = 1, 2, 3, 4$. We also use throughout the text the notation T^a (whose first three components correspond to T_L^a and the second three to T_R^a) and $T^{\hat{a}}$ (instead of T_B^k). Now lets introduce the following notation and parametrization of the Goldstone bosons

$$\Phi = U\Phi_0, \qquad U = e^{i\Pi/f}, \qquad \Pi = -\sqrt{2}h^{\hat{a}}T^{\hat{a}}, \qquad \Phi_0^T = (0,0,0,0,1). \tag{2.12}$$

The effective Lagrangian at quadratic order and in momentum space then reads

$$\mathcal{L}_{\text{eff}} = \frac{1}{2}P_{\mu\nu}^T\left[\Pi_0^X(p^2)X^\mu X^\nu + \Pi_0(p^2)\text{Tr}[A^\mu A^\nu] + \Pi_1(p^2)\Phi^T A^\mu A^\nu\Phi\right]. \tag{2.13}$$

The form factors parametrize the strong sector which is integrated out. By considering this Lagrangian in the $SO(4)$ preserving vacuum one can conclude that the form factors Π_0^X and Π_0 vanish at zero momentum, while Π_1 does not. This follows from local $SO(4)$ invariance and also by Large N arguments [27] and the fact that there is a massless excitation with the quantum numbers of the broken generators (see also [28]). Expanding U in its exponent, one can easily write

$$\Phi = \left(c_h + i\frac{s_h}{h}\Pi\right)\Phi_0, \qquad s_h \equiv \sin(h/f),\ \ c_h \equiv \cos(h/f), \qquad h \equiv \sqrt{h^{\hat{a}}h^{\hat{a}}}. \tag{2.14}$$

Using this, the Lagrangian (2.13) is written as

$$\begin{aligned}\mathcal{L}_{\text{eff}} &= \frac{1}{2}P_{\mu\nu}^T\Big[\Pi_0^X(p^2)X^\mu X^\nu + \Pi_0(p^2)(A_L^{a\mu}A_L^{a\nu} + A_R^{3\mu}A_R^{3\nu}) \\ &+ \frac{s_h^2}{4}\Pi_1(p^2)(A_L^{a\mu}A_L^{a\nu} + A_R^{3\mu}A_R^{3\nu}) + \frac{s_h^2}{2h^2}\Pi_1(p^2)A_{L\mu}^a A_{R\nu}^3 H^\dagger\sigma^a H\Big]\end{aligned} \tag{2.15}$$

where

$$H \equiv \begin{pmatrix} -ih^1 - h^2 \\ h^4 + ih^3 \end{pmatrix}. \tag{2.16}$$

After setting to zero the spurious fields and keeping only the physical fields, $A_{L\mu} = W_\mu$ and $A^3_{R\mu} = X_\mu = B_\mu$, and adding possible bare kinetic terms for them, the above Lagrangian will lead to

$$\begin{aligned}\mathcal{L}_{\text{eff}} = \frac{1}{2}P^T_{\mu\nu}\Bigg[&\left(-\frac{p^2}{g_0'^2} + \Pi^X_0(p^2) + \Pi_0(p^2) + \frac{s_h^2}{4}\Pi_1(p^2)\right)B^\mu B^\nu \\ &+\left(-\frac{p^2}{g_0^2} + \Pi_0(p^2) + \frac{s_h^2}{4}\Pi_1(p^2)\right)W^{a\mu}W^{a\nu} + \frac{s_h^2}{h^2}\Pi_1(p^2)W^a_\mu B_\nu H^\dagger \frac{\sigma^a}{2}H\Bigg].\end{aligned} \tag{2.17}$$

Expanding this Lagrangian in momentum and keeping terms of at most second order gives, after EWSB

$$\begin{aligned}\mathcal{L}_{\text{eff}} = \frac{1}{2}P^T_{\mu\nu}\Bigg[&\frac{s_h^2}{4}\Pi_1(0)\left(W^{a\mu}W^{a\nu} + B^\mu B^\nu - 2W^{3\mu}B^\nu\right) \\ &+p^2\left(-\frac{1}{g_0'^2} + \Pi'^X_0(0) + \Pi'_0(0) + \frac{s_h^2}{4}\Pi'_1(0)\right)B^\mu B^\nu \\ &+p^2\left(-\frac{1}{g_0^2} + \Pi'_0(0) + \frac{s_h^2}{4}\Pi'_1(0)\right)W^{a\mu}W^{a\nu} - p^2\frac{s_h^2}{2}\Pi'_1(0)W^3_\mu B_\nu\Bigg]\end{aligned} \tag{2.18}$$

from which one can identify the gauge couplings of W^a_μ and B_μ

$$\frac{1}{g^2} = \frac{1}{g_0^2} - \Pi'_0(0) - \frac{s_h^2}{4}\Pi'_1(0), \qquad \frac{1}{g'^2} = \frac{1}{g_0'^2} - \Pi'^X_0(0) - \Pi'_0(0) - \frac{s_h^2}{4}\Pi'_1(0). \tag{2.19}$$

This low energy (second order) effective Lagrangian could also be found, in coordinate space and including the derivatives of U, using the ingredients introduced in the previous section. The result is

$$\begin{aligned}\mathcal{L} \;=\; &-\frac{1}{4g_0^2}W^2_{\mu\nu} - \frac{1}{4g_0'^2}B^2_{\mu\nu} - \frac{1}{4\tilde{g}_+^2}\text{Tr}\left[F^+_{\mu\nu}F^{+\mu\nu}\right] - \frac{1}{4\tilde{g}_-^2}\text{Tr}\left[F^-_{\mu\nu}F^{-\mu\nu}\right] - \frac{1}{4\tilde{g}'^2}X^2_{\mu\nu} \\ &+\frac{f^2}{4}\text{Tr}\left[d_\mu d^\mu\right], \qquad A_\mu = W^a_\mu T^a_L + B_\mu T^3_R, \quad X_\mu = B_\mu\end{aligned} \tag{2.20}$$

where id_μ, iE_μ are the projections of $U^\dagger(\partial_\mu - iA_\mu)U$ along the broken and unbroken generators, and $F^\pm_{\mu\nu}$ are projections of $U^\dagger F_{\mu\nu}U$ along the broken and unbroken generators respectively. In fact we did not need to use $E_{\mu\nu}$ (which includes, apart from terms of first derivative order, also terms with two derivatives) as our building block of constructing the

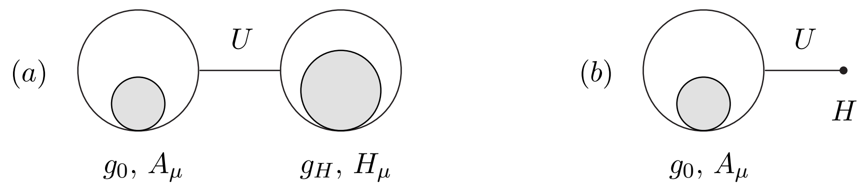


Figure 2.2: (a) 2-site moose diagram. Both sites have global symmetry G with the gray blobs representing the gauged subgroups H_0 on the left site and H on the right site. (b) 1-site diagram. The line with a small black circle on the end means that the NLSM field U parametrizes G/H

Lagrangian, because the special quotient space we are considering here, $SO(5)/SO(4)$, is a symmetric space, so $E_{\mu\nu}$ is related to $F^+_{\mu\nu}$ and d_μ through the equation (see e.g. [29])

$$E_{\mu\nu} = F^+_{\mu\nu} + i[d_\mu, d_\nu]. \tag{2.21}$$

Now, one can write (after EWSB) the two kinetic terms involving $F^\pm_{\mu\nu}$ as

$$\begin{aligned} \mathrm{Tr}\left[F^+_{\mu\nu}F^{+\mu\nu}\right] &= \left(W^2_{\mu\nu} + B^2_{\mu\nu}\right)\left(1 - s_h^2/2\right) + W_{\mu\nu}B^{\mu\nu}s_h^2 \\ \mathrm{Tr}\left[F^-_{\mu\nu}F^{-\mu\nu}\right] &= \left(W^2_{\mu\nu} + B^2_{\mu\nu}\right)s_h^2/2 - W_{\mu\nu}B^{\mu\nu}s_h^2. \end{aligned} \tag{2.22}$$

These relations show that the kinetic terms of the two Lagrangians (2.15) and (2.20) have exactly the same structure, as they should, and one can make the following identifications

$$\Pi'^X_0(0) = -\frac{1}{\tilde{g}'^2}, \quad \Pi'_0(0) = -\frac{1}{\tilde{g}'^2_+}, \quad \Pi'_1(0) = -\frac{2}{\tilde{g}'^2_-} + \frac{2}{\tilde{g}'^2_+}. \tag{2.23}$$

The term on the second line of (2.20) can be written in a more transparent way by adopting the "physical" basis for the Goldstone bosons, defined in the following way

$$\Phi = e^{i\vec{\chi}\cdot\vec{t}/v} r(\theta + h/f)\Phi_0 = \begin{pmatrix} \sin(\theta + \frac{h}{f})\widetilde{\Sigma}\phi_0 \\ \cos(\theta + \frac{h}{f}) \end{pmatrix}, \tag{2.24}$$

where we have defined

$$r(\theta) \equiv e^{-i\sqrt{2}\theta T^{\hat{4}}}, \qquad \widetilde{\Sigma} \equiv e^{i\vec{\chi}\cdot\vec{t}/v}, \qquad \vec{t} \equiv \vec{T}_L - \vec{T}_R, \qquad \phi_0^T \equiv (0,0,0,1) \tag{2.25}$$

t^a are the generators of $SO(4)$ that are broken after EWSB, the three fields χ^a are the Goldstone bosons that are eaten to give mass to the gauge fields, and h is the physical Higgs field with VEV θf. Using this parametrization one can show that (see [29])

$$\frac{f^2}{4}\mathrm{Tr}\left[d_\mu d^\mu\right] = \frac{f^2}{2}D_\mu\Phi^T D^\mu\Phi = \frac{1}{2}\partial_\mu h\partial^\mu h + \frac{f^2}{4}\mathrm{Tr}\left[(D_\mu\Sigma)^\dagger(D^\mu\Sigma)\right]\sin^2\left(\theta + \frac{h}{f}\right), \tag{2.26}$$

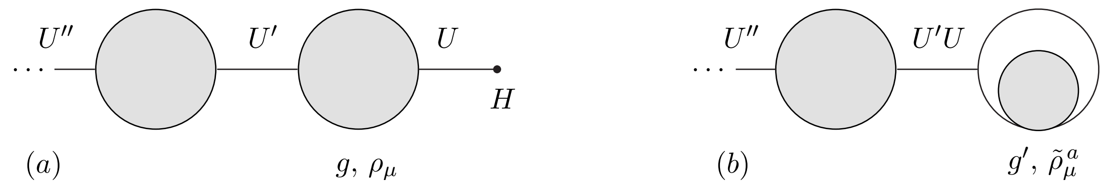


Figure 2.3: Diagram (b) results from performing a gauge transformation by U^{-1} and integrating out the gauge components along the broken generators in the rightmost site of diagram (a). The field U parametrizes G/H.

where $\Sigma \equiv e^{i\vec{\chi}\cdot\vec{\sigma}/v}$ and $D_\mu\Sigma = \partial_\mu\Sigma - i\vec{W}_\mu \cdot \vec{\sigma}/2\ \Sigma + iB_\mu\Sigma\ \sigma^3/2$. This expression also reproduces the mass term of eq.(2.15) with the identification

$$\Pi_1(0) = f^2. \tag{2.27}$$

By expanding the factor $\sin^2(\theta + h/f)$, one can find the definition of the weak scale in terms of the parameters of the theory $v = f\sin\theta$, as well as the Higgs couplings with gauge bosons which are modified with respect to the SM couplings

$$\sin^2\left(\theta + \frac{h}{f}\right) = \sin^2\theta\left[1 + 2\cos\theta\ \frac{h}{v} + \cos 2\theta\ \frac{h^2}{v^2} + \cdots\right]. \tag{2.28}$$

The two parameters a, b appearing in the low energy parametrization of Higgs interactions with gauge fields introduced in [30] are then given by

$$a = \cos\theta = \sqrt{1 - \frac{v^2}{f^2}}, \qquad b = \cos 2\theta = 1 - 2\,\frac{v^2}{f^2}. \tag{2.29}$$

For $\theta = 0$ or equivalently $f \to \infty$ the SM couplings are recovered, while $\theta = \pi/2$ ($f = v$) gives the Technicolor limit.

2.4 Composite Higgs and deconstructed models

As we mentioned in the introduction, certain extra dimensional theories are holographically dual to models of composite Higgs. It might be instructive to see this, first in the context of deconstructed models [31] before moving to theories with extra dimensions in the next chapter. In fact these theories can be considered as the simplest calculable versions of composite Higgs models. We do not aim to give an introduction to deconstructed models or composite Higgs models in this context [4] but rather address the above mentioned specific aspect of such theories as a warm up and introduction to the subject of the next chapter which will also be addressed there in a different way.

Consider a theory described by a moose diagram of $N+2$ sites, with the middle sites representing N copies of a gauge theory with gauge group G while the leftmost and rightmost sites are gauge theories based on the two subgroups H_0 and H_1 respectively, and with all the sites linked together through NLSM fields U_i, $i = 1, \cdots, N+1$. The gauge fields in middle sites are denoted by ρ_μ and on the leftmost and rightmost sites by A_μ and H_μ respectively. This setup is shown schematically by the diagram of fig.(2.4a), which is a discretized version of a theory with an extra dimension. Our aim is to start from the diagram of fig.(2.4a) and integrate out the fields on all the sites but the leftmost one. As the first step, consider the diagram of fig.(2.2a) which consists of only two sites. The Lagrangian of this 2-site model to second derivative order is given by

$$\mathcal{L} = -\frac{1}{4g_0^2}\mathrm{Tr}\,A_{\mu\nu}^2 - \frac{1}{4g_H^2}\mathrm{Tr}\,H_{\mu\nu}^2 + \frac{f^2}{2}\mathrm{Tr}\big(D_\mu U^\dagger D^\mu U\big), \qquad D_\mu U = \partial_\mu U - iA_\mu U + iUH_\mu \tag{2.30}$$

which includes the kinetic terms of the two gauge fields on the left and right site and the nonlinear sigma model (NLSM) field U, making a link between them. It is useful to write the kinetic term for U in a different way by the following simple manipulation

$$\begin{aligned}\mathrm{Tr}\big(D_\mu U^\dagger D^\mu U\big) &= -\mathrm{Tr}\big(U^\dagger D^\mu U\big)^2 = -\mathrm{Tr}\big[U^\dagger(\partial_\mu - iA_\mu)U + iH_\mu\big]^2 \\ &= \mathrm{Tr}\big(d_\mu + E_\mu + H_\mu\big)^2 = \mathrm{Tr}\,d_\mu^2 + \mathrm{Tr}\big(E_\mu + H_\mu\big)^2\end{aligned} \tag{2.31}$$

where we have used the definitions of d_μ and E_μ given by $U^\dagger(\partial_\mu - iA_\mu)U \equiv id_\mu + iE_\mu$. Doing this, one can now easily integrate out the gauge field H_μ on the right site to get

$$\mathcal{L} = -\frac{1}{4g_0^2}\mathrm{Tr}\,A_{\mu\nu}^2 - \frac{1}{4g_H^2}\mathrm{Tr}\,E_{\mu\nu}^2 + \frac{f^2}{2}\mathrm{Tr}\,d_\mu^2 \tag{2.32}$$

which represents the low energy Lagrangian of a theory with global symmetry G, whose subgroup H_0 is gauged, and is broken to its subgroup H. This is schematically shown in fig.(2.2) by the diagram on the right. The above argument shows that starting with the theory of fig.(2.4a), integrating out the field H_μ on its rightmost site leads to the diagram of fig.(2.3a), of which the Lagrangian of the rightmost site along with the NLSM field U is given by

$$\mathcal{L} = -\frac{1}{4g^2}\mathrm{Tr}\,\rho_{\mu\nu}^2 - \frac{1}{4g_H^2}\mathrm{Tr}\,E_{\mu\nu}^2 + \frac{f^2}{2}\mathrm{Tr}\,d_\mu^2 \tag{2.33}$$

where now d_μ and E_μ are defined by $U^\dagger(\partial_\mu - i\rho_\mu)U \equiv id_\mu + iE_\mu$. As the second step, we

[4] For recent work on composite Higgs models in the context of deconstructed theories see [32]

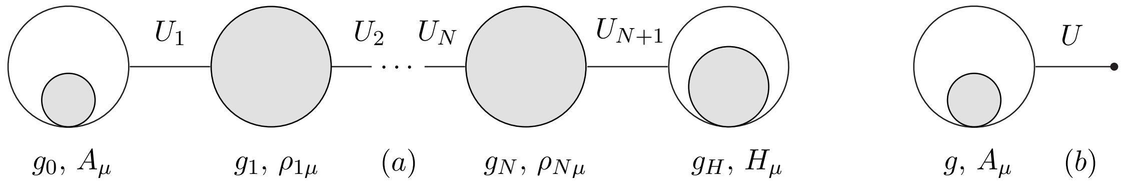


Figure 2.4: (a)An $N+2$ site moose diagram, with gauge symmetry G on the middle sites and a global G with gauged subgroups H_0 and H_1, respectively on the left and right site. (b) 1-site diagram resulting after integrating out all the sites on the right. Here $U \equiv U_1 \cdots U_{N+1}$.

perform a gauge transformation by the element $g = U^{-1}$ of the gauge group on the last site on the right and arrive at

$$\mathcal{L} = -\frac{1}{4g^2}\mathrm{Tr}\,\tilde{\rho}_{\mu\nu}^2 - \frac{1}{4g_H^2}\left(\tilde{\rho}_\mu^a\right)^2 + \frac{f^2}{2}\left(\tilde{\rho}_\mu^{\hat{a}}\right)^2, \qquad \tilde{\rho}_\mu \equiv U^\dagger(\rho_\mu + i\partial_\mu)U \tag{2.34}$$

In the process, the field U' is also multiplied by U on the right. The above Lagrangian shows that the field components along the broken generators $\rho_\mu^{\hat{a}}$ are massive, with a mass squared equal to $f^2 g^2$. Integrating out these fields, which is done trivially, will lead us to the Lagrangian

$$\mathcal{L} = -\frac{1}{4g'^2}\left(\tilde{\rho}_\mu^a\right)^2, \qquad \frac{1}{g'^2} \equiv \frac{1}{g^2} + \frac{1}{g_H^2}, \tag{2.35}$$

which corresponds to the rightmost site of the diagram in fig.(2.3b). This brings us back to a diagram of the same type as the one we started with, but with one site less and with modified fields and couplings

$$\frac{1}{g_N^2} \to \frac{1}{g_N^2} + \frac{1}{g_H^2}, \qquad U_N \to U_N U_{N+1}, \qquad \rho_N \to \tilde{\rho}_N^a, \tag{2.36}$$

where $\tilde{\rho}_N = U_{N+1}^\dagger(\rho_\mu + i\partial_\mu)U_{N+1}$. Repeating this procedure, one can integrate out all the N sites, which leaves us with a theory described schematically by fig.(2.4b) with Lagrangian

$$\mathcal{L} = -\frac{1}{4g_0^2}\mathrm{Tr}\,A_{\mu\nu}^2 - \frac{1}{4}\left(\sum_{k=1}^{N+1}\frac{1}{g_k^2}\right)\mathrm{Tr}\,E_{\mu\nu}^2 + \frac{f_1^2}{2}\mathrm{Tr}\,d_\mu^2, \qquad g_{N+1} = g_H \tag{2.37}$$

where now d_μ and E_μ are defined in the usual way $U^\dagger(\partial_\mu - iA_\mu)U \equiv id_\mu + iE_\mu$ but with $U \equiv U_1 \cdots U_{N+1}$. The Lagrangian (2.37) gives the low energy description of a theory with the approximate global symmetry G, which is broken explicitly by gauging a subgroup H_0, and is broken at low energies to its subgroup H.

Chapter 3

5D Theories and Holography

3.1 Gauge fields in 5D

This section is devoted to the study of Gauge fields in 5 dimensions, with an emphasis on the holographic approach [33, 34]. We will try to be general at the beginning and restrict to special cases of our interest as we go on. Consider a 5D space which consists of the usual 4D space-time along with a spatial interval extra dimension $\mathbb{R}^4 \times [z_0, z_1]$. The 4D spaces located at $z = z_0$ and $z = z_1$ will be called UV and IR boundaries respectively. The 5D coordinates are labelled by capital Latin indices $M = (\mu, z)$ where $\mu = 0, \cdots, 3$ represent the usual 4D Lorentz indices and z is the fifth component. The metric on this space is taken to be

$$ds^2 = a(z)^2 \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right) \equiv a(z)^2 \eta_{MN} dx^M dx^N \tag{3.1}$$

with the Minkowski metric $\eta_{\mu\nu}$ having signature $\eta = (+, -, -, -)$ and $a(z)$ assumed to be positive. Consider a gauge field F^a_{MN} in the bulk, associated with a simple group G. The Yang-Mills Lagrangian is

$$\mathcal{L}_{Bulk} = -\frac{a(z)}{4g_5^2} F^a_{MN} F^{aMN} \tag{3.2}$$

where the capital Latin indices are raised and lowered with the metric η_{MN}. We also introduce the gauge fixing term

$$\mathcal{L}^{G.F}_{Bulk} = -\frac{a(z)}{2\xi g_5^2} \left(\partial_\mu A^{a\mu} - \xi a(z)^{-1} \partial_z (a(z) A^a_z)\right)^2 \tag{3.3}$$

which is chosen such that the mixing term between A_μ and A_z cancels that of the bulk. By varying the bulk Lagrangian

$$g_5^2 \delta\mathcal{L}_{Bulk} = \partial_M \left(a(z) F^{aMN}\right) \delta A^a_N - a(z) f^{cba} F^{cMN} A^b_M \delta A^a_N - \partial_z \left(a(z) F^{az\mu} \delta A^a_\mu\right) + \partial_\mu(\cdots), \tag{3.4}$$

and the gauge fixing term

$$\begin{aligned}\delta\mathcal{L}^{G.F}_{Bulk} &= \frac{a(z)}{\xi g_5^2}\partial_\mu\left(\partial_\nu A^{a\nu} - \xi a(z)^{-1}\partial_z(a(z)A^a_z)\right)\delta A^a_\mu \\ &- \frac{a(z)}{\xi g_5^2}\partial_z\left(\partial_\nu A^{a\nu} - \xi a(z)^{-1}\partial_z(a(z)A^a_z)\right)\delta A^a_z \\ &+ \frac{1}{\xi g_5^2}\partial_z\left[\left(\partial_\nu A^{a\nu} - \xi a(z)^{-1}\partial_z(a(z)A^a_z)\right)a(z)\delta A^a_z\right],\end{aligned} \tag{3.5}$$

one can find the bulk e.o.m for A_μ and A_z

$$\begin{cases}\partial_\nu\partial^\nu A^a_\mu + \left(\xi^{-1}-1\right)\partial_\mu\partial_\nu A^{a\nu} - a(z)^{-1}\partial_z\left(a(z)\partial_z A^a_\mu\right) = 0\\ \partial_\nu\partial^\nu A^a_z - \xi\,\partial_z\left(a(z)^{-1}\partial_z\left(a(z)A^a_z\right)\right) = 0.\end{cases} \tag{3.6}$$

The first equation above can be further decomposed into two equations for the transverse and longitudinal components defined by $A^a_{T\mu} = \left(\eta_{\mu\nu} - \partial_\mu\partial_\nu/\partial^2\right)A^{a\nu}$ and $A^a_{L\mu} = \partial_\mu\partial_\nu/\partial^2\, A^{a\nu}$

$$\begin{cases}\partial_\nu\partial^\nu A^a_{T\mu} - \partial_\mu\partial_\nu A^{a\nu}_T - a(z)^{-1}\partial_z\left(a(z)\partial_z A^a_{T\mu}\right) = 0\\ \xi^{-1}\partial_\mu\partial_\nu A^{a\nu}_L - a(z)^{-1}\partial_z\left(a(z)\partial_z A^a_{L\mu}\right) = 0.\end{cases} \tag{3.7}$$

Also the boundary e.o.m can be read off from the last terms in the variations (3.4) and (3.5)

$$\begin{cases}F^{az\mu}\delta A^a_\mu = 0\\ \left(\partial_\mu A^{a\mu} - \xi a(z)^{-1}\partial_z\left(a(z)A^a_z\right)\right)\delta A^a_z = 0.\end{cases} \tag{3.8}$$

There are two possibilities to satisfy the first equation, to keep A^a_μ fixed on the boundary or to let it satisfy $F^a_{\mu z} = 0$. The first condition breaks the gauge symmetry along the corresponding generator on the boundary, while the second condition will preserve it. A set of consistent b.c that we will be using throughout this work is

$$\begin{cases}0 = A^a_\mu & (-)\\ 0 = \partial_z(a(z)A^a_z) & (+)\end{cases}, \qquad \begin{cases}0 = \partial_z A^a_\mu & (+)\\ 0 = A^a_z & (-)\end{cases} \tag{3.9}$$

Note that the condition $\partial_z A^a_\mu = 0$ is equivalent to $F^a_{\mu z} = 0$ when accompanied by $A_z = 0$. So we will choose the b.c on the left for the field components corresponding to the generators that are broken on the boundary, and the conditions on the right for the components along the unbroken generators.

In the Feynman gauge $\xi \to 1$, If we denote the mass spectrum of the transverse and longitudinal modes by m_n and the mass spectrum of the fifth component by $\tilde{m}_n$, which of course depend on the boundary conditions, then in a general gauge the spectrum for the transverse mode, the longitudinal mode, and the fifth component will be m_n, $m_n\sqrt{\xi}$ and $\tilde{m}_n\sqrt{\xi}$ respectively. This shows that the spectrum of the longitudinal modes and the fifth component of the gauge field will become infinitely heavy in the unitary gauge $\xi \to \infty$ and fade away from the theory, unless there is a zero mode. To seek the condition under which a zero mode exists for the the fifth component of the gauge field, we concentrate on the massless solution of the second equation in (3.6)

$$\partial_z \left(a(z) A^a_z \right) = c\, a(z) \tag{3.10}$$

where c is a constant. According to the b.c (3.9), if A_z has $(+)$ b.c on one end of the extra dimension, it must have $(+)$ b.c on the other end as well because of the positivity of $a(z)$. But also the solution to (3.10) cannot vanish on both ends because $a(z)A_z(a)$ is a monotonically increasing or decreasing function of z, depending on the sign of c. So out of the four b.c $(\pm,\pm)$ and $(\pm,\mp)$, it is only $(+,+)$ that gives rise to a non vanishing solution, which is

$$f^{(0)}_5(z) = \frac{N}{a(z)}, \qquad N\int_{z_0}^{z_1} \frac{dz}{a(z)} = 1 \tag{3.11}$$

where N is the normalization factor. A similar argument is valid for A^a_μ whose zero mode has a flat profile. Note that in this case also the ghost fields will have a zero mode.

3.1.1 Holography for gauge fields

A standard way to treat extra dimensional theories is to expand the fields in a complete set of mass eigenstates, the so called Kaluza-Klein approach. One can then integrate out heavy fields to get a low energy effective theory. However, if we are concerned with S-matrix elements with external states which are non vanishing on a boundary, one can take an alternative approach and take boundary values of the fields as external states, this does not affect S-matrix elements. In this case all we need is an effective action in terms of the boundary fields, so one can integrate out the bulk and arrive at a 4D effective action which we will call the holographic action. We will discuss this approach for gauge fields in this section. Next section will be devoted to fermions.

Consider a gauge symmetry G in the bulk which is broken to H on the IR and to H_0 on the UV boundary. According to the above definition, the holographic action as a functional of the UV boundary value C_μ is computed by the integrating over the bulk field while keeping its UV value fixed at C_μ and with the additional IR constraints compatible with the symmetry breaking pattern

$$e^{iS_{Hol}[C_\mu]} \equiv \int_{A^{\rm uv}_\mu = C_\mu} DA_M\, e^{iS[A_M]}, \qquad F^a_{\mu z}\Big|_{IR} = 0, \quad A^{\hat{a}}_\mu\Big|_{IR} = 0. \tag{3.12}$$

This holographic action is invariant under all gauge group elements g such that $g_{IR} \in H$ and $g_{UV} = 1$. According to the argument of [35] one can add extra d.o.f on the IR brane and at the same time enlarge the symmetry by removing the constraint $g_{IR} \in H$. This is done by introducing the Goldstone field Σ on the IR brane, which parametrizes G/H, and with the usual transformation law (2.4) under G. In this way we get an equivalent action

$$e^{iS_{\text{hol}}[C_\mu,\Sigma]} \equiv \int_{A_\mu^{\text{uv}}=C_\mu} DA_M \; e^{iS[A_M]}, \qquad \left(F^{\Sigma^{-1}}\right)^a_{\mu z}\Big|_{IR} = 0, \quad \left(A^{\Sigma^{-1}}\right)^{\hat{a}}_{\mu}\Big|_{IR} = 0. \tag{3.13}$$

The IR constraints are now G invariant. In fact under the group element g

$$F \to F^g, \qquad \Sigma \to \Sigma^g, \qquad F^{\Sigma^{-1}} \to (F^g)^{(g\Sigma)^{-1}} = F^{(g\Sigma)^{-1}g} = F^{\Sigma^{-1}}, \tag{3.14}$$

with a similar transformation for $A_\mu^{\hat{a}}$. Now, by applying

$$g = P\exp\left(-i\int_{z_{\text{uv}}}^{z} dz\, A_z\right) \tag{3.15}$$

which is equal to the identity on the UV brane, the z-component of the gauge field is set to zero everywhere $A_z = 0$. By renaming the dummy variable $A_\mu \to A_\mu^\Sigma$ we arrive at the action 3.12 but with the UV b.c $A_\mu|_{UV} = C_\mu^{\Sigma^{-1}}$ and a vanishing A_z.

$$e^{iS_{\text{hol}}[C_\mu,\Sigma]} = \int_{A_\mu^{\text{uv}}=C_\mu^{\Sigma^{-1}}} DA_\mu \; e^{iS[A_\mu,A_z=0]}, \qquad F^a_{\mu z}\Big|_{IR} = 0, \quad A^{\hat{a}}_\mu\Big|_{IR} = 0 \tag{3.16}$$

3.1.2 Flat Space Example

We will illustrate the method described in the previous section with the simple example of a gauge field on a flat extra dimension, that we choose to be $[0, L]$ throughout this work, by computing the holographic Lagrangian at tree level and at quadratic order in the fields [20]. We start with the Yang-Mills action in the axial gauge $A_z = 0$

$$S = -\frac{1}{4g_5^2}\int dz\,\text{Tr}\left[F_{MN}F^{MN}\right] \stackrel{A_z=0}{\longrightarrow} -\frac{1}{4g_5^2}\int dz\,\text{Tr}\left[F_{\mu\nu}F^{\mu\nu} - 2\partial_z A_\mu \partial_z A^\mu\right]. \tag{3.17}$$

In this case the bulk e.o.m in momentum space read

$$(p^2 + \partial_z^2)A_T^\mu = 0, \qquad \partial_z^2 A_L^\mu = 0. \tag{3.18}$$

For a fixed value $A_\mu(p,0)$ of the gauge field at $z=0$, depending on the $(+)$ or $(-)$ b.c at $z=L$ the solutions are

$$\begin{aligned}
A^a_{\mu T}(p,z) &= G^+_T(p,z)A^a_{\mu T}(p,0) & G^+_T(p,z) &= \cos pz + \tan pL \sin pz \\
A^{\hat a}_{\mu T}(p,z) &= G^-_T(p,z)A^{\hat a}_{\mu T}(p,0) & G^-_T(p,z) &= \cos pz - \cot pL \sin pz \\
A^a_{\mu L}(p,z) &= G^+_L(p,z)A^a_{\mu L}(p,0) & G^+_L(p,z) &= 1 \\
A^{\hat a}_{\mu L}(p,z) &= G^-_L(p,z)A^{\hat a}_{\mu L}(p,0) & G^-_L(p,z) &= 1-\tfrac{z}{L}
\end{aligned} \tag{3.19}$$

plugging these solutions in (3.17) we find the holographic action

$$\begin{aligned}
S &= -\frac{1}{g_5^2}\int dz\, \mathrm{Tr}\left[A_{\mu T}(p^2+\partial_z^2)A^\mu_T + A_{\mu L}\partial_z^2 A^\mu_L - \partial_z(A_\mu \partial_z A^\mu)\right] \\
&\xrightarrow{\text{on shell}} -\frac{1}{2g_5^2}\mathrm{Tr}\left[A_\mu\partial_z A^\mu\right]|_{z=0} = -\frac{1}{2g_5^2}\mathrm{Tr}\left[A_{\mu T}\partial_z A^\mu_T + A_{\mu L}\partial_z A^\mu_L\right]|_{z=0},
\end{aligned} \tag{3.20}$$

where we have used the fact that $A_\mu\partial_z A^\mu = 0$ on the IR brane because either $A_\mu = 0$ or $\partial_z A_\mu = 0$. Using the definitions (3.19) this can be written as

$$\begin{aligned}
S_{Hol} = -\frac{1}{2g_5^2}\mathrm{Tr}\Big[&G^+_T\partial_z G^+_T A^a_{\mu T}A^{\mu a}_T + G^-_T\partial_z G^-_T A^{\hat a}_{\mu T}A^{\mu\hat a}_T \\
&+G^+_L\partial_z G^+_L A^a_{\mu L}A^{\mu a}_L + G^-_L\partial_z G^-_L A^{\hat a}_{\mu L}A^{\mu\hat a}_L\Big]\Big|_{z=0}.
\end{aligned} \tag{3.21}$$

Assuming $\langle\pi^{\hat a}\rangle = 0$ we have $A_\mu = C_\mu^{\Sigma^{-1}} = \Sigma(C_\mu - i\partial_\mu)\Sigma^\dagger = C_\mu - \frac{\sqrt2}{f_\pi}\partial_\mu\pi$ which in momentum space becomes $C_\mu - i\frac{\sqrt2}{f_\pi}p_\mu\pi$, where $C_\mu = C^a_\mu T^a + C^{\hat a}_\mu T^{\hat a}$ and $\pi = \pi^{\hat a}T^{\hat a}$. So to linear order in $\pi^{\hat a}$

$$\begin{aligned}
A^a_{\mu T}(p,0) &= C^a_{\mu T}(p) & A^a_{\mu L}(p,0) &= C^a_{\mu L}(p) \\
A^{\hat a}_{\mu T}(p,0) &= C^{\hat a}_{\mu T}(p)\ , & A^{\hat a}_{\mu L}(p,0) &= C^{\hat a}_{\mu L}(p) - i\tfrac{\sqrt2}{f_\pi}p_\mu\pi
\end{aligned} \tag{3.22}$$

we further fix the gauge to $C_{L\mu} = 0$ and plug these solutions in eq.(3.21) to get

$$S_{Hol} = \frac{1}{g_5^2 f_\pi^2 L}p^2\,\pi^{\hat a}\pi^{\hat a} - \frac{P_T^{\mu\nu}}{2g_5^2}\left(\Pi^+_g(p)\,C^a_{\mu T}C^{\mu a}_T + \Pi^-_g(p)\,C^{\hat a}_{\mu T}C^{\mu\hat a}_T\right) \tag{3.23}$$

where we have defined

$$\Pi^+_g = G^+_T\partial_z G^+_T\big|_{z=0} = p\tan pL, \qquad \Pi^-_g = G^-_T\partial_z G^-_T\big|_{z=0} = -p\cot pL \tag{3.24}$$

and we have also used $G^-_L\partial_z G^-_L\big|_{z=0} = -1/L$. choosing $\pi^{\hat a}$ to be canonically normalized, fixes f_π to be $f_\pi = \frac{1}{g_5}\sqrt{\frac{2}{L}}$.

