ISAS - INTERNATIONAL SCHOOL
FOR ADVANCED STUDIES

CP violation for leptogenesis

CANDIDATE

Laura Covi

SUPERVISORS

Dr. Esteban Roulet

Prof. Antonio Masiero

Thesis submitted for the degree of "Doctor Philosophiæ"

High Energy Physics Sector

Academic Year 1996/97

SISSA - SCUOLA
INTERNAZIONALE
SUPERIORE
DI STUDI AVANZATI
TRIESTE
Strada Costiera 11

TRIESTE

Contents

Introduction

The overabundance of matter over antimatter in the universe is one of the most amazing puzzles of modern physics and at the same time one of the few observational evidences that we have of the physics of very high energies, much higher than those experimentally accessible in the accelerators. In spite of the fact that the CPT theorem ensures that the laws of physics must be symmetric for particles and anti-particles, the world around us seems unaware of this and the existence of antimatter went unnoticed until the discovery of the positron in 1933.

The problem of explaining such phenomenon is one of the issues of cosmology where particle physics plays a fundamental role. At the basis of the dynamical generation of the baryon asymmetry, i.e. of the creation of the present matter abundance from a symmetric initial state, lies the existence of an elementary particle theory containing baryon number violation and C and CP violation. From the start it has been therefore clear that the evidence of matter–antimatter asymmetry could be a hint of physics beyond the Standard Model (SM in the following), based on the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$, and the usual perturbative expansion: baryon number seemed to be conserved in such a picture and the known CP violation in the SM is too small to be helpful in generating the observed baryon asymmetry.

Since the pioneering work of Sakharov in 1967 [1], various scenarios for the baryogenesis have been put forward with different particle physics content, based on extensions of the Standard Model, like Grand Unified Theories (GUT's), Supersymmetric GUT's or Superstrings, or on non-perturbative effects of the SM itself (for an inclusive review see [2, 3]). None of such proposal for the baryogenesis has prevailed until now and this is still an open question after 30 years of study.

A very attractive mechanism, proposed by Fukugita and Yanagida in 1986 [4], is based on the production of a lepton asymmetry by the out of equilibrium decays of heavy electroweak singlet neutrinos. Their decays into light leptons and Higgs bosons can violate CP if the Yukawa couplings involved have unremovable phases, and can then lead to the production of an excess of anti-leptons over leptons in the final state. The lepton asymmetry so produced is then partially converted into a baryon asymmetry by anomalous electroweak processes [5, 6], which are in equilibrium at temperatures larger than the electroweak phase transition one, and as a result, the amount of baryons present in the Universe, $n_B/n_\gamma \simeq 5 \times 10^{-10}$, can be accounted for [7, 8, 9].

To postulate the existence of right–handed 'sterile' neutrinos constitutes one of the simplest and most economical extensions of the Standard Model, being strongly motivated by Grand Unified theories such as SO(10). It also allows to implement the see–saw mechanism which naturally accounts for non–zero, but still very small, light neutrino masses. In this context, the values of the light neutrino masses suggested by the MSW solution to the solar neutrino problem, the atmospheric neutrino anomaly and the hot dark matter scenarios [10], point towards an intermediate scale, $M \sim 10^9$–10^{13} GeV, for the right–handed neutrino masses. This further encourages the consideration of scenarios where the heavy neutrino decays act as sources for the generation of the baryon asymmetry of the Universe, since for these large masses it becomes easier to achieve the out of equilibrium conditions required for the efficient generation of an asymmetry, and also because temperatures comparable to the intermediate scale are more likely to be produced as a result of the re-heating process at the end of inflation than the temperatures required in the conventional baryogenesis GUT scenarios.

In addition to the non–supersymmetric version originally considered by Fukugita and Yanagida, the extension of this scenario to the supersymmetric case has been studied by Campbell, Davidson and Olive [11], and also a related scenario where the asymmetry is produced in the decay of heavy scalar neutrinos produced non–thermally by the coherent oscillations of the scalar field at the end of inflation has been discussed by Murayama et al. [12]. A key ingredient for all these scenarios is the one–loop CP violating asymmetry involved in the heavy (s)neutrino decay, and the reconsideration of this quantity will be the main issue of this work. As will be showed, there are contributions to the asymmetry that have not been included previously, so we will compute the total CP violation both in the non-supersymmetric and in the supersymmetric case and discuss the different results present in the literature on this subject, clarifying some common misconceptions [13].

We will also consider the possibility of using the baryon asymmetry to get an insight into the problem of neutrino masses and discuss the limiting cases of strong hierarchy and mass degeneracy of the mass matrix. We will see that a considerable enhancement of the CP asymmetries is achieved when the masses of the mixed states are comparable, and the enhancement is maximal for mass splittings of the order of the widths of the decaying particles. Applying the results to the leptogenesis scenarios, we will find that large enhancements of the asymmetries may become possible as a consequence of the oscillations among the nearly degenerate mixed states [14]. This can be helpful to obtain the required lepton number even for low neutrino masses since a partial erasure by the L violating processes which are still active after the decay can be compensated by an enhanced L overproduction in the decay.

Finally we will take into account the effect of the thermal bath on the baryon number generated and compute the finite temperature corrections to the CP asymmetry [15]. The thermal bath dynamics is usually treated as a classical problem and the Boltzmann equation for the particle number density is integrated, considering the particles as classical objects with Boltzmann statistics. Although such approximation appears to be reasonable due to the high temperatures and low densities involved, we will show that in general quantum thermal effects can modify strongly the size of the CP violation. Interesting cancellations are anyway to be found for our model and in particular for any supersymmetric model in such a way to leave validity to the T = 0 result.

Throughout we will use units such that $\hbar = c = k_B = 1$. In this system masses, energies and temperatures are all measured in GeV, while time and distance are expressed in GeV^{-1}. For comparison with the usual units, remember that 1 GeV $= 1.8 \times 10^{-27} kg = 1.2 \times 10^{13} K$ and 1 GeV^{-1} $= 2.0 \times 10^{-16} m = 6.6 \times 10^{-25} s$.

Chapter 1

Overview on baryogenesis

1.1 Experimental evidence of baryon asymmetry

It is an undeniable experimental fact that on earth and in the solar system antimatter is very rare. Apart from the interplanetary explorations of the last years, where probes have landed on most of the planets without annihilating, we know also that the solar wind is made of matter and would make very bright in our sky any anti-planet present in its reach.

Even if the evidence of local baryon asymmetry is very strong, it becomes weaker for what concerns bigger scales. Cosmic rays, also constituted mainly by particles (the ratio of anti-proton to proton content is 10^{-4}, as expected by the anti-proton production in spallation processes, and no anti-nucleus has ever been detected), are an indication that nearby galaxies are made of matter, but for further distances we lack direct evidence and we are obliged to rely on indirect hints. The possibility that we live in a baryon symmetric universe with large matter and antimatter domains has not been ruled out, but recently a lower limit on the domain size comparable to the current size of the universe has been derived [16] from comparison of the cosmic diffuse gamma flux with the expected red-shifted annihilation photons flux from the matter–antimatter boundary regions. No physical mechanism able to explain baryons anti-baryons segregation on such a large scale has been found yet.

Another indirect, but very strong argument in favor of a baryonic asymmetric universe is the successful model of nucleosynthesis, which is able to predict the observed abundances for light elements. The basic parameter of the model is exactly the baryon-to-photon ratio $\eta = \frac{n_B}{n_\gamma}$, which is constrained by the experimental data on the abundances to be of the order of 10^{-10}, more precisely [17]

$$4 \times 10^{-10} \leq \eta \leq 7 \times 10^{-10}. \tag{1.1}$$

While nowadays the world seems extremely baryon-asymmetric, with no antimatter around, this η amounts to a very small asymmetry if we consider the universe before the quark-gluon plasma phase transition at T $\simeq$ 150 – 200 MeV, when quark and antiquarks were free and approximately as many as the photons:

$$\frac{n_q - n_{\bar{q}}}{n_q} \simeq 10^{-9}. \tag{1.2}$$

It is therefore plausible to be able to explain such a number dynamically, as an initial very small fluctuation in the particle numbers.

1.2 Dynamical generation of the Baryon Number

The three conditions necessary for the dynamical generation of the Baryon Number (denoted from now on by B) have been found by Sakharov in his article of 1967 [1]. Analyzing the general picture he concluded that the following ingredients were necessary for any theory in order to build up B from the symmetric $B = 0$ state:

- obviously B violation; otherwise B would stay at the initial value, $B = 0$;

- C and CP violation in such a way to produce an asymmetry between particles and anti-particles carrying B-number; on the contrary in case of C and CP conservation the same number of baryons and anti-baryons would be created so that they would annihilate almost completely; notice moreover that B is odd under C and CP transformations and therefore to generate any baryon number this symmetries must be broken;

- a departure from thermal equilibrium; since the maximal entropy state is characterized by zero chemical potential, it is impossible to create dynamically a state with $\mu \neq 0$ maintaining thermal equilibrium.

The first two conditions concern directly the elementary particle models, while the third links particle physics with cosmology.

Notice that the Standard Model presents strong C violation through the chirality of the interaction and very small CP violation due to the Cabibbo-Kobayashi-Maskawa (CKM) matrix phase, but no B violation, at least at the perturbative level. At the beginning, therefore, only extensions of the SM, like GUT's, seemed viable to solve the puzzle of the baryon asymmetry of the Universe (BAU). Later on it was realized that B violating processes are possible in the SM as non perturbative effects, and the possibility has been proposed to generate the B-number through electroweak processes.

In the following sections we will first discuss the general picture of Standard Cosmology where the generation of matter–antimatter asymmetry should have taken place and review some generalities on the thermodynamics of the Early Universe. Then we will describe two models for baryogenesis, fulfilling in different way

Sakharov's conditions: electroweak and GUT baryogenesis. We will concentrate in particular on the mechanism of out-of-equilibrium decays, first proposed in the context of GUT baryogenesis, since it is at the basis also of the Fukugita-Yanagida model.

1.3 The Standard Cosmology

The hot big bang model (for a more detailed discussion see [18]) that constitutes the standard cosmology describes an isotropic and homogeneous universe beginning with a space-time singularity and expanding through its history until the present time. The space-time evolution of the universe can be described through the Friedmann-Robertson-Walker metric

$$ds^2 = -dt^2 + R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + sin^2\theta d\phi^2)\right], \tag{1.3}$$

where $k = 1, 0, -1$ in the case of positively curved and finite, flat and infinite, negatively curved and infinite universe respectively. The basic quantity is then the cosmic scale factor $R(t)$, related to the distance of two points on the comoving frame. The time evolution of $R(t)$ is given by the Hubble parameter $H(t)$, which measures the expansion rate:

$$H(t) = \frac{\dot{R}(t)}{R(t)}. \tag{1.4}$$

The evolution of the universe due to the gravitational interaction is then ruled by the Friedmann equation:

$$H^2 = \frac{8\pi G\rho}{3} - \frac{k}{R^2} \tag{1.5}$$

with ρ denoting the total energy density of the universe, and by the first law of thermodynamics for the perfect cosmic fluid:

$$dU + PdV = d(\rho R^3) + pd(R^3) = 0 \tag{1.6}$$

where p is the pressure and is given by the spatial part of the stress-energy tensor, $T_i^i = 3p$. The relation between the energy density and the pressure for any particle species is connected to the equation of state; so for non-relativistic matter $p = 0$ and for radiation $p = \rho/3$. It is then clear from eq. (1.6) that the energy density of non relativistic particles is decreasing in the expanding universe as $1/R^3$, while that of radiation as $1/R^4$. Nowadays we are in a matter dominated epoch, but in the earlier times the energy density of the universe was mainly stored into radiation, i.e. relativistic particles. It is in that period that dynamical generation of the baryon asymmetry should have taken place.

1.3.1 Thermodynamics in the early universe

Let us consider how it is possible to define a temperature in the standard cosmology picture. We have seen that in the earlier epoch the universe was radiation dominated and therefore large numbers of relativistic particles were present and continuously interacting among themselves through the electroweak (EW) or strong interactions. We can then suppose that if the interaction rates of those forces were much larger than the expansion rate of the universe, the particles could reach the thermal equilibrium in a short time compared to the expansion and then the thermal bath so formed would adjust itself to the universe evolution passing through a succession of nearly equilibrium states. The single species number densities would therefore be given by the Bose-Einstein or Fermi-Dirac distribution computed at the equilibrium temperature T:

$$n = \frac{g}{(2\pi)^3} \int d^3p \frac{1}{e^{(E-\mu)/\mathrm{T}} \pm 1} \simeq \frac{g}{\pi^2}\mathrm{T}^3 \qquad (1.7)$$

where g is the number of degrees of freedom and μ the chemical potential. The last approximate expression is valid for $\mu, M \ll \mathrm{T}$.

Due to the adiabatic expansion, anyway, the temperature decreased as $1/R$: in the isolated system of the universe the entropy is constant and its density in the radiation dominated epoch was given by the relativistic species densities and pressure:

$$\frac{S}{R^3} = \frac{\rho + p}{\mathrm{T}} = \frac{2\pi^2}{45} g_* \mathrm{T}^3 \tag{1.8}$$

where

$$g_* = \Sigma_{\mathrm{bosons}}\, g + \frac{7}{8} \Sigma_{\mathrm{fermions}}\, g \tag{1.9}$$

is the effective number of relativistic bosonic and fermionic degrees of freedom that were in equilibrium. So for constant g_*, the temperature must change as $1/R$.

The equilibrium condition for any process is then the comparison of its rate with the Hubble parameter: since $\dot{\mathrm{T}}/\mathrm{T} = -H$, as long as $\Gamma \gg H(t)$, the number densities can adjust theirselves to the temperature change, otherwise, for $\Gamma \ll H(t)$ the interactions are no more able to maintain the thermal equilibrium and the particle distributions fall out of it. Anyway in the case of massless particles, as are photons of the Cosmic Background Radiation, the number density is not distorted and remains that of a thermal distribution, with a temperature red-shifted by the universe expansion. For this reason the CBR spectrum resembles closely that of a black body.

In case of a massive species the problem is more complex. At temperatures of the order of the mass M, the particles become non-relativistic and their equilibrium distribution falls rapidly to zero; approximating with the Boltzmann distribution, one obtains:

$$n \simeq g \left(\frac{M\mathrm{T}}{2\pi} \right)^{3/2} e^{-(M-\mu)/\mathrm{T}}. \tag{1.10}$$

So we see that a chemical potential is needed for preserving a net number of heavy particles at low temperature; for $\mu = 0$ the number density tends exponentially to zero.

Moreover for T $\simeq M$ some processes involving the species, like scattering processes or inverse decay, fall naturally out of equilibrium due to the fact that very few of the lighter particles in the thermal bath have enough energy to create one heavy particle. Other processes, like the decay, are still active, but if they happen to be slower than the expansion rate of the universe, out of equilibrium conditions can be reached.

To follow the density distribution for massive particles out of equilibrium, the unique tool is the classical Boltzmann equation for their number density [19]:

$$\dot{n}_X + 3Hn_X = \Lambda \tag{1.11}$$

where the term proportional to H takes into account the dilution due to the universe expansion, while Λ indicates the collision integral for the species X, usually involving the number densities of other species. To solve the equation, one has to compute the collision integral explicitly and integrate the equation numerically [2, 18, 7]. Only if the particle are way out of equilibrium, i.e. the decay process is dominant and $\Gamma_D << H$, it is possible to find an approximate analytical solution.

In that case one can neglect all the scattering processes and solve the approximate equation, obtained assuming thermal equilibrium for all the decay products of the particle X:

$$\dot{n}_X + 3Hn_X = -(n_X - n_X^{eq})\langle\Gamma_D\rangle \tag{1.12}$$

where n_X^{eq} is the equilibrium density for the species X and $\langle\Gamma_D\rangle$ its thermally averaged decay rate.

1.4 EW baryogenesis

We know from the experiments that C and CP are not good symmetries of the Standard Model, but since now no evidence of baryon number violation has been observed (e.g. proton decay, neutron anti-neutron oscillation, etc...) in the laboratories. In fact the electroweak lagrangian automatically conserves baryon number, even if such symmetry has not been explicitly imposed. However in 1976, 't Hooft discovered [20] that quantum correction break this symmetry for any non Abelian gauge theory, due to the non trivial vacuum structure of such theories. Vacuum field configurations can belong to different homotopy classes and also have different $B+L$ number, while $B-L$ is still conserved.

We have in fact that the currents satisfy the equations [5]:

$$\partial_\mu J^\mu_{B+L} = \frac{g^2}{16\pi^2}\left(F^a_{\mu\nu}\widetilde{F}^{\mu\nu}_a + f_{\mu\nu}\widetilde{f}^{\mu\nu}\right) \tag{1.13}$$

$$\partial_\mu J^\mu_{B-L} = 0 \tag{1.14}$$

where $F^{\mu\nu}_a, f^{\mu\nu}$ are the gauge field strengths for the $SU(2)_L$ and $U(1)_Y$ respectively and $\widetilde{F}^{\mu\nu}, \widetilde{f}^{\mu\nu}$ their dual tensors. It is possible to write the r.h.s. of eq. (1.13) as a total derivative and define a conserved current if we assign a $B+L$ number to the vacuum field configurations; changing vacuum background implies then a change in the total $B+L$ charge of the particle content. The vacua are separated by a potential barrier so that at zero temperature the probability of tunneling between the different field configurations is strongly suppressed and the baryon number is effectively conserved.

At temperatures larger than zero, another possibility of transition arises, i.e. passing over the barrier. Such processes are called sphaleronic, since they proceed through the semi-classical unstable field configuration called sphaleron, i.e. the configuration at the top of the energy barrier between the two topologically inequivalent vacua [5].

For temperatures larger than 100 GeV, these baryon-number violating interactions are not only active, but in thermal equilibrium with respect to the universe expansion [6]. Baryon number is then violated in the thermal bath, but surprisingly the presence of a net conserved $B - L$ number prevents the vanishing of the total baryon number as we will see in the next paragraph.

Since both the first two Sakharov's conditions are fulfilled by the Standard Model, the question arises if it is possible to generate the baryon number within this model. We need the third ingredient, i.e. departure from thermal equilibrium, and this may be realized in the EW phase transition. Since sphalerons are in equilibrium up to the EW scale, to explain the present day B-asymmetry it is necessary for the phase transition to be strongly of the first order so that in the broken phase the anomalous processes are no more active. The basic picture [6, 3, 21] is the nucleation of bubbles of broken phase that quickly expand in the symmetric phase; the bubble wall is the place of out of thermal equilibrium processes able to build up a baryonic asymmetry inside the bubble, where the baryon number is conserved. At the same time, outside the bubble sphaleron processes are still active and wash out the anti-baryon number.

The minimal model, where the CP violation in the bubble wall is given only by the CKM matrix phase, has been shown to give a too small B number [22], but more complex scenarios, like the Minimal Supersymmetric Standard Model, are still viable. In general, all these models need a strongly first order phase transition and therefore require a small mass for the Higgs field; their higher bounds are not far from the lower bounds coming from LEP.

