SCUOLA INTERNAZIONALE SUPERIORE DI STUDI AVANZATI

On the Two-Step Moduli Stabilization

by

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Declaration of Authorship

I, Diego GALLEGO, declare that this thesis titled, 'On the Two-Step Moduli Stabilization' and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at SISSA.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
- The original work presented in the thesis is based on the research presented in references [1, 2, 3] done in collaboration with Marco Serone.

Diego M. Gallego M.

Trieste, Settember 2009.

“...[Mais devant cette grosse patte rugueuse, ni l’ignorance ni le savoir n’avaient d’importance: le monde des explications et des raisons n’est pas celui de l’existence. Un cercle n’est pas absurde, il s’explique très bien par la rotation d’un segment de droite autour d’une de ses extrémités. Mais aussi un cercle n’existe pas. Cette racine, au contraire, existait dans la mesure où je ne pouvais pas l’expliquer. Noueuse, inerte, sans nom, elle me fascinait, m’emplissait les yeux, me ramenait sans cesse à sa propre existence. J’avais beau répéter: C’est une racine ça ne prenait plus.]...”

From “La Nausée” by Jean-Paul Sartre

“Fue sin querer queriendo”

El Chavo del Ocho

Abstract

High Energy Physics Sector
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Doctor of Philosophy

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We analyze the circumstances under which part of the information in the full Lagrangian of certain supersymmetric theories can be neglected. More precisely, we study when heavy moduli can be frozen out rather than being properly integrated out, and still get a reliable low energy effective action around nearly supersymmetric solutions.

The procedure, usually known as Two-Step moduli stabilization, is studied for a generic class of $\mathcal{N}=1$ supersymmetric theories in four dimensions, described by a superpotential for the moduli of the form $W=W_0(H)+\epsilon W_1(H,L)$, with $\epsilon \ll 1$, and an arbitrary but regular Kähler potential. We find that the simplified description, where the H superfields are frozen out, is a reliable description as far as their scalar components be solutions of the leading F-flatness conditions. For generic regular Kähler potential, a mass hierarchy of $\mathcal{O}(\epsilon)$ between the two sets of fields is also required. This last condition, in supergravity, is a requirement on the expectation value for the superpotential, $\langle W_0\rangle \sim \mathcal{O}(\epsilon)$. For nearly factorizable Kähler potentials, $K=K_H(H)+K_L(L)+\epsilon K_{mix}(H,L)$, the condition of a mass hierarchy is relaxed and W_0 can take arbitrary values at the vacuum. In presence of matter and gauge interactions the previous conditions continue to hold with a further comment: only neutral chiral multiplets can be frozen. In the case of broken gauge symmetry further states becoming massive through the breaking should be properly integrated out. The higher order terms not reliable in the simple approach, induced by the presence of $\mathcal{O}(1)$ couplings in the matter sector, are also figured out.

Relaying on these results, we use the simplified system to study how two light moduli can be stabilized in a Minkowski/de Sitter vacuum for a wide class of string-inspired supergravity models, with an effective Fayet-like SUSY breaking. It is shown under which conditions this mechanism can be made natural and how it can give rise to an interesting spectrum of soft masses, with a relatively small mass difference between scalar and gaugino masses. In absence of a constant superpotential term, the model becomes completely natural and gives rise to a dynamical explanation of supersymmetry breaking. Some specific type-IIB and Heterotic string inspired models are considered in detail.

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To My Family

Chapter 1

Introduction

Last century have been of great excitement for fundamental physics. This enterprize had its peak with the theoretical establishment, and later experimental test, of the Standard Model of electroweak interactions. Indeed, the level of precision the theory has been tested [4] makes rather difficult not to expect that the Large Hadron Collider (LHC) will find the last piece of the puzzle, the Higgs boson. Still, the Standard Model[1] was from the beginning understood to be an effective description at low energies of some underlying theory. In fact, although the situation was not as clear as in the case of Fermi theory, where one is dealing with a non-renormalizable Lagrangian, and new effects were in any case expected at scales where unitarity breaks down, or where its coupling become non-perturbative, there are some insights telling us that there is still a lot to be understood. Some of them are quite ambitious and motivated mainly theoretically, starting from the explanation for the value for the parameters in the Lagrangian, and numbers like the triplication of families, going to more formal and fundamental questions like the quantization of charge, or an unification of the fundamental interactions, which contain a quantum description of gravity. There are other hints, ambitious as well, more immediate in the sense that deal with the stability of the theory under quantum corrections. More precisely, the corrections to the mass term for the Higgs bosons in the Lagrangian diverge as the cut-off scale squared,

$$m^2 \sim m_0^2 + \frac{1}{16\pi^2}\Lambda_{cut-off}^2 \,. \tag{1.1}$$

Since all the known massive particles acquire masses that are proportional to this parameter, its value should be around the weak scale, $M_W \sim 100 GeV$. However, the