3.2 Gauge sector of the $SO(5)/SO(4)$ model in 5D flat space

3.2.1 The Lagrangian

We describe here, the gauge sector Lagrangian of a 5D model in flat space with $SO(5) \times U(1)_X$ symmetry in the bulk, which is broken to an $SO(4) \times U(1)_X$ subgroup on the IR boundary, and to the SM gauge group $SU(2)_L \times U(1)_Y$ on the UV boundary. The hypercharge generator is defined to be $Y \equiv T^3_R + X$. The bulk Lagrangian is

$$\mathcal{L}_{5g} = -\int dz \left(\frac{1}{4g_5^2} \mathrm{Tr}\left[F_{\mu\nu}F^{\mu\nu} - 2F_{\mu z}F^{\mu}{}_{z}\right] + \frac{1}{4g_{5X}^2} \mathrm{Tr}\left[F_{X,\mu\nu}F_X^{\mu\nu} - 2F_{X,\mu z}F^{\mu}_{Xz}\right]\right) . \quad (3.25)$$

We also adopt BKT for gauge fields associated with the unbroken generators on the UV brane

$$\mathcal{L}_{4g,\mathrm{UV}} = -\frac{\theta L}{4g_5^2}\, W^a_{\mu\nu} W^{a,\mu\nu} - \frac{\theta' L}{4g_{5X}^2}\, B_{\mu\nu}B^{\mu\nu} \,, \quad (3.26)$$

where B_μ is the field associated with the hypercharge generator Y. To define it we proceed as follows. Restricting to the $W^3_{\mu R}, A_{\mu X}$ subset of bulk fields, temporarily we absorb the gauge couplings, appearing in the bulk gauge kinetic terms, into the gauge fields so that the bulk gauge kinetic terms are canonical

$$-\frac{1}{4} W^3_{\mu\nu R} W^{3,\mu\nu}_R - \frac{1}{4} A_{\mu\nu X} A^{\mu\nu}_X + \cdots \,, \qquad D_\mu = \partial_\mu - i g_5 W^3_{\mu R} T^3_R - i g_{5X} A_{\mu X} X - \cdots \,, \quad (3.27)$$

where here the Abelian part of the gauge field strengths are understood. We want to rotate the fields $W^3_{\mu R}, A_{\mu X}$

$$\begin{pmatrix} W^3_{\mu R} \\ A_{\mu X} \end{pmatrix} = R \begin{pmatrix} B_\mu \\ Z'_\mu \end{pmatrix}, \qquad R \in O(2) \quad (3.28)$$

and bring the gauge connection in the form

$$g_5 W^3_{\mu R} T^3_R + g_{5X} A_{\mu X} X = \tilde{g}_5 B_\mu \left(T^3_R + X\right) + \cdots \,, \quad (3.29)$$

with $\tilde{g}_5$ to be specified later. In order to find the appropriate rotation, we write the gauge connection as

$$(B_\mu, Z'_\mu) R^T \begin{pmatrix} g_5 & 0 \\ 0 & g_{5,X} \end{pmatrix} O^{-1} O \begin{pmatrix} T^3_R \\ X \end{pmatrix}, \qquad O = \begin{pmatrix} 1 & 1 \\ a & b \end{pmatrix}, \quad (3.30)$$

so the requirement is that

$$R^T \begin{pmatrix} g_5 & 0 \\ 0 & g_{5,X} \end{pmatrix} O^{-1} \quad (3.31)$$

be diagonal, and the diagonal elements define the gauge couplings. The necessary condition for this is that the two columns of the matrix

$$\begin{pmatrix} g_5 & 0 \\ 0 & g_{5,X} \end{pmatrix} O^{-1} = \begin{pmatrix} g_5 & 0 \\ 0 & g_{5,X} \end{pmatrix} \frac{1}{b-a} \begin{pmatrix} b & -1 \\ -a & 1 \end{pmatrix} = \frac{1}{b-a} \begin{pmatrix} g_5\, b & -g_5 \\ -g_{5X}\, a & g_{5X} \end{pmatrix} \tag{3.32}$$

be orthogonal, which means $g_5^2\, b + g_{5X}^2\, a = 0$, so $b = -a\,(g_{5X}^2/g_5^2)$. This leaves the parameter a free. With the choice $a = g_5/g_{5X}$, the matrix O and the rotation R whose left action on 3.32 puts it in a diagonal form are

$$O = \begin{pmatrix} 1 & 1 \\ \frac{g_5^2}{g_{5X}^2} & \frac{g_{5X}^2}{g_5^2} \end{pmatrix}, \qquad R = \frac{1}{\sqrt{g_5^2 + g_{5X}^2}} \begin{pmatrix} g_{5X} & g_5 \\ g_5 & -g_{5X} \end{pmatrix}, \tag{3.33}$$

so the redefined fields will be

$$\begin{pmatrix} B_\mu \\ Z'_\mu \end{pmatrix} \equiv R^T \begin{pmatrix} W^3_{\mu R} \\ A_{\mu X} \end{pmatrix}, \qquad B_\mu = \frac{g_{5X} W^3_{\mu R} + g_5 A_{\mu X}}{\sqrt{g_5^2 + g_{5X}^2}}, \quad Z'_\mu = \frac{g_X W^3_{\mu R} - g_{5X} A_{\mu X}}{\sqrt{g_5^2 + g_{5X}^2}}, \tag{3.34}$$

which after restoring the absorbed gauge couplings $W^3_{\mu R} \to W^3_{\mu R}/g_5$, $A_{\mu X} \to A_{\mu X}/g_{5X}$ become

$$B_\mu = \frac{1}{g_5 g_{5X}} \frac{g_{5X}^2 W^3_{\mu R} + g_5^2 A_{\mu X}}{\sqrt{g_5^2 + g_{5X}^2}}, \quad Z'_\mu = \frac{W^3_{\mu R} - A_{\mu X}}{\sqrt{g_5^2 + g_{5X}^2}}. \tag{3.35}$$

The couplings are the diagonal elements of

$$R^T \begin{pmatrix} g_5 & 0 \\ 0 & g_{5,X} \end{pmatrix} O^{-1} = \frac{g_5 g_{5X}}{\sqrt{g_5^2 + g_{5X}^2}} \mathbf{1} \equiv \tilde{g}_5 \mathbf{1}. \tag{3.36}$$

Note that the fields B_μ and Z'_μ are in the canonical form, the fields in the non canonical form are given by 3.35 multiplied by their couplings 3.36. Here, for simplicity of notation, we only chose B_μ to be in non-canonical form, and Z'_μ will remain in its canonical form

$$B_\mu = \frac{g_{5X}^2 W^3_{\mu R} + g_5^2 A_{\mu X}}{g_5^2 + g_{5X}^2}, \qquad Z'_\mu = \frac{W^3_{\mu R} - A_{\mu X}}{\sqrt{g_5^2 + g_{5X}^2}}, \tag{3.37}$$

or equivalently

$$W^3_{R,\mu} = B_\mu + \frac{g_5^2}{\sqrt{g_5^2 + g_{5X}^2}} Z'_\mu, \qquad X_\mu = B_\mu - \frac{g_{5X}^2}{\sqrt{g_5^2 + g_{5X}^2}} Z'_\mu. \tag{3.38}$$

In this basis the covariant derivative reads

$$D_\mu = \partial_\mu - i\vec{W}_{\mu L}\vec{T}_L - iW^1_{\mu R}T^1_R - iW^2_{\mu R}T^2_R - iB_\mu Y - i\tilde{g}_5 Z'_\mu T_Z, \quad T_Z = \frac{g_5^2}{g_{5X}^2} T^3_R + \frac{g_{5X}^2}{g_5^2} X. \tag{3.39}$$

3.2.2 Holographic Analysis

We apply the tools of holography for gauge fields described in section (3.1.1) to the 5D flat model introduced in the previous section. Adopting the axial gauge $A_z = 0$, the Lagrangian (3.25) will be

$$\mathcal{L}_{5g} = -\int dz \left(\frac{1}{4g_5^2} \mathrm{Tr}\left[F_{\mu\nu}F^{\mu\nu} - 2\partial_z A_\mu \partial_z A^\mu\right] + \frac{1}{4g_{5X}^2} \mathrm{Tr}\left[F_{X,\mu\nu}F_X^{\mu\nu} - 2\partial_z A_{X,\mu}\partial_z A_X^\mu\right] \right) . \tag{3.40}$$

Ignoring the fluctuations of the Goldstone fields around their VEV in the 4D sigma model field and using the unbroken $SO(4)$ symmetry to align the VEV along the fourth broken generator

$$\Sigma = \exp\left(-i\sqrt{2}\alpha T^{\hat{4}}\right), \qquad \alpha \equiv \langle\pi\rangle/f_\pi, \tag{3.41}$$

the tree level holographic Lagrangian in momentum space and at quadratic order in the fields can be written as

$$\begin{aligned}\mathcal{L}_{Hol} = & -\frac{P_T^{\mu\nu}}{2}\left[\frac{\theta L p^2}{g_5^2} W^a_{L\mu} W^a_{L\nu} + \frac{\theta' L p^2}{g_{5X}^2} B_\mu B_\nu\right] \\ & -\frac{P_T^{\mu\nu}}{2}\left[\frac{1}{g_5^2}\Pi_g^+ (C^{\Sigma^{-1}})^a_\mu (C^{\Sigma^{-1}})^a_\nu + \frac{1}{g_{5X}^2}\Pi_g^+ (C_X^{\Sigma^{-1}})_\mu (C_X^{\Sigma^{-1}})_\nu + \frac{1}{g_5^2}\Pi_g^- (C^{\Sigma^{-1}})^{\hat{a}}_\mu (C^{\Sigma^{-1}})^{\hat{a}}_\nu\right],\end{aligned} \tag{3.42}$$

with the holographic field being

$$C_\mu = \vec{W}_{L\mu}\cdot\vec{T}_L + B_\mu T_R^3, \qquad C_{X\mu} = B_\mu, \tag{3.43}$$

in which the fields with $(-)$ b.c on the UV are set to zero. The components of the rotated holographic field

$$C_\mu^{\Sigma^{-1}} = \left(\Sigma^{-1}\right)^\dagger C_\mu \Sigma^{-1} = \left(W_L^{\Sigma^{-1}}\right)^a_\mu T_L^a + \left(W_R^{\Sigma^{-1}}\right)^a_\mu T_R^a + \left(A_B^{\Sigma^{-1}}\right)^i_\mu T_B^i \tag{3.44}$$

are given by

$$\begin{aligned}\left(W_L^{\Sigma^{-1}}\right)^{1,2} &= W_L^{1,2}\cos^2\tfrac{\alpha}{2} & \left(W_L^{\Sigma^{-1}}\right)^3 &= W_L^3\cos^2\tfrac{\alpha}{2} + B\sin^2\tfrac{\alpha}{2} \\ \left(W_R^{\Sigma^{-1}}\right)^{1,2} &= W_R^{1,2}\sin^2\tfrac{\alpha}{2} & \left(W_R^{\Sigma^{-1}}\right)^3 &= W_R^3\sin^2\tfrac{\alpha}{2} + B\cos^2\tfrac{\alpha}{2}\end{aligned} \tag{3.45}$$

for the components along the unbroken generators, and

$$\left(A_B^{\Sigma^{-1}}\right)^{1,2} = W_L^{1,2}\frac{\sin\alpha}{\sqrt{2}}, \quad \left(A_B^{\Sigma^{-1}}\right)^3 = \left(W_L^3 - B\right)\frac{\sin\alpha}{\sqrt{2}} \quad \left(A_B^{\Sigma^{-1}}\right)^4 = 0 \tag{3.46}$$

for the components along the broken generators. Plugging these into (3.42) we arrive at the Lagrangian

$$\mathcal{L}_{Hol} = -\frac{P_T^{\mu\nu}}{2}\left[\Pi_{ab}W^a_{L\mu}W^b_{L\nu} + \Pi_{YY}B_\mu B_\nu + 2\Pi_{3Y}W^3_{L\mu}B_\nu\right], \tag{3.47}$$

in which the introduced form factors are given by

$$\begin{aligned}
\Pi_{ab} &= \frac{\delta_{ab}}{2g_5^2}\left[2\Pi_g^+ + \sin^2\alpha\left(\Pi_g^- - \Pi_g^+\right) + 2p^2\theta L\right] \\
\Pi_{3Y} &= \frac{1}{2g_5^2}\sin^2\alpha\left(\Pi_g^+ - \Pi_g^-\right) \\
\Pi_{YY} &= \frac{1}{g_{5X}^2}\left(\Pi_g^+ + p^2\theta' L\right) + \frac{1}{2g_5^2}\left[2\Pi_g^+ + \sin^2\alpha\left(\Pi_g^- - \Pi_g^+\right)\right],
\end{aligned} \tag{3.48}$$

where $\Pi_g^\pm$ are defined in eq.(3.24). The roots of Π_{11} will give a tower of KK masses which includes the $W^\pm$ mass, while the Z boson mass is included in the roots of

$$\begin{aligned}
2g_5^2 g_{5X}^2\left(\Pi_{33}\Pi_{YY} - \Pi_{3Y}^2\right) &= 2(1+\eta)\Big(\Pi_g^+ + p^2\tilde{\theta}L\Big)\Big(\Pi_g^+ + p^2\theta L\Big) \\
&+ \sin^2\alpha\left(\Pi_g^- - \Pi_g^+\right)\Big[(1+2\eta)\Pi_g^+ + p^2(\tilde{\theta} + \eta(\theta+\tilde{\theta}))L\Big],
\end{aligned} \tag{3.49}$$

where we have defined

$$\eta \equiv g_{5X}^2/g_5^2, \qquad \tilde{\theta} \equiv \theta' \frac{g_5^2}{g_5^2 + g_{5X}^2}. \tag{3.50}$$

By expanding these form factors in momentum

$$\begin{aligned}
\Pi_{ab} &= -\frac{\delta_{ab}}{2g_5^2 L}\sin^2\alpha + p^2\frac{\delta_{ab}L}{g_5^2}\left(1 + \theta - \frac{1}{3}\sin^2\alpha\right) + \mathcal{O}(p^4) \\
\Pi_{3Y} &= \frac{1}{2g_5^2 L}\sin^2\alpha + p^2\frac{L}{3g_5^2} + \mathcal{O}(p^4) \\
\Pi_{YY} &= -\frac{1}{2g_5^2 L}\sin^2\alpha + p^2 L\left[\frac{1+\theta'}{g_{5X}^2} + \frac{3-\sin^2\alpha}{3g_5^2}\right] + \mathcal{O}(p^4)
\end{aligned} \tag{3.51}$$

one can easily read off the $SU(2)_L$ and $U(1)_Y$ gauge couplings and the Higgs VEV

$$\begin{aligned}
\frac{1}{g^2} &= \Pi_{11}'(0) = \frac{L(3 + 3\theta - \sin^2\alpha)}{3g_5^2} \\
\frac{1}{g'^2} &= \Pi_{YY}'(0) = \frac{L(3 + 3\theta')}{g_{5X}^2} + \frac{L(3 - \sin^2\alpha)}{3g_5^2} \\
v^2 &= -4\Pi_{11}(0) = \frac{2\sin^2\alpha}{g_5^2 L} = f_\pi^2\sin^2\alpha.
\end{aligned} \tag{3.52}$$

As expected, at tree level, the custodial symmetry leads to $\Pi_{ab} \propto \delta_{ab}$ which results in a vanishing T parameter. Also the S parameter, written in terms the couplings (3.52) is found to be

$$S = 16\pi\Pi_{3Y}'(0) = 16\pi\frac{L\sin^2\alpha}{3g_5^2} = 16\pi\frac{\sin^2\alpha}{g^2(3 + 3\theta - \sin^2\alpha)}. \tag{3.53}$$

By looking at the holographic Lagrangian expanded up to second order in momentum

$$\begin{aligned}\mathcal{L}_{Hol} &= P_T^{\mu\nu}\frac{\sin^2\alpha}{4g_5^2 L}\left(W^a_{L\mu}W^a_{L\nu}+B_\mu B_\nu-2W^3_{L\mu}B_\nu\right)\\ &- \frac{P_T^{\mu\nu}}{2}p^2\left[\frac{1}{g^2}W^a_{L\mu}W^a_{L\nu}+\frac{1}{g'^2}B_\mu B_\nu+\frac{\sin^2\alpha}{g^2(3+3\theta-\sin^2\alpha)}W^3_{L\mu}B_\nu\right] \quad (3.54)\end{aligned}$$

one can see that the fields W_L^3 and B are related to the canonically normalized Z and γ fields in the following way

$$\begin{pmatrix} W_L^3 \\ B \end{pmatrix} = \frac{1}{\sqrt{g^2+g'^2}}\begin{pmatrix} g^2 & gg' \\ -g'^2 & gg' \end{pmatrix}\begin{pmatrix} Z \\ \gamma \end{pmatrix}. \quad (3.55)$$

In finding the transformation to the Z,γ basis we have ignored the $W_L^3 B$ mixing, because we treat this term as a contribution to the S parameter. So it is the above definition of Z,γ that we use for computing the corrections to the $Z\bar{b}b$ vertex, since we are interested only in the non-universal corrections.

From the Lagrangian (3.47), and by using the definitions (3.48), one can easily compute the gauge contribution to the Coleman-Weinberg potential [36]

$$\begin{aligned}V_g &= \frac{3}{2}\int\frac{d^4p}{(2\pi)^4}\left[2\log\left(1+s_\alpha^2\frac{\Pi_g^- - \Pi_g^+}{2(\Pi_g^+ + p^2\theta L)}\right)\right.\\ &\left.+\log\left(1+s_\alpha^2\frac{\Pi_g^- - \Pi_g^+}{2(\Pi_g^+ + p^2\theta L)}+s_\alpha^2\frac{\tilde{g}_5^2}{g_5^2}\frac{\Pi_g^- - \Pi_g^+}{2(\Pi_g^+ + p^2\tilde{\theta}L)}\right)\right]. \quad (3.56)\end{aligned}$$

Fig.(3.1) shows the 1-loop diagrams that contribute to the potential. After a Wick rotation, the combination $\Pi_g^- - \Pi_g^+$ falls off exponentially with momentum, while Π_g^+ grows linearly. So the ratios appearing in front of s_α^2 in V_g fall off exponentially and at leading order in these ratios, this potential is proportional to s_α^2 which means that it tends to align the vacuum in a direction that preserves G_{SM}, in accordance with [37].

3.2.3 Comparison with the analysis in the KK basis

Our aim in this section is to make a comparison between the analysis of section (3.2.2) based on the holographic approach and the more standard KK approach. We will see that with more effort we will be able to reproduce the mass spectra and the precision electroweak observables S and T. According to the analysis of section (3.2.1) the kinetic terms of the 5D Lagrangian (3.25) can be written in the following form

$$\begin{aligned}\mathcal{L}_{5g} &= -\int dz\left[\frac{1}{4g_5^2}\left(W^a_{L\mu\nu}\right)^2+\frac{1}{4g_5^2}\left(W^1_{R\mu\nu}\right)^2+\frac{1}{4g_5^2}\left(W^2_{R\mu\nu}\right)^2\right.\\ &\left.+\frac{1}{4g_5^2}\left(A^i_{B\mu\nu}\right)^2+\frac{1}{4\tilde{g}_5^2}\left(B_{\mu\nu}\right)^2+\frac{1}{4}\left(Z'_{\mu\nu}\right)^2\right] \quad (3.57)\end{aligned}$$

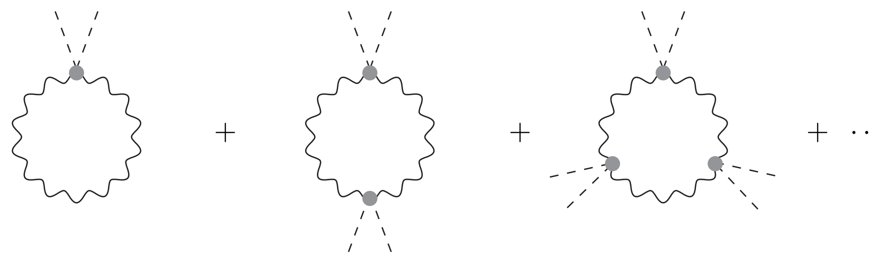


Figure 3.1: 1-loop gauge contribution to the Higgs potential. Each external line stands for a s_α factor. These contributions consist of diagrams in which either W^1_μ or W^2_μ run in the loops or W^3_μ and B_μ appear in all possible combinations.

while we rewrite the UV action in terms of the parameters $\tilde{\theta}$ and $\tilde{g}_5$ by using the identity $\theta'/g^2_{5X} = \tilde{\theta}/\tilde{g}^2_5$ which is clear from their definitions

$$\mathcal{L}_{4g,\mathrm{UV}} = -\frac{\theta L}{4g_5^2}\, W^a_{\mu\nu} W^{a,\mu\nu} - \frac{\tilde{\theta} L}{4\tilde{g}_5^2}\, B_{\mu\nu} B^{\mu\nu}\,. \tag{3.58}$$

The Lagrangian (3.25) also includes interaction terms between the fields in (3.57) as well as the two terms $(2g_5^2)^{-1}\mathrm{Tr}\,[F_{\mu z}F^\mu_z]$ and $(2g_5^2)^{-1}\mathrm{Tr}\left[F_{X,\mu z}F^\mu_{Xz}\right]$. As discussed above, in the unitary gauge which we adopt here, among all the KK modes of $A_{X,z}$ and different components of A_z, the zero modes of A_{Bz}, the components along the broken generators, will remain in the spectrum. So from now on we will set $A_{X,z} = 0$ and $A_z = A_{Bz}$, where, by A_{Bz} the zero mode is meant. Doing this we can write $(2g_5^2)^{-1}\mathrm{Tr}\left[F_{X,\mu z}F^\mu_{Xz}\right] = (2g_5^2)^{-1}\mathrm{Tr}\left[\partial_z A_{X,\mu}\partial_z A^\mu_X\right]$. To express the second term $(2g_5^2)^{-1}\mathrm{Tr}\,[F_{\mu z}F^\mu_z]$ mentioned above in a more transparent way, we write the field strength as

$$F_{\mu z} = D_\mu A_z - i\,[A_{B\mu}, A_z] - \partial_z A_\mu, \quad D_\mu A_z = \partial_\mu A_z - i\,[A_{L\mu}, A_z] - i\,[A_{R\mu}, A_z] \tag{3.59}$$

and use it to write

$$\begin{aligned}\mathrm{Tr}\,[F_{\mu z}F^\mu_z] \;\; = \;\; & \mathrm{Tr}\big[\partial_z A_\mu \partial_z A^\mu + D_\mu A_z D^\mu A_z - [A_{B\mu}, A_z][A^\mu_B, A_z] \\ & -2\partial_z A_\mu D^\mu A_z + 2i\partial_z A_\mu [A^\mu_B, A_z] - 2iD_\mu A_z [A^\mu_B, A_z]\big]\,. \end{aligned}\tag{3.60}$$

The last term in eq.(3.60) vanishes because $D_\mu A_z$ lies in the broken subspace of the generators while the commutator of two broken generators $[A^\mu_B, A_z]$ is an unbroken generator. In order to proceed we find it more convenient to express the fields in a two by two matrix notation and define $A_{L,R\mu} = W^a_{L,R\mu}\sigma^a/2$, $A_{B\mu} = A^4_{B\mu} - i\vec{A}_{B\mu}\cdot\vec{\sigma}$ (where the same symbols have been used) and also

$$A^i_{Bz} \equiv h^i, \qquad \Omega \equiv h^4 - i\vec{h}\cdot\vec{\sigma} = (H^c, H) \tag{3.61}$$

Using these definitions one can write

$$\begin{aligned}
\frac{1}{2g_5^2}\mathrm{Tr}\left[F_{\mu z}F_z^{\mu}\right] &= \frac{1}{4g_5^2}\mathrm{Tr}\left[\partial_z A_{L\mu}\partial_z A_L^{\mu} + \partial_z A_{R\mu}\partial_z A_R^{\mu} + \partial_z A_{B\mu}\partial_z A_B^{\mu}\right] \\
&+ \frac{1}{4g_5^2}\mathrm{Tr}\left[(D_\mu\Omega)^\dagger D^\mu\Omega\right] - \frac{1}{2g_5^2}\mathrm{Tr}\left[\partial_z A_{B\mu}^\dagger D^\mu\Omega\right] \\
&- \frac{1}{64g_5^2}\mathrm{Tr}\left[(A_{B\mu}\Omega^\dagger - \Omega A_{B\mu}^\dagger)^2 + (A_{B\mu}^\dagger\Omega - \Omega^\dagger A_{B\mu})^2\right] \\
&+ \frac{i}{4g_5^2}\mathrm{Tr}\left[\partial_z A_{L\mu}(A_B^{\mu}\Omega^\dagger - \Omega A_B^{\mu\dagger}) + \partial_z A_{R\mu}(A_B^{\mu\dagger}\Omega - \Omega^\dagger A_B^{\mu})\right]
\end{aligned} \tag{3.62}$$

In the mas basis the term $(2g_5^2)^{-1}\mathrm{Tr}\left[\partial_z A_{X,\mu}\partial_z A_X^{\mu}\right]$ together with the first line of (3.62) have the same structure as (3.57) with the field strengths replaced by the squared values of the z-derivatives of the fields. These terms along with (3.57) and (3.58) give the quadratic term of the Lagrangian which we use to find the linear e.o.m.

KK wave-functions and mass spectra before EWSB

In order to solve the wave equations we need to specify the boundary conditions of the fields. These b.c are chosen according to the symmetry breaking pattern on the two boundaries. For the fields associated with the broken generators we choose Dirichlet $(-)$ b.c, and for the fields associated with the unbroken generators we choose Neumann $(+)$ b.c

$$W_{L\mu}^a,\ B_\mu \quad (++) \qquad\qquad W_{R\mu}^{1,2},\ Z'_\mu \quad (-+) \qquad\qquad A_{B\mu}^i \quad (--) \tag{3.63}$$

where $a = 1,2,3$ and $i = 1,2,3,4$ and the first(second) entry in the parentheses denotes the b.c on the UV(IR) brane. The z components have the opposite b.c.

The bulk e.o.m for a gauge field, which we generally call A_μ is

$$\left(\partial^2\delta_\mu^\nu - \partial_\mu\partial^\nu\right)A_\nu + \partial_z^2 A_\mu = 0 \tag{3.64}$$

we plug the ansantz $F(x)f(z)$ in this equation and divide the equation by the ansatz to get

$$\frac{1}{F}\left(\partial^2 - \partial_\mu\partial\nu\right)F_\nu(x) + \frac{1}{f}\partial_z^2 f(z) = 0 \tag{3.65}$$

the two terms are functions of different variables so their are both constant. choosing $\left(\partial^2 - \partial_\mu\partial^\nu\right)F_\nu = -m^2 F_\mu$ the bulk profile will satisfy

$$m^2 f + f''(z) = 0 \tag{3.66}$$

where a prime denotes a derivative with respect to z. This equation admits the solution

$$f(z) = A\cos mz + B\sin mz \tag{3.67}$$

We still need to impose the b.c, so we now concentrate on different fields separately.

The $SU(2)_L$ triplet $W^a_{L\mu}$

The UV and IR b.c for W^a_L, taking into account, of course, the variation of the boundary action 3.26, are

$$\theta L m^2 f(0) + f'(0) = 0, \qquad f'(L) = 0. \tag{3.68}$$

After inserting the solution 3.67, the IR b.c will give $B = A\tan mL$, so that the bulk profile is now $f(z) = A(\cos mz + \tan mL \sin mz)$, and the overall factor A is fixed by the normalization condition

$$\theta L f^2(0) + \int_0^L dz\ f^2(z) = g_5^2 \tag{3.69}$$

The UV b.c imposes a second constraint which gives the mass equation

$$m(\tan mL + \theta mL) = 0 \tag{3.70}$$

the mass equation 3.70 has a zero mode, the non zero modes are functions of θ given by

$$m_n = \frac{2n-1}{2}\frac{\pi}{L}\left(1 + \frac{4}{\pi^2(2n-1)^2}\frac{1}{\theta} + \mathcal{O}\left(\frac{1}{\theta^2}\right)\right), \qquad n \geq 1 \tag{3.71}$$

So the 5D wave-function in the mixed (p, z) basis is

$$W^{a(n)}_{L\mu}(p,z) = g_5\sqrt{\frac{2}{L}}\left(\frac{1}{\theta + \sec^2 m_n L}\right)^{1/2}(\cos m_n z + \tan m_n L\ \sin m_n z)\ \widetilde{W}^{a(n)}_{\mu L}(p) \tag{3.72}$$

which reduces to

$$W^{a(0)}_{L\mu}(p,z) = \frac{g_5}{\sqrt{L(1+\theta)}}\widetilde{W}^{a(0)}_{\mu L}(p) \tag{3.73}$$

for the zero mode.

The $W^{1,2}_{R\mu}$ fields

For the fields $W^{1,2}_{R\mu}$ the UV and IR the b.c and the normalization conditions are

$$f(0) = 0, \qquad f'(L) = 0, \qquad \int_0^L dz\ f^2(z) = g_5 \tag{3.74}$$

so the mass spectra are given by $\cos mL = 0$, which means

$$m_n = \frac{2n-1}{2}\frac{\pi}{L}, \qquad n \geq 1 \tag{3.75}$$

and the 5D wave-function in the mixed (p, z) basis is

$$W^{1,2(n)}_{R\mu}(p,z) = g_5\sqrt{\frac{2}{L}}\,\sin m_n z\; \widetilde{W}^{1,2(n)}_{\mu R}(p) \tag{3.76}$$

The B_μ and Z'_μ fields

For the extra dimensional profile of Z' the b.c and normalization condition are the same as that of the fields $W^{1,2}_{R\mu}$ with a missing overall g_5 factor

$$f(0) = 0, \qquad f'(L) = 0, \qquad \int_0^L dz\; f^2(z) = 1 \tag{3.77}$$

so the mass spectrum is

$$m_n = \frac{2n-1}{2}\frac{\pi}{L}, \qquad n \geq 1 \tag{3.78}$$

and the wave-function in the mixed (p, z) basis is

$$Z'_\mu(p,z) = \sqrt{\frac{2}{L}}\,\sin m_n z\; \widetilde{Z}'_\mu(p) \tag{3.79}$$

From the way the Lagrangians (3.57) and (3.58) are written, it is obvious that the situation for B_μ is exactly the same as $W^a_{L\mu}$ but with the replacements

$$\theta \to \tilde{\theta}, \qquad g_5 \to \tilde{g}_5 \tag{3.80}$$

The fields $A^i_{B\mu}$ associated with the broken generators $SO(5)/SO(4)$

Finally, the b.c and normalization condition for the fields $A^i_{B\mu}$ are

$$f(0) = 0, \qquad f(L) = 0, \qquad \int_0^L dz\; f^2(z) = g_5^2 \tag{3.81}$$

which result in a mass spectrum

$$m_n = n\frac{\pi}{L}, \quad n \geq 1 \tag{3.82}$$

and a bulk wave-function

$$A_{B\mu}(p,z) = g_5\sqrt{\frac{2}{L}}\,\sin m_n z\; \widetilde{A}_{B\mu}(p) \tag{3.83}$$

As discussed in section 3.1 in the unitary gauge $\partial_z A_z = 0$, this is the only field whose z component has a non zero wave-function

$$A_{Bz}(p,z) = g_5\sqrt{\frac{1}{L}}\,\widetilde{A}_{Bz}(p) \tag{3.84}$$

KK wave-functions and mass spectra after EWSB

we choose the direction of the VEV to lie along T^4_B. For simplicity one can perform the gauge transformation

$$A_M \to U^\dagger(A_M - i\partial_M)U, \qquad U = e^{-i\sqrt{2}\alpha T^4 z/L} \tag{3.85}$$

to eliminate the Higgs VEV in the bulk. The parameter α must satisfy $\langle h\rangle = \sqrt{2}\alpha/L$. Note that U is the identity matrix on the UV brane, but on the IR brane

$$U|_{IR} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\alpha & \sin\alpha \\ 0 & 0 & 0 & -\sin\alpha & \cos\alpha \end{pmatrix} \equiv \Sigma \tag{3.86}$$

so the IR b.c which used to be $f'^a_L(L) = 0$, $f'^a_R(L) = 0$, and $f^i_B(L) = 0$ are now modified. To find the new b.c one must perform the transformation

$$\vec{f}_L\cdot\vec{T}_L + \vec{f}_R\cdot\vec{T}_R + \vec{f}_B\cdot\vec{T}_B \to \Sigma^\dagger\left(\vec{f}_L\cdot\vec{T}_L + \vec{f}_R\cdot\vec{T}_R + \vec{f}_B\cdot\vec{T}_B\right)\Sigma \tag{3.87}$$

in the old b.c. So on the IR brane the $(-)$ b.c become

$$\begin{cases} (f^1_L - f^1_R)\sin\alpha + \sqrt{2}f^1_B\cos\alpha = 0 \\ (f^2_L - f^2_R)\sin\alpha + \sqrt{2}f^2_B\cos\alpha = 0 \\ (f^3_L - f^3_R)\sin\alpha + \sqrt{2}f^3_B\cos\alpha = 0 \\ f^4_B = 0 \end{cases} \tag{3.88}$$

while the $(+)$ b.c are now

$$\begin{cases} f'^1_L + f'^1_R = 0 \\ (f'^1_R - f'^1_L)\cos\alpha + \sqrt{2}f'^1_B\sin\alpha = 0 \\ f'^2_L + f'^2_R = 0 \\ (f'^2_R - f'^2_L)\cos\alpha + \sqrt{2}f'^2_B\sin\alpha = 0 \\ f'^3_L + f'^3_R = 0 \\ (f'^3_R - f'^3_L)\cos\alpha + \sqrt{2}f'^3_B\sin\alpha = 0 \end{cases} \tag{3.89}$$

the functions f_L^1, f_R^1 and f_B^1 are mixed together through the b.c on the UV and IR branes. We now concentrate on these set of functions with b.c

$$UV: \begin{cases} \theta L m^2 f_L^1 + f_L'^1 = 0 \\ f_R^1 = 0 \\ f_B^1 = 0 \end{cases} \qquad IR: \begin{cases} (f_L^1 - f_R^1)\sin\alpha + \sqrt{2} f_B^1 \cos\alpha = 0 \\ (f_R'^1 - f_L'^1)\cos\alpha + \sqrt{2} f_B'^1 \sin\alpha = 0 \\ f_L'^1 + f_R'^1 = 0. \end{cases} \tag{3.90}$$

The same b.c are valid for f_L^2, f_R^2 and f_B^2 of course, because of the $U(1)_{em}$ symmetry. Inserting the bulk solutions

$$\begin{aligned} f_L^1(z) &= A_L^1 \cos mz + B_L^1 \sin mz \\ f_R^1(z) &= A_R^1 \cos mz + B_R^1 \sin mz \\ f_B^1(z) &= A_B^1 \cos mz + B_B^1 \sin mz \end{aligned} \tag{3.91}$$

the UV b.c give

$$\begin{aligned} f_L^1(z) &= A_L^1 (\cos mz - \theta L m \sin mz) \\ f_R^1(z) &= B_R^1 \sin mz \\ f_B^1(z) &= B_B^1 \sin mz \end{aligned} \tag{3.92}$$

plugging these into the IR constraints gives a set of three coupled linear homogeneous equations in A_L^1, A_R^1 and A_B^1. The requirement that these linear equations have a non trivial solution means that the matrix of coefficients must have zero determinant, which in this case is

$$\cos(mL)\left[2\sin^2(mL) + \theta m L \sin(2mL) - \sin^2\alpha\right] = 0. \tag{3.93}$$

This is the mass equation. A subset of the mass spectra is given by the condition that the overall factor $\cos(mL)$ vanishes

$$m_n = \frac{2n-1}{2}\frac{\pi}{L}, \quad n \geq 1 \tag{3.94}$$

in this case the solution to the linear equations is $A_L^1 = 0$ and $B_B^1 = B_R^1 \tan\alpha/\sqrt{2}$, so the wave-functions are

$$\begin{aligned} f_L^1(z) &= 0 \\ f_R^1(z) &= N \sin mz \\ f_B^1(z) &= N \frac{\tan\alpha}{\sqrt{2}} \sin mz \end{aligned} \tag{3.95}$$

with a normalization factor

$$N = g_5 \sqrt{\frac{2}{L}} \left[1 + \frac{1}{2}\tan^2\alpha\right]^{-\frac{1}{2}} \tag{3.96}$$

chosen such that the normalization condition

$$\theta L \left(f_L^1(0)\right)^2 + \int_0^L dz \left[\left(f_L^1(z)\right)^2 + \left(f_R^1(z)\right)^2 + \left(f_B^1(z)\right)^2\right] = g_5^2 \tag{3.97}$$

is satisfied. For the other tower of masses

$$\left[2\sin^2(mL) + \theta mL \sin(2mL) - \sin^2\alpha\right] = 0 \tag{3.98}$$

which are modified after EWSB, the solution to the linear equations will be

$$B_R^1 = (mL\theta + \tan mL)A_L^1 = \frac{\sin^2\alpha}{\sin mL}A_L^1 \tag{3.99}$$

$$B_B^1 = -\sqrt{2}\cot\alpha(mL\theta + \tan mL)A_L^1 = -\frac{\sin 2\alpha}{\sqrt{2}\sin mL}A_L^1. \tag{3.100}$$

So in this case the wave-functions are

$$\begin{aligned} f_L^1(z) &= N(\cos mz - \theta Lm \sin mz) \\ f_R^1(z) &= N\frac{\sin^2\alpha}{\sin mL}\sin mz \\ f_B^1(z) &= -N\frac{\sin 2\alpha}{\sqrt{2}\sin mL}\sin mz \end{aligned} \tag{3.101}$$

with the normalization factor

$$N = g_5\sqrt{\frac{2}{L}}\left[\theta + 2\,\frac{1-\cos(2mL)\cos^2\alpha}{\sin^2(2mL)}\right]^{-\frac{1}{2}}. \tag{3.102}$$

The mass of the W boson is the lightest excitation of the tower of masses (3.98)

$$m_W = \frac{\sin\alpha}{\sqrt{2L}\sqrt{1+\theta}}\left(1 + \frac{1+2\theta}{12(1+\theta)^2}\sin^2\alpha + \mathcal{O}(\sin^4\alpha)\right) \tag{3.103}$$

for which the normalization facto is

$$N = \frac{g_5}{\sqrt{L}\sqrt{1+\theta}}\left(1 + \frac{\theta}{6(1+\theta)^2}\sin^2\alpha + \mathcal{O}(\sin^4\alpha)\right). \tag{3.104}$$

Finally, for completeness, we briefly mention the case for the coupled set of wave functions f_L^3, f_R^3, f_B^3 and f_X, and write down the b.c

$$UV: \begin{cases} \theta L m^2 f_L^3 + f_L'^3 = 0 \\ \tilde{\theta}(1+\eta)Lm^2 f_R^3 + \eta f_R'^3 + f_X' = 0 \\ f_B^3 = 0 \\ f_X = f_R^3 \end{cases} \qquad IR: \begin{cases} (f_L^3 - f_R^3)\sin\alpha + \sqrt{2}f_B^3\cos\alpha = 0 \\ (f_R'^3 - f_L'^3)\cos\alpha + \sqrt{2}f_B'^3\sin\alpha = 0 \\ f_L'^3 + f_R'^3 = 0 \\ f_X' = 0 \end{cases} \tag{3.105}$$

where we have defined $\eta \equiv g_{5X}^2/g_5^2$. Proceeding as before, the tower of masses is given by the roots of the mass equation

$$(1+\eta)\sin(2mL)\Big(Lm\theta\cos mL + \sin mL\Big)\Big(Lm\tilde{\theta}\cos mL + \sin mL\Big)$$
$$-\sin^2\!\alpha\,\cos mL\,\Big(Lm(\tilde{\theta}+\eta(\theta+\tilde{\theta}))\cos mL + (1+2\eta)\sin mL\Big) = 0. \qquad (3.106)$$

This equation, whose roots include the mass of the Z boson as the lightest excitation, is in complete agreement with the result (3.49) obtained using holographic methods.

Computation of the S parameter at tree level

We continue our study in KK basis by computing the electroweak S parameter. At leading order in the Higgs VEV, the contributions to the S parameter come from coupling deviations of light fermions to gauge bosons. The light fermion wave-functions are highly localized towards the UV brane, so one can approximate the bulk gauge-fermion interactions, by setting the gauge wave-functions to their UV values, which gives, due to the normalization of the fermion wave-functions

$$\int dz f_A(z) f_\psi^2(z) \longrightarrow f_A(UV) \qquad (3.107)$$

This makes the gauge-fermion couplings independent of the light fermion species, and hence universal. In this case the relation between gauge-fermion coupling deviations and electroweak precision observables [38] is discussed in section (3.2.4). Using (3.101) the UV value of the W boson bulk profile, which is the W coupling, is given by (3.104). So the relative coupling deviation will be

$$\frac{\delta g}{g} = \frac{\theta}{6(1+\theta)^2}\sin^2\alpha \qquad (3.108)$$

Alternatively, one can find this result by using the gauge wave-functions before EWSB and inserting Higgs VEVs on the gauge propagator. Fig.(3.2) shows the mixing between a gauge zero mode, which is the wave-function of the SM W_L^1 boson, and a massive KK mode X^n, through a Higgs insertion. To find the value of this mixing, we give a look at the relevant gauge-Higgs interaction terms discussed in section (3.2.3)

$$\frac{1}{4g_5^2}\mathrm{Tr}[D\Omega^\dagger D\Omega] \to \frac{\langle h\rangle^2}{8g_5^2}\left(W_{\mu L}^a W_L^{a\mu} + W_{\mu R}^a W_R^{a\mu} - 2W_{\mu L}^a W_R^{a\mu}\right) \qquad (3.109)$$

where $W_{L,R}^a$ are the 5D wave-functions. Using this, the diagram (3.2) can be computed

$$iI_{1W_L^1}^{(n)} = i\frac{\langle h\rangle^2}{4g_5^2}\int_0^L dz\; f_L^{1(0)}(z) f_L^{1(n)}(z), \qquad n>0 \qquad (3.110)$$

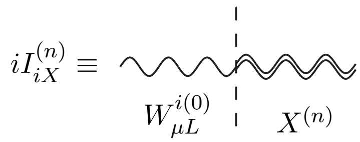


Figure 3.2: Mixing between W_L^i and a nth KK mode, through interactions with the Higgs.

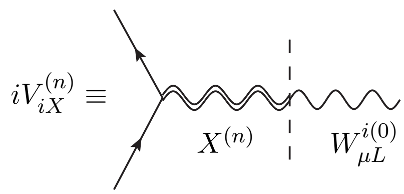


Figure 3.3: Tree level corrections to the gauge-fermion vertex through the exchange of massive KK modes.

where

$$\int_0^L dz\, f_L^{1(0)}(z) f_L^{1(n)}(z) = -\frac{2\sqrt{2}\, g_5^2}{(2n-1)\pi\sqrt{1+\theta}}\left(1 - \frac{6}{(2n-1)^2\pi^2\theta} + \mathcal{O}\left(\frac{1}{\theta^2}\right)\right) \tag{3.111}$$

The diagram (3.3) shows how the exchange of massive KK modes will give, at tree level, the leading correction to the gauge coupling. These diagrams can be easily computed using $I_{1W_L^1}^{(n)}$ evaluated above, and the fact that the UV value of the nth KK mode is

$$f_L^{1(n)}(0) = \sqrt{\frac{2}{L}}\frac{2g_5}{(2n-1)\theta\pi} \tag{3.112}$$

By summing over diagrams (3.3) in which all possible KK modes are exchanged we find the gauge coupling deviation to be

$$\begin{aligned} i\delta g &= \sum_{n=1}^{\infty} iV_{1W_L^1}^{(n)} = i f_L^{1(n)}(0)\left(\frac{-i}{-m_n^2}\right) iI_{1W_L^1}^{(n)} = \sum_{n=1}^{\infty} i\frac{\langle h\rangle^2}{4g_5^2}\sqrt{\frac{2}{L}}\frac{4\sqrt{2}g_5^3}{\pi^4\theta\sqrt{1+\theta}}\frac{4L^2}{(2n-1)^4} + \cdots \\ &= \sum_{n=1}^{\infty} i\frac{16\alpha^2 g_5}{\sqrt{L}\pi^4\theta\sqrt{1+\theta}}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^4} + \cdots = i\frac{\alpha^2 g_5}{6\sqrt{L}\theta\sqrt{1+\theta}} + \cdots \end{aligned} \tag{3.113}$$

where we have used

$$\sum_{n=1}^{\infty} \frac{1}{(2n-2)^4} = \frac{\pi^4}{96}. \tag{3.114}$$

Dividing this deviation by the coupling before EWSB

$$g = \frac{g_5}{\sqrt{L(1+\theta)}}, \tag{3.115}$$

the relative coupling deviation is found to be

$$\frac{\delta g}{g} = \frac{\alpha^2}{6\theta} + \cdots , \tag{3.116}$$

in agreement with the result (3.108). Using this result, the arguments of section (3.2.4) show that the S parameter is

$$\delta S = \frac{a}{2\pi} = 32\pi\tilde{a} = \frac{32\pi}{g^2}\tilde{x} = \frac{16\pi}{g^2}\frac{\alpha^2}{3\theta}, \tag{3.117}$$

where in the last equation we have used the fact that

$$\tilde{x} = \frac{\delta g}{g} = \frac{\alpha^2}{6\theta}, \tag{3.118}$$

in agreement with the result (3.53), obtained using the holographic approach.