1.4.1 Sphalerons and preexisting B-asymmetry

It has been computed [6] that in the early universe, for temperature larger than 100 GeV, baryon-number violating interactions are in equilibrium. We will have then that sphaleronic processes would modify any $B+L$ number produced at early stages and so menace any baryogenesis model at high energies. Relating among themselves the chemical potentials for all the particles of the SM taking part to interactions in thermal equilibrium before the EW phase transition, sphaleronic processes included, the following relations have been obtained [23]:

$$B+L = -\frac{6N_f+5N_H}{22N_f+13N_H}(B-L) \tag{1.15}$$

$$B = \frac{8N_f+4N_H}{22N_f+13N_H}(B-L) \tag{1.16}$$

$$L = -\frac{14N_f+9N_H}{22N_f+13N_H}(B-L) \tag{1.17}$$

where N_f is the number of flavors and N_H the number of Higgs doublets of the model. We see clearly from the relations that no baryonic asymmetry will survive if $B-L$ is zero, so that any baryogenesis model which conserves $B-L$, like the $SU(5)$ GUT, is ineffective, but at the same time, even if the sphaleronic processes are at equilibrium, they cannot cancel completely $B+L$ if $B-L \neq 0$. The preexistence of a net $B-L$ number will be translated into a final matter-antimatter asymmetry.

1.5 GUT baryogenesis

The first attempts to explain the presence of a net baryon number were those based on Grand Unifying Theories, precisely the simplest one with unifying group $SU(5)$ [24]. B-number violating interactions are in this case mediated by the exchange of exotic gauge bosons transforming quarks and leptons in the same multiplet among themselves or by exchange of the colored, fractionally charged Higgs bosons.

C violation is naturally present like in the Standard Model, while the breaking of the CP symmetry is given by the unremovable phases in the Yukawa couplings. In particular CP violations appears in the decay channel given by such complex couplings from the interference of the tree level and one loop decay amplitudes. So if the decay also violates B, a net baryon number is generated in the process. In case of thermal equilibrium, such number is immediately washed out by other B-violating processes, maintaining the chemical potential of the baryons at zero; otherwise, if thermal equilibrium is broken, a non-zero baryon number can survive to the present epoch[1].

The condition of departure from thermal equilibrium is fulfilled thanks to the expansion of the universe, as we have seen. All the processes involving a massive species X, except the decay, fall quite naturally out of equilibrium when the temperature is of the order of the mass, since the kinetic energy of the lighter particles in the thermal bath is no more sufficient to create new X particles; then if the decay rate happens to be smaller than the Hubble parameter, their number density drifts away from the equilibrium distribution and the particles decay out of equilibrium and can generate a net baryon number.

1.5.1 One loop CP violation

Let us now describe the basic ingredients necessary to have CP violation at the one loop level. We can define as the CP asymmetry of any decay process $X \to f$,

[1]Neglecting obviously the sphaleronic processes!

as the ratio:

$$\epsilon_{CP} = \frac{\Gamma(X \to f) - \Gamma(\bar{X} \to \bar{f})}{\Gamma(X \to f) + \Gamma(\bar{X} \to \bar{f})}. \tag{1.18}$$

Due to the hermiticity of the lagrangian, it is not possible to have CP violation at tree level; consider in fact the generic example of the Yukawa interactions of a family of bosonic particles X_a:

$$\mathcal{L}_Y = h_{ajk} X_a \bar{\psi}_j \psi_k + h.c. \tag{1.19}$$

At tree level we have that the amplitudes for the process $X_a \to \psi_j \bar{\psi}_k$ and the conjugate one are proportional respectively to h_{ajk} and h^*_{ajk}, but, since the phase space is the same for particle and anti-particles[2], the transition rate is equal and proportional to $|h_{ajk}|^2$.

So to have any CP violation we must go beyond the tree approximation and consider loop effects. This descends directly from the unitarity relation for the S matrix; considering $S = 1 - iT$, we have

$$1 = S^\dagger S = 1 + i(T^\dagger - T) + T^\dagger T \tag{1.20}$$

that yields

$$T = T^\dagger - iT^\dagger T. \tag{1.21}$$

Taking the matrix element between the initial state i and the final state f and computing the modulus squared on both sides, we obtain:

$$|T_{fi}|^2 = |T^*_{if}|^2 + 2\,\mathrm{Im}\left[(T^\dagger T)_{fi} T_{if}\right] + |(T^\dagger T)_{fi}|^2. \tag{1.22}$$

But $|T^*_{if}|^2 = |T_{if}|^2$ and from CPT $T_{if} = T_{\bar{f}\bar{i}}$ so that we have

$$|T_{fi}|^2 - |T_{\bar{f}\bar{i}}|^2 = 2\,\mathrm{Im}\left[(T^\dagger T)_{fi} T_{if}\right] + |(T^\dagger T)_{fi}|^2. \tag{1.23}$$

[2] CPT imposes that particles and anti-particles have the same mass.

At lowest order in the coupling constant only the first term on the r.h.s. is important. We see therefore that the CP asymmetry is given by considering all intermediate real physical states n in the sum $\sum_n T^*_{nf} T_{ni}$, i.e. the absorptive part of all the possible loop diagrams connecting initial and final state. Moreover, since in eq. (1.23) the contribution comes from the imaginary part of the transition amplitudes products, fundamental to have CP violation are complex coupling constants.

Let us exemplify this in the case of the lagrangian (1.19) and the decay $X_a \to \psi_j \bar{\psi}_k$; the intermediate state can be any pair of particles $\psi_l \bar{\psi}_m$, so that T^*_{nf} is the amplitude for the elastic scattering process $\psi_l \bar{\psi}_m \to \psi_j \bar{\psi}_k$. If the leading contributions to the 2-particle scattering is again given by the same Yukawa interaction (1.19), the process takes place with the exchange of a virtual X_b particle in the s- or t-channel, as shown in Fig. 1.1.

Notice that in this case the virtual intermediate particle X_b must be different from the initial one to have a net CP asymmetry; otherwise, apart from the real phase space factors, we would have $T^*_{nf} \propto h^*_{alm} h_{ajk}$ and so:

$$\mathrm{Im}\left[(T^\dagger T)_{fi} T_{if}\right] \propto \mathrm{Im}\left[\sum_{lm} h_{alm} h^*_{alm} h_{ajk} h^*_{ajk}\right]; \tag{1.24}$$

then the total CP asymmetry would be zero even for complex Yukawa couplings:

$$\epsilon_{CP} \propto \mathrm{Im}\left[\sum_{lm} |h_{alm}|^2 |h_{ajk}|^2\right] = 0. \tag{1.25}$$

The t-channel contribution is the one that is traditionally considered and will be denoted in the following as vertex contribution. The s-channel part is instead connected to the mixing between the heavy states due to the 1-loop off-diagonal contributions to the mass matrix; the renormalization procedure permits to absorb the real divergent parts into the mass and wave function renormalization constants, but, as we will see, it is not possible to re-absorb the finite imaginary part of the mixing

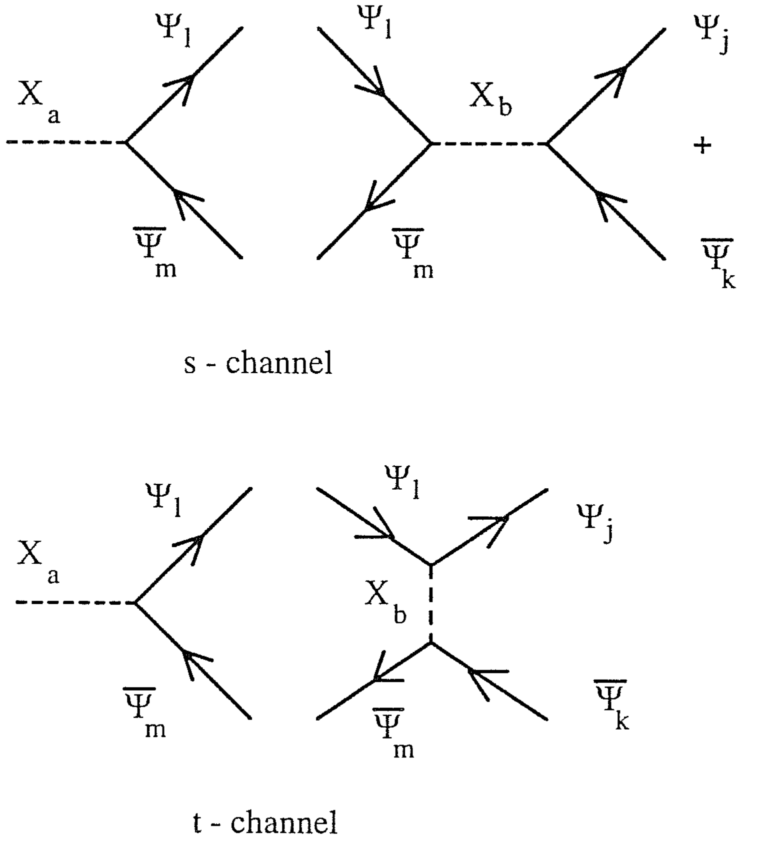


Figure 1.1: Contributions in the s-channel and t-channel for $T^*_{nf}T_{ni}$; the initial and final states are fixed, $X_a \to \psi_j\bar{\psi}_k$, while any intermediate on shell state n, in this case $\psi_l\bar{\psi}_m$, must be considered.

diagram without spoiling the orthonormality of the particle states. The analysis of the s-channel or wave function contribution will be in particular the subject of chapter 3.

Notice that to have also B-violation in the decay process, two decay channels with different baryon number must be open for the heavy particle; otherwise it would be possible to assign a specific B value to X_a itself and no violation of the baryon number would be present.

We will consider in the next subsection a specific example of GUT baryogenesis and compute the C and CP asymmetry for the decay of the heavy triplet in $SU(5)$, even if this model is not viable for explaining baryogenesis since it conserves $B-L$.

1.5.2 An $SU(5)$ example of B and CP violation

Let us begin to consider as a basic example of GUT baryogenesis the out of equilibrium decay of the heavy Higgs boson triplet in $SU(5)$. In such model there exist also the possibility to generate a baryon number through the decay of exotic gauge bosons, but in this case, since the gauge coupling is quite large, $\alpha_5 = g_5^2/(4\pi) \simeq 1/50$ at unification scale, the out of equilibrium condition [18]

$$\Gamma_D \propto \alpha_5 M_X \leq H(T = M_X) \simeq g_*^{1/2} M_X^2 / M_{Pl}, \tag{1.26}$$

where $M_{Pl} = 1.22 \times 10^{19}$ GeV is the Planck mass, is less likely to be satisfied for gauge than for Higgs bosons. Moreover the constraint for the Higgs mass coming from the proton lifetime are less severe so that the Higgs bosons are usually thought to be lighter than the exotic gauge bosons; then B violating processes involving the Higgses would still be active after the gauge bosons decay and diminish the baryon asymmetry produced.

The Higgs scalar particle content in $SU(5)$ is larger than in the Standard Model; in the minimal case we have one Higgs field in the 5 dimensional irreducible representation of the group and another Higgs in the adjoint (24 dimensional) representation. This last field is responsible for the breaking to the SM group at high energies (of the order of 10^{15} GeV). The fiveplet Higgs field Φ decomposes under the SM group $SU(3)_c \times SU(2)_L \times U(1)_Y$ as a colored triplet T and an EW doublet H, to be identified with the SM Higgs doublet. Since the colored triplet Higgs can mediate proton decay, a strong bound on its mass is given by the experiments. A fine tuning of the scalar potential parameter is then needed for providing doublet-triplet mass splitting. Consistency with the proton lifetime of about 10^{31} years is ensured for a triplet mass greater than 10^{10} GeV [25].

Notice that in order to have CP violation at one loop in the $SU(5)$ GUT it is necessary to have more than one heavy state, so we have to add to the minimal $SU(5)$ GUT at least one Higgs fiveplet. In the minimal picture CP violation would appear only at three loops level and therefore would be very suppressed [26].

For simplicity we will consider the case in which the masses of the heavy triplet states T_i are significantly split, since the study of the CP violation in the near degenerate situation present additional complications [27, 14]. We will see such case in detail later on.

One generally expects that only the decay of the lightest of the heavy states, T_1, will be the one leading to a net B, since any asymmetry produced at earlier times through the decay of heavier states would be erased by B violating processes which could still be in equilibrium. Hence, one has to compute the asymmetry resulting from the T_1 decay, and in the case we will consider in which there is a hierarchy among the masses of the heavy states ($M_k \gg M_1$, with $k > 1$), it will be natural to assume that the heavier states T_k have already decayed when the lighter one is falling out of equilibrium.

Let us consider the $SU(5)$ lagrangian involving several scalar five-plets $\Phi_i = (T_i, H_i)$:

$$\mathcal{L} = f_i \Phi_{i\alpha} \, (\bar{\Psi}^{\alpha\beta} \, \chi_\beta) + \frac{g_i}{8} \Phi_{i\alpha} \, (\epsilon^{\alpha\beta\gamma\delta\epsilon} \, \bar{\Psi}^c_{\beta\gamma} \, \Psi_{\delta\epsilon}) + h.c., \tag{1.27}$$

where the gauge indices are denoted by greek letters. The matter fields are in the decuplet and the fiveplet representations as usual, $\Psi = (q, u^c, e^c)$ and $\chi = (d, l^c)$. Since they are vectors in flavor space, the Yukawa couplings f_i and g_i should be thought as 3×3 matrices, but for simplicity the flavor indices are not displayed.

The CP violating B asymmetry arising from the decay of a T_1 and $\bar{T}_1$ pair is

$$\epsilon = \frac{\sum_f B_f [\Gamma(T_1 \to F_f) - \Gamma(\bar{T}_1 \to \bar{F}_f)]}{\sum_f [\Gamma(T_1 \to F_f) + \Gamma(\bar{T}_1 \to \bar{F}_f)]}, \tag{1.28}$$

with B_f the baryon number of the final states $F_f = \bar{q}\ell^c, \; \bar{u}e^c, \; \bar{u}^c d, \; \bar{q}^c q$. From the interactions in eq. (1.27) we have

$$\epsilon = \frac{4 \sum_k \left[-\mathrm{Im}\left\{ \mathrm{Tr}\left(g_k^\dagger g_1 \, f_k f_1^\dagger \right) \right\} \mathrm{Im}\{ I_t(x_k) \} + \frac{1}{1-x_k} \mathrm{Im}\left\{ \mathrm{Tr}\left(f_k f_1^\dagger \right) \mathrm{Tr}\left(g_k^\dagger g_1 \right) \right\} \mathrm{Im}\{ I_s \} \right]}{\left[4\mathrm{Tr}\left(f_1 f_1^\dagger \right) + 3\mathrm{Tr}\left(g_1^\dagger g_1 \right) \right]}, \tag{1.29}$$

where x_k is the heavy-to-light ratio of squared masses:

$$x_k \equiv \frac{M_k^2}{M_1^2} \tag{1.30}$$

Notice that, as we already pointed out, the asymmetry is null if there is a single triplet field. In this case, the result (1.29) depends on the hermitian matrices $g_1^\dagger g_1$ and $f_1 f_1^\dagger$; therefore their trace has no imaginary part, and the trace of the product is equal to half the trace of the anticommutator, again an hermitian matrix.

The terms in the denominator correspond to the tree level decay rate; those in the numerator to the interference between the tree level and the absorptive part of the loop amplitudes[3]. In particular, I_t arises from the loop integral in the 'vertex' contribution, in which the T_k is exchanged in the t–channel (see Fig. 1.1), and I_s comes from the loop integral in the 'wave' contribution, in which the T_k is exchanged in the s–channel, i.e.

$$\bar{u}(p_1)v(p_2)I_t(x_k) = \bar{u}(p_1)\int \frac{d^4q}{(2\pi)^4} S(p-q,0)S(-q,0)D(q-p_2,M_k)v(p_2), \qquad (1.31)$$

$$M_1^2 I_s = -\frac{i}{2}\int \frac{d^4q}{(2\pi)^4}\mathrm{Tr}(S(-q,0)S(p-q,0))\,, \qquad (1.32)$$

with $S(p,m)$ $(D(p,m))$ being the propagator of a fermion (scalar) with mass m and momentum p.

From these expressions one gets the following absorptive parts:

$$\mathrm{Im}\{I_t(x)\} = \frac{1}{16\pi}\left[1 - x\,\ln\left(1+\frac{1}{x}\right)\right], \qquad (1.33)$$

and

$$\mathrm{Im}\{I_s\} = \frac{1}{16\pi}. \qquad (1.34)$$

In the limit of very large mass hierarchy, $x \to \infty$, the first expression reduces to

$$\mathrm{Im}\{I_t(x)\} = \frac{1}{32\pi x}. \qquad (1.35)$$

We have therefore that the B asymmetry defined in eq. (1.29) is:

$$\epsilon \simeq -\frac{\sum_k \frac{1}{x_k}\left[\mathrm{Im}\left\{\mathrm{Tr}\left(g_k^\dagger g_1\, f_k f_1^\dagger\right)\right\} + 2\,\mathrm{Im}\left\{\mathrm{Tr}\left(f_k f_1^\dagger\right)\,\mathrm{Tr}\left(g_k^\dagger g_1\right)\right\}\right]}{8\pi\left[4\,\mathrm{Tr}\left(f_1 f_1^\dagger\right) + 3\,\mathrm{Tr}\left(g_1^\dagger g_1\right)\right]}. \qquad (1.36)$$

The way out of equilibrium condition $\Gamma_D \ll H(\mathrm{T})$ for $\mathrm{T} = M_1$ is in the Higgs triplet case:

$$\left[4\,\mathrm{Tr}\left(f_1 f_1^\dagger\right) + 3\,\mathrm{Tr}\left(g_1^\dagger g_1\right)\right] \ll 8\pi g_*^{1/2} M_1/M_{Pl}, \qquad (1.37)$$

[3]The apparent difference with respect to refs. [28, 29] in the denominator is just due to a different normalization of the field Ψ, and hence of the Yukawa couplings f_i. However, our result for the numerator differs from ref. [30] in the sign of the first term (in agreement with [31, 32]) and a factor 2 in the second one.

or equivalently for $g_* \simeq 100$

$$4\,\mathrm{Tr}\left(f_1 f_1^\dagger\right) + 3\,\mathrm{Tr}\left(g_1^\dagger g_1\right) \ll 2.5 \times 10^2 M_1/M_{Pl}. \tag{1.38}$$

If this inequality is satisfied, as can well happen for reasonable values of the Yukawa couplings and of the M_1 mass, it is simple to estimate the baryon number generated in the decay process. Neglecting inverse decay and all the other scattering processes, we can consider that every couple of T_1 and $\bar{T}_1$ produces ϵ baryon number. Since before decoupling the Higgs triplets are as common as the photons, we have, using eqs. (1.7) and (1.8),

$$\frac{n_B}{s} \leq \left[\frac{\epsilon}{s}\frac{g}{2}n_\gamma\right]_{\mathrm{T}=M_1} = \epsilon\,\frac{135}{\pi^4 g_*} \simeq .01\,\epsilon \tag{1.39}$$

where g is the number of degrees of freedom of the decaying particle (6 in the case of the complex colored triplet T_1). Since the baryon number per entropy density is constant in the expansion of the universe, we can compute (1.39) at the production temperature $\mathrm{T} \simeq M_1$ in order to obtain the present value. Obviously we are still neglecting the effect of sphaleronic or other processes that could modify the baryon number at later times.

We would like to stress that also in the case $\Gamma_D \geq H(\mathrm{T})$ baryogenesis takes place, but the final B number is smaller than given by eq. (1.39) and to compute it the Boltzmann equations for the species involved must be solved [2, 18, 7].