[1] As is commonly used we will call by Standard Model the theory of electroweak interactions together with the theory of strong interactions, Quantum Cromodynamics (QCD). Here we extend even more this definition by allowing masses for the neutrinos, another signature of incompleteness of the Standard Model, since these can be "easily" accommodated without dramatically affecting the original framework.

cut-off scale is naturally expected to be near the Planck scale, $M_{Planck} \sim 10^{19} GeV$, so that seems to be in action a very unnatural cancellation, fine tuned at least in one part out of 10^{28}, between the bare mass and the quantum corrections. Trying to explain the puzzled small ratio M_W/M_{Plank}, known as the "hierarchy problem", is probably the main suggestion of physics beyond the Standard Model in the TeV region, a range of energies we will be able to explore with the LHC, filling the present era of fundamental physics with a lot of expectations an relevant excitement.

Probably the simplest and best candidate to solve this problem is Supersymmetry (SUSY), a symmetry between fermions and bosons. The first consequences of the SUSY algebra are that in each supermultiplet there are as many fermionic degrees of freedom as bosonic ones, with identical masses and quantum numbers. Therefore, the quantum corrections from each sector exactly coincide, and having opposite sings cancel each other. Of course, if indeed SUSY is realized in nature it should be spontaneously broken at a scale M_S so to explain why we have not found any of the predicted superpartners. Once it is broken the degeneracy on the masses is splitted by approximately the scale of SUSY breaking, and the correction to the Higgs bosons mass parameter is roughly[2],

$$\Delta m^2 \sim \frac{1}{16\pi^2} M_S^2 \,. \tag{1.2}$$

Then the scale of SUSY breaking is expected to be in TeV region. A further motivation for SUSY at TeV energies is the unification of the Standard Model couplings at $M_{GUT} \sim 10^{16} GeV$ [5], suggesting a Grand Unified Theory (GUT) scenario [6].

Besides the fact that SUSY is the only extension of the Poincarè algebra consistent with a relativistic quantum theory of fields [7], probably one of the first theoretical appealing facts of SUSY is that, due to the mixings in the algebra with the usual Poincarè generators, once the SUSY transformations are performed locally, i.e. space-time dependent, gravity naturally arises. This SUSY invariant version of gravity, know as Supergravity (SUGRA), brings also the resolutions to some problematic issues present in the global SUSY case. Indeed, the unobserved massless fermion, the golsdtino, product of the spontaneous breaking of global SUSY, in SUGRA is understood as a would-be Goldstone fermion that is eaten up by the gravitino, the graviton superpartner, becoming massive throw a super-Higgs mechanism [7]. Also the fact that the energy density in global SUSY theories is positive definite, vanishing only for SUSY vacua, makes rather difficult to explain the observed almost vanishing cosmological constant of the universe. As we will see in chapter 2, in SUGRA theories the energy density can take negative

[2] This shows why SUSY cannot be an explanation for the smallness of the cosmological constant, the "Big Hierarchy Problem", where the original tuning is far more unnatural.

values and, interestingly enough, appealing vacua whose cosmological constant vanishes in general present SUSY spontaneously broken.

In general SUSY theories share the property of being better behaved under quantum corrections making them very appealing in contexts like quantum gravity. Indeed there are strong indications that enough symmetries, $\mathcal{N} = 8$, render SUGRA a finite theory [8]. Also SUSY Yang-Mills theories compared to the non-SUSY ones are rather easy to study due to the restrictions imposed by the symmetries and the dualities among them, making them a fruitful playground of Toy Models in a first approximation to understand the dynamics of strong coupled systems like QCD (for an introductory review see [9]). In this sense SUSY is not only the best motivated scenarios for physics beyond the Standard Model but also very suggestive from the theoretical point of view and deep understanding of fundamental physics.

Interestingly enough, one of the first papers where SUSY was realized as a new kind of symmetry [10] was inspired by a generalized Lagrangian with fermionic degrees of freedom in the world-sheet of a string, the fundamental object that now seems to explain many of the issues listed above. At that time string theory was born as a possible theory for strong interactions, but with the successful description by QCD this perspective was soon abandoned. Still this very simple idea of regarding strings and not point particles as the fundamental objects, and interpreting its resonance modes as the observed particles, has dramatic consequences when is looked at from another point of view. This was soon after noticed by Michael Green and John Schwarz who pointed out that the massless spin two particle of the string spectrum can be interpreted as the graviton, and string theory might actually be a quantum description of gravity with the extra ingredient of encoding simultaneously the already well understood Yang-Mills theories. However, the theory at its state of art was plagued with inconsistencies like gravitational and gauge anomalies, so very few people consider this idea as a viable one.