Computation of the T parameter at tree level

We next move to the tree level computation of the T parameter. There are two possible sources of custodial breaking. The first is the exchange of heavy KK modes between two $W_L^{1(0)}$ or $W_L^{3(0)}$ zero modes. And the the second, similar to the case of the S parameter, is the deviation of gauge coupling to fermions. In order to find the first contribution to the T parameter we need to compute diagrams such as fig.(3.4). The relevant diagrams of this sort are the ones with $W_L^{1(0)}$ and $W_L^{3(0)}$ as external lines. For external $W_L^{3(0)}$ lines, massive towers of $B^{(n)}$, $Z'^{(n)}$ and $W_L^{3(n)}$ can be exchanged.The exchange of $W_L^{3(n)}$ will cancel the $W_L^{1(n)}$ exchange between two $W_L^{1(0)}$ due to $SU(2)_L$ symmetry. The associated diagrams of $\Pi_{33Z'}$ and Π_{33B} involve the mixings

$$iI_{3B}^{(n)} = -i\frac{\langle h\rangle^2}{4g_5^2}\int_0^L dz\, f_L^{3(0)}(z) f_B^{(n)}(z), \qquad n > 0 \tag{3.119}$$

$$iI_{3Z}^{(n)} = -i\frac{\langle h\rangle^2}{4g_5^2}\frac{g_5^2}{\sqrt{g_5^2+g_{5X}^2}}\int_0^L dz\, f_L^{3(0)}(z) f_Z^{(n)}(z), \qquad n > 0 \tag{3.120}$$

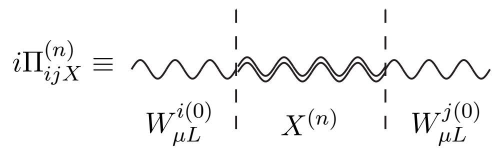


Figure 3.4: Tree level corrections to the mixing between W^i_L and W^j_L through the exchange of massive KK modes.

where

$$\int_0^L dz\ f_L^{3(0)}(z) f_{Z'}^{(n)}(z) = \frac{g_5}{\sqrt{L(1+\theta)}} \cdot \sqrt{\frac{2}{L}} \int_0^L dz\ \sin m_n z = \frac{2\sqrt{2}\, g_5}{\pi\sqrt{1+\theta}} \frac{1}{2n-1} \tag{3.121}$$

$$\begin{aligned}\int_0^L dz\ f_L^{3(0)}(z) f_B^{(n)}(z) &= -\frac{\sqrt{2} g_5 \tilde{g}_5 \tilde{\theta}}{\sqrt{1+\theta}} \left(\frac{1}{1+\tilde{\theta}+\tilde{\theta}^2 L^2 m_n^2}\right)^{\frac{1}{2}} \\ &= -\frac{2\sqrt{2} g_5 \tilde{g}_5 \tilde{\theta}}{\pi\sqrt{1+\theta}} \frac{1}{2n-1} \left(1 - \frac{6}{(2n-1)^2 \pi^2 \tilde{\theta}} + \mathcal{O}\left(\frac{1}{\tilde{\theta}^2}\right)\right). \end{aligned} \tag{3.122}$$

The mixing between $W_L^{3(0)}$, $B^{(n)}$ and $Z'^{(n)}$ is found using eq.(3.109) and the first equation in (3.38). Using these results, the computation of Π_{33B} and $\Pi_{33Z'}$ goes as follows

$$\begin{aligned} i\Pi_{33B} &\equiv \sum_{n=1}^{\infty} i\Pi_{33B}^{(n)} = -i \sum_{n=1}^{\infty} \frac{1}{m_n^2} \left(I_{3B}^{(n)}\right)^2 \\ &= -i \frac{\alpha^4}{4L^4 g_5^4} \sum_{n=1}^{\infty} \frac{4L^2}{\pi^2 (2n-1)^2} \left(1 - \frac{8}{\pi^2 (2n-1)^2} \frac{1}{\tilde{\theta}} \mathcal{O}\left(\frac{1}{\tilde{\theta}^2}\right)\right) \\ &\quad \cdot \left(\frac{-2\sqrt{2} g_5 \tilde{g}_5}{\pi\sqrt{1+\theta}}\right)^2 \frac{1}{(2n-1)^2} \left(1 - \frac{12}{\pi^2 (2n-1)^2} \frac{1}{\tilde{\theta}} \mathcal{O}\left(\frac{1}{\tilde{\theta}^2}\right)\right) \\ &= -i \frac{\alpha^4}{4L^4 g_5^4} \sum_{n=1}^{\infty} \frac{32 L^2 g_5^2 \tilde{g}_5^2}{\pi^4 (1+\theta)(2n-1)^4} \left(1 - \frac{20}{\pi^2 \tilde{\theta} (2n-1)^2}\right) \\ &= -i \frac{\alpha^4}{4L^4 g_5^4} \frac{32 L^2 g_5^2 \tilde{g}_5^2}{\pi^4 (1+\theta)} \left(\frac{\pi^4}{96} - \frac{20}{\pi^2 \tilde{\theta}} \frac{\pi^6}{960}\right) = -i \frac{\alpha^4 L^2 g_5^{-2} \tilde{g}_5^2}{12 L^2 (1+\theta)} \left(1 - \frac{2}{\tilde{\theta}}\right) \\ &= -i \frac{g_{5X}^2}{g_5^2 + g_{5X}^2} \frac{\alpha^4 L^2}{12 L^2 (1+\theta)} \left(1 - \frac{2}{\tilde{\theta}}\right) \end{aligned} \tag{3.123}$$

$$
\begin{aligned}
i\Pi_{33Z'} &\equiv \sum_{n=1}^{\infty} i\Pi_{33Z}^{(n)} = -i\sum_{n=1}^{\infty}\frac{1}{m_n^2}\left(I_{3Z}^{(n)}\right)^2 \\
&= -i\frac{\alpha^4}{4L^4 g_5^4}\left(\frac{g_5^2}{\sqrt{g_5^2+g_{5X}^2}}\right)^2 \sum_{n=1}^{\infty}\frac{4L^2}{\pi^2(2n-1)^2}\left(\frac{2\sqrt{2}\,g_5}{\pi\sqrt{1+\theta}}\frac{1}{2n-1}\right)^2 \\
&= -i\frac{8\alpha^4}{L^2(1+\theta)}\frac{g_5^2}{g_5^2+g_{5X}^2}\frac{1}{\pi^4}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^4} = -i\frac{g_5^2}{g_5^2+g_{5X}^2}\frac{\alpha^4}{12L^2(1+\theta)}
\end{aligned}
\tag{3.124}
$$

The final vacuum polarization amplitude will be the sum of three terms

$$
g^2\Pi_{33}(0) = \Pi_{33B} + \Pi_{33Z'} + \Pi_{33W_L^3} \tag{3.125}
$$

with the sum of the first two being

$$
\Pi_{33B} + \Pi_{33Z'} = -\frac{\alpha^4}{12L^2(1+\theta)} + \frac{g_{5X}^2}{g_5^2+g_{5X}^2}\frac{\alpha^4 L^2}{12L^2(1+\theta)}\frac{2}{\tilde{\theta}}. \tag{3.126}
$$

On the other hand, the massive KK modes that contribute to Π_{11} are $W_R^{1(n)}$ and $W_L^{1(n)}$. The mixing between $W_L^{1(0)}$ and $W_R^{1(n)}$ is given by

$$
iI_{1W_R^1}^{(n)} = -i\frac{\langle h\rangle^2}{4g_5^2}\int_0^L dz\; f_L^{1(0)}(z) f_R^{(n)}(z), \qquad n>0 \tag{3.127}
$$

where

$$
\int_0^L dz\; f_L^{1(0)}(z) f_R^{1(n)}(z) = \frac{g_5}{\sqrt{L(1+\theta)}}\cdot g_5\sqrt{\frac{2}{L}}\int_0^L dz\;\sin m_n z = \frac{2\sqrt{2}\,g_5^2}{\pi\sqrt{1+\theta}}\frac{1}{2n-1}. \tag{3.128}
$$

So the contribution of $W_R^{1(n)}$ to the vacuum polarization amplitude is

$$
\begin{aligned}
i\Pi_{11W_R^1} &\equiv \sum_{n=1}^{\infty} i\Pi_{11W_R^1}^{(n)} = -i\sum_{n=1}^{\infty}\frac{1}{m_n^2}\left(I_{1W_R^1}^{(n)}\right)^2 \\
&= -i\frac{\alpha^4}{4L^4 g_5^4}\sum_{n=1}^{\infty}\frac{4L^2}{\pi^2(2n-1)^2}\left(\frac{2\sqrt{2}\,g_5^2}{\pi\sqrt{1+\theta}}\frac{1}{2n-1}\right)^2 \\
&= -i\frac{8\alpha^4}{L^2\pi^4(1+\theta)}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^4} = -i\frac{\alpha^4}{12L^2\pi^4(1+\theta)}
\end{aligned}
\tag{3.129}
$$

Finally, the total vacuum polarization amplitude is the sum of two terms

$$
g^2\Pi_{11}(0) = \Pi_{11W_R^1} + \Pi_{11W_L^1}. \tag{3.130}
$$

So the custodial breaking combination of $\Pi_{33}(0)$ and $\Pi_{11}(0)$ is

$$\begin{aligned} g^2\left(\Pi_{11}(0)-\Pi_{33}(0)\right) &= \Pi_{11W_R^1}+\Pi_{11W_L^1}-\Pi_{33B}-\Pi_{33Z}-\Pi_{33W_L^3} \\ &= -\frac{g_{5X}^2}{g_5^2+g_{5X}^2}\frac{\alpha^4 L^2}{12L^2(1+\theta)}\frac{2}{\tilde{\theta}} \end{aligned} \quad (3.131)$$

As mentioned before, because of the $SU(2)_L$ symmetry, the two quantities $\Pi^{(n)}_{11W_L^1}$ and $\Pi^{(n)}_{33W_L^3}$ are equal and cancel out in the above combination. This gives the first contribution to the T parameter

$$T_1=\frac{4\pi}{s_\theta^2 c_\theta^2 m_Z^2}\left(\Pi_{11}(0)-\Pi_{33}(0)\right)=\frac{4\pi}{e^2 m_W^2}\,g^2\left(\Pi_{11}(0)-\Pi_{33}(0)\right)=-\frac{4\pi}{e^2}\frac{g_{5X}^2}{g_5^2+g_{5X}^2}\frac{\alpha^2}{3\tilde{\theta}} \quad (3.132)$$

where in the last equation we have used the value of the W mass which can be derived using (3.109) in the following way

$$m_W^2=2\frac{\langle h\rangle^2}{8g_5^2}\int_0^L dz\,\left(f_L^{1(0)}(z)\right)^2=\frac{\langle h\rangle^2}{4g_5^2}\frac{g_5^2}{L(1+\theta)}L=\frac{\langle h\rangle^2}{4(1+\theta)}=\frac{\alpha^2}{2L^2(1+\theta)}, \quad (3.133)$$

where $\langle h\rangle=\sqrt{2}\alpha/L$ has been used. The second contribution is obtained, again, using the results of section (3.2.4) and the gauge coupling deviation found in the previous section

$$T_2=\frac{8\pi\tilde{a}}{e^2}=\frac{8\pi\tilde{x}}{e^2}\left(\frac{g'}{g}\right)^2=\frac{8\pi}{e^2}\frac{\alpha^2}{6\theta}\frac{\tilde{g}_5^2}{g_5^2}\frac{\theta}{\tilde{\theta}}=\frac{8\pi}{e^2}\frac{\alpha^2}{6\theta}\frac{g_{5X}^2}{g_5^2+g_{5X}^2}\frac{\theta}{\tilde{\theta}}=\frac{4\pi}{e^2}\frac{g_{5X}^2}{g_5^2+g_{5X}^2}\frac{\alpha^2}{3\tilde{\theta}}. \quad (3.134)$$

It is clearly seen that these two contributions sum up to zero, as expected, because of the custodial symmetry.

$$T=T_1+T_2=0 \quad (3.135)$$

3.2.4 Vertex corrections interpreted as oblique corrections

In this section, following [38], we argue that if the dimension 6 operators contributing to the gauge fermion coupling appear with certain coefficients, they can be translated into oblique corrections. In this case we also find the explicit form of the electroweak observables S and T. For this purpose consider the dimension 6 operators including fermions

$$\mathcal{L}^{(6)}_{fermions}=-\frac{ix}{16\pi^2 v^2}\bar{\psi}\gamma^\mu\frac{\sigma^a}{2}\psi D_\mu H^\dagger\frac{\sigma^a}{2}H-\frac{iy}{16\pi^2 v^2}\bar{\psi}\gamma^\mu\psi D_\mu H^\dagger H+\frac{V}{16\pi^2 v^2}\bar{\psi}\psi\bar{\psi}\psi+h.c \quad (3.136)$$

concentrating on the first two terms, after EWSB this Lagrangian gives rise to

$$\begin{aligned}
\mathcal{L}^{(6)}_{fermions} &\to -\frac{ix}{64\pi^2 v^2}\bar{\psi}\gamma^\mu\sigma^a\psi\langle H\rangle^\dagger\left(g\vec{W}_\mu\cdot\frac{\vec{\sigma}}{2}+g'B_\mu Y_H\right)\sigma^a\langle H\rangle \\
&\quad -\frac{iy}{16\pi^2 v^2}\bar{\psi}\gamma^\mu\psi\langle H\rangle^\dagger\left(g\vec{W}_\mu\cdot\frac{\vec{\sigma}}{2}+g'B_\mu Y_H\right)\langle H\rangle + h.c \\
&= \frac{x}{128\pi^2}\left(\bar{\psi}\gamma^\mu\sigma^a\psi g W^b_\mu\frac{1}{2}(\sigma^a\sigma^b+\sigma^b\sigma^a)-\bar{\psi}\gamma^\mu\sigma^3\psi 2g'B_\mu Y_H\right) \\
&\quad +\frac{y}{32\pi^2}\bar{\psi}\gamma^\mu\psi\left(-gW^3_\mu+2g'B_\mu Y_H\right) \\
&= \frac{x}{64\pi^2}\left(\bar{\psi}\gamma^\mu\vec{W}_\mu\cdot\frac{\vec{\sigma}}{2}\psi-\bar{\psi}\gamma^\mu\sigma^3\psi g'B_\mu Y_H\right)+\frac{y}{32\pi^2}\bar{\psi}\gamma^\mu\psi\left(-gW^3_\mu+2g'B_\mu Y_H\right)
\end{aligned} \tag{3.137}$$

which leads to a modification of gauge fermion interactions

$$\bar{\psi}i\not{D}\psi+\mathcal{L}^{(6)}_{fermions}\to\bar{\psi}i\not{\partial}\psi+\bar{\psi}\gamma^\mu\left(\cdots\right)_\mu\psi, \tag{3.138}$$

where the terms in the parenthesis are given by

$$\begin{aligned}
\cdots &= g\left(W^1\frac{\sigma^1}{2}+W^2\frac{\sigma^2}{2}\right)\left(1+\frac{x}{64\pi^2}\right)+\frac{\sigma^3}{2}\left[gW^3\left(1+\frac{x}{64\pi^2}\right)-g'B(2Y_H)\frac{x}{64\pi^2}\right] \\
&\quad +g'B\left(Y+2(2Y_H)\frac{y}{64\pi^2}\right)-2gW^3\frac{y}{64\pi^2}.
\end{aligned} \tag{3.139}$$

For simplicity of notation one can set $2Y_H = 1$ and make the transformation $g'\to g'(2Y_H)$ and $Y\to Y/(2Y_H)$ to restore it whenever needed. We also define a, b by $x = ag^2$ and $y = bg'^2$ and the tilded quantities $\tilde{x}$, $\tilde{y}$, $\tilde{a}$ and $\tilde{b}$ through $\tilde{x}=\frac{x}{64\pi^2}$, $\tilde{y}=\frac{y}{64\pi^2}$, $\tilde{a}=\frac{a}{64\pi^2}$ and $\tilde{b}=\frac{b}{64\pi^2}$. The redefinition of the gauge fields needed to bring the gauge-fermion couplings in the canonical form are given by

$$W^{1,2}\to\frac{1}{1+\tilde{a}g^2}W^{1,2} \tag{3.140}$$

and

$$\begin{pmatrix} W^3 \\ B\end{pmatrix}\to\frac{1}{1+\tilde{a}g^2+\tilde{b}g'^2}\begin{pmatrix}1+\tilde{b}g'^2 & \tilde{b}gg' \\ \tilde{a}gg' & 1+\tilde{a}g^2\end{pmatrix}\begin{pmatrix} W^3 \\ B\end{pmatrix}. \tag{3.141}$$

Lets consider these field redefinitions in the Z_μ, A_μ basis. As a simple computation shows

$$\begin{aligned}
gW^3_\mu - g'B_\mu &\to \frac{1}{1+\tilde{a}g^2+\tilde{b}g'^2}\left(g(1+g'^2\tilde{b})W^3+g^2g'\tilde{a}B_\mu-gg'^2\tilde{b}W^3_\mu-g'(1+g^2\tilde{a})B_\mu\right) \\
&= \frac{1}{1+\tilde{a}g^2+\tilde{b}g'^2}\left(gW^3_\mu-g'B_\mu\right)\qquad\Rightarrow\qquad Z_\mu\to\frac{1}{1+\tilde{a}g^2+\tilde{b}g'^2}Z_\mu
\end{aligned} \tag{3.142}$$

so the field Z_μ is only rescaled, as it should, because no mass terms for A_μ must be generated due to $U(1)_{em}$ gauge invariance. But lets now figure out the transformation of A_μ

$$\begin{aligned} g'W^3_\mu + gB_\mu &\rightarrow \frac{1}{1+\tilde{a}g^2+\tilde{b}g'^2}\left(g'(1+g'^2\tilde{b})W^3 + gg'^2\tilde{a}B_\mu + g^2g'\tilde{b}W^3_\mu + g(1+g^2\tilde{a})B_\mu\right) \\ &= \frac{1}{1+\tilde{a}g^2+\tilde{b}g'^2}\left(g'(1+(g^2+g'^2)\tilde{b})W^3 + g(1+(g^2+g'^2\tilde{a})B_\mu\right). \end{aligned} \quad (3.143)$$

If we require only oblique corrections , then the analysis by [39] shows that no mixing between Z_μ and A_μ must be generated, this implies that A_μ must also be only rescaled, which is equivalent to having $\tilde{a} = \tilde{b}$. In fact in this case the scaling factor is 1, $A_\mu \rightarrow A_\mu$. For this special case, after these field transformations, the modified vacuum polarization amplitudes and their derivatives at $q^2 = 0$ are ($s_W \equiv \sin\theta_W$, $c_W \equiv \cos\theta_W$)

$$\Pi'_{W^3B}(0) = s_W c_W \left[\frac{1}{(1+\tilde{a}(g^2+g'^2))^2} - 1\right] = -2s_W c_W \tilde{a}(g^2+g'^2))^2 + \mathcal{O}(\tilde{a}^2), \quad (3.144)$$

which lead to the contribution

$$S = -\frac{16\pi}{gg'}\Pi'_{W^3B}(0) = 32\pi\tilde{a} = \frac{a}{2\pi} \quad (3.145)$$

to the S parameter, and

$$\Pi_{W^1W^1}(0) = \frac{v^2}{4}\frac{g^2}{(1+\tilde{a}g^2)^2}, \qquad \Pi_{W^3W^3}(0) = \frac{v^2}{4}\frac{g^2}{(1+\tilde{a}(g^2+g'^2))^2}, \quad (3.146)$$

which give rise to the T parameter

$$\begin{aligned} T &= \frac{4\pi}{s_W c_W^2 m_Z^2 g^2}\left(\Pi_{W^1W^1}(0) - \Pi_{W^3W^3}(0)\right) \\ &= \frac{4\pi}{s_W c_W^2 m_Z^2 g^2}\frac{v^2g^2}{4}\left[\frac{1}{(1+\tilde{a}g^2)^2} - \frac{1}{(1+\tilde{a}(g^2+g'^2))^2}\right] \\ &= \frac{4\pi}{s_W c_W^2 m_Z^2 g^2}\frac{v^2g^2}{4}\left(2\tilde{a}g'^2\right) + \mathcal{O}(\tilde{a}^2) = \frac{8\pi g'^2}{e^2}\tilde{a} + \mathcal{O}(\tilde{a}^2) = \frac{ag'^2}{8\pi e^2} + \mathcal{O}(\tilde{a}^2). \end{aligned} \quad (3.147)$$

3.3 Fermions in 5D

Along the lines of section 3.1 on gauge fields, we will briefly discuss some basic facts regarding fermion fields in 5D. Taking the space-time metric to be of the general form

(3.1), the fermion Lagrangian will be

$$\mathcal{L} = \int_{z_0}^{z_1} dz\ a^5 \left[\frac{i}{2} \bar{\psi}\, \Gamma^M D_M \psi\, \frac{1}{a} - \frac{i}{2} \overline{D_M \psi}\, \Gamma^M \psi\, \frac{1}{a} - M \bar{\psi}\psi \right], \tag{3.148}$$

where the 5D gamma matrices satisfying the Clifford algebra $[\Gamma^M, \Gamma^N] = 2\eta^{MN}\mathbf{1}$ and the covariant derivative are defined by

$$\Gamma^\mu = \gamma^\mu, \quad \Gamma^5 = -i\gamma^5, \qquad\qquad D_5 = \partial_5, \quad D_\mu = \partial_\mu + \frac{1}{4}\frac{a'}{a}\Gamma_\mu \Gamma^5. \tag{3.149}$$

The bulk e.o.m are easily found to be

$$\begin{aligned} \Big(\partial_z + Ma + 2\,\frac{a'}{a}\Big)\psi_L &= i\,\partial\!\!\!/\,\psi_R \\ \Big(\partial_z - Ma + 2\,\frac{a'}{a}\Big)\psi_R &= -i\,\partial\!\!\!/\,\psi_L, \end{aligned} \tag{3.150}$$

while the UV and IR boundary e.o.m are

$$\Big(\bar{\psi}_L \delta\psi_R + \delta\bar{\psi}_R \psi_L - \bar{\psi}_R \delta\psi_L - \delta\bar{\psi}_L \psi_R\Big)\Big|_{UV,IR} = 0. \tag{3.151}$$

We will call $\psi_{L,R} = 0$ Dirichlet or $(-)$ b.c while the constraint on the other chirality imposed by the bulk eqs.(3.150) will be called Neuman or $(+)$

$$\begin{aligned} 0 &= \Big(\partial_z + Ma + 2\,\frac{a'}{a}\Big)\psi_L\,\Big|_{UV,IR} \\ 0 &= \Big(\partial_z - Ma + 2\,\frac{a'}{a}\Big)\psi_R\,\Big|_{UV,IR}, \end{aligned} \tag{3.152}$$

In the next section we will move to the discussion of the holographic method for fermions.

3.3.1 Holography for fermions

Just like the case of gauge fields, one can find a holographic action for fermions by integrating out the bulk, keeping the UV boundary value of the fermion fields fixed [40]. As in the case of gauge fields, after moving to the axial gauge the fixed UV boundary values of the fermions will be transformed by the group element Σ^{-1}

$$e^{iS_{Hol}[\psi,\Sigma]} \equiv \int_{\Psi|_{uv} = \psi^{\Sigma^{-1}}} D\psi\, e^{iS[\Psi]}, \tag{3.153}$$

where the IR constraints are implicit. Of course one can only take one of the chiralities to be fixed on the boundary, since the b.c for the other chirality will automatically follow by the bulk e.o.m, being a first order differential equation. We demonstrate here the main features of how in practice this is achieved in the case of a single fermion in a flat extra dimension. The bulk fermion action is

$$\mathcal{L}_{Bulk} = \int_0^L dz \left[\frac{i}{2}\bar{\psi}\gamma^M\partial_M\psi - \frac{i}{2}\partial_M\bar{\psi}\gamma^M\psi - m\bar{\psi}\psi\right] \tag{3.154}$$

as mentioned above, we wish to take one of the chiralities as the holographic field to be fixed on the UV boundary. To see if these choices are compatible with the boundary constraints we consider the UV piece of the variation of this Lagrangian

$$\delta\mathcal{L}_{Bulk}|_{UV} = -\frac{1}{2}\left[\bar{\psi}_L\delta\psi_R + \delta\bar{\psi}_R\psi_L - \bar{\psi}_R\delta\psi_L - \delta\bar{\psi}_L\psi_R\right]_{z=0} \tag{3.155}$$

Depending on whether we choose the LH field or the RH field as the holographic field, the second or the first two terms in the above Lagrangian will vanish, leaving us with some extra constraints which are undesirable. So we are forced to add a UV boundary term to the Lagrangian whose variation cancels this unwanted piece. For ψ_L taken as the holographic field the appropriate Lagrangian is

$$\mathcal{L} = \frac{1}{2}(\bar{\psi}_L^0\psi_R^0 + \bar{\psi}_R^0\psi_L^0) + \mathcal{L}_{Bulk}, \tag{3.156}$$

while taking ψ_R to be holographic will lead us to

$$\mathcal{L} = -\frac{1}{2}(\bar{\psi}_L^0\psi_R^0 + \bar{\psi}_R^0\psi_L^0) + \mathcal{L}_{Bulk}. \tag{3.157}$$

Now the variation of these Lagrangians vanishes. For either of these cases one can add a function of the holographic field on the UV boundary, since their variation vanishes. So the most general Lagrangians are

$$\mathcal{L}_{UV}(\psi_L^0) + \frac{1}{2}(\bar{\psi}_L^0\psi_R^0 + \bar{\psi}_R^0\psi_L^0) + \mathcal{L}_{\text{Bulk}} \tag{3.158}$$

$$\mathcal{L}_{UV}(\psi_R^0) - \frac{1}{2}(\bar{\psi}_L^0\psi_R^0 + \bar{\psi}_R^0\psi_L^0) + \mathcal{L}_{\text{Bulk}}. \tag{3.159}$$

After extremizing the holographic Lagrangian, these two options lead to (+) b.c for ψ_L and ψ_R respectively. In other words, allowing the holographic fields on the UV boundary to vary and requiring that the variations of the two Lagrangians above vanish, gives rise to the constraints

$$\frac{\delta\mathcal{L}_{UV}(\psi_L^0)}{\delta\bar{\psi}_L^0} + \psi_R^0 = 0 \tag{3.160}$$

$$\frac{\delta\mathcal{L}_{UV}(\psi_R^0)}{\delta\bar{\psi}_R^0} - \psi_L^0 = 0 \tag{3.161}$$

respectively for the Lagrangians (3.158) and (3.159). Consider the first equation, it is clear that in the case of no UV Lagrangian $\mathcal{L}_{UV}$, this will give $\psi_R^0 = 0$. So we also refer to the more general condition (3.158) as a $(-)$ b.c for ψ_R, and by definition, what follows from the e.o.m is called a $(+)$ b.c for ψ_L. In the same way, eq.(3.159) leads to a $(-)$ b.c for ψ_L and a $(+)$ b.c for ψ_R. One might wonder if boundary constraints can give rise to a $(-)$ b.c for the holographic field. The answer is that this is simply done by adding a Lagrange multiplyer term to the Lagrangian. The Lagrangian (3.158) gives rise to a $(+)$ b.c for the holographic ψ_L, but modifying it as

$$\mathcal{L}_{UV}(\psi'_R) + (\bar{\psi}'_R\psi_L + \bar{\psi}_L\psi'_R) + \frac{1}{2}(\bar{\psi}^0_L\psi^0_R + \bar{\psi}^0_R\psi^0_L) + \mathcal{L}_{Bulk} \tag{3.162}$$

will lead to a $(-)$ b.c for the holographic ψ_L. This is easily seen by integrating out the Lagrange multiplyer ψ'_R by varying ψ_L, which brings us to (3.159), the Lagrangian that gives $(+)$ b.c for ψ_R. But a $(+)$ b.c for ψ_R means a $(-)$ b.c for ψ_L. For the other case, namely having a holographic $\psi_R(-)$, the Lagrangian (3.159) can be modified to

$$\mathcal{L}_{UV}(\psi'_L) + (\bar{\psi}'_L\psi_R + \bar{\psi}_R\psi'_L) - \frac{1}{2}(\bar{\psi}^0_L\psi^0_R + \bar{\psi}^0_R\psi^0_L) + \mathcal{L}_{Bulk}. \tag{3.163}$$

The Lagrangians in the Holographic and KK approaches are the same. In the holographic approach we first choose ψ_L or ψ_R as the holographic field and extremize the action with the constraint that the holographic field is fixed at the UV boundary (integrating out the bulk), doing this we arrive at a 4D action in terms of the holographic field. Then one can extremize this 4D action and get an extremum which is of course the extremum of the original 5D action. In the KK approach one extremizes the 5D action directly without first taking the UV value of one of the chiralities fixed. No matter which route we take, we arrive at the same extremum as far as the original 5D Lagrangians are the same. The only difference is that in the KK approach we don't need the Lagrange multiplyer terms. This was forced to us in the holographic approach by the requirement that the $(-)$ b.c for the holographic field be generated dynamically, otherwise we have to set it to zero by hand in which case there will be no action functional of it and we might loose possible zero modes coming from the other chirality. It is worth mentioning that in the KK approach when $\mathcal{L}_{UV} = 0$ we really don't need the term $\frac{1}{2}(\bar{\psi}^0_L\psi^0_R + \bar{\psi}^0_R\psi^0_L)$ also, because even without it the b.c s can be satisfied, in fact the boundary e.o.m in this case is eq.(3.155) equal to zero, which is satisfied when ψ_L and ψ_R are proportional to each other on the UV.

3.3.2 Flat Space Example

To demonstrate the holographic approach for fermions, we consider a single fermion on a flat extra dimension [20] with the bulk action (3.154). Taking the LH chirality ψ_L to be the holographic field with UV value χ_L the solutions to the e.o.m, respectively for $\psi_L(+)$ and $\psi_L(-)$ b.c on the IR brane are

$$\psi_{L,R} = \Pi^+_{L,R}\,\chi_L \qquad \psi_R(L) = 0 \tag{3.164}$$

$$\psi_{L,R} = \Pi^-_{L,R}\,\chi_L \qquad \psi_L(L) = 0 \tag{3.165}$$

where we have defined the bulk to boundary propagators

$$\begin{aligned} \Pi^+_L(z,m) &= \frac{F_+(z,m)}{F_+(m)} & \Pi^-_L(z,m) &= \frac{F_-(z,m)}{F_-(m)} \\ \Pi^+_R(z,m) &= \frac{F_-(z,m)}{F_+(m)}\frac{\not p}{p} & \Pi^-_R(z,m) &= -\frac{F_+(z,-m)}{F_-(m)}\frac{\not p}{p} \end{aligned} \tag{3.166}$$

using the definitions

$$\begin{aligned} F_+(z,m) &= \cos\omega(L-z) + \frac{m}{\omega}\sin\omega(L-z) & F_+(m) &= F_+(0,m) \\ F_-(z,m) &= \frac{p}{\omega}\sin\omega(L-z) & F_-(m) &= F_-(0,m) \end{aligned} \tag{3.167}$$

with $\omega \equiv \sqrt{p^2 - m^2}$.

When the RH chirality is taken as the holographic field with boundary value χ_R, the solutions for $\psi_R(+)$ and $\psi_R(-)$ b.c on the IR brane become

$$\psi_{L,R} = \widetilde{\Pi}^+_{L,R}\,\chi_R \qquad \psi_L(L) = 0 \tag{3.168}$$

$$\psi_{L,R} = \widetilde{\Pi}^-_{L,R}\,\chi_R \qquad \psi_R(L) = 0, \tag{3.169}$$

with the bulk to boundary propagators defined by

$$\begin{aligned} \widetilde{\Pi}^+_L(z,m) &= -\frac{F_-(z,m)}{F_+(-m)}\frac{\not p}{p} & \widetilde{\Pi}^-_L(z,m) &= \frac{F_+(z,m)}{F_-(m)}\frac{\not p}{p} \\ \widetilde{\Pi}^+_R(z,m) &= \frac{F_+(z,-m)}{F_+(-m)} & \widetilde{\Pi}^-_R(z,m) &= \frac{F_-(z,m)}{F_-(m)}. \end{aligned} \tag{3.170}$$

As we discussed previously, the Lagrangians we must deal with in the two cases above with ψ_L or ψ_R taken as holographic fields are given respectively by (3.158) and (3.159) Plugging the solutions (3.164) and (3.168) into the corresponding Lagrangians we find that the non vanishing contributions come from the boundary terms since the bulk action vanishes on-shell. The resulting holographic Lagrangians are

$$\begin{aligned} \mathcal{L}^\pm_L &= \bar\chi_L\,\Pi^\pm_R(m)\,\chi_L & \Pi^\pm_L(m) &\equiv \Pi^\pm_L(0,m) \\ \mathcal{L}^\pm_R &= \bar\chi_R\,\widetilde{\Pi}^\pm_L(m)\,\chi_R & \widetilde{\Pi}^\pm_L(m) &\equiv \widetilde{\Pi}^\pm_L(0,m) \end{aligned} \tag{3.171}$$

where the index L, R on $\mathcal{L}$ refers to the holographic chirality, and $\pm$ refers to the IR b.c of the holographic field. Finally in the case where the fields $\chi_{L,R}$ carry a nontrivial representation of the bulk gauge group, they must be replaced with $\Sigma^{-1}\chi_{L,R}$ and the components with $(-)$ UV b.c must be put to zero or one should add Lagrange multiplyers for them.

Chapter 4

Simple Composite Higgs models in Flat Extra Dimensions

In this chapter we will introduce three composite-Higgs/GHU models [41] based on the minimal symmetry breaking pattern $SO(5)/SO(4)$. This pattern of symmetry breaking was originally introduced in [13] in the context of warped extra dimensions, here we will adopt a flat extra dimensional set up but with boundary gauge kinetic terms. The three models we are about to discuss share the same gauge sector which was introduced in section 3.2 and differ in the way fermions are embedded in complete representations of $SO(5)$. In [13] the SM fermions were embedded in spinorial representations of $SO(5)$. This choice lead to large corrections to the $Z\bar{b}_L b_L$ vertex. In [42] it was shown that the same symmetry that protects the T parameter from large corrections, when accompanied by a $\mathbb{Z}_2$ symmetry that interchanges left and right, can in fact also protect the couplings of fermions to the Z boson. For this to occur, the quantum numbers of the fermion must satisfy either $T_L = T_R$ and $T_L^3 = T_R^3$ or $T_L^3 = T_R^3 = 0$. It turns out that the minimal choices to embed b_L are the fundamental and the adjoint representations of $SO(5)$. This paved the way to constructing more realistic models [14]. We will now move to the discussion of our models.

4.1 Model I: Modified MCHM$_5$

As the first model we embed the third generation SM fermions in four fundamental representations of $SO(5)$. An extra gauged $U(1)_X$ is added in order to fix the fermion hypercharges to their correct values. We choose two multiplets ξ_1 and ξ_u to have X charge 2/3 and the other two ξ_2 and ξ_d to have X charge $-1/3$. A fundamental of $SO(5)$ is decomposed under its $SO(4) = SU(2)_L \times SU(2)_R$ subgroup as $\mathbf{5} = (\mathbf{2}, \mathbf{2}) + (\mathbf{1}, \mathbf{1})$. Using this decomposition the embeddings of the SM fermions are shown in the following way

$$\xi_1 = \begin{pmatrix} (2,2)^1_L = \begin{bmatrix} q'_{1L}(-+) \\ q_{1L}(++) \end{bmatrix} & (2,2)^1_R = \begin{bmatrix} q'_{1R}(+-) \\ q_{1R}(--) \end{bmatrix} \\ (1,1)^1_L(--) & (1,1)^1_R(++) \end{pmatrix}_{2/3} \tag{4.1}$$

$$\xi_2 = \begin{pmatrix} (2,2)^2_L = \begin{bmatrix} q_{2L}(++) \\ q'_{2L}(-+) \end{bmatrix} & (2,2)^2_R = \begin{bmatrix} q_{2R}(--) \\ q'_{2R}(+-) \end{bmatrix} \\ (1,1)^2_L(--) & (1,1)^2_R(++) \end{pmatrix}_{-1/3} \tag{4.2}$$

$$\xi_u = \begin{pmatrix} (2,2)^u_L(+-) & (2,2)^u_R(-+) \\ (1,1)^u_L(-+) & (1,1)^u_R(+-) \end{pmatrix}_{2/3}, \qquad \xi_d = \begin{pmatrix} (2,2)^d_L(+-) & (2,2)^d_R(-+) \\ (1,1)^d_L(-+) & (1,1)^d_R(+-) \end{pmatrix}_{-1/3} \tag{4.3}$$

where L, R denote the chiralities and the first(second) entries in the parentheses are the b.c on the UV(IR) boundaries. These b.c are chosen in such a way to give rise to zero modes only for the SM fields, while respecting the symmetries on the boundaries. To embed the RH top and bottom singlets we need both ξ_1 and ξ_2, but as shown above, the LH doublet is embedded in both of them. To get rid of the extra doublet zero mode one can add a RH $SU(2)_L$ doublet that mixes with the combination $q_{1L} - q_{2L}$ on the UV boundary so that it leads to $q_{1L} = q_{2L}$. The bulk Lagrangian is

$$\mathcal{L}_{5f} = \int_0^L dz \sum_{j=1,2,u,d} \left[\frac{i}{2}\bar{\xi}_j \gamma^M \partial_M \xi_j - \frac{i}{2}\partial_M \bar{\xi}_j \gamma^M \xi_j - m_j \bar{\xi}_j \xi_j \right], \tag{4.4}$$

while the most general Lagrangian on the IR boundary, compatible with the symmetry $O(4) \times U(1)_X$, is

$$\mathcal{L}_{4f,IR} = \tilde{m}_u\, \bar{\xi}^b_{1L}\, \xi^b_{uR} + \tilde{m}_d\, \bar{\xi}^b_{2L}\, \xi^b_{dR} + \tilde{M}_u\, \bar{\xi}^s_{1R}\, \xi^s_{uL} + \tilde{M}_d\, \bar{\xi}^s_{2R}\, \xi^s_{dL} + h.c \tag{4.5}$$

where by $b(s)$ a bidoublet(singlet) subrepresentation is meant. Notice that because the fermion fields in 5D have mass dimension 2, the mass mixing parameters introduced above are dimensionless. Taking ξ_{1L}, ξ_{2L}, ξ_{uR} and ξ_{dR} to be holographic, According to the discussion of section 3.3.1 a UV Lagrangian has to be added

$$\mathcal{L}_{4f,UV} = \frac{1}{2} \sum_{j=1,2} \left(\bar{\xi}_{jL}\, \xi_{jR} + \bar{\xi}_{jR}\, \xi_{jL} \right) - \frac{1}{2} \sum_{j=u,d} \left(\bar{\xi}_{jL}\, \xi_{jR} + \bar{\xi}_{jR}\, \xi_{jL} \right). \tag{4.6}$$

We have not added Lagrange multiplyers here for the holographic components which vanish at the UV boundary because the possible zero modes are already captured by the presence of the IR mixing terms.