Chapter 2

CP violating decays in leptogenesis scenarios

We have considered in the previous chapter the consequences of the baryon number violating anomalous processes on any preexisting asymmetry, and have seen that a sufficient $B-L$ number is necessary to explain the present day overabundance of matter, if it has been generated at high energy scales. Therefore the possibility arises of generating the present day baryon asymmetry not through B violating processes like in the case of the $SU(5)$ GUT discussed previously, but through lepton number (L) violating interactions. Any L built up at high energies, contributes to the $B-L$ number and is transformed by sphaleronic processes in equilibrium into a final baryon number. In such a picture baryogenesis and neutrino masses and mixings are tightly related to each other.

2.1 The Fukugita-Yanagida model

In this section we will concern ourselves with the first model for leptogenesis, proposed by Fukugita and Yanagida in 1986 [4] as a minimal extension of the Standard Model.

Apart for the usual SM pieces, the new terms of the Lagrangian are given by

$$\mathcal{L} = \mathcal{L}_{SM} - \frac{1}{2} M_j \overline{N_j^c} N_j - \lambda_{ij} \epsilon_{\alpha\beta} \overline{N_j} P_L \ell_i^\alpha H^\beta + h.c. \ , \tag{2.1}$$

where $\ell_i^T = (\nu_i \ l_i^-)$ and $H^T = (H^+ \ H^0)$ are the lepton and Higgs doublets ($\epsilon_{\alpha\beta} = -\epsilon_{\beta\alpha}$, with $\epsilon_{12} = +1$), and, without loss of generality, we are taking the N_j to be the Majorana mass eigenstate fields, of mass M_j. The scale of this singlet neutrinos mass should be quite large, of the order $\geq 10^8$ GeV in order to be able to explain the small light neutrino's masses through the see-saw mechanism. We will see this later on.

Since M_j are so much larger than the electroweak scale, it is reasonable to work directly in the symmetric phase where all particles except the heavy Majorana neutrinos N_j are massless, and all components of the neutral and charged complex Higgs fields are physical.

The heavy neutrinos are Majorana particles so that they can decay either in a lepton and a Higgs or in an anti-lepton and anti-Higgs breaking lepton number. The basic quantity we are interested in is the L-violating CP asymmetry,

$$\epsilon_\ell^N \equiv \frac{\Gamma_{N\ell} - \Gamma_{N\bar{\ell}}}{\Gamma_{N\ell} + \Gamma_{N\bar{\ell}}}, \tag{2.2}$$

where $\Gamma_{N\ell} \equiv \sum_{\alpha,\beta} \Gamma(N \to \ell^\alpha H^\beta)$ and $\Gamma_{N\bar{\ell}} \equiv \sum_{\alpha,\beta} \Gamma(N \to \bar{\ell}^\alpha H^{\beta\dagger})$ are the N decay rates (in the N rest frame), summed over the neutral and charged leptons (and Higgs

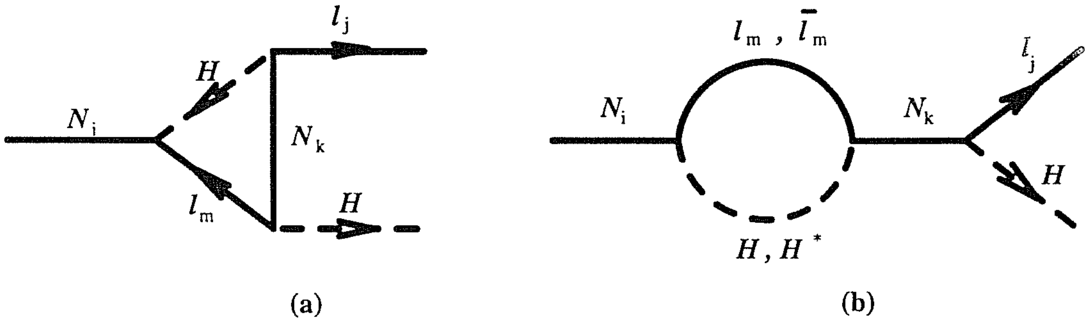


Figure 2.1: Diagrams contributing to the vertex (Fig. 2.1a) and wave function (Fig. 2.1b) CP violation in the heavy singlet neutrino decay.

fields) which appear as final states in the N decays[1]. Note that at tree level

$$\Gamma_{N_i\ell} = \Gamma_{N_i\bar{\ell}} = \frac{(\lambda^\dagger\lambda)_{ii}}{16\pi} M_i. \tag{2.3}$$

As we have remarked no asymmetry is present at this level.

The asymmetry ϵ_ℓ^N arises from the interference of the one–loop diagrams depicted in Fig. 2.1 with the tree level coupling. The vertex correction in Fig. 2.1a is the one that is usually considered, but it has however been pointed out [33, 30, 32] that the wave function piece in Fig. 2.1b also contributes to the asymmetry, in an amount which is typically comparable to the vertex contribution. We have computed these CP violating asymmetries and we obtain for the vertex contribution to ϵ_ℓ^N

$$\epsilon_\ell^{N_i}(\text{vertex}) = 2\sum_k \text{Im}\,\{J_t\,(x_k)\}\mathcal{I}_{ki}\ , \tag{2.4}$$

where $x_k \equiv M_k^2/M_i^2$ and we have defined

$$\mathcal{I}_{ki} \equiv \frac{\text{Im}\left[\left(\lambda^\dagger\lambda\right)^2_{ki}\right]}{(\lambda^\dagger\lambda)_{ii}}. \tag{2.5}$$

[1]For the Majorana light neutrinos, one should think of ℓ and $\bar{\ell}$ as being the left and right–handed helicities of ν.

$\mathrm{Im}\,\{J_t\}$ is the absorptive part of the loop integral in Fig. 2.1a, defined as

$$\bar{u}(p-k)P_R u(p) J_t(x_k) = \bar{u}(p-k)P_R \int \frac{d^4q}{(2\pi)^4} S(q-k, M_k) S(q,0) D(p-q,0) P_L u(p), \tag{2.6}$$

where p is the singlet neutrino momentum and k the final Higgs boson momentum. This quantity results to be, as computed in appendix A.1,

$$\mathrm{Im}\,\{J_t(x)\} = \frac{1}{16\pi}\sqrt{x}\left(1-(1+x)\ln\left[\frac{1+x}{x}\right]\right). \tag{2.7}$$

For the wave function piece we find instead

$$\epsilon_{\ell_j}^{N_i}(\mathrm{wave}) = 4\sum_{k\neq i} \mathrm{Im}\,\{J_s\} \frac{M_i}{M_k^2-M_i^2} \frac{\mathrm{Im}\left\{\left[M_k\left(\lambda^\dagger\lambda\right)_{ki} + M_i\left(\lambda^\dagger\lambda\right)_{ik}\right]\lambda_{jk}^*\lambda_{ji}\right\}}{(\lambda^\dagger\lambda)_{ii}}. \tag{2.8}$$

In this case the loop integral is shown in Fig. 2.1b and is defined by

$$\bar{u}(p-k)P_R u(p) M_1 J_s = -i\bar{u}(p-k)P_R \int \frac{d^4q}{(2\pi)^4} S(q,0) D(p-q,0) u(p), \tag{2.9}$$

so that the absorptive part is, as shown in appendix A.1,

$$\mathrm{Im}\{J_s\} = -\frac{1}{32\pi}. \tag{2.10}$$

This result satisfies the unitarity relation we have found in chapter 1, (1.23),

$$|T_{fi}|^2 - |T_{if}|^2 \simeq -2\ \mathrm{Im}\left[\sum_n T_{ni}T_{nf}^*T_{if}\right],$$

where we are retaining only the leading order terms; i and f are the initial and final states, and n are the possible intermediate states connecting them. T_{fn} are the transition amplitudes between the intermediate and final states, which in the present case arise from both the s and t–channel N_k exchanges, which correspond to wave and vertex contributions respectively.

The result in eq. (2.4) coincides with the one recently obtained in ref. [9], differing by factors of 2, 4 and 8 from those in ref. [34], [7] and [8] respectively[2]. The wave function piece, which in this scenario was considered previously by Liu and Segrè [32], is a factor of two larger than their result due to the fact that their computation applies to the case in which the scalar field is real and only the neutral lepton runs in the loop. Adding the charged lepton loop contribution the result is doubled. In eq. (2.8) we do not include in the sum over the flavor of the intermediate neutrinos the case $k = i$, since in general states degenerate with the initial one do not contribute to the CP violating asymmetry [27]. We also assumed that $|M_k - M_i| \gg \Gamma_{N_k}$ in the computation. Notice that the contribution from $k = i$ in the vertex piece, being proportional to $\mathrm{Im}\left[(\lambda^\dagger\lambda)^2_{ii}\right] = 0$, also vanishes. Hence, both sums in eqs. (2.4) and (2.8) may be restricted to $k \neq i$. In eq. (2.8) we have not summed over the final lepton flavor, but after summing over it and substituting $\mathrm{Im}\{J_t\}$, one gets

$$\epsilon_\ell^{N_i}(\text{wave}) = -\frac{1}{8\pi}\sum_{k\neq i}\frac{M_i M_k}{M_k^2 - M_i^2}\,\mathcal{I}_{ki}, \tag{2.11}$$

so that the piece proportional to M_i in the square brackets in eq. (2.8) actually gives a null contribution to the total lepton number asymmetry (although it may generate an asymmetry in the individual leptonic numbers L_j). This same quantity has been computed employing a different formalism in ref. [35] and their results, initially in disagreement with ours, have been corrected and they now agree with eq.(2.11) in the limit of strong hierarchy.

In ref. [32] it was pointed out that there should also be in the vertex contribution, in addition to the result in eq. (2.4), a term like the one just discussed appearing in eq. (2.8). This term appears however only when a single real scalar Higgs field is allowed to run in the loop, but it can be shown that the new contri-

[2]The normalization of the ϵ parameters are sometimes different (and not always explicit).

butions arising from the two real scalars in the decomposition of the standard model Higgs fields $H^\alpha = (H_1^\alpha + iH_2^\alpha)/\sqrt{2}$ actually cancel each other, leaving only the term proportional to $\mathcal{I}_{ki}$ present in eq. (2.4).

In the particular case in which a hierarchy among the right–handed neutrino masses is considered, i.e. for $x_k \gg 1$, we have that the wave function contribution becomes twice as large as the vertex one, and hence the total asymmetry produced in the decay of the lightest heavy neutrino[3], N_1, becomes

$$\epsilon_\ell^{N_1} \equiv \epsilon_\ell^{N_1}(\text{vertex}) + \epsilon_\ell^{N_1}(\text{wave}) \simeq -\frac{3}{16\pi} \sum_{k\neq 1} \frac{1}{\sqrt{x_k}} \mathcal{I}_{k1}. \tag{2.12}$$

The inclusion of the wave function piece then increases by a factor of three the CP violating asymmetry in the case of large mass hierarchies.

2.1.1 Baryon number generation

Let us try now to estimate the lepton number generated in the decay of the lightest singlet neutrino and the subsequent baryon number.

If the departure from equilibrium in N_1 decay is large, i.e.

$$\Gamma_{N_1} = (\lambda^\dagger\lambda)_{11}/(8\pi)M_1 \ll H(\mathrm{T} = M_1) \simeq g_*^{1/2} M_1^2/M_{Pl}, \tag{2.13}$$

the lepton number asymmetry produced, per unit entropy, is

$$\frac{n_L}{s} \simeq \frac{\epsilon_\ell^{N_1}}{s} \frac{g_N \mathrm{T}^3}{\pi^2}, \tag{2.14}$$

where $g_N = 2$ are the spin degrees of freedom of the Majorana neutrino N_1, so that their number density (assuming Maxwell–Boltzmann statistics) before they decay is

[3]In the case of large hierarchies, the asymmetries produced by the heavier neutrino decays are usually erased before the lightest one decays.

$g_N T^3/\pi^2$ if we assume that they were in chemical equilibrium before becoming non–relativistic (see below). Using eq.(1.8), where g_*, defined in eq.(1.9), is the effective number of relativistic degrees of freedom contributing to the entropy, ($g_* = 106.75$ in the standard non–supersymmetric version of the model), we get

$$\frac{n_L}{s} \simeq 4 \times 10^{-3} \epsilon_\ell^{N_1}. \tag{2.15}$$

This lepton asymmetry will then be reprocessed by anomalous electroweak processes, leading to a baryon asymmetry given by eq.(1.16):

$$n_B = \left(\frac{24 + 4N_H}{66 + 13N_H}\right) n_{B-L} \simeq -\frac{28}{79} n_L. \tag{2.16}$$

Here N_H is the number of Higgs doublets ($N_H = 1$ in the standard scenario considered now, while $N_H = 2$ in the supersymmetric version to be discussed below if the scalar Higgs bosons are assumed to be lighter than the electroweak symmetry breaking scale). Combining eqs.(2.15) and (2.16) and assuming that the Universe expansion is adiabatic, the present baryon asymmetry which results is

$$\frac{n_B}{s} \simeq -1.5 \times 10^{-3} \epsilon_\ell^{N_1}. \tag{2.17}$$

Values of $\epsilon_\ell^N \simeq -5 \times 10^{-8}$ are then required to account for the value $n_B/s \simeq 0.6$–1×10^{-10} inferred from the successful theory of primordial nucleosynthesis [17]. In the case in which there is a hierarchy in the heavy neutrino masses, the Yukawa parameters need then to satisfy

$$\sum_{k \neq 1} \frac{M_1}{M_k} \mathcal{I}_{k1} \simeq 0.9 \times 10^{-6}. \tag{2.18}$$

Let us also briefly mention that the lepton asymmetry produced by N decays is smaller than the value in eq. (2.14) if the departure from equilibrium is not large during the decay epoch, as is the case if the decay rate $\Gamma_1 \equiv \Gamma_{N_1 \ell} + \Gamma_{N_1 \bar{\ell}}$ is comparable or larger than the expansion rate of the Universe when N_1 becomes non relativistic, i.e. $\Gamma_1 \geq H(\mathrm{T} = M_1)$ (see references [7, 9]).

We also want to emphasize that a problem of these scenarios is that the Yukawa interactions are not effective in establishing an equilibrium population of heavy neutrinos. To see this, note that before N_1 becomes non–relativistic, i.e. for $x \equiv M_1/\mathrm{T} \ll 1$, the thermally averaged rate scales as T^{-1}, since $\langle\Gamma_1\rangle = K_1(x)/K_2(x)\Gamma_1 \simeq (x/2)\Gamma_1 \propto M_1^2/\mathrm{T}$, where $K_{1,2}$ are Bessel functions [7]. Taking into account that $H \propto \mathrm{T}^2/M_{Pl}$ and supposing that $\Gamma_1 \ll H(\mathrm{T} = M_1)$ (so that the departure from equilibrium during the decay is significant), one concludes that $\langle\Gamma_1\rangle \ll H$ holds true also for all temperatures larger than M_1. Hence, the inverse decays are unable to establish chemical equilibrium for N_1 at any temperature $\mathrm{T} > M_1$. The pair production of N_1 from Higgs boson or light lepton scattering may do somewhat better than inverse decays at high temperatures, since the relevant rate scales as $\langle\sigma v\rangle \propto \mathrm{T}$, but it is anyhow insufficient to achieve chemical equilibrium for N_1 because the rates need to be quite suppressed at $\mathrm{T} \simeq M_1$ in order not to erase any lepton asymmetry generated during the decay, and consequently are also suppressed with respect to the expansion rate at higher temperatures. Hence, additional interactions of the heavy neutrinos are required for them to have a thermal distribution prior to the decay, so that the leptogenesis scenario can be successful. This point was recently remarked in ref. [9], where it was shown that gauge interactions which are naturally present in GUT scenarios (new Z' or $\mathrm{SU}(2)_R$ gauge bosons) can easily do the job of producing a thermal population of heavy neutrinos at $\mathrm{T} > M_1$.

2.2 Supersymmetric extension of the model

Turning now to the supersymmetric version of the Fukugita–Yanagida model, the interactions of the heavy (s)neutrino field can be derived from the superpotential

$$W = \frac{1}{2} M_i N_i N_i + \lambda_{ij} \epsilon_{\alpha\beta} L_i^\alpha H^\beta N_j. \tag{2.19}$$

Supersymmetry breaking terms are of no relevance for the mechanism of lepton number generation; even the mass difference between the particles and their superpartners, in the range of $10^3 - 10^4$ GeV, is negligible at the high temperatures of the order of the singlet neutrino mass, so in the following we will consider also the superparticles as massless.

From eq. (2.19) we are then left with the following trilinear couplings in the Lagrangian, in terms of four component spinors,

$$\mathcal{L} = -\lambda_{ij} \epsilon_{\alpha\beta} \left\{ M_j \tilde{N}_j^* \tilde{L}_i^\alpha H^\beta + \overline{N_j} P_L \ell_i^\alpha H^\beta + \overline{(\tilde{h}^\beta)^c} P_L \ell_i^\alpha \tilde{N}_j + \overline{(\tilde{h}^\beta)^c} P_L N_j \tilde{L}_i^\alpha \right\} + h.c. \; . \tag{2.20}$$

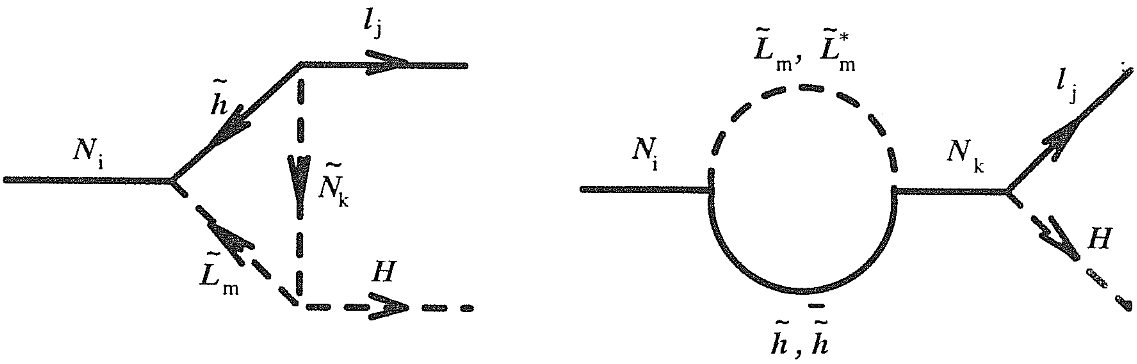
From these couplings we obtain the tree level relations

$$\Gamma_{N_i \ell} + \Gamma_{N_i \bar{\ell}} = \Gamma_{N_i \tilde{L}} + \Gamma_{N_i \tilde{L}^*} = \Gamma_{\tilde{N}^* \ell} = \Gamma_{\tilde{N} \tilde{L}} = \frac{(\lambda^\dagger \lambda)_{ii}}{8\pi} M_i. \tag{2.21}$$

There are now new diagrams, like the ones in Figure 2.2, contributing to the generation of a leptonic asymmetry. We will denote the corresponding asymmetry parameters in the supersymmetric case with a tilde, so that for instance $\tilde{\epsilon}_\ell^N$(vertex) will receive contributions from the 'standard' diagram in Figure 2.1a as well as from the one in Figure 2.2a which involves superpartners in the loop[4]. We also define the slepton asymmetry associated to N decays as

$$\tilde{\epsilon}_{\tilde{L}}^N \equiv \frac{\Gamma_{N\tilde{L}} - \Gamma_{N\tilde{L}^*}}{\Gamma_{N\tilde{L}} + \Gamma_{N\tilde{L}^*}}, \tag{2.22}$$

[4]For later convenience, we define $\tilde{\epsilon}_\ell^N$ as in (2.2), rather than normalizing it to the total N decay rate which also includes the slepton final states.



