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Aspects of type I string theory in 4d with $N = 2$ supersymmetry

CANDIDATE

Marco Serone

SUPERVISORS

Profs. Edi Gava and

Kumar S. Narain

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SISSA - SCUOLA
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Strada Costiera 11

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Introduction

At present, the only consistent quantum theory of gravitation is superstring theory. After the discovery of the Green-Schwarz mechanism [1] and the subsequent discovery of the heterotic string [2], this theory received a new interest and was studied in greater detail. Among all the five known perturbative string theories, type I, type IIA and IIB, heterotic $SO(32)$ and $E_8 \times E_8$, the last one received most of the attentions, because of its more interesting phenomenological implications. During the last two years, however, the new developments in string theory that led to the exploration for the first time of non-perturbative phenomena, revealed the existence of several dualities relating one theory with an other and also with eleven-dimensional supergravity. It is becoming clearer and clearer that there are not five distinct string theories, but on the contrary one single theory that reveals itself as a given string theory or 11d sugra in different perturbative regimes. Due to these relations, it is clear that the heterotic string is now on the same footing of the other perturbative strings, that are presently analyzed much more than what done in the past.

Aim of this thesis, in particular, is to present some results concerning models with $N = 2$ supersymmetry in four space-time dimensions, obtained by compactifications of type I string theory. Our first result is a test of the conjectured type I- heterotic duality [3]. This duality is believed to be a strong-weak duality in ten dimensions, but after suitable compactifications of both theories in four dimensions, it can be shown that exists a region of the moduli space of the theory in which both descriptions are weakly coupled. In this region, then, it is meaningful to compare perturbative amplitudes of both models. We test the aforementioned duality by comparing [4] a class of higher derivative gravitational couplings of the form, in superspace notation, $F_g W^{2g}$ where W is the gravitational superfield of the $N = 2$ supergravity, that appear on both theories at one-loop level. The study of higher derivative terms is crucial if one wants to establish a string-duality beyond the low-energy effective action level. The duality predicts the heterotic F_g couplings to be equal to the type I ones in a given limit. We compute the F_g's at one-loop in the type I model, showing that they receive contributions only from the $N = 2$ BPS states of the theory and that, in the appropriate limit, they coincide with the heterotic couplings already computed in [5, 6], in agreement with the given duality. These higher derivative F-terms have been studied in greater detail in the past. In Calabi-Yau compactifications of the type II string the F_g's represent the partition function at genus g of the twisted Calabi-Yau sigma-model [7]. Moreover, they also played a crucial role in establishing a test of $N = 2$ type II-heterotic duality in four dimensions [5]. Their properties and application will be reviewed in chapter 2. We then analyze more general one-loop threshold corrections in $K3 \times T^2$ compactifications of type I string theory [8]. These corrections can be written in general in terms of an $N = 2$ supersymmetric index [9, 10, 11], analogously to what has been previously showed to happen in $N = 2$ four dimensional compactifications of the heterotic string [12]. Studying the superconformal algebra underlying these heterotic models, it has been found [13] that the mentioned $N = 2$ supersymmetric index is determined purely in terms of the BPS states of the theory and, in particular, it counts the difference beteween the number of BPS hyper and vectormultiplets present in the string spectrum. In a similar fashion we find that exactly the same thing happens in $N = 2$ $K3 \times T^2$ type I compactifications, again in agreement with heterotic-type I duality. This result, however, is shown to be valid also for more general type I models, now called orientifolds, by an explicit one-loop computation of the mentioned F_g gravitational couplings, where it is reproduced the BPS dependence of these amplitudes. It is important to note that this result is independent on any string duality statement and indeed all these orientifold models but one, cannot have a weak heterotic dual theory.

The present thesis is organized as follows. In chapter one we review the construction, the spectrum and the basic properties of four dimensional $N = 2$ theories arising from the type I string compactified on the six-manifold $K3 \times T^2$. We then briefly review what D-branes and orientfold are, in order to construct general type I compactifications involving these objects. In chapter two, as already anticipated, a survey on higher derivative F-terms of the form $F_g W^{2g}$ is given, pointing out their importance in string dualities. Chapter 3 is devoted to type I-heterotic duality, where this is analyzed for $d \leq 10$ and in particular for $d = 4$. A dual pair of four dimensional models is presented and then it is performed the one-loop computation of the F_g's in type I, to test the aforementioned duality. The structure of threshold corrections in type I $K3 \times T^2$ compactifications and their relation with the $N = 2$ supersymmetric index is presented in chapter four, where it is then shown the purely dependence of this index on the BPS states of the theory. After that, we show the one-loop moduli dependence of the F_g couplings for orientifold models, pointing out their BPS dependence. Some comments and remarks are finally given in the conclusions, while technical details about the one-loop computations are reported in the appendix.

Chapter 1

Compactifications of Type I string in 4d with $N = 2$ Susy

Type I string theory is a theory of unoriented closed and open strings. It contains both gravity, in the closed string spectrum, and massless gauge mesons, in the open one. It is also possible to realize non-abelian gauge symmetries by considering Chan-Paton factors [14], that is non-dynamical degrees of freedom to the ends of the open string; consistency with tree-level unitarity and factorization of amplitudes shows that the only allowed Chan-Paton gauge groups are $SO(n)$ and $USp(2n)$ [15]. Unoriented strings means simply that there are not "arrows" on the strings, and that the world-sheet parity operator $\Omega : \sigma \to \pi - \sigma$, where $0 \leq \sigma \leq \pi$ parametrizes the spatial coordinate of the world-sheet (as usual, for closed strings, $0 \leq \sigma \leq 2\pi$), is a global symmetry. In particular, all the states in the string spectrum are required to be Ω-invariant, in order to be physical. At the massless level, in the open string sector this simply fixes how Ω acts on the Chan-Paton degrees of freedom, while in the closed string sector, where Ω exchanges the left and right-moving modes, it projects out all the Ω-odd states. The low-energy effective field theory describing the interactions of only the massless string states is the ten dimensional chiral $N = 1$ supergravity coupled to $SO(n)$ or $USp(2n)$ super Yang-Mills theory. All these theories are in general plagued with gauge and gravitational anomalies that make them quantum-mechanically inconsistent. It was however discovered by Green and Schwarz [1] a mechanism that allowed an anomaly-free coupling of pure $N = 1$ sugra with super Yang-Mills theory for the groups $SO(32)$, $E_8 \times E_8, E_8 \times U(1)^{248}, U(1)^{496}$. While the first two theories are known to be realized in terms of strings (type I and heterotic for $SO(32)$ and heterotic only for $E_8 \times E_8$), no string model is known for the last two. The only consistent theory of open and closed string is then $SO(32)$ type I string theory. We will see in the next sections compactifications of this model down to four dimensions, making also use of non-perturbative stringy solitons, i.e. D-branes. In particular in section one we review the standard geometric construction of type I models that present $N = 2$ susy in four dimensions; in sections two and three D-branes and orientifolds are introduced and their basic properties reviewed. Finally, in the last section, we construct more general four dimensional type I vacua, making use of D-branes and orientifolds [1].

[1]There exist other constructions of type I vacua in four dimensions, based on suitable products of rational conformal field theories [16], that will not be discussed in the present thesis.

1.1 Standard geometric compactification

All known string theories are consistent in a flat space-time background only in ten dimensions. In order to construct "realistic" models, we have to deal with six extra dimensions. The common way to do this is to consider them as Kaluza-Klein like directions, forming all together an internal six-manifold K_6. According to the usual Kaluza-Klein reduction, massless fields in four dimensions will be in one-to-one correspondence with zero modes of K_6 and the number of unbroken supersymmetries in 4d will depend on the number of covariantly constant spinors on K_6. In ten dimensions type I theory presents $N = 1$ supersymmetry, that is 16 conserved supercharges; in order to obtain a model that has $N = 2$ in four dimensions, that is 8 supercharges, we then need a manifold that break precisely half of the supersymmetries. Since a model $N = 2$ in 4d can be considered as a toroidal compactification of a $N = 1$ six-dimensional theory, we take K_6 to be a direct product $K_6 = K_4 \times T^2$ and try to find a suitable four-manifold that gives $N = 1$ in six dimensions. Luckily, the answer is unique and is given by the Kummer surface called $K3$. It is a Ricci-flat compact Kähler manifold, topologically unique, whose hodge numbers are $h^{0,0} = h^{2,2} = 1, h^{1,0} = h^{1,2} = h^{2,1} = h^{0,1} = 0, h^{2,0} = h^{0,2} = 1, h^{1,1} = 20$. In order to understand which is the four-dimensional low-energy effective field theory arising from this compactification on $K_6 = K3 \times T^2$, let us analyze the massless spectrum of type I $SO(32)$ theory in ten dimensions. Its bosonic content is given by the graviton g_{MN} and the dilaton Φ, coming from the Neveu-Schwarz Neveu-Schwarz (NS-NS) sector of the closed string, the antisymmetric tensor field B_{MN}, arising from the Ramond Ramond (R-R) sector and the vector fields A^a_M, coming from the open string sector. All capital indices M, N run from zero to nine, while a runs over the adjoint representation of $SO(32)$. We then have the fermions: a left-handed gravitino Ψ_M, a right-handed spinor λ and left-handed gluinos ψ^a, all being 16-components Majorana-Weyl spinors. These states combine together in two supermultiplets: the gravitational $(g_{MN}, B_{MN}, \Phi, \Psi_M, \lambda)$ and gauge (A^a_M, ψ^a) multiplets. Before analyzing the massless spectrum that arises in four dimensions, it is useful to consider, as first step, the six-dimensional theory compactified on $K3$. We will then subsequently compactify this model down to four dimensions on T^2. [2] We only consider bosonic states, since supersymmetry will automatically give us the fermionic partners. According to the hodge numbers given above, the antisymmetric tensor field B_{MN} produces a six-dimensional $B_{\tilde{\mu}\tilde{\nu}}$ and $b_2 = 22$ scalars, the dilaton Φ_{10} gives an other dilaton Φ_6 and the vector fields A^a_M give simply rise to $A^a_{\tilde{\mu}}$ vectors. The massless modes corresponding to g_{MN} are a six-dimensional graviton $g_{\tilde{\mu}\tilde{\nu}}$ and a number of scalars equal to the dimension of the moduli space of the $K3$ metric deformations, that is known to be 58. We then have a total of 81 scalars, neutral under the gauge group, one of which is the dilaton and the remaining 80 parametrize the full moduli space of $K3$. In terms of the $N = 1$ supermultiplets in six dimensions, the massless spectrum contains one gravitational multiplet, that in 6d is composed by the graviton $g_{\tilde{\mu}\tilde{\nu}}$, one right-handed gravitino and the two-form $B^+_{\tilde{\mu}\tilde{\nu}}$, that is the part of

[2] From now on $\tilde{\mu}, \tilde{\nu} = 0, 1, 2, 3, 4, 5$, $\mu, \nu = 0, 1, 2, 3$ run respectively over the six and four space-time directions, while $i, j = 6, 7, 8, 9$ represent $K3$ indices.

the antisymmetric tensor field whose field strength is self-dual, one tensor multiplet composed by $B^{-}_{\bar{\mu}\bar{\nu}}$ (the anti-self dual part), one left-handed spinor and a scalar, 20 hypermultiplets $(\lambda^-, 4\phi)$ and n_V vectormultiplets $(A^{\underline{a}}_{\bar{\mu}}, \psi^{a+})$ in the adjoint of the gauge group, of course. This is not, however, the end of the story. There is a well-known relation [17] that has to be satisfied in order to have a consistent model. The coupling of the ten-dimensional sugra with super Yang-Mills and the Green-Schwarz mechanism for cancelling the anomalies modify the field strength of the antisymmetric field to be

$$H_{MNP} = d_{[M}B_{NP]} + \frac{\alpha'}{4}(\omega^{(L)}_{[MNP]} - (\omega^{(Y)}_{[MNP]}) \tag{1.1}$$

where $\omega^{(L,Y)}$ are respectively the Chern-Simons three-form associated to the Lorentz and Yang-Mills gauge groups. This implies that

$$\int_{K3} dH = \frac{\alpha'}{4}\int_{K3}(trR\wedge R - trF\wedge F) = 0 \tag{1.2}$$

since H has to be globally defined. The integral of the Euler class $R\wedge R$ gives the Euler characteristic of the corresponding manifold, and for $K3$ is equal to 24. We then need to define our theory on a non-trivial background inserting a total of 24 instantons inside $K3$. When relation (1.2) is satisfied, moreover, the six-dimensional theory is also anomaly-free [18]. Since the number of massless states is given by the topology of the internal manifold, as we have seen, for six-dimensional theories the requirement of anomaly-cancellation is often written as a constraint on the number of supermultiplets. In particular, if n_V, n_H and n_T are respectively the number of vector, hyper and tensormultiplets of the model, cancellation of anomalies or, more precisely, the cancellation of the coefficient in the anomaly polynomial proportional to

$\mathrm{Tr}R^4$, require that

$$n_H - n_V = 244 - 29(n_T - 1) \tag{1.3}$$

The simplest way to satisfy (1.2) is to embed the $SU(2)$ spin connection inside the $SO(32)$ gauge group. Since $SO(32) \supset SO(28) \otimes SO(4) \cong SO(28) \otimes SU(2) \otimes SU(2)$, with this choice of background the unbroken gauge group will be $SO(28) \otimes SU(2)$. In addition to the massless spectrum derived above, we obtain also charged and further neutral hypermultiplets, the last one corresponding to the deformations of the gauge bundle. This content can be easily derived by using various index theorems, relating the net number of chiral fermions to topological quantities (see e.g.[19]). In particular we obtain 10 $(\underline{28}, \underline{2})$ and 45 $(\underline{1}, \underline{1})$ hypermultiplets, where we have denoted respectively their representations under $SO(28) \otimes SU(2)$.

Summarizing, the six-dimensional massless spectrum arising from $K3$ compactifications with the spin connection embedded in the gauge group, consists of one gravitational and tensormultiplet, 65 neutral and 10 $(\underline{28}, \underline{2})$ hypermultiplets and 381 vectormultiplets forming the adjoint of $SO(28) \otimes SU(2)$. As has to be, this spectrum verifies relation (1.3). It is important to remember that there are in general different ways to solve the topological condition (1.2) that lead to different massless spectra and unbroken gauge groups in six dimensions. In any case, however, the number of tensormultiplets that can arise in this kind of compactifications is always fixed to be one, altough it was shown by Sagnotti [20] that supersymmetric anomaly-free six-dimensional theories with more than one tensormultiplet can be constructed through a generalization of the Green-Schwarz mechanism. We will see in the next sections that these low-energy models are actually realized in type

I string theory by considering more general vacua involving D-branes and orientifolds.

Finally, the further compactification on T^2 gives rise to a four-dimensional graviton $g_{\mu\nu}$, two vector fields $g_{\mu 4(5)}$ and three scalars g_{44}, g_{55}, g_{45} from $g_{\tilde{\mu}\tilde{\nu}}$, a dilaton Φ_4 from Φ_6 and an antisymmetric tensor field $B_{\mu\nu}$, two vectors $B_{\mu 4(5)}$ and one scalar B_{45} from $B_{\tilde{\mu}\tilde{\nu}}$. Finally $A^a_{\tilde{\mu}}$ gives four-dimensional gauge bosons and two scalars ϕ^a, in the adjoint of the gauge group. In terms of $N = 2$ supermultiplets, the massless spectrum is formed by a gravitational multiplet $(g_{\mu\nu}, 2\Psi_\mu, A_\mu)$, 65 neutral and 10 $(\underline{28}, \underline{2})$ hypermultiplets $(\lambda, 4\phi)$ and $(n_V + 3)$ vectormultiplets $(A^a_\mu, \psi^a, 2\phi^a)$ where $(n_V + 3)$ is the number of vectors present in six dimensions (in our case 381) plus the three more, coming from the torus compactification; Ψ_μ, λ and ψ^a are, of course, the gravitinos, the fermion matter and the gluinos. Note that, among the four vectors arising from T^2, a linear combination A_μ belongs to the gravitational multiplet. For this reason, this gauge field is also called the graviphoton.

1.2 D-branes

String theory is a perturbative first-quantized theory defined only on-shell. For this reason, it has been a long-standing problem how to take into account non-perturbative states, like monopoles or dyons, or more generally, non-perturbative effects in this theory. In particular, non-trivial solutions of various supergravity theories, called p-branes, a sort of soliton extended in p-spatial directions, were known (for a review on this subject see [21] and references therein) and it was a problem how to interpret them in string theory up to two years ago, when in a seminal paper [22] Polchinski suggested that the type II (and type I) p-branes, charged under the antisymmetric tensor fields coming from the Ramond-Ramond closed string sector are simply described in string theory as Dirichlet p-branes (commonly denoted D-branes), i.e. p-dimensional hyperplanes where open strings can end. So, type II string theories are theories of only closed strings at the perturbative level: in order to describe solitonic sectors one has to introduce open strings as well. In a modern language, the usual type I string theory is a theory of Dirichlet 9-branes.