Keeping the holographic fields fixed on the UV boundary and subject to the IR b.c, we solve the e.o.m and plug in (4.6) to find the holographic Lagrangian [1]

$$\begin{aligned}
\mathcal{L}_{Hol} &= \sum_{i=1L,uR} \Pi_i^b \left(\Sigma^{-1}\bar{\chi}_i\right)^b \frac{\not p}{p}\left(\Sigma^{-1}\chi_i\right)^b + \Pi_{1u}^b\Big[\left(\Sigma^{-1}\bar{\chi}_{1L}\right)^b\left(\Sigma^{-1}\chi_{uR}\right)^b + h.c\Big] \\
&+ \sum_{i=2L,dR} \Pi_i^b \left(\Sigma^{-1}\bar{\chi}_i\right)^b \frac{\not p}{p}\left(\Sigma^{-1}\chi_i\right)^b + \Pi_{2d}^b\Big[\left(\Sigma^{-1}\bar{\chi}_{2L}\right)^b\left(\Sigma^{-1}\chi_{dR}\right)^b + h.c\Big] \\
&+ \sum_{i=1L,uR} \Pi_i^s \left(\Sigma^{-1}\bar{\chi}_i\right)^s \frac{\not p}{p}\left(\Sigma^{-1}\chi_i\right)^s + \Pi_{1u}^s\Big[\left(\Sigma^{-1}\bar{\chi}_{1L}\right)^s\left(\Sigma^{-1}\chi_{uR}\right)^s + h.c\Big] \\
&+ \sum_{i=2L,dR} \Pi_i^s \left(\Sigma^{-1}\bar{\chi}_i\right)^s \frac{\not p}{p}\left(\Sigma^{-1}\chi_i\right)^s + \Pi_{2d}^s\Big[\left(\Sigma^{-1}\bar{\chi}_{2L}\right)^s\left(\Sigma^{-1}\chi_{dR}\right)^s + h.c\Big] \quad (4.7)
\end{aligned}$$

Where $(\cdots)^b$ and $(\cdots)^s$ are meant to denote projections on the bidoublet and singlet subrepresentations respectively. Using the following identities in which the $\not p/p$ factors have been omitted (throughout this chapter we use Σ instead of U, so that $\Phi = \Sigma\Phi_0$)

$$\begin{aligned}
\left(\Sigma^{-1}\bar{\chi}_{1L}\right)^b\left(\Sigma^{-1}\chi_{1L}\right)^b &= \bar{\chi}_{1L}\left(1-\Phi\Phi^T\right)\chi_{1L} = \bar{q}_L\big(1-\frac{s_h^2}{2h^2}H^cH^{c\dagger}\big)q_L \\
\left(\Sigma^{-1}\bar{\chi}_{uR}\right)^b\left(\Sigma^{-1}\chi_{uR}\right)^b &= \bar{\chi}_{uR}\left(1-\Phi\Phi^T\right)\chi_{uR} = \bar{t}_R\left(1-c_h^2\right)t_R = s_h^2\bar{t}_Rt_R \\
\left(\Sigma^{-1}\bar{\chi}_{1L}\right)^b\left(\Sigma^{-1}\chi_{uR}\right)^b &= \bar{\chi}_{1L}\left(1-\Phi\Phi^T\right)\chi_{uR} = -\frac{s_hc_h}{\sqrt{2}h}\bar{q}_LH^ct_R \\
\left(\Sigma^{-1}\bar{\chi}_{2L}\right)^s\left(\Sigma^{-1}\chi_{2L}\right)^s &= \bar{\chi}_{2L}\Phi\Phi^T\chi_{2L} = \bar{q}_L\big(1-\frac{s_h^2}{2h^2}HH^\dagger\big)q_L \\
\left(\Sigma^{-1}\bar{\chi}_{dR}\right)^s\left(\Sigma^{-1}\chi_{dR}\right)^s &= \bar{\chi}_{dR}\Phi\Phi^T\chi_{dR} = c_h^2\bar{b}_Rb_R \\
\left(\Sigma^{-1}\bar{\chi}_{2L}\right)^s\left(\Sigma^{-1}\chi_{dR}\right)^s &= \bar{\chi}_{2L}\Phi\Phi^T\chi_{dR} = \frac{s_hc_h}{\sqrt{2}h}\bar{q}_LHb_R, \quad (4.8)
\end{aligned}$$

the Lagrangian (4.7) can be rewritten as

$$\begin{aligned}
\mathcal{L}_{Hol} &= \bar{q}_L\frac{\not p}{p}\left[\Pi^q + \frac{s_h^2}{h^2}\Big(\Pi^t\,H^cH^{c\dagger} + \Pi^b\,HH^\dagger\Big)\right]q_L + \sum_{a=t,b}\bar{a}_R\frac{\not p}{p}\left(\Pi_0^a + s_h^2\Pi_1^a\right)a_R \\
&+ \frac{s_hc_h}{\sqrt{2}h}\Big(\Pi_M^t\,\bar{q}_LH^ct_R + \Pi_M^b\,\bar{q}_LHb_R + h.c\Big) \quad (4.9)
\end{aligned}$$

in which the form factors are related to those of (4.7) through

$$\Pi^q = \Pi_{1L}^b + \Pi_{1L}^s, \qquad \begin{aligned}\Pi^t &= \tfrac{1}{2}\left(\Pi_{1L}^s - \Pi_{1L}^b\right)\\ \Pi^b &= \tfrac{1}{2}\left(\Pi_{2L}^s - \Pi_{2L}^b\right)\end{aligned} \quad (4.10)$$

[1]For a generalization of eqs.(3.171) to the case of 2 fermions with a mixing term on the IR brane, and also for the definitions of the above form factors refer to appendices B and C of [20].

$$\begin{array}{lll}\Pi_0^t &=& \Pi_{uR}^s \\ \Pi_0^b &=& \Pi_{dR}^s\end{array}, \qquad \begin{array}{lll}\Pi_1^t &=& \Pi_{uR}^b - \Pi_{uR}^b \\ \Pi_1^b &=& \Pi_{dR}^b - \Pi_{dR}^s\end{array}, \qquad \begin{array}{lll}\Pi_M^t &=& \Pi_{1u}^s - \Pi_{1u}^b \\ \Pi_M^b &=& \Pi_{2d}^s - \Pi_{2d}^b\end{array}. \tag{4.11}$$

The top and bottom quark masses are approximately given by

$$\frac{M_t^2}{M_W^2} \simeq \frac{\theta|\tilde{m}_u - \tilde{M}_u^{-1}|^2 e^{2L(m_u - m_1)}}{N_L N_{uR}}, \qquad \frac{M_b^2}{M_W^2} \simeq \frac{\theta|\tilde{m}_d - \tilde{M}_d^{-1}|^2 e^{2L(m_d - m_2)}}{N_L N_{dR}}, \tag{4.12}$$

where $N_L L$, $N_{uR} L$ and $N_{dR} L$ are, respectively, the coefficients of the kinetic terms of q_L, t_R and b_R, given by the following expressions

$$\begin{aligned} N_L &= \lim_{p\to 0} \frac{\Pi_0^q}{pL} = \frac{1}{L} \sum_{i=u,d,q_1,q_2} \int_0^L dy f_{iL}^2(y)\,, \\ N_{uR} &= \lim_{p\to 0} \frac{\Pi_0^u}{pL} = \frac{1}{L} \int_0^L dy \Big(f_{uR}^2(y) + f_{q_1 R}^2(y) \Big)\,, \\ N_{dR} &= \lim_{p\to 0} \frac{\Pi_0^d}{pL} = \frac{1}{L} \int_0^L dy \Big(f_{dR}^2(y) + f_{q_2 R}^2(y) \Big)\,, \end{aligned} \tag{4.13}$$

with $f_{iL,iR}(y)$ the "holographic" wave functions of the LH/RH top and bottom quarks before EWSB which read

$$\left\{\begin{array}{lll} f_{1L} &=& e^{-m_1 y} \\ f_{1R} &=& \frac{1}{\tilde{M}_u} e^{m_u L - m_1 (L-y)} \end{array}\right., \qquad \left\{\begin{array}{lll} f_{2L} &=& e^{-m_2 y} \\ f_{2R} &=& \frac{1}{\tilde{M}_d} e^{m_d L - m_2 (L-y)} \end{array}\right.,$$

$$\left\{\begin{array}{lll} f_{uL} &=& -\tilde{m}_u e^{-m_1 L + m_u (L-y)} \\ f_{uR} &=& e^{m_u y} \end{array}\right., \qquad \left\{\begin{array}{lll} f_{dL} &=& -\tilde{m}_d e^{-m_2 L + m_d (L-y)} \\ f_{dR} &=& e^{m_d y} \end{array}\right.. \tag{4.14}$$

The relations (4.13) will be proved at the end of this section. The spectrum of fermion resonances beyond the SM, before EWSB, is given by KK towers of states in the $\mathbf{2}_{7/6}$, $\mathbf{2}_{-5/6}$, $\mathbf{2}_{1/6}$, $\mathbf{1}_{2/3}$ and $\mathbf{1}_{-1/3}$ of $SU(2)_L \times U(1)_Y$.

4.1.1 The effective potential

Using the holographic Lagrangian (4.9) one can easily compute the effective potential. There are two contributions, one from the 2/3 charge sector V_t and one from the $-1/3$ charge sector V_b

$$V_i = -2N_c \int \frac{d^4p}{(2\pi)^4} \log\left[\left(1 + s_\alpha^2 \frac{\Pi^i}{\Pi^q}\right)\left(1 + s_\alpha^2 \frac{\Pi_1^i}{\Pi_0^i}\right) - s_{2\alpha}^2 \frac{(\Pi_M^i)^2}{8\Pi^q \Pi_0^i}\right], \qquad i = t, b \tag{4.15}$$

where $s_\alpha^2 \equiv \sin^2\alpha$ and $\alpha \equiv \langle h\rangle/f$. The factor of 2 is there to count the spin index. So the total effective potential will be

$$V = V_t + V_b + V_g \tag{4.16}$$

where V_g is given in (3.56). The form factors Π^i, Π_1^i and Π_M^i fall off exponentially with momentum. Expanding the Logarithms in the potential to leading order in the small ratios appearing in front of s_α^2, one finds that it has the general structure

$$V_f = a\, s_\alpha^2 - b\, s_\alpha^4, \qquad\qquad a, b = \text{const.} \tag{4.17}$$

which can cause EWSB. In fact it is the top quark loop that triggers EWSB by contributing most significantly to the constant b.

4.1.2 Computation of the $Z\bar{b}_L b_L$ vertex at tree level in Model I

Suppose b_L is the only $-1/3$ charge field embedded in the LH multiplet ξ, in the fundamental of $SO(5)$ which is nonvanishing on the UV boundary, an assumption which is valid for model I. Taking the LH field χ to be the UV value of ξ, this field can be written, after moving to the axial gauge and at zero momentum, in the following way

$$\xi = f_b(z)\left(\Sigma^{-1}\chi\right)_b + f_s(z)\left(\Sigma^{-1}\chi\right)_s \tag{4.18}$$

where, again the indices b and s denote projections on the bidoublet and singlet components. Notice that at non zero momentum there are other terms appearing on the RHS of (4.18) which are proportional to RH fields on UV. Concentrating on the $-1/3$ charge sector, b_L is the only field that appears in χ, so it always appears as an overall factor and we omit it for simplicity, treating χ as a vector. We also assume that it is normalized to unity $\chi^\dagger\chi = 1$.

To find the coefficient of the vertex $Z\bar{b}_L b_L$ we need to concentrate on the gauge interaction term of the holographic Lagrangian. The relevant part of this term, which involves $W^3_{L\mu}$, B_μ and X_μ is given by

$$\begin{aligned}\Sigma^\dagger\left[W_L^3 T_L^3 + B\left(T_R^3 + X\right)\right]\Sigma = &\left(W_L^3\cos^2\frac{\alpha}{2} + B\sin^2\frac{\alpha}{2}\right)T_L^3 \\ &+\left(W_L^3\sin^2\frac{\alpha}{2} + B\cos^2\frac{\alpha}{2}\right)T_R^3 + BX - \left(B - W_L^3\right)\frac{\sin\alpha}{\sqrt{2}}\,T_B^3\end{aligned} \tag{4.19}$$

in which the zero momentum bulk to boundary propagators, $G_T^+(0,z) = 1$ and $G_T^-(0,z) = 1 - z/L$, must be understood in front of the unbroken and broken generators respectively, that is

$$\begin{aligned}&\left(W_L^3\cos^2\frac{\alpha}{2} + B\sin^2\frac{\alpha}{2}\right)T_L^3 + \left(W_L^3\sin^2\frac{\alpha}{2} + B\cos^2\frac{\alpha}{2}\right)T_R^3 + BX - \left(1 - \frac{z}{L}\right)\left(B - W_L^3\right)\frac{\sin\alpha}{\sqrt{2}}T_B^3 \\ &= \Sigma^\dagger\left[W_L^3 T_L^3 + B T_R^3\right]\Sigma - \left(B - W_L^3\right)\frac{\sin\alpha}{\sqrt{2}}T_B^3 + BX - \left(1 - \frac{z}{L}\right)\left(B - W_L^3\right)\frac{\sin\alpha}{\sqrt{2}}T_B^3.\end{aligned} \tag{4.20}$$

Since we are interested in the Z couplings, we can omit the photon field and set, using (3.55)

$$W_L^3 \to \frac{g^2}{\sqrt{g^2+g'^2}} Z + \cdots , \qquad B \to -\frac{g'^2}{\sqrt{g^2+g'^2}} Z + \cdots , \tag{4.21}$$

where the dots represent the terms proportional to the photon field in which we are not interested here. Doing this, we find the relevant gauge connection term to be

$$\begin{aligned}
&\frac{Z}{\sqrt{g^2+g'^2}} \left[\Sigma^\dagger \left[g^2 T_L^3 - g'^2 T_R^3 \right] \Sigma + \left(g'^2 + g^2 \right) \frac{\sin\alpha}{\sqrt{2}} T_B^3 - g'^2 X + \left(1 - \frac{z}{L} \right) \left(g'^2 + g^2 \right) \frac{\sin\alpha}{\sqrt{2}} T_B^3 \right] \\
&= \frac{gZ}{\cos\theta_W} \left[\Sigma^\dagger \left[\cos^2\theta_W T_L^3 - \sin^2\theta_W T_R^3 \right] \Sigma + \frac{\sin\alpha}{\sqrt{2}} T_B^3 - \sin^2\theta_W X + \left(1 - \frac{z}{L} \right) \frac{\sin\alpha}{\sqrt{2}} T_B^3 \right].
\end{aligned} \tag{4.22}$$

We denote the four terms appearing in the brackets by $\Gamma_{\text{I}}, \Gamma_{\text{II}}, \Gamma_{\text{III}}$ and Γ_{IV}, and their sum by Γ. It is clear from eq (4.19) that the sum of the first two terms is a linear combination of $T_{L,R}^3$ and so it only has a bidoublet-bidoublet component. So its contribution to the $Z\bar{b}_L b_L$ vertex can be found in the following way

$$\begin{aligned}
\bar{\xi} \left(\Gamma_{\text{I}} + \Gamma_{\text{II}} \right) \xi &= f_b^2(z) \left(\Sigma^{-1} \chi \right)_b^\dagger \left(\Gamma_{\text{I}} + \Gamma_{\text{II}} \right) \left(\Sigma^{-1} \chi \right)_b = f_b^2(z) \left(\Sigma^{-1} \chi \right)^\dagger \left(\Gamma_{\text{I}} + \Gamma_{\text{II}} \right) \left(\Sigma^{-1} \chi \right) \\
&= f_b^2(z) \chi^\dagger \left(\cos^2\theta_W T_L^3 - \sin^2\theta_W T_R^3 - \frac{\sin\alpha}{\sqrt{2}} \Sigma^\dagger T_B^3 \Sigma \right) \chi \\
&= f_b^2(z) \chi^\dagger \left(\cos^2\theta_W T_L^3 - \sin^2\theta_W T_R^3 - \frac{\sin 2\alpha}{2\sqrt{2}} T_B^3 - \frac{\sin^2\alpha}{2} \left(T_L^3 - T_R^3 \right) \right) \chi \\
&= f_b^2(z) \chi^\dagger \left(\cos^2\theta_W T_L^3 - \sin^2\theta_W T_R^3 - \frac{\sin^2\alpha}{2} \left(T_L^3 - T_R^3 \right) \right) \chi \\
&= f_b^2(z) \left(\cos^2\theta_W T_L^3 - \sin^2\theta_W T_R^3 - \frac{\sin^2\alpha}{2} \left(T_L^3 - T_R^3 \right) \right),
\end{aligned} \tag{4.23}$$

where in the third equation we have used the relation

$$\Sigma^\dagger T_B^3 \Sigma = \cos\alpha \, T_B^3 + \frac{\sin\alpha}{\sqrt{2}} \left(T_L^3 - T_R^3 \right) \tag{4.24}$$

and in the second last equation we have used $\chi^\dagger T_B^3 \chi = 0$. Also in the last line $T_{L,R}^3$ are meant to be the eigenvalues of $T_{L,R}^3$ associated with χ. Similarly the fourth term will give

$$\begin{aligned}
\bar{\xi} \, \Gamma_{\text{IV}} \, \xi &= f_b(z) f_s(z) \left(\Sigma^{-1} \chi \right)^\dagger \Gamma_{\text{IV}} \left(\Sigma^{-1} \chi \right) \\
&= f_b(z) f_s(z) \left(1 - \frac{z}{L} \right) \frac{\sin\alpha}{\sqrt{2}} \chi^\dagger \left(\cos\alpha \, T_B^3 + \frac{\sin\alpha}{\sqrt{2}} \left(T_L^3 - T_R^3 \right) \right) \chi \\
&= f_b(z) f_s(z) \left(1 - \frac{z}{L} \right) \frac{\sin^2\alpha}{2} \left(T_L^3 - T_R^3 \right).
\end{aligned} \tag{4.25}$$

In order to find the contribution of the third term we first compute

$$\begin{aligned}
\bar{\xi}\,\xi &= f_b^2(z)\left(\Sigma^{-1}\chi\right)_b^\dagger\left(\Sigma^{-1}\chi\right)_b + f_s^2(z)\left(\Sigma^{-1}\chi\right)_s^\dagger\left(\Sigma^{-1}\chi\right)_s \\
&= f_b^2(z)\big(1-\left(\Sigma^{-1}\chi\right)_s^\dagger\left(\Sigma^{-1}\chi\right)_s\big) + f_s^2(z)\left(\Sigma^{-1}\chi\right)_s^\dagger\left(\Sigma^{-1}\chi\right)_s \\
&= f_b^2(z) + \left(f_s^2(z)-f_b^2(z)\right)\left(\Sigma^{-1}\chi\right)_s^\dagger\left(\Sigma^{-1}\chi\right)_s \\
&= f_b^2(z) - \frac{1}{2}\left(f_s^2(z)-f_b^2(z)\right)\left(T_L^3-T_R^3\right)\sin^2\alpha
\end{aligned} \tag{4.26}$$

in which we have used

$$\left(\Sigma^{-1}\chi\right)_s^\dagger\left(\Sigma^{-1}\chi\right)_s = -\frac{1}{2}\left(T_L^3-T_R^3\right)\sin^2\alpha. \tag{4.27}$$

Using this the contribution of the third term reads

$$\bar{\xi}\,\Gamma_{\rm III}\,\xi = -s_W^2\,X\,\bar{\xi}\,\xi = -s_W^2\,X f_b^2(z) + \frac{1}{2}s_W^2 X\left(f_s^2(z)-f_b^2(z)\right)\left(T_L^3-T_R^3\right)s_\alpha^2 \tag{4.28}$$

where $s_W^2 \equiv \sin^2\theta_W$ and $s_\alpha^2 \equiv \sin^2\alpha$. Putting the three pieces together we find

$$\int_0^L dz\,\xi^\dagger(z)\,\Gamma\,\xi(z) = N_b\left(T_L^3 - s_W^2 Q\right) - \frac{\sin^2\alpha}{2}\left(T_L^3-T_R^3\right)\left[N_b - s_W^2 X(N_s-N_b) - N_{bs}\right] \tag{4.29}$$

where we have made use of the following definitions

$$N_b = \int_0^L dz\,f_b^2(z), \qquad N_s = \int_0^L dz\,f_s^2(z), \qquad N_{bs} = \int_0^L dz\,f_b(z)f_s(z)\left(1-\frac{z}{L}\right). \tag{4.30}$$

Also using (4.26) we can write

$$\int_0^L dz\,\xi^\dagger(z)\,\xi(z) = N_b - \frac{\sin^2\alpha}{2}\left(T_L^3-T_R^3\right)(N_s-N_b) \tag{4.31}$$

Finally, the expression for the $Z\bar{b}_L b_L$ coupling is given by

$$\begin{aligned}
g_L &= \frac{\int_0^L dz\, \xi^\dagger(z)\,\Gamma\,\xi(z)}{\int_0^L dz\, \xi^\dagger(z)\xi(z)} \\
&= \left(T_L^3 - s_W^2 Q\right) + \frac{\sin^2\alpha}{2N_b}\left(T_L^3 - T_R^3\right)\left[(N_s - N_b)\left(T_L^3 - s_W^2 Q\right) - N_b + s_W^2 X (N_s - N_b) + N_{bs}\right] \\
&= \left(T_L^3 - s_W^2 Q\right) + \frac{\sin^2\alpha}{2N_b}\left(T_L^3 - T_R^3\right)\left[(N_s - N_b)\left(c_W^2 T_L^3 - s_W^2 T_R^3\right) - N_b + N_{bs}\right] \\
&= \left(T_L^3 - s_W^2 Q\right) + \frac{\sin^2\alpha}{2N_b}\left(T_L^3 - T_R^3\right)\left[-(N_s - N_b)/2 - N_b + N_{bs}\right] \\
&= \left(T_L^3 - s_W^2 Q\right) + \frac{\sin^2\alpha}{4N_b}\left(T_R^3 - T_L^3\right)\left[N_s + N_b - 2N_{bs}\right] \\
&= \left(T_L^3 - s_W^2 Q\right) + \frac{\sin^2\alpha}{4N_b}\left(T_R^3 - T_L^3\right)\int_0^L dz\, \left[(\delta f(z))^2 + 2\frac{z}{L} f_b(z) f_s(z)\right]
\end{aligned} \tag{4.32}$$

where $\delta f(z) = f_s(z) - f_b(z)$. To go from the third equation to the fourth, in the expression $c_W^2 T_L^3 - s_W^2 T_R^3$ we have set $T_R^3 = -T_L^3 = 1/2$, because otherwise it gives no contribution, being multiplied by $T_L^3 - T_R^3$. The generalization to the case where b_L is embedded in several multiplets is obvious, one adds an index i to $f_b(z)$, $f_s(z)$ and T_R^3 and sums over it. In this case the $Z\bar{b}_L b_L$ coupling deviation is

$$\delta g = \frac{\sin^2\alpha}{4N}\sum_i \left(T_{Ri}^3 - T_L^3\right)\int_0^L dz\, \left[(\delta f_i(z))^2 + \frac{z}{L} f_{bi}(z) f_{si}(z)\right] + \mathcal{O}(\sin^4\alpha), \tag{4.33}$$

where now

$$N = \int_0^L dz \sum_i f_{bi}^2(z). \tag{4.34}$$

For model I the index i runs over 1, 2, u and d, and the wave functions $f_{bi}(z)$ are equal to $f_{iL}(z)$ defined in (4.14), while $f_{si}(z)$ are equal to $f_{iL}(z)$ in which the replacement $\tilde{m}_{u,d} \to 1/\tilde{M}_{u,d}$ has been made.

In the following we argue that $\int_0^L dz\, \xi^\dagger(z)\xi(z)$ is in fact equal to the coefficient of the kinetic term of χ_L at zero momentum. For this purpose, consider the bulk e.o.m for fermions

$$\begin{aligned}
\not{p}\,\psi_R &= (\partial_5 + m)\psi_L \qquad (4.35)\\
-\not{p}\,\psi_L &= (\partial_5 - m)\psi_R \qquad (4.36)
\end{aligned}$$

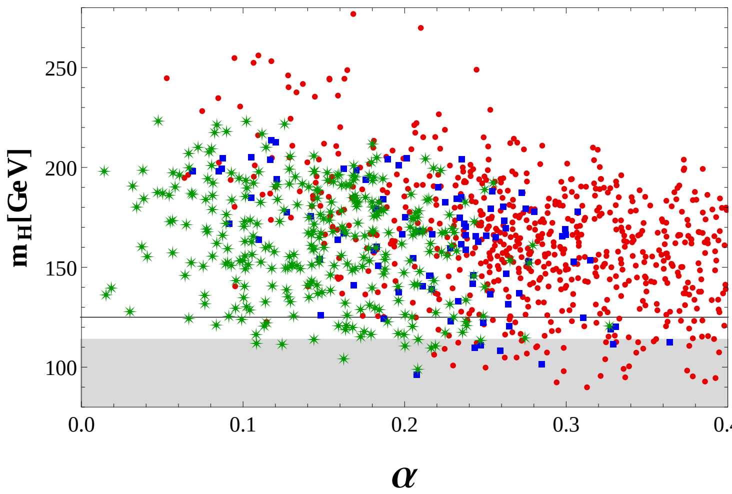


Figure 4.1: *Scatter plot of points obtained from a scan over the parameter space of model I. Small red dots represent points which don't pass EWPT at* 99%*C.L., square blue dots represent points which pass EWPT at* 99%*C.L. but not at* 90% *C.L., and star shape green dots represent points which pass EWPT at* 90%*C.L.. The region below the LEP bound* (m_H < 114 GeV) *is shaded. The mass of the recently discovered Higgs-like particle at* 125 GeV *is shown by the black line.*

and multiply the first(second) equation on the left by $\bar{\psi}_{R(L)}$, this leads to

$$\bar{\psi}_R \not{p}\, \psi_R = \bar{\psi}_R \partial_5 \psi_L + m\bar{\psi}_R \psi_L \quad (4.37)$$

$$-\bar{\psi}_L \not{p}\, \psi_L = \bar{\psi}_L \partial_5 \psi_R - m\bar{\psi}_R \psi_R. \quad (4.38)$$

Adding the first equation to the conjugate of the second equation gives

$$\bar{\psi}_L \not{p} \psi_L - \bar{\psi}_R \not{p} \psi_R = -\partial_5(\bar{\psi}_R \psi_L) = -\frac{1}{2}\partial_5(\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R) \quad (4.39)$$

which, upon integration over the extra dimension, leads to

$$\int_0^L dz\, (\bar{\psi}_L \not{p} \psi_L - \bar{\psi}_R \not{p} \psi_R) = \frac{1}{2}(\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R)|_{UV} - \frac{1}{2}(\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R)|_{IR}. \quad (4.40)$$

For a single fermion the term on the IR brane vanishes, since one of the chiralities is chosen to have (−) b.c. For two fermions with mixed b.c the IR term vanishes when summed over the two fermion fields

$$\sum_{i=1,2} (\bar{\psi}_{iR} \psi_{iL} + \bar{\psi}_{iL} \psi_{iR})|_{IR} = 0 \quad (4.41)$$

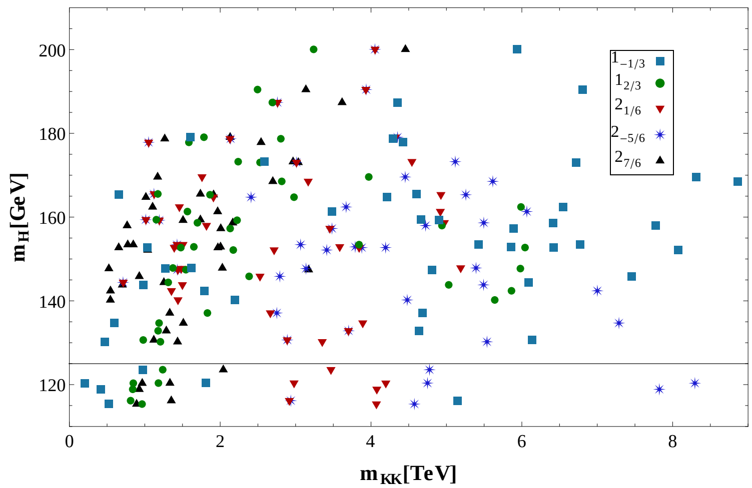


Figure 4.2: *Higgs mass m_H versus the mass of the first KK resonances (before EWSB) for the points of model I with $m_H > 114$ GeV and $\alpha \in [0.26, 0.34]$. The mass of the recently discovered Higgs-like particle at 125 GeV is shown by the black line.*

this is due to the b.c on the IR brane (see appendix B of [20])

$$\begin{array}{rcl} \psi_{1R} & = & \tilde{m}\,\psi_{2R} \\ -\psi_{2L} & = & \tilde{m}\,\psi_{1L} \end{array} \qquad \text{or} \qquad \begin{array}{rcl} \psi_{2R} & = & \tilde{m}\,\psi_{1R} \\ -\psi_{1L} & = & \tilde{m}\,\psi_{2L}. \end{array} \tag{4.42}$$

As a result

$$\sum_{i=1,2} \int_0^L dz\,(\bar{\psi}_{iL}\partial\!\!\!/\psi_{iL} - \bar{\psi}_{iR}\partial\!\!\!/\psi_{iR}) = \frac{1}{2}\sum_{i=1,2}(\bar{\psi}_{iR}\psi_{iL} + \bar{\psi}_{iL}\psi_{iR})|_{UV}. \tag{4.43}$$

If for both fermions the LH chiralities are chosen as holographic fields, the RHS of the above equation is exactly the holographic Lagrangian, if instead the RH chiralities are chosen to be holographic, then the RHS will be minus the holographic Lagrangian. For the mixed case where the LH chirality for one fermion and the RH chirality for the other are holographic, there is no such identification, but as far as we are concerned with the terms of the holographic Lagrangian up to first order in momentum, there is still something to say in this special example. In the following argument we will make use of some properties of the wave-functions given explicitly in appendix B of [20]. At leading order in momentum, the first term on the LHS gives a term of the form $\bar{\chi}_L p\!\!\!/ \chi_L$ with χ_L being the LH holographic field while the second term gives a term of the form $\bar{\chi}_R p\!\!\!/ \chi_R$, with χ_R being the RH holographic field. This can be seen from the fact that at zero momentum the bulk fermion fields are proportional to the holographic fields of the their same chirality, while in general they are linear combinations of holographic fields with both chiralities. On the other hand, the term involving $\psi_{1\,L,R}$ on the RHS expanded up to $\mathcal{O}(p)$ includes the kinetic term for χ_L, while the term involving $\psi_{2\,L,R}$ when expanded up to $\mathcal{O}(p)$ includes minus the kinetic term for χ_R. There are actually also mass terms in both of them, but of course they must sum up to zero, as can also be checked in the RHS explicitly using the expressions for $\psi_{iL,R}(z), i = 1, 2$ in appendix B of [20]. Consequently, equating the coefficients of $\bar{\chi}_L \not{p} \chi_L$ and $\bar{\chi}_R \not{p} \chi_R$ in eq.(4.43) gives the identities

$$\sum_{i=1,2} \int_0^L dz\, f_{iL,R}^2(z) = \lim_{p\to 0} \frac{\Pi_{L,R}(p)}{p} = \text{ coefficient of the kinetic term of } \chi_{L,R} \tag{4.44}$$

where the general form of the kinetic terms in the holographic Lagrangian are defined to be $\bar{\chi}_{L,R}\, \Pi_{L,R}(p) p^{-1} \not{p}\, \chi_{L,R}$.

4.1.3 Results

The results of our numerical scan are summarized in figs. 4.1, 4.2 and 4.3. The details of the χ^2 fit, which has been done with 4 d.o.f, are explained in section B.3. The fermion sector of this model has 8 parameters, 4 bulk masses and 4 IR masses. The randomly chosen input parameters are m_u, m_d, m_1, m_2, $\tilde{m}_u$, $\tilde{m}_d$, θ and θ'. The remaining two parameters $\tilde{M}_u$ and $\tilde{M}_d$ are fixed by the top and bottom mass formulas. Demanding a small δg_b at tree-level requires $m_2 L \gtrsim 1$, as can be verified by using eq. (4.33). We have scanned the parameter space over the region $m_u L \in [-3, 3]$, $m_d L \in [-5, 2.5]$, $m_1 L \in [-2, 2]$, $m_2 L \in [2.2, 4.5]$, $\tilde{m}_u \in [-2.3, 4.1]$, $\tilde{m}_d \in [-3.5, 4]$, $\theta \in [17, 27]$, $\theta' \in [14, 26]$. As can be seen in fig.4.1, at 90% C.L. EWPT constraints allow roughly a value up to $\alpha \simeq 1/3$ for the EWSB parameter, while at 99% C.L. this is reduced to $\alpha \simeq 1/4$. This is still valid when we restrict the Higgs mass to be around 125 GeV which is the mass of the recently discovered Higgs-like particle. There is no definite pattern for the lightest exotic particles for general Higgs mass. As expected, b_R is always mostly elementary, while q_L and t_R typically show a sizable degree of compositeness. Depending on the region in parameter space, q_L can be semi-composite and t_R mostly composite or vice versa, with q_L mostly composite and t_R semi-composite.

4.2 Model II: Fermions in two fundamentals of $SO(5)$

In this section we introduce an alternative model by taking the same setup as model I, with the same gauge sector, but embedding the SM fermions in two fundamentals of $SO(5)$, rather than four, with 2/3 and $-1/3$ charges under $U(1)_X$. In order to study this

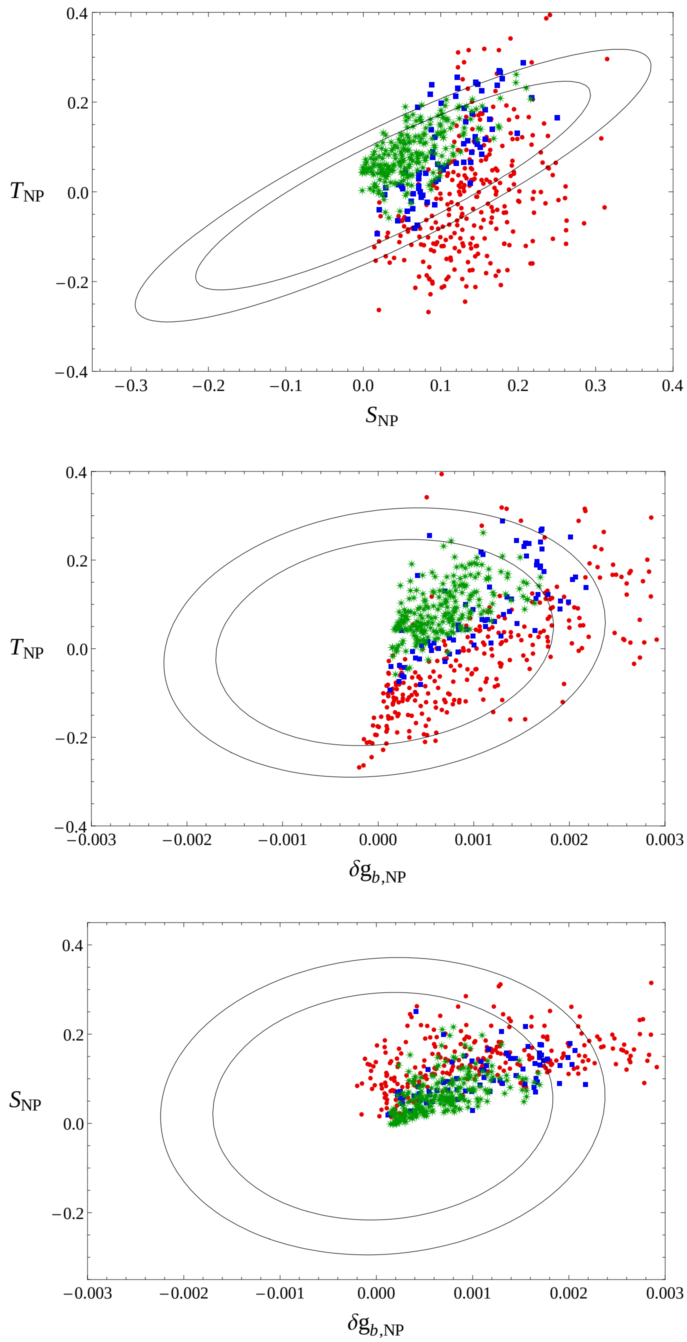


Figure 4.3: *Scatter plot of points in model I with* $m_H > 114$ GeV *and projected on the* T_{NP}-S_{NP}, T_{NP}-$\delta g_{b,NP}$ *and* S_{NP}-$\delta g_{b,NP}$ *planes. We have set* $M_{H,eff} = 120$ GeV. *Small red dots represent points which don't pass EWPT at* 99% *C.L., square blue dots represent points which pass EWPT at* 99%*C.L. but not at* 90% *C.L., and star shape green dots represent points which pass EWPT at* 90% *C.L.. The big and small ellipses correspond to* 99% *and* 90% *C.L. respectively.*

model using the holographic approach, we take the LH chiralities of the two multiplets as the holographic fields. The SM fermions are embedded in the following way

$$\xi_{tL} = \begin{pmatrix} (2,2)^t_L = \begin{bmatrix} q'_{1L}(-+) \\ q_{1L}(++) \end{bmatrix} \\ (1,1)^t_L = u_L(--) \end{pmatrix}_{2/3} \quad , \qquad \xi_{bL} = \begin{pmatrix} (2,2)^b_L = \begin{bmatrix} q_{2L}(++) \\ q'_{2L}(-+) \end{bmatrix} \\ (1,1)^b_L = d_L(--) \end{pmatrix}_{-1/3} \quad , \tag{4.45}$$

The boundary conditions are chosen such as to give rise to zero modes only for the SM fields, namely q_L, u_R and d_R, and to respect the $O(4)$ symmetry on the IR boundary. Note that ξ_{tL} and ξ_{bL} both include a LH $SU(2)$ doublet with 1/6 hypercharge, q_{1L} and q_{2L}. In order to avoid extra zero modes we impose a $(-)$ UV b.c on a linear combination of them. The $O(4)$ symmetry imposed on the IR boundary forbids any term made out of the above fields, $(2,2)_{2/3}$, $(2,2)_{-1/3}$, $(1,1)_{2/3}$ and $(1,1)_{-1/3}$. From the point of view of the KK approach, the most general Lagrangian compatible with the symmetries includes kinetic term for all the fields with $(+)$ b.c on the UV brane. This translates in the holographic approach, to Lagrange multiplyers with UV boundary kinetic terms for the holographic fields with $(-)$ b.c. So in the holographic approach the most general Lagrangian can be written in the following way

$$\begin{aligned} \mathcal{L}_{Hol} &= Z_q(\cos\beta\bar{q}_{1L} + \sin\beta\bar{q}_{2L})\not{D}(\cos\beta q_{1L} + \sin\beta q_{2L}) \\ &+ Z_t\,\bar{t}_R\,\not{D}\,t_R + Z_b\,\bar{b}_R\,\not{D}\,b_R + \sum_{i=1,2} Z_{iR}\,\bar{q}'_{iR}\,\not{D}\,q'_{iR} + Z_g\,\bar{\gamma}_R\,\not{D}\,\gamma_R \\ &+ (\bar{t}_R u_L + \bar{u}_L t_R) + \left(\bar{b}_R d_L + \bar{d}_L b_R\right) + \sum_{i=1,2}\left(\bar{q}'_{iR}q'_{iL} + \bar{q}'_{iL}q'_{iR}\right) \\ &+ \bar{\gamma}_R(-\sin\theta q_{1L} + \cos\theta q_{2L}) + (-\sin\theta\bar{q}_{1L} + \cos\theta\bar{q}_{2L})\gamma_R \\ &+ \frac{1}{2}\sum_{i=t,b}\left(\bar{\xi}_{iL}\xi_{iR} + \bar{\xi}_{iR}\xi_{iL}\right) \end{aligned} \tag{4.46}$$

where the term on the last line is the 4D Lagrangian resulted from integrating out the bulk fields. The variable β parametrizes the linear combination of q_{1L} and q_{2L} on which we impose a $(-)$ b.c, with γ_R being the associated Lagrange multiplyer. t_R, b_R and q'_{iR} are the Lagrange mutiplyers for u_L, d_L and q'_{iL} respectively. To simplify the model we set $Z_g = 0$ and $\beta = \pi/4$. We also integrate out the Lagrange multiplyer γ_R which imposes the constraint $q_{1L} = q_{2L} \equiv q_L$ on the UV boundary, while making the replacement $Z_q \to Z_q/2$

for convenience. Doing this we arrive at the following Lagrangian

$$\begin{aligned}\mathcal{L}_{Hol} &= Z_q \bar{q}_L \not{D}\, q_L + Z_t \bar{t}_R \not{D}\, t_R + Z_b \bar{b}_R \not{D}\, b_R + \sum_{i=1,2} Z_{iR}\, \bar{q}'_{iR} \not{D}\, q'_{iR} \\ &+ (\bar{t}_R u_L + \bar{u}_L t_R) + \left(\bar{b}_R d_L + \bar{d}_L b_R\right) + \sum_{i=1,2} \left(\bar{q}'_{iR} q'_{iL} + \bar{q}'_{iL} q'_{iR}\right) \\ &+ \frac{1}{2} \sum_{i=t,b} \left(\bar{\xi}_{iL} \xi_{iR} + \bar{\xi}_{iR} \xi_{iL}\right) |_{UV}. \end{aligned} \tag{4.47}$$