(a) (b)

Figure 2.2: Supersymmetric contribution to the diagrams leading to CP violation in the $N \to \ell H$ decay.

which arises from diagrams similar to those in Figures 2.1 and 2.2 but with $\tilde L \tilde h$ in the final state. We similarly define the asymmetry parameters associated to sneutrino decays as

$$\tilde\epsilon_\ell^{\tilde N^*} \equiv \frac{\Gamma_{\tilde N^*\ell} - \Gamma_{\tilde N \bar\ell}}{\Gamma_{\tilde N^*\ell} + \Gamma_{\tilde N \bar\ell}} \quad , \quad \tilde\epsilon_{\tilde L}^{\tilde N} \equiv \frac{\Gamma_{\tilde N \tilde L} - \Gamma_{\tilde N^* \tilde L^*}}{\Gamma_{\tilde N \tilde L} + \Gamma_{\tilde N^* \tilde L^*}}. \qquad (2.23)$$

After a direct computation of these quantities we obtain

$$\tilde\epsilon_\ell^{N_i}(\text{vertex}) = 2\sum_k \text{Im}\,\{\tilde J_t\,(x_k)\}\mathcal{I}_{ki}\ , \qquad (2.24)$$

where the loop integral has in this case contributions coming not only from the diagrams in Fig. 2.1a, but also from the one in Fig. 2.2a, with superpartners in the intermediate state. The diagram absorptive part is

$$\text{Im}\,\{\tilde J_t(x)\} = -\frac{1}{16\pi}\sqrt{x}\ \ln\left[(1+x)/x\right]. \qquad (2.25)$$

For the wave function piece in the supersymmetric model, there is also an additional contribution with superpartners intermediate states, so that we have

$$\tilde\epsilon_\ell^{N}(\text{wave}) = 2\epsilon_\ell^{N}(\text{wave}). \qquad (2.26)$$

For the remaining channels we get similar results

$$\tilde{\epsilon}_{\tilde{L}}^{N}(\text{vertex}) = \tilde{\epsilon}_{\ell}^{\tilde{N}^*}(\text{vertex}) = \tilde{\epsilon}_{\tilde{L}}^{\tilde{N}}(\text{vertex}) = \tilde{\epsilon}_{\ell}^{N}(\text{vertex}), \tag{2.27}$$

and also the wave function contributions are equal

$$\tilde{\epsilon}_{\tilde{L}}^{N}(\text{wave}) = \tilde{\epsilon}_{\ell}^{\tilde{N}^*}(\text{wave}) = \tilde{\epsilon}_{\tilde{L}}^{\tilde{N}}(\text{wave}) = \tilde{\epsilon}_{\ell}^{N}(\text{wave}). \tag{2.28}$$

The vertex pieces were computed previously in ref. [11] and our results are proportional. The sneutrino decay asymmetry was also considered in ref. [12] in the limit in which all heavy (s)neutrinos are degenerate and, specialized to that case, our results are in agreement (except for the overall sign). The wave function contributions were not considered previously and, as in the non–supersymmetric case, they are non–negligible. Let us also note that there are additional one loop diagrams involving the four–scalar couplings appearing in the F–terms of the scalar potential. Although helpful to cure the divergences in the loops, they do not contribute to the asymmetry generation.

For the study of the decays of thermal populations of heavy neutrinos and sneutrinos, it is convenient to introduce the total asymmetry defined as

$$\tilde{\epsilon}_i \equiv \tilde{\epsilon}_{\ell}^{N_i} + \tilde{\epsilon}_{\tilde{L}}^{N_i} + \tilde{\epsilon}_{\ell}^{\tilde{N}_i^*} + \tilde{\epsilon}_{\tilde{L}}^{\tilde{N}_i} = 4\tilde{\epsilon}_{\ell}^{N}, \tag{2.29}$$

resulting in

$$\tilde{\epsilon}_i = -\frac{1}{2\pi}\sum_{k\neq i}\sqrt{x_k}\left[\ln\left(1+\frac{1}{x_k}\right)+\frac{2}{x_k-1}\right]\mathcal{I}_{ki}. \tag{2.30}$$

In particular, in the case of hierarchical masses ($x_k \gg 1$), we find that again the wave contribution becomes twice as large as the vertex one, giving

$$\tilde{\epsilon}_1 \simeq -\frac{3}{2\pi}\sum_{k\neq 1}\frac{1}{\sqrt{x_k}}\,\mathcal{I}_{k1}. \tag{2.31}$$

With the definition in eq. (2.29), the resulting lepton asymmetry is[5]

$$\frac{n_L}{s} = \frac{\tilde{\epsilon}_1}{s}\frac{T^3}{\pi^2} \simeq 1 \times 10^{-3}\tilde{\epsilon}_1, \tag{2.32}$$

where we have used that the effective number of degrees of freedom in the supersymmetric scenario is approximately doubled, i.e. $g_*^{SUSY} \simeq 2\ g_*^{SM}$.

Assuming the sleptons to be heavier than 300 GeV and the lepton number to be conserved in their decay process, so that they can all decay into leptons before the EW phase transition, we can still use eq. (1.16) for computing the baryon number. Hence, to account for the baryon asymmetry of the Universe we need now

$$\sum_{k\neq 1} \frac{M_1}{M_k}\ \mathcal{I}_{k1} \simeq 0.7 \times 10^{-6}. \tag{2.33}$$

Comparing this with eq. (2.18), we see that the required parameters are similar in the supersymmetric and standard scenarios.

2.3 The see-saw mechanism

Let us consider now the consequences of the lagrangian (2.1) on the mass spectrum of the neutrino's. At the electroweak transition one neutral component of the Higgs field takes a non-vanishing vacuum expectation value $v/\sqrt{2}$, so that the Yukawa interaction gives rise to a Dirac mass term involving the LH and RH neutrinos similarly to what happens to the other standard model particles. We have therefore the mass terms:

$$\mathcal{L}_{\rm m} = -\frac{1}{2}M_j\overline{N_j^c}N_j - \lambda_{ij}\frac{v}{\sqrt{2}}\overline{N_j}P_L\nu_i + h.c. \tag{2.34}$$

$$= -\frac{1}{2}M_j\overline{N_j^c}N_j - M_{ij}^D\overline{N_j}P_L\nu_i + h.c.; \tag{2.35}$$

[5]Had we normalized $\epsilon^N_{\ell,\tilde{L}}$ to the total N decay rate, their contribution to n_L would have to be multiplied by $g_N = 2$ as in eq. (2.14).

due to the fact that M^D is of the order of the EW scale and therefore $\ll M_j$, this lagrangian displays the so called see-saw mechanism [36, 37] that ensures a very small, but non zero mass for the light neutrinos.

Let us for the moment neglect the family structure of the theory and consider the simpler case of only one family. Defining the field $n^T = (\nu, N^c)$ we can rewrite the mass terms as

$$\mathcal{L}_{\rm m} = -\frac{1}{2}\overline{n^c}\mathcal{M}P_L n + h.c. \tag{2.36}$$

where now $\mathcal{M}$ is a mass matrix:

$$\mathcal{M} = \begin{pmatrix} 0 & M^D \\ M^D & M \end{pmatrix} \tag{2.37}$$

Then the physical states corresponding to the mass matrix eigenvectors are Majorana fermions with different CP parities and the mass eigenvalues are

$$m_{1,2} = \frac{M}{2} \pm \frac{1}{2}\sqrt{M^2 + 4(M^D)^2}. \tag{2.38}$$

We see that in case $M^D \ll M$ the two eigenvalues are very split:

$$m_1 \simeq M \qquad m_2 \simeq \frac{(M^D)^2}{M}. \tag{2.39}$$

The first eigenvector is constituted mainly by the N component, while the second by the ν component.

In the case of more than one family, the structure is more involved and the light states present an effective Majorana mass

$$m \simeq (M^D)^T M^{-1} M^D \tag{2.40}$$

given by the diagrams with intermediate heavy states. Such a mass matrix would be in general non-diagonal and give rise to light neutrino mixing [38].

The picture described above is typical of the $SO(10)$ unification group [36]; in this model all the 15 fermions belonging to one generation can be accommodated into the 16-dimensional spinor representation of that group. The additional 16th particle, behaves exactly as a RH SM singlet neutrino. Its Majorana mass term could be then the remnant of the spontaneous breaking of $SO(10)$ and therefore be quite large.

2.4 Neutrino's masses and mixings

We have seen in the previous section, that the same terms in the lagrangian can give rise to the BAU and the light masses and mixings for the neutrinos. The same Yukawa couplings are involved in both cases, so that it seems an interesting question whether it is possible to account at one time for all the experimental evidence on neutrinos and baryogenesis. Unfortunately it seems that this would not be an easy task since the baryon number is related to the *phases* of the mass matrix and no informations are nowadays available regarding the phases in the neutrino mass matrix, since every effect of such phases is suppressed by the small mass itself and it is therefore well below the experimental bounds [34]. Anyway it is still possible to use the BAU and other cosmological constraints to get an insight into the mass structure of neutrinos [7, 39, 9].

Let us illustrate this in a toy model. Consider the Dirac mass matrix M^D to be equal to the one of the up-type quarks, as it would be in the case of $SO(10)$ and suppose that its hierarchical structure would be maintained also in the basis where

M is diagonal. So we would have

$$\lambda \simeq \begin{pmatrix} \eta^2 A & \eta^2 B & \eta^2 C \\ \eta^2 B' & \eta D & \eta E \\ \eta^2 C' & \eta E' & 1 \end{pmatrix} \frac{\sqrt{2}\, m_t}{v} \tag{2.41}$$

where η is a small parameter $\eta \simeq m_c/m_t \simeq m_u/m_c \simeq 10^{-2}$ with m_t, m_c, m_u equal to the top, charm, up quark mass respectively and A, B, B',etc.. are complex numbers of modulus of order 1. We are considering here the case where both the heavy Majorana mass matrix and the leptonic EW charged interactions are flavor diagonal; therefore no freedom is left for diagonalizing the λ matrix or absorbing its phases.

For this Yukawa matrix, the mass of the ν_τ would be, in the case that the hierarchy between the RH neutrino is smaller than that between the quarks families, i.e. $M_1/M_2, M_2/M_3 \leq \eta$,

$$m_{\nu_\tau} \simeq \frac{m_t^2}{M_3} \ll m_t; \tag{2.42}$$

requiring this to be around 10 eV, so that tau neutrinos would be good candidates for the hot dark matter [10], imposes the constraint $M_3 \simeq 10^{12}$ GeV.

Regarding the baryon asymmetry, under the same assumption on the hierarchies, the dominant part of ϵ_1 in eq.(2.12) would be given by the exchange of an N_3 in the loop:

$$\epsilon_1 \simeq -\frac{3}{16\pi}\frac{M_1}{M_3}\mathcal{I}_{31}; \tag{2.43}$$

keeping only the leading terms in η we get

$$\epsilon_1 \simeq -\frac{3}{16\pi}\frac{M_1}{M_3}\frac{2m_t^2}{v}\sin(2\delta) \tag{2.44}$$

where δ is the phase of the relevant Yukawa coupling (C').

Comparing this number to the observed asymmetry and using $2m_t^2/v \simeq 1$ gives the bound

$$\frac{M_1}{M_3}\sin(2\delta) \simeq 0.9 \times 10^{-6} \tag{2.45}$$

We can see then that also a small phase could generate the BAU if the hierarchy $M_1/M_3 \leq 10^{-2}$.

What we have considered here is the maximal baryon asymmetry generated in the way out of equilibrium conditions. Let us check now the bound (2.13) on the Yukawa matrix,

$$(\lambda^\dagger\lambda)_{11}/(8\pi) \ll g_*^{1/2} M_1/M_{Pl}, \tag{2.46}$$

this yields in our toy model:

$$M_1/M_{Pl} > 4 \times 10^{-3}\eta^4 = 4 \times 10^{-11} \tag{2.47}$$

corresponding to a mass scale

$$M_1 > 5 \times 10^8 \text{GeV}. \tag{2.48}$$

We have seen therefore in a simple toy model that it is possible, in the case of mass hierarchy, to generate the BAU at a scale T $\leq M_1 \simeq 10^8 - 10^9$ GeV; such a scale is lower than the scale of GUT baryogenesis and could possibly be reached also in the inflationary models after re-heating.

Chapter 3

The case of quasi-degenerate singlet sneutrinos

We will consider in this chapter the wave function contribution to the CP violating asymmetries produced in the decay of heavy particles, studying the effects of heavy particle mixing for arbitrary mass splittings in the case of bosonic particles.

In the previous chapter we have seen that apart from the one loop contribution which is usually taken into account, i.e. the vertex one, in which two light particles produced in the decay of a heavy one exchange another heavy particle in the t-channel, a second possibility is present, i.e. the wave function contribution [33, 30, 32, 35, 13]. In this case the intermediate heavy particle is exchanged in the s-channel so that the diagram contains a loop of light particles that just mixes the initial state Φ_a with another different heavy state Φ_b, and this later decays to the final state as shown in Fig. 3.1c. This wave contribution turns out to be comparable to the vertex one when the heavy states have large mass splittings, as we have seen, and may be significantly enhanced for nearly degenerate states due to the resonance effect

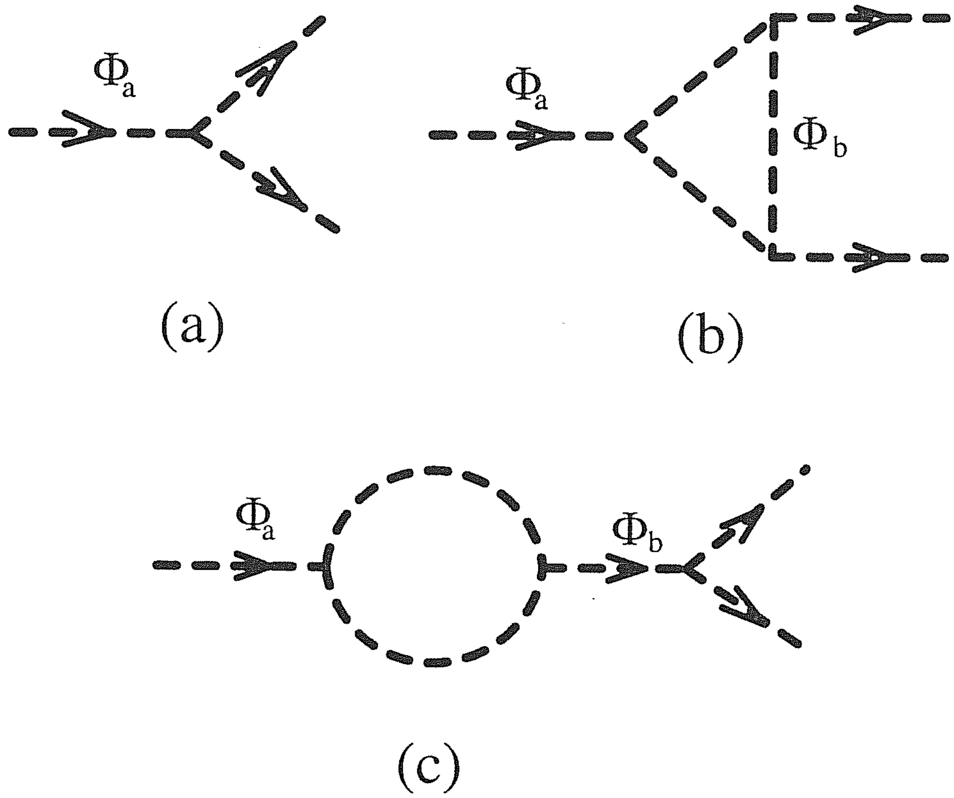


Figure 3.1: Diagrams which interfere to produce the CP violation in the heavy particle decay. Fig. 3.1b gives the so called vertex contribution while Fig. 3.1c gives the wave function one.

in the s-channel. To define appropriately the CP asymmetry for any mass splitting we will use the formalism describing unstable particle oscillations [27].

3.1 General formalism

The asymmetry in a global quantum number N (for instance B or L) produced in the decay of a pair made of particle Φ_a and its antiparticle $\bar{\Phi}_a$ is given by

$$\epsilon_a = \sum_f \epsilon_{fa}, \tag{3.1}$$

where the quantity of interest to us is the partial rate asymmetry per decay into final state f (of global charge N_f), given by

$$\epsilon_{fa} = N_f \left[\mathrm{BR}(\Phi_a \to f) - \mathrm{BR}(\bar{\Phi}_a \to \bar{f}) \right], \tag{3.2}$$

where $\mathrm{BR}(\Phi_a \to f)$ is just the branching ratio for the decay of particle Φ_a into the final state f. This definition is the most general; it coincides with twice eq. (1.28) in chapter 1 and is equal to eq. (2.2) in chapter 2 in the case of negligible mixing. In fact, if the state Φ_a is nearly an eigenstate of the one loop self energy matrix, we have

$$\mathrm{BR}(\Phi_a \to f) \equiv \frac{\Gamma(\Phi_a \to f)}{\Gamma(\Phi_a \to \text{anything})}. \tag{3.3}$$

Otherwise, for a state which is strongly mixed, it is not possible to define its decay rate and eq. (3.2) defines the asymmetry.

The wave function contribution to this quantity behaves, in the limit of large mass splittings, as $\epsilon_{fa}(\text{wave}) \propto (M_a^2 - M_b^2)^{-1}$, due to the propagator of the intermediate state Φ_b, and hence is expected to be enhanced in the limit $M_b \to M_a$. A general phenomenological approach to study this quantity for arbitrary mass splittings has been considered in ref. [27]. It is based on the work of Weisskopf and Wigner [40] and describes the behavior of mixed unstable particles introducing an effective hamiltonian and solving the corresponding Schrödinger equation.

It is our purpose here to extend the formalism developed in ref. [27] and apply the results to the study of leptogenesis scenarios, for which a computation of the CP violation for large mass splittings has been discussed in the previous chapter [13].

To be specific, we will consider the case in which the heavy decaying particles are scalars, and ignore the vertex CP violation effects, which can be studied separately. The wave function mixing will have the effect of inducing an absorptive part in the heavy particle propagators, which will be responsible for the generation of the asymmetry. The effect of the one–loop self–energy diagrams in the propagators will be to modify the squared scalar mass matrix as follows

$$M_a^{(0)2}\delta_{ab} \to H_{ab}^2 = M_{ab}^2 - i\Gamma_{ab}^2, \tag{3.4}$$

where the renormalized mass matrix M^2 includes the (divergent) dispersive part of the loops while the matrix Γ^2 arises from the (finite) absorptive part alone. The matrices M^2 and Γ^2 are hermitian, but H^2 is not, it is a general complex matrix. Let us consider in the following such matrix to be diagonalizable; otherwise it would not be possible to decouple the Schrödinger equations for the system. Hence, H^2 will be diagonalized by a non–unitary transformation matrix V [27], i.e.

$$(VH^2V^{-1})_{ab} = \omega_a^2\delta_{ab}. \tag{3.5}$$

This matrix V will then transform the initial 'flavor' states $|\Phi_a\rangle$ into the 'propagation' eigenstates[1] $|\Phi'_c\rangle$, i.e.

$$|\Phi'_c\rangle = V_{ac}^{-1}|\Phi_a\rangle. \tag{3.6}$$

Similarly, for the antiparticle states $|\bar{\Phi}_a\rangle$, one will have

$$|\bar{\Phi}'_c\rangle = V_{ca}|\bar{\Phi}_a\rangle. \tag{3.7}$$

These propagation eigenstates are the ones that will evolve as

$$|\Phi'_c(t)\rangle = \mathrm{e}^{-i\omega_c t}|\Phi'_c(0)\rangle. \tag{3.8}$$

[1]The appearance of V^{-1} in eq. (3.5) ensures that the kinetic term remains canonical, but the fact that $V^{-1} \neq V^\dagger$ implies that the propagation eigenstates are not orthonormal.