Altough D-branes were discovered historically [23] by analyzing the behaviour of open string theory compactified on a circle of radius $R \rightarrow 0$, we will briefly review here their characteristics following a different path and pointing out some properties more than others, according to the use we will do of them. Let us consider for simplicity the bosonic string; in a flat background the two-dimensional action is simply

$$S = \frac{1}{4\pi\alpha'} \int_M d^2\sigma \, \partial_\alpha X^\mu \partial^\alpha X_\mu \tag{1.4}$$

where μ runs from 0 to 25 and M represents the two-dimensional world-sheet. Taking the variation of the action with respect to X, we have

$$\begin{aligned} \delta S &= \frac{1}{2\pi\alpha'} \int_M d^2\sigma \, \delta(\partial_\alpha X^\mu) \partial^\alpha X_\mu = \\ &= -\frac{1}{2\pi\alpha'} \int_M d^2\sigma \, \delta X^\mu \Box X_\mu + \frac{1}{2\pi\alpha'} \int_{\partial M} d\Sigma^\alpha \delta X_\mu \partial_\alpha X^\mu \end{aligned} \tag{1.5}$$

While the vanishing of the first term in (1.5) gives us the equations of motion for X^μ, the vanishing of the second fixes the possible consistent boundary conditions for the fields. If we are dealing with closed strings, there is no boundary on the world-sheet and the second term in (1.5) automatically vanishes (at the "boundaries" $\tau = \pm\infty$ we always take $\delta X = 0$); in the presence of open strings we have to make it vanishing. The usual way to do this is to choose Neumann boundary conditions (b.c.) $\partial_n X^\mu|_{\partial M} = 0$ where n is the direction perpendicular to the boundary; with this choice we obtain the usual open string whose ends are free to move in all the space. An equally good choice is to take Dirichlet boundary conditions $\delta X^\mu|_{\partial M} = 0$ or, more generally Neumann b.c. for some directions and Dirichlet b.c. for the remaining. This second possibility was ignored in the past because it breaks manifestly the Lorentz invariance of the theory, as a topological defect in general does. Parameterizing the world-sheet as usual with τ and $0 \leq \sigma \leq \pi$, the Neumann b.c. fix the mode expansions of X to be

$$X_N^\mu(\tau,\sigma) = x^\mu + p^\mu \tau + \frac{i}{2}\sum_{n\neq 0}\frac{\alpha_n^\mu}{n}(e^{-in\sigma^+} + e^{-in\sigma^-}) \tag{1.6}$$

where we set $\alpha' \equiv 1/2$ and $\sigma_\pm = \tau \pm \sigma$, whereas the Dirichlet b.c.

$$\begin{aligned} X(\sigma = 0, \tau) &= x_0 \\ X(\sigma = \pi, \tau) &= x_\pi \end{aligned} \tag{1.7}$$

give

$$X_D^\mu(\tau,\sigma) = x_0^\mu + \frac{x_\pi^\mu - x_0^\mu}{\pi}\sigma + \frac{i}{2}\sum_{n\neq 0}\frac{\alpha_n^\mu}{n}(e^{-in\sigma^+} - e^{-in\sigma^-}) \tag{1.8}$$

As we can see from (1.7) and (1.8), in the directions corresponding to Dirichlet b.c., the ends of the string are fixed and the total momentum is vanishing. In both Dirichlet and Neumann b.c., however, the canonical quantization rules for the α_n are the usual $[\alpha_n^\mu, \alpha_m^\nu] = n\eta^{\mu\nu}\delta_{n+m}$. If we impose Neumann b.c. for p+1 directions (we single out the time to be always Neumann) and

Dirichlet for the remaining 25-p, the ends of the strings are constrained to live in a p-spatial dimensional hyperplane that we call a Dirichlet p-brane or simply a (D)p-brane. We will see in the following that this hyperplane has all the properties to be identified as an extended soliton in string theory; in this way we interpret the open strings living on the D-brane as excitations of it. For a flat space-time background, the conditions (1.7) are equivalently written as

$$\partial_\tau X(\sigma = 0, \pi; \tau) = 0 \tag{1.9}$$

similarly to the Neumann b.c.

$$\partial_\sigma X(\sigma = 0, \pi; \tau) = 0 \tag{1.10}$$

We see then that Dirichlet $\leftrightarrow$ Neumann for $\partial\tau \leftrightarrow \partial\sigma$. Actually, there is a symmetry in closed string theory that sends $\partial\tau \leftrightarrow \partial\sigma$ and is just T-duality. More precisely, if we take one of the spatial directions, say X^{25}, to be a circle of radius R, then under the T-duality trasformation sending

$$R \rightarrow \frac{\alpha'}{R}, \quad m \leftrightarrow n$$

$$\alpha_n^{25} \rightarrow \alpha_n^{25}, \quad \tilde{\alpha}_n^{25} \rightarrow -\tilde{\alpha}_n^{25} \tag{1.11}$$

where m, n are the Kaluza-Klein and winding modes on R, the theory is invariant. In open string theory, T-duality is not a symmetry but it can always be considered as the trasformation of variables

$$X_L^{25} \rightarrow X_L^{25}, \quad X_R^{25} \rightarrow -X_R^{25} \tag{1.12}$$

under which

$$\partial_\sigma X^{25} = \partial X_L - \bar{\partial} X_R \rightarrow \partial X_L + \bar{\partial} X_R = \partial_\tau X^{25} \tag{1.13}$$

and viceversa. In this way, indeed, [23] found that bosonic open string theory at radius R is equivalent to a theory of (D)25-branes on a radius α'/R, where the Kaluza-Klein modes of the original open strings are mapped to strings whose ends wrap around the circle before ending on the D-brane [3].

As in any topological defect, it is interesting to consider zero modes of D-branes, that is the spectrum of massless open strings living on them. A generic (D)p-brane breaks the Lorentz group $SO(25,1)$ in $SO(p,1) \otimes SO(25-p)$, so that in the p+1 world-volume of the D-brane the 26 massless open string states $\alpha^{M}_{-1}|0>$ are splitted into a vector field and 25-p scalars:

$$\begin{aligned} \alpha^{\mu}_{-1}|0> \quad \mu &= 0,...,p \\ \alpha^{i}_{-1}|0> \quad i &= p+1,...,25 \end{aligned} \tag{1.14}$$

The p+1 dimensional classical action describing the interactions of these states between each other and with the usual "bulk" closed states was derived by [24] at all orders in α' for a constant field strenght $F_{\mu\nu}$. It is given by

$$S_p = -T_p \int d^{p+1}\sigma e^{-\phi} \sqrt{det[G_{\mu\nu} + B_{\mu\nu} + 2\pi\alpha' F_{\mu\nu}]} \tag{1.15}$$

where $\mu, \nu = 0, ..., p+1$ and $G_{\mu\nu}, B_{\mu\nu}$ are the pull-back of the graviton and the antisymmetric tensor field to the (D)p-brane. T_p is the D-brane tension and $e^{-\phi}$ is the right dilaton dependence coming from a disk computation in the string frame, where the bulk action is weighted with $e^{-2\phi}$. In the string frame the tension of ordinary strings is $\sim g_S^0$, where $g_S = e^{<\phi>}$ represents the string coupling constant; on the other hand the effective D-brane tension is $\sim T_p/g_S$, as shown by eq.(1.15), that is a typical dependence in the coupling

[3]Strictly speaking, the dual radius is the orbifold S^1/Z_2 where we have orientifold planes at the fixed points. See the next section for a survey of orientifolds.

characteristic of a non-perturbative object, altough slightly different for a typical soliton mass that is of order $1/g^2$. For this reason D-branes are sometimes called "stringy"-solitons[4]. The dynamics of configurations of multiple D-branes can be equally well described. In this case we have to in general to consider, among the strings whose ends live on a given D-brane, those stretched between two different D-branes; all these strings are however massive, having a non-vanishing tension, and then the low-energy world-volume action of n D-branes is simply given by n copies of (1.15) with total gauge group $U(1)^n$. On the other hand, in the limit in which two D-branes overlap each other, the lowest stretched modes become massless and the effective world-volume theory gets enhanced to a $U(2)$ gauge theory, where these new massless modes correspond to the $A^{\pm}$ charged gluons. For n-coincident D-branes the world-volume action is of course enhanced to $U(n)$.

Let us now turn to the superstring, the case we are actually interested in. All the properties we have decribed up to now for D-branes coming from the bosonic string are valid in presence of supersymmetry with simple generalizations. If we denote with $\psi_{L,R}$ the left(right)-moving world-sheet fermions in the R-NS formalism, by supersymmetry and (1.12), Dirichlet b.c. can be implemented simply by sending

$$\psi_L \to \psi_L, \quad \psi_R \to -\psi_R \tag{1.16}$$

[4]It is worth while to point out that this $1/g$-dependence was already predicted several years ago by Shenker [25] that argued that the large order behaviour of the perturbative expansion in string theory allowed for non-perturbative effects at weak coupling of order e^{-1/g_S}.

in the usual Neumann b.c.; since it is the relative sign between the two boundaries that is relevant, the mode expansions is unaltered, except a sign between left and right moving modes, exactly as in (1.8) [5]. The low-energy effective action for n (D)p-branes is the supersymmetric generalization of the bosonic one and it is [26] the dimensional reduction of $N = 1$ $U(n)$ ten dimensional super Yang-Mills theory down to p+1 dimensions. In the supersymmetric case, however, D-branes enjoy two more fundamental properties: they are the sources of the R-R antisymmetric tensor fields and they are BPS-saturated states. Let us discuss these two properties in some more detail.

R-R antisymmetric tensor fields arise in both type IIA,B and type I theories; due to the different chiralities of their supersymmetries, type IIA contains p-forms with p odd, type IIB even p-forms and type I, as we saw in the last section, the antisymmetric two-form B_{MN} and its Hodge dual six-form. All these tensor fields present a $U(1)$ local gauge symmetry, but at the perturbative level, in every string theory, there does not exist any state charged under any of these fields. This can be verified by computing three-point amplitudes between R-R forms and any string state, but it can be argued easily by noting that in general a p+1-form couples minimally to a p-extended object, generalizing the usual minimal coupling between a vector field (1-form) and a charged particle (0-brane). Actually, even the R-R two form does not couple minimally to perturbative strings. On the contrary, it was realized by Polchinski [22] that (D)p-branes are the sources for the R-R p+1

[5]Note that in the heterotic string D-branes do not exist because they require left and right moving sector of the string to be both bosonic or supersymmetric as can be easily deduced, for instance, from (1.16).

tensor fields and, amazingly, with the minimum charge allowed by Dirac-Nepomechie-Teitelboim's [27] quantization condition. It follows that type IIA(B) theory has only even(odd) D-branes whereas type I contains 1,5 and 9-branes. In the same work [22], the force between two D-branes has been computed through a one-loop annulus computation and it has been shown to vanish; moreover, all the D-branes charge densities are always equal to their tensions. These two important facts show that D-branes are indeed BPS-saturated states where these properties hold. In particular they break exactly one half of the supersymmetries. There are several ways to see this; we will use here a conformal field theory approach that is useful to explicitly construct the unbroken supesymmetries in terms of vertex operators, also in more complicated D-brane backgrounds, as those we will consider in the next sections.

Let us start from type I, i.e. 9-branes. In this case the linear combination of unbroken supesymmetries is just $Q_L^\alpha + Q_R^\alpha$ (as can be deduced for instance by requiring Ω-invariance) where Q_L, Q_R are just the 16-components Majorana-Weyl left(right)-supercharges that in the R-NS formalism are given by

$$\begin{aligned} Q_L^\alpha &= \int dz\, e^{-\frac{\phi_L}{2}} S_L^\alpha(z) \\ Q_R^\alpha &= \int d\bar{z}\, e^{-\frac{\phi_R}{2}} S_R^\alpha(\bar{z}) \end{aligned} \quad (1.17)$$

where $\phi_{L,R}$ and $S_{L,R}^\alpha$ are respectively the bosonization of the superghosts and the space-time spin-field operators for both sectors [6]. If we take one direction, say X^9, to be a circle, we can then construct 8-branes, according

[6] We use here and throughout all the present thesis the formalism originally introduced by [28].

to (1.16), simply by sending

$$\psi_L^9 \to \psi_L^9, \quad \psi_R^9 \to -\psi_R^9 \tag{1.18}$$

while leaving invariant the remaining directions. Since the fermion fields $\psi_{L,R}$ act on the spin-field operators $S_{L,R}^\alpha$ essentially as gamma matrices, it is easy to see that (1.18) require for consistency that

$$S_L^\alpha \to S_L^\alpha, \quad S_R^\alpha \to (\gamma^9\gamma^{11})^{\alpha\beta} S_{R,\beta} \tag{1.19}$$

We can then argue that a (D)8-brane fixed in the 9^{th} direction leaves unbroken the linear combinations of supercharges given by

$$Q_L^\alpha + (\gamma^9\gamma^{11})^{\alpha\beta} Q_{R,\beta} \tag{1.20}$$

The generalization is strightforward: a (D)p-brane fixed in the directions $X^{p+1}, ..., X^9$ leaves unbroken the combinations

$$Q_L^\alpha + \prod_{m=p+1}^{9} (\gamma^m\gamma^{11})^{\alpha\beta} Q_{R,\beta} \tag{1.21}$$

for p even and

$$Q_L^\alpha + \prod_{m=p+1}^{9} (\gamma^m\gamma^{11})^\alpha_\beta Q_R^\beta \tag{1.22}$$

for p odd, according to the different chiralities of type IIA and IIB theories. It is also possible to consider more general backgrounds where different or non-parallel (D)p-branes are present. Consider for instance a (D)p-brane extended in the directions $X^0, X^1, ..., X^p$ and a (D)q-brane, with q>p, extended in $X^0, X^1, ..., X^q$. Among the usual open strings living on the same brane, consider those stretched between the two. For these kind of strings, we have Neuamnn b.c. on both ends for directions $X^0, X^1, ..., X^p$ (call it N-N b.c.), Dirichlet-Dirichlet (D-D) b.c. for $X^{q+1},...,X^9$ but we have now also mixed b.c. D-N or N-D for the $X^{p+1},...,X^q$ directions, i.e. open strings in which one end is fixed to a D-brane while the other is free to move. These mixed conditions D-N and N-D b.c.

$$\begin{aligned} X(\sigma=0,\tau) &= x_0 & \partial_\sigma X(\sigma=0,\tau) &= 0 \\ \partial_\sigma X(\sigma=\pi,\tau) &= 0 & X(\sigma=\pi,\tau) &= x_\pi \end{aligned} \tag{1.23}$$

fix the mode expansions for x to be

$$X^\mu_{DN,ND} = x^\mu_{0,\pi} + \frac{i}{2} \sum_{r\in Z+1/2} \frac{\alpha^\mu_r}{r}(e^{-ir\sigma_+} \mp e^{-ir\sigma_-}) \tag{1.24}$$

In this case the bosonic oscillators α_r have half-integer modes, while in the fermions this $\mathbf{Z}_2$-twist shift the NS and R modes to be respectively integer and half-integer. What about supersymmetry in these backgrounds ? The total number of conserved supercharges will be the intersection between the linear combinations

$$Q^\alpha_L + \prod_{n=q+1}^{9} (\gamma^n\gamma^{11})^{\alpha\beta} Q_{R,\beta} \tag{1.25}$$

and

$$Q^\alpha_L + \prod_{m=p+1}^{9} (\gamma^m\gamma^{11})^{\alpha\beta} Q_{R,\beta} = Q^\alpha_L + \prod_{n=q+1}^{9} (\gamma^n\gamma^{11})^{\alpha\beta} \prod_{m=p+1}^{q+1} (\gamma^m\gamma^{11})^{\beta\delta} Q_{R,\delta} \tag{1.26}$$

The number of unbroken supersymmetries is then basically given by the number of $+1$ eigenvalues of the matrix

$$M \equiv \prod_{m=p+1}^{q+1} (\gamma^m\gamma^{11}) \tag{1.27}$$

When q-p=2 mod 4, $M^2 = -I$ and all eigenvalues are purely imaginary, meaning that a system with p and p+2, or p+6, D-branes breaks all the supersymmetries. On the contrary if q-p=0 mod 4, $M^2 = I$ and since M is traceless, the eigenvalues will be ± 1 in equal number. This implies that the system p-p+4 and p-p+8 is supersymmetric, altough in this case the number of conserved supercharges is one-quarter of the original supersymmetries. The same argument also applies when we have D-branes intersecting at right-angles [7]; in all the cases we will have unbroken supersymmetries if the sum of the ND and DN directions is a multiple of four. The class of $N = 2$ 4d type I models we will construct in the end of this chapter, for instance, involves both (D)9 and 5-branes. The presence in type I string theory of a world-sheet parity symmetry, however, allows and requires the presence of other non-dynamical objects, the orientifold planes, that will be the topic of next section.

1.3 Orientifolds

We mentioned at the beginning of this chapter that in type I theory the string spectrum is required to be Ω-invariant. This operator exchanges left and right-moving modes on the world-sheet

$$\Omega X_{L,R} \Omega^{-1} = X_{R,L} \tag{1.28}$$

but it obviously leaves invariant the sum $X = X_L + X_R$. This means that it acts locally on space-time. If we denote with $|x\rangle$ the lowest string state localized in x, Ω-invariance requires, for instance, that among the closed

[7]The case of branes intersecting at generic angles has been considered by [29].

string states

$$(\alpha^{\mu}_{L,-1}\alpha^{\nu}_{R,-1} + \alpha^{\mu}_{R,-1}\alpha^{\nu}_{L,-1})|x\rangle \tag{1.29}$$

only the symmetric combination is physical, whereas the antisymmetric one is projected out from the spectrum in all space-time. Consider now, as usual, one direction, say X^{25} in bosonic string theory, to be compactfied on a circle S^1 of radius R and perform the trasformation (1.12); the spatial-coordinate associated with the circle in the new variables is

$$X'^{25} = X^{25}_L - X^{25}_R \tag{1.30}$$

that changes sign under an Ω-trasformation. This phenomenon drastically alters the action of Ω in this dual theory. There are not anymore states projected out from the string spectrum, but on the other hand states located at different points are not anymore independent, due to relation (1.30); in particular Ω-invariance now implies that for any propagating state localized in $|x\rangle$ there exists an "image" state in $|-x\rangle$ with the same wave function. This means that the dynamics of this model is completely fixed once we know it on "half" of the circle $S^{1\prime}$ of radius $1/R$, i.e. on the segment $S^{1\prime}/\mathbf{Z}_2$. The two hyperplanes corresponding to the fixed points $X'^{25} = 0, \pi$ are known as orientifold planes. In brief, if we start from the bosonic theory of unoriented closed and open strings (25-branes) defined on $R^{1,24} \times S^1$ and perform the trasformation (1.12), the dual model we get is a theory of oriented closed strings and 24-branes on $R^{1,24} \times S^{1\prime}/\mathbf{Z}_2$ plus two orientifold planes. Unlike D-branes, orientifold planes do not carry any physical degree of freedom and are not dynamical objects; on the other hand, they can act as sources absorbing or emitting closed strings. The same considerations performed here for the bosonic case are also valid for the superstring and are easily generalized for a generic torus compactification T^n. Type I $SO(32)$ string theory compactified on T^n, for instance, is equivalent to a theory of $(9-n)$-branes moving on $T^n/\mathbf{Z}_2$ with 2^n orientifold $(9-n)$ hyperplanes, i.e. hyperplanes of dimension $(9-n)$. Similarly to (D)p-branes, p-orientifolds are also sources for the R-R p+1 antisymmetric tensor fields with a charge Q_p^O that is [30]

$$Q_p^O = -2^{p-5} Q_p^D \tag{1.31}$$

where Q_p^D is the (D)p-brane R-R charge. We saw that when n D-branes overlap each other, there is an enhancement of the world-volume gauge theory up to $U(n)$. In presence of orientifolds, there is also the possibility for n D-branes to overlap each other to the fixed points; in this case there are additional massless string states corresponding to strings stretched between a D-brane and the image of another brane, with the result of a further symmetry enhancement from $U(n)$ up to $SO(2n)$.