Using the bulk to boundary propagators for fermions, the terms on the last line can be written in the following way

$$\begin{aligned}\frac{1}{2} \mathrm{Tr}\left(\bar{\xi}_{tL} \xi_{tR} + \bar{\xi}_{tR} \xi_{tL}\right) &= \Pi_t^+ \left(\Sigma^\dagger \chi_{tL}\right)^b \frac{\not{p}}{p} \left(\Sigma^\dagger \chi_{tL}\right)^b + \Pi_t^- \left(\Sigma^\dagger \chi_{tL}\right)^s \frac{\not{p}}{p} \left(\Sigma^\dagger \chi_{tL}\right)^s \\ &= \Pi_t^+\, \bar{\chi}_{tL} \frac{\not{p}}{p} \chi_{tL} + \left(\Pi_t^- - \Pi_t^+\right)\, \bar{\chi}_{tL} \frac{\not{p}}{p} \Phi \Phi^T \chi_{tL} \end{aligned} \tag{4.48}$$

with a similar expression for ξ_{bL}. For simplicity of notation we have defined

$$\Pi_t^\pm = \Pi_R^\pm(0, m_t), \qquad \Pi_b^\pm = \Pi_R^\pm(0, m_b) \tag{4.49}$$

one can further write the term (4.48) and its analogue for ξ_{bL}, in terms of the $SU(2)_L$ multiplets by using

$$\begin{aligned}\bar{\chi}_{tL} \chi_{tL} &= \bar{q}_L q_L + \bar{q}'_{1L} q'_{1L} + \bar{u}_L u_L \\ \bar{\chi}_{bL} \chi_{bL} &= \bar{q}_L q_L + \bar{q}'_{2L} q'_{2L} + \bar{d}_L d_L \end{aligned} \tag{4.50}$$

and the relation

$$\begin{aligned}\bar{\chi}_{tL} \Phi \Phi^T \chi_{tL} &= \frac{s_h^2}{2h^2} (\bar{q}_L H^c + \bar{q}'_{1L} H)(H^{c\dagger} q_L + H^\dagger q'_{1L}) + c_h^2 \bar{u}_L u_L \\ &- \frac{s_h c_h}{\sqrt{2} h} (\bar{q}_L H^c + \bar{q}'_{1L} H) u_L - \frac{s_h c_h}{\sqrt{2} h} \bar{u}_L (H^{c\dagger} q_L + H^\dagger q'_{1L}) \end{aligned} \tag{4.51}$$

to find the analogue formula for ξ_{bL} one just has to make the replacements $q_L \to q'_{2L}$, $q'_{1L} \to q_L$ and $u_L \to d_L$ in the above expression. Plugging the result into (4.47) gives the holographic Lagrangian in terms of $SU(2)_L \times U(1)_Y$ multiplets, which we have chosen to write in momentum space and ignore the gauge interactions

$$\begin{aligned}
\mathcal{L}_{Hol} &= Z_q \bar{q}_L \not{p}\, q_L + Z_t\, \bar{t}_R \not{p}\, t_R + Z_b\, \bar{b}_R \not{p}\, b_R + \sum_{i=1,2} Z_{iR}\, \bar{q}'_{iR} \not{p}\, q'_{iR} \\
&+ \left(\bar{t}_R u_L + \bar{u}_L t_R\right) + \left(\bar{b}_R d_L + \bar{d}_L b_R\right) + \sum_{i=1,2} \left(\bar{q}'_{iR} q'_{iL} + \bar{q}'_{iL} q'_{iR}\right) \\
&+ \Pi_t^+ (\bar{q}_L q_L + \bar{q}'_{1L} q'_{1L}) + \Pi_t^-\, \bar{u}_L u_L + \Pi_b^+ (\bar{q}_L q_L + \bar{q}'_{2L} q'_{2L}) + \Pi_b^-\, \bar{d}_L d_L \\
&+ \left(\Pi_t^- - \Pi_t^+\right) \Big[\frac{s_h^2}{2h^2} (\bar{q}_L H^c + \bar{q}'_{1L} H)(H^{c\dagger} q_L + H^\dagger q'_{1L}) - s_h^2 \bar{u}_L u_L \\
&- \frac{s_h c_h}{\sqrt{2} h} (\bar{q}_L H^c + \bar{q}'_{1L} H) u_L - \frac{s_h c_h}{\sqrt{2} h} \bar{u}_L (H^{c\dagger} q_L + H^\dagger q'_{1L})\Big] \\
&+ \left(\Pi_b^- - \Pi_b^+\right) \Big[\frac{s_h^2}{2h^2} (\bar{q}'_{2L} H^c + \bar{q}_L H)(H^{c\dagger} q'_{2L} + H^\dagger q_L) - s_h^2 \bar{d}_L d_L \\
&- \frac{s_h c_h}{\sqrt{2} h} (\bar{q}'_{2L} H^c + \bar{q}_L H) d_L - \frac{s_h c_h}{\sqrt{2} h} \bar{d}_L (H^{c\dagger} q'_{2L} + H^\dagger q_L)\Big]. \qquad (4.52)
\end{aligned}$$

By integrating out the heavy fields we arrive at the following low energy effective Lagrangian which we have written up to $\mathcal{O}(s_\alpha^2)$ terms

$$\mathcal{L}_H = \bar{q}_L \frac{\not{p}}{p} \Pi^q q_L + \sum_{a=t,b} \bar{a}_R \frac{\not{p}}{p} \Pi^a a_R + \frac{s_\alpha}{h} \Big(\Pi_M^t \bar{q}_L H^c t_R + \Pi_M^b \bar{q}_L H b_R + h.c.\Big)\,, \qquad (4.53)$$

where

$$\Pi^q = p Z_q + \frac{1}{2}(\Pi^+(m_t) + \Pi^+(m_b))\,, \qquad (4.54)$$

$$\Pi^{t,b} = p Z_{t,b} - \frac{1}{\Pi^-(m_{t,b})}\,, \qquad (4.55)$$

$$\Pi_M^{t,b} = \frac{\Pi^-(m_{t,b}) - \Pi^+(m_{t,b})}{2\Pi^-(m_{t,b})}\,. \qquad (4.56)$$

From eq.(4.53) the masses of the top and bottom quarks at zero momentum can easily be read off. Using the expression for the mass of the W boson (3.103), the top and bottom mass to W mass ratios are

$$\frac{M_t^2}{M_W^2} \simeq \frac{\theta + 1}{2 N_L N_{tR}}\,, \qquad \frac{M_b^2}{M_W^2} \simeq \frac{\theta + 1}{2 N_L N_{bR}}\,, \qquad (4.57)$$

where

$$\begin{aligned}
N_L &= \lim_{p\to 0} \frac{\Pi_0^q}{pL} = \frac{Z_q}{L} + \frac{1 - e^{-2Lm_t}}{4Lm_t} + \frac{1 - e^{-2Lm_b}}{4Lm_b}\,, \\
N_{tR,bR} &= \lim_{p\to 0} \frac{\Pi_0^{t,b}}{pL} = \frac{Z_{t,b}}{L} + \frac{e^{2Lm_{t,b}} - 1}{2Lm_{t,b}}\,. \qquad (4.58)
\end{aligned}$$

4.2.1 Computation of the $Z\bar{b}_L b_L$ vertex at tree level in Model II

We present here in some detail the computation of the $Z\bar{b}_L b_L$ coupling deviation at tree level in the model with two multiplets in the fundamental of $SO(5)$. The bulk fermion wave functions for the two multiplets ξ_{tL} and ξ_{tR} are

$$\begin{aligned}\xi_{tL} &= \Pi_L^+(z,m_t)\big(\Sigma^\dagger\chi_{tL}\big)^b + \Pi_L^-(z,m_t)\big(\Sigma^\dagger\chi_{tL}\big)^s \\ \xi_{tR} &= \Pi_R^+(z,m_t)\big(\Sigma^\dagger\chi_{tL}\big)^b + \Pi_R^-(z,m_t)\big(\Sigma^\dagger\chi_{tL}\big)^s\end{aligned} \tag{4.59}$$

with a similar expression for ξ_{bL} and ξ_{bR}

$$\begin{aligned}\xi_{bL} &= \Pi_L^+(z,m_b)\big(\Sigma^\dagger\chi_{bL}\big)^b + \Pi_L^-(z,m_b)\big(\Sigma^\dagger\chi_{bL}\big)^s \\ \xi_{bR} &= \Pi_R^+(z,m_b)\big(\Sigma^\dagger\chi_{bL}\big)^b + \Pi_R^-(z,m_b)\big(\Sigma^\dagger\chi_{bL}\big)^s.\end{aligned} \tag{4.60}$$

As we are interested in the $Z\bar{b}_L b_L$ coupling, we can restrict to the $-1/3$ charge sector, and write the two LH holographic fields as

$$\chi_{tL} = \frac{1}{\sqrt{2}}\begin{pmatrix} ib_L \\ b_L \\ 0 \\ 0 \\ 0\end{pmatrix}, \qquad \chi_{bL} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 0 \\ i(q_{2L}^{u\prime} - b_L) \\ q_{2L}^{u\prime} + b_L \\ \sqrt{2}d_L\end{pmatrix}. \tag{4.61}$$

Finally we need to integrate out the fields d_L, $q_{2L}^{u\prime}$ and $q_{2R}^{u\prime}$. Varying the Lagrangian (4.52) with respect to these fields, we find the following e.o.m

$$\begin{aligned}\delta\bar{d}_L :&\quad 0 = b_R + \Pi_b^- d_L + (\Pi_b^- - \Pi_b^+)\big[-\sin^2\!\alpha d_L - \frac{\sin\alpha\cos\alpha}{\sqrt{2}}(q_{2L}^{u\prime} + b_L)\big] \\ \delta\bar{q}_{2R}^{u\prime} :&\quad 0 = Z_{2R}\not{p}\, q_{2R}^{u\prime} + q_{2L}^{u\prime} \\ \delta q_{2L}^{u\prime} :&\quad 0 = q_{2R}^{u\prime} + (\Pi_b^- - \Pi_b^+)\big[\frac{\sin^2\!\alpha}{2}(q_{2L}^{u\prime} + b_L) - \frac{\sin\alpha\cos\alpha}{\sqrt{2}} d_L\big] + \Pi_b^+ q_{2L}^{u\prime}\end{aligned} \tag{4.62}$$

which can be solved to give, at leading order in momentum

$$\begin{aligned}q_{2R}^{u\prime} &= -\frac{\tan\alpha}{\sqrt{2}} b_R - \frac{1 - e^{-2m_bL}}{4m_b}\tan^2\!\alpha\,\not{p}\, b_L + \mathcal{O}(p^2) \\ q_{2L}^{u\prime} &= \frac{\tan\alpha}{\sqrt{2}} Z_{2R}\not{p}\, b_R + \mathcal{O}(p^2) \\ d_L &= \frac{\tan\alpha}{\sqrt{2}} b_L + \left[\frac{e^{2m_bL} - 1}{2m_b} + \left(\frac{e^{2m_bL} - 1}{2m_b} + \frac{1}{2}Z_{2R}\right)\tan^2\!\alpha\right]\not{p}\, b_R + \mathcal{O}(p^2).\end{aligned} \tag{4.63}$$

From the Lagrangian (4.47) it is clear that the boundary contribution to the $Z\bar{b}_L b_L$ vertex, at zero momentum, is $Z_q\left(T_L^3 - \sin^2\theta_W Q\right)$, while the zero momentum boundary contribution to the b_L kinetic term is Z_q. Of course there are also contributions to the $\bar{b}_L b_L$ term coming from the terms $Z_{iR}\,\bar{q}^{u\prime}_{2R}\,\not{D}\,q^{u\prime}_{2R}$ and $\bar{q}^{u\prime}_{2R}q^{u\prime}_{2L} + \bar{q}^{u\prime}_{2L}q^{u\prime}_{2R}$, but they vanish at zero momentum, as can be seen from eqs(4.63).

To find the effect of the bulk on the coupling, we need to compute the bulk contribution

$$\int_0^L dz\,\left(\bar{\xi}_t\,\Gamma\,\xi_t + \bar{\xi}_b\,\Gamma\,\xi_b\right) \tag{4.64}$$

to the vertex, where Γ is defined in section 4.1.2, and the bulk contribution to the b_L kinetic term, which according to the arguments of section 4.1.2 can be written as

$$\int_0^L dz\,\left(\bar{\xi}_{tL}\,\xi_{tL} + \bar{\xi}_{bL}\,\xi_{bL}\right) \tag{4.65}$$

The first term in the integrand of (4.64) has four pieces $\bar{\xi}_{tL,R}\,\Gamma\,\xi_{tL,R}$ and $\bar{\xi}_{tL,R}\,\Gamma\,\xi_{tR,L}$, among which only $\bar{\xi}_{tL}\,\Gamma\,\xi_{tL}$ gives a non vanishing contribution as we will argue. The computation of $\bar{\xi}_{tL,R}\,\Gamma\,\xi_{tL,R}$ follows exactly the same lines as that of model I, but the embedding of b_L in ξ_{tL} is such that $T_L^3 = T_R^3$, so using eq.(4.29), the final result will be very simple

$$\bar{\xi}_{tL,R}\,\Gamma\,\xi_{tL,R} = \Pi^{+2}_{L,R}\left(T_L^3 - \sin^2\theta_W Q\right) \tag{4.66}$$

from this result it is clear that the contribution of ξ_{tR} vanishes at zero momentum due to the small momentum behaviour $\Pi_R^+ \sim p$. To show that the cross terms $\bar{\xi}_{tL,R}\,\Gamma\,\xi_{tR,L}$ also give no contribution at zero momentum, following the same lines which resulted in (4.23) one can write the contributions of Γ_I and Γ_{II} as

$$\bar{\xi}_{tL}\left(\Gamma_{\rm I} + \Gamma_{\rm II}\right)\xi_{tR} = \Pi_L^+\Pi_R^+\left(\cos^2\theta_W T_L^3 - \sin^2\theta_W T_R^3 - \frac{\sin^2\alpha}{2}\left(T_L^3 - T_R^3\right)\right)\bar{b}_L b_L \tag{4.67}$$

which vanishes at zero momentum for the same reason, namely that $\Pi_R^+ \sim p$ for small momenta. On the other hand the contribution of the term Γ_{IV} is

$$\bar{\xi}_{tL}\Gamma_{\rm IV}\xi_{tR} + \bar{\xi}_{tR}\Gamma_{\rm IV}\xi_{tL} = \left(\Pi_L^+(z,m_t)\Pi_R^-(z,m_t) + \Pi_L^-(z,m_t)\Pi_R^+(z,m_t)\right)\left(\Sigma\bar{\chi}_{tL}\right)\Gamma_{\rm IV}\left(\Sigma\chi_{tL}\right) \tag{4.68}$$

but the expression $\left(\Sigma\bar{\chi}_{tL}\right)\Gamma_{\rm IV}\left(\Sigma\chi_{tL}\right)$ vanishes according to (4.25). Finally $\bar{\xi}_{tL,R}\xi_{tL,R} = \Pi^{+2}_{L,R}$ and $\bar{\xi}_{tL,R}\xi_{tR,L} = \Pi_L^+\Pi_R^+$ from which the kinetic term and the contribution of Γ_{III} follow. Also for this term only $\bar{\xi}_{tL}\xi_{tL}$ is non vanishing at zero momentum.

The second term in the integrand of (4.64) is slightly more involved since it depends on the fields $q^{u\prime}_{2L,R}$ and d_L which must be replaced with their solutions (4.63). Similar to the case of ξ_{tR}, this term is also the sum of the four terms $\bar{\xi}_{bL,R}\,\Gamma\,\xi_{bL,R}$ and

$\bar{\xi}_{bL,R}\,\Gamma\,\xi_{bR,L}$, which, except $\bar{\xi}_{bL}\,\Gamma\,\xi_{bL}$, all vanish at zero momentum. Lets first consider the term $\bar{\xi}_{bL,R}\,(\Gamma_{\rm I}+\Gamma_{\rm II})\,\xi_{bL,R}$. The computation of this quantity follows the computation leading to (4.23) with the difference that $\bar{\chi}_{bL}T^3_B\chi_{bL}$ is not zero here but rather given by

$$\bar{\chi}_{bL}\,T^3_B\,\chi_{bL} = \frac{1}{2}(\bar{q}^{u\prime}_{2L} - \bar{b}_L)d_L + h.c \;\rightarrow\; -\left(\frac{\tan\alpha}{\sqrt{2}} + \mathcal{O}(p^2)\right)\bar{b}_L b_L + \cdots \tag{4.69}$$

the expression in front of the arrow represents the result after the substitution of (4.63). Using this we find

$$\begin{aligned}
&\bar{\xi}_{bL,R}\,(\Gamma_{\rm I}+\Gamma_{\rm II})\,\xi_{bL,R}\\
&= \Pi^{+2}_{L,R}(z,m_b)\,\bar{\chi}_{bL}\left(\cos^2\theta_W T^3_L - \sin^2\theta_W T^3_R + \frac{\sin 2\alpha}{2\sqrt{2}}T^3_B - \frac{\sin^2\alpha}{2}\left(T^3_L - T^3_R\right)\right)\chi_{bL}\\
&= \Pi^{+2}_{L,R}(z,m_b)\,\bar{q}^{u\prime}_{2L}\left(\cos^2\theta_W T^3_L - \sin^2\theta_W T^3_R - \frac{\sin^2\alpha}{2}\left(T^3_L - T^3_R\right)\right)q^{u\prime}_{2L}\\
&\;+ \Pi^{+2}_{L,R}(z,m_b)\,\bar{b_L}\left(\cos^2\theta_W T^3_L - \sin^2\theta_W T^3_R - \frac{\sin^2\alpha}{2}\left(T^3_L - T^3_R\right)\right)b_L\\
&\;+ \Pi^{+2}_{L,R}(z,m_b)\,\frac{s_\alpha c_\alpha}{2\sqrt{2}}(\bar{q}^{u\prime}_{2L} - \bar{b}_L)d_L + h.c\\
&\rightarrow \Pi^{+2}_{L,R}(z,m_b)\,\bar{b_L}\left(\cos^2\theta_W T^3_L - \sin^2\theta_W T^3_R - \frac{\sin^2\alpha}{2}\left(T^3_L - T^3_R + 1\right) + \mathcal{O}(p^2)\right)b_L + \cdots\\
&= \Pi^{+2}_{L,R}(z,m_b)\,\bar{b_L}\left(\cos^2\theta_W T^3_L - \sin^2\theta_W T^3_R + \mathcal{O}(p^2)\right)b_L + \cdots
\end{aligned} \tag{4.70}$$

where we have used the fact that for the embedding of b_L in χ_{bL}, $T^3_L - T^3_R + 1 = 0$. Again, from the expression of $\Pi^+_R(z,m_b)$ it is seen that $\bar{\xi}_{bR}\,(\Gamma_{\rm I}+\Gamma_{\rm II})\,\xi_{bR} = 0$. For the same reason, the following expression also vanishes at zero momentum

$$\bar{\xi}_{bL}\,(\Gamma_{\rm I}+\Gamma_{\rm II})\,\xi_{bR} = \Pi^+_L(z,m_b)\Pi^+_R(z,m_b)\,\bar{b_L}\left(\cos^2\theta_W T^3_L - \sin^2\theta_W T^3_R + \mathcal{O}(p^2)\right)b_L + \cdots \tag{4.71}$$

Now lets consider the contribution of the Γ_{IV} term. A similar analysis to that of (4.25) shows

$$
\begin{aligned}
\left(\Sigma\bar{\chi}_{bL}\right)\Gamma_{\rm IV}\left(\Sigma\chi_{bL}\right) &= -\left(1-\frac{z}{L}\right)\frac{\sin\alpha}{\sqrt{2}}\bar{\chi}_{bL}\left(\cos\alpha\, T^3_B - \frac{\sin\alpha}{\sqrt{2}}\left(T^3_L - T^3_R\right)\right)\chi_{bL} \\
&\quad + \left(1-\frac{z}{L}\right)\frac{\sin^2\alpha}{2}\,\bar{q}^{u\prime}_{2L}\left(T^3_L - T^3_R\right)q^{u\prime}_{2L} \\
&\quad + \left(1-\frac{z}{L}\right)\frac{\sin^2\alpha}{2}\,\bar{b}_L\left(T^3_L - T^3_R\right)b_L \\
&\quad - \frac{s_\alpha c_\alpha}{2\sqrt{2}}\left(\bar{q}^{u\prime}_{2L} - \bar{b}_L\right)d_L + h.c \\
&\to + \left(1-\frac{z}{L}\right)\frac{\sin^2\alpha}{2}\,\bar{b}_L\,\left(T^3_L - T^3_R + 1 + \mathcal{O}(p^2)\right)\,b_L \\
&= + \left(1-\frac{z}{L}\right)\frac{\sin^2\alpha}{2}\,\mathcal{O}(p^2)\,\bar{b}_L b_L
\end{aligned}
\tag{4.72}
$$

where again the expression in front of the arrow represents the result after the substitution of (4.63). Using this we can immediately write

$$
\begin{aligned}
\bar{\xi}_{bL,R}\,\Gamma_{\rm IV}\,\xi_{bL,R} &= \Pi^+_{L,R}(z,m_b)\Pi^-_{L,R}(z,m_b)\left(\Sigma\bar{\chi}_{bL}\right)\Gamma_{\rm IV}\left(\Sigma\chi_{bL}\right) \\
&= \Pi^+_{L,R}(z,m_b)\Pi^-_{L,R}(z,m_b)\,\mathcal{O}(p^2)\,\bar{b}_L b_L
\end{aligned}
\tag{4.73}
$$

which is vanishing due to the low momentum behaviour of $\Pi^+_{L,R}(z,m_b)\Pi^-_{L,R}(z,m_b)$. The vanishing of $\bar{\xi}_{bL}\,\Gamma_{\rm IV}\,\xi_{bR} + \bar{\xi}_{bR}\,\Gamma_{\rm IV}\,\xi_{bL}$ also follows as a result of (4.72)

$$
\begin{aligned}
&\bar{\xi}_{bL}\,\Gamma_{\rm IV}\,\xi_{bR} + \bar{\xi}_{bR}\,\Gamma_{\rm IV}\,\xi_{bL} \\
&= \left(\Pi^+_L(z,m_b)\Pi^-_R(z,m_b) + \Pi^-_L(z,m_b)\Pi^+_R(z,m_b)\right)\left(\Sigma\bar{\chi}_{bL}\right)\Gamma_{\rm IV}\left(\Sigma\chi_{bL}\right) \\
&= \left(\Pi^+_L(z,m_b)\Pi^-_R(z,m_b) + \Pi^-_L(z,m_b)\Pi^+_R(z,m_b)\right)\mathcal{O}(p^2)\,\bar{b}_L b_L \stackrel{p\to 0}{\to} 0
\end{aligned}
\tag{4.74}
$$

Finally we compute $\bar{\xi}_{bL,R}\,\xi_{bL,R}$ which is proportional to the contribution of Γ_{III}

$$
\begin{aligned}
&\bar{\xi}_{bL,R}\,\xi_{bL,R} \\
&= \Pi^{+2}_{L,R}(z,m_b)\,\bar{\chi}_{bL}\chi_{bL} - \left(\Pi^{+2}_{L,R}(z,m_b) - \Pi^{-2}_{L,R}(z,m_b)\right)\bar{\chi}_{bL}\Phi\Phi^T\chi_{bL} \\
&= \Pi^{+2}_{L,R}(z,m_b)\,\left(\bar{q}^{u\prime}_{2L}q^{u\prime}_{2L} + \bar{b}_L b_L + \bar{d}_L d_L\right) - \left(\Pi^{+2}_{L,R}(z,m_b) - \Pi^{-2}_{L,R}(z,m_b)\right)\bar{\chi}_{bL}\Phi\Phi^T\chi_{bL} \\
&= \Pi^{+2}_{L,R}(z,m_b)\,\left(1 + \frac{\tan^2\alpha}{2} + \mathcal{O}(p^2)\right)\bar{b}_L b_L - \left(\Pi^{+2}_{L,R}(z,m_b) - \Pi^{-2}_{L,R}(z,m_b)\right)\mathcal{O}(p^4)\bar{b}_L b_L \\
&= \Pi^{+2}_{L,R}(z,m_b)\,\left(1 + \frac{\tan^2\alpha}{2} + \mathcal{O}(p^2)\right)\bar{b}_L b_L
\end{aligned}
\tag{4.75}
$$

in which we have used

$$\bar{\chi}_{bL}\Phi\Phi^T\chi_{bL} = \frac{s_\alpha^2}{2}(\bar{b}_L + \bar{q}_{2L}^{u\prime})(b_L + q_{2L}^{u\prime}) + c_\alpha^2 \bar{d}_L d_L - \frac{s_\alpha c_\alpha}{\sqrt{2}}(\bar{b}_L + \bar{q}_{2L}^{u\prime})d_L + h.c \;\rightarrow\; \mathcal{O}(p^4)\,\bar{b}_L b_L + \cdots \tag{4.76}$$

The vanishing of $\bar{\xi}_{bL}\,\xi_{bR}$ follows from a similar computation

$$\bar{\xi}_{bL}\,\xi_{bR} = \Pi_L^+(z,m_b)\Pi_R^+(z,m_b)\left(1 + \frac{\tan^2\alpha}{2} + \mathcal{O}(p^2)\right)\bar{b}_L b_L \overset{p\to 0}{\to} 0. \tag{4.77}$$

In summary, the term $\bar{\xi}_{tL}\,\Gamma\,\xi_{tL}$ is given exactly by the same expression as $\bar{\xi}_{tL}\,\Gamma\,\xi_{tL}$ computed in section 4.1.2 for model I but with $T_L^3 - T_R^3 = 0$. On the other hand, to find $\bar{\xi}_{bL}\,\Gamma\,\xi_{bL}$ one can simply take the expression for $\bar{\xi}_{tL}\,\Gamma\,\xi_{tL}$ of model I and make the replacement $T_L^3 - T_R^3 \rightarrow T_L^3 - T_R^3 + 1 = 0$, with the only difference that $\bar{\xi}_{bL}\,\xi_{bL}$ has an extra term proportional to $\tan^2\alpha$ as shown in (4.75). So to find the total bulk contributions and the contributions to $\bar{b}_L$ kinetic term, we can use the result of section 4.1.2 with $T_L^3 - T_R^3 = 0$, and add the extra piece coming from $\bar{\xi}_{bL}\,\xi_{bL}$. This means that the total bulk contribution to the $Z\bar{b}_L b_L$ vertex is

$$\bar{\xi}_{tL}\,\Gamma\,\xi_{tL} + \bar{\xi}_{bL}\,\Gamma\,\xi_{bL} = \left(\Pi_{Lt}^{+2} + \Pi_{Lb}^{+2}\right)\left(T_L^3 - \sin^2\theta_W Q\right) + \frac{1}{3}\sin^2\theta_W\,\Pi_{Lb}^{+2}\,\frac{\tan^2\alpha}{2} \tag{4.78}$$

while the contribution to the kinetic term is given by

$$\bar{\xi}_{tL}\,\xi_{tL} + \bar{\xi}_{Lb}\,\xi_{Lb} = \left(\Pi_{Lt}^{+2} + \Pi_{Lb}^{+2}\right) + \Pi_{Lb}^{+2}\,\frac{\tan^2\alpha}{2} \tag{4.79}$$

where $\Pi_{Lb,t}^+ \equiv \lim_{p\to 0}\Pi_L^+(z,m_{b,t})$. Adding the boundary contributions for the vertex and kinetic terms, which are $Z_q\left(T_L^3 - \sin^2\theta_W Q\right)$ and Z_q respectively as mentioned at the beginning of this section, and dividing the two, we find the total $Z\bar{b}_L b_L$ coupling deviation at tree level and at zero momentum to be

$$\delta g_b = \frac{\int_0^L dz\,\Pi_{Lb}^{+2}}{Z_q + \int_0^L dz\left(\Pi_{Lt}^{+2} + \Pi_{Lb}^{+2}\right)}\,\frac{\tan^2\alpha}{4} = \frac{\frac{1 - e^{-2m_b L}}{2m_b}}{Z_q + \frac{1 - e^{-2m_t L}}{2m_t} + \frac{1 - e^{-2m_b L}}{2m_b}}\,\frac{\tan^2\alpha}{4} \tag{4.80}$$

4.2.2 The effective potential

We now turn to the computation of the effective potential. Given a tree level Lagrangian, it is strait forward to compute the effective potential, however the low energy Lagrangian

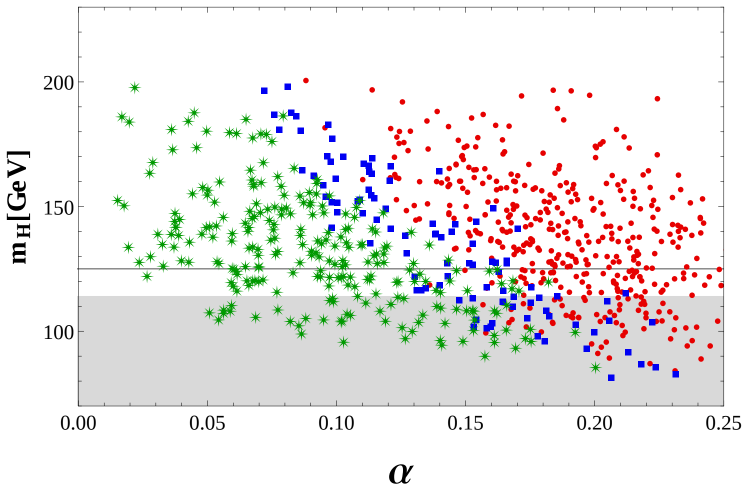


Figure 4.4: *Scatter plot of points obtained from a scan over the parameter space of model II. Small red dots represent points which don't pass EWPT at* 99%*C.L., square blue dots represent points which pass EWPT at* 99%*C.L. but not at* 90%*C.L., and star shape green dots represent points which pass EWPT at* 90%*C.L.. The region below the LEP bound (*$m_H < 114$ GeV*) is shaded. The mass of the recently discovered Higgs-like particle at* 125 GeV *is shown by the black line.*

(4.53) is not the correct Lagrangian to use. This is first of all because this Lagrangian is not exact in α but more importantly, even if it were exact in α, to arrive at the Lagrangian (4.53) which is written in terms of the light fields, we had to integrate out the LH fields u_L, d_L and $q_{1,2L}$, but doing this we miss some α dependent mixing terms between them, appearing in the Lagrangian (4.52), and hence all the loop contributions in which only the fields u_L, d_L and $q_{1,2L}$ run in the loops. Instead there are no α dependent mixing terms between the RH fields t_R, b_R and $q_{1,2R}$, so one can integrate them out

$$t_R = -\frac{1}{Z_t p^2} \not{p}\, u_L, \qquad b_R = -\frac{1}{Z_b p^2} \not{p}\, d_L, \qquad q_{iR} = -\frac{1}{Z_{iR} p^2} \not{p}\, q_{iL}, \tag{4.81}$$

leaving an effective Lagrangian in terms of LH fields only

$$\begin{aligned}
\mathcal{L}^L_{Hol} &= Z_q \bar{q}_L \not{p}\, q_L - \frac{1}{Z_t}\,\bar{u}_L \not{p}\, u_L - \frac{1}{Z_b}\,\bar{d}_L \not{p}\, d_L - \sum_{i=1,2} \frac{1}{Z_{iR}}\,\bar{q}'_{iL} \not{p}\, q'_{iL} \\
&+ \Pi_t^+ (\bar{q}_L q_L + \bar{q}'_{1L} q'_{1L}) + \Pi_t^-\, \bar{u}_L u_L + \Pi_b^+ (\bar{q}_L q_L + \bar{q}'_{2L} q'_{2L}) + \Pi_b^-\, \bar{d}_L d_L \\
&+ \left(\Pi_t^- - \Pi_t^+\right) \Big[\frac{s_h^2}{2h^2}(\bar{q}_L H^c + \bar{q}'_{1L} H)(H^{c\dagger} q_L + H^\dagger q'_{1L}) - s_h^2 \bar{u}_L u_L \\
&- \frac{s_h c_h}{\sqrt{2}h}(\bar{q}_L H^c + \bar{q}'_{1L} H) u_L - \frac{s_h c_h}{\sqrt{2}h}\bar{u}_L (H^{c\dagger} q_L + H^\dagger q'_{1L})\Big] \\
&+ \left(\Pi_b^- - \Pi_b^+\right) \Big[\frac{s_h^2}{2h^2}(\bar{q}'_{2L} H^c + \bar{q}_L H)(H^{c\dagger} q'_{2L} + H^\dagger q_L) - s_h^2 \bar{d}_L d_L \\
&- \frac{s_h c_h}{\sqrt{2}h}(\bar{q}'_{2L} H^c + \bar{q}_L H) d_L - \frac{s_h c_h}{\sqrt{2}h}\bar{d}_L (H^{c\dagger} q'_{2L} + H^\dagger q_L)\Big]
\end{aligned} \tag{4.82}$$

Since there are no loop diagrams with only RH fields, in this case the effect of loop contributions with RH fields running in the loops is captured by the effective vertices in terms of LH fields. Using the Lagrangian (4.82) we can readily compute the effective potential. As before, we subtract a constant term from the potential such that $V_f(0) = 0$. The top sector contribution is

$$\begin{aligned}
V_t &= -2N_c \int \frac{d^4p}{(2\pi)^4} \ln\left[1 + \sin^2\alpha \frac{\Pi_+(m_t) - \Pi_-(m_t)}{2(pZ_t\Pi_-(m_t) - 1)} \left(pZ_t + p\frac{Z_{1R} - Z_t}{pZ_{1R}\Pi_+(m_t) - 1}\right.\right. \\
&\qquad \left.\left. - \frac{pZ_t\Pi_+(m_t) - 1}{2pZ_q + \Pi_+(m_t) + \Pi_+(m_b)}\right)\right],
\end{aligned} \tag{4.83}$$

while the bottom sector contribution V_b is obtained from V_t by the replacements $t \leftrightarrow b$ and $Z_{1R} \to Z_{2R}$. There are two other contributions from the exotic 5/3 and $-4/3$ charge towers which are independent of α and so eliminated when subtracting the constant piece. The total Higgs potential is finally

$$V_{tot} = V_g + V_t + V_b\,, \tag{4.84}$$

with V_g given in eq.(3.56). Also in this model the complicated expression in front of $\sin^2\alpha$ in the Logarithm falls off exponentially with momentum, but in this case the leading order term in an expansion of the Logarithm is proportional to $\sin^2\alpha$ just like the gauge potential, so one has to resort to higher order terms in this expansion in order to achieve EWSB.

4.2.3 Results

The results of our numerical scan are summarized in figs. 4.4, 4.5 and 4.6. The details of the χ^2 fit, which has been done with 4 d.o.f, are explained in section B.3. The fermion

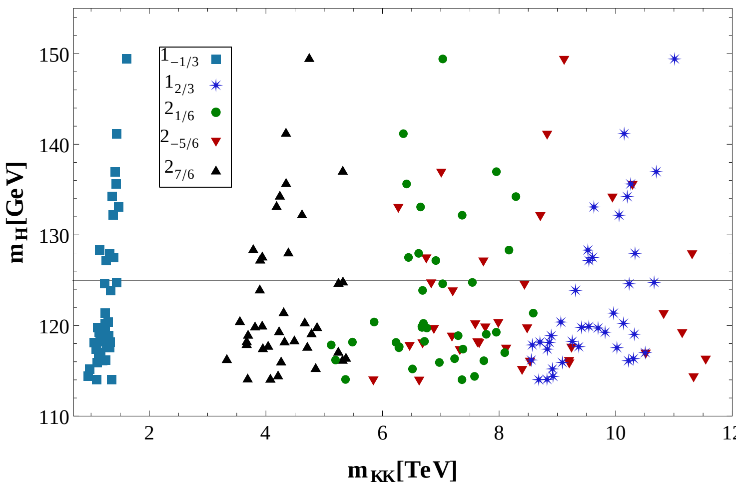


Figure 4.5: *Higgs mass m_H versus the mass of the first KK resonances (before EWSB) for the points of model II with $m_H > 114$ GeV and $\alpha \in [0.16, 0.22]$. The mass of the recently discovered Higgs-like particle at 125 GeV is shown by the black line.*

sector of this model has 7 parameters, 2 bulk masses and 5 coefficients for the BKT. The randomly chosen input parameters are m_t, m_b, Z_{1R}, Z_{2R}, Z_q, θ and θ'. The remaining two parameters Z_b and Z_t are fixed by the top and bottom mass formulas. For stability reasons, we take positive coefficients for all the BKT and $m_bL \gtrsim 1$ in order to suppress δg_b, as given by eq.(4.80). More precisely, we have taken $m_tL \in [0.1, 1.3]$, $m_bL \in [2, 2.5]$, $Z_{1R}/L \in [0.1, 1.6]$, $Z_{2R}/L \in [0, 1]$, $Z_q/L \in [0.5, 2]$, $\theta \in [15, 25]$, $\theta' \in [15, 25]$. As can be seen in fig.4.4, at 90% C.L. the EWPT constrain $\alpha \simeq 1/5$, with a very light Higgs mass. The latter increases only for more tuned configurations with $\alpha < 0.15$. For $m_H = 125$ GeV the value $\alpha = 0.16$ is also achieved at 99% C.L.. Interestingly enough, the lightest exotic particle is always a fermion singlet with $Q = -1/3$, see fig.4.5. Its mass is of order 1 TeV, significantly lighter than the gauge KK modes (~ 5 TeV) and the other fermion resonances, with masses starting from around 4 TeV. The doublet q_L is generically semi-composite, the singlet t_R is mostly composite and b_R is mostly elementary.