Considering then the transition amplitude from the state $|\Phi_a\rangle$ to a final state $|f\rangle$, we have

$$T_{fa} = \langle f|H_{int}|\Phi_a\rangle, \tag{3.9}$$

where H_{int} describes the interactions of Φ_a with the final state particles. From the superposition principle and using eqs. (3.4),(3.5), one has

$$\begin{aligned} T_{fa}(t) = & \sum_{b,c} T_{fb} V_{bc}^{-1} V_{ca} \mathrm{e}^{-i\omega_c t} \\ \bar{T}_{fa}(t) = & \sum_{b,c} T_{fb}^{*} V_{cb} V_{ac}^{-1} \mathrm{e}^{-i\omega_c t}. \end{aligned} \tag{3.10}$$

If one starts with a state made of pairs of ϕ_a and $\bar{\phi}_a$ flavor eigenstates, the differential partial decay rate asymmetry arising from particle mixing will be proportional to the quantity[2]

$$\Delta_{fa}(t) \equiv |T_{fa}(t)|^2 - |\bar{T}_{fa}(t)|^2. \tag{3.11}$$

It is interesting to notice that in the limit of degenerate propagation eigenstates, i.e. $\omega_c = \omega$, this asymmetry vanishes, as can be seen from eqs. (3.10),(3.11) [3].

To continue we will concentrate in the case of mixing between just two particles, for which the matrix V can be parameterized in terms of two complex mixing angles, θ and ϕ, as follows

$$V = \begin{pmatrix} \cos\theta & -\sin\theta \mathrm{e}^{i\phi} \\ \sin\theta \mathrm{e}^{-i\phi} & \cos\theta \end{pmatrix}. \tag{3.12}$$

In this case the trigonometric functions of a complex argument are defined through their expansion series; the trigonometric relations like

$$\sin^2\theta + \cos^2\theta = 1 \tag{3.13}$$

[2]If instead of starting with 'pure states', the initial state consisted of mixed states (as would be the case for a thermal population), the computation of the asymmetry would actually require to follow the evolution of the density matrix describing the initial state.

[3]As we will see, this is true only for a diagonalizable hamiltonian.

are still valid, but we have that $|\sin\theta|, |\cos\theta|$ are not bounded.

Replacing V in eq. (3.5) it is easy to obtain

$$e^{2i\phi} = \frac{H^2_{12}}{H^2_{21}} \quad ; \quad (\mathrm{tg}2\theta)^2 = \frac{4H^2_{12}H^2_{21}}{(H^2_{11} - H^2_{22})^2}, \tag{3.14}$$

where we recall that $H^2_{21} = M^{2*}_{12} - i\Gamma^{2*}_{12}$. Notice that the matrix (3.12) becomes the identity if the hamiltonian is already diagonal as it should be.

The eigenvalues of H^2 are then

$$\omega^2_{1,2} = \frac{1}{2}\left\{H^2_{11} + H^2_{22} \pm \sqrt{(H^2_{11} - H^2_{22})^2 + 4H^2_{12}H^2_{21}}\right\}. \tag{3.15}$$

We see that for

$$(H^2_{11} - H^2_{22})^2 + 4H^2_{12}H^2_{21} = 0, \tag{3.16}$$

the two eigenvalues are equal and the transformation (3.12) is no more well-defined: in fact we have $(\mathrm{tg}2\theta)^2 = -1$, i.e. $\sin^2 2\theta = -\cos^2 2\theta$, that cannot be satisfied by any θ. Two possibilities are then present in this case: either the hamiltonian is already diagonal, i.e. $H_{12} = H_{21} = 0$, or it is non–diagonalizable. In this last case we cannot apply our formalism, since there are no eigenvectors evolving independently according to (3.8). We will see later than the degeneracy occurs anyway only in a very particular situation.

3.1.1 The CP asymmetry

Let us go back to the diagonalizable case. After an explicit computation we then get for the asymmetry the expression:

$$\Delta_{fa}(t) = 2\mathrm{Re}\left\{T_{f1}T^*_{f2}\left[U_{1a}U^*_{2a} - U_{a2}U^*_{a1}\right]\right\} + |T_{fb}|^2\left\{|U_{ba}|^2 - |U_{ab}|^2\right\}, \tag{3.17}$$

where we have defined

$$U_{ab} \equiv V_{ac}^{-1} W_c V_{cb}, \tag{3.18}$$

with

$$W_c \equiv \mathrm{e}^{-i\omega_c t}. \tag{3.19}$$

In the case in which the initial state under consideration is an eigenstate of the (renormalized) mass matrix, i.e. for $M_{ab}^2 = M_a^2 \delta_{ab}$, all these expressions simplify considerably and we have in fact

$$\mathrm{e}^{2i\phi} = \frac{(\Gamma_{12}^2)^2}{|\Gamma_{21}^2|^2} \quad ; \quad (\mathrm{tg}2\theta)^2 = \frac{-4|\Gamma_{12}^2|^2}{[M_1^2 - M_2^2 - i(\Gamma_{11}^2 - \Gamma_{22}^2)]^2}; \tag{3.20}$$

we can see that in this case ϕ is real and corresponds to the phase of Γ_{12}^2.

Also eq. (3.17) is simpler since the second term on the r.h.s. vanishes; we have then

$$\Delta_{f1}(t) = 4\frac{\mathrm{Im}\left\{T_{f1}T_{f2}^*\Gamma_{12}^2\right\}}{|\omega_1^2 - \omega_2^2|^2}\mathrm{Re}\left\{(\omega_1^2 - \omega_2^2)(W_2^* - W_1^*)(\cos^2\theta W_1 + \sin^2\theta W_2)\right\}, \tag{3.21}$$

and a similar result holds for Δ_{f2} with the substitution $W_2 \leftrightarrow W_1$.

We will then compute in detail the integrated rate asymmetry in this case. The branching ratios entering in eq. (3.2) are just

$$\mathrm{BR}(\Phi_a \to f) = \int d\Omega_a \int_0^\infty dt\ |T_{fa}(t)|^2, \tag{3.22}$$

with $d\Omega_a$ the phase space element of particle Φ_a. We have then

$$\epsilon_{fa}(\text{wave}) = N_f \Omega_a \int_0^\infty dt \Delta_{fa}(t), \tag{3.23}$$

where $\Omega_a = M_a/16\pi$ in our case of two body scalar decay.

Integrating over time we find

$$\epsilon_{fa}(\text{wave}) = 2N_f\Omega_a\,\mathrm{Im}\left\{T_{f1}T_{f2}^*\Gamma_{12}^2\right\}F_a, \tag{3.24}$$

with

$$F_1 = \frac{1}{|\omega_1^2 - \omega_2^2|^2} \left\{ \text{Re}\{\omega_1^2 - \omega_2^2\} \left[\frac{1}{\gamma_2} - \frac{1}{\gamma_1} \right] - (M_1^2 - M_2^2) \left[\frac{1}{\gamma_1} + \frac{1}{\gamma_2} - \frac{\gamma_1 + \gamma_2}{|\omega_1^* - \omega_2|^2} \right] \right.$$

$$\left. + 2 \, \text{Im}\{\omega_1^2 - \omega_2^2\} \frac{(m_1 - m_2)}{|\omega_1^* - \omega_2|^2} \right\}, \tag{3.25}$$

where we defined $\omega_a \equiv m_a - i\gamma_a/2$. The result for F_2 is similar with the replacements $\gamma_1 \leftrightarrow \gamma_2$ and $m_1 \leftrightarrow m_2$ in the expression for F_1.

In the case $M_2^2 \gg M_1^2 \gg |\Gamma_{ab}^2|$, i.e. for negligible mixing, one has

$$F_a \to \frac{2}{\gamma_a(M_2^2 - M_1^2)}, \tag{3.26}$$

where in this limit $\gamma_a \simeq \Gamma_{aa}^2/M_a$ becomes just the total width of particle Φ_a. Hence, the results obtained in the previous chapter in the limit of large mass splittings can be recovered.

For decreasing mass splittings, the function F_1 reaches a maximum, which for $|\Gamma_{12}^2| \ll |\Gamma_{22}^2 - \Gamma_{11}^2|$ takes place for

$$M_2^2 - M_1^2 \simeq \Gamma_{11}^2 + \Gamma_{22}^2. \tag{3.27}$$

The value of F_1 at the maximum is $F_1 \simeq M_1/[\Gamma_{11}^2(\Gamma_{11}^2 + \Gamma_{22}^2)]$.

On the other hand, for $|\Gamma_{11}^2 - \Gamma_{22}^2| \gg |M_1^2 - M_2^2|, |\Gamma_{12}^2|$, i.e. in the limit of small mass splittings and small mixing, we get

$$F_1 \simeq \frac{(M_2^2 - M_1^2)}{|\Gamma_{11}^2 - \Gamma_{22}^2|^2} \frac{2M_1}{\Gamma_{11}^2} \left\{ 1 - \frac{4\Gamma_{11}^2 \Gamma_{22}^2}{(\Gamma_{11}^2 + \Gamma_{22}^2)^2} \right\} - \frac{1}{M_1(\Gamma_{11}^2 + \Gamma_{22}^2)}. \tag{3.28}$$

This result coincides with the one in ref. [27] only in the limit $\Gamma_{11}^2 \ll \Gamma_{22}^2$ (or $\Gamma_{11}^2 \gg \Gamma_{22}^2$) and if we neglect the last term, which although small is non–vanishing and survives in the limit $M_2^2 \to M_1^2$ (in which the propagation eigenstates are not degenerate due to $\Gamma_{11}^2 \neq \Gamma_{22}^2$).

Another interesting example is for the case $\Gamma^2_{11} = \Gamma^2_{22}$, for which we have

$$\omega_2^2 - \omega_1^2 = \sqrt{(M_2^2 - M_1^2)^2 - 4|\Gamma^2_{12}|^2} \tag{3.29}$$

We see that here for $|M_2^2 - M_1^2| = 2|\Gamma^2_{12}|$ the two propagation eigenstates become degenerate, and the matrix V singular. Notice that for $\Gamma^2_{11} \neq \Gamma^2_{22}$ the eigenvalues are never degenerate and the effective hamiltonian can always be diagonalized. In the specific singular case $\Gamma^2_{11} = \Gamma^2_{22}$ and $|M_2^2 - M_1^2| = 2|\Gamma^2_{12}|$, the hamiltonian can be put only in the Jordan form and the Schrödinger equations for the two eigenstates are not decoupled, as observed in ref. [41]. Away from this very particular values, anyway, the formalism described here is well defined and consent to evaluate correctly the CP violating asymmetry rising from particle mixing.

In particular we can compute the limit $M_2^2 \to M_1^2$ since for $|M_2^2 - M_1^2| < 2|\Gamma^2_{12}|$ the formalism is valid and we have $F_1 \simeq -1/(2M_1\Gamma^2_{11})$. The 'degenerate' situation $\Gamma^2_{11} = \Gamma^2_{22}$ and $M_1^2 = M_2^2$ actually is present in well known cases such as $K^0\bar{K}^0$ or $B^0\bar{B}^0$ mixing, where those constraints are imposed by CPT relations, and for which the integrated CP violating asymmetries are non–vanishing [42].

Let us also emphasize that a crucial ingredient in all this computation is the proper specification of the initial state. The asymmetry of course depends on the starting basis for Φ_a considered, and hence on the process which produces the initial state, so that ignoring the mixing at production would lead to incorrect results. For instance, in the case in which $M^2 \propto \mathbf{1}$ (or more generally whenever the matrices M^2 and Γ^2 commute), it is possible to change basis with a unitary transformation to make M^2 and Γ^2 both diagonal. In fact it is apparent that for $M_1 = M_2$, the second equation in (3.20) becomes

$$(\mathrm{tg}2\theta)^2 = \frac{4|\Gamma^2_{12}|^2}{(\Gamma^2_{11} - \Gamma^2_{22})^2} \tag{3.30}$$

so that θ is real and the matrix V unitary and diagonalizes both M^2 and Γ^2. Hence, the asymmetry computed in the new basis will vanish, since $\Gamma_{12}^2 = 0$ now. However, these new states may not be the quantum states generated in the production process, and therefore the new basis may not be the appropriate one to compute the resulting asymmetry.

3.2 The leptogenesis scenario

Let us now consider the particular example of lepton number generation in the out of equilibrium decay of heavy scalar neutrinos, i.e. the supersymmetric version of the Fukugita and Yanagida scenario [4]. The study of the CP violation in these models, considering both the vertex part as well as the wave contribution in the limit of large mass splittings, was carried out in the previous chapter. We now consider the effects of mixing for arbitrary masses using the formalism introduced above[4].

The Lagrangian for the scalar neutrinos is, in a basis in which the mass matrix is diagonal,

$$\mathcal{L} = -\lambda_{ia}\epsilon_{\alpha\beta}\left\{M_a\tilde{N}_a^*\tilde{L}_i^\alpha H^\beta + \overline{(\tilde{h}^\beta)^c}P_L\ell_i^\alpha\tilde{N}_a\right\} + h.c. \tag{3.31}$$

where $\ell_i^T = (\nu_i \; l_i^-)$ and $H^T = (H^+ \; H^0)$ are the lepton and Higgs doublets ($i = e, \mu, \tau$, and $\epsilon_{\alpha\beta} = -\epsilon_{\beta\alpha}$, with $\epsilon_{12} = +1$).

Since we are interested in the possible implications of small mass splittings, we will assume that the right handed neutrino masses consist of two almost degenerate

[4]There has been a recent attempt to study the asymmetry in heavy Majorana neutrino decays in the limit of small mass splittings [43], but those results are however at variance with ours. For instance, we find a dependence on H_{12} through $H_{12}H_{21}$, and not through $(H_{12}+H_{21})^2$ as in ref. [43], although we expect similar results for neutrino and sneutrino decays in the supersymmetric model considered here.

states, with the third one being much heavier and hence effectively decoupled from the mixing mechanism. In this case, the effects of the third scalar neutrino can be included independently, and the mixing effects can be studied with the two flavor formalism discussed before. It is particularly interesting that scenarios with this type of spectrum have been widely considered in the literature [44], and can naturally arise in $SO(10)$ models.

We will assume that sneutrinos are produced out of equilibrium by a certain unspecified mechanism (e.g. if sneutrinos are inflaton decay products [11] or the inflaton itself [12]), and for simplicity consider that the states produced initially correspond to one of the eigenstates[5], say $\tilde{N}_1$, of the mass matrix (so that $M_{12}^2 = 0$) and its charge conjugated state, $\tilde{N}_1^*$. Hence, the asymmetry will be given by eqs. (3.24),(3.25), where in this model a direct computation of the absorptive part of the sneutrino propagator leads to

$$\Gamma_{ab}^2 = \frac{1}{8\pi}\left[(\lambda^\dagger\lambda)_{ba}M_a M_b + (\lambda^\dagger\lambda)_{ab}s\right], \tag{3.32}$$

where the square of the four momentum will just be $s \equiv p^2 = M_1^2$ by the on–shell condition. The first contribution to the r.h.s. of eq. (3.32) is given by the slepton and Higgs loop, while the second by the lepton and Higgsino one.

As final states, we need to consider two possibilities, i.e. $f = \tilde{L}_i^\alpha H^\beta$ as well as $f = \bar{\ell}_i^\alpha \tilde{h}^{c\beta}$. For the final state with sleptons, we have $T_{fa} = -i\epsilon_{\alpha\beta}\lambda_{ai}^* M_a/\sqrt{s}$, so that only the second term in the r.h.s. of eq. (3.32) contributes to the total asymmetry, and we have

$$\sum_{i,\alpha,\beta} L_f \text{Im}\left\{T_{f1}T_{f2}^*\Gamma_{12}^2\right\} = \frac{M_1 M_2}{4\pi}\text{Im}\left\{(\lambda^\dagger\lambda)_{12}^2\right\} = -\frac{M_1 M_2}{4\pi}\text{Im}\left\{(\lambda^\dagger\lambda)_{21}^2\right\}, \tag{3.33}$$

[5]If both $\tilde{N}_1$ and $\tilde{N}_2$ are simultaneously, but incoherently, produced, one needs just to add the asymmetries from both decays. The case of an initial thermal population of sneutrinos is different and must be treated instead using the density matrix formalism; in this case, due to the mixing between the flavors, the thermal density matrix is non-diagonal in the $\tilde{N}_1/\tilde{N}_2$ basis.

where $L_f = +1$ is the lepton number of the final state. On the other hand, for the decay $\tilde{N}_a \to \bar{\ell}_i^{\alpha} \tilde{h}^{c\beta}$, one has $T_{fa} = -i\epsilon_{\alpha\beta}\lambda_{ai}$, and only the first term in the r.h.s. of eq. (3.32) contributes to the total asymmetry. Since now $L_f = -1$, we end up with the same contribution as the one coming from the slepton channel. (Notice that the asymmetry in a given final state channel results from the mixing generated by a loop involving the particles of the other final state channel.)

So, summing the contributions from both final states we get

$$\bar{\epsilon}_1^{\tilde{N}}(\text{wave}) = -\frac{M_1^2 M_2}{16\pi^2}\text{Im}\left\{(\lambda^\dagger\lambda)_{21}^2\right\} F_1, \tag{3.34}$$

where F_1 is given in eq. (3.25). If we use that $\Gamma_{11}^2 = (\lambda^\dagger\lambda)_{11} M_1^2/4\pi$, and the asymptotic expressions discussed previously for $M_2^2 \gg M_1^2 \gg |\Gamma_{ab}^2|$, one can see that in this limit

$$\bar{\epsilon}_1^{\tilde{N}}(\text{wave}) = -\frac{1}{2\pi}\frac{M_1 M_2}{M_2^2 - M_1^2}\frac{\text{Im}\left\{\left(\lambda^\dagger\lambda\right)_{21}^2\right\}}{\left(\lambda^\dagger\lambda\right)_{11}}, \tag{3.35}$$

which coincides with the expression (2.11) obtained before (see also (2.28)), apart from a factor 4: one factor 2 comes from the different normalization as discussed after eq. (3.2), while the other from considering the total asymmetry given by both final states.