It is quite interesting to understand which is the constraint that singles out the $SO(32)$ gauge group as one of the 10d anomaly-free Yang-Mills couplings with $N=1$ supergravity, in the dual theory of D-branes and orientifolds. The argument is very simple. Orientifolds and D-branes are sources for R-R tensor fields; while in uncompactified spaces there is no constraint to the number of D-branes one can have, in compact spaces this number is constrained to give vanishing R-R fluxes, that otherwise could not spread off to infinity. This implies that the total R-R charge of p-orientifolds and (D)p-branes has to sum up to zero for each p. Taking only one compact direction, this means, because of (1.31), that we need 16 8-branes plus the corresponding images. In the $R \to 0$ limit (uncompactified 10d space-time in the dual theory) all branes collaps together on the fixed point giving rise to the $SO(32)$ gauge group as the only possibility. We will see in the next section that consistency conditions similar to the present one allows to construct consistent type I $N = 2$ models in four dimensions not directly reachable by the standard geometric compactification reviewed in section one. It is worth while to remember that, altough the description we gave here about orientifolds follow closely [30], they were effectively considered also in [31, 32], but from a slightly different point of view.

1.4 Orientifold compactifications

One of the most important consistency conditions for oriented closed string theories is modular-invariance of one-loop amplitudes; when this property holds, the theory is simultaneously anomaly-free and ultraviolet finite; on the other way, for theories based on open and closed unoriented strings this important property is lost. It was however noted in [1, 33] that in this case the cancellation of divergencies that appear in one-loop amplitudes plays the same role of modular invariance in oriented closed strings, implying simultaneously UV finiteness and cancellation of anomalies. Later on, [34, 35] argued that the presence of one-loop divergencies in type I theory can be interpreted as inconsistencies in the equations of motion of some massless closed string states, according to the usual UV-IR relation between dual string channels. This last interpretation is indeed easily extendable to the case of D-branes and orientifolds. We learned in last sections that type I string theory in a modern language is a theory of unoriented closed strings interacting with

(D)9-branes, where the number of branes is fixed by requiring the total R-R flux to be zero. For 9-branes the corresponding R-R tensor field is a 10-form A_{10}, an unphysical field whose kinetic term automatically vanishes; it is present in the action only with the term

$$iq \int A_{10} \tag{1.32}$$

Its "equation of motion" then trivially requires $q = 0$, where q is the total charge carried by 9-branes and orientifolds. In case of a compact space, there are more consistency conditions analogous to (1.32), because we can have several R-R fluxes in the compact directions.

Type I string theory, according to considerations that dates back to [31], can be constructed starting from type IIB theory and taking the quotient under Ω. This point of view is very useful to construct compactified orientifold models. After this operation, type IIB loses its modular invariance, because we are now obliged to consider unoriented closed surfaces in the world-sheet, like the Klein bottle, that spoils this property. The theory is no more UV finite and consistent, but introducing boundaries on the world-sheet, i.e. open strings, one can hope to cancel the divergencies carried by the Klein bottle, as indeed happens. The interpretation of this mechanism in our by now familiar language, is that the quotient of Ω in type IIB created 9-orientifolds filling all space, and inducing a non-vanishing charge in (1.32) that has to be balanced by introducing an appropriate number of (D)9-branes. According to this point of view, we can then construct type I vacua starting in ten dimensions from type IIB theory and taking into account the Ω-projection. Since we are interested to models that present $N = 2$ susy in 4d, that is

$N = 1$ in 6d, we consider then IIB orientifolds on $K3$ and afterwards we compactify them down to four dimensions on a T^2 torus. In order to have also an explicit construction of these models, we take $K3$ in its orbifold limits $T^4/\mathbf{Z}_N$ with $N = 2, 3, 4, 6$. All these models have been already constructed in [36, 37, 38], and we will then simply review their constructions, following in particular [37] [8]. The total discrete group by which we mod out the IIB theory is then a combination of the space-time symmetry group $\mathbf{Z}_N$, acting on T^4, and the world-sheet parity operator Ω. The structure of the elements of the total group is fixed by requiring the closure of the group. In particular there are two possible choices [37]:

$$\mathbf{Z}_N^A = \{1, \Omega, \alpha_N^k, \Omega_j\} \quad k, j = 1, 2, ..., N-1; \quad (N = 2, 3, 4, 6) \qquad (1.33)$$

and

$$\mathbf{Z}_N^B = \{1, \alpha_N^{2k-2}, \Omega_{2j-1}\} \quad k, j = 1, 2, ..., N/2; \quad (N = 4, 6) \qquad (1.34)$$

where, $\alpha_N = \exp(2\pi i/N)$ is the generator of the discrete group $\mathbf{Z}_N$, that acts on the two complex T^4 coordinates z_1, z_2 as

$$(z_1, z_2) \rightarrow (\alpha_N \, z_1, \alpha_N^{-1} \, z_2); \qquad (\bar{z}_1, \bar{z}_2) \rightarrow (\alpha_N^{-1} \, \bar{z}_1, \alpha_N \, \bar{z}_2) \qquad (1.35)$$

and $\Omega_j \equiv \Omega \cdot \alpha_N^j$. We will call the corresponding models respectively $\mathbf{Z}_N^A$ and $\mathbf{Z}_N^B$ models. As we saw in last section, each fixed point of the orbifold can be considered as an orientifold plane and then in general it will require the presence of suitable D-branes to cancel its R-R flux. It has been found

[8] It is amazing that several years ago in a complete different approach, based on more abstract CFT constructions, many of the properties of these models were already discovered by [39, 40].

Model	Neutral hypermultiplets	Tensor multiplets
$\mathbf{Z}_2^A$	20	1
$\mathbf{Z}_3^A$	11	10
$\mathbf{Z}_4^A$	16	5
$\mathbf{Z}_6^A$	14	7
$\mathbf{Z}_4^B$	12	9
$\mathbf{Z}_6^B$	11	10

Table 1.1: *Massless closed string spectrum for the various orientifold models.*

in [37] that for the A-models we always need 32 (D)-9 branes and 32 (D)-5 branes, excluding the case of $\mathbf{Z}_3^A$ where we do not have (D)-5 branes. On the other hand, the $\mathbf{Z}_4^B$ model does not have open sectors at all, while the $\mathbf{Z}_6^B$ has only 32 (D)-5 branes. These (D)5-branes are fixed on the orbifold, so their world-volumes span all the six-dimensional space-time. Note that we are not counting the number of independent dynamical D-branes in the various models, but the total number of them, including all the images under the discrete group. The total gauge group of these models depends on the location of the (D)5-branes; analogously to what seen in last section, the maximum gauge group is achieved when all the (D)5-branes coincide at a fixed point. When the gauge group involves $U(1)$ factors, however, there can be further anomalies in the theory due to the presence of new terms in the anomaly polynomial. They are cancelled [41] by a generalization of the Green-Schwarz mechanism that make these $U(1)$ gauge fields massive, similarly to a mechanism cancelling $U(1)$ anomalies in four-dimensional models

[42]. Let us consider now the massless string spectra of these models in this particular enhancement point. In the open sector, strings whose ends lie on

Model	Gauge group	Charged hypermultiplets
$\mathbf{Z}_2^A$	99: $U(16)$ 55: $U(16)$	99: 2× **120** 55: 2× **120** 59+95: **(16,16)**
$\mathbf{Z}_3^A$	99: $U(8) \otimes SO(16)$	99: **(28,1)**; **(8,16)**
$\mathbf{Z}_4^A$	99: $U(8) \otimes U(8)$ 55: $U(8) \otimes U(8)$	99: **(28,1)**; **(1,28)**; **(8,8)** 55: **(28,1)**; **(1,28)**; **(8,8)** 59+95: **(8,1;8,1)**; **(1,8;1,8)**
$\mathbf{Z}_6^A$	99: $U(4) \otimes U(4) \otimes U(8)$ 55: $U(4) \otimes U(4) \otimes U(8)$	99: **(6,1,1;)**; **(1,6,1)** **(4,1,8)**; **(1,4,8)** 55: **(6,1,1;)**; **(1,6,1)** **(4,1,8)**; **(1,4,8)** 59+95: **(4,1,1;4,1,1)** **(1,4,1;1,4,1)**; **(1,1,8;1,1,8)**
$\mathbf{Z}_4^B$	-	-
$\mathbf{Z}_6^B$	55: $U(8) \otimes SO(16)$	55: **(28,1)**; **(8,16)**

Table 1.2: *Massless open string spectrum for the various orientifold models.*

9-branes and 5-branes only (9-9 and 5-5 strings) give rise to the vectormultiplets and charged hypermultiplets of the corresponding gauge groups, while strings lying between 9- and 5-branes (5-9 and 9-5 strings) produce additional hypermultiplets, charged under both the gauge groups of 9- and 5-branes. In the closed sector we always obtain the states filling one $N = 1$ gravitational and tensormultiplet, plus neutral hypermultiplets and further tensormultiplets. We report in tables 1.1 and 1.2 the massless closed and open string spectra for each model; you can easily verify that the anomaly-free condition (1.3) is always satisfied. As already noted previously, the appearance of more tensormultiplets is the clearest indication that these type I vacua are really different from the usual $K3$ compactifications where we always have only one tensormultiplet [9].

All the four dimensional models we want to consider are obtained by a further compactification on a T^2 torus. The charged hypermultiplets and the vectormultiplets in 6d give rise to their equivalents in 4d, while the reduction of the $N = 1$ gravitational multiplet and of the n_T tensormultiplets down to four dimensions give the $N = 2$ gravitational multiplet plus $n_T + 3$ further vectormultiplets.

[9] See [43] for a discussion of the relation between these orientifold models and more standard type I compactifications.

Chapter 2

Higher derivative $F_g W^{2g}$ couplings in string theory

Every string theory that presents space-time supersymmetry is well approximated at low-energies by an effective supergravity quantum field theory. These theories are in general full of bad UV divergencies, that do not represent a real problem if one consider these models as describing only physics at distances bigger than the string length, fixed approximately by $\sqrt{\alpha'}$. On the contrary, string theories are believed to be finite theories at all scales and so the common belief is that the "correct" sugra theories describing the interactions of the massless string states are given by an infinite series of terms of higher dimensions, that arise from integrating out of the spectrum all the tower of massive string states. Altough the control of these terms is very difficult, they are crucial to establish results in string theory that are not valid only at the level of the low-energy effective sugra field theories. There are, however, a class of these higher derivative terms that have been discovered in four dimensional $N = 2$ models arising from compactifying type II theories on Calaby-Yau manifolds [44, 7, 45]. They are chiral F-terms involving the gravitational $N = 2$ multiplet W, and they are commonly denoted, in superspace language, $F_g(X)W^{2g}$ where X denotes generically chiral vector superfields of the theory. Aim of this chapter is to review the main properties of this class of higher derivative couplings, pointing out the important role they played in testing string dualities in four dimensions. In section one we consider them from the type II string point of view, where they were originally found, and then in the second section we present these couplings in the heterotic string. For a particular class of heterotic vacua, we will show in some detail a one-loop string computation involving the F_g's that has been used to test the 4d $N = 2$ type II-heterotic duality [5] and that is very useful to establish also a test of type I-heterotic duality, as we will see in detail in chapter three.

2.1 F_g's couplings in type II string theory

Type II (A and B indefferently) string theories have $N = 2$ susy in ten dimensions, that is 32 supercharges; in order to construct a model with $N = 2$ in four dimensions, we need to compactify these theories in six-dimensional Kähler manifolds with $SU(3)$-holonomy, the so called Calabi-Yau manifolds. It is a well-known result that the two-dimensional world-sheet field theory corresponding to such compactifications is a $N = (2, 2)$ superconformal field theory (SCFT); by introducing a suitable background gauge field coupled to the $U(1)$ $N = 2$ current, that is *twisting* the theory, one can obtain a topological field theory, i.e. a theory whose correlation functions do not depend on the details of the target space M, but only on its topology. The F_g couplings were then originally defined in [44, 7] to be simply the partition function at genus g of this twisted Calabi-Yau σ-model. The $N = 2$ algebra, moreover, constraints the F_g to be the sum of a holomorphic and anti-holomorphic functions of the moduli of the SCFT; due to anomalies, arising from the boundaries of the moduli space of genug g-surfaces, however, these partition functions acquire also non-holomorphic pieces and it has been shown in detail in [7] that this holomorphic anomaly allows to write a very useful recursion relation for the F_g's. The interpretation of these topological partition functions as higher derivative F-terms appearing in the four-dimensional $N = 2$ effective action was performed in [45], where it was found that they correspond to moduli-dependent couplings associated to chiral terms of the form

$$I_g = \tilde{F}_g(X) W^{2g} \tag{2.1}$$

where W is the Weyl superfield

$$W^{ij}_{\mu\nu} = F^{ij}_{\mu\nu} - R_{\mu\nu\lambda\rho}\,\theta^i \sigma_{\lambda\rho} \theta^j + \ldots \tag{2.2}$$

that is anti-self-dual in its Lorentz indices and antisymmetric in the indices i, j labeling the two supersymmetries; $W^2 \equiv \epsilon_{ij}\epsilon_{kl} W^{ij}_{\mu\nu} W^{kl}_{\mu\nu}$. $R_{\mu\nu\lambda\rho}$ is the anti-self-dual Riemann tensor, while $F^{ij}_{\mu\nu}$ is the (anti-self-dual) graviphoton field strength. $\tilde{F}_g(X)$ is an analytic function of the $N = 2$ chiral superfields X^I:

$$X^I = \hat{X}^I + \frac{1}{2} \widehat{F}^I_{\lambda\rho} \epsilon_{ij} \theta^i \sigma_{\lambda\rho} \theta^j + \ldots, \tag{2.3}$$

where $\hat{X}^I$, $\widehat{F}^I_{\lambda\rho}$ are the scalar components and the anti-self-dual vector field strengths of X^I. We are following here the formalism of [46] in which $N = 2$

Poincarè supergravity is constructed strarting from a superconformal theory and then imposing gauge fixing constraints. In particular a weight w is associated to any chiral superfield Φ, according to the transformation

$$\Phi(z^\mu,\theta^i) \rightarrow e^{w\zeta^*}\Phi(z^\mu, e^{-\zeta/2}\theta^i) \quad i=1,2 \tag{2.4}$$

where $z^\mu = x^\mu + \bar{\theta}^i\gamma^\mu\theta_i$ and $\zeta = \zeta_W + i\zeta_A$ with ζ_W, ζ_A the parameters of the dilatations and chiral transformations, respectively. The chiral superfields W and X have both $w=1$. The uncostrained physical scalars of the vector multiplets are given by $Z^A \equiv X^A/X^0$ where X^0 is given, in the string frame, by

$$X^0 = \frac{1}{g_S}e^{K/2} \tag{2.5}$$

where $K(Z,\bar{Z})$ is the Kähler potential of the theory. According to the conformal transformation of the θ-parameters given by (2.4), we argue that any chiral F-term in the action, being integrated on only half superspace, has to have $w=2$. Since in a product of two or more chiral superfields, the corresponding weights add up, it then follows that $\tilde{F}_g$ is a homogenous function of X^I's of degree $w = 2-2g$. Its lowest component can then be written as

$$\tilde{F}_g(X) = (X^0)^{2-2g}F_g(Z) = (g_S)^{g-1}e^{(1-g)K}F_g(Z) \tag{2.6}$$

In type II Calabi-Yau compactifications the dilaton belongs to a hypermultiplet, and it is a well-known result that there are no gauge neutral interactions between hyper and vectormultiplets in $N=2$ sugra; it then follows that the string coupling constant-dependence given in (2.6) is exact. This means that the F_g's in type II are determined at genug g and should not receive any further perturbative or non-perturbative corrections[1]. This was indeed explicitly

[1]Note that $F_0(Z)$ coincides with the prepotential of the $N=2$ theory.

confirmed at the perturbative-level in [45] by a direct genus-g computation on Calabi-Yau orbifolds, involving $2g-2$ graviphotons and 2 gravitons.
The holomorphic anomalies found in [7] correspond then to non-local interactions caused by massless particles, whose infrared singular contributions may lead to singular D-terms in the action, that look like chiral F-terms.
There are also very interesting connections [47] between the F_g's near a singular Calabi-Yau manifold and the free partition function of the $c=1$ string compactified on the self-dual radius $R=\sqrt{\alpha'}$, that will not be discussed in the present thesis.