4.3 Model III: Fermions in an adjoint of $SO(5)$

In the previous section we introduced a model in which the SM fermions were embedded in two fundamentals of $SO(5)$, to embed them in one multiplet the minimal choice is an adjoint representation. This is the model we are about to discuss in this section. This is also the simplest possible construction so far. The adjoint of $SO(5)$ is decomposed under

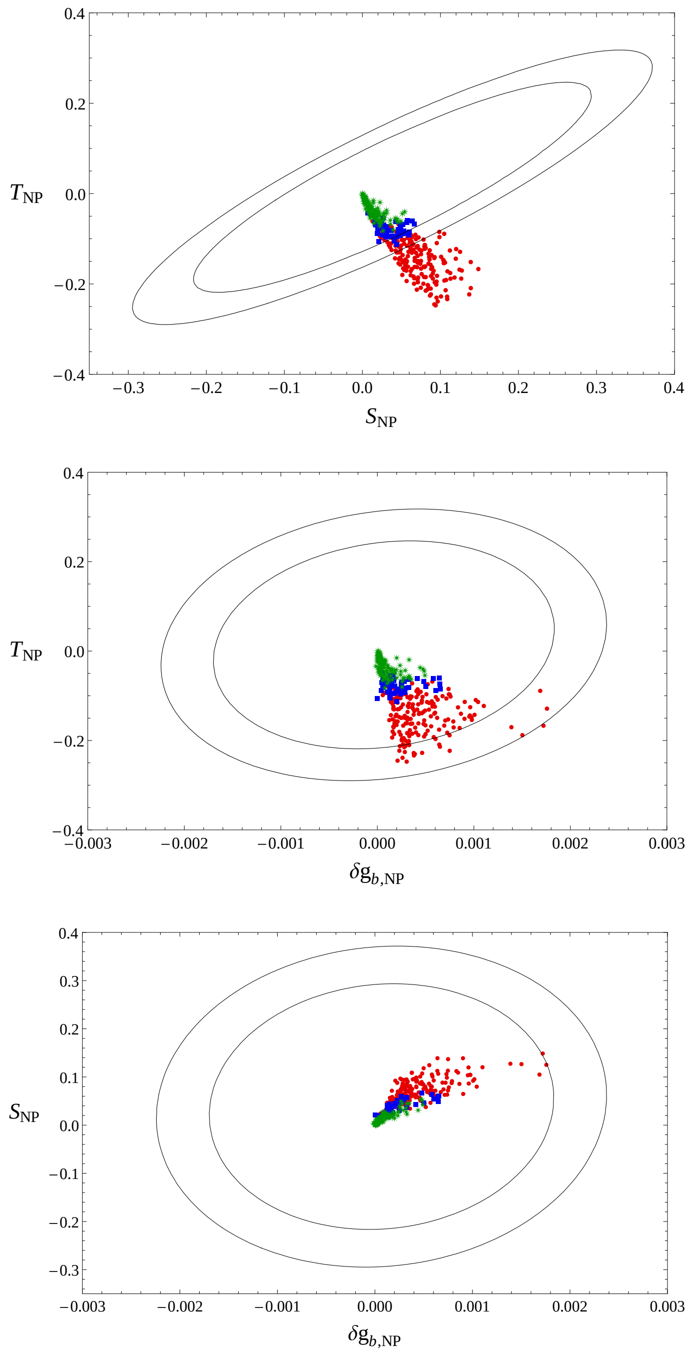


Figure 4.6: *Scatter plot of points in model II with* $m_H > 114$ GeV *and projected on the* T_{NP}*-*S_{NP}*,* T_{NP}*-*$\delta g_{b,NP}$ *and* S_{NP}*-* $\delta g_{b,NP}$ *planes. We have set* $M_{H,eff} = 120$ GeV*. Small red dots represent points which don't pass EWPT at* 99% *C.L., square blue dots represent points which pass EWPT at* 99%*C.L. but not at* 90% *C.L., and star shape green dots represent points which pass EWPT at* 90% *C.L.. The big and small ellipses correspond to* 99% *and* 90% *C.L. respectively.*

$SO(4) = S(2)_L \times SU(2)_R$ as $\mathbf{10} = (\mathbf{2},\mathbf{2}) + (\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{3})$. The third generation SM fields are embedded in a multiplet carrying the representation $\mathbf{10}_{2/3}$ of $SO(5) \times U(1)_X$ whose LH chirality is taken as the holographic field. The embeddings and the b.c are depicted in the following way

$$\xi_L = \left(\begin{array}{c} \left\{ \begin{array}{l} x_L\,(+-) \\ u_L\,(--) \qquad T_L\,(+-) \\ d_L\,(--) \end{array} \right. \\ [\,q_L\,(++)\,,\, q'_L\,(-+)\,] \end{array} \right)_{\frac{2}{3}} . \tag{4.85}$$

As before the boundary conditions are chosen in a way to respect the symmetries on the two branes and to give rise to zero modes only for the SM fields. This leads to the above unique assignment of b.c. so the field content of this model will include three vector-like singlets $x = \mathbf{1}_{5/3}$, $u = \mathbf{1}_{2/3}$ and $d = \mathbf{1}_{-1/3}$, two vector-like doublets $q = \mathbf{2}_{1/6}$ and $q' = \mathbf{2}_{7/6}$, and one vector-like triplet $T = \mathbf{3}_{2/3}$. In addition, because of the adopted b.c, the fields q_L, u_R and d_R have massless modes, which are identified with the SM chiral fields. However, As discussed in section 3.3.1 in the holographic approach, in order to capture the possible massless modes coming from the opposite chirality of the holographic fields with $(-)$ UV b.c, we need to introduce the Lagrange multiplyers t_R, b_R and ρ_R respectively for the fields u_L, d_L and q'_L, among which the zero modes of t_R and b_R are identified with the SM RH top and bottom quarks respectively, hence the symbol. The most general holographic Lagrangian can be parametrized as

$$\begin{aligned} \mathcal{L}_{Hol} &= Z_q\,\bar{q}_L\,\not{D}\,q_L + Z_x\,\bar{x}_L\,\not{D}\,x_L + 2Z_T\,\mathrm{Tr}\left(\bar{T}_L\,\not{D}\,T_L\right) \\ &+ Z_t\,\bar{t}_R\,\not{D}\,t_R + Z_b\,\bar{b}_R\,\not{D}\,b_R + Z_{q'}\,\bar{\rho}_R\,\not{D}\,\rho_R \\ &+ \left(\bar{t}_R u_L + \bar{u}_L t_R\right) + \left(\bar{b}_R d_L + \bar{d}_L b_R\right) + \left(\bar{\rho}_R q'_L + \bar{q}'_L \rho_R\right) \\ &+ \frac{1}{2}\mathrm{Tr}\left(\bar{\xi}_L \xi_R + \bar{\xi}_R \xi_L\right)|_{UV}. \end{aligned} \tag{4.86}$$

In terms of the holographic field, which we call χ_L here, the boundary values of the bulk wave-functions $\xi_{L,R}$ appearing in the final line of (4.86) are written as

$$\xi_L|_{UV} = \Sigma^\dagger \chi_L \Sigma, \qquad \xi_R|_{UV} = \Pi^+ \frac{\not{p}}{p} \left(\Sigma^\dagger \chi_L \Sigma\right)^b + \Pi^- \frac{\not{p}}{p} \left(\Sigma^\dagger \chi_L \Sigma\right)^t \tag{4.87}$$

in which by $(\cdots)^b$ and $(\cdots)^t$ we mean projections on the bidoublet and triplet subrepresentations respectively, and $\Pi^\pm$ are defined in terms of the functions (3.166)

$$\Pi^\pm \equiv \Pi_R^\pm(0,m). \tag{4.88}$$

Using (4.87) the last term in (4.86), which results from integrating out the bulk Lagrangian, can be evaluated

$$\begin{aligned}\frac{1}{2}\mathrm{Tr}\left(\bar{\xi}_L\xi_R+\bar{\xi}_R\xi_L\right) &= \Pi^+\mathrm{Tr}\big[\big(\Sigma^\dagger\chi_L\Sigma\big)^b\frac{\not p}{p}\big(\Sigma^\dagger\chi_L\Sigma\big)^b\big]+\Pi^-\mathrm{Tr}\big[\big(\Sigma^\dagger\chi_L\Sigma\big)^t\frac{\not p}{p}\big(\Sigma^\dagger\chi_L\Sigma\big)^t\big] \\ &= \Pi^-\ \mathrm{Tr}\big(\bar{\chi}_L\frac{\not p}{p}\chi_L\big)+2\left(\Pi^+-\Pi^-\right)\ \Phi^T\bar{\chi}_L\frac{\not p}{p}\chi_L\Phi \end{aligned} \tag{4.89}$$

in which we have used

$$\mathrm{Tr}\big[\big(\Sigma^\dagger\chi_L\Sigma\big)^b\big(\Sigma^\dagger\chi_L\Sigma\big)^b\big] = 2\Phi^T\bar{\chi}_L\chi_L\Phi \tag{4.90}$$

$$\mathrm{Tr}\big[\big(\Sigma^\dagger\chi_L\Sigma\big)^t\big(\Sigma^\dagger\chi_L\Sigma\big)^t\big] = \mathrm{Tr}\left(\bar{\chi}_L\chi_L\right)-2\Phi^T\bar{\chi}_L\chi_L\Phi \tag{4.91}$$

where

$$\mathrm{Tr}\left(\bar{\chi}_L\chi_L\right)=2\mathrm{Tr}\left(\bar{T}_L\,T_L\right)+\bar{x}_L\,x_L+\bar{u}_L\,u_L+\bar{d}_L\,d_L+\bar{q}_L\,q_L+\bar{q}'_L\,q'_L \tag{4.92}$$

and also

$$\begin{aligned}&\Phi^T\bar{\chi}_L\chi_L\Phi \\ =&\ \frac{s_h^2}{4}\left(\bar{x}_L\,x_L+\bar{u}_L\,u_L+\bar{d}_L\,d_L\right)+\frac{s_h^2}{2}\mathrm{Tr}\left(\bar{T}_L\,T_L\right) \\ -&\ \frac{s_h^2}{2\sqrt{2}h^2}\bar{x}_LH^{c\dagger}T_LH-\frac{s_h^2}{4h^2}\bar{u}_L(H^{c\dagger}T_LH^c-H^\dagger T_LH)-\frac{s_h^2}{2\sqrt{2}h^2}\bar{d}_LH^\dagger T_LH^c+h.c \\ -&\ i\frac{s_hc_h}{2\sqrt{2}h}\bar{x}_LH^{c\dagger}q'_L-i\frac{s_hc_h}{4h}\bar{u}_L(H^{c\dagger}q_L-H^\dagger q'_L)-i\frac{s_hc_h}{2\sqrt{2}h}\bar{d}_LH^\dagger q_L+h.c \\ +&\ i\frac{s_hc_h}{2h}H^{c\dagger}\bar{T}_Lq_L+i\frac{s_hc_h}{2h}H^\dagger\bar{T}_Lq'_L+h.c \\ +&\ \frac{s_h^2}{4h^2}\left(\bar{q}_LH^cH^{c\dagger}q_L+\bar{q}'_LHH^\dagger q'_L+\bar{q}_LH^cH^\dagger q'_L+\bar{q}'_LHH^{c\dagger}q_L\right)+\frac{c_h^2}{2}\bar{q}_Lq_L+\frac{c_h^2}{2}\bar{q}'_Lq'_L.\end{aligned} \tag{4.93}$$

Note that in the two equations above, we haven't written the $\not p$ for more clarity. Using these results the holographic Lagrangian in momentum space and at quadratic order reads

$$\begin{aligned}
\mathcal{L}_{Hol} =& Z_q\,\bar{q}_L \not p\, q_L + Z_x\,\bar{x}_L \not p\, x_L + 2 Z_T\,\mathrm{Tr}\left(\bar{T}_L \not p\, T_L\right) \\
&+ Z_t\,\bar{t}_R \not p\, t_R + Z_b\,\bar{b}_R \not p\, b_R + Z_{q'}\,\bar{\rho}_R \not p\, \rho_R \\
&+ \left(\bar{t}_R u_L + \bar{u}_L t_R\right) + \left(\bar{b}_R d_L + \bar{d}_L b_R\right) + \left(\bar{\rho}_R q'_L + \bar{q}'_L \rho_R\right) \\
&+ \Pi^- \left[2\mathrm{Tr}\left(\bar{T}_L\,T_L\right) + \bar{x}_L\,x_L + \bar{u}_L\,u_L + \bar{d}_L\,d_L\right] + \Pi^+ \frac{\not p}{p}\left[\bar{q}_L\,q_L + \bar{q}'_L\,q'_L\right] \\
&+ 2\left(\Pi^+ - \Pi^-\right)\left[\frac{s_h^2}{4}\left(\bar{x}_L\,x_L + \bar{u}_L\,u_L + \bar{d}_L\,d_L\right) + \frac{s_h^2}{2}\mathrm{Tr}\left(\bar{T}_L\,T_L\right)\right. \\
&- \frac{s_h^2}{2\sqrt{2}h^2}\bar{x}_L H^{c\dagger} T_L H - \frac{s_h^2}{4h^2}\bar{u}_L (H^{c\dagger} T_L H^c - H^\dagger T_L H) - \frac{s_h^2}{2\sqrt{2}h^2}\bar{d}_L H^\dagger T_L H^c + h.c \\
&- i\frac{s_h c_h}{2\sqrt{2}h}\bar{x}_L H^{c\dagger} q'_L - i\frac{s_h c_h}{4h}\bar{u}_L (H^{c\dagger} q_L - H^\dagger q'_L) - i\frac{s_h c_h}{2\sqrt{2}h}\bar{d}_L H^\dagger q_L + h.c \\
&+ i\frac{s_h c_h}{2h} H^{c\dagger} \bar{T}_L q_L + i\frac{s_h c_h}{2h} H^\dagger \bar{T}_L q'_L + h.c \\
&\left. + \frac{s_h^2}{4h^2}\left(\bar{q}_L H^c H^{c\dagger} q_L + \bar{q}'_L H H^\dagger q'_L + \bar{q}_L H^c H^\dagger q'_L + \bar{q}'_L H H^{c\dagger} q_L\right) - \frac{s_h^2}{2}\bar{q}_L q_L - \frac{s_h^2}{2}\bar{q}'_L q'_L\right].
\end{aligned} \tag{4.94}$$

In order to find the low energy effective Lagrangian we must integrate out the heavy fields u_L, d_L, q'_L, ρ_R, T_L and x_L. varying the Lagrangian (4.94) with respect to these fields we get the equations of motion

$$
\begin{aligned}
\delta\bar{u}_L: \quad & 0 = t_R + \Pi^- u_L + 2\left(\Pi^+ - \Pi^-\right)\big[i\frac{s_h c_h}{4h}(H^\dagger q'_L - H^{c\dagger} q_L) + \frac{s_h^2}{4}u_L \\
& \quad -\frac{s_h^2}{4h^2}(H^{c\dagger}T_L H^c - H^\dagger T_L H)\big] \\
\delta\bar{d}_L: \quad & 0 = b_R + \Pi^- d_L + 2\left(\Pi^+ - \Pi^-\right)\big[-i\frac{s_h c_h}{2\sqrt{2}h}H^\dagger q_L + \frac{s_h^2}{4}d_L - \frac{s_h^2}{2\sqrt{2}h^2}H^\dagger T_L H^c\big] \\
\delta\bar{q}'_L: \quad & 0 = \rho_R + \Pi^+ q'_L + 2\left(\Pi^+ - \Pi^-\right)\big[i\frac{s_h c_h}{2\sqrt{2}h}H^c x_L - i\frac{s_h c_h}{4h}H u_L - i\frac{s_h c_h}{2h}T_L H \\
& \quad +\frac{s_h^2}{4h^2}HH^\dagger q'_L + \frac{s_h^2}{4h^2}HH^{c\dagger}q_L - \frac{s_h^2}{2}q'_L\big] \qquad (4.95)\\
\delta\bar{\rho}_R: \quad & 0 = Z_{q'}\not{p}\rho_R + q'_L \\
\delta\bar{T}_L: \quad & 0 = \left(Z_T\not{p} + \Pi^-\right)\vec{T}_L + 2\left(\Pi^+ - \Pi^-\right)\big[i\frac{s_h c_h}{2h}H^{c\dagger}\frac{\vec{\sigma}}{2}q_L + i\frac{s_h c_h}{2h}H^\dagger\frac{\vec{\sigma}}{2}q'_L + \frac{s_h^2}{4}\vec{T}_L \\
& \quad -\frac{s_h^2}{2\sqrt{2}h^2}H^\dagger\frac{\vec{\sigma}}{2}H^c x_L - \frac{s_h^2}{4h^2}(H^{c\dagger}\frac{\vec{\sigma}}{2}H^c - H^\dagger\frac{\vec{\sigma}}{2}H)u_L - \frac{s_h^2}{2\sqrt{2}h^2}H^{c\dagger}\frac{\vec{\sigma}}{2}Hd_L\big] \\
\delta\bar{x}_L: \quad & 0 = \left(Z_x\not{p} + \Pi^-\right)x_L + 2\left(\Pi^+ - \Pi^-\right)\big[-i\frac{s_h c_h}{2\sqrt{2}h}H^\dagger q_L + \frac{s_h^2}{4}x_L - \frac{s_h^2}{2\sqrt{2}h^2}H^{c\dagger}T_L H\big].
\end{aligned}
$$

In (4.94) and the above equations, a factor $\not{p}/p$ in front of $\Pi^\pm$ must be understood, which we have avoided writing, to make the equations more transparent. Inserting the solutions to the above e.o.m (4.95) into the Lagrangian (4.94) and also making the redefinition $t_R \to -it_R$ and $b_R \to -ib_R$ to get rid of the i factors which are otherwise present in the mass terms, we arrive at the effective Lagrangian (4.53) in terms of the light fields q_L, t_R and b_R, written up to $\mathcal{O}(s_\alpha^2)$, with the form factors defined by

$$
\Pi^q = pZ_q + \Pi^+\,, \qquad \Pi^{t,b} = pZ_{t,b} - \frac{1}{\Pi^-}\,, \qquad \Pi^b_M = \sqrt{2}\Pi^t_M = \frac{\Pi^- - \Pi^+}{\sqrt{2}\Pi^-}. \qquad (4.96)
$$

Also in this case we find simple formulas for the ratio of the top and bottom mass to the W mass

$$
\frac{M_t^2}{M_W^2} \simeq \frac{\theta+1}{2N_L N_{tR}}\,, \qquad \frac{M_b^2}{M_W^2} \simeq \frac{\theta+1}{N_L N_{bR}}\,, \qquad (4.97)
$$

where

$$
\begin{aligned}
N_L &= \lim_{p\to 0}\frac{\Pi_0^q}{pL} = \frac{Z_q}{L} + \frac{1}{mL(\coth mL + 1)}, \\
N_{tR,bR} &= \lim_{p\to 0}\frac{\Pi_0^{t,b}}{pL} = \frac{Z_{t,b}}{L} + \frac{1}{mL(\coth mL - 1)}\,. \qquad (4.98)
\end{aligned}
$$

4.3.1 Computation of the $Z\bar{b}_L b_L$, $Z\bar{b}_R b_R$ and $W^+\bar{t}_R b_R$ vertices at tree level in Model III

The embedding of the LH bottom quark b_L is such that $T_L = T_R = 1/2$ and $T^3_L = T^3_R = -1/2$, so the mixing of the elementary b_L with the bulk respects the symmetry $U(1)_V \times P_{LR}$ which, according to the argument of [42] , protects its coupling to the Z boson from tree level corrections from the bulk. However the term $\bar{T}_L \not{D} T_L$ on the UV boundary is part of the beyond SM sector which does not respect this symmetry, so if it were not for this term, the $Z\bar{b}_L b_L$ coupling deviation would have vanished. To find the coupling deviation we restrict to the $-1/3$ charge sector b_L, b_R, d_L and T^-_L and finally integrate out d_L and T^-_L by solving the e.o.m, which are the second and fifth equations of (4.95) after EWSB,

$$\delta\bar{T}_L : \quad 0 = \left(Z_T \not{p} + \Pi^-\right)T^-_L + 2\left(\Pi^+ - \Pi^-\right)\left[i\frac{s_\alpha c_\alpha}{2\sqrt{2}} b_L + \frac{s^2_\alpha}{4} T^-_L - \frac{s^2_\alpha}{4} d_L\right] \tag{4.99}$$

$$\delta\bar{d}_L : \quad 0 = b_R + \Pi^- d_L + 2\left(\Pi^+ - \Pi^-\right)\left[-i\frac{s_\alpha c_\alpha}{2\sqrt{2}} b_L + \frac{s^2_\alpha}{4} d_L - \frac{s^2_\alpha}{4} T^-_L\right] \tag{4.100}$$

The solutions to these equations are

$$T^-_L = i\frac{\tan\alpha}{\sqrt{2}} b_L + \frac{\tan^2\alpha}{2}\frac{1 - e^{2mL}}{2m} \not{p}\, b_R + \mathcal{O}(p^2) \tag{4.101}$$

$$\bar{d}_L = -i\frac{\tan\alpha}{\sqrt{2}} b_L + \left(1 + \frac{\tan^2\alpha}{2}\right)\frac{1 - e^{2mL}}{2m} \not{p}\, b_R + \mathcal{O}(p^2). \tag{4.102}$$

The relevant piece of the fermion-gauge interaction vertex of the bulk is

$$\int_0^L dz\ \mathrm{Tr}\left(\bar{\xi}\ \left[\Sigma\left[\not{W}^3_L T^3_L + \not{B}\left(T^3_R + X\right)\right]\Sigma^\dagger, \xi\right]\right) \tag{4.103}$$

plugging the solutions (4.101) into this term and using (4.21) to pick up the term proportional to the Z field we find the following $Z\bar{b}_L b_L$ and $Z\bar{b}_R b_R$ terms from the bulk

$$\begin{aligned} &\left[\frac{2 + \cos 2\theta_W}{6}\frac{1 - e^{-2mL}}{2m}\left(1 + \tan^2\alpha\right)\right]\bar{b}_L \not{Z} b_L + \\ &\left[\frac{\cos 2\theta_W - 1}{6}\frac{e^{2mL} - 1}{2m} + \frac{2 + \cos 2\theta_W}{6}\frac{e^{2mL} - 1}{2m}\frac{\tan^2\alpha}{2}\right]\bar{b}_R \not{Z} b_R. \end{aligned} \tag{4.104}$$

Note that we have dropped the factor $g/\cos\theta_W$, so that the SM $Z\bar{b}_L b_L$ coupling is $-(2 + \cos 2\theta_W)/6$. By plugging the solutions (4.101) into the boundary fermion-gauge interactions given in the first two lines of (4.86) we can easily find the boundary contributions to the $Z\bar{b}_L b_L$ and $Z\bar{b}_R b_R$ vertices

$$\left[-Z_q \frac{2+\cos 2\theta_W}{6} - Z_T \frac{5+\cos 2\theta_W}{6}\frac{\tan^2\alpha}{2}\right]\bar{b}_L \not{Z} b_L + \left[\frac{1}{3} Z_b \sin^2\theta_W\right]\bar{b}_R \not{Z} b_R \qquad (4.105)$$

also, integrating out d_L and T_L^- from the Lagrangian (4.94) will give us the total kinetic terms for b_L and b_R

$$\begin{aligned} &\left[Z_b + \frac{1-e^{-2mL}}{2m}\left(1+\tan^2\alpha\right) + \frac{1}{2} Z_T \tan^2\alpha\right]\bar{b}_L \not{p} b_L + \\ &\left[Z_b + \frac{e^{2mL}-1}{2m}\left(1+\frac{\tan^2\alpha}{2}\right)\right]\bar{b}_R \not{p} b_R. \end{aligned} \qquad (4.106)$$

Note that these results are exact in α. Dividing the first(second) equation of (4.104) by the first(second) equation in (4.106) we find the $Z\bar{b}_L b_L$ and $Z\bar{b}_R b_R$ couplings whose deviations from their SM values are

$$\delta g_{b,L} = \frac{e^{2mL} m Z_T}{1-e^{2mL}(1+2mZ_q)}\frac{\tan^2\alpha}{2} \qquad (4.107)$$

$$\delta g_{b,R} = -\frac{e^{2mL}-1}{e^{2mL}-1+2mZ_b}\frac{\tan^2\alpha}{4}. \qquad (4.108)$$

As expected the coupling deviation $\delta g_{b,L}$ is proportional to Z_T.

We next move to the computation of the $W^+\bar{t}_R b_R$ vertex which is non vanishing in this model. There is no boundary contribution to this vertex and the relevant term of the bulk is

$$\int_0^L dz \ \mathrm{Tr}\left(\bar{\xi}\left[\Sigma \not{W}_L^+ T_L^+ \Sigma^\dagger, \xi\right]\right), \qquad (4.109)$$

which after integrating out the heavy fields gives rise to the following contribution, which is again exact in α

$$\left[\frac{1-e^{2mL}}{4\sqrt{2}m}\tan^2\alpha\right]\frac{1}{\sqrt{2}}\bar{t}_R \not{W}_L^+ b_R, \qquad (4.110)$$

while for the t_R kinetic term, integrating out the heavy fields from (4.94), we will get

$$\left[Z_t + \frac{e^{2mL}-1}{2m}\left(1+\frac{\tan^2\alpha}{2}\right) + \frac{1}{4} Z_{q'}\tan^2\alpha + \mathcal{O}(\tan^4\alpha)\right]\bar{t}_R \not{p} t_R. \qquad (4.111)$$

After normalization of the t_R and b_R kinetic terms, the $W^+\bar{t}_R b_R$ coupling will be

$$g_{tb,R} = -\frac{e^{2mL}-1}{m\sqrt{Z_t + \frac{e^{2mL}-1}{2m}}\sqrt{Z_b + \frac{e^{2mL}-1}{2m}}}\frac{\tan^2\alpha}{4\sqrt{2}}. \qquad (4.112)$$

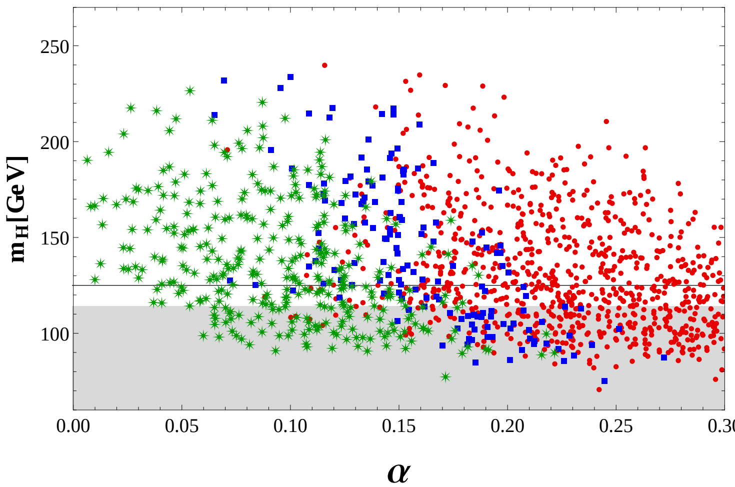


Figure 4.7: *Scatter plot of points obtained from a scan over the parameter space of model III. Small red dots represent points which don't pass EWPT at* 99%*C.L., square blue dots represent points which pass EWPT at* 99%*C.L. but not at* 90% *C.L., and star shape green dots represent points which pass EWPT at* 90%*C.L.. The region below the LEP bound* ($m_H < 114$ GeV) *is shaded. The mass of the recently discovered Higgs-like particle at* 125 GeV *is shown by the black line.*

In the limit of no boundary kinetic terms $Z_t = Z_b = 0$ for t_R and b_R, this coupling will reduce to

$$g_{tb,R} = -\frac{\tan^2\alpha}{2\sqrt{2}}. \tag{4.113}$$

4.3.2 The effective potential

For the same reasons explained in section 4.2.1 for model II, to find the effective potential we cannot use the low energy Lagrangian in terms of the light fields, but instead we need to integrate out the RH fields from the Lagrangian to get an effective Lagrangian in terms of LH fields only. Doing this we arrive at a Lagrangian which is obtained from (4.94) by replacing its second and third lines with

$$-\frac{1}{Z_t p^2}\not{p}\, u_L - \frac{1}{Z_b p^2}\not{p}\, d_L - \frac{1}{Z_{q'} p^2}\not{p}\, q'_L. \tag{4.114}$$

The effective potential has contributions from the top sector V_t, the bottom sector V_b and the exotic 5/3 charge sector V_{ex}. The exact expressions are too lengthy to write here and we report in the following the explicit form of the Higgs potential only in the relevant

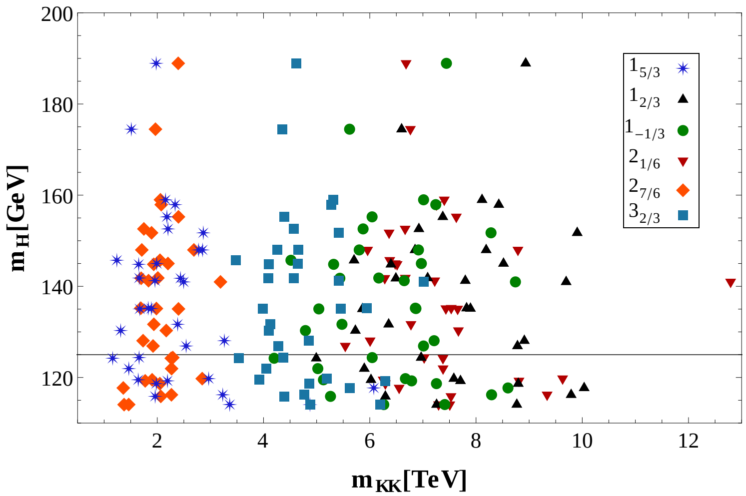


Figure 4.8: *Higgs mass m_H versus the mass of the first KK resonances (before EWSB) for the points of model III with $m_H > 114$ GeV and $\alpha \in [0.16, 0.23]$. The mass of the recently discovered Higgs-like particle at 125 GeV is shown by the black line.*

region in parameter space where $Z_T, Z_q, Z_x \ll 1$ and $Z_b \gg 1$. Neglecting Z_T, Z_q and Z_x, we get

$$\begin{aligned} V_t &\simeq -2N_c \int \frac{d^4p}{(2\pi)^4} \ln\left(1 + s_\alpha^2 \frac{(\Pi^- - \Pi^+)\Big(2p\Pi^+\Pi^-(Z_t - Z_{q'}) + \Pi^- - \Pi^+\Big)}{4\Pi^+\Pi^-(pZ_{q'}\Pi^+ - 1)(pZ_t\Pi^- - 1)}\right) , \\ V_b &\simeq -2N_c \int \frac{d^4p}{(2\pi)^4} \ln\left(1 + s_\alpha^2 \frac{\Pi^+ - \Pi^-}{2pZ_b\Pi^+\Pi^-}\right) , \\ V_{ex} &\simeq -2N_c \int \frac{d^4p}{(2\pi)^4} \ln\left(1 + s_\alpha^2 \frac{\Pi^- - \Pi^+}{\Pi^-(pZ_{q'}\Pi^+ - 1)}\right) , \end{aligned} \tag{4.115}$$

where we have omitted the mass dependence of the form factors $\Pi^\pm$. The total Higgs potential is finally

$$V_{tot} = V_g + V_t + V_b + V_{ex} \,, \tag{4.116}$$

with V_g given in eq.(3.56). Expanding the Logarithms in the small quantities appearing in front of $\sin^2\alpha$ which fall off exponentially with momentum, this potential has the same structure as that of model II.

4.3.3 Results

The results of our numerical scan are summarized in figs. 4.7, 4.8 and 4.9. The details of the χ^2 fit, which has been done with 5 d.o.f, are explained in section B.3. The fermion sector of this model has 7 parameters, 1 bulk masses and 6 coefficients for the BKT. The randomly chosen input parameters are m, Z_q, $Z_{q'}$, Z_x, Z_T, θ and θ'. The remaining two parameters Z_t and Z_b are fixed by the top and bottom mass formulas. For stability reasons, we take positive coefficients for all the BKT. We have scanned the parameter space over the region $mL \in [-1.5, 0.5]$, $Z_q/L \in [0, 1.5]$, $Z_{q'}/L \in [0, 2]$, $Z_x/L \in [0, 6]$, $Z_T/L \in [0, 1.5]$, $\theta \in [20, 30]$ and $\theta' \in [15, 25]$.

As can be seen in fig.4.7, at 90% C.L. EWPT constrain $\alpha \lesssim 1/5$ to allow for a 125 GeV Higgs mass. This reduces to $\alpha \lesssim 0.15$ at 90% C.L.. The lightest exotic particles are fermion $SU(2)_L$ singlets with $Y = 5/3$ and $SU(2)_L$ doublets with $Y = 7/6$, see fig.4.8. After EWSB, these multiplets give rise to 5/3 and 2/3 charged fermions. Their mass is of order $1 \div 2$ TeV, significantly lighter than the gauge KK modes (~ 5 TeV). The doublet q_L and the singlet t_R have typically a sizable and comparable degree of compositeness, while b_R is mostly elementary. When $mL \lesssim -1$, q_L turns out to be even more composite than t_R.

4.4 Conclusions

We have constructed three different composite Higgs/GHU models in flat space with large BKT, based on the minimal custodially-symmetric $SO(5) \times U(1)_X$ gauge group, and we have shown that EWSB and EWPT are compatible in these models. We stress that model building in this context is significantly simpler than in warped space.

The Higgs is predicted to be light with a mass $m_H \leq 200$ GeV, which is consistent with the recently found 125 GeV mass. The lightest new-physics particles are colored fermions with a mass as low as about 500 GeV in model I and 1 TeV in models II and III. Their electroweak quantum numbers depend on the model and on the region in parameter space, but they are always particles with electric charges -1/3, +2/3 or +5/3.

The next step in constructing fully realistic models would be the addition of the two light quark generations, leptons, and flavour in general. We expect that the typical known patterns of flavour physics in warped space, such as the so-called RS-GIM, should also be captured by our effective flat space description. Indeed, in presence of large BKT, the cut-off of the theory becomes effectively a function of the position in the internal space and is maximal at the UV brane, with the SM fields becoming more elementary (peaked at the UV brane at $y = 0$) and the KK states more composite (peaked at the IR brane at $y = L$). In this way, otherwise too large flavour-changing violating operators might be naturally suppressed. It will be very interesting to study this issue in detail and see whether and to what extent this expectation is valid.

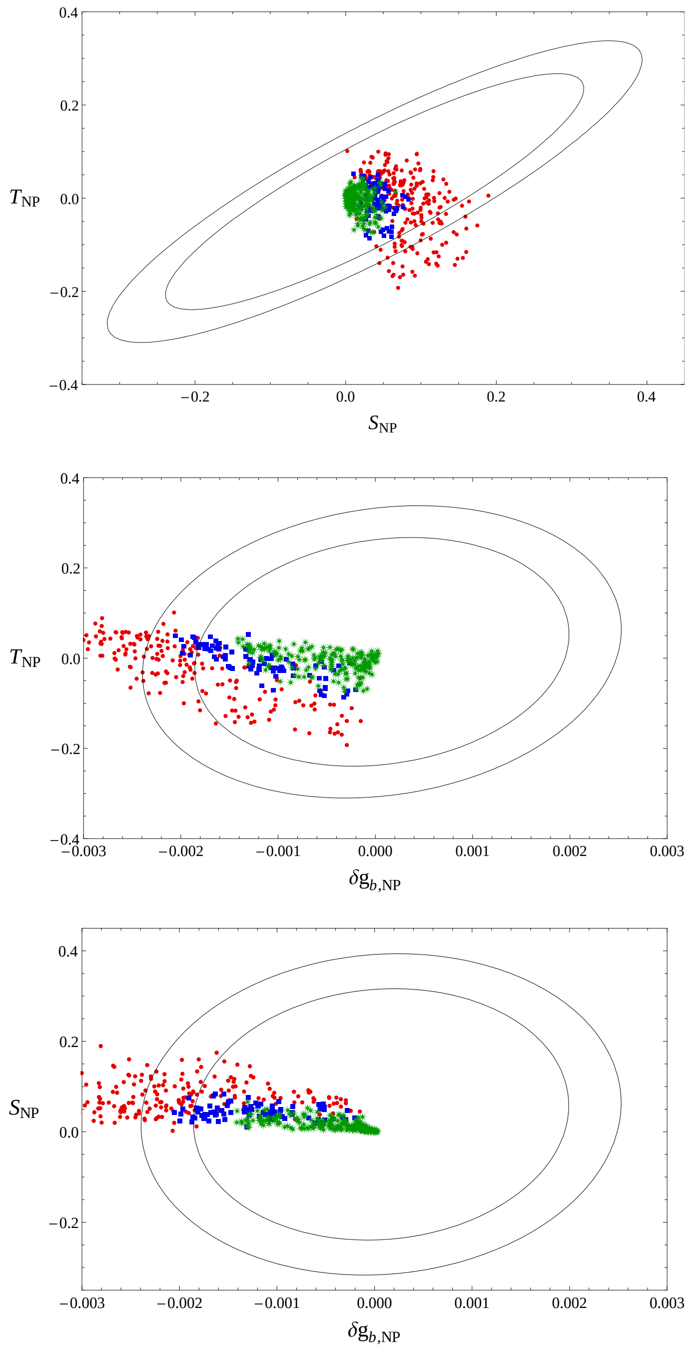


Figure 4.9: *Scatter plot of points in model III with* $m_H > 114$ GeV *and projected on the* T_{NP}-S_{NP}*,* T_{NP}-$\delta g_{b,NP}$ *and* S_{NP}-$\delta g_{b,NP}$ *planes. We have set* $M_{H,eff} = 120$ GeV*. Small red dots represent points which don't pass EWPT at* 99% *C.L., square blue dots represent points which pass EWPT at* 99%*C.L. but not at* 90% *C.L., and star shape green dots represent points which pass EWPT at* 90% *C.L.. The big and small ellipses correspond to* 99% *and* 90% *C.L. respectively.*

The very broad collider signatures of our models completely fall into those of composite Higgs/warped GHU models. The correct EWSB pattern in all composite Higgs/GHU models constructed so far (warped or flat, with $SO(5)$ or $SU(3)$ gauge groups) seems to indicate that the lightest (below TeV) new physics states beyond the SM should be fermionic colored particles, with model-dependent $SU(2)_L \times U(1)_Y$ quantum numbers. Of course, this generic prediction cannot be seen as a “signature” of composite Higgs/GHU models. More specific predictions are the expected sizable deviations to the Higgs-gauge couplings or to the top couplings, which will be tested at the future stages of the LHC run.

Appendix A

1-Loop Computation of the $Z\bar{b}_L b_L$ Vertex

A.1 Notation and Feynman rules

We present here some of the Feynman rules relevant for the 1-loop $Z\bar{b}_L b_L$ vertex computation [43] of the next section. Taking a basis $T^T_{L,R} = (t^1_{L,R}, t^2_{L,R}, \cdots)$ for the particles in the top sector and a basis $B^T_{L,R} = (b^1_{L,R}, b^2_{L,R}, \cdots)$ for the particles in the bottom sector, the relevant part af the Lagrangian includes the mass terms of the top and bottom sectors and the $W^\pm$ and $\phi^\pm$ interactions

$$\begin{aligned}\mathcal{L} \;\supset\; & -\bar{T}_R M_t T_L - \bar{B}_R M_b B_L + \frac{\sqrt{2}}{v}\bar{T}_L V_W W^+ B_L + h.c \\ & + \frac{\sqrt{2}}{v}\bar{T}_R M^L_\phi \phi^+ B_L - \frac{\sqrt{2}}{v}\bar{T}_L M^R_\phi \phi^+ B_R + h.c \end{aligned} \tag{A.1}$$

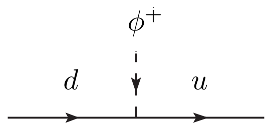
By performing the rotations $T_{L,R} \to U^t_{L,R} T_{L,R}$ and $B_{L,R} \to U^b_{L,R} B_{L,R}$ in (A.1) we diagonalize the matrices $M_{t,b}$ and move to the mass basis in which the Lagrangian reads

$$\begin{aligned} & -\bar{T}_R M^D_t T_L - \bar{B}_R M^D_b B_L + \frac{\sqrt{2}}{v}\bar{T}_L V W^+ B_L + h.c \\ & + \frac{\sqrt{2}}{v}\bar{T}_R M^D_t \widetilde{V}_L B_L - \frac{\sqrt{2}}{v}\bar{T}_L \widetilde{V}_R M^D_b B_R + h.c \end{aligned} \tag{A.2}$$

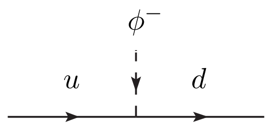
where the matrices appearing in the above Lagrangian are defined through the following relations

$$\begin{aligned} M^D_{t,b} &= U^{t,b\dagger}_R \, M_{t,b} \, U^{t,b}_L \\ V &= U^{t\dagger}_L \, V_W \, U^b_L \end{aligned} \;, \qquad \begin{aligned} M^D_t \, \widetilde{V}_L &= U^{t\dagger}_R \, M^L_\phi \, U^b_L \\ \widetilde{V}_R \, M^D_b &= U^{t\dagger}_L \, M^R_\phi \, U^b_R \end{aligned} \tag{A.3}$$

From the Lagrangian (A.2) one can easily read off the Feynman rules for the $\phi^{\pm}$ interactions

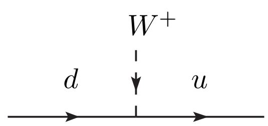


$$= -\frac{ie}{\sqrt{2} s_W M_W} \left[(\widetilde{V}_R)_{ud}\, m_d\, P_R - m_u\, (\widetilde{V}_L)_{ud}\, P_L \right]$$



$$= \frac{ie}{\sqrt{2} s_W M_W} \left[(\widetilde{V}_L^{\dagger})_{du}\, m_u\, P_R - m_d\, (\widetilde{V}_R^{\dagger})_{du}\, P_L \right],$$

where $m_{u,d}$ are entries on the diagonal of $M_{t,b}$, and also $W^{\pm}$ interactions



$$= \frac{ie}{\sqrt{2} s_W M_W} V_{ud}.$$

Similarly for the conjugate vertex in which W^+ is replaced with W^- and u, d are interchanged, V_{ud} has to be replaced with $V^{\dagger}_{du}$.

The Feynman diagrams that contribute to the $Z\bar{b}_L b_L$ at 1-loop are shown in fig.(A.1). Now we have all the ingredients to compute these diagrams. We will write the expression for the Feynman diagram (i) as

$$J_I = \int \frac{d^d k}{(2\pi)^d}\, K_i = i \frac{\alpha}{2\pi} \left(\frac{g}{c_W} \right)\, I_i\, \bar{b}_L \gamma^{\mu} b_L \tag{A.4}$$

defining in this way the two quantities K_i and I_i. We isolate the effect of the top quark from other radiative corrections by subtracting a constant piece from I_i and defining

$$F(r) \equiv I(r) - I(0). \tag{A.5}$$

where $r \equiv m_t^2/M_W^2$. Using this definition, the effective $Z\bar{b}_L b_L$ coupling is written as

$$g_b = \frac{\alpha}{2\pi} \left(\frac{g}{c_W} \right) F. \tag{A.6}$$

Out of the two matrices $\widetilde{V}_{L,R}$, only $\widetilde{V}_L$ will appear in the expressions for the Feynman diagrams, and we will finally call it $\widetilde{V}$ for conciseness.

It is worth mentioning that, assuming no mixing in the bottom sector, whenever the elements of the RH top sector T_R belong to the representations $\mathbf{1}_{2/3}$, $\mathbf{2}_{1/6}$, $\mathbf{2}_{7/6}$ or $\mathbf{3}_{5/3}$, then the matrix element $\widetilde{V}_{ib}$, which represents the mixing between the ith element of the RH top sector with the LH bottom quark of the SM though ϕ^+, is equal to the mixing V_{ib} between the ith element of the LH top sector with the SM LH bottom quark though W^+. To see this, we concentrate on the top sector mass mixing term and ϕ interaction of the left handed top and right handed bottom sectors

$$\mathcal{L} \supset -\bar{T}_R^T M_t T_L + \frac{\sqrt{2}}{v}\bar{T}_R^T M_\phi \phi^+ B_L + h.c. \tag{A.7}$$

For t_R^i belonging to the representation $\mathbf{1}_{2/3}$, the part of the above Lagrangian including t_L and b_L appears as

$$\bar{q}_L H^c t_R^i \supset \frac{v}{\sqrt{2}}\bar{t}_L t_R^i - \bar{b}_L \phi^- t_R^i \tag{A.8}$$

which shows that the relation

$$(M_t)_{ib} = (M_\phi^L)_{ib} \tag{A.9}$$

is satisfied. on the other hand, if t_R^i belongs to the representations $\mathbf{2}_{1/6}$, $\mathbf{2}_{7/6}$ or $\mathbf{3}_{5/3}$, there could be no Yukawa interaction with q_L, which means that there will be no contribution to M_t and M_ϕ^L and so eq.(A.9) is still valid. A t_R^i belonging to the representations $\mathbf{3}_{2/3}$ or $\mathbf{3}_{-1/3}$ for example, will spoil the relation (A.9).