In Fig. 3.2 we plot the total asymmetry $\bar{\epsilon}_1^{\tilde{N}}(\text{wave})$ for arbitrary mass splittings, normalized to the vertex contribution $\bar{\epsilon}_1^{\tilde{N}}(\text{vertex})$ arising from the exchange of the second state $\tilde{N}_2$, (2.24) and (2.27),

$$\bar{\epsilon}_1^{\tilde{N}}(\text{vertex}) = -\frac{1}{4\pi}\frac{M_2}{M_1}\ln\left[\frac{M_1^2 + M_2^2}{M_2^2}\right]\frac{\text{Im}\left\{\left(\lambda^\dagger\lambda\right)_{21}^2\right\}}{\left(\lambda^\dagger\lambda\right)_{11}}. \tag{3.36}$$

Notice that $\bar{\epsilon}_1^{\tilde{N}}(\text{vertex})$ contains actually the same combination of Yukawas

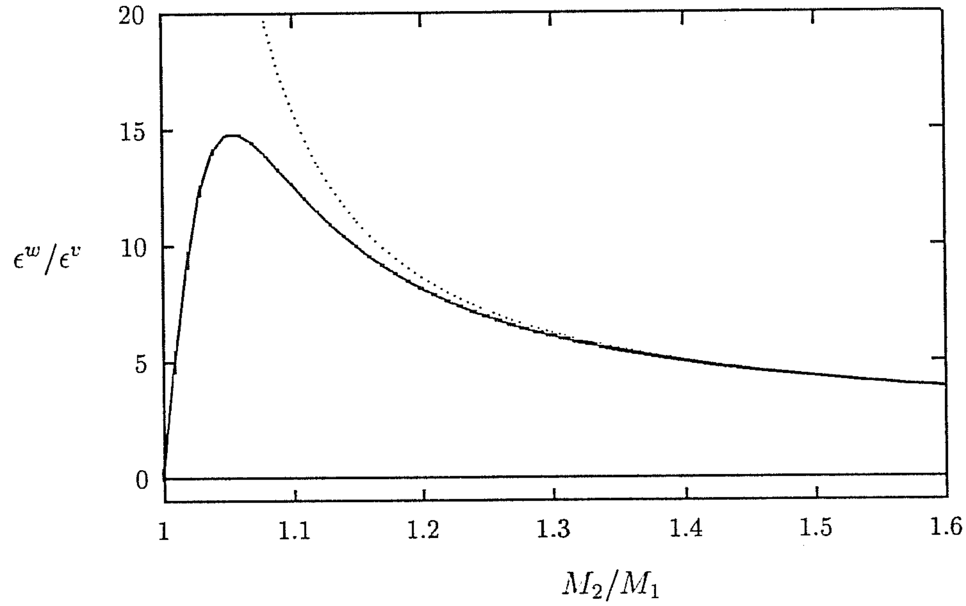


Figure 3.2: Wave function contribution to the CP asymmetry, normalized to the vertex one, as a function of M_2/M_1, assuming $M_{12}^2 = 0$ and taking $\Gamma_{22}^2 = M_1^2/10$, $\Gamma_{11}^2 = |\Gamma_{12}^2| = \Gamma_{22}^2/10$.

appearing in the wave function contribution $\bar{\epsilon}_1^{\tilde{N}}$(wave) (i.e. the factor $\text{Im}\,\{(\lambda^\dagger\lambda)_{21}^2\}$), so that the CP violating phase cancels in the ratio.

In Fig. 3.2 we adopted for definiteness $\Gamma_{22}^2 = M_1^2/10$, $\Gamma_{11}^2 = |\Gamma_{12}^2| = \Gamma_{22}^2/10$, plotting the result as a function of $x \equiv M_2/M_1$. In the limit of large mass splittings, the wave contribution approaches twice the value of the vertex one, as expected. For decreasing mass splittings ($x \to 1$), the enhancement in the wave contribution due to the mixing of the states is apparent, and reaches a maximum value $\simeq M_1^2/(\Gamma_{22}^2 \ln 2)$ for $M_2^2 - M_1^2 \simeq \Gamma_{22}^2$ (corresponding to $x \simeq 1.05$ in this case) as discussed in eq. (3.27).

The dotted line corresponds to the asymptotic expression for the wave contribution in eq. (3.35), and gives a reasonable approximation to the result for

$M_2^2 - M_1^2 > 4\Gamma_{22}^2$. For smaller values of Γ_{22}^2/M_1^2, the enhancement in the wave contribution is larger (and can be in principle of many orders of magnitude), and the asymptotic expression is valid down to smaller values of x. On the other hand, for $M_2^2 - M_1^2 < \Gamma_{22}^2$, $\bar{\epsilon}_1^{\tilde{N}}(\text{wave})$ decreases significantly, and for $M_2 \to M_1$ one has

$$\frac{\bar{\epsilon}_1^{\tilde{N}}(\text{wave})}{\bar{\epsilon}_1^{\tilde{N}}(\text{vertex})} \to -\frac{\Gamma_{11}^2}{(\Gamma_{11}^2 + \Gamma_{22}^2)\ \ln 2}, \tag{3.37}$$

which is tiny. The results are quite insensitive to the actual values of Γ_{11}^2 and $|\Gamma_{12}^2|$, as long as this last stays much smaller than Γ_{22}^2, so that the mixing angle θ is small. If $|\Gamma_{12}^2| \sim \Gamma_{22}^2$, the maximum enhancement is somewhat smaller but the general behavior remains similar.

It is important to keep in mind that the vertex and wave contribution arising from the exchange of the heavier third scalar neutrino $\tilde{N}_3$ may however be larger than the one coming from the exchange of the second one $\tilde{N}_2$, even taking into account the possible enhancements for small mass splittings, due to the probably larger Yukawa couplings involved and the unknown size of the CP violating phases appearing in both channels (for three families, there are actually three independent CP violating phases entering in the asymmetry [45]).

Chapter 4

Finite temperature effects

In this chapter we will compute the finite temperature correction to the CP violating decay asymmetries relevant for baryogenesis scenarios involving the out of equilibrium decays of heavy particles, including the effects arising from the background of light thermal particles which are present during the decay epoch. Thermal effects can modify the size of CP violation by a sizeable fraction in the decay of scalar particles, but we find interesting cancellations in the finite temperature corrections affecting the asymmetries in the decays of fermions, as well as in the decay of scalars in supersymmetric theories. We also estimate the effects which arise from the motion of the decaying particles with respect to the background plasma and from considering the thermal masses of the light particles.

4.1 Introduction

The scenarios for the generation of the baryon asymmetry of the Universe, that we have been considering, are based on the fact that very massive particles fall out of equilibrium as the temperature of the Universe drops below their mass and their equilibrium density becomes Boltzmann suppressed. As we have shown, the CP violation in the decay results from a one–loop diagram, whose interference with the tree level process allows the phases of the complex coupling constants to show up in the decay asymmetries.

In addition to the complex couplings, to have a non–zero partial decay rate asymmetry requires the loop integral to develop an absorptive part, i.e. the intermediate particles must be produced on–shell and are therefore real particles. In the proposed baryogenesis scenarios of this kind, the light particles in the loop are just standard quarks, leptons or Higgs bosons, so that the appearance of a non–vanishing absorptive part is guaranteed.

At the high temperatures at which the heavy particles decay, the light standard particles are in equilibrium with the hot plasma present, and hence a question arises on whether the existence of the background particles has any effect in the evaluation of the CP violating asymmetries. Indeed, some time ago Takahashi [29] showed that the thermal effects could modify the predictions for baryogenesis in $SU(5)$ models by up to $\sim 40\%$ with respect to the $T = 0$ results, and hence these effects may need to be taken into account in the proper computation of the resulting baryon asymmetry in specific models.

The main effect of the background can be taken into account by employing finite temperature propagators in the computation of the loop. In this way it is possible to consider simultaneously both the 'direct' propagation of a particle between two vertices in the loop and the absorption by the medium of a particle from the first vertex combined with the emission of another one towards the second. These two alternatives are actually indistinguishable in a thermal bath.

Another implication of the finite density background is the modification of the final state phase space density distributions, which take into account the stimulation of the decays into bosons and the Pauli blocking of the decays into fermions, and this may eventually also affect the rates.

4.2 Real Time Formalism (RTF)

We will review in this section the basic elements of the RTF that we will need in our computations. In thermal field theory [46], two formalisms are given to perform perturbative computation at finite temperature. Historically the first was the Imaginary Time Formalism (ITF), introduced by Matsubara in 1955, which links thermal field theory with an Euclidean field theory periodic in time [47]. In this formalism the Green functions in momentum space are defined only for discrete frequencies (the Matsubara frequencies).

The RTF instead consent to work with real momenta, at the price of doubling the degrees of freedom so to cancel ill–defined expressions (i.e. products of delta functions).

We choose to work in RTF mainly for two reasons. First the RTF consent to separate directly the T = 0 result and the T dependent part. Moreover the Green's functions computed in RTF are directly the time ordered ones, unlike in the ITF where different analytical extensions to real momenta lead to different Green's functions (retarded, advanced, etc.) [48].

The RTF involves the introduction of a ghost field dual to each physical field, and leads to a doubling of the degrees of freedom. We have then physical vertices, defined as in the zero temperature case, and ghost vertices, differing from the first in the sign. The thermal propagator has moreover a 2×2 matrix structure: the (11) component refers to the physical field, the (22) component to the corresponding ghost field and the off-diagonal (12) and (21) components mix them.

But, since we are working only at one loop and the external legs are physical, i.e. type–1 fields, we will need only the (11) component of the propagators. For what concerns the vertex diagram this is clear since no internal vertices are present and therefore only physical propagator will appear. Regarding instead the wave function piece, the internal vertex can in principle be also of type 2 giving rise to an additional part involving off diagonal propagators; but it can be shown that such contribution disappears in the case of strong hierarchy between the families of decaying heavy particles. Then in fact the CP asymmetry will arise only from the decay of the lighter heavy state, when all the others have already disappeared from the thermal bath. For such particles, with zero number density, physical and ghost degrees of freedom are completely decoupled because the off–diagonal term of the propagator is proportional to the number density. Therefore for the heavy intermediate particles it will be sufficient to use the T = 0 propagator, while for the lighter particles we will need the physical component of the thermal propagator, which is, for fermions and bosons respectively,

$$S_{11}(p,m) = (\not{p} + m)\left[\frac{i}{p^2 - m^2 + i0^+} - 2\pi n_F(p\cdot u)\delta(p^2 - m^2)\right], \qquad (4.1)$$

$$D_{11}(p,m) = \left[\frac{i}{p^2 - m^2 + i0^+} + 2\pi n_B(p\cdot u)\delta(p^2 - m^2)\right], \qquad (4.2)$$

with u the 4–velocity of the medium ($u = (1,0,0,0)$ in the medium rest frame), and

$$n_{F,B}(x) = \frac{1}{\exp\left(|x|/T\right) \pm 1}. \qquad (4.3)$$

We will drop the (11) subindex in the propagators from now on.

4.3 $SU(5)$ triplet decays

In order to discuss these issues, let us start by reanalyzing the scalar decays into two fermions, which is relevant in the case of heavy Higgs boson triplet decays in $SU(5)$ that we discussed in chapter 1. We have there generalized the computation for T = 0 given in ref. [29] by including also the CP violating diagrams arising from mixing among different heavy states [33, 30, 32, 13]. In this chapter we will consider also the finite temperature corrections for both vertex and wave contributions.

Let us consider the $SU(5)$ lagrangian (1.27) involving several scalar five-plets $\Phi_i = (T_i, H_i)$, containing, together with Higgs doublets H_i, the heavy color triplets T_i :

$$\mathcal{L} = f_i \Phi_{i\alpha} \, (\bar{\Psi}^{\alpha\beta} \, \chi_\beta) + \frac{g_i}{8} \Phi_{i\alpha} \, (\epsilon^{\alpha\beta\gamma\delta\epsilon} \, \bar{\Psi}^c_{\beta\gamma} \, \Psi_{\delta\epsilon}) + h.c., \tag{4.4}$$

where the gauge indices are denoted by greek letters. The matter fields are in the decuplet and the fiveplet representations as usual, $\Psi = (q, u^c, e^c)$ and $\chi = (d, l^c)$. The flavor indices of the Yukawa couplings are not displayed for simplicity.

As in chapter 1, we will consider the case of strong hierarchy and consider therefore the CP asymmetry generated in the decay of the lightest colored Higgses. The CP violating B asymmetry arising from the decay of a T_1 and $\bar{T}_1$ pair is defined in eq. (1.28) as

$$\epsilon = \frac{\sum_f B_f [\Gamma(T_1 \to F_f) - \Gamma(\bar{T}_1 \to \bar{F}_f)]}{\sum_f [\Gamma(T_1 \to F_f) + \Gamma(\bar{T}_1 \to \bar{F}_f)]}, \tag{4.5}$$

with B_f the baryon number of the final states $F_f = \bar{q}\ell^c, \; \bar{u}e^c, \; \bar{u}^c d, \; \bar{q}^c q$.

We consider now the finite temperature effects on the CP violating asymmetry ϵ. The main effect comes from using instead of the usual $T = 0$ propagators, the finite T ones.

As discussed previously, we neglect the background density of the heavy triplets [1] T_k (assuming $M_k \gg M_1$), and as a first approximation we assume that the decaying particle T_1 is at rest (particle motion effects will be considered in section 4). Using the well known property,

$$\frac{1}{x \pm i0^+} = \mathcal{P}\left(\frac{1}{x}\right) \mp i\pi\delta(x) \tag{4.6}$$

we get for the absorptive part of the vertex loop integral [2]

$$\mathrm{Im}\left\{I_t^T(x_k)\right\} = \mathrm{Im}\{I_t(x_k)\}\left[1 - 2\bar{n}_F + 2\bar{n}_F^2\right] - \frac{x_k}{8\pi}\int_{x_k}^{\infty}\frac{du}{u+1}n_F\left(\frac{M_1 u}{2}\right), \tag{4.7}$$

where

$$\bar{n}_{F,B} \equiv n_{F,B}\left(\frac{M_1}{2}\right). \tag{4.8}$$

This is similar to the result in ref. [29], except for the relative sign between the $T = 0$ part and the finite temperature corrections, implying that the temperature effects tend to reduce (instead of enhancing) the CP violation. The integral term in the r.h.s. of eq. (4.7) arises from the absorption of particles from the background which are energetic enough so as to put the intermediate state T_k on–shell and hence make it contribute to the absorptive part. This term becomes then extremely small in the case in which the heavy masses have a significant hierarchy, i.e. for $M_k \gg M_1$, since the amount of background particles which are energetic enough is Boltzmann suppressed.

For the finite temperature wave contribution we obtain

$$\mathrm{Im}\left\{I_s^T\right\} = \mathrm{Im}\{I_s\}\left[1 - 2\bar{n}_F + 2\bar{n}_F^2\right], \tag{4.9}$$

[1] Notice that the exchange of T_1 in the one–loop diagrams does not contribute to ϵ, so that only $k > 1$ are relevant.

[2] We have checked that the same result can be obtained by applying the Cutkosky cutting rules at finite temperature in the real time formalism [49].

so that the temperature dependence is similar to the one in the leading term of the vertex part. To give a quantitative idea of the effect, we notice that if the temperature is taken to be 1, 1/3 or 1/10 of the lightest triplet mass, M_1, the overall factor in (4.9) including the temperature effects is 0.53, 0.70 or 0.99 respectively. The physical interpretation of this effect is simple: due to the thermal background, the two light fermions exchanged in the loop are subject to a Pauli blocking, and this leads to a reduction of the amount of CP violation.

In this scenario there is no effect on ϵ resulting from the final state blocking, since this just leads to overall factors $(1-\bar{n}_F)^2$ multiplying the rates, and hence these factors cancel in the ratio in eq. (1.28). Other thermal effects, such as thermal masses or wave function renormalization, are higher order in the coupling constants and hence we neglect them.

4.4 Leptogenesis scenarios

The $SU(5)$ model discussed in the previous section, in spite of being the prototype for the 'out of equilibrium decay' scenarios of baryogenesis, has the drawback that it generates no net $B-L$ asymmetry (a characteristic of $SU(5)$), and hence the B generation is vulnerable to the anomalous B violating processes of the Standard Model [5, 6] (which only leave $B-L$ unaffected), with the consequence that all asymmetries generated within this model will be eventually erased.

A very interesting way out to this problem was introduced in chapter 2 and is based on the generation of a lepton (L) asymmetry at early times, by the out of equilibrium decay of heavy isosinglet neutrinos (the usual ones appearing in see–saw models for neutrino masses and naturally present in GUT models such as $SO(10)$).

In this section we discuss temperature effects in this kind of models, first under the minimal assumption that the standard model spectrum is enlarged to include heavy right-handed neutrinos, and then, considering the supersymmetric extension of the model.

4.4.1 Non-supersymmetric case

The interactions of the heavy neutrinos N_i, in the basis in which their mass matrix is diagonal, are given by the following Lagrangian

$$\mathcal{L} = -\lambda_{ai}\epsilon_{\alpha\beta}\bar{N}_i P_L \ell_a^\alpha H^\beta + h.c. \tag{4.10}$$

where $\ell_a = (\nu_a,\ l_a^-)$ and $H = (H^+,\ H^0)$ are the lepton and Higgs Standard Model doublets ($a = e, \mu, \tau$, $i = 1, 2, 3$, and $\epsilon_{\alpha\beta} = -\epsilon_{\beta\alpha}$, with $\epsilon_{12} = +1$).

The complete $T = 0$ CP violating L asymmetry was computed for this model in chapter 2 resulting in

$$\epsilon = 2\sum_{k>1} \mathcal{I}_{k1} \left[\mathrm{Im}\{J_t(x_k)\} + 2\frac{\sqrt{x_k}}{x_k - 1}\mathrm{Im}\{J_s\} \right], \tag{4.11}$$

where x_k is defined analogously to eq. (1.30) and

$$\mathcal{I}_{k1} \equiv \frac{\mathrm{Im}\left\{ \left(\lambda^\dagger\lambda\right)_{k1}^2 \right\}}{(\lambda^\dagger\lambda)_{11}}. \tag{4.12}$$

Notice that, in full analogy with the $SU(5)$ case, a single right handed neutrino would be unable to generate any asymmetry.

The loop integrals $J_t(x_k)$ and J_s at zero temperature are given in equations (2.7) and (2.10).

The computation of the finite temperature contribution to these quantities is similar to the one in the previous section, and leads to

$$\begin{aligned}\mathrm{Im}\left\{J_t^T(x_k)\right\} =& \ \mathrm{Im}\{J_t(x_k)\}\left[1-\bar{n}_F+\bar{n}_B-2\bar{n}_F\bar{n}_B\right] \\ &+\frac{\sqrt{x_k}}{16\pi}\int_{x_k}^{\infty}\frac{du}{u+1}\left[n_F\left(\frac{M_1 u}{2}\right)(u-x_k)+n_B\left(\frac{M_1 u}{2}\right)(x_k+1)\right], \\ \mathrm{Im}\left\{J_s^T\right\} =& \ \mathrm{Im}\{J_s\}\left[1-\bar{n}_F+\bar{n}_B-2\bar{n}_F\bar{n}_B\right],\end{aligned} \tag{4.13}$$

where for instance in this last expression the different contributions in the r.h.s. clearly separate into the pieces coming from the $T=0$ propagators, the one from the thermal correction to the fermion (ℓ) propagator, the one from the thermal piece of the boson (H) propagator and the product of these two corrections. However, it is easy to check that the Bose and Fermi distributions satisfy

$$n_B(E)-n_F(E)=2n_B(E)n_F(E), \tag{4.14}$$

so that the main temperature correction cancels out (only the small integral term in $\mathrm{Im}\{J_t\}$ survives). This is due to the opposite effects resulting from the Pauli blocking of the loop fermion line and the stimulation of the bosonic loop line. On the other hand, here again the final state statistical factors $(1-\bar{n}_F)(1+\bar{n}_B)$ cancel in the expression for ϵ, and as a result no significant temperature dependent effect is found.