2.2 F_g's couplings in heterotic string theory

Analogously to the case of type I string theory, 4d $N=2$ models arising from compactifications of the heterotic string require the compact six-manifold to be $K3\times T^2$. We will consider a particular class of these models, that give rise in four dimensions generically to an abelian gauge group $U(1)^{n_V+1}$ with n_V vectormultiplets plus the graviphoton gauge field. The only thing we need to know for what follows about such compactifications is that the n_V complex scalars parametrizing the vector moduli space are the complex dilaton $S=a+ie^{-2\phi}$ (a is the axion field), the Kähler class and complex structure of the torus T,U and n_V-3 Wilson lines y_α. We will see that we do not need to know the details of the hypermultiplet spectrum. Since the dilaton belongs to a vectormultiplet, the vector moduli space is subject now to quantum corrections. This modifies substantially the string-coupling dependence of the F_g's given in (2.6). In particular the Kähler potential is now $\sim \ln g_S^2$, implying that

$$\tilde{F}_g(X) \sim F_g(Z) \tag{2.7}$$

The Peccei-Quinn symmetry

$$a(x) \to a(x) + constant. \tag{2.8}$$

and holomorphicity uniquely fix the F_g's to be g_S-independent, with the exception of the prepotential F_0 and F_1, the gravitational R^2 coupling [2]. They both have a linear dependence on the field S at tree-level that is allowed, giving a total derivative under the P-Q transformation (2.8). We then expect to find in the heterotic theory all the F_g's already at one-loop. Consider then the amplitude involving two gravitons and $(2g-2)$ graviphotons. The relevant terms in the action are obtained by expanding $F_g W^{2g}$ in terms of component fields with the result:

$$S_{eff} = gF_g(R^2)(F^2)^g + 2g(g-1)F_g(RF)^2(F^2)^{g-2} \tag{2.9}$$

with $R^2 = R_{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$, $F^2 = F_{\mu\nu}F_{\mu\nu}$ and $(RF)^2 = (R_{\mu\nu\rho\sigma}F_{\rho\sigma})(R_{\mu\nu\eta\kappa}F_{\eta\kappa})$, and where again $R_{\mu\nu\rho\sigma}$ and $F_{\mu\nu}$ represent the anti-self-dual parts of the Riemann tensor and graviphoton field strengths respectively. We want then to compute a $2g$-point one-loop string amplitude, involving two graviton and $2g-2$ graviphoton vertex operators. In the following we will closely follow the computation reported in [5] and its generalization performed in [6]. The graviphoton vertex operator V_γ can be obtained starting from the graviton

[2]Note that F_1 should be evaluated by a three point function [12] but, as showed by [5], the computation we will summarize here gives the right result even for this case. The same will happen in Type I theory.

operator V_g and applying two supersymmetries; in this way one obtains:

$$V_g(p) = \xi_{\mu\nu}(p)\,(\partial X_\mu + ip\cdot\psi\psi_\mu)\,\bar\partial X_\nu\, e^{ip\cdot X}$$
$$V_\gamma(p) = \epsilon_\mu(p)\,(\partial X^+ + ip\cdot\psi\Psi^+)\,\bar\partial X_\mu\, e^{ip\cdot X} \tag{2.10}$$

where X_μ, ψ_μ are the world-sheet scalar and fermion fields associated to the euclidean four-dimensional directions $\mu = 1,2,3,4$, while X, Ψ are the complex fields

$$X^+ = (X^5 + iX^6)/\sqrt{2}, \quad \Psi^+ = (\Psi^5 + i\Psi^6)/\sqrt{2} \tag{2.11}$$

associated to the internal T^2-torus. The anti-self duality conditions are easily solved in the gauge in which $\epsilon_4 = \xi_{\mu 4} = \xi_{4\mu} = 0$ and in complex coordinates where

$$Z_1^\pm = (X^4 \pm iX^3)/\sqrt{2} \qquad Z_2^\pm = (X^1 \pm iX^2)/\sqrt{2} \tag{2.12}$$

and similarly for the left moving fermions

$$\chi_1^\pm = (\psi^4 \pm i\psi^3)/\sqrt{2} \qquad \chi_2^\pm = (\psi^1 \pm i\psi^2)/\sqrt{2} \tag{2.13}$$

and for the four-momentum p^μ. In the kinematical configuration where $p_2^\pm = p_1^- = 0$, it is easy to see that the anti-self duality and trasversality condition

$$\epsilon_{[\mu}p_{\nu]} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\epsilon_\rho p_\sigma, \qquad \epsilon\cdot p = 0 \tag{2.14}$$

imply that $\epsilon_1^\pm = \epsilon_2^- = 0$ for the graviphoton polarization and analogously $\epsilon_1^\pm = \epsilon_2^+ = 0$ when $p_2^\pm = p_1^+ = 0$. The polarizations of the gravitons can be easily recovered from those of the graviphoton by chosing $\xi_{ij} = \epsilon_i\epsilon_j$, where $i,j = 1,2,3$. In this kinematical configuration the vertex operators are then the following:

$$V_g(p_1^\pm) \;=\; (\partial Z_2^\mp + ip_1^\pm\chi_1^\mp\chi_2^\mp)\,\bar\partial Z_2^\mp\, e^{ip_1^\pm Z_1^\mp}$$
$$V_\gamma(p_1^\pm) \;=\; (\partial X^+ + ip_1^\pm\chi_1^\mp\Psi^+)\,\bar\partial Z_2^\mp\, e^{ip_1^\pm Z_1^\mp} \tag{2.15}$$

Consider now an amplitude A_{2g} involving one graviton and $(g-1)$ graviphotons with $p_2^{\pm} = p_1^- = 0, p_1^+ \neq 0$ and the remaining graviton and $(g-1)$ graviphotons with $p_2^{\pm} = p_1^+ = 0, p_1^- \neq 0$. This amplitude gets contribution from both the terms in eq.(2.9) and it is easy to show that

$$
\begin{aligned}
A_{2g} &= \langle V_g(p_1^+)V_g(p_1^-)\prod_{i=1}^{g-1} V_\gamma(p_1^{+(i)})V_\gamma(p_1^{-(i)})\rangle \\
&= (p_1^+)^2(p_1^-)^2\prod_{i=1}^{g-1} p_1^{+(i)}p_1^{-(i)}(g!)^2F_g. \qquad (2.16)
\end{aligned}
$$

In general this amplitude receives contribution from all the spin structures and one must sum over all the spin structures weighted by a factor half associated to GSO projection. However one can show that the sum over even spin structures gives the same contribution as the odd one. Thus the full amplitude can be evaluated in the odd spin structure without the factor of half. In the odd spin structure one of the vertex operators must be inserted in (-1)-ghost picture due to the presence of a Killing spinor on the world sheet torus, and one must also insert a picture changing operator to take care of the world-sheet gravitino zero mode. It is convenient to take one of the graviphoton vertices in the (-1)-ghost picture which comes with a fermion Ψ^+. Recalling that in the odd-spin structure the space-time fermions $\chi_{1,2}^{\pm}$ as well as the internal fermions $\Psi^{\pm}$ associated with the torus T^2 have one zero-mode each, one concludes that the only contribution of the picture changing operator comes from the term $e^{\phi}\Psi^-\partial X^+$. Moreover the space-time fermion zero modes are soaked by the fermionic part of the graviton vertices. From the remaining $(2g-3)$ graviphoton vertices in the (0)-ghost picture only the terms involving ∂X^+ survive. Together with the ∂X^+ appearing in the picture changing operator they provide a total of $(2g-2)$ ∂X^+'s which contribute only through their zero modes. Finally we are left with the correlation functions of space-time bosons. The structure of the terms in the action (2.9) imply that every graviphoton contributes with one power of momentum; it then follows that in the contraction of the exponentials $< e^{ip_1^{(i)+}Z_1^-} e^{ip_1^{(j)+}Z_1^-} >$, the relevant term is that linear in both momenta $p_1^{(i)+}, p_1^{(j)-}$. In this way the momentum structure of this amplitude matches with that of eq.(2.16) and F_g's are given by the following expression:

$$F_g = -\frac{(4\pi i)^{g-1}}{4\pi^2} \frac{1}{(g!)^2} \int \frac{d^2\tau}{\tau_2^3} \frac{1}{\bar{\eta}^4} \langle \prod_{i=1}^{g} \int d^2x_i Z_1^+ \bar{\partial} Z_2^+(x_i) \prod_{j=1}^{g} \int d^2y_j Z_1^- \bar{\partial} Z_2^-(y_j) \rangle$$
$$\sum_C C_C(\bar{\tau}) \sum_{(P_L,P_R)\in\Gamma_C} (e^{K_0/2} P_L)^{2g-2} q^{\frac{1}{2}|P_L|^2} \bar{q}^{\frac{1}{2}|P_R|^2} \qquad (2.17)$$

where τ is the Teichmüller parameter of the world-sheet torus, q is $e^{2\pi i\tau}$ and $1/\bar{\eta}^4$ accounts for the partition function of the two space-time and torus right moving bosons in light-cone. The normalization constant has been fixed as in [5]. K_0 is the Kähler potential modded out of the dilaton dependence and comes from the change of variables performed in (2.6), P_L and P_R are the left and right moving momenta sitting in the $n+2$ real dimensional lattice Γ_c. Since this lattice is not in general self-dual, world sheet modular invariance implies that the vectors in the dual lattice must also appear in the spectrum. This dual lattice splits into several conjugacy classes labelled by Γ_c. Each of these classes are coupled to different blocks of the remaining conformal field theory ($c = 6, \bar{c} = 22 - n$) where $C_c(\bar{\tau}) = \mathrm{Tr}_c(-1)^F q^{L_0 - c/24} \bar{q}^{\bar{L}_0 - \bar{c}/24}$ is τ-independent, as has been argued in [5]. In order to evaluate these correlation functions it is convenient to introduce the generating function

$$G(\lambda, \tau, \bar{\tau}) = \sum_{g=1}^{\infty} \frac{1}{(g!)^2} (\frac{\lambda}{\tau_2})^{2g} \langle \prod_{i=1}^{g} \int d^2x_i Z_1^+ \bar{\partial} Z_2^+(x_i) \prod_{j=1}^{g} \int d^2y_j Z_1^- \bar{\partial} Z_2^-(y_j) \rangle$$

$$\equiv \sum_{g=1}^{\infty} \lambda^{2g} G_g(\tau, \bar{\tau}) \tag{2.18}$$

whose coefficients G_g times τ_2^{2g} appear in the expression (2.17) for the F_g's. In this way we can exponentiate the bosonic correlators in (2.17) reducing the amplitude to the computation of a determinant of a λ-twisted free field action. The result is:

$$G(\lambda, \tau, \bar{\tau}) = \left(\frac{2\pi i \lambda \bar{\eta}^3}{\bar{\Theta}_1(\lambda, \bar{\tau})} \right)^2 e^{-\frac{\pi \lambda^2}{\tau_2}} \tag{2.19}$$

where $\Theta_1(z, \tau)$ is the odd theta-function

$$\Theta_1(z, \tau) = 2 \prod_{n=1}^{\infty} (1 - q^n) \sin \pi z \prod_{n=1}^{\infty} (1 - 2q^n \cos 2\pi z + q^{2n}) \tag{2.20}$$

We can write the result of the computation in a closed way for all the F_g's by defining an analogous generating function:

$$F(\lambda) \equiv \sum_{g=1}^{\infty} g^2 \lambda^{2g} F_g \tag{2.21}$$

It is then not difficult to show that:

$$F(\lambda) = \frac{\lambda^2}{\pi^2} \int \frac{d^2\tau}{\tau_2} \frac{1}{\bar{\eta}^4(\bar{q})} \sum_C C_C(\bar{q}) \sum_{(P_L, P_R) \in \Gamma_C} q^{\frac{1}{2}|P_L|^2} \bar{q}^{\frac{1}{2}|P_R|^2} \times$$

$$\frac{1}{2} \frac{d^2}{d\tilde{\lambda}^2} \left[\left(\frac{2\pi i \tilde{\lambda} \bar{\eta}^3(\bar{q})}{\bar{\theta}_1(\tilde{\lambda}, \bar{\tau})} \right)^2 e^{-\frac{\pi \tilde{\lambda}^2}{\tau_2}} \right] \tag{2.22}$$

where $\tilde{\lambda} = e^{K_0/2} P_L \tau_2 \lambda$. For the particular model with $n_V = 2$, i.e. two only vector moduli, S and T, [5] showed that derivating (2.22) with respect to $\bar{T}$, one obtains recursive holomorphic anomaly equations for the F_g's that agree exactly to those found in the conjectured type II dual model, after having taken the appropriate type II limit corresponding to the heterotic $S \to \infty$ limit (weak coupling limit).

As we will see in greater detail in next chapters, the topological origin of these higher derivative couplings in type II string theory is manifested in the heterotic theory in the fact that the amplitude (2.22) actually depends on the internal six-dimensional manifold only through a supersymmetric index [9]. It then follows [13] that these effective higher dimensional couplings do not depend on the form of the whole massive string spectrum, but they receive contributions only from the $N = 2$ BPS-saturated states of the theory. Before concluding this brief survey about the F_g's, I would like to mention that these couplings have been also crucial in establishing important checks on the description and interpretation of certain singularities in Calabi-Yau moduli spaces, known as conifold singularities. In particular the one-loop computation of [5] reported before, led to a check of the conjecture that the physics near the conifold is governed by the c=1 string at the self-dual radius, while the one-loop computation performed in [48] provided a check of the description of the conifold in terms of intersecting D-branes [49]. Both works, moreover, can be considered as a check of the Strominger proposal for the resolution of the conifold singularity [50].

Chapter 3

Type I-heterotic duality

We already noted in the first chapter that the ten-dimensional $N = 1$ sugra with gauge group $SO(32)$ admit two string constructions, in terms of heterotic and type I strings. This fact can be considered a first hint of a possible equivalence of these two string theories. However, the equivalence should not simply relate the perturbative regimes of the two theories, because, altough the low-energy effective theories match up, this is not the case for all the perturbative massive string spectrum. The heterotic gauge group is not properly $SO(32)$, but $Spin(32)/Z_2$ that admits also states transforming as spinors of $SO(32)$, absent in the type I string spectrum. It has been argued in [51] that the two low-energy effective theories are related by a strong-weak duality, inverting their string coupling constants. Consider, indeed, the form of the ten-dimensional heterotic effective action, in the string frame and without taking care of numerical coefficients:

$$\Gamma_H \sim \int d^{10}x\sqrt{g}e^{-2\phi}[R + (\partial\phi)^2 + F^2 + (dB)^2] + ... \tag{3.1}$$

where ϕ, R, F and B are respectively the dilaton, the scalar curvature, the vector field strengths and the antisymmetric tensor field. Since the terms given in eq.(3.1) come from a tree-level sphere computation, they are all proportional to $e^{-2\phi}$. Consider now the form of the ten-dimensional type I effective action, again in the string frame:

$$\Gamma_I \sim \int d^{10}x\sqrt{g}\{e^{-2\phi}[R+(\partial\phi)^2]+e^{-\phi}F^2+(dB)^2\}+... \qquad (3.2)$$

with the same fields as before. The dilaton dependence is now given by the sphere, as in (3.1), for both R and ϕ and by a disk computation for the vector field strengths, belonging to the open string spectrum. The scaling of the R-R antisymmetric tensor field B is a bit more subtle; being a closed string state, it should naively scale with $e^{-2\phi}$ but it was shown by [35] that the R-R field strengths satisfying in general the Bianchi identities and Maxwell equations derive from the rescaled p-potentials $C_p \rightarrow e^{\phi}C_p$. In this way, the R-R kinetic term $(dB)^2$ scales trivially with ϕ. Since two terms in both actions have already the same ϕ-dependence, it is clear that if we want to match up (3.1) with (3.2), we have to rescale also the ten-dimensional metrics, or in other words, the measure of lenghts in the theories. It is then very easy to see that after the identifications

$$\phi_I = -\phi_H, \qquad g^{(I)}_{\mu\nu} = e^{-\phi_H}g^{(H)}_{\mu\nu} \qquad (3.3)$$

the two actions exactly match up. The first relation in (3.3) states that if the two string theories are actually equivalent, they are strong-weak dual, in the sense that the strong coupling behaviour of one of the two is described by the weak coupling of the other. Due to the second relation in (3.3), it is not excluded, however, that after having compactified the two theories on compact manifolds of suitable sizes, the purely strong-weak duality in 10d could appear as a perturbative weak-weak duality in lower dimensions. This up to now rather weak evidence for type I-heterotic duality was strongly reinforced in [3]. In this reference, the implications of heterotic T-duality in type I theory were studied and it was found that possible inconsistencies of the dual map (3.3) are avoided in a very non-trivial way. As further check, they found that the (D)1-brane of type I string theory has the same world-sheet structure as the heterotic string. This is an important statement, because when the type I string coupling constant is very large, the (D)1-brane becomes a light state and, according to (3.3), should be dual to an elementary perturbative heterotic string.

The study of the four dimensional consequences of type I-heterotic duality has been started only recently in ref.[11], where it has been shown that indeed there exist type I and heterotic models that are weak-weak dual in four dimensions, once compactified in suitable manifolds. They studied the 1-loop corrections to the prepotential for the $\mathbf{Z}_2^A$ model considered in chapter one and in particular they showed that the perturbative prepotential for a rank four model that admits type I, type II and heterotic descriptions, agree. Aim of this chapter is to present an other test [4] of the aforementioned duality that involves the higher derivative F_g-terms analyzed in chapter two. We compute them at 1-loop for the $\mathbf{Z}_2^A$ model, that is the only orientifold model, among those we analyzed, having one tensormultiplet in six-dimensions and then admitting a perturbative heterotic dual [1] and then compare to the het-

[1]The computation does not really depend on the details of the various models, and indeed we will generalize it to all the orientifolds in next chapter.

erotic couplings (2.22) computed for the corresponding dual model. In this way we show the complete agreement of the moduli-dependence of the F_g's in both models, providing a further check for the given duality. This study has been generalized in [52] where, among other things, it has been shown the agreement of the F_g's of the same type I model with that of an heterotic orbifold dual model, in the Coulomb phase with the insertion of arbitrary Wilson lines.

In section one we construct the pair of heterotic and type I models dual to each other and consider in detail the structure of the dual map. In section two, starting from the results obtained in last chapter for the heterotic F_g's couplings and according with the dual map found, we perform the limit in which duality predicts that both couplings have to agree. Finally, in section three, we compute in detail the F_g's couplings in the type I dual model showing that they are in complete agreement with the heterotic one's, providing an other important test of the conjectured type I-heterotic duality.

3.1 The dual pair

It is known that $N = 2$ models deriving from heterotic theory require the internal six-manifold to be $K3 \times T^2$. Analogously to the type I case discussed in the first chapter, in order to construct a consistent model we have to satisfy the topological condition (1.2). Consider the $E_8 \times E_8$ heterotic theory on $K3 \times T^2$; if we solve the condition (1.2) by putting 12 instantons on an $SU(2)$ subgroup of each E_8, by using index theorems it is not difficult to see [53] that the resulting four-dimensional massless spectrum contains, among the

$N = 2$ gravitational superfield, 266+3 vectormultiplets forming the adjoint representation of the unbroken gauge group $E_7 \times E_7 \times U(1)^3$, 62 neutral $(\mathbf{1}, \mathbf{1}, \mathbf{1})$ and 4 charged $(\mathbf{1}, \mathbf{56}, \mathbf{1}) + (\mathbf{56}, \mathbf{1}, \mathbf{1})$ hypermultiplets. We can now give non-vanishing vacuum expectation values to the charged hypermultiplets higgsing the gauge group and leaving unbroken the abelian $U(1)^3$ factor only. In this way we obtain $112 \times 4 - 266 = 182$ further neutral hypermultiplets, for a total of $62 + 182 = 244$ hypers and three vectors. The vector moduli space is then parametrized by three complex scalars $S - T - U$, where

$$S_H = a + ie^{-2\phi_H}, \quad T_H = B_{45} + i\sqrt{G}, \quad U_H = (G_{45} + i\sqrt{G})/G_{44} \qquad (3.4)$$

denote the complex dilaton, the Kähler and complex structure of the torus, respectively [2].