It was not until the eighties with the consistency proof of $\mathcal{N} = 1$ ten dimensional SUGRA with gauge groups $SO(32)$ or $E_8 \times E_8$ [11], and the construction of the corresponding underlaying superstring description, the Heterotic string [12], that string theory starts to show up as a strong candidate for a unified quantum description of fundamental interactions. Now we know that only five consistent theories can be constructed, Heterotic (with its two possible gauge groups), type-I, type-IIB and type-IIA. Moreover, all these are nothing but different limits of the same theory, M-theory, a quite appealing result in the aim of a unique theory of everything!

Sadly enough consistency also requires these five theories to live in ten, and not four, dimensions [13]. The good point is that at least the number of space-time dimensions is larger than four, so still we have the possibility to extract a four dimensional ($4D$) theory by compactifying the six extra dimensions. This is not a new idea from string theory, as it was already proposed in the twenties by Kaluza and Klein in a unified description of gravity and electromagnetism from gravity in five dimensions [14]. However, in doing so the acclaimed uniqueness of the theory is completely lost as the Einstein equations have an incredible huge number of solutions turning into probably equal number of different $4D$ theories, which moreover seem to be dynamically equally favored. In any case the situation seems to be very likely to reproduce the Standard Model or at least it minimal SUSY extension (MSSM). One of the early hints pointing suggesting this is so, is the fact the the gauge group of Heterotic strings $E_8 \times E_8$, easily accommodates standard GUT models plus a hidden sector necessary for SUSY breaking. Therefore, the analysis of these compactified theories is far from being pointless.

Since we are interested in physics at rather low energies, with Compton wavelength larger than the fundamental string length and the extra-dimension size[3] we restrict ourselves to study only the SUSY Effective Field Theory (EFT) describing the physics of the states that at first approximation look massless, i.e. a SUGRA theory in $4D$. In this context we now know that out of the many $4D$ solutions only a very small set can realize our world [15]. However, is still rather "easy" to construct models with almost or the exact spectrum[4] and gauge group of the Minimal Supersymmetric Standard Model (MSSM) [16]. On the other hand, it is expected that a theory of everything explains dynamically the way this extra dimensions are compactified, and what one usually finds, instead, is a continuous set of unequivalent vacua parametrized by massless modes (moduli), which characterize, among others, the size and shape of the compact manifold, and the couplings of the $4D$ theory! At the moment there are in the market various ways to generated non-trivial potentials for the moduli uplifting these flat directions, like background fluxes or non-perturbative effects. In the field space what we have then is a very complicated scalar potential with a profile that looks like a huge multidimensional landscape, and whose depressions are the solutions to the equations of motion (e.o.m.). This analogy was first introduced by Susskind [17] describing the moduli space of SUSY vacua one gets by compactifying on a Calabi-Yau (CY) 3-fold, but then was generalize to the full space of solutions coming from the low energy EFT of string theory. Already in the eighties, dealing only with the Heterotic string, it was known that this space was very large, even after imposing phenomenological conditions like chirality [18]. Now, with the developing of D-brane models and flux compactifications [19] the extension of such Landscape has increased tremendously, so to open even more room for the Standard

[3]A small curvature is also required in order to make trustable the first orders in the expansion on the metric.

[4]By spectrum we refer to the particle content and quantum numbers, being at this stage still massless fields.

Model to lie on it, or interesting mechanisms to realize a small cosmological constant [20], but also to make more worrisome matters on the predictive power of the theory.

The strong motivations we have for requiring SUSY breaking at low energies suggest a first approach compactifying on manifolds which preserve SUSY in $4D$, like CY 3-folds, and break it later on by low energy effects. This has the extra advantage of dealing with SUSY theories in $4D$ that are far more restricted and robust than the non-SUSY ones, allowing more control on the mechanisms implemented. Indeed, although preserving SUSY in the compactification process is tightly related to the appearance of the moduli spaces mentioned above, the up-lifting of these degeneracies better to be performed before, or meanwhile, SUSY is broken so to have complete control on the prediction of couplings and masses[5]. Moreover, generally the moduli fields play an important rôle in breaking SUSY and generation of soft masses in the visible sector. This mechanism of moduli mediated SUSY breaking, contrary to other mechanisms, is always present so a precise and complete computation of the soft terms require a controlled stabilization of the moduli.