4.4.2 Supersymmetric case

This scenario has also received considerable attention within a supersymmetric framework [11, 12, 13], in particular because the scalar partner of the heavy neutrino $\tilde{N}_1$ is a good candidate for being the inflaton field, in which case the L asymmetry could be produced during the process of reheating of the Universe as $\tilde{N}_1$ decays [12]. In this case, as we will now show, another interesting cancellation is found in the asymmetry produced by $\tilde{N}_1$ decay.

The scalar neutrino can decay either into two scalars ($\tilde{L}H$) or into two fermions ($\ell\tilde{h}$), and the contribution to CP violation in one channel is obtained from the loop involving the particles of the other channel [13]. The thermal effects will modify the asymmetries corresponding to each channel, in a way similar as they did in the case of $SU(5)$. For instance, in the $\tilde{L}H$ channel, if we ignore the small integral piece coming from the vertex (equivalent to the last term in the r.h.s. of eq. (4.7)), the asymmetry will be

$$\epsilon^T(\tilde{N} \to \tilde{L}H) \simeq \epsilon^{T=0}(\tilde{N} \to \tilde{L}H)\left[1 - 2\bar{n}_F(1-\bar{n}_F)\right], \tag{4.15}$$

and similarly

$$\epsilon^T(\tilde{N} \to \ell\tilde{h}) \simeq \epsilon^{T=0}(\tilde{N} \to \ell\tilde{h})\left[1 + 2\bar{n}_B(1+\bar{n}_B)\right]. \tag{4.16}$$

However, due to the effects of the final state phase space factors $(1 \mp \bar{n}_{F,B})$ entering into the partial decay rates, the branching ratios of the two different channels will no longer be equal (as is the case at $T = 0$). One has instead that

$$BR(\tilde{N} \to \ell\tilde{h}) = \frac{(1-\bar{n}_F)^2}{(1-\bar{n}_F)^2 + (1+\bar{n}_B)^2} = 1 - BR(\tilde{N} \to \tilde{L}H). \tag{4.17}$$

The total asymmetry produced in the $\tilde{N}$ decay is

$$\epsilon^T_{\tilde{N}} = BR(\tilde{N} \to \ell\tilde{h})\epsilon^T(\tilde{N} \to \ell\tilde{h}) + BR(\tilde{N} \to \tilde{L}H)\epsilon^T(\tilde{N} \to \tilde{L}H), \tag{4.18}$$

with the surprising result that the main corrections arising from thermal effects actually cancel out, leading to

$$\epsilon^T_{\tilde{N}} \simeq \epsilon^{T=0}_{\tilde{N}}. \tag{4.19}$$

Notice that there are also new supersymmetric diagrams contributing to the CP violating asymmetry in the heavy neutrino decays. However, since the particles in the loop as well as the external ones are always one fermion and one boson, both massless, the cancellation found in the previous subsection will also occur in the new channels, and therefore there are no significant thermal corrections to the zero temperature result.

4.5 Effects of particle motion

In the discussion so far we have always considered the decay rate of a particle at rest in the thermal bath. However, the decaying particle will in general be moving through the background with non zero velocity $\vec{\beta} = \vec{v}/c$. Since now the Lorentz symmetry is explicitly broken by the plasma, this motion can in principle affect the thermal corrections to the decay asymmetry. When the leading thermal corrections cancel, as in the leptogenesis scenarios discussed in Section 4.4, the effects of the motion of the decaying particle will provide one of the main thermal corrections, and hence it is worth to quantify them. To estimate the size of these effects, we will consider here the case of the heavy neutrino decay in the non-supersymmetric model. The other cases can be analyzed similarly.

It is convenient to compute the decay rate asymmetries in the rest frame of the decaying particle, where the medium will be characterized by a non trivial 4-velocity $u = (\gamma, -\gamma\vec{\beta})$, with $\gamma = 1/\sqrt{1-\beta^2}$ as usual. In this system, the effect of the motion will reflect in a modification of the equilibrium distributions appearing in the thermal propagators, eqs. (4.1–4.2). The decay rate will also depend on $\vec{\beta}$ through the final state phase space factors, which for the case of fermion decays is

$$[1 - n_F(k_\ell \cdot u)]\,[1 + n_B(k_H \cdot u)] = \frac{2}{1 - \exp(-M_1\gamma/T)} P_\beta(\cos\theta), \tag{4.20}$$

where θ is the angle between $\vec{\beta}$ and the momentum of the final state lepton, and we have introduced

$$P_\beta(z) \equiv \frac{1}{2}\left\{1 - \sinh\left(\frac{z\beta\gamma M_1}{2T}\right) \Big/ \sinh\left(\frac{\gamma M_1}{2T}\right)\right\}^{-1}. \tag{4.21}$$

We see that now there is a privileged direction selected by the plasma spatial velocity and therefore the decay process is no more isotropic: the rate depends on the angle of the decay product trajectories with respect to this direction. Due to statistics the fermions (bosons) are preferentially emitted in the direction anti-parallel (parallel) to the plasma velocity $(-\gamma\vec{\beta})$, which corresponds to the less (more) occupied region of the thermal distribution. Anyway, since we are interested in the total decay rate, the angular dependence is integrated out and we are left only with a β^2 dependence[3].

Let us define ϵ_β as the integrated asymmetry generated in the decay of a heavy neutrino moving with velocity β. The decaying particles will actually have a distribution of velocities with occupation numbers $n(E)$, where $E = M_1\gamma$. To estimate the overall effect of the particle motion we may just approximate this distribution with the Fermi Dirac distribution, $n(E) = n_F(E)$, and compute the average asymmetry

$$\langle\epsilon\rangle = \frac{1}{N_1}\int \frac{d^3p}{(2\pi)^3}\; n(\gamma M_1)\epsilon_\beta = \frac{1}{N_1}\int_0^1 d\beta \frac{dN_1}{d\beta}\epsilon_\beta, \tag{4.22}$$

where N_1 is the particle's volume density and

$$\frac{dN_1}{d\beta} = \frac{M_1^3}{2\pi^2}\beta^2\gamma^5\; n(\gamma M_1). \tag{4.23}$$

It is clear that $\langle\epsilon\rangle$, computed for a given temperature T, is just the asymmetry that would result if the initial thermal population of heavy neutrinos went out of equilibrium and decayed all simultaneously at temperature T. We will use the asymmetry $\langle\epsilon\rangle$ as an indicator of the possible effects of the particle motion, although clearly to

[3]The integrated decay rate can depend only on the Lorentz invariants $p^2 = M_1^2$ and $p \cdot u = M_1\gamma$ [50].

obtain the exact impact of this into the final lepton asymmetry would require to integrate the whole Boltzmann equations, a task beyond our scopes.

To obtain the decay asymmetry at a fixed velocity ϵ_β, it is convenient to separate the angular dependence arising from the final state phase space factor $P_\beta(\cos\theta)$ and the one arising from the one loop integrals (via the anisotropic background density in the particle rest frame). This last will be included in the factor $L_\beta(\cos\theta, x_k)$ defined through

$$\epsilon^c_\beta = \frac{2}{\int_{-1}^{1} dz P_\beta(z)} \sum_k \mathcal{I}_{k1} \int_{-1}^{1} dz L^c_\beta(z, x_k) P_\beta(z). \tag{4.24}$$

Here the supra-index c labels the two different contributions to the CP asymmetry in the decay, i.e. the wave (ϵ^s) and the vertex (ϵ^t) pieces. Clearly at zero velocity we have, according to eq. (4.11),

$$L^s_0(z, x_k) = \frac{2\sqrt{x_k}}{x_k - 1} \mathrm{Im}\{J_s\}\,, \tag{4.25}$$

$$L^t_0(z, x_k) = \mathrm{Im}\{J_t(x_k)\}\,, \tag{4.26}$$

which are actually independent of z as expected.

At $\beta \neq 0$ we find after direct computation of the interference terms

$$L^s_\beta(z, x_k) = L^s_0(z, x_k)[f^{(0)}_\beta - z f^{(1)}_\beta], \tag{4.27}$$

where the loop functions $f^{(n)}_\beta$

$$f^{(n)}_\beta = \int_{-1}^{1} dy\; y^n\; P_\beta(y) \tag{4.28}$$

arise after integrating over the angle of the momenta in the loop. Notice that the phase space factor P_β appears also in these integrals, arising from the statistical factors in the thermal loops. In the limit of zero velocity, we have $f^{(0)}_0 = 1$ and $f^{(1)}_0 = 0$.

Now we can write in a simple form the final expression for ϵ^s_β; namely, putting together eqs. (4.22), (4.24) and (4.27) we get:

$$\langle \epsilon^s \rangle = \epsilon^s_0 \frac{1}{N_1} \int_0^1 d\beta \, \frac{dN_1}{d\beta} \, f^{(0)}_\beta \left[1 - \left(\frac{f^{(1)}_\beta}{f^{(0)}_\beta} \right)^2 \right] . \tag{4.29}$$

It is not possible to integrate analytically this expression, but expanding the loop functions to first order in β^2 we obtain

$$f^{(0)}_\beta \simeq 1 + \frac{1}{3}\,\beta^2 \left(\frac{M_1/2T}{\sinh\left(M_1/2T\right)} \right)^2 , \tag{4.30}$$

$$f^{(1)}_\beta \simeq \frac{1}{3}\,\beta \, \frac{M_1/2T}{\sinh\left(M_1/2T\right)} . \tag{4.31}$$

Substituting in eq. (4.29), we get

$$\langle \epsilon^s \rangle \simeq \epsilon^s_0 \left[1 + \frac{2}{9} \langle \beta^2 \rangle \left(\frac{M_1/2T}{\sinh\left(M_1/2T\right)} \right)^2 \right] . \tag{4.32}$$

For $M_1/T = 1, 3, 10$ we have $\langle \epsilon^s \rangle / \epsilon^s_0 \simeq 1.168, 1.057, 1.000$; we see therefore that the effect is small compared to the $\beta = 0$ piece. In the previous evaluation we have used for the average velocity just a simple approximation, which is accurate to the 10% level, writing $\langle \beta^2 \rangle \simeq \langle \beta \rangle^2_{Boltz}$, with the average velocity for a Boltzmann distribution of massive relativistic particles being (with $x \equiv M/T$)

$$\langle \beta \rangle_{Boltz} = \frac{2(1+x)\exp(-x)}{x^2 K_2(x)} . \tag{4.33}$$

Computing numerically eq. (4.29) we find, for the same reference values of M_1/T as before, that $\langle \epsilon^s \rangle / \epsilon^s_0 = 1.231, 1.054$ and 1.000, so that the approximate result is acceptable.

Let us now consider the "vertex" contribution. In this case the loop integration is more involved and we get the following expression (neglecting the small integral piece which is Boltzmann suppressed):

$$L^t_\beta(z,x_k) = \frac{\sqrt{x_k}}{16\pi}\left[f^{(0)}_\beta - g_\beta(x_k,z)\right], \tag{4.34}$$

where

$$g_\beta(x_k,z) = \int_{-1}^{1} dy\; P_\beta(y)\frac{2(1+x_k)}{\sqrt{[y+z(1+2x_k)]^2+4x_k(x_k+1)(1-z^2)}}. \tag{4.35}$$

For $\beta = 0$ the function g actually does not depend on z and is

$$g_0(x_k) = (1+x_k)\ln\left(1+\frac{1}{x_k}\right), \tag{4.36}$$

so that we recover exactly eq. (4.26).

Substituting the result of the loop integration in eq. (4.24) we finally obtain

$$\epsilon^t_\beta = 2\sum_k \mathcal{I}_{k1}\frac{\sqrt{x_k}}{16\pi}\left[f^{(0)}_\beta - \frac{\int_{-1}^{1} dz\; g_\beta(x_k,z)P_\beta(z)}{f^{(0)}_\beta}\right]. \tag{4.37}$$

Again we can evaluate this function analytically only for small β, by using the expansion

$$g_\beta(x_k,z) \simeq g_0(x_k) + \beta z\frac{M_1/2T}{\sinh(M_1/2T)}h(x_k) + \beta^2\left(\frac{M_1/2T}{\sinh(M_1/2T)}\right)^2\left[j(x_k) - z^2 l(x_k)\right], \tag{4.38}$$

where

$$h(x) = 2(1+x) - (1+2x)g_0(x), \tag{4.39}$$

$$j(x) = (1+x)(1+2x) - 2x(1+x)g_0(x), \tag{4.40}$$

$$l(x) = 3(1+x)(1+2x) - (1+6x+6x^2)g_0(x). \tag{4.41}$$

So, the average CP violation is given by

$$\langle\epsilon^t\rangle \simeq 2\sum_k \mathcal{I}_{k1}\mathrm{Im}\{J_t(x_k)\}\left[1+\frac{1}{3}\langle\beta^2\rangle\left(\frac{M_1/2T}{\sinh(M_1/2T)}\right)^2\left(\frac{1}{g_0(x_k)-1} - 2x_k\right)\right]. \tag{4.42}$$

We then see that the effect due to the particle motion is again to increase the vertex CP asymmetry, as in the case of the wave part. However, the effect depends also on x_k, i.e. on the hierarchy between the particle masses. In the limit of large x_k we can use the expansion $(g_0(x_k) - 1)^{-1} \simeq 2x_k + 2/3$, to obtain

$$\langle \epsilon^t \rangle \simeq \epsilon_0^t \left[1 + \frac{2}{9} \langle \beta^2 \rangle \left(\frac{M_1/2T}{\sinh\left(M_1/2T\right)} \right)^2 \right], \tag{4.43}$$

so that the overall factor coincides with the one in eq. (4.32) for $\langle \epsilon^s \rangle$.

To estimate the goodness of the approximate result in eq. (4.42) we evaluated the term in square brackets there, taking $x_k = 5$ for definiteness, and we obtained 1.163, 1.055 and 1.000 for $M_1/T = 1$, 3 and 10 respectively. A numerical evaluation of the exact correction due to the velocity, using eq. (4.37), leads for the same factor the values 1.222, 1.052, 1.000 respectively, so that again the accuracy is reasonable, and we see that for $T < M_1$ the velocity dependent correction to ϵ is not large.

4.6 Effects of the thermal masses

In this last section we will proceed to estimate the effects of the presence of thermal masses for the light particles considered massless so far in the non supersymmetric scenario.

Regarding the singlet neutrinos, such corrections are obviously negligible, because at temperatures T $\simeq M_1$ they are much smaller than the mass itself since they arise from loop effects and are suppressed by the Yukawa couplings. In fact the singlet particles do not feel any gauge interactions and are unaffected by the presence in the thermal bath of $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge bosons.

Different is the case of the leptons, which acquire a thermal mass squared proportional to $g^2\mathrm{T}^2/8$ in the thermal bath [46], where g is their gauge coupling with the $SU(2)_L \times U(1)_Y$ group, $g^2 = 3g_L^2 + g_Y^2$. Yukawa interactions for the leptons are negligible due to the smallness of the couplings. We see that the mass corrections for the leptons is small compared to the energy of the decay process, but probably not negligible: $g\mathrm{T}/(2\sqrt{2}) \leq gM_1/(2\sqrt{2}) < M_1$ during the out of equilibrium decay.

Considering instead the Higgs bosons, the computation of the thermal mass is more involving and we will have to consider not only the gauge interactions, but also the self-interactions of the fields and the Yukawa interaction with the top quark. We are assuming here that the temperature we are considering is such that the gauge symmetry is restored and that the tree level (negative) mass of the Higgs particle is negligible with respect to the thermal mass. It is easy to see that such limit is realized for temperatures $\mathrm{T} \gg T_c$. We can compute then the effective masses for every real component of the Higgs doublet and obtain (see also [51] for the computation):

$$m_H^2(\mathrm{T}) \simeq \frac{\mathrm{T}^2}{8}\left(g^2 + 14a + 4y_t^2\right) \tag{4.44}$$

where a is the Higgs boson self–coupling and y_t the Yukawa coupling of the Higgs with the top quarks. Since the leading contributions to the thermal masses are CP invariant (the only CP violating piece can possibly come from the Yukawa interactions), the effect of such masses on the decay process and the CP violating asymmetry is mainly to lower the decay rate reducing the phase space available and to spoil the symmetric distribution of the energy in the final particles state.

Therefore, on one hand the way out of equilibrium condition for the heavy singlet states may be somewhat easier to be achieved in the thermal bath, while on the other hand the cancellation found in section 4.4 is not complete.

In the case the most important piece is due to the gauge bosons, it is evident that the leptons and Higgs masses are very close to each other and this effect is small. It is easy to quantify the effect in the expression (4.14). We will then have the different final states energies in the center of mass frame:

$$E_H = \frac{M_1}{2} + \frac{m_H^2 - m_\ell^2}{2M_1} \tag{4.45}$$

$$E_\ell = \frac{M_1}{2} - \frac{m_H^2 - m_\ell^2}{2M_1}, \tag{4.46}$$

so that

$$n_B(E_H) - n_F(E_\ell) - 2n_B(E_H)n_F(E_\ell) \simeq \tag{4.47}$$

$$\simeq n_B(E_H)n_F(E_\ell)\exp(E_\ell/\mathrm{T})\frac{m_\ell^2 - m_H^2}{\mathrm{T}M_1} = \tag{4.48}$$

$$= \frac{M_1/2\mathrm{T}}{\sinh(M_1/2T)}\frac{m_\ell^2 - m_H^2}{M_1^2}. \tag{4.49}$$

The effect of the mass difference is then similar to the effect of the decaying particle motion, and will be in general small due to the suppression factor given by the mass difference.

Using the expressions above to estimate the mass difference, we have that the thermal contribution to ϵ is

$$\epsilon \simeq \epsilon_0\left[1 - \frac{M_1/2\mathrm{T}}{\sinh(M_1/2T)}\frac{7a + y_t^2}{4}\frac{\mathrm{T}^2}{M_1^2}\right]; \tag{4.50}$$

the effect of the thermal mass seems therefore opposite to that of the decaying particle motion, but is much more sensitive on the temperature. Assuming that the most important contribution is due to the top quark and that y_t is of order 1, for $M_1/\mathrm{T} = 1, 3, 10$ we have $\epsilon/\epsilon_0 \simeq 0.760, 0.980, 1$.

We have then that for temperatures $\mathrm{T} = M_1$, the two next order effects tend to cancel each other, while for decreasing temperature, the decaying particle motion is the main contribution, but tends to vanish.

Another thermal correction that could in principle change the $\mathrm{T} = 0$ value of the CP asymmetry and that affects in general the tree level rates is the finite temperature renormalization of the wave functions [50, 52] for the light leptons. At zero temperature, the wave function renormalization constant is obtained from the on-shell self-energy and is independent of the momentum, while at finite temperature Lorentz invariance is broken and the renormalization "constant" is momentum dependent. Two different proposals have been put forward for taking into account this effect [53, 54] (for a comparison see [55]): with both methods the modification of the decay rates we are considering amounts only on a common constant factor, which cancels out in the computation of the CP asymmetry.

We have considered the next order thermal corrections to the CP asymmetry; the effects of the decaying particle motion and of the thermal masses for the light particles change only slightly the value of ϵ, since their contributions are opposite to each other. The finite temperature renormalization of the spinors changes the rates, but does not affect the ratio of rates. The zero temperature result can therefore be safely used for the computation of the CP asymmetry in the thermal background.