Performing the rotation $T_{L,R} \to U_{L,R}^t T_{L,R}$, we diagonalize M_t, $U_R^{t\dagger} M_t U_L^t = M_t^D$. Assuming no mixing in the bottom sector, the mixing through ϕ^+ of the left handed bottom sector with the right handed top sector in the mass basis will be given by $U_R M_\phi$. The ib component of this matrix can be written in terms of the top masses and the matrix element V_{ib}

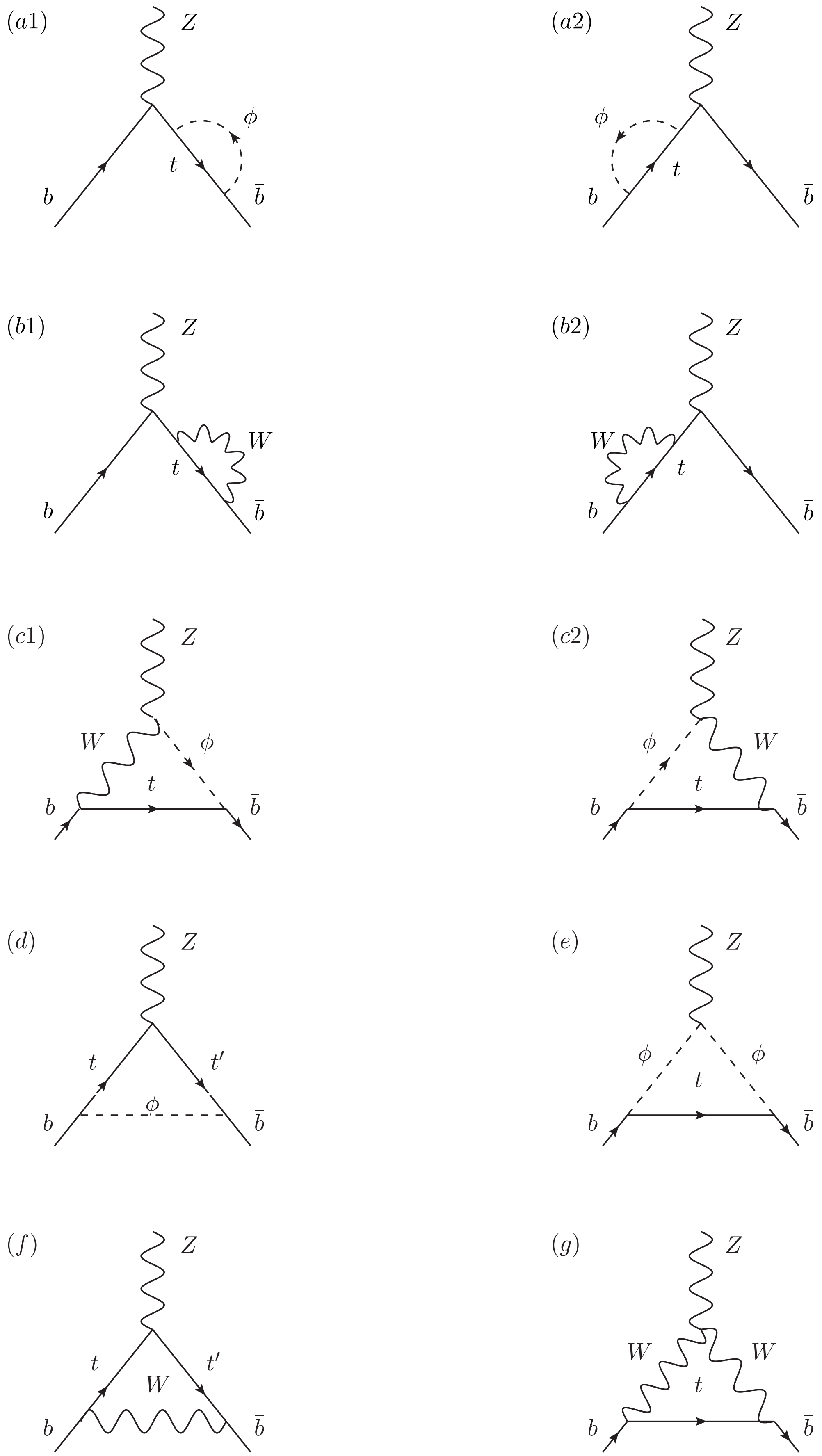
$$[U_R^\dagger M_\phi^L]_{i1} = [U_R^\dagger M_t]_{ib} = [U_R^\dagger M_t U_L^t U_L^t]_{ib} = [M_t^D U_L^t]_{ib} = m_{ti} V_{ib}, \tag{A.10}$$

with no summation in the last term. Using the definitions (A.3), this implies that $\widetilde{V}_{ib} = V_{ib}$

A.2 Details of the 1-loop computation

The diagrams that contribute to the $Z\bar{b}_L b_L$ vertex at 1-loop are shown in fig.(A.1). In the following we compute these diagrams ignoring the bottom mass and setting the external momenta to zero[1] (see [44] for a more general computation).

[1]The non SM fermions are significantly lighter that the non SM vector mesons so we have neglected diagrams in which a massive vector resonance is exchanged.



(a1)
Z
φ
t
b
b̄
(a2)
Z
φ
t
b
b̄
(b1)
Z
W
t
b
b̄
(b2)
Z
W
t
b
b̄
(c1)
Z
W
φ
t
b
b̄
(c2)
Z
φ
W
t
b
b̄
(d)
Z
t
t′
φ
b
b̄
(e)
Z
φ
φ
t
b
b̄
(f)
Z
t
t′
W
b
b̄
(g)
Z
W
W
t
b
b̄

Figure A.1: 1-loop contributions to the $Z\bar{b}_L b_L$ vertex.

Computing diagrams a_1 and a_2

The computation of diagram ($a1$) goes as follows

$$\begin{aligned}
K_{a1} &= \bar{b}_L i\gamma^\mu \left(\frac{g}{c_W}\right)(g_L^b P_L + g_R^b P_R)\,\frac{i(\not{p}+m_b)}{p^2-m_b^2} \\
&\quad \times \frac{ie}{\sqrt{2}s_W M_W}\left[(\widetilde{V}_L^\dagger)_{bt}\,m_t\,P_R - m_b\,(\widetilde{V}_R^\dagger)_{bt}\,P_L\right]\,\frac{i(\not{k}+\not{p}+m_t)}{(k+p)^2-m_t^2} \\
&\quad \times \frac{-ie}{\sqrt{2}s_W M_W}\left[(\widetilde{V}_R)_{tb}\,m_b\,P_R - m_t\,(\widetilde{V}_L)_{tb}\,P_L\right]\,\frac{i}{k^2-m_W^2} b_L \\
&= -\left(\frac{e}{\sqrt{2}s_W M_W}\right)^2\left(\frac{g}{c_W}\right)|\widetilde{V}_{tb}|^2\,\bar{b}_L\gamma^\mu g_L^b P_L\,\frac{\not{p}+m_b}{p^2-m_b^2}\,m_t P_R\,\frac{\not{k}+\not{p}+m_t}{(k+p)^2-m_t^2} \\
&\quad \times m_t P_L\,\frac{1}{k^2-m_W^2} b_L \\
&= -\left(\frac{e m_t}{\sqrt{2}s_W M_W}\right)^2\left(\frac{g}{c_W}\right)|\widetilde{V}_{tb}|^2\,\frac{1}{p^2-m_b^2}\,\frac{1}{(k+p)^2-m_t^2}\,\frac{1}{k^2-m_W^2} g_L^b \\
&\quad \times \bar{b}_L\gamma^\mu P_L\,(\not{p}+m_b)\,P_R\,(\not{k}+\not{p}+m_t)\times\;P_L\,b_L \\
&= -\left(\frac{e m_t}{\sqrt{2}s_W M_W}\right)^2\left(\frac{g}{c_W}\right)|\widetilde{V}_{tb}|^2\,\frac{1}{p^2-m_b^2}\,\frac{1}{(k+p)^2-m_t^2}\,\frac{1}{k^2-m_W^2} g_L^b \\
&\quad \times \bar{b}_L\gamma^\mu\,\not{p}\,(\not{k}+\not{p})\,P_L\,b_L
\end{aligned} \tag{A.11}$$

Adding Diagram ($a2$) which is its conjugate and using Feynman parametrization we get

$$\begin{aligned}
& K_{a1}+K_{a2} \\
&= -\left(\frac{e m_t}{\sqrt{2}s_W M_W}\right)^2|\widetilde{V}_{tb}|^2\left(\frac{g}{c_W}\right)\frac{g_L^b}{p^2-m_b^2}\int_0^1 dx\frac{\bar{b}_L[\gamma^\mu\not{p}(\not{k}+\not{p})+(\not{k}+\not{p})\not{p}\gamma^\mu]P_L b_L}{[(k+xp)^2+x(1-x)p^2-xm_t^2-(1-x)m_W^2]^2} \\
&= -\left(\frac{e m_t}{\sqrt{2}s_W M_W}\right)^2|\widetilde{V}_{tb}|^2\left(\frac{g}{c_W}\right)\frac{g_L^b}{p^2-m_b^2}\int_0^1 dx\frac{2p^2(1-x)\bar{b}_L\gamma^\mu b_L}{[k^2-x(1-x)p^2-xm_t^2-(1-x)m_W^2]^2} \\
&\xrightarrow{m_b,p\to 0} -\left(\frac{e m_t}{\sqrt{2}s_W M_W}\right)^2|\widetilde{V}_{tb}|^2\left(\frac{g}{c_W}\right)g_L^b\int_0^1 dx\,\frac{2(1-x)}{[k^2-xm_t^2-(1-x)m_W^2]^2}\,\bar{b}_L\,\gamma^\mu\,b_L
\end{aligned} \tag{A.12}$$

where in the second equation we have used

$$\begin{aligned}
\gamma^\mu\,\not{p}\,(\not{k}+\not{p})+(\not{k}+\not{p})\,\not{p}\,\gamma^\mu &= 2\gamma^\mu\,p^2+\gamma^\mu\,\not{p}\,\not{k}+\not{k}\,\not{p}\,\gamma^\mu \\
&\xrightarrow{k\to k-xp} 2\gamma^\mu\,p^2(1-x)+\gamma^\mu\,\not{p}\,\not{k}+\not{k}\,\not{p}\,\gamma^\mu
\end{aligned} \tag{A.13}$$

and in the third equation we have sent $m_b \to 0$ and then $p \to 0$. So integrating over the momenta, the expression for the sum of the two Feynman diagrams $(a1)$ and $(a2)$ is

$$\begin{aligned}
J_a &= J_{a1} + J_{a2} \\
&= -\left(\frac{e m_t}{\sqrt{2} s_W M_W}\right)^2 |\widetilde{V}_{tb}|^2 \left(\frac{g}{c_W}\right) g_L^b \int_0^1 dx \int \frac{d^d k}{(2\pi)^d} \frac{2(1-x)}{[k^2 - x m_t^2 - (1-x) m_W^2]^2} \bar{b}_L \gamma^\mu b_L \\
&= i \left(\frac{e m_t}{\sqrt{2} s_W M_W}\right)^2 |\widetilde{V}_{tb}|^2 \left(\frac{g}{c_W}\right) g_L^b \frac{1}{(4\pi)^2} \left[\mathbb{D} + \frac{r^2 \log r}{(r-1)^2} - \frac{1}{r-1}\right] \bar{b}_L \gamma^\mu b_L \\
&= i \frac{\alpha}{2\pi} \left(\frac{g}{c_W}\right) |\widetilde{V}_{tb}|^2 \frac{r}{12} \left(1 - \frac{3}{2 s_W^2}\right) \left[\mathbb{D} + \frac{r^2 \log r}{(r-1)^2} - \frac{1}{r-1}\right] \bar{b}_L \gamma^\mu b_L \\
&\equiv i \frac{\alpha}{2\pi} \left(\frac{g}{c_W}\right) I_a(r) \, \bar{b}_L \gamma^\mu b_L
\end{aligned} \tag{A.14}$$

where

$$-\mathbb{D} \equiv \frac{2}{\epsilon} - \gamma - \log \frac{M_W^2}{4\pi \mu^2} + \frac{3}{2} \tag{A.15}$$

and the function $I_a(r)$, which is equal to $F_a(r)$ here, is given by

$$I_a(r) = |\widetilde{V}_{tb}|^2 \frac{r}{12} \left(1 - \frac{3}{2 s_W^2}\right) \left[\mathbb{D} + \frac{r^2 \log r}{(r-1)^2} - \frac{1}{r-1}\right] \tag{A.16}$$

Computing diagrams b_1 and b_2

The expression for diagram $(b1)$ is

$$\begin{aligned}
K_{b1} &= \bar{b}_L i \gamma^\mu \left(\frac{g}{c_W}\right) (g_L^b P_L + g_R^b P_R) \frac{i(\not{p} + m_b)}{p^2 - m_b^2} \\
&\quad \times \frac{i e \gamma^\alpha}{\sqrt{2} s_W} V_{bt}^\dagger \frac{i(\not{k} + \not{p} + m_t)}{(k+p)^2 - m_t^2} \frac{i e \gamma_\alpha}{\sqrt{2} s_W} V_{tb} \frac{-i}{k^2 - m_W^2} b_L \\
&= \left(\frac{e}{\sqrt{2} s_W}\right)^2 \left(\frac{g}{c_W}\right) |V_{tb}|^2 g_L^b \frac{\bar{b}_L \gamma^\mu \big[-(d-2)\not{p}(\not{k}+\not{p}) + d m_b m_t\big] b_L}{[(k+p)^2 - m_t^2](k^2 - m_W^2) p^2 - m_b^2}
\end{aligned} \tag{A.17}$$

where we have used

$$\begin{aligned}
\gamma^\mu P_L (\not{p} + m_b) \gamma^\alpha (\not{k} + \not{p} + m_t) \gamma_\alpha P_L &= \gamma^\mu P_L (\not{p} + m_b) \big[-(d-2)(\not{k} + \not{p}) + d m_t\big] P_L \\
&= \gamma^\mu \big[-(d-2)\not{p}(\not{k} + \not{p}) + d m_b m_t\big] P_L
\end{aligned} \tag{A.18}$$

in the second equation . Adding the contribution of K_{b2} and using Feynman parametrization along with eq.(A.13) one finds

$$\begin{aligned}
&K_{b1}+K_{b2}\\
&= \left(\frac{e}{\sqrt{2}s_W}\right)^2\left(\frac{g}{c_W}\right)|V_{tb}|^2\, g_L^b\int_0^1 dx\frac{2\big[-(d-2)p^2(1-x)+dm_bm_t\big]\bar{b}_L\gamma^\mu b_L}{[k^2-x(1-x)p^2-xm_t^2-(1-x)m_W^2]^2}\frac{1}{p^2-m_b^2}\\
&\xrightarrow{m_b,p\to 0}\left(\frac{e}{\sqrt{2}s_W}\right)^2\left(\frac{g}{c_W}\right)|V_{tb}|^2\, g_L^b\int_0^1 dx\frac{-2(d-2)(1-x)\bar{b}_L\gamma^\mu b_L}{[k^2-xm_t^2-(1-x)m_W^2]^2}
\end{aligned}\tag{A.19}$$

which after integrating over momenta becomes

$$\begin{aligned}
J_b &\equiv J_{b1}+J_{b2}\\
&= \left(\frac{e}{\sqrt{2}s_W}\right)^2\left(\frac{g}{c_W}\right)|V_{tb}|^2\, g_L^b\int_0^1 dx\int\frac{d^dk}{(2\pi)^d}\frac{-2(d-2)(1-x)\bar{b}_L\gamma^\mu b_L}{[k^2-xm_t^2-(1-x)m_W^2]^2}\\
&= 2i\left(\frac{e}{\sqrt{2}s_W}\right)^2\left(\frac{g}{c_W}\right)|V_{tb}|^2\, g_L^b\frac{1}{(4\pi)^2}\Big(\mathbb{D}+1+\frac{r^2\log r}{(r-1)^2}-\frac{1}{r-1}\Big)\bar{b}_L\gamma^\mu b_L\\
&= i\frac{\alpha}{2\pi}\left(\frac{g}{c_W}\right)|V_{tb}|^2\,\frac{g_L^b}{2s_W^2}\Big(\mathbb{D}+1+\frac{r^2\log r}{(r-1)^2}-\frac{1}{r-1}\Big)\bar{b}_L\gamma^\mu b_L\\
&= i\frac{\alpha}{2\pi}\left(\frac{g}{c_W}\right)|V_{tb}|^2\frac{1}{6}\left(1-\frac{3}{2s_W^2}\right)\Big(\mathbb{D}+1+\frac{r^2\log r}{(r-1)^2}-\frac{1}{r-1}\Big)\bar{b}_L\gamma^\mu b_L
\end{aligned}\tag{A.20}$$

so the function $F(r)$ defined in (A.5) is

$$F_b(r)=I_b(r)-I_b(0)=|V_{tb}|^2\,\frac{1}{6}\left(1-\frac{3}{2s_W^2}\right)\left[\frac{r^2\log r}{(r-1)^2}-\frac{r}{r-1}\right].\tag{A.21}$$

Computing diagrams c_1 and c_2

Diagram $(c1)$ is computed as follows

$$\begin{aligned}
K_{c1} &= \bar{b}_L\left(\frac{ie}{\sqrt{2}s_WM_W}\right)\widetilde{V}_{bt}^\dagger m_tP_R\frac{i(\not{k}+m_t)}{k^2-m_t^2}\\
&\quad\times\left(\frac{ie\gamma_\nu}{\sqrt{2}s_W}\right)V_{tb}\big(-igM_Zs_W^2\eta_{\mu\nu}\big)\frac{1}{(k^2-m_W^2)^2}b_L\\
&= -\bar{b}_L\left(\frac{e}{\sqrt{2}s_WM_W}\right)\widetilde{V}_{bt}^\dagger V_{tb}\frac{m_t^2}{k^2-m_t^2}\left(\frac{e\gamma_\nu}{\sqrt{2}s_W}\right)\big(gM_Zs_W^2\eta_{\mu\nu}\big)\frac{1}{(k^2-m_W^2)^2}b_L\\
&= -\left(\frac{e^2}{2}\right)\left(\frac{g}{c_W}\right)\widetilde{V}_{bt}^\dagger V_{tb}\frac{m_t^2}{(k^2-m_t^2)(k^2-m_W^2)^2}\bar{b}_L\gamma^\mu b_L\\
&= -\left(\frac{e^2}{2}\right)\left(\frac{g}{c_W}\right)\widetilde{V}_{bt}^\dagger V_{tb}rm_W^2\int_0^1 dx\frac{2(1-x)}{[k^2-m_W^2(1-x)-m_t^2x]^3}\bar{b}_L\gamma^\mu b_L
\end{aligned}\tag{A.22}$$

which leads to the following expression after integrating over the momenta

$$\begin{aligned}
J_{c1} &= -\left(\frac{e^2}{2}\right)\left(\frac{g}{c_W}\right)\widetilde{V}^\dagger_{bt}V_{tb}rm_W^2\int_0^1 dx\int\frac{d^dk}{(2\pi)^d}\frac{2(1-x)}{[k^2-m_W^2(1-x)-m_t^2x]^3}\,\bar{b}_L\gamma^\mu b_L \\
&= -\left(\frac{e^2}{2}\right)\left(\frac{g}{c_W}\right)\widetilde{V}^\dagger_{bt}V_{tb}rm_W^2v\int_0^1 dx\frac{-i}{2(4\pi)^2}\frac{2(1-x)}{m_W^2(1-x)+m_t^2x}\,\bar{b}_L\gamma^\mu b_L \\
&= -\left(\frac{e^2}{2}\right)\left(\frac{g}{c_W}\right)\widetilde{V}^\dagger_{bt}V_{tb}r\int_0^1 dx\frac{-i}{(4\pi)^2}\frac{1-x}{1-x+xr}\,\bar{b}_L\gamma^\mu b_L \\
&= \frac{i}{(4\pi)^2}\left(\frac{e^2}{2}\right)\left(\frac{g}{c_W}\right)\widetilde{V}^\dagger_{bt}V_{tb}r\left[\frac{r\log r}{(r-1)^2}-\frac{1}{r-1}\right]\bar{b}_L\gamma^\mu b_L \\
&= i\frac{\alpha}{2\pi}\left(\frac{g}{c_W}\right)\widetilde{V}^\dagger_{bt}V_{tb}\frac{r}{4}\left[\frac{r\log r}{(r-1)^2}-\frac{1}{r-1}\right]\bar{b}_L\gamma^\mu b_L. \qquad \text{(A.23)}
\end{aligned}$$

This expression along with its conjugate sum up to the following result

$$J_c = J_{c1}+J_{c2} = i\frac{\alpha}{2\pi}\left(\frac{g}{c_W}\right)\left(\widetilde{V}^\dagger_{bt}V_{tb}+V^\dagger_{bt}\widetilde{V}_{tb}\right)\frac{r}{4}\left[\frac{r\log r}{(r-1)^2}-\frac{1}{r-1}\right]\bar{b}_L\gamma^\mu b_L \qquad \text{(A.24)}$$

which means that the function $I_c(r)$, here equal to $F_c(r)$, is

$$I_c = \left(\widetilde{V}^\dagger_{bt}V_{tb}+V^\dagger_{bt}\widetilde{V}_{tb}\right)\frac{r}{4}\left[\frac{r\log r}{(r-1)^2}-\frac{1}{r-1}\right]. \qquad \text{(A.25)}$$

Notice that we have included a $\frac{1}{2}$ factor for each external leg correction coming from the $\sqrt{Z}=\sqrt{1+\delta Z}=1+\frac{1}{2}\delta Z$ factors appearing in the formula for S-matrix elements.

Computing diagram d

Next we find the expression for diagram (d)

$$\begin{aligned}
K_d &= \bar{b}_L\frac{ie}{\sqrt{2}s_WM_W}\widetilde{V}^\dagger_{bt'}m_tP_R\frac{i(\not{k}+m_t)}{k^2-m_t^2}i\,\gamma^\mu\left(\frac{g}{c_W}\right)(g_L^{tt'}P_L+g_R^{tt'}P_R)\frac{i(\not{k}+m_{t'})}{k^2-m_{t'}^2} \\
&\quad\times\frac{-ie}{\sqrt{2}s_WM_W}\widetilde{V}_{tb}(-m_{t'}P_L)\frac{i}{k^2-m_W^2}b_L \\
&= -\sqrt{rr'}\left(\frac{e}{\sqrt{2}s_W}\right)^2\left(\frac{g}{c_W}\right)\widetilde{V}^\dagger_{bt'}\widetilde{V}_{tb}\frac{\bar{b}_LP_R(\not{k}+m_t)\gamma^\mu(g_L^{tt'}P_L+g_R^{tt'}P_R)(\not{k}+m_{t'})P_Lb_L}{(k^2-m_t^2)(k^2-m_{t'}^2)(k^2-m_W^2)} \\
&= -\sqrt{rr'}\left(\frac{e}{\sqrt{2}s_W}\right)^2\left(\frac{g}{c_W}\right)\widetilde{V}^\dagger_{bt'}\widetilde{V}_{tb}\frac{\bar{b}_LP_R(\not{k}+m_t)\gamma^\mu(g_L^{tt'}P_L+g_R^{tt'}P_R)(\not{k}+m_{t'})P_Lb_L}{(k^2-m_t^2)(k^2-m_{t'}^2)(k^2-m_W^2)} \\
&= -2\sqrt{rr'}\left(\frac{e}{\sqrt{2}s_W}\right)^2\left(\frac{g}{c_W}\right)\widetilde{V}^\dagger_{bt'}\widetilde{V}_{tb}\int_0^1 dx\int_0^{1-x}dy\frac{\left[g_L^{tt'}m_tm_t'-\frac{d-2}{d}k^2g_R^{tt'}\right]\bar{b}_L\gamma^\mu b_L}{[k^2-m_t^2(1-x-y)-m_{t'}^2y-m_W^2x]^3}
\end{aligned}$$

(A.26)

where we have made use of

$$P_R(\not{k}+m_t)\gamma^\mu(g_L^{tt'}P_L+g_R^{tt'}P_R)(\not{k}+m_{t'})P_L=\big[g_L m_t m_t' - \frac{d-2}{d}k^2 g_R\big]\gamma^\mu P_L \tag{A.27}$$

The result (A.26) gives upon integration over the momenta

$$\begin{aligned}J_d &= -2\sqrt{rr'}\left(\frac{e}{\sqrt{2}s_W}\right)^2\left(\frac{g}{c_W}\right)\widetilde{V}_{bt'}^\dagger\widetilde{V}_{tb}\int_0^1 dx\int_0^{1-x}dy\int\frac{d^dk}{(2\pi)^d}\frac{\big[g_L^{tt'}m_t m_t' - \frac{d-2}{d}k^2 g_R^{tt'}\big]\bar{b}_L\gamma^\mu b_L}{[k^2-m_t^2(1-x-y)-m_{t'}^2 y-m_W^2 x]^3}\\
&= i\frac{\alpha}{2\pi}\left(\frac{g}{c_W}\right)\widetilde{V}_{bt'}^\dagger\widetilde{V}_{tb}\frac{\sqrt{rr'}}{4s_W^2}\Bigg[g_L^{tt'}\frac{\sqrt{rr'}}{r'-r}\left(\frac{r'\log r'}{r'-1}-\frac{r\log r}{r-1}\right)\\
&- g_R^{tt'}\frac{1}{2}\left(\mathbb{D}+1+\frac{1}{r'-r}\left(\frac{r'^2\log r'}{r'-1}-\frac{r^2\log r}{r-1}\right)\right)\Bigg]\bar{b}_L\gamma^\mu b_L\end{aligned} \tag{A.28}$$

and consequently

$$\begin{aligned}I_d(r,r') &= \widetilde{V}_{bt'}^\dagger\widetilde{V}_{tb}\frac{\sqrt{rr'}}{4s_W^2}\Bigg[g_L^{tt'}\frac{\sqrt{rr'}}{r'-r}\left(\frac{r'\log r'}{r'-1}-\frac{r\log r}{r-1}\right)\\
&\qquad -g_R^{tt'}\frac{1}{2}\left(\mathbb{D}+1+\frac{1}{r'-r}\left(\frac{r'^2\log r'}{r'-1}-\frac{r^2\log r}{r-1}\right)\right)\Bigg].\end{aligned} \tag{A.29}$$

In the limit $r'\to r$ and $t'=t$ the above expression reduces to

$$I_d(r)=-|\widetilde{V}_{tb}|^2\frac{r}{4s_W^2}\left[g_L^t\left(\frac{r\log r}{(r-1)^2}-\frac{r}{r-1}\right)+g_R^t\frac{1}{2}\left(\mathbb{D}+\frac{2r-1}{r-1}+\frac{r(r-2)\log r}{(r-1)^2}\right)\right]\bar{b}_L\gamma^\mu b_L. \tag{A.30}$$

which is equal to $F_d(r)$.

Computing diagram e

Diagram (e) is computed by integrating over momenta of the following expression

$$\begin{aligned}
K_e &= \bar{b}_L \frac{ie}{\sqrt{2}s_W m_W}\widetilde{V}^\dagger_{bt} m_t P_R \frac{i(\not{k}+m_t)}{k^2-m_t^2} \\
&\quad \times \frac{-ie}{\sqrt{2}s_W M_W}\widetilde{V}_{tb}(-m_t P_L)\frac{-1}{(k^2-m_W^2)^2} 2iek_\mu \cot 2\theta_W\, b_L \\
&= \left(\frac{iem_t}{\sqrt{2}s_W m_W}\right)^2 |\widetilde{V}_{tb}|^2\, \bar{b}_L \frac{i(\not{k}+m_t)}{k^2-m_t^2}\, b_L \frac{-1}{(k^2-m_W^2)^2} 2iek_\mu \cot 2\theta_W \\
&= -\left(\frac{em_t}{\sqrt{2}s_W m_W}\right)^2 |\widetilde{V}_{tb}|^2 \frac{k_\alpha k_\mu}{(k^2-m_W^2)^2(k^2-m_t^2)}\, 2e\cot 2\theta_W\, \bar{b}_L\, \gamma_\alpha\, b_L \\
&= -\left(\frac{em_t}{\sqrt{2}s_W m_W}\right)^2 |\widetilde{V}_{tb}|^2 \int_0^1 dx\, \frac{2k_\alpha k_\mu (1-x)}{[k^2 - xm_t^2 - (1-x)m_W^2]^3}\, 2e\cot 2\theta_W\, \bar{b}_L\, \gamma_\alpha\, b_L
\end{aligned} \tag{A.31}$$

which gives

$$\begin{aligned}
J_e &= -\left(\frac{em_t}{\sqrt{2}s_W m_W}\right)^2 |\widetilde{V}_{tb}|^2 \int_0^1 dx \int \frac{d^d k}{(2\pi)^d} \frac{2k_\alpha k_\mu (1-x)}{[k^2 - xm_t^2 - (1-x)m_W^2]^3}\, 2e\cot 2\theta_W\, \bar{b}_L\, \gamma^\alpha\, b_L \\
&= -\left(\frac{em_t}{\sqrt{2}s_W m_W}\right)^2 |\widetilde{V}_{tb}|^2 \frac{1}{d}\int_0^1 dx \int \frac{d^d k}{(2\pi)^d} \frac{2k^2 (1-x)}{[k^2 - xm_t^2 - (1-x)m_W^2]^3}\, 2e\cot 2\theta_W\, \bar{b}_L\, \gamma_\mu\, b_L \\
&= \left(\frac{em_t}{\sqrt{2}s_W m_W}\right)^2 |\widetilde{V}_{tb}|^2 (\, 2e\cot 2\theta_W\,) \frac{i}{4(4\pi)^2}\left(\mathbb{D} + \frac{r^2 \log r}{(r-1)^2} - \frac{1}{r-1} + \mathcal{O}(\epsilon)\right)\bar{b}_L \gamma_\mu b_L \\
&= i\frac{\alpha}{2\pi}\left(\frac{g}{c_W}\right)|\widetilde{V}_{tb}|^2 \frac{\cos 2\theta_W\, r}{16 s_W^2}\left(\mathbb{D} + \frac{r^2 \log r}{(r-1)^2} - \frac{1}{r-1} + \mathcal{O}(\epsilon)\right)\bar{b}_L\, \gamma_\mu\, b_L \\
&= i\frac{\alpha}{2\pi}\left(\frac{g}{c_W}\right)|\widetilde{V}_{tb}|^2 \frac{1}{8}\left(\frac{1}{2s_W^2} - 1\right) r\left(\mathbb{D} + \frac{r^2 \log r}{(r-1)^2} - \frac{1}{r-1} + \mathcal{O}(\epsilon)\right)\bar{b}_L\, \gamma_\mu\, b_L,
\end{aligned} \tag{A.32}$$

and consequently $I_e(r) = F_e(r)$ is given by

$$I_e(r) = |\widetilde{V}_{tb}|^2 \frac{1}{8}\left(\frac{1}{2s_W^2} - 1\right) r\left(\mathbb{D} + \frac{r^2 \log r}{(r-1)^2} - \frac{1}{r-1} + \mathcal{O}(\epsilon)\right)\bar{b}_L\, \gamma_\mu\, b_L. \tag{A.33}$$

Computing diagram f

Diagram f is computed in the following way

$$\begin{aligned} K_f =& \left(\frac{ie}{\sqrt{2}s_W}\gamma^\alpha P_L\right) V_{bt}^\dagger \frac{i(\not k + m_t)}{k^2 - m_t^2} i\gamma^\mu \left(\frac{g}{c_W}\right)(g_L^{tt'} P_L + g_R^{tt'} P_R)\frac{i(\not k + m_{t'})}{k^2 - m_{t'}^2} \\ &\times \left(\frac{ie}{\sqrt{2}s_W}\gamma_\alpha P_L\right) V_{t'b} \frac{-i}{k^2 - m_W^2} \\ =& V_{bt}^\dagger V_{t'b} \left(\frac{e}{\sqrt{2}s_W}\right)^2 \left(\frac{g}{c_W}\right) \frac{-(d-2)\left[g_R^{tt'}\, m_t\, m_{t'} - \frac{d-2}{d}\, k^2\, g_L^{tt'}\right]\bar{b}_L \gamma^\mu\, b_L}{(k^2 - m_t^2)(k^2 - m_{t'}^2)(k^2 - m_W^2)} \\ =& 2V_{bt}^\dagger V_{t'b} \left(\frac{e}{\sqrt{2}s_W}\right)^2 \left(\frac{g}{c_W}\right) \int_0^1 dx \int_0^{1-x} dy \frac{-(d-2)\left[g_R^{tt'} m_t m_{t'} - \frac{d-2}{d} k^2 g_L^{tt'}\right]\bar{b}_L \gamma^\mu\, b_L}{[k^2 - m_t^2(1-x-y) - m_{t'}^2 y - m_W^2 x]^3} \end{aligned} \tag{A.34}$$

in which the identity

$$\gamma^\alpha P_L(\not k + m_t)\gamma^\mu (g_L^{tt'} P_L + g_R^{tt'} P_R)(\not k + m_t')\gamma_\alpha P_L = -(d-2)\left[g_R^{tt'} m_t m_{t'} - \frac{d-2}{d} k^2 g_L^{tt'}\right]\gamma^\mu P_L \tag{A.35}$$

has been used. This gives, after integrating over the momenta

$$\begin{aligned} J_f =& 2V_{bt}^\dagger V_{t'b} \left(\frac{e}{\sqrt{2}s_W}\right)^2 \left(\frac{g}{c_W}\right) \int_0^1 dx \int_0^{1-x} dy \int \frac{d^d k}{(2\pi)^d} \frac{-(d-2)\left[g_R^{tt'} m_t m_{t'} - \frac{d-2}{d} k^2 g_L^{tt'}\right]\bar{b}_L \gamma^\mu\, b_L}{[k^2 - m_t^2(1-x-y) - m_{t'}^2 y - m_W^2 x]^3} \\ =& i\left(\frac{\alpha}{2\pi}\right)\left(\frac{g}{c_W}\right)\frac{V_{bt}^\dagger V_{t'b}}{2s_W^2}\left[g_R^{tt'} \frac{\sqrt{rr'}}{r'-r}\left(\frac{r'\log r'}{r'-1} - \frac{r\log r}{r-1}\right)\right. \\ &\left. - g_L^{tt'}\frac{1}{2}\left(\mathbb{D} + 1 + \frac{1}{r'-r}\left(\frac{r'^2\log r'}{r'-1} - \frac{r^2 \log r}{r-1}\right)\right)\right] \end{aligned} \tag{A.36}$$

which leads to the following expression for the function $F_f(r,r') = I_f(r,r') - I_f(0,0)$, now depending on r and r'

$$F_f(r,r') = \frac{V_{bt}^\dagger V_{t'b}}{2s_W^2}\left[g_R^{tt'}\frac{\sqrt{rr'}}{r'-r}\left(\frac{r'\log r'}{r'-1} - \frac{r\log r}{r-1}\right) - g_L^{tt'}\frac{1}{2}\left(\frac{1}{r'-r}\left(\frac{r'^2\log r'}{r'-1} - \frac{r^2\log r}{r-1}\right)\right)\right]. \tag{A.37}$$

In the limit $r' \to r$ and $t' = t$ this becomes

$$F_f(r) = -\frac{|V_{tb}|^2}{2s_W^2}\left[g_R^t\left(\frac{r\log r}{(r-1)^2} - \frac{r}{r-1}\right) + \frac{g_L^t}{2}\left(\frac{r(r-2)\log r}{(r-1)^2} + \frac{r}{r-1}\right)\right] \tag{A.38}$$

Computing diagram g

Finally we compute diagram (g) starting from the integrand K_g

$$\begin{aligned}
K_g &= \bar{b}_L\left(\frac{ie\gamma_\lambda}{\sqrt{2}s_W}\right)V_{bt}^\dagger \frac{i(\not{k}+m_t)}{k^2-m_t^2}\left(\frac{ie\gamma_\nu}{\sqrt{2}s_W}\right)V_{tb}\left[igc_W\left(k^\lambda\eta^{\mu\nu}-2k^\mu\eta^{\nu\lambda}+k^\nu\eta^{\lambda\mu}\right)\right]\frac{-1}{(k^2-m_W^2)^2}b_L \\
&\to -\left(\frac{e}{\sqrt{2}s_W}\right)^2|V_{tb}|^2\frac{gc_W}{(k^2-m_W^2)^2(k^2-m_t^2)}\bar{b}_L\left(\not{k}\not{k}\gamma^\mu - 2k^\mu\gamma^\nu\not{k}\gamma_\nu + \gamma^\mu\not{k}\not{k}\right)b_L \\
&= -\left(\frac{e}{\sqrt{2}s_W}\right)^2|V_{tb}|^2\frac{gc_W}{(k^2-m_W^2)^2(k^2-m_t^2)}\bar{b}_L\left(2k^2\gamma^\mu + 2(d-2)k^\mu\not{k}\right)b_L \\
&\to -\left(\frac{e}{\sqrt{2}s_W}\right)^2|V_{tb}|^2\frac{d-1}{d}\frac{4k^2gc_W}{(k^2-m_W^2)^2(k^2-m_t^2)}\bar{b}_L\gamma^\mu b_L \\
&= -\left(\frac{e}{\sqrt{2}s_W}\right)^2|V_{tb}|^2\frac{d-1}{d}\int_0^1 dx\frac{8(1-x)k^2gc_W}{[k^2-m_W^2(1-x)-m_t^2x]^3}\bar{b}_L\gamma^\mu b_L
\end{aligned} \tag{A.39}$$

and integrating over the momenta

$$\begin{aligned}
J_g &= -\left(\frac{e}{\sqrt{2}s_W}\right)^2|V_{tb}|^2\frac{d-1}{d}\int_0^1 dx\int\frac{d^dk}{(2\pi)^d}\frac{8(1-x)k^2gc_W}{[k^2-m_W^2(1-x)-m_t^2x]^3}\bar{b}_L\gamma^\mu b_L \\
&= \frac{3igc_W}{(4\pi)^2}\left(\frac{e}{\sqrt{2}s_W}\right)^2|V_{tb}|^2\left[\frac{r^2\log r}{(r-1)^2}-\frac{1}{r-1}+\mathbb{D}+\frac{2}{3}\right]\bar{b}_L\gamma^\mu b_L \\
&= i\left(\frac{\alpha}{2\pi}\right)\left(\frac{g}{c_W}\right)|V_{tb}|^2\frac{3}{4}\frac{c_W^2}{s_W^2}\left[\frac{r^2\log r}{(r-1)^2}-\frac{1}{r-1}+\mathbb{D}+\frac{2}{3}\right]\bar{b}_L\gamma^\mu b_L.
\end{aligned} \tag{A.40}$$

This gives the function $I_g(r)$

$$I_g(r) = |V_{tb}|^2\frac{3}{4}\frac{c_W^2}{s_W^2}\left[\frac{r^2\log r}{(r-1)^2}-\frac{1}{r-1}+\mathbb{D}+\frac{2}{3}\right], \tag{A.41}$$

which leads to the following expression for the function $F_g(r)$

$$F_g(r) = I_g(r) - I_g(0) = |V_{tb}|^2\frac{3}{4}\frac{c_W^2}{s_W^2}\left[\frac{r^2\log r}{(r-1)^2}-\frac{r}{r-1}\right]. \tag{A.42}$$

Final result

We define the tilded quantities $\tilde{F}_k(r_i)$ to be equal to $F_k(r_i)$ with the matrices V and $\widetilde{V}$ set to identity. For the case of the SM, the full result will be the sum of all diagrams computed in the previous section, with $V_{tb} = \widetilde{V}_{tb} = 1$, which gives the finite expression

$$F_{SM} = \sum_{i=a_1}^{g}\tilde{F}_i(r) = \frac{r\left(r^2-7r+6+(2+3r)\log r\right)}{8s_W^2(r-1)^2}. \tag{A.43}$$

For the general case the full result, including the exchange of one 2/3 charge fermion resonance, can be written as

$$\begin{aligned} F &= \sum_i \sum_{k\neq d,f} F_k(r_i) + \sum_{i,j} \sum_{k=d,f} F_k(g^{ij}_{L,R}, r_i, r_j) \\ &= \sum_i \sum_{k=a_1}^{g} F_k(r_i) + \sum_{i,j} \sum_{k=d,f} F_k(\delta g^{ij}_{L,R}, r_i, r_j), \end{aligned} \tag{A.44}$$

where in the second equation we have added $\sum_{k=d,f} F_k(r_i)$ (in which $g^i_{L,R}$ have been set to their SM values $g^t_{L,R}$) to the first term and subtracted it from the second term. $\delta g^{ij}_{L,R}$ is thus equal to $g^{ij}_{L,R} - g^t_{L,R}\,\delta^{ij}$. This can be further written as

$$\begin{aligned} F &= \sum_i |V_{ib}|^2 (\tilde{F}_b(r_i) + \tilde{F}_f(r_i) + \tilde{F}_g(r_i)) + \sum_i \widetilde{V}^\dagger_{bi} V_{ib}\, \tilde{F}_{c1}(r_i) + \sum_i V^\dagger_{bi} \widetilde{V}_{ib}\, \tilde{F}_{c2}(r_i) \\ &+ \sum_i |\widetilde{V}_{ib}|^2 (\tilde{F}_a(r_i) + \tilde{F}_d(r_i) + \tilde{F}_e(r_i)) + \sum_{i,j} \widetilde{V}^\dagger_{bi} \widetilde{V}_{jb}\, \tilde{F}(\delta g^{ij}_{L,R}, r_i, r_j), \end{aligned} \tag{A.45}$$

where in the last equation we have defined

$$\tilde{F}(\delta g^{ij}_{L,R}, r_i, r_j) \equiv \sum_{k=d,f} \tilde{F}_k(\delta g^{ij}_{L,R}, r_i, r_j). \tag{A.46}$$

The total $Z\bar{b}b$ coupling and the new physics contribution are given by

$$g_b = \frac{\alpha}{2\pi}\left(\frac{g}{c_W}\right) F, \qquad \delta g_b = g_b - g_{SM} = \frac{\alpha}{2\pi}\left(\frac{g}{c_W}\right) F_{NP}, \qquad F_{NP} = F - F_{SM} \tag{A.47}$$

It might be worth mentioning that for the special case where $\widetilde{V}_{ib} = V_{ib}$, the function (A.45) will simplify to

$$F = \sum_i |V_{ib}|^2 F_{SM}(r_i) + \sum_{i,j} V^\dagger_{bi} V_{jb}\, \tilde{F}(\delta g^{ij}_{L,R}, r_i, r_j) \tag{A.48}$$

so that

$$F_{NP} = F - F_{SM} = \sum_i |V_{ib}|^2 \left(F_{SM}(r_i) - F_{SM}(r)\right) + \sum_{i,j} V^\dagger_{bj} V_{ib}\, \tilde{F}(\delta g^{ji}_{L,R}, r_i, r_j) \tag{A.49}$$

where we have used the fact that $\sum_i |V_{ib}|^2 = 1$. Note that according to the discussions at the end of section A.1, in the models presented in this thesis, the only situation where the relation $\widetilde{V}_{ib} = V_{ib}$ is spoiled is in model III where there is a vector-like triplet $\mathbf{3}_{2/3}$.