As already mentioned before, the type I dual is constructed starting from the six-dimensional model on the $K3$ orbifold $T^4/\mathbf{Z}_2$. The maximal gauge group of this model is (see table 1.2) $U(16) \otimes U(16)$, when all the (D)5-branes with their images are overlapped together on one of the 16 fixed points. This model presents also $U(1)$ gauge anomalies that are non-vanishing even when relation (1.2) is satisfied. It has been shown by [41] that they are cancelled adding a counterterm to the langrangian that induces an anomalous $U(1)$ charge to neutral scalar fields, corresponding to R-R twisted closed string states. In this way, the would-be anomalous $U(1)$ gauge fields become massive, through an Higgs mechanism, eating the R-R twisted states; because of supersymmetry, that remains unbroken, the whole vectormultiplet, containing the $U(1)$ gauge field and the whole hypermultiplet, containing the R-R

[2] From now on a subscript H or I refers to heterotic and type I quantities.

scalars become massive and disappear from the low-energy effective action. It has also been shown in [41] that in a generic (D)5-brane configuration with 16 or fewer $U(1)$'s, all of them are broken, while if there are more than 16, exactly 16 are broken. Put now two (D)5-branes at each fixed point of the $T^4/\mathbf{Z}_2$ orbifold. The gauge group should be then $U(16)$, coming from the 9-9 strings, times $U(1)^{16}$ from the 5-branes, but according to the previous analysis, it is reduced to $SU(16) \otimes U(1)$. In the closed string spectrum, moreover, we are left with only four neutral hypermultiplets. By giving non-vanishing vacuum expectation values to scalars belonging to charged hypermultiplets, it is possible [11] to break completely the gauge group, remaining only with 244 $N = 1$ hypermultiplets, 4 of which coming from the closed string sector and the remaining 240 from the open string sector. After the further compactification on T^2, the massless spectrum consists of 4 $U(1)$'s, 3 complex scalars and the 244 $N = 2$ hypermultiplets, of course.

Let us analyze the dual map between the states of both theories, according to the conjectured duality. We will mainly focus here on the precise identifications of the heterotic and type I string coupling constants, showing how a perturbative duality can arise in four dimensions. For this purpose it will be sufficient in the following to write only the R and kinetic energy terms of gauge fields of both heterotic and type I effective actions. Compactification of the heterotic action (3.1) on a generic 10-d dimensional manifold gives simply

$$\Gamma_H^{(d)} \sim \int d^d x e^{-2\phi_H^{(d)}} (R_d + F^2 + \ldots) \tag{3.5}$$

with

$$e^{-2\phi_H^{(d)}} = e^{-2\phi_H^{(10)}} V_{10-d} \tag{3.6}$$

and where V_{10-d} represent the volume of the internal manifold. The type I ten-dimensional action (3.2) gets modified, in presence of (D)5-branes, for additional terms living on the 5-brane world-volume. Schematically we have now, for the R and kinetic gauge terms:

$$\Gamma_I^{(10)} \sim \int d^{10}x \left[e^{-2\phi_I^{(10)}}(R_{10}+...)+e^{-\phi_I^{(10)}}F^2+e^{-\phi_I^{(10)}}(F')^2\delta^{(4)}(x)+...\right] \tag{3.7}$$

that under compactification on $K3$ gives

$$\Gamma_I^{(6)} \sim \int d^6x \left[e^{-2\phi_I^{(6)}}(R_6+...)+e^{-\phi_I^{(6)}}\omega_I^2F^2+e^{-\phi_I^{(6)}}\omega_I^{-2}(F')^2+...\right] \tag{3.8}$$

due to eq.(3.6), valid also for type I, where we have denoted with ω_I^4 the volume of the $K3$ orbifold. The ten-dimensional relations (3.3) imply that in six-dimensions

$$e^{-2\phi_H^{(6)}}=\omega_I^4, \qquad e^{-2\phi_I^{(6)}}=\omega_H^4 \tag{3.9}$$

Note in particular that in 6d the heterotic dilaton $\phi_H^{(6)}$, as well as the Kähler modulus ω_I^4 in type I, belong to a tensormultiplet, while on the other hand the type I dilaton $\phi_I^{(6)}$ and the heterotic Kähler modulus ω_H^4 are part of an hypermultiplet. From eqs.(3.8) and (3.9) it follows that the gauge fields arising from the 5-branes in type I are non-perturbative in the heterotic string, where it is believed they arise from small instantons [54, 41]. Compactifying further eq.(3.8) down to four dimensions, we obtain

$$\Gamma_I^{(4)} \sim \int d^4x \left[e^{-2\phi_I^{(4)}}(R_4+...)+e^{-\phi_I^{(4)}}\omega_I^2\,G_I^{1/4}F^2+e^{-\phi_I^{(4)}}\omega_I^{-2}\,G_I^{1/4}(F')^2+...\right] \tag{3.10}$$

where $\sqrt{G_I}$ is the volume of the type I torus. If we denote respectively with $\mathrm{Im}\,S_I, \mathrm{Im}\,S_I'$ the gauge coupling constants of the F, F' field strenghts, relations (3.3) and (3.9) imply that in 4d

$$\begin{aligned}\mathrm{Im}\, S_H &= e^{-2\phi_H^{(4)}} = e^{-\phi_I^{(4)}}\omega_I^2\, G_I^{1/4} = \mathrm{Im}\, S_I \\ \mathrm{Im}\, T_H &= \sqrt{G_H} = e^{-\phi_I^{(10)}}\sqrt{G_I} = e^{-\phi_I^{(4)}}\omega_I^{-2}\, G_I^{1/4} = \mathrm{Im}\, S_I'\end{aligned} \tag{3.11}$$

Weakly coupled type I theory, corresponding to the limit of large $\mathrm{Im}\, S_I, \mathrm{Im}\, S_I'$, is then equivalent to weakly coupled heterotic theory, provided that the volume of the heterotic T^2 torus is large. It is not difficult to show that the entire duality mapping in 4d for the vector moduli space is [11]

$$S_H = S_I, \quad T_H = S_I', \quad U_H = U_I \tag{3.12}$$

where

$$S_I = a + ie^{-\phi}G^{1/4}\omega^2, \quad S_I' = B_{45} + ie^{-\phi}G^{1/4}\omega^{-2}, \quad U_I = (G_{45} + i\sqrt{G})/G_{44} \tag{3.13}$$

omitting a subscript I to all quantities. While the four-dimensional heterotic dilaton $\phi_H^{(4)}$ belongs entirely to a vectormultiplet, the type I dilaton $\phi_I^{(4)}$ is a combination of fields appearing in hyper and vectormultiplets:

$$e^{-2\phi^{(4)}} = e^{-\phi^{(6)}}(\mathrm{Im}\, S\ \mathrm{Im}\, S')^{1/2} \tag{3.14}$$

This implies that in general vector and hyper-moduli spaces can both receive quantum corrections in these type I 4d vacua. On the other hand the type I low-energy effective action is invariant under the two Peccei-Quinn symmetries

$$\begin{aligned} a(x) &\rightarrow a(x) + const. \\ B_{45}(x) &\rightarrow B_{45}(x) + const.\end{aligned} \tag{3.15}$$

since the scalar field B_{45} comes from the R-R closed string sector. This implies the perturbative independence of chiral amplitudes from S_I and S'_I [3]. The heterotic effective action presents instead only the usual P-Q symmetry (2.8). Given the relations (3.12), it then follows that for every chiral amplitude A:

$$\lim_{T_2 \to \infty} A_H(T,U) = A_I(U)|_{S_2 > S'_2} \tag{3.16}$$

where $T_2 = \mathrm{Im}\, T$ and the same for S, S'. The restriction $S_2 > S'_2$ is due to the fact that in the heterotic theory we take first the perturbative large S_2 limit, and then the large T_2 limit. It simply means in type I to take a $K3$ volume bigger than 1, in α' units. Since the tree-level type I Kähler potential is $\sim \ln g_S^2$ like the corresponding heterotic one, from relation (3.16) we expect that

$$\lim_{T_2 \to \infty} F_{g,H}^{1-loop}(T,U) = F_{g,I}^{1-loop}(U) \tag{3.17}$$

This is indeed true for $g \geq 2$, but F_1 has also a tree-level contribution that is [52]

$$F_{1,H}^{tree} = 4\pi S_2 \tag{3.18}$$

for the heterotic model and

$$F_{1,I}^{tree} = 4\pi(S_2 + S'_2) \tag{3.19}$$

for the type I model. This means that, according to duality,

$$\lim_{T_2 \to \infty} F_{1,H}^{1-loop}(T,U) = 4\pi T_2 + F_{1,I}^{1-loop}(U) \tag{3.20}$$

where the first term is mapped to the corresponding tree-level type I coupling.

[3] With the exception of a linear dependence in S, S' for F_0 and F_1, as already seen in chapter two.

3.2 Heterotic F_g couplings in the $T_2 \to \infty$ limit

Given the relation (3.16) found above, in this section we will simply consider the heterotic F_g couplings (2.22) reported in last chapter, and take their $T_2 \to \infty$ limit. For the particular heterotic model under consideration, eq.(2.22) assumes the following form:

$$F_H(\lambda, T, U) = \frac{\lambda^2}{8\pi^2} \int \frac{d^2\tau}{\tau_2} \frac{1}{\bar{\eta}^4(\bar{q})} C(\bar{q}) \sum_{\substack{n_1,n_2 \\ m_1,m_2}} q^{\frac{1}{2}|P_L|^2} \bar{q}^{\frac{1}{2}|P_R|^2} \frac{1}{2} \frac{d^2}{d\tilde{\lambda}^2} \left[\left(\frac{2\pi i \tilde{\lambda} \bar{\eta}^3(\bar{q})}{\bar{\theta}_1(\tilde{\lambda}, \bar{\tau})} \right)^2 e^{-\frac{\pi\tilde{\lambda}^2}{\tau_2}} \right] \tag{3.21}$$

where

$$\begin{aligned} P_L &= \frac{1}{\sqrt{2T_2U_2}} (n_1 + n_2\bar{U} + m_1\bar{T} + m_2\bar{T}\bar{U}) \\ P_R &= \frac{1}{\sqrt{2T_2U_2}} (n_1 + n_2\bar{U} + m_1 T + m_2 T\bar{U}) \end{aligned} \tag{3.22}$$

$\tilde{\lambda} = P_L \tau_2 \lambda / \sqrt{2T_2U_2}$ and we fixed the overall normalization in order to reproduce the tree-level heterotic coupling (3.18). Since the lattice of momenta is self-dual for this model, there are no conformal blocks and $C(\bar{q})$ is actually the partition function of $K3$ in the odd spin structure. In particular, $C(\bar{q})$ has the following expansion [5]:

$$C(\bar{q}) = \bar{q}^{-5/6}(1 - 244\,\bar{q} + ...) \tag{3.23}$$

where the first factor accounts for the tachyon and 244 is the number of massless hypermultiplets of the model. In order to show more explicitly the T_2 dependence of $F_H(\lambda)$ rescale

$$\tau_2 \to T_2 \tau_2 / 2 \tag{3.24}$$

It is now evident that all the lattice momenta with $m_1, m_2 \neq 0$ are exponentially suppressed with T_2. After the rescaling (3.24), the integration on the fundamental domain $\mathcal{F}$ of the torus becomes the strip $-1/2 \leq \tau_1 \leq 1/2$, $0 \leq \tau_2 \leq \infty$; bringing the limit inside the integral and expanding in $\bar{q}$, the non-vanishing result will be given by the $\bar{q}^0$ coefficient of the expansion:

$$\lim_{T_2\to\infty} F_H(\lambda, T, U) = \frac{\lambda^2}{16\pi^2}\int_0^\infty \frac{d\tau_2}{\tau_2}\sum_{n_1,n_2} e^{-\frac{\pi\tau_2|P|^2}{2U_2}}\left[240\frac{d^2}{d\bar{\lambda}^2}\left(\frac{\bar{\lambda}\pi}{\sin\bar{\lambda}}\right)^2 + 16\pi^2\right] \tag{3.25}$$

where $P \equiv n_1 + n_2\bar{U}$ and $\bar{\lambda} = \lambda\pi\tau_2 P/4U_2$. The second term in square brackets comes from the tachyon contribution together with the linear term in $\bar{q}$ deriving from the expansion of the eta and theta-function and gives a contribution only to F_1. From eq.(3.25) it is evident that all the F_g are convergent, with the exception of the gravitational coupling F_1 that presents both IR and UV divergencies. This last one comes from the large T_2 limit and can be regulated going back to eq.(3.21) and picking up the divergence piece of F_1. The total F_1 contribution is obviously given by taking the λ^0 coefficient in the expansion of the square bracket in (3.21); it is given by [52]

$$F_{1,H}(T,U) = \frac{1}{16\pi^2}\int\frac{d^2\tau}{\tau_2}\frac{1}{\bar{\eta}^4(\bar{q})}C(\bar{q})\sum_{\substack{n_1,n_2\\ m_1,m_2}} q^{\frac{1}{2}|P_L|^2}\bar{q}^{\frac{1}{2}|P_R|^2}\left(\frac{2\pi}{\tau_2} - 8i\pi\partial_{\bar{\tau}}\ln\bar{\eta}(\bar{q})\right) \tag{3.26}$$

We can now perform a Poisson resummation of the lattice sum in n_1, n_2, putting $m_1 = m_2 = 0$:

$$\sum_{n_1,n_2} e^{-\frac{\pi\tau_2}{T_2U_2}|n_1+n_2\bar{U}|^2} = T_2\sum_{l_1,l_2} e^{-\frac{\pi T_2\tau_2}{U_2}|l_1-l_2\bar{U}|^2} \tag{3.27}$$

The remaining integral on the fundamental domain $\mathcal{F}$ can be explicitly performed; the ending result for the divergent piece of F_1 is then just [52]:

$$F_{1,H}^{div} = 4\pi T_2 \tag{3.28}$$

reproducing the tree-level type I coupling. We can rewrite eq.(3.25) more appropriately as

$$\lim_{T_2\to\infty} F_H(\lambda,T,U) = 4\pi\lambda^2 T_2 + \frac{\lambda^2}{16\pi^2}\int_0^{\infty\prime}\frac{d\tau_2}{\tau_2}\sum_{n_1,n_2} e^{-\frac{\pi\tau_2|P|^2}{2U_2}}\left[240\frac{d^2}{d\lambda^2}\left(\frac{\lambda\pi}{\sin\bar{\lambda}}\right)^2 + 16\pi^2\right] \tag{3.29}$$

where the prime in the integral indicates the subtraction of the UV divergent piece. The IR F_1-divergence, on the other hand, is present before the large T^2 limit and is the result of the one-loop renormalization of the gravitational coupling constant due to massless particles. We will see in next section how this amplitude is reproduced in type I.

3.3 Computation of F_g in the type I model

We compute the F_g couplings in the type I model by considering the same amplitude taken into account in section 2.2, involving two gravitons and $2g-2$ graviphotons; the graviton vertex operator is the usual

$$V_g^{\mu\nu}(p) = (\partial X^\mu + ip\cdot\psi\psi^\mu)(\bar{\partial}X^\nu + ip\cdot\psi\psi^\nu)e^{ip\cdot X} \tag{3.30}$$

The graviphoton vertex operator is obtained applying the two supersymmetric charges to V_g:

$$V_\gamma(p) = (Q_1^{(L)} + Q_1^{(R)})(Q_2^{(L)} + Q_2^{(R)})V_g(p) \tag{3.31}$$

The explicit form of $Q_{1,2}^{(L,R)}$ can be obtained starting from the free orbifold case, using eq.(1.27). They are given by

$$\begin{aligned} Q_{\alpha,1}^{(L)} + Q_{\alpha,1}^{(R)} &= \oint dz e^{-\frac{\phi}{2}} S_\alpha \Sigma e^{i\frac{H_5}{2}}(z) + \oint d\bar{z} e^{-\frac{\bar{\phi}}{2}} \tilde{S}_\alpha \tilde{\Sigma} e^{i\frac{\bar{H}_5}{2}}(\bar{z}) \\ Q_{\alpha,2}^{(L)} + Q_{\alpha,2}^{(R)} &= \oint dz e^{-\frac{\phi}{2}} S_\alpha \bar{\Sigma} e^{i\frac{H_5}{2}}(z) + \oint d\bar{z} e^{-\frac{\bar{\phi}}{2}} \tilde{S}_\alpha \tilde{\bar{\Sigma}} e^{i\frac{\bar{H}_5}{2}}(\bar{z}) \end{aligned} \tag{3.32}$$

where $e^{-\frac{\phi}{2}}$, $e^{i\frac{H_5}{2}}$ are the bosonization of the superghosts and of the complex fermion associated to the internal torus respectively, S_α is the four dimensional space-time spin field operator and Σ and its complex conjugate are the $K3$ internal spin field operators; they can be explicitly written by reminding that the conformal field theory associated to $K3$ is a $N = (4,4)$ SCFT, that presents an $SU(2)$ current algebra. Bosonizing the $U(1)$ Cartan current of the $SU(2)$ algebra as $J_3 = i\sqrt{2}H$, Σ can then be expressed as $\Sigma = e^{i\frac{\sqrt{2}}{2}H}$. The same of course for the right-moving sector. We choose exactly the same kinematic configuration taken in section 2.2 to compute the heterotic F_g's.