Conclusions

The baryon asymmetry of the Universe is probably the most important manifestation of the existence of CP violation. In the classic scenarios for baryogenesis through the out of equilibrium decay of heavy particles in the early stages of rapid expansion of the Universe, the asymmetries in the B (or L) violating decay rates to conjugate final states arise at the one loop level, involving the virtual exchange of light particles, such as quarks, leptons or Higgs bosons.

We have studied in this thesis the CP violating asymmetries necessary for baryon number generation, specifying in the case of leptogenesis.

After a basic review on the subject and an explicit example of $SU(5)$ baryogenesis in chapter 1, we have considered in chapter 2 the Fukugita–Yanagida scenario and computed all the contributions to the CP violating asymmetries arising at one–loop in the decays of heavy (s)neutrinos, both in the standard non–supersymmetric and in the supersymmetric versions. In these type of models [7, 8, 11, 12, 9], the decay of the electroweak singlet (s)neutrinos, with masses $M \gg$ TeV, produces a lepton asymmetry. This is then partially converted into a baryon asymmetry [23] by the effects of the anomalous $B + L$ violation in the SM [5, 6], which is in equilibrium at temperatures larger than the electroweak phase transition one ($\simeq 10^2$ GeV).

We have discussed the different results present in the literature and showed that the contribution from wave function mixing is relevant in the computation of the CP violating asymmetries. In the case of strong hierarchy among the heavy masses it increases by a factor three the amount of asymmetry. The baryon number generated in both non–supersymmetric and supersymmetric scenarios was also obtained for the case of way out of equilibrium decay. We have also showed with a toy example that the right amount of baryon number could be generated even with a small CP violating phase and at scales around $10^8 - 10^9$ GeV or lower. In this case, leptogenesis could have easily taken place after inflation during reheating.

In chapter 3, we have considered in detail the integrated CP violating asymmetries arising from heavy particle mixing, and studied the effects that appear when the mass splittings are of the order of the particle widths. The large enhancements which can be achieved can be helpful to explain the observed baryon asymmetry of the Universe, as we have exemplified with the study of a scenario for leptogenesis. For quasi-degenerate masses the wave function contribution to the CP asymmetry would dominate and consent a L number production greater by several orders of magnitude than the usual vertex part.

We have studied then in chapter 4 the effects of the thermal background of standard particles present during the decay epoch in the evaluation of the CP violating asymmetries. We first reconsidered the triplet scalar decay in $SU(5)$, finding that the asymmetry is reduced (contrary to an earlier result), as could be expected on the basis of the Pauli exclusion principle applied to the virtual fermionic lines in the loop. We also included the CP violation produced by the mixing among different heavy states. Confronting with the $T = 0$ results, the modification produced by the thermal effects could be as large as 50% for $T \simeq M_1$, but diminishes with decreasing temperature, becoming negligible for $T < M_1/10$.

For the leptogenesis scenario, we showed that the leading thermal corrections cancel among themselves, due to the opposing effects produced by the bosons and fermions involved in the loop. Also in the supersymmetric version of leptogenesis the thermal corrections to the heavy scalar neutrino decay were shown to vanish, once the thermal modification of the branching ratios to the different final states are included. In view of this, we studied the correction due to the fact that in general the decaying particle is not at rest in the thermal bath and the effect of the thermal masses acquired by the light particles. We showed that the CP asymmetry depends on the particle motion, since the background density distribution, and hence the thermal corrections, are now modified in the rest frame of the decaying particle. This effect can however increase the decay asymmetries only by at most $\sim 20\%$ with respect to the usual $T = 0$ results. These last are also modified by the fact that thermal masses are different for leptons and Higgs bosons; such effect is comparable, but opposite to the previous so that they tend to cancel out. Hence the zero temperature results can be safely employed.

As a summary, the realization that standard model anomalous B and L violating processes are in equilibrium at temperatures above the electroweak phase transition one, has made the $SU(5)$ scenario just of academic interest, since no net $B - L$ asymmetry (the only one unaffected by sphaleronic processes) is generated within it. However, the same anomalous processes have allowed some new very attractive possibilities, including the baryogenesis at the electroweak scale itself (although its practical implementation faces several difficulties). The most simple and promising scenario seems to be the leptogenesis, in which heavy right handed neutrinos generate a lepton asymmetry in their decay, which is then reprocessed into a baryon asymmetry by the Standard Model anomalous processes. We have shown in this context that many new interesting physical processes need to be considered in the proper computation of the CP asymmetry, which is the crucial quantity determining the final outcome of the baryon number generation.

Acknowledgments

I would like to thank my supervisors Dr. Esteban Roulet and Prof. Antonio Masiero for their invaluable support and encouragement during my studies. I would also like to thank Dr. Francesco Vissani and Dr. Nuria Rius with whom part of this work has been done.

I am indebted with Marco Serone, Mario Trigiante, Qiu Yu Liu and all my other colleagues and friends at SISSA for many interesting and useful discussions in these 3 years, not always on the subject of this thesis; I feel very grateful to SISSA and in particular to the High Energy Sector for giving me the opportunity to work in such a friendly and lively environment.

Appendix A

CP asymmetry for the Fukugita-Yanagida model

We reproduce here as an example the computations of the CP asymmetry in the minimal extension of the Standard Model with heavy singlet neutrinos discussed in chapter 2. The Feynman rules for the lagrangian (2.1) are standard, apart from the fact than the heavy neutrinos are Majorana particles. For the Feynman rules for Majorana spinors see [7]. We will compute the CP asymmetry in the center of mass frame of the decaying particle, whose 4-momentum is $p = (M_i, \vec{0})$ using Cutkosky rules at T = 0 [56].

A.1 The vertex contribution

Let us compute explicitly the CP asymmetry due to the vertex contribution in the minimal extension of the SM.

The one-loop amplitude for the diagram in Fig. 2.1a, where $N_i \to \ell_j^\alpha H^\beta$ with an intermediate N_k, is given by

$$\begin{aligned} \mathcal{A}_t &= i\lambda^\dagger_{kj}(\lambda^\dagger\lambda)_{ki}\epsilon_{\alpha\beta}\bar{u}^\alpha_j(p-k)P_R\int\frac{d^4q}{(2\pi)^4}S(q-k,M_k)S(q,0)D(p-q,0)P_L u_i(p) = \\ &= i\lambda^\dagger_{kj}(\lambda^\dagger\lambda)_{ki}\epsilon_{\alpha\beta}J_t(x_k)\bar{u}^\alpha_j(p-k)P_R u_i(p), \qquad \text{(A.1)} \end{aligned}$$

where $x_k \equiv M_k^2/M_i^2$.

It can be shown that $J_t(x_k)$ is related to the loop integral by the equation:

$$J_t(x_k) = \frac{1}{2M_i}\int\frac{d^4q}{(2\pi)^4}\,\mathrm{Tr}\left[(\not{p}-\not{k})S(q-k,M_k)S(q,0)D(p-q,0)\right]. \qquad \text{(A.2)}$$

We compute the imaginary part of this function using the Cutkosky rules [56] for the lepton and boson propagators, i.e. substituting

$$\frac{1}{k^2 - m^2 + i\epsilon} \to -i2\pi\theta(k_0)\delta(k^2 - m^2) \tag{A.3}$$

and we have

$$2\,\mathrm{Im}\,\{J_t(x_k)\} = 2\frac{M_k}{M_i}\int \frac{d^4q}{(2\pi)^2}\frac{q\cdot(p-k)}{(q-k)^2 - M_k^2}\theta(q_0)\delta(q^2)\theta(p_0-q_0)\delta((p-q)^2). \tag{A.4}$$

Notice that the heavy particle in the loop can never be on shell and so no contribution arises from the imaginary part of its propagator.

After integrating the delta functions we arrive at the result

$$2\,\mathrm{Im}\,\{J_t(x_k)\} = \frac{\sqrt{x_k}}{16\pi}\int_{-1}^{1} dz\left[1 + \frac{2(1+x_k)}{z - 1 - 2x_k}\right] = \tag{A.5}$$

$$= \frac{\sqrt{x_k}}{8\pi}\left(1 - (1+x_k)\ln\left[\frac{1+x_k}{x_k}\right]\right), \tag{A.6}$$

i.e. eq. (2.7).

The tree level amplitude is instead

$$\mathcal{A}_0 = -i\lambda^\dagger_{ij}\epsilon_{\alpha\beta}\bar{u}^\alpha_j(p-k)P_R u_i(p) \tag{A.7}$$

so that the interference term between the tree and the one loop vertex correction amplitudes, averaging over the initial spin state and summing over the final spin and flavor state, is

$$2\,\mathrm{Re}\,\{\mathcal{A}_0^*\mathcal{A}_t\} = -\sum_k 2\,\mathrm{Re}\,\left\{(\lambda^\dagger\lambda)^2_{ki}J_t(x_k)\right\}; \tag{A.8}$$

the interference term for the decay into antiparticles is identical, apart from containing the conjugated couplings.

Factorizing out the common phase space factor Ω, the tree level total decay rate is

$$\Gamma = 2(\lambda^\dagger\lambda)_{ii}\Omega = (\lambda^\dagger\lambda)_{ii}\frac{M_i}{8\pi} \tag{A.9}$$

so that the CP asymmetry results to be

$$\epsilon_\ell^{N_i}(\mathrm{vertex}) = -\sum_k \frac{\mathrm{Re}\,\left\{J_t(x_k)\left[(\lambda^\dagger\lambda)^2_{ki} - \left((\lambda^\dagger\lambda)^2_{ki}\right)^*\right]\right\}}{(\lambda^\dagger\lambda)_{ii}} = \tag{A.10}$$

$$= 2\sum_k \frac{\mathrm{Im}\,\left\{(\lambda^\dagger\lambda)^2_{ki}\right\}}{(\lambda^\dagger\lambda)_{ii}}\,\mathrm{Im}\,\{J_t(x_k)\} = \tag{A.11}$$

$$= 2\sum_k \mathrm{Im}\,\{J_t\,(x_k)\}\mathcal{I}_{ki}\,, \tag{A.12}$$

where we have recovered eq. (2.4).

A.2 The wave function contribution

The one-loop amplitude for the wave function diagram in Fig. 2.1b relevant for the L violating CP asymmetry[1], is given by

$$\begin{aligned} \mathcal{A}_s &= 2i\lambda^\dagger_{kj}(\lambda^\dagger\lambda)_{ki}\epsilon_{\alpha\beta}\bar{u}^\alpha_j(p-k)P_R\int\frac{d^4q}{(2\pi)^4}S(p,M_k)P_RS(q,0)D(p-q,0)P_Lu_i(p) = \\ &= 2i\lambda^\dagger_{kj}(\lambda^\dagger\lambda)_{ki}\epsilon_{\alpha\beta}\frac{M_iM_k}{M_k^2-M_i^2}J_s\bar{u}^\alpha_j(p-k)P_Ru_i(p) \end{aligned} \tag{A.13}$$

where J_s is given by

$$J_s = -\frac{i}{4p^2}\int\frac{d^4q}{(2\pi)^4}\,\mathrm{Tr}\,[\not{p}S(q,0)]\,D(p-q,0). \tag{A.14}$$

The imaginary part of this function is easy computed with the Cutkosky rules; we have

$$2\,\mathrm{Im}\,\{J_s\} = -\frac{1}{p^2}\int\frac{d^4q}{(2\pi)^2}p\cdot q\theta(q_0)\delta(q^2)\theta(p_0-q_0)\delta((p-q)^2). \tag{A.15}$$

Integrating the delta functions in the center of mass frame, we get

$$2\,\mathrm{Im}\,\{J_s\} = -\frac{1}{16\pi} \tag{A.16}$$

as in eq. (2.7).

The computation of $\epsilon^{N_i}_\ell$(wave) is analogous to the one in the vertex case; we have

$$2\,\mathrm{Re}\,\{\mathcal{A}_0^*\mathcal{A}_s\} = -\sum_k\frac{M_kM_i}{M_k^2-M_i^2}2\,\mathrm{Re}\,\left\{(\lambda^\dagger\lambda)^2_{ki}J_s\right\} \tag{A.17}$$

so that

$$\begin{aligned} \epsilon^{N_i}(\text{wave}) &= -\sum_k\frac{4M_kM_i}{M_k^2-M_i^2}\frac{\mathrm{Im}\,\left\{(\lambda^\dagger\lambda)^2_{ki}\right\}}{(\lambda^\dagger\lambda)_{ii}}\mathrm{Im}\,\{J_t(x_k)\} & \text{(A.18)} \\ &= -\sum_k\frac{1}{8\pi}\frac{M_kM_i}{M_k^2-M_i^2}\mathcal{I}_{ki}. & \text{(A.19)} \end{aligned}$$

[1]The other part arises considering on shell particles instead of antiparticles propagating in the loop and can be computed similarly.

Bibliography

[1] A. D. Sakharov, JETP Lett. **5** (1967) 24.

[2] E. W. Kolb and M. S. Turner, Ann. Rev. Nucl. Part. Sci. 33 (1983) 645.

[3] A. D. Dolgov, Phys. Rep. 222 (1992) 309.

[4] M. Fukugita and T. Yanagida, Phys. Lett. **B174** (1986) 45.

[5] N. S. Manton, Phys. Rev. **D28** (1983) 2019; F. R. Klinkhammer and N. S. Manton, Phys. Rev. **D30** (1984) 2212.

[6] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. **B155** (1985) 36; S. Khlebnikov and M. Shaposhnikov, Nucl. Phys. **B308** (1988) 883.

[7] M. A. Luty, Phys. Rev. **D45** (1992) 455.

[8] C. E. Vayonakis, Phys. Lett. **B286** (1992) 92.

[9] M. Plümacher, Z. Phys. **C74** (1997) 549.

[10] G. Gelmini and E. Roulet, Rep. Prog. Phys. **58** (1995) 1207.

[11] B. A. Campbell, S. Davidson and K. A. Olive, Nucl. Phys. **B399** (1993) 111.

[12] H. Murayama, H. Suzuki, T. Yanagida and J. Yokoyama, Phys. Rev. Lett. **70** (1993) 1912.

[13] L. Covi, E. Roulet and F. Vissani, Phys. Lett **B384** (1996) 169.

[14] L. Covi and E. Roulet, Phys. Lett. **B399** (1997) 113.

[15] L. Covi, N. Rius, E. Roulet and F. Vissani, hep-ph/9704366, submitted to Phys. Rev. D.

[16] A. G. Cohen, A. De Rújula and S. L. Glashow, astro-ph/9707087.

[17] C. J. Copi, D. N. Schramm and M. S. Turner, Science **267** (1995) 192.

[18] E. W. Kolb and M. S. Turner, *The Early Universe*, Addison-Wesley, Redwood City, California, 1990.

[19] E. W. Kolb and S. Wolfram, Nucl. Phys. **B 172** (1980) 224.

[20] G. 't Hooft, Phys. Rev. Lett. **37** (1976) 8; Phys. Rev. **D14** (1976) 3432.

[21] A. G. Cohen, D. B. Kaplan and A. E. Nelson, Annu. Rev. Nucl. Part. Sci. **43** (1993) 27.

[22] M. B. Gavela, P. Hernandez, J. Orloff, O. Pène and C. Quimbay, Nucl. Phys. **B430** (1994) 382.

[23] J. A. Harvey and M. S. Turner, Phys. Rev. **D42** (1990) 3344.

[24] R. N. Mohapatra, *Unification and Supersymmetry*, Springer Verlag, Heidelberg, 1986.

[25] J. Ellis, M. K. Gaillard and D. V. Nanopoulos, Phys. Lett. **B80** (1979) 360.

[26] J. A. Harvey, E. W. Kolb, D. B. Reiss and S. Wolfram, Nucl. Phys. **B201** (1982) 16.

[27] J. Liu and G. Segrè, Phys. Rev. **D49** (1994) 1342.

[28] A. Yildiz and P. H. Cox, Phys. Rev. **D21** (1980) 906.

[29] K. Takahashi, Phys. Rev. **D29** (1984) 632.

[30] F. J. Botella and J. Roldan, Phys. Rev. **D44** (1991) 966.

[31] D.V. Nanopoulos and S. Weinberg, Phys. Rev. **D20** (1979) 2484.

[32] J. Liu and G. Segrè, Phys. Rev. **D48** (1993) 4609.

[33] A. Yu. Ignatiev, V. A. Kuzmin and M. E. Shaposhnikov, JETP Lett. **30** (1979) 688.

[34] M. Fukugita and T. Yanagida, *Physics of Neutrinos,* in: *Physics and Astrophysics of Neutrinos*, Eds. M. Fukugita and A. Suzuki, (Springer–Verlag, Tokyo, 1994).

[35] M. Flanz, E. A. Paschos and U. Sarkar, Phys. Lett. **B345** (1995) 248; Erratum: ibidem **B382** (1996) 447 and **B384** (1996) 487.

[36] M. Gell–Mann, P. Ramond and R. Slansky, *Complex Spinors and Unified Theories* in: *Supergravity*, Eds. P. van Nieuwenhuizen and D. Z. Freedman, (North–Holland, Amsterdam, 1979).

[37] T. Yanagida in: *Proceedings of the Workshop on Unified Theories and Baryon Number in the Universe*, Eds. A. Sawada and A. Sugamoto (KEK, Tsukuba).

[38] S. M. Bilenky and S. T. Petcov, Rev. Mod. Phys. **59** (1987) 671.

[39] W. Buchmüller and M. Plümacher, Phys. Lett. **B389** (1996) 73.

[40] W. Weisskopf and E. P. Wigner, Z. Phys. **63** (1930) 54; Z. Phys. **65** (1930) 18.

[41] A. Pilaftsis, hep-ph/9707235.

[42] See e.g. I. Dunietz and J. L. Rosner, Phys. Rev. **D34** (1986) 1404.

[43] M. Flanz, E. A. Paschos, U. Sarkar and J. Weiss, Phys. Lett. **B389** (1996) 693.

[44] J. A. Harvey, D. B. Reiss and P. Ramond, Nucl. Phys. **B199** (1982) 223; S. Dimopoulos, L. Hall and S. Raby, Phys. Rev. **D47** (1993) R3697; Y. Achiman and Th. Greiner, Nucl. Phys. **B443** (1995) 3.

[45] A. Acker, H. Kikuchi, E. Ma and U. Sarkar, Phys. Rev. **D48** (1993) 5006.

[46] M. Le Bellac, *Thermal Field Theory*, Cambridge University Press, 1996; N. P. Landsman and Ch. G. van Weert, Phys. Rep. **145** (1987) 141.

[47] J. I. Kapusta, *Finite-Temperature Field Theory*, Cambridge University Press, 1989.

[48] R. Kobes, Phys. Rev. Lett. **67** (1991) 1384; M. E. Carrington and U. Heinz, hep-th/9606055.

[49] R. Kobes and G. W. Semenoff, Nucl. Phys. **B272** (1986) 329; F. Gelis, hep-ph/9701410.

[50] H. A. Weldon, Phys. Rev. **D26** (1982) 2789.

[51] M. E. Carrington, Phys. Rev. **D45** (1992) 2933.

[52] H. A. Weldon, Phys. Rev. **D40** (1989) 2410.

[53] J. F. Donoghue and B. R. Holstein, Phys. Rev. **D28** (1983) 340, Phys. Rev. **D29** (1984) 3004.

[54] R. F. Sawyer, Phys. Rev. **D53** (1996) 4232.

[55] I. A. Chapman, hep-ph/9701403.

[56] C. Itzykson and J.-B. Zuber, *Quantum Field Theory*, Mc Graw - Hill, New York, U.S.A., 1980.