In the next section we will prove that the function F in (A.45) is finite.

A.3 Proof of finiteness of F

In this subsection we aim to argue that new physics contributions to the $Z\bar{b}b$ coupling at 1-loop is finite as far as we neglect the mixing in the bottom sector, which is what we actually did in this work. To do this, we adopt the same notation mentioned at the beginning of this chapter and choose a basis $T_{L,R}^T = (t_{L,R}^1, t_{L,R}^2, \cdots)$ for the particles in the top sector and a basis $B_L^T = (b_L^1, b_L^2, \cdots)$ for the particles in the bottom sector. The divergent piece $\mathbb{D}$ appears only in the diagrams $(a1)$, $(a2)$, (d) and (e). This means that we only need to concentrate on the following terms of the expression (A.45)

$$\sum_i |\widetilde{V}_{ib}|^2 (\tilde{F}_a(r_i) + \tilde{F}_d(r_i) + \tilde{F}_e(r_i)) + \sum_{i,j} \widetilde{V}_{bi}^\dagger \widetilde{V}_{jb}\, \tilde{F}_d(\delta g_{L,R}^{ij}, r_i, r_j), \tag{A.50}$$

but since we have used the SM coupling g_R^t in $\tilde{F}_d(r_i)$, the divergences in the sum $\tilde{F}_a(r_i) + \tilde{F}_d(r_i) + \tilde{F}_e(r_i)$ cancel out for the same reason that they cancelled out in F_{SM}. So we just need to show that the coefficients of $\mathbb{D}$ in

$$\sum_{i,j} \widetilde{V}_{bi}^\dagger \widetilde{V}_{jb}\, \tilde{F}_d(\delta g_{L,R}^{ij}, r_i, r_j) \tag{A.51}$$

sum up to zero. For this purpose we can write the deviations, from the coupling of the SM RH top quark, of the matrix of Z couplings of the RH top sector as

$$\delta g_R^{ji} = U_R^t \begin{pmatrix} g_R^1 & & \\ & g_R^2 & \\ & & \ddots \end{pmatrix} U_R^{t\dagger} - g_R^t \mathbb{I} = U_R^t \begin{pmatrix} T_{3L}^1 & & \\ & T_{3L}^2 & \\ & & \ddots \end{pmatrix} U_R^{t\dagger}, \tag{A.52}$$

where $g_R^i = T_{3L}^i - \frac{2}{3} s_W^2$ and $g_R^t = -\frac{2}{3} s_W^2$. Using the definitions (A.3) with no mixing in the bottom sector $U_{L,R}^b = \mathbf{1}$, the coefficient of $\mathbb{D}$ in expression A.51 is proportional to

$$\begin{aligned} \sum_{i,j} m_{t^i} m_{t^j}\, \widetilde{V}_{bj}^\dagger \widetilde{V}_{ib}\, \delta g_R^{ji} &= \sum_{i,j} [M_\phi^{L\dagger} U_R^t]_{bj} [U_R^{t\dagger} M_\phi^L]_{ib}\, \delta g_R^{ji} = [M_\phi^{L\dagger} U_R^t\, \delta g_R^{ji}\, U_R^{t\dagger} M_\phi^L]_{bb} \\ &= \sum_i [M_\phi^{L\dagger}]_{bi}\, T_{3L}^i\, [M_\phi^L]_{ib} = 0. \end{aligned} \tag{A.53}$$

which vanishes. This is because either $T_{3L}^i = 0$ or there could be no interaction between b_L and t_R^i through ϕ, and so $[M_\phi^L]_{ib} = 0$.

Appendix B

Electroweak Precision Tests

B.1 Introduction

In this section we briefly review general aspects of electroweak radiative corrections and their effect on precision electroweak measurements. We are interested in the corrections which are independent of the fermion species, that is, the corrections to the gauge propagators, usually called oblique. These corrections are encoded in the vacuum polarization amplitudes

$$i\,\Pi_{XY}(q^2)\,\eta^{\mu\nu} + q^\mu q^\nu\,\bar{\Pi}_{XY}(q^2) \equiv \int dx\, e^{-iq\cdot x}\langle J^\mu_X(x) J^\nu_Y(0)\rangle \tag{B.1}$$

where X, Y refer to the electroweak gauge bosons.

Being interested in low energy processes, out of the infinite terms appearing in a Taylor expansion of Π_{XY} in momentum squared, only the first two terms are relevant, the remaining terms are of order q^4/Λ^2, where Λ is the scale of new physics, and decouple at low energies as far as we consider 1-loop contributions. Mass dependent couplings due to Higgs exchange can occur at higher loops. We choose as a basis for Π_{XY}, the vacuum polarization amplitudes Π_{ij}, $(i,j) = (Q,Q), (3,Q), (3,3), (1,1)$, defined through

$$e^2\Pi_{QQ} = \Pi_{AA}, \quad ge\Pi_{3Q} = \Pi_{W^3A}, \quad g^2\Pi_{ab} = \Pi_{W^aW^b}, \quad a,b = 1,2,3. \tag{B.2}$$

Note that $\Pi_{11} = \Pi_{22}$, and all other combinations vanish by global $U(1)_{em}$ invariance. We write their Taylor expansions as

$$\begin{aligned}
\Pi_{QQ}(q^2) &= \Pi'_{QQ}(0)\,q^2 + \cdots \\
\Pi_{3Q}(q^2) &= \Pi'_{3Q}(0)\,q^2 + \cdots \\
\Pi_{33}(q^2) &= \Pi_{33}(0) + \Pi'_{33}(0)\,q^2 + \cdots \\
\Pi_{11}(q^2) &= \Pi_{11}(0) + \Pi'_{11}(0)\,q^2 + \cdots
\end{aligned} \tag{B.3}$$

where a prime denotes a derivative with respect to momentum squared and $\Pi_{QQ}(0) = \Pi_{3Q}(0) = 0$ because of $U(1)_{em}$ gauge invariance. So we are left with six parameters out of which three combinations are used to fix the three parameters of the theory, namely e^2, s^2 (or equivalently g, g') and v. To do this, three observables are needed. The natural choice is α, G_F and m_Z as the most accurately measured parameters.

So the effect of oblique corrections can be parametrized by α, G_F and m_Z and three other combinations of the six parameters in (B.3). For a renormalizable theory all the divergences are absorbed in g, g' and v, through the definition of α, G_F and m_Z. So the other three combinations of the the six parameters in (B.3) can be chosen to be finite. In fact, one natural choice is given by the S, T and U parameters [17] defined by

$$\begin{aligned} \alpha S &= 4e^2 \left(\Pi'_{33}(0) - \Pi'_{3Q}(0)\right) \\ \alpha T &= \frac{e^2}{s^2 c^2 m_Z^2} \left(\Pi_{11}(0) - \Pi_{33}(0)\right) \\ \alpha U &= 4e^2 \left(\Pi'_{11}(0) - \Pi'_{33}(0)\right) . \end{aligned} \tag{B.4}$$

So all the oblique corrections are expected to be functions of the six parameters α, G_F, m_Z, S, T and U. This is indeed the case.

In the next section we introduce the notion of custodial symmetry and also express the S and T parameters in terms of the coefficients of some higher dimensional operators which appear in the EW effective Lagrangian.

B.1.1 Operator Analysis

Lets extend the gauge symmetry of the SM to the global symmetry $SU(2)_L \times SU(2)_R$ by adding two spurious fields $W^{1,2}_{R\mu}$ which along with $W^3_{R\mu} \equiv B_\mu$ form a triplet of $SU(2)_R$. Also, by collecting the components of the Higgs field into the two by two matrix $\Omega = (H^c, H)$ as in eq.(3.61), its transformation can be generalized to a bidoublet of $SU(2)_L \times SU(2)_R$

$$\Omega \to L\,\Omega\,R^\dagger, \qquad L \in SU(2)_L,\ R \in SU(2)_R. \tag{B.5}$$

Now lets consider $SU(2)_L \times SU(2)_R$ invariant operators constructed out of the gauge fields $W^a_{L,R\mu}$ and the Higgs. Obviously any such operator is $SU(2)_L \times U(1)_Y$ invariant even when we put back the two extra gauge fields $W^{1,2}_{R\mu}$ to zero. But the converse is not true, not all $SU(2)_L \times U(1)_Y$ invariant operators are reproduced in this way. Whenever possible, we will also call such operators $SU(2)_L \times SU(2)_R$ invariant by abuse of language.

After EWSB the field Ω will be proportional to the identity matrix, $\Omega \to \langle h\rangle \mathbf{1}$, and so the symmetry group $SU(2)_L \times SU(2)_R$ will break to its diagonal subgroup $SU(2)_c$, usually called custodial symmetry [45], whose only gauged generator is $T^3_L + T^3_R$ which generates $U(1)_{em}$. So after EWSB the gauge sector of an $SU(2)_L \times SU(2)_R$ invariant Lagrangian, including $W^{1,2}_{R\mu}$, at quadratic level and in momentum space can be written as

$$\mathcal{L} = P_T^{\mu\nu}\left[\frac{1}{2}\Pi_{LL}\, W^a_{L\mu}W^a_{L\nu} + \frac{1}{2}\Pi_{RR}\, W^a_{R\mu}W^a_{R\nu} + \Pi_{LR}\, W^a_{L\mu}W^a_{R\nu}\right]. \tag{B.6}$$

This form of the Lagrangian is dictated by global $SU(2)_c$ invariance . When we set the fields $W^{1,2}_{R\mu}$ to zero, the Lagrangian (B.6) will reduce to

$$\mathcal{L} = P_T^{\mu\nu}\left[\frac{1}{2}\Pi_{LL}\, W^a_{L\mu}W^a_{L\nu} + \frac{1}{2}\Pi_{RR}\, W^3_{R\mu}W^3_{R\nu} + \Pi_{LR}\, W^3_{L\mu}W^3_{R\nu}\right]. \tag{B.7}$$

This Lagrangian, clearly, will give no contribution to the T parameter because of global $SU(2)_c$ symmetry which fixes the structure of the quadratic terms in $W^a_{L\mu}$ and leads, in particular, to $\Pi_{W^3W^3} = \Pi_{W^1W^1} = \Pi_{LL}$.

It turns out that operators constructed out of $W^a_{L,R\mu}$ and the Higgs, of dimension four or less, which include the gauge and Higgs kinetic terms and the Higgs potential, are $SU(2)_L\times SU(2)_R$ invariant. In other words custodial symmetry is an accidental symmetry of the SM. This is easily seen by noticing that the manifestly $SU(2)_L \times SU(2)_R$ invariant Lagrangian

$$\mathcal{L} = \frac{1}{2}\mathrm{Tr}\, D_\mu\Omega^\dagger D^\mu\Omega - V(\mathrm{Tr}(\Omega^\dagger\Omega)), \qquad D_\mu\Omega = \partial_\mu\Omega - i\,\vec{W}_L\!\cdot\!\vec{\tau}\,\Omega + i\,\Omega\,\vec{W}_R\!\cdot\!\vec{\tau} \tag{B.8}$$

with $\tau^i = \sigma^i/2$, reduces to the Higgs sector Lagrangian of the SM when $W^{1,2}_{R\mu} = 0$.

At dimension 5, there are no $SU(2)_L \times U(1)_Y$ invariant operators made of only gauge fields and the Higgs. In fact, including fermions, there is only one such operator, which is actually responsible for neutrino masses [46]. So the custodial breaking operators, which contribute to the T parameter, arise at dimension 6 or higher. These operators which must include W^a_L and not their derivatives, are of the form

$$(H^\dagger H)^n\, |HD_\mu H|^2. \tag{B.9}$$

After EWSB, the operator $HD_\mu H$ will reduce to

$$HD_\mu H \to \frac{v^2}{4}(gW^3_\mu - g'B_\mu) = \frac{v^2}{4}\,\frac{e}{sc}\,Z_\mu, \tag{B.10}$$

so (B.9) will contribute to the mass of the Z_μ boson.

In an $SU(2)_L \times U(1)_Y$ invariant theory, after EWSB, the general form of the Lagrangian at quadratic level and in momentum space is

$$\mathcal{L} = P_T^{\mu\nu}\left[\Pi_{WW}\, W^+W^- + \frac{1}{2}\Pi_{W^3W^3}\, W^3_\mu W^3_\nu + \frac{1}{2}\Pi_{BB}\, B_\mu B_\nu + \Pi_{W^3B}\, W^3_\mu B_\nu\right]. \tag{B.11}$$

Through the text and in the following we will also occasionally make use of the definitions

$$g'^2\Pi_{YY} = \Pi_{BB}, \qquad gg'\Pi_{3Y} = \Pi_{W^3B}. \tag{B.12}$$

If the operator $O_T = |HD_\mu H|^2$ appears with a coefficient a_T in the effective Lagrangian ,then from (B.10) one can easily find its contribution to the vacuum polarization amplitudes. In particular we have

$$\begin{aligned}\Pi_{33}(0) &= \Pi_{33}^{d\leq 4}(0) + \Pi_{33}^{d>4}(0) = \Pi_{33}^{d\leq 4}(0) + \frac{2}{g^2}\left(\frac{a_T v^4 g^2}{16}\right) \\ \Pi_{11}(0) &= \Pi_{11}^{d\leq 4}(0) + \Pi_{11}^{d>4}(0) = \Pi_{11}^{d\leq 4}(0)\end{aligned} \tag{B.13}$$

where $\Pi^{d\leq 4}$ and $\Pi^{d>4}$ represent the contributions to the vacuum polarization amplitudes from operators of dimension ≤ 4 and > 4 respectively. According to the discussion above $\Pi_{33}^{d\leq 4}(0) = \Pi_{11}^{d\leq 4}(0)$, so

$$\alpha T = \frac{e^2}{s^2c^2m_Z^2}\left(\Pi_{11}(0) - \Pi_{33}(0)\right) = \frac{4}{v^2}\left(\Pi_{11}(0) - \Pi_{33}(0)\right) = \frac{4}{v^2}\left(-\frac{a_T v^4}{8}\right) = -\frac{a_T v^2}{2} \tag{B.14}$$

which shows the relation between the coefficient of the operator $|HD_\mu H|^2$ and the T parameter.

The lowest order operator that contributes to the S parameter is $O_S = H^\dagger\sigma^a H W^a_{\mu\nu}B^{\mu\nu}$. After EWSB this operator reduces to $\frac{v^2}{2}W^3_{\mu\nu}B^{\mu\nu}$ which in momentum space becomes $-v^2q^2P_T^{\mu\nu}W^3_\mu B_\nu$. This means that the operator O_S appearing with a coefficient a_S in the effective Lagrangian will have the contribution to the vacuum polarization amplitudes

$$\Pi'_{3Y}(0) = \frac{1}{gg'}\left(-v^2 a_S\right), \tag{B.15}$$

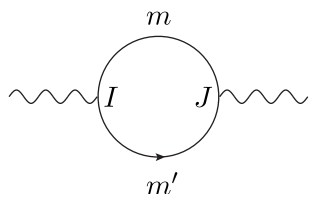
which in turn leads to the following contribution to the S parameter

$$\alpha S = -4e^2\Pi'_{3Y}(0) = -\frac{4e^2}{gg'}\left(-v^2 a_S\right) = 4s\,c\,v^2\,a_S. \tag{B.16}$$

B.2 One-loop fermion contributions to the electroweak S and T parameters

In this section we give some of the details regarding the computation of the S and T parameters in our models at 1-loop. The basic quantity entering in all expressions is the 1-loop fermion contribution to the gauge boson propagators (in which the couplings have been omitted) where two fermions with masses m and m' run in the loop

$$i\,\Pi_{IJ}(q^2, m, m') \equiv$$



with $I, J = L, R$ denoting the chirality of the two currents and q the momentum of the gauge bosons. Using this quantity and the definition (B.4), for a number of fermions with masses m_i, the T parameter is expressed in the following way

$$\begin{aligned} T &= \frac{4\pi N_c}{s_\theta^2 c_\theta^2 M_Z^2 g^2} \sum_{i,j} \left[\left((g_{L+}^{ij})^2 + (g_{R+}^{ij})^2 \right) \Pi_{LL}(m_i, m_j) + 2\, g_{L+}^{ij}\, g_{R+}^{ij}\, \Pi_{LR}(m_i, m_j) \right] \\ &- \frac{4\pi N_c}{s_\theta^2 c_\theta^2 M_Z^2 g^2} \sum_{i,j} \left[\left((g_{L3}^{ij})^2 + (g_{R3}^{ij})^2 \right) \Pi_{LL}(m_i, m_j) + 2\, g_{L3}^{ij}\, g_{R3}^{ij}\, \Pi_{LR}(m_i, m_j) \right] \end{aligned} \quad \text{(B.17)}$$

where N_c is the number of colors, and g_{L+}^{ij} and g_{R+}^{ij} are the $W^+\bar{q}_L^i q_L^j$ and $W^+\bar{q}_R^i q_R^j$ couplings respectively, while g_{L3}^{ij} and g_{R3}^{ij} are the $W^3\bar{q}_L^i q_L^j$ and $W^3\bar{q}_R^i q_R^j$ couplings. We have also defined $\Pi_{IJ}(m_i, m_j) \equiv \Pi_{IJ}(0, m_i, m_j)$. Also, by the definition (B.4), the S parameter can be expressed as

$$S = -\frac{16\pi N_c}{g g'} \sum_{i,j} \left[\left(g_{L3}^{ij}\, g_{LB}^{ij} + g_{R3}^{ij}\, g_{RB}^{ij} \right) \Pi'_{LL}(m_i, m_j) + \left(g_{L3}^{ij}\, g_{RB}^{ij} + g_{R3}^{ij}\, g_{LB}^{ij} \right) \Pi'_{LR}(m_i, m_j) \right] \quad \text{(B.18)}$$

where similarly g_{LB}^{ij} and g_{RB}^{ij} are the $B\bar{q}_L^i q_L^j$ and $B\bar{q}_R^i q_R^j$ couplings respectively, and we have also defined $\Pi'_{IJ}(m_i, m_j) \equiv \frac{d}{dq^2} \Pi_{IJ}(q^2, m_i, m_j)|_{q^2=0}$.

Making use of the definition

$$K \equiv \frac{2}{\epsilon} - \gamma + \log \frac{4\pi}{M^2}, \quad \text{(B.19)}$$

explicit expressions for this 1-loop self energies and their derivatives at zero momentum are given in the following

$$\Pi_{LL}(m,m') = \frac{(1+2K)(m^2+m'^2)}{32\pi^2} - \frac{m^4\log\frac{m^2}{M^2} - m'^4\log\frac{m'^2}{M^2}}{16\pi^2(m^2-m'^2)} \tag{B.20}$$

$$\Pi'_{LL}(m,m') = -\frac{(1+3K)(m^2-m'^2)^2 - 6m^2m'^2}{72\pi^2(m^2-m'^2)} + \frac{3m^4(m^2-3m'^2)\log\frac{m^2}{M^2} - 3m'^4(m'^2-3m^2)\log\frac{m'^2}{M^2}}{72\,\pi^2(m^2-m'^2)^3} \tag{B.21}$$

$$\Pi_{LR}(m,m') = -\frac{m\,m'(1+K)}{8\pi^2} + \frac{m^2\log\frac{m^2}{M^2} - m'^2\log\frac{m'^2}{M^2}}{8\pi^2(m^2-m'^2)} \tag{B.22}$$

$$\Pi'_{LR}(m,m') = -\frac{m\,m'\left(m^4 - m'^4 - 2\,m^2\,m'^2\,\log\frac{m^2}{m'^2}\right)}{16\,\pi^2\,(m^2-m'^2)^3}. \tag{B.23}$$

Note that $\Pi_{RR} = \Pi_{LL}$ and $\Pi_{RL} = \Pi_{LR}$. In the limit $m' \to m$ these amplitudes reduce to

$$\begin{aligned} \Pi_{LL}(m) &= \frac{m^2\left(K - \log\frac{m^2}{M^2}\right)}{8\pi^2}, & \Pi_{LR}(m) &= -\frac{m^2\left(K - \log\frac{m^2}{M^2}\right)}{8\pi^2} \\ \Pi'_{LL}(m) &= \frac{1-2K+2\log\frac{m^2}{M^2}}{48\pi^2}, & \Pi'_{LR}(m) &= -\frac{1}{48\pi^2} \end{aligned} \tag{B.24}$$

For several beyond SM fermions running in the loops the analytic expressions for S and T parameters are quite involved. In the following we find analytic expressions for one-loop fermion contribution to T and S, along with δg_b (which for coherence we have included in this appendix) in particular limits where relatively simple analytic expressions are available. This is motivated by the fact that often in our models one or two fermion states are significantly lighter than the others and dominate the loop corrections. The SM quantum numbers of these light fermion states vary along the parameter space and thus it can be useful to list the single fermion contribution to T, S and δg_b. The contribution of the first two light quark generations, including their KK towers, given their light masses and small Yukawa couplings with the KK modes, is expected to be negligible. We have actually checked that even the fermion mixing in the bottom sector is negligible, so that only the charge $+2/3$ states mixing with the top quark should be considered. In what follows we define T_{NP}, S_{NP} and $\delta g_{b,NP}$, the fermion one-loop contributions given by new physics only, with the SM contribution subtracted, by

$$T_{NP} = T - T_{SM}\,, \quad S_{NP} = S - S_{SM}, \quad \delta g_{b,NP} = \delta g_b - \delta g_{b,SM}\,, \tag{B.25}$$

where

$$\begin{aligned} g_{b,SM} &= -\frac{1}{2}+\frac{1}{3}s_W^2\,, \quad T_{SM}\simeq\frac{N_c r}{16\pi s_W^2}\,, \qquad S_{SM}=\frac{N_c}{18\pi}\left(3+\log\left(\frac{M_b^2}{M_t^2}\right)\right)\,, \\ \delta g_{b,SM} &= \frac{\alpha_{em}}{16\pi s_W^2}\frac{r(r^2-7r+6+(2+3r)\log r)}{(r-1)^2}\,, \quad r\equiv\frac{M_t^2}{M_W^2}\,, \end{aligned} \tag{B.26}$$

$s_W \equiv \sin\theta_W$, and M_t is the pole top mass, $M_t = 173.1$ GeV [47].

We do not exploit the full $SO(5)$ symmetry underlying our model and classify the new fermion states by their SM quantum numbers. In this way, the explicit $SO(5)$ symmetry breaking effects due to the UV b.c., that can be sizable, are taken into account and more reliable expressions are obtained. For simplicity, we take in the following all Yukawa couplings to be real, the extension to complex ones being straightforward.

B.2.1 Singlet with $Y = 2/3$

The simplest situation arises when the top quark mixes with just one SM singlet vector-like fermion X with hypercharge $Y = 2/3$. The two possible Yukawa couplings are

$$\mathcal{L} \supset y_t\bar{q}_L H^c t_R + y_X\bar{q}_L H^c X_R + h.c. \to \lambda_t\bar{t}_L t_R + \lambda_X\bar{t}_L X_R + h.c.\,, \tag{B.27}$$

where here and in the following we use the notation that $\lambda_i = y_i v/\sqrt{2}$ is the mass parameter corresponding to the Yukawa coupling y_i. The λ_i are assumed to be small with respect to the vector-like mass M_X of the new exotic fermions. By using standard techniques and keeping the leading order terms in the λ_i/M_X expansion, we get

$$T_{NP} = \frac{N_c\lambda_X^2\Big(2\lambda_t^2\log\big(\frac{M_X^2}{\lambda_t^2}\big)+\lambda_X^2-2\lambda_t^2\Big)}{16\pi s_W^2 M_W^2 M_X^2}\,, \tag{B.28}$$

$$S_{NP} = \frac{N_c\lambda_X^2\Big(2\log\big(\frac{M_X^2}{\lambda_t^2}\big)-5\Big)}{18\pi M_X^2}\,, \tag{B.29}$$

$$\delta g_{b,NP} = \frac{\alpha_{em}\lambda_X^2\Big(2\lambda_t^2\log\big(\frac{M_X^2}{\lambda_t^2}\big)+\lambda_X^2-2\lambda_t^2\Big)}{16\pi s_W^2 M_W^2 M_X^2}\,, \tag{B.30}$$

in agreement with [48, 49]. For simplicity, in eq.(B.30) we have only reported the leading order terms in the limit $\lambda_i/M_W \gg 1$. The top mass is given by

$$M_t \simeq \lambda_t\Big(1-\frac{\lambda_X^2}{2M_X^2}\Big)\,. \tag{B.31}$$

As can be seen from eqs.(B.28)-(B.30), for a sufficiently large M_X, T_{NP} and $\delta g_{b,NP}$ are closely related and positive (like S_{NP}).

B.2.2 Doublet with $Y = 1/6$

The two possible Yukawa couplings mixing the top with a new doublet Q_1 with hypercharge $Y = 1/6$ are

$$\mathcal{L} \supset y_t \bar{q}_L H^c t_R + y_1 \bar{Q}_{1L} H^c t_R + h.c. \to \lambda_t \bar{t}_L t_R + \lambda_1 \bar{Q}_{1uL} t_R + h.c.\,. \tag{B.32}$$

We find

$$T_{NP} = \frac{N_c \lambda_1^2 \Big(6\lambda_t^2 \log\big(\frac{M_1^2}{\lambda_t^2}\big) + 2\lambda_1^2 - 9\lambda_t^2\Big)}{24\pi s_W^2 M_W^2 M_1^2}, \tag{B.33}$$

$$S_{NP} = \frac{N_c \lambda_1^2 \Big(4\log\big(\frac{M_1^2}{\lambda_t^2}\big) - 7\Big)}{18\pi M_1^2}, \tag{B.34}$$

$$\delta g_{b,NP} = \frac{\alpha_{em} \lambda_1^2 \lambda_t^2 \log\big(\frac{M_1^2}{\lambda_t^2}\big)}{32\pi s_W^2 M_W^2 M_1^2}, \tag{B.35}$$

in agreement with [48]. The top mass is given by

$$M_t \simeq \lambda_t \Big(1 - \frac{\lambda_1^2}{2M_1^2}\Big)\,. \tag{B.36}$$

As can be seen from eqs.(B.33)-(B.35), for a sufficiently large M_1, T_{NP}, $\delta g_{b,NP}$ and S_{NP} are all positive.

B.2.3 Doublet with $Y = 7/6$

The two possible Yukawa couplings mixing the top with a new doublet Q_7 with hypercharge $Y = 7/6$ are

$$\mathcal{L} \supset y_t \bar{q}_L H^c t_R + y_7 \bar{Q}_{7L} H t_R + h.c. \to \lambda_t \bar{t}_L t_R + \lambda_7 \bar{Q}_{7dL} t_R + h.c.\,. \tag{B.37}$$

We find

$$T_{NP} = -\frac{N_c \lambda_7^2 \Big(6\lambda_t^2 \log\big(\frac{M_7^2}{\lambda_t^2}\big) - 2\lambda_7^2 - 9\lambda_t^2\Big)}{24\pi s_W^2 M_W^2 M_7^2}, \tag{B.38}$$

$$S_{NP} = -\frac{N_c \lambda_7^2 \Big(4\log\big(\frac{M_7^2}{\lambda_t^2}\big) - 15\Big)}{18\pi M_7^2}, \tag{B.39}$$

$$\delta g_{b,NP} = -\frac{\alpha_{em} \lambda_7^2 \lambda_t^2 \log\big(\frac{M_7^2}{\lambda_t^2}\big)}{32\pi s_W^2 M_W^2 M_7^2}, \tag{B.40}$$

in agreement with [48]. The top mass is given by

$$M_t \simeq \lambda_t\Big(1 - \frac{\lambda_7^2}{2M_7^2}\Big)\,. \tag{B.41}$$

As can be seen from eqs.(B.38)-(B.40), for a sufficiently large M_7, T_{NP}, $\delta g_{b,NP}$ and S_{NP} are all negative.

The contributions to T, S and δg_b of the doublets with $Y = 1/6$ and $Y = 7/6$ are almost the same in magnitude, but opposite in sign. When present together, then, there tends to be a partial cancellation among these two contributions. In the $SO(4)$ invariant limit in which $M_1 = M_7$ and $\lambda_1 = \lambda_7$, their contributions to δg_b precisely cancel.

B.2.4 Triplet with $Y = 2/3$

The two possible Yukawa couplings mixing the top with a new triplet T with hypercharge $Y = 2/3$ are

$$\mathcal{L} \supset y_t \bar{q}_L H^c t_R + \sqrt{2} y_T \bar{q}_L T_R H^c + h.c. \to \lambda_t \bar{t}_L t_R + \lambda_T \bar{t}_L T_{0R} + h.c.\,, \tag{B.42}$$

where $T_{0,R}$ is the triplet component with $T_{3L} = 0$. We find

$$T_{NP} = \frac{N_c \lambda_T^2\Big(18\lambda_t^2 \log\big(\frac{M_T^2}{\lambda_t^2}\big) + 19\lambda_T^2 - 30\lambda_t^2\Big)}{48\pi s_W^2 M_W^2 M_T^2}\,, \tag{B.43}$$

$$S_{NP} = -\frac{N_c \lambda_T^2\Big(4\log\big(\frac{M_T^3 \lambda_t}{\lambda_b^4}\big) - 29\Big)}{18\pi M_T^2}\,, \tag{B.44}$$

$$\delta g_{b,NP} = -\frac{\alpha_{em} \lambda_T^2\Big(2\lambda_t^2 \log\big(\frac{M_T^2}{\lambda_t^2}\big) - \lambda_T^2\Big)}{16\pi s_W^2 M_W^2 M_T^2}\,. \tag{B.45}$$

The top mass is given by

$$M_t \simeq \lambda_t\Big(1 - \frac{\lambda_T^2}{2M_T^2}\Big)\,. \tag{B.46}$$

As can be seen from eqs.(B.43)-(B.45), $T_{NP} > 0$ and $\delta g_{b,NP} < 0$. Contrary to the previous cases, the bottom quark mixing cannot consistently be neglected, since the same Yukawa coupling in eq.(B.42) mixing the top with the $T_{3L} = 0$ triplet component gives also a mixing between the bottom and the $T_{3L} = -1$ triplet component. This mixing is at the origin of the log term involving the bottom Yukawa coupling λ_b in eq.(B.44), which enhances the fermion one-loop contribution to S with respect to the previous cases and gives $S_{NP} < 0$.

B.2.5 Doublet with $Y = 7/6$ mixing with singlet with $Y = 5/3$

The two Yukawa couplings mixing a vector-like singlet X with hypercharge $Y = 5/3$ with a vector-like doublet Q_7 with $Y = 7/6$ are

$$\mathcal{L} \supset y_{XL}\bar{Q}_{7R}H^c X_L + y_{XR}\bar{Q}_{7L}H^c X_R + h.c. \to \lambda_{XL}\bar{Q}_{7uR}X_L + \lambda_{XR}\bar{Q}_{7uL}X_R + h.c.\,. \quad \text{(B.47)}$$

In the limit in which $M_7 = M_X$, we have

$$T_{NP} = \frac{N_c\Big(13\lambda_{XL}^4 + 2\lambda_{XL}^3\lambda_{XR} + 18\lambda_{XL}^2\lambda_{XR}^2 + 2\lambda_{XL}\lambda_{XR}^3 + 13\lambda_{XR}^4\Big)}{480\pi s_W^2 M_W^2 M_X^2}, \quad \text{(B.48)}$$

$$S_{NP} = \frac{N_c\Big(12\lambda_{XL}^2 + 79\lambda_{XL}\lambda_{XR} + 12\lambda_{XR}^2\Big)}{90\pi M_X^2}. \quad \text{(B.49)}$$

Of course, δg_b vanishes, since there is no coupling between the bottom and these states. Being given by vector-like states, eqs.(B.48) and (B.49) do not contain "large" log's of the form $\log M/\lambda_t$. Assuming equality of masses and Yukawa's, the contribution to T in eq.(B.48) is suppressed with respect to the other contributions previously determined.

B.3 χ^2 Fit

Any theory beyond the SM would lead to some modifications of the low energy effective theory which in turn is constrained by experimental data. In this way one can put bounds on the parameters of the theory beyond the SM. This is done systematically by performing a χ^2 fit. Consider a set of observables $\mathcal{O}_i$ which are computed in terms of the parameters, a_i, of the theory beyond the SM, and denote by $\mathcal{O}_i^{exp}$ the measured values of these observables. A measure of how well the theoretical values $\mathcal{O}_i$ fit the experimental data, is given by the quantity χ^2 defined by

$$\chi^2 = (\mathcal{O}_i - \mathcal{O}_i^{exp})(\sigma)_{ij}^{-1}(\mathcal{O}_j - \mathcal{O}_j^{exp}), \qquad \sigma_{ij} = \sigma_i\rho_{ij}\sigma_j. \quad \text{(B.50)}$$

where ρ is the correlation matrix and σ_i are the errors . Using this quantity one can define the region of parameter space a_i compatible with data to the desired level of accuracy, by allowing it to lie within some distance of its minimum value, $\chi^2 \leq \chi^2_{min} + \Delta$. Table.(B.1) shows different values of Δ for different numbers of degrees of freedom and associated with different confidence levels (C.L.), which is the probability that a normally distributed set of variables lie within some distance $R = \sqrt{\Delta}$ of their average.

In the present work, following [50], we have tested our models by performing a combined χ^2 fit expressed in terms of the ϵ_i parameters [51]

$$\epsilon_1 = \alpha_{em}T, \qquad \epsilon_2 = -\frac{\alpha_{em}}{4\sin^2\theta_W}U, \qquad \epsilon_3 = \frac{\alpha_{em}}{4\sin^2\theta_W}S. \quad \text{(B.51)}$$

	99%	95%	90%	70%
1	6.63632	3.84173	2.70571	1.07423
2	9.21034	5.99146	4.60517	2.40795
3	11.3449	7.81473	6.25139	3.66487
4	13.2767	9.48773	7.77944	4.87843
5	15.0863	11.0705	9.23636	6.06443
6	16.8119	12.5916	10.6446	7.23114

Table B.1: Some sample values of Δ in terms of C.L. (top row), and the number of d.o.f (left column)

We use the following theoretical values for the ϵ_i parameters

$$\begin{aligned}
\epsilon_1 &= \big(5.64 - 0.86\; lh\big) \times 10^{-3} + \alpha_{em} T_{NP}\,, \\
\epsilon_2 &= \big(-7.10 + 0.16\; lh\big) \times 10^{-3}\,, \\
\epsilon_3 &= \big(5.25 + 0.54\; lh\big) \times 10^{-3} + \frac{\alpha_{em}}{4\sin^2\theta_W} S_{NP}\,, \\
\epsilon_b &= -6.47 \times 10^{-3} - 2\delta g_{b,NP}\,,
\end{aligned} \tag{B.52}$$

where the first terms are the SM values but with the modified Higgs couplings, and T_{NP}, S_{NP} and $\delta g_{b,NP}$, defined in eq.(B.25), encode the new physics contribution without the SM one and $lh \equiv \log M_{H,eff}/M_Z$, with the effective Higgs mass $M_{H,eff}$ defined as [49]

$$M_{H,eff} = M_H \Big(\frac{1}{M_H L}\Big)^{\sin^2\alpha}\,, \tag{B.53}$$

with L being the length of the extra dimension. This modification of the Higgs mass is due to the fact that in composite Higgs models the Higgs couplings, in particular to the gauge bosons, is different from that of the SM (see eq.(2.29)), as a result the theory is not renormalizable and the S and T parameters need not be finite. In fact contrary to the SM case, the cut-off does not cancel among different diagrams contributing to these parameters through Higgs exchange and leads to the following deviations from the finite SM values

$$\Delta S = \frac{1}{12\pi}(1-a^2)\log\frac{\Lambda^2}{M_H^2} \tag{B.54}$$

$$\Delta T = -\frac{3}{16\pi c_W^2}(1-a^2)\log\frac{\Lambda^2}{M_H^2} \tag{B.55}$$

where in our case $\Lambda = L^{-1}$ and a is given by eq.(2.29), so $1 - a^2 = \sin^2\alpha$.

The experimental values of the ϵ_i, as obtained by LEP1 and SLD data [52], are

$$\begin{aligned} \epsilon_1^{exp} &= (5.03 \pm 0.93) \times 10^{-3}\,, \\ \epsilon_2^{exp} &= (-7.73 \pm 0.95) \times 10^{-3}\,, \\ \epsilon_3^{exp} &= (5.44 \pm 0.87) \times 10^{-3}\,, \\ \epsilon_b^{exp} &= (-6.36 \pm 1.3) \times 10^{-3}\,. \end{aligned} \tag{B.56}$$

Finally, the χ^2 fit has been performed using the following correlation matrix and error values

$$\rho = \begin{pmatrix} 1 & 0.72 & 0.87 & -0.29 \\ 0.72 & 1 & 0.46 & -0.26 \\ 0.87 & 0.46 & 1 & -0.18 \\ -0.29 & -0.26 & -0.18 & 1 \end{pmatrix}\,, \qquad \sigma = \begin{pmatrix} 0.932 \\ 0.953 \\ 0.868 \\ 1.313 \end{pmatrix} \times 10^{-3}\,. \tag{B.57}$$

The bound on $g_{bt,R}$ in model III is included by adding in quadratures to the χ^2 (B.50) the result coming from $b \to s\gamma$ decay [19]:

$$g_{bt,R} = (9 \pm 8) \times 10^{-4}\,. \tag{B.58}$$

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