The 1-loop amplitude involves now a sum over the torus, Klein bottle, annulus and Möbius strip surfaces. Since the $2g-2$ graviphoton vertex operators so constructed are in the (-1)-ghost picture, we need to include in the correlation function $2g-2$ picture changing vertex operators. The non-vanishing contribution of the picture changing operators will come only from the part $(e^{\phi}e^{-iH_5}\partial Z_5^+ + \text{right-moving})$[4], where Z_5^+ is the complex scalar associated to the torus direction, that cancels the $+1$ charge carried by V_γ in the torus direction. The $2g-2$ ∂Z_5^+ and $\bar{\partial} Z_5^+$ cannot contract with anything in the correlation function, so that only their zero modes part give a non-vanishing contribution. We can use the method of images, as described in [55], in order to compute the boson and fermion propagators on all the surfaces starting from those on the torus. We take into account of the different N-N, D-D, N-D, D-N boundary conditions according to the expansions given in eqs.(1.6),(1.8),(1.24) valid for the strip. The only relevant thing, due to

[4]The discussion done here and in what follows also applies to the torus contribution to the amplitude, reminding that in this case left and right sectors are unrelated, of course.

eq.(1.24), is that all the propagators in the D-N and N-D directions, those corresponding to the $K3$ orbifold, are $\mathbf{Z}_2$-twisted. Bosonizing fermions and superghosts, we can then compute each term using the results of [56]. The important point to note is that we have always a cancellation between the contributions of the superghosts and the fermions of the torus direction. We are left in this way with a sum over the spin structures for the remaining eight directions. Given the structure of the graviton and graviphoton vertex operators, the spin structure dependent part of each term contributing to the amplitude is always of the form, for generic arguments a_1, a_2:

$$\sum_{g_1,g_2=0,-1/2}\sum_{\alpha,\beta=0,1/2}(-)^{4\alpha\beta}\theta^2\begin{bmatrix}\alpha\\ \beta\end{bmatrix}(a_1)\theta\begin{bmatrix}\alpha-g_1\\ \beta-g_2\end{bmatrix}(a_2)\theta\begin{bmatrix}\alpha-g_1\\ \beta-g_2\end{bmatrix}(a_2) \tag{3.33}$$

where

$$\theta\begin{bmatrix}\alpha\\ \beta\end{bmatrix}(z)\equiv\theta\begin{bmatrix}\alpha\\ \beta\end{bmatrix}(z;\tau)=\sum_{n\in\mathbf{Z}}e^{i\pi(n+\frac{1}{2})^2\tau+2\pi i(n+\alpha)(z+\beta)} \tag{3.34}$$

The first sum is over the twisted sectors of the orbifold, the second is over the spin structures, the first two theta functions refer to the space-time coordinates and the remaining two to the internal $K3$ directions. For the open string sector, $g_1 = 0$ for N-N and D-D directions and $g_1 = -1/2$ for N-D and D-N. We can now perform the spin structure sum using the Riemann identity:

$$\sum_{g_1,g_2=0,-1/2}\sum_{\alpha,\beta=0,1/2}(-)^{4\alpha\beta}\theta^2\begin{bmatrix}\alpha\\ \beta\end{bmatrix}(a_1)\theta\begin{bmatrix}\alpha-g_1\\ \beta-g_2\end{bmatrix}(a_2)\theta\begin{bmatrix}\alpha-g_1\\ \beta-g_2\end{bmatrix}(a_2)=$$

$$\sum_{g_1,g_2=0,-1/2}\theta\begin{bmatrix}1/2\\ 1/2\end{bmatrix}(\tilde{a}_1)\theta\begin{bmatrix}1/2\\ 1/2\end{bmatrix}(\tilde{a}_2)\theta\begin{bmatrix}1/2-g_1\\ 1/2-g_2\end{bmatrix}(0)\theta\begin{bmatrix}1/2-g_1\\ 1/2-g_2\end{bmatrix}(0) \tag{3.35}$$

where $\tilde{a}_{1,2} = a_1 \pm a_2$. We can reinterpret the result (3.35) of the spin-structure sum as the amplitude in the odd spin structure of new vertex operators obtained from the original one through the $SO(8)$ triality rotation defined by the Riemann identity. As shown in [48], the graviton vertices are left invariant by the map while the graviphoton operators are transformed to:

$$V_\gamma(p_1^\mp) \to \left[(\partial+\bar\partial)Z_2^\pm + ip_1^\mp(\psi_1^\pm - \tilde\psi_1^\pm)(\psi_2^\pm - \tilde\psi_2^\pm)\right] e^{ip_1^\mp Z_1^\pm} \tag{3.36}$$

The remarkable fact is that the correlation functions now depend only on the space-time coordinates, the internal $K3$ part entering through its partition function in the odd spin structure. After having extracted the terms linear in the momentum of each graviphoton, the F_g couplings are given by the following amplitude in the odd spin structure:

$$F_{g,I}(U) = \frac{1}{(g!)^2 32\pi^2} \sum_{\substack{\alpha=\mathrm{T,K}\\ \mathrm{M,A}}} \int [dM]_\alpha \, C_\alpha([t]) \sum_{n_1,n_2} e^{-\frac{\pi[t]|P|^2}{U_2\sqrt{G}}} \left(\frac{P}{2U_2\sqrt{G}}\right)^{2g-2} \langle V_g^+ V_g^- \prod_{i=1}^{g-1}\prod_{j=1}^{g-1}$$

$$\left[\int d^2x_i (Z_1^+(\partial+\bar\partial)Z_2^+ + (\psi_1^+ - \tilde\psi_1^+)(\psi_2^+ - \tilde\psi_2^+)\right]\left[\int d^2y_j (Z_2^-(\partial+\bar\partial)Z_1^- + (\psi_1^- - \tilde\psi_1^-)(\psi_2^- - \tilde\psi_2^-)\right]\rangle_\alpha$$

where

$$V_g^\pm = \int d^2x (Z_{1,2}^\pm \partial Z_{2,1}^\pm + \psi_1^\pm \psi_2^\pm)(Z_{1,2}^\pm \bar\partial Z_{2,1}^\pm + \tilde\psi_1^\pm \tilde\psi_2^\pm) \tag{3.37}$$

$[dM]_\alpha$ denotes the measure of the moduli integration for each surface α with $[t]$ the corresponding coordinate, $C_\alpha([t])$ is the partition function in the odd spin structure of the internal sector and $P = n_1 + n_2\bar U$ are the discrete momenta corresponding to the T^2 torus. Again, the normalization of the amplitude has been fixed according to eq.(3.19). Define now, analogously to eq.(2.21):

$$F_I(\lambda, U) \equiv \sum_{g=1}^{\infty} g^2 \lambda^{2g} F_{g,I}(U) \tag{3.38}$$

Exponentiating we obtain

$$F_I(\lambda, U) = \frac{\lambda^2}{32\pi^2} \sum_{\substack{\alpha=\mathrm{T,K}\\ \mathrm{M,A}}} \int [dM]_\alpha \, C_\alpha([t]) \sum_{n_1,n_2} e^{-\frac{\pi t|P|^2}{U_2\sqrt{G}}} \langle e^{-S_0+\tilde\lambda S} V_g^+ V_g^- \rangle_\alpha \tag{3.39}$$

where $\tilde{\lambda} = \lambda t P/\sqrt{2U_2}G^{1/4}$, S_0 is the free action for the space-time bosons and fermions and

$$S = \int \frac{d^2x}{t}[Z_1^+(\partial+\bar{\partial})Z_2^+ + (\psi_1^+ - \tilde{\psi}_1^+)(\psi_2^+ - \tilde{\psi}_2^+) + Z_2^-(\partial+\bar{\partial})Z_1^- + (\psi_1^- - \tilde{\psi}_1^-)(\psi_2^- - \tilde{\psi}_2^-)] \tag{3.40}$$

Because of the four zero modes $\psi_1^\pm = \tilde{\psi}_1^\pm$ =const., $\psi_2^\pm = \tilde{\psi}_2^\pm$ =const. of the new action, it is easy to check that

$$\langle e^{-S_0+\tilde{\lambda}S} V_g^+ V_g^- \rangle_\alpha = \frac{t^2}{2}\frac{d^2}{d\tilde{\lambda}^2}\langle e^{-S_0+\tilde{\lambda}S}\rangle_\alpha \tag{3.41}$$

The amplitude is then reduced to the evaluation of determinants of space-time bosons and fermions; before computing them, however, note that $C([t])$ is actually an index on all the surfaces. In order to see this, it is better to go to the operatorial formalism. For the torus and annulus, $C(t)$ coincides with the Witten index [57] $\mathrm{Tr}_{\mathrm{R-R}}(-)^{F_L+F_R} q^{L_0}\bar{q}^{\bar{L}_0}$ and $\mathrm{Tr}_{\mathrm{R}}(-)^F q^{L_0}$. For the Möbius strip and Klein bottle, $C_\alpha(t)$ is respectively $\mathrm{Tr}_{\mathrm{R}}\Omega(-)^F q^{L_0}$ and $\mathrm{Tr}_{\mathrm{R-R}}\Omega\, q^{L_0}\bar{q}^{\bar{L}_0}$. It is easy to see that these are still indices. In the Möbius strip Ω will act simply by multiplying each multiplet by a common eigenvalue, while in the Klein bottle it is possible to check that each multiplet entering in the evaluation of the trace has equal number of states with opposite eigenvalues of Ω. Since the $C_\alpha([t])$ are all indices, their values are invariant for small perturbations of the theory and then it is possible to compute them directly from the spectrum of the free theory summarized in tables 1.1 and 1.2 and from its stringy origin [36]. By a simple counting in the closed string spectrum, we easily derive that $C_{\mathrm{K}} = 0$ and $C_{\mathrm{T}} = 8$ [5]. Since in the torus the fermion and boson determinants cancel, leaving a λ-independent constant,

[5] Note that the twisted closed states in the orbifold limit are all massive [41].

we have a non-vanishing contribution only for the coupling $F_{1,I}$ coming from the odd-odd and even-even spin structures [52]. It can be shown that the sum of the two contributions can be extracted by the odd-odd part in the limit $\sqrt{G} \to \infty$ [52]. Its contribution is:

$$F_{1,I}|^{\text{torus}}_{odd-odd} = \frac{1}{32\pi^2} \int d^2\tau \, C_{\text{T}} \sum_{n_1,n_2} e^{-\frac{\pi\tau_2 |P|^2}{U_2\sqrt{G}}} \langle V_g^+ V_g^- \rangle_{\text{T}} \tag{3.42}$$

The only non-vanishing result comes when we take all the eight fermions of the two graviton vertex operators to soak up the eight zero-modes present in the odd-odd spin structure of the torus. The result gives simply:

$$F_{1,I}|^{\text{torus}}_{odd-odd} = \frac{1}{8} \int \frac{d^2\tau}{\tau_2} C_{\text{T}} \sum_{n_1,n_2} e^{-\frac{\pi\tau_2 |P|^2}{U_2\sqrt{G}}} \tag{3.43}$$

Rescaling $\tau_2 \to \sqrt{G}\,\tau_2/2$, we obtain, for $\sqrt{G} \to \infty$:

$$F^{\text{torus}}_{1,I} = \int_0^\infty \frac{d\tau_2}{\tau_2} \sum_{n_1,n_2} e^{-\frac{\pi\tau_2 |P|^2}{2U_2}} \tag{3.44}$$

The remaining contribution for the F_g's comes from the annulus and Möbius strip surfaces. The corresponding determinants can be simply computed using the corresponding mode expansions of the fields reported in Appendix A. For $n \neq 0$, boson and fermion determinants always cancel. For $n = 0$, in the annulus and Möbius strip the λ-dependent term in the fermion part of the action drops out, while the bosonic contribution reduce to:

$$\langle e^{-S_0 + \tilde{\lambda} S} \rangle_{\text{A}} = \langle e^{-S_0 + \tilde{\lambda} S} \rangle_{\text{M}} = \frac{1}{t^3} \prod_{m=1}^{\infty} \left(1 - \frac{\tilde{\lambda}^2}{m^2}\right)^{-2} = \frac{1}{t^3} \left(\frac{\tilde{\lambda}\pi}{\sin\tilde{\lambda}\pi}\right)^2 \tag{3.45}$$

Looking to the open massless string spectrum [36], it turns out that $C_{\text{A}} + C_{\text{M}} = 32^2 - 2 \cdot 32 = 4 \cdot 240$. Putting everything together and rescaling $t \to \sqrt{G}\, t/2$, we finally obtain:

$$F_I(\lambda, U) = \frac{\lambda^2}{16\pi^2} \int_0^\infty \frac{dt}{t} \sum_{n_1,n_2} e^{-\frac{\pi t |P|^2}{2U_2}} \left[240 \frac{d^2}{d\lambda^2} \left(\frac{\lambda\pi}{\sin\lambda}\right)^2 + 16\pi^2\right] \tag{3.46}$$

where $\bar{\lambda} = \lambda \pi t P / 4U_2$. Eq.(3.46) reproduces, according to the type I-heterotic duality, the one-loop part of the generating function (3.29). As in the heterotic case, all the F_g with $g \geq 2$ are convergent whereas F_1 presents both an IR, and an apparent UV divergence that disappears when the contributions of the different surfaces are appropriately regularized [10]. On the other hand, as can be seen comparing eqs.(3.29) and (3.46), the IR divergences match up. Moreover, both amplitudes receive contributions only from the $N = 2$ BPS-saturated states, property that will be studied in greater detail in next chapter.

Chapter 4

BPS states and the supersymmetric index

During the recent developments achieved in string and quantum field theories with extended supersymmetry, it has become evident that a prominent role in governing the dynamics is played by BPS saturated states. These special states provide important clues in the exploration of the strong coupling regimes of the corresponding theories. This is due to the fact that their properties are usually fixed exactly at tree level, since supersymmetry protects them to receive quantum corrections. The appeareance of solitonic massless BPS states in $\mathcal{N} = 2$ supersymmetric Yang-Mills theories [58], the prediction of U-duality in type II string theories of the existence of BPS states carrying Ramond-Ramond charges [59], subsequently identified with D-branes [22], and the resolution of the conifold singularity via a solitonic charged BPS state [50] are only few examples of the vast number of results supporting their importance.

An interesting connection between BPS states and generalized Kac-Moody currents has been found in [13]. In this reference the authors study threshold and gravitational corrections in $N = 2$ four-dimensional compactifications of the heterotic string. These corrections can be written in terms of the $N = 2$ supersymmtric index defined in [9] and are determined purely in terms of BPS states. In particular the index is shown to count the difference between the number of vector- and hyper- multiplets in the effective four-dimensional theory. The scope of this chapter is to show that an analogous result holds in $N = 2$ type I string compactifications. The $N = 2$ supersymmetric index is realized in type I compactifications and then related to the corresponding threshold and gravitational corrections. For a generic compactification we compute this index as a function of the BPS spectrum, finding again that BPS contributions enter only through the difference between vector- and hyper-multiplets[1].

We then extend these results to the orientifold models discussed in the first chapter, generalizing the one-loop computation of the F_g's couplings performed in last chapter for the $\mathbf{Z}_2^A$ model and showing explicitly the BPS dependence of these amplitudes. This index encodes the compactification model-dependence of these couplings, while an universal part coming from the spacetime correlations complete its structure.

In section one we review the definition and the main properties of the supersymmetric index [9], as well as those of the Witten index [57], pointing out their differences. In section two we realize the supersymmetric index in

[1]Strictly speaking this is not the case for the F_1-coupling, see below.

$N=2$ type I compactifications, by reporting two simple examples of one loop amplitudes whose expressions are given by it. We show in section three how for a generic $K3\times T^2$ type I string compactifications, this index is determined purely as a function of the BPS spectrum of the four dimensional theory. In last section we finally extend the computation of the F_g's to all the orientifold models reviewed in chapter one.

4.1 The Witten and supersymmetric indices

In this section we briefly report the most important properties of the Witten and supersymmetric indices, starting from the former one.

For any supersymmetric field theory defined on a compact space of arbitrary dimension that does not break supersymmetry itself, the quantity $\mathrm{Tr}\,(-)^F$ [57], where $(-)^F$ is the operator whose eigenvlues are +1 on the bosons and -1 on the fermions is an index, in the sense that its value is invariant for finite perturbations of the theory. Consider, indeed, a generic bosonic state $|b\rangle$ with a non-vanishing energy E. If supersymmetry is unbroken, for each of these states, there exists the supersymmetric partner

$$\frac{1}{\sqrt{E}}\,Q|b\rangle \tag{4.1}$$

with the same energy but opposite fermion number, since $\{(-)^F, Q\}=0$. This means that the index above, called the Witten index, depends only on states with vanishing energy, i.e. the vacua of the theory; in particular it is equal to

$$\mathrm{Tr}\,(-)^F = n_B^{E=0} - n_F^{E=0} \tag{4.2}$$

where $n_B^{E=0}, n_F^{E=0}$ are respectively the bosonic and fermionic states with vanishing energy. The important observation is that this difference cannot change if we vary the parameters of our theory, because if states with $E \neq 0$ are driven to $E = 0$ or viceversa, it always has to happen for pairs $|b\rangle, |f\rangle$. Altough the net number of $n_B^{E=0}, n_F^{E=0}$ states can change, their difference is then invariant. It is natural that whenever the Witten index is different from zero, supersymmetry is unbroken, whereas it is not valid the contrary. The property of being invariant for finite perturbations of the theory makes the Witten index an important tool to study supersymmetry breaking, since it often allows to do an exact prediction on SBS by a simple perturbative computation. For two dimensional supesymmetric non-linear σ-model with target space $\mathcal{M}$, it has also been shown in [57] that the Witten index equals the Euler characteristic of the manifold $\mathcal{M}$. It is now straightforward to extend the previous results for more general cases. Whenever we have a conserved operator K that commute with supersymmetry, the combination $\mathrm{Tr}\, K\, (-)^F$ is again invariant for finite perturbations of the theory and define new topological indices. Note that the indices discussed in section (3.3) are precisely of this kind with $K = \Omega$, the world-sheet parity operator.

Let us now turn to the supersymmetric index [9]. Contrary to the case of the Witten index, this new index is defined only for two dimensional theories with at least $N = 2$ supersymmetry. It is given by $\mathrm{Tr}\, F\, (-)^F$ where F is the fermion number charge, associated to the $U(1)$ current present in the $N = 2$ algebra. It has been shown in [9] that the quantity $\mathrm{Tr}\, F\, (-)^F$ is invariant for any finite perturbation of the two dimensional theory that can be written as a D-term, i.e. a term that in superspace language is integrated on the whole superspace $d^4\theta$, whereas it depends in general on F-deformations of the model. This means that strictly speaking $\mathrm{Tr}\, F\, (-)^F$ is not a topological index, a pure number, but really a function of the moduli of the theory that parametrize the F-deformations of the model. The abuse of language is due to the fact that in most of the practical cases, D-deformations represent almost all the possible deformations. It is also interesting to understand which states contribute to the supersymmetric index. This can be easily done for compact spaces, where we do not have to deal with a continous spectrum and with density of states. Since a two dimensional $N = 2$ theory presents four supercharges, we can have either non-reduced multiplets with four states or BPS-saturated states [2] with two states, in addition to the trivial representation of the vacua, of course. If we split the supercharges in creation and annihilation operators $Q^-_{1,2}$ and $Q^+_{1,2}$ respectively, a generic non-reduced multiplet will be composed as follows:

$$|\alpha\rangle, \quad Q_1^-|\alpha\rangle, \quad Q_2^-|\alpha\rangle, \quad Q_1^-Q_2^-|\alpha\rangle \tag{4.3}$$

where $|\alpha\rangle$ is a generic state, eigenvector of the operator F with eigenvalue f_α. A BPS saturated state will be instead given by:

$$|\alpha\rangle, \quad Q^-|\alpha\rangle \tag{4.4}$$

with Q^- a given combination of Q_1^- and Q_2^-. It follows from the definition of the index that the contribution of any non-reduced multiplet is proportional to $f_\alpha - 2(f_\alpha + 1) + (f_\alpha + 2) = 0$, so that only the short multiplets (4.4) give a non-vanishing contribution. It has been shown in [9] that in this case the

[2] These BPS states of the $N = 2$ two dimensional theory should not be confused with those of the 4d space-time effective action, that we will consider later.

ground states of the theory, that determine completely the Witten index, do not play any role for the supersymmetric index.

4.2 One-loop corrections in Type I string compactifications

We consider here two relevant examples of one-loop amplitudes that are related to a realization of the supersymmetric index discussed above. The analysis will be restricted to the main features of the involved one-loop corrections in order to allow a straightforward generalization to a wider class of terms. The important point in the considered amplitudes is that the internal theory, entering simply through an index, receive contributions only from its ground states. As we have seen in last chapter, the space-time part of the correlation function can be reduced to a supersymmetric partition function in the odd-odd spin structure, where a cancellation between fermion and boson determinants holds and again only ground states give a non-vanishing result. In this way we reduce the amplitude to a sum of BPS states contributions.

One loop amplitudes in type I compactifications involve a sum over the spin structures carried by the involved surfaces. Using the Riemann identity we can relate the sum over spin structures to an odd-odd spin structure correlation of new operators obtained by a triality rotation of the original ones, as performed in last chapter in order to compute the F_g's couplings. Let us now briefly work it out two simple examples of relevant odd-odd spin structure correlations in $K3 \times T^2$ type I compactifications, in order to understand better the general structure of the kind of amplitudes involved in our discussion.

a) *F_1 gravitational coupling*

In this first example we discuss an amplitude, relevant to find the moduli dependence of F_1. This coupling can be extracted [12] from the on-shell three-point function of two gravitons and a generic modulus field, or directly from the computation of the F_g we performed, that is valid also for $g = 1$ (see footnote of p.35). We follow here the first way. It is convenient to treat separately the torus contribution to the amplitude from those coming from the other one-loop surfaces, Klein bottle, annulus and Möbius strip, that we will denote respectively by a superscript T, K, A, M. Indicating with T the modulus field and with Δ_T the derivative of the odd-odd part of F_1 with respect to T, the torus contribution can be written, once extracted appropriately the kinematical structure, as

$$\Delta_T^T \sim \int_\Gamma \frac{d^2\tau}{\tau_2} \int \prod_{i=1}^{3} d^2 z_i \langle V_h^{\mu\alpha}(p_1, z_1)\, V_h^{\nu\beta}(p_2, z_2) \times V_T^{(-1,-1)}(p_3, z_3)\, T_F(z)\, \tilde{T}_F(\bar{z})\rangle_{\rm odd} \tag{4.5}$$

where $\tau = \tau_1 + i\tau_2$ is the modular parameter of the world-sheet torus, Γ its fundamental domain and $V_T^{(-1,-1)}$ is the vertex operator in the $(-1,-1)$ left-right ghost picture of the closed string state, associated with the T-modulus:

$$V_T^{(-1,-1)}(p) = e^{-(\phi+\tilde{\phi})}\Psi_+(z,\bar{z})\, e^{ip\cdot X} \tag{4.6}$$

$\Psi(z,\bar{z})$ being a primary field of dimension $(1/2, 1/2)$. The $(-1,-1)$ ghost picture takes into account the left-right Killing spinors on the world-sheet torus while the picture changing operators $T_F(z)$ and $\tilde{T}_F(\bar{z})$ soak the gravitino zero modes. The V_h's are the usual graviton vertex operators given in eq.(3.30). The analysis of this amplitude is just a left-right symmetric version of that performed in [12]. The OPE of the $N = 2$ internal superconformal algebras are given by

$$\tilde{T}_F^{\mp}(\bar{w})T_F^{\mp}(w)\Psi_{\pm}(z,\bar{z}) = \mp\tilde{J}(\bar{w})J(w)\Phi_{\pm}(z,\bar{z}) + ... \tag{4.7}$$

where $J, \tilde{J}$ are the U(1) currents associated to the $N = 2$ superconformal algebras, $\Phi_{\pm}$ is the upper component of $\Psi_{\pm}$ with dimension (1,1) and ... represent contour integrals that give vanishing contribution to the amplitude. Since $\Phi_{\pm}$ are the fields associated to the marginal deformations of the T-modulus, we can write explicitly Δ_T^T as the T-derivative of the odd-odd component of F_1, coming form the torus contribution:

$$\Delta_T^T = i\partial_T F_1^T|_{odd-odd}$$

with

$$F_1|_{odd-odd} = -i\int_\Gamma \frac{d^2\tau}{\tau_2} C_T \tag{4.8}$$

and

$$C_T \equiv \mathrm{Tr}_{RR}(-1)^{F_L+F_R}\, F_L F_R\, q^{\Delta}\bar{q}^{\bar{\Delta}} \tag{4.9}$$

where $q = e^{2\pi i\tau}$, the trace is restricted to the Ramond-Ramond sector and contains the momenta lattice sum in the torus direction, and Δ is the conformal dimension of the state propagating around the loop. F_L, F_R are the fermionic numbers, i.e. the zero modes of the $U(1)$ currents J_L, J_R, which soak the four zero modes of the free fermions associated to the torus direction and then are necessary to get a non-vanishing result. The space-time part contributes only trough the eight zero modes needed in the torus for the odd-odd spin structure. We already noticed in last chapter how the total moduli dependence of F_1, given also by the even-even spin structure, can actually be extracted from eq.(4.8) by taking the limit of infinite volume for the compactification torus. In the last section we will consider in detail the whole tower of F_g couplings, which include, beside the two gravitons, an additional bunch of $(2g-2)$ spacetime operators, obtained from the graviphotons through a triality rotation induced by the spin-structure sum. The internal structure is therefore untouched and only the space-time part of the amplitude will be modified. In this section we will concentrate in this internal part C_T (and similar quantities for the rest of the surfaces) which realize the $N=2$ supersymmetric index. The contributions given by the other surfaces can be analyzed in an analogous way. Proceeding along the same lines followed for the torus, we relate the amplitudes to an integration in the corresponding worldsheet moduli of a spacetime correlation and an internal contribution through an index written as:

$$\begin{aligned} C_K &\equiv \mathrm{Tr}_{RR}\frac{(F_L+F_R)}{2}(-)^{F_L+F_R}\,\Omega\, q^{\Delta}\bar{q}^{\bar{\Delta}} \\ C_A &\equiv \mathrm{Tr}_R(-1)^F\, F\, q^{\Delta}\bar{q}^{\bar{\Delta}} \\ C_M &\equiv \mathrm{Tr}_R(-1)^F\, F\,\Omega\, q^{\Delta}\bar{q}^{\bar{\Delta}} \end{aligned} \tag{4.10}$$

Again the F insertions provide the correct number of zero modes (two in this case) that we need to soak in order to get a non-vanishing result. The next section is devoted to the study of these quantities; as we will see they are indices in the sense that they cannot be changed by small deformations of the parameters of the theory.

b) *Threshold corrections to gauge couplings*

The second example refers to moduli dependence of one-loop threshold corrections to gauge couplings. More general corrections of this kind have been studied in [10]. We restrict ourselves to show the connection of these amplitudes with the considered index. We consider here the CP-odd theta angle of a generic term of the form $F \wedge F$, whose moduli dependence can again be extracted from a three-point function where now, instead of gravitons, we insert the gauge field vertex operators:

$$V_A^{\mu,a}(p,z) = (\partial_\tau X^\mu + ip_\tau \cdot \psi \, \psi^\mu) e^{ip \cdot X} \lambda^a \tag{4.11}$$

with a an index in the adjoint of the gauge group and λ^a the corresponding Chan-Paton matrix. The relevant amplitude is then given by

$$\begin{aligned} \epsilon^{\mu\nu\lambda\rho} p_{1\lambda} p_{2\rho} \delta^{ab} \, \Theta_T^{A,M} &= \int \frac{dt}{t} \int \prod_{i=1}^{2} dt_i \, d^2 z \\ \langle V_A^{\mu,a}(p_1, t_1) \, V_A^{\nu,b}(p_2, t_2) \, & V_T^{(-1)}(p_3, z) \, \hat{T}_F(z_0) \rangle_{\text{odd}}^{A,M} \end{aligned} \tag{4.12}$$

where V_T^{-1} is the vertex operator of the closed string state associated to the T modulus:

$$V_T^{(-1)}(p) = (e^{-\phi} \Psi(z, \bar{z}) + e^{-\tilde{\phi}} \tilde{\Psi}(\bar{z}, z)) e^{ip \cdot X} \tag{4.13}$$

and Ψ, $\tilde{\Psi}$ are respectively the components of dimensions (1/2,1) and (1,1/2) of an $N = 2$ superfield and $\hat{T}_F \equiv T_F + \tilde{T}_F$ is the left-right symmetric picture changing operator.

The four spacetime zero modes required in the odd spin structure come from the fermion part of the gauge field vertices reproducing the correct kinematic factor in (4.12). As before the $N = 2$ OPE allows us to write the internal contribution as a derivative with respect to T of a trace in the

Ramond sector

$$\Theta_T^A = i\partial_T \Delta^A$$

with

$$\Delta^A \sim \int \frac{dt}{t} C_A \tag{4.14}$$

and

$$C_A \equiv \mathrm{Tr}_R (-1)^F F \, q^{\Delta} \bar{q}^{\bar{\Delta}} \tag{4.15}$$

The Möbius strip contribution on the other hand is just given by an Ω insertion in this trace. We recognize again the indices found in the previous example. In this way we have expressed some one-loop corrections to four-dimensional effective actions arising from compactifications of type I strings, in terms of a realization of the $N = 2$ supersymmetric index in these theories.

4.3 BPS states and $N = 2$ Supersymmetric indices

In the beginning of last section we argued that the quantities (4.9,4.10) can be written as a sum over BPS contributions. Aim of the present section is to determine them in terms of this spectrum for generic $K3 \times T^2$ type I compactifications. Threshold and gravitational one-loop corrections for analogous compactifications of the heterotic string are also written in terms of this $N = 2$ supersymmetric index. Exploiting the representation properties of the internal superconformal algebra for these compactifications, Harvey and Moore [13] found that the index counts the difference between the number of BPS vector- and hyper-multiplets at each level of mass. We will follow the lines of this reference to find similar results for the relevant indices involved in type I compactifications. In order to maintain the analysis performed as easy as possible, we will consider in the following type I compactifications in which no Wilson lines on the T^2 torus are turned on. Our considerations are however general, and the modifications brought by Wilson lines inclusions will be briefly pointed out.

The internal superconformal theory associated to $K3 \times T^2$ compactifications of type I theory is a sum of two pieces, corresponding to the open and closed string sectors. The open sector is realized with a $(c = 3, N = 2) \oplus (c = 6, N = 4)$ Super Conformal Field Theory (SCFT) while the closed sector is associated to the conformal theory that arises after an Ω-projection of the $[(c = 3, N = 2) \oplus (c = 6, N = 4)]_L \otimes [(\tilde{c} = 3, N = 2) \oplus (\tilde{c} = 6, N = 4)]_R$ SCFT, where Ω is the world-sheet parity operator. In order to relate (4.9,4.10) to a counting of four-dimensional BPS states let us review the structure and superconformal content of these states in type I compactifications.

String states in the open sector satisfy the mass condition (in the Neveu-Schwarz sector):

$$\frac{1}{2}M^2 = \frac{1}{2}p^2 + (N - \frac{1}{2}) + h_{int} \tag{4.16}$$

with N the oscillator number associated to the space-time and torus directions, h_{int} the conformal weight in the $N = 4$ SCFT and p the Kaluza-Klein momentum coming from the torus. By considering the four dimensional $N = 2$ action as the reduction on T^2 of the $N = 1$ 6d theory, it is easily seen that the Kaluza Klein momentum is effectively the central charge of the 4d $N = 2$ algebra. The BPS bound $M^2 = Z^2$ is then simply replaced by $M^2 = p^2$, that is satisfied only by the six-dimensional massless Neveu-Schwarz states ($N = 1/2, h_{int} = 0$) and ($N = 0, h_{int} = 1/2$), which after a further torus compactification generate all the four dimensional vector- and hyper- BPS multiplets. Note that this is not what happens in $N = 2$ heterotic models where the BPS condition (in the Neveu-Schwarz sector) reads

$$\frac{1}{8}M^2 = \frac{1}{2}p_R^2 = \frac{1}{2}p_L^2 + (h-1) \tag{4.17}$$

where p_L, p_R include now winding and Kaluza-Klein modes on the torus direction, and h is the conformal weight of the states in the $c = 26$ CFT. In this case, the number of BPS states depends on the level of mass, since for a fixed p_R^2 each point in the lattice $p_R^2 - p_L^2 = 2\,nm > 0$ defines additional BPS states with $h = nm + 1$ besides the six-dimensional massless one $h = 1$.

The structure of the type I open Ramond sector is simply obtained by that of the two massless Neveu-Schwarz representations seen before by spectral flow. We remind that in $N = 2$ SCA there is a free continous parameter, whose value fix the different boundary conditions of the supercharges. All this family of $N = 2$ algebras are however isomorphic, being related by a unitary operator, whose action is usually called spectral flow. The conformal content is displayed in the present table:

Sector	**Vectormultiplets**	**Hypermultiplets**
Ramond	$(1/8, \pm 1/2) \otimes (1/4, 1/2)$	$2 \times (1/8, \pm 1/2) \otimes (1/4, 0)$

where, following the notation of [13], we denote with $(h, q) \otimes (h', I)$ a state with conformal weight h and $U(1)$ charge q of the $c = 3, N = 2$ theory and weight h' and representation I of the $SU(2)$ current of the $c = 6, N = 4$ theory. We are now ready to compute the indices (4.10) associated to the open string sector.

The fermionic numbers decompose as $F = F_1 + F_2$, F_1 and F_2 being the fermionic currents associated to the $U(1)$ currents J_1 and J_2 of the $N = 2$ and $N = 4$ superconformal algebras respectively ($J_2 = 2J^3$, where J^3 is the Cartan element of the $N = 4$ $SU(2)$ current). Since in $SU(2)$ representations the eigenvalues of J^3 come always in pairs,

$$\mathrm{Tr}_{N=4}\, F_2\, (-)^{F_2} q^{\Delta} \bar{q}^{\bar{\Delta}} = 0 \tag{4.18}$$

leaving as the only non-vanishing contribution to the indices the F_1 insertion. This can be easily understood from the examples of last section; in order to have a non-vanishing contribution to our amplitudes, we needed to soak the zero modes of the free fermions associated to the T^2 torus, that means to insert a fermionic F_1 current. We are then left with the trace

$$C_{\mathrm{A}} + C_{\mathrm{M}} = 2\, \mathrm{Tr}_{N=2}\, F_1\, (-)^{F_1}\, \mathrm{Tr}_{N=4}\, (-)^{F_2}\, q^{\Delta}, \tag{4.19}$$

where the trace now runs over the Ω-invariant states [3], including the Chan-Paton degrees of freedom. As can be seen from the table above, the $N = 2$ conformal content of both vector and hypermultiplet states is equal, while the $N = 4$ SCA enters only through the Witten index. This important fact allows us to conclude that our amplitudes are completely independent of any deformation performed on the internal $N = 4$ SCA, including deformations

[3]Note that the Ω-projection in the open string sector gives constraints only on the Chan-Paton degrees of freedom.

that can be written as F-terms. Following ref.[60], the only $N=4$ multiplets whose Witten index is different from zero are:

$$\begin{aligned} \mathrm{Tr}_{(1/4,0)}(-)^{F_2} &= 1 \\ \mathrm{Tr}_{(1/4,1/2)}(-)^{F_2} &= -2 \end{aligned} \tag{4.20}$$

We can then finally write

$$\begin{aligned} C_{\mathrm{A}}+C_{\mathrm{M}} &= 2\sum_{p\in\Gamma}(\frac{1}{2}e^{i\frac{\pi}{2}}-\frac{1}{2}e^{-i\frac{\pi}{2}})(\mathrm{Tr}_{I=0}(-)^{F_2}+\mathrm{Tr}_{I=1/2}(-)^{F_2})e^{-\pi t|p|^2}= \\ &= 4i(n_H^{open}-n_V^{open})\sum_{p\in\Gamma}e^{-\pi t|p|^2} \end{aligned} \tag{4.21}$$

where Γ represents the lattice momenta sum and n_V^{open}, n_H^{open} are respectively the number of massless four dimensional vector and hypermultiplets in the open string sector. We have extracted the overall factor $(n_H^{open}-n_V^{open})$, corresponding to the common degeneration to all levels of mass of the BPS number, as was already discussed before. More general backgrounds including Wilson lines on the torus can be analyzed. In this case, the T^2 momenta lattice will be shifted by the included gauge field expectation values, and a given number of massless vector- and hyper- multiplets will get masses. All the previous considerations are however left invariant, and eq.(4.21) continue to be valid, with the aforementioned modifications.

Let us now turn to the closed string spectrum. The bosonic BPS content in this sector arise from a tensor product of the NS-NS (R-R) massless representations symmetrized (antisymmetrized) under Ω. Let us first notice that for each R-R ground state in the $N=4$ SCFT we can construct four states, taking into account the multiplicities coming from the $N=2$ space-time and torus algebra representations. The Ω-even contribution to this trace takes into account the spectral flown NS-NS states kept by the Ω-projection. In this way we are then really counting all the BPS multiplets even if the trace is performed only in the R-R sector. The counting of R-R ground states in the $N = 4$ theory is simply given by the geometrical structure of $K3$: 20 states $(1/4, 0) \otimes (1/4, 0)$, corresponding to the $h^{1,1} = 20$ cohomologically distinct $(1, 1)$ differential forms on $K3$ and one multiplet $(1/4, 1/2) \otimes (1/4, 1/2)$ counting the four forms given by $h^{0.0}, h^{2,0}, h^{0,2}, h^{2,2}$. As is clear, there are no states $(1/4, 1/2) \otimes (1/4, 0)$ or viceversa because $h^{1,0} = h^{0,1} = h^{2,1} = h^{1,2} = 0$. The index associated to the torus can then be easily computed to be:

$$C_{\rm T} = {\rm Tr}_{RR}^{N=2\oplus N=4} F_L F_R (-)^{F_L+F_R} q^{\frac{\Delta}{2}} \bar{q}^{\frac{\bar{\Delta}}{2}} = -(n_V^{closed} + n_H^{closed}) \sum_{p\in\Gamma} e^{-\pi\tau_2|p|^2} \tag{4.22}$$

where $n_V^{closed} + n_H^{closed}$ are the 24 massless four dimensional vector- and hypermultiplets [4], corresponding to the Witten index [57] ${\rm Tr}_{RR}(-)^{F_2^{(L)}+F_2^{(R)}} = \chi(K3)$, the Euler characteristic of $K3$. The $N = 2$ part enters only through the lattice sum and an overall (-1) factor.

The last involved quantity is the index related to the Klein Bottle. If we call Ω_{int} the worldsheet parity operator restricted to the $K3$ part, we can observe that vectors, being constructed from left-right symmmetric spacetime+torus combinations of states, are counted by Ω_{int} with a minus sign in the RR sector. This observation allows us to write finally:

$$C_{\rm K} = {\rm Tr}_{RR}^{N=2\oplus N=4} \frac{F_L + F_R}{2} (-)^{F_L+F_R}\, \Omega\, q^{\Delta+\bar{\Delta}} = i(n_V^{closed} - n_H^{closed}) \sum_{p\in\Gamma} e^{-\pi t|p|^2} \tag{4.23}$$

where again the overall factor (i) comes from the $N = 2$ superconformal

[4]We include in this counting the gravitational multiplet as a vectormultiplet.

content of the multiplets.

We achieved the final goal of this section. The contributions associated to the Klein bottle, annulus and Möbius strip depend only on the difference between the number of hyper and vector BPS states of a given compactification. As noted in [13], this dependence ensures the smoothness of amplitudes in the moduli space, since chaotic BPS states in $N=2$ theories always appear and disappear in hypermultiplet-vectormultiplet pairs. Notice, however, that C_{T} enters in a different way, but being associated to corrections in $N=4$ supersymmetric theories, it does not contribute in general. Among the couplings we analyzed, it gives a non-trivial contribution only to the gravitational coupling F_1, as has been shown in [61] to happen in four dimensional string compactifications with $N=4$ supersymmetry.

4.4 F_g terms on $K3$ orientifolds

In last section we gave a general argument to establish that all one-loop corrections to gauge and gravitational couplings which are related to the supersymmetric index receive contributions only from the BPS-saturated states of the four dimensional effective action. Our result, however, is valid for 4d models arising from $K3\times T^2$ geometric Type I string compactifications, since it was based on a analysis of the Super Conformal Algebra underlying this kind of compactifications. We discuss now the case of the orientifold models treated in chapter one, by generalizing the one-loop computation of the gravitational couplings F_g, performed for the $\mathbf{Z}_2^A$ model in last chapter. As we already noted, these amplitudes are actually proportional to the supersymmetric index and then, if the considerations of last section apply to the orientifolds as well, they have to receive a non-vanishing contributions only from BPS states and in the by now familiar combination given by the difference between the numbers of hyper and vectormultiplet BPS states. We will see that our computation displays exactly this purely BPS dependence, strongly supporting the generalization of the consideration of last chapter to this wider class of four dimensional vacua.

All these orientifold models, having in general abelian subgroups, have potentially dangerous $U(1)$ anomalies, that, exactly as for the $\mathbf{Z}_2^A$ case, are removed by a Higgs mechanism that give mass to the corresponding $U(1)$ gauge field. These amplitudes, however, do not depend on this Higgs mechanism that give mass to an hyper-vector multiplet pair, being proportional, as we wll see, to the difference of hyper and vector BPS states. In the following we will then simply ignore these potential $U(1)$ anomalies.

The generalization of the F_g computation performed in section 3.3 is easily done since all the considerations of last chapter are equally valid now. In particular, after the spin structure sum we get again eq.(3.35), where g_1, g_2 run over the twisted sectors of the corresponding orbidolf, of course, and still the $K3$ part enters simply through its partition function in the odd spin structure. The only difference among the various models is entirely encoded in the indices $C_\alpha(t)$. The Klein bottle contribution is now in general non-vanishing, so that we need now to compute the Klein bottle determinant, in addition to those of the annulus and Möbius strip, already considered in section 3.3 and given by eq.(3.45). For the Klein bottle, besides the bosonic contribution there is also, for $n = 0$, a fermionic contribution for m =odd leading to:

$$\langle e^{-S_0+\tilde{\lambda}S}\rangle_{\rm K} = \frac{4}{t^3}\left[\prod_{m=1}^{\infty}\left(1-\frac{\tilde{\lambda}^2}{m^2}\right)^{-2}\right]\left[\prod_{k=0}^{\infty}\left(1-\frac{4\tilde{\lambda}^2}{(2k+1)^2}\right)^{2}\right] = \frac{4}{t^3}\left(\frac{\tilde{\lambda}\pi}{\sin\tilde{\lambda}\pi}\right)^2\cos^2\tilde{\lambda}\pi \tag{4.24}$$

where the factor four between the contribution of the Klein bottle with that of the annulus and Möbius strip is due to the different modular parameter of the covering tori (see Appendix A). Note that the space-time contribution to the amplitude for all the surfaces is given by the string states with $n = 0$, i.e. with oscillation number zero. This observation, together with the analysis performed in last section for the internal part, allows us to conclude that only BPS states are contributing to the considered correlation functions. Putting all the results together, we finally have:

$$F(\lambda) = \frac{\lambda^2}{16}\int_0^{\infty}\frac{dt}{t}\sum_{p\in\Gamma}e^{-\pi t|p|^2}\left[\left(\frac{C_{\rm A}^{(0)}+C_{\rm M}^{(0)}}{4}+C_{\rm K}^{(0)}\right)\frac{d^2}{d\bar{\lambda}^2}\left(\frac{\bar{\lambda}}{\sin\bar{\lambda}}\right)^2+2(C_{\rm T}^{(0)}-C_{\rm K}^{(0)})\right] \tag{4.25}$$

where $C_\alpha^{(0)}$ represent the $K3$ part of the indices and the rest of notation follows that of section 3.3. Comparing the open and closed massless spectrum reported in tables 1.1 and 1.2 with the values of $C_\alpha^{(0)}$ for the various surfaces, given in Appendix B (table B.1), we can explicitly check that this index reproduces separately for all the sectors the difference between the number of four-dimensional hyper- and vector-multiplets:

$$\frac{C_{\rm A}^{(0)}+C_{\rm M}^{(0)}}{4}+C_{\rm K}^{(0)} = n_H^{total}-n_V^{total} \tag{4.26}$$

whereas

$$C_{\rm T}^{(0)}-C_{\rm K}^{(0)} = 2\,n_V^{closed} \tag{4.27}$$

This gives strong evidence to the suggestion that the results of last section are equally valid for these models. These are, of course, particular free models with no background fields; in general we will have a mixing between the sectors, but such that the total contribution $(C_{\rm A}+C_{\rm M})/4+C_{\rm K}$ will always count the number of BPS (hypers-vectors).

It is worth while to point out that the results obtained here for the $C^{(0)}_{\alpha}$ are not in contradiction with what found in chapter three for the $\mathbf{Z}_2^A$ model. The values obtained before, that is $C^{(0)}_{\rm T}=8, C^{(0)}_{\rm K}=0, C^{(0)}_{\rm A}+C^{(0)}_{\rm M}=4\cdot 240$ take into account the U(1) anomalies that give masses to 16 hypermultiplets in the closed string spectrum and 16 vectormultiplets to the open one.

Conclusions

In the present thesis some results concerning $N = 2$ type I string compactifications in four dimensions have been presented. Our main points regard a test of the conjectured type I-heterotic duality and the realization that a class of amplitudes in these type I compactifications is related to some indices, whose values are determined by the BPS-saturated states of the 4d theory. Both these results are in accordance with the general belief that all the perturbative string theories, including eleven-dimensional supergravity, or better, M-theory, are really a single unified theory whose properties are up to now almost completely unknown. Particular limits of this underlying non-perturbative theory, corners in its moduli space, are sometimes accessible perturbatively and reveals a given string theory or 11d sugra. It is certain that BPS-saturated states play a very important role in trying to understand the strongly coupled region of a given theory, at least when enough supersymmetry is left unbroken. In particular, our result about the analogous role that BPS states play in different string theories, like heterotic and type I, suggest perhaps that there is a geometrical structure underlying these states, in the same spirit of the analysis of ref.[13]. The BPS dependence found in last chapter for the F_g's in orientifold models is a significative example of this phenomenon. Altough most of them cannot have a weak heterotic dual pair, the structure of the amplitudes computed is analogous to that found for more standard type I and heterotic compactifications. As last remark, I would like to remind the importance of the study of the higher derivative F-terms in general, as a clear distinction between results valid in full string theory in contrast to low-energy supergravity effects, and in particular of the F_g's couplings that have been used constantly in the present work and already revealed their relevance in string theory in previous works [7, 45, 5].

Appendix

Appendix A

Mode expansions

We present here the mode expansions of the bosonic and fermionic fields in the annulus, Möbius strip and Klein bottle surfaces, needed to evaluate the corresponding determinants. We take as fundamental region of each surface $0 \leq \tau \leq t, 0 \leq \sigma \leq 1$, where t is the corresponding modulus. Following Burgess and Morris [55], we can consider each surface as a torus with points identified under a given projection; in this way we can solve the boundary or crosscap conditions by considering suitable identifications of the bosonic and fermionic fields that take values on the covering torus [5]. The mode expansion for the fields in the annulus are then given by:

$$\begin{aligned} x_{\mathrm{A}}(\tau,\sigma) &= \sum_{\substack{m=-\infty \\ n\geq 0}}^{+\infty} \alpha_{m,n} e^{2i\pi m\tau} \cos \pi n\sigma \\ \psi_{\mathrm{A}}(\tau,\sigma) &= \sum_{m,n=-\infty}^{+\infty} d_{m,n} e^{2i\pi m\tau} e^{i\pi n\sigma} \end{aligned} \tag{A.1}$$

[5]Note that since we need to compute determinants in the odd spin structure, we will consider in the following fermionic fields on this spin structure only.

$$\tilde{\psi}_{\mathrm{A}}(\tau,\sigma) = \sum_{m,n=-\infty}^{+\infty} d_{m,n} e^{2i\pi m\tau} e^{-i\pi n\sigma}$$

after having extended the fields to the torus $0 \leq \tau \leq t, 0 \leq \sigma \leq 2$ and identified

$$x(\tau,\sigma) = x(\tau, 2-\sigma), \qquad \psi(\tau,\sigma) = \tilde{\psi}(\tau, 2-\sigma) \tag{A.2}$$

For the Möbius strip we extend the field to the torus $0 \leq \tau \leq 2t, 0 \leq \sigma \leq 2$ by identifying

$$\begin{aligned} x(\tau,\sigma) &= x(1+\tau, 1-\sigma), \qquad x(\tau,\sigma) = x(\tau,2-\sigma) \\ \psi(\tau,\sigma) &= \tilde{\psi}(1+\tau,1-\sigma), \qquad \psi(\tau,\sigma) = \tilde{\psi}(\tau,2-\sigma) \end{aligned} \tag{A.3}$$

Then the mode expansion is:

$$\begin{aligned} x_{\mathrm{M}}(\tau,\sigma) &= \sum_{\substack{m=-\infty \\ n\geq 0}}^{+\infty} \alpha_{m,n} e^{i\pi m\tau} \cos \pi n\sigma \quad m+n = even \\ \psi_{\mathrm{M}}(\tau,\sigma) &= \sum_{m,n=-\infty}^{+\infty} d_{m,n} e^{i\pi m\tau} e^{i\pi n\sigma} \quad m+n = even \\ \tilde{\psi}_{\mathrm{M}}(\tau,\sigma) &= \sum_{m,n=-\infty}^{+\infty} d_{m,n} e^{i\pi m\tau} e^{-i\pi n\sigma} \quad m+n = even \end{aligned} \tag{A.4}$$

The bosonic and fermionic mode expansion in the Klein bottle are:

$$\begin{aligned} x_{\mathrm{K}}(\tau,\sigma) &= \frac{1}{2} \sum_{\substack{m=-\infty \\ n\geq 0}}^{+\infty} \alpha_{m,n} e^{i\pi m\tau} (e^{2i\pi n\sigma} + (-)^m e^{-2i\pi n\sigma}) \\ \psi_{\mathrm{K}}(\tau,\sigma) &= \sum_{m,n=-\infty}^{+\infty} d_{m,n} e^{i\pi m\tau} e^{2i\pi n\sigma} \\ \tilde{\psi}_{\mathrm{K}}(\tau,\sigma) &= \sum_{m,n=-\infty}^{+\infty} d_{m,n} (-)^m e^{i\pi m\tau} e^{-2i\pi n\sigma} \end{aligned} \tag{A.5}$$

resulting from the identification

$$x(\tau,\sigma) = x(1+\tau, 1-\sigma), \qquad \psi(\tau,\sigma) = \tilde{\psi}(1+\tau, 1-\sigma) \tag{A.6}$$

on the torus $0 \leq \tau \leq 2t, 0 \leq \sigma \leq 1$. Note that the modular parameter of the covering tori for the three surfaces with the aforementioned projections is respectively $\tau = it/2, it, 2it$. The factor of two between the annulus and Möbius strip parameters is due to the fact that the corresponding tori cover two times the annulus and four the Möbius surface.

Appendix B

C_α for $K3$ orientifolds

In this appendix we report the value of the indices $C_\alpha^{(0)}$, i.e. of the indices (4.9,4.10) restricted to the $K3$ part, for the orientifold models considered in chapter one. In order to avoid an heavy notation, we will omit in the following the superscript (0) in $C_\alpha^{(0)}$.

$C_{\rm T} = {\rm Tr}_{\rm RR}\,(-)^{F_L+F_R}$ is the Witten index [57], whose value gives the Euler characteristic of $K3$, that is +24, independently of the model[6]. The value of $C_{\rm K} = {\rm Tr}_{\rm RR}\,\Omega\,(-)^{F_L+F_R}$[7] can be written for any A-model as

$$C_{\rm K}(\mathbf{Z}_N^A) = -\frac{1}{N}\sum_{k=0}^{N-1} 4\sin^2\frac{2\pi k}{N} + n_{(\frac{N}{2},\frac{N}{2})} \qquad \text{(B.1)}$$

where the first factor is due to the untwisted sector while $n_{(\frac{N}{2},\frac{N}{2})}$ is the contribution of the sector twisted by $\alpha_N^{N/2}$ (when it exists, i.e. for $N \neq 3$) and equals the number of fixed points invariant under α_N present in that sector. The other twisted sectors give a vanishing contribution because of the

[6] From now on it will be understood that the trace is performed on the α_N-invariant states on all the sectors, twisted and untwisted.

[7] Note that due to the world-sheet parity operator Ω, $F_L = F_R$ so that $(-)^{F_L+F_R}$ is completely irrelevant.

world-sheet parity operator Ω. For the B-models $C_{\rm K} = {\rm Tr}_{\rm RR}\,\Omega\,\alpha_N\,(-)^{F_L+F_R}$ and its value is

$$C_{\rm K}(\mathbf{Z}_N^B) = -\frac{2}{N}\sum_{k=1}^{N/2} 4\sin^2\frac{2\pi(2k-1)}{N} + n_{(\frac{N}{2},\frac{N}{2})} \tag{B.2}$$

where $n_{(\frac{N}{2},\frac{N}{2})}$ is again the contribution of the $\alpha_N^{N/2}$-twisted sector, but it now counts the number of fixed points invariant under α_N^2, weighted by their eigenvalues under α_N [8].

Let us now turn our attention to the open string indices $C_{\rm A} = {\rm Tr}_{\rm R}(-)^F$ and $C_{\rm M} = {\rm Tr}_{\rm R}\,\Omega\,(-)^F$, considering separately the 99, 55 and 95+59 sectors. Taking into account the results of [37] for the open massless spectrum, we can write for all the A-models in the 99 sector:

$$\begin{aligned} C_{\rm A}^{99}(\mathbf{Z}_N^A) &= -\frac{1}{N}\sum_{k=0}^{N-1} 4\sin^2\frac{\pi k}{N}({\rm Tr}\gamma_{k,9})^2 \\ C_{\rm M}^{99}(\mathbf{Z}_N^A) &= +\frac{1}{N}\sum_{k=0}^{N-1} 4\sin^2\frac{\pi k}{N}{\rm Tr}(\gamma_{\Omega_k,9}^{-1}\gamma_{\Omega_k,9}^{t}) \end{aligned} \tag{B.3}$$

following the same notation of [37], where the minus sign in $C_{\rm A}$ is due to the fermionic charges of the two spin fields in the Ramond sector. The B-models do not have D9 branes at all. In the 55 sector

$$\begin{aligned} C_{\rm A}^{55}(\mathbf{Z}_N^A) &= -\frac{1}{N}\sum_{k=0}^{N-1} 4\sin^2\frac{\pi k}{N}({\rm Tr}\gamma_{k,5})^2 \\ C_{\rm M}^{55}(\mathbf{Z}_N^A) &= +\frac{1}{N}\sum_{k=0}^{N-1} 4\cos^2\frac{\pi k}{N}{\rm Tr}(\gamma_{\Omega_k,5}^{-1}\gamma_{\Omega_k,5}^{t}) \end{aligned} \tag{B.4}$$

where, since $\Omega\psi_0^{3,4}\Omega^{-1} = -\psi_0^{3,4}$ in the 5-sector, we have $\sin^2 \to \cos^2$ in $C_{\rm M}$. For the $\mathbf{Z}_6^B$ model

$$C_{\rm A}^{55}(\mathbf{Z}_6^B) = -\frac{2}{N}\sum_{k=0}^{2} 4\sin^2\frac{\pi 2k}{N}({\rm Tr}\gamma_{2k,5})^2$$

[8]Remember that in these models there are only the sectors twisted by an even number of α_N's, so that $n_{(\frac{N}{2},\frac{N}{2})} = 0$ for the $\mathbf{Z}_6^B$ model.

Model	$(C_{\rm A}+C_{\rm M})/4$	$C_{\rm K}$
$\mathbf{Z}_2^A$	99: -16 55: -16 95+59: +256	+16
$\mathbf{Z}_3^A$	99: -28	-2
$\mathbf{Z}_4^A$	99: -8 55: -8 59+95: +128	+8
$\mathbf{Z}_6^A$	99: -20 55: -20 59+95: +48	+8
$\mathbf{Z}_4^B$	-	0
$\mathbf{Z}_6^B$	55: -28	-2

Table B.1: *Values of the indices C_α for the various surfaces in the open and closed string sectors.*

$$C_{\rm M}^{55}(\mathbf{Z}_6^B) \quad = \quad +\frac{2}{N}\sum_{k=1}^{3} 4\cos^2\frac{\pi(2k-1)}{N}{\rm Tr}(\gamma_{\Omega_{2k-1},5}^{-1}\gamma_{\Omega_{2k-1},5}^{t}) \qquad \text{(B.5)}$$

Finally, in the 95+59 sector:

$$C_{\rm A}^{95+59}(\mathbf{Z}_N^A) = +\frac{2}{N}\sum_{k=0}^{N-1}({\rm Tr}\gamma_{k,9})({\rm Tr}\gamma_{k,5}) \qquad \text{(B.6)}$$

Given the solution for the matrices γ's representing the orientifold group [37], we can explicitly compute the values of these indices for all the models, that are reported in table B.1.

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