Cosmological observations also impose non-trivial condition on the way the moduli are stabilized. The first one, is that the vacuum should realize a nearly vanishing cosmological constant [21]. It turns out that although the original moduli space has exactly zero cosmological constant, once non-trivial dynamics are generated the system likely relaxes to a SUSY or nearly SUSY vacuum now with a deep negative cosmological constant. The second one, known as the moduli "problem", points out that decays from light moduli may generate entropy that spoils features like big-bang nucleosynthesis, or if stable affect the relict energy density we observe. This sets a lower bound in the value for the mass of the moduli $m_{mod} \geq \mathcal{O}(10)TeV$ [22]. Stabilizing all moduli with heavy enough masses in a nearly Minkowski space, predicting the value of the observed couplings and a low SUSY breaking is one of the mayor challenges in string phenomenology.

In this context a tremendous progress has been realized during recent years, mostly in type-IIB string theories. There, a combination of fluxes for Ramond–Ramond (RR) tensor field strengths and for the Neveu Schwarz–Neveu Schwarz (NSNS) field strength H has been shown to stabilize the complex structure moduli of the unperturbed compactified space [23]. Subsequently a "scenario" with the Kähler structure moduli stabilized on a Minkowski/de Sitter (dS) vacuum with SUSY breaking has been introduced by Kachru, Kallosh, Linde and Trivedi (KKLT) [24], assuming a complete decoupling between complex structure and Kähler structure moduli. In this scenario all moduli are originally stabilized in a SUSY Anti-de Sitter (AdS) point, the vacuum is then up-lifted

[5]There are some axionic-like direction that can be stabilized even after SUSY breaking by quantum corrections without consequences on the couplings neither on cosmology.

by an explicit SUSY breaking effect due to the introduction of $\overline{D3}$-branes. Although still it is possible to perform some quantitative analysis of the soft-term sector [25], it is nevertheless desirable to have a fully satisfactory spontaneous SUSY breaking mechanism, motivated also by the fact that SUSY is a local symmetry at the SUGRA level and hence an explicit breaking should be avoided.

So far we have spoken about the low energy EFT of string theory in a very hypothetical way. Indeed, the huge number of degrees of freedom (actually infinite) one is dealing with makes the task of formally computing the EFT Lagrangian practically impossible. The EFT approach is still desirable since it provides precisely the necessary information we need at low energies, with tools like the *decoupling theorem* [26], simplifying dramatically the analysis. To get rid of the string and Kaluza–Klein resonances the approach is a consistent truncation eliminating the heavy states using symmetry considerations on a well known $4D$ theory Lagrangian [27], like the one obtained in toroidal compactifications. At first approximation this is fine, since such states are easily detected and sufficiently heavy. Despite such a huge simplification, which leads us to the $4D$ SUGRA description, the theory typically contains hundreds of fields with complicated dynamics, making an explicit full study of these theories a formidable task. Indeed, as already mentioned, the scalar potential admits in general a "landscape" of possible vacua, where even determining the location of the solution in the full set-up might be impossible.

With the developing of flux compactification we have understood how to generate nontrivial dynamics at tree level for some of the moduli. This, being a tree level effect and generated by quantized fluxes, induce a large superpotential and in principle large masses for the moduli involved, compared to the non-perturbative dynamics responsible for the stabilization of the rest of the moduli. So, in the aim of the EFT low energy description we are pursuing, these modes might be integrated out as well. However, the situation seems still so complicated that the common approach is to neglect the dynamics of these modes by simply freezing them at some approximate Vacuum Expectation Value (VEV). Notice that the situation is more delicate than in the case of the string and Kaluza–Klein resonances mainly due to the fact that the moduli are not in general expected to have polynomial potentials neither vanishing VEV, moreover many couplings may depend them. Indeed, is not immediately clear that their VEV decouple from the dynamics of the remaining fields, a first condition to have a reliable description by freezing.

Asking when and up to what extend this approach is valid is a highly crucial question as by now we are only able to solve system under this set-up. After the acclaimed work of KKLT there have been some works addressing this issue mainly restricted to this particular scenario [28, 25, 29, 30, 31, 32]. In particular Choi *et. al.* found in [25] that the large flux induced superpotential not necessarily implies a large mass for the moduli, so that the dynamics of the light fields might be enriched. Among these works there is one in particular claiming that the procedure is in general incorrect, so that a proper integration of the heavy fields leads to completely different conclusions, like the possibility of having a dS vacuum without the introduction on the $\overline{D3}$-branes [29]. This result turns out to be in contradiction with our findings and the ones in independent works as we will comment in the analysis.

Other approach was taken by Achucarro *et. al.* [33, 34, 35], following a proposal by Binetruy *et. al.* in [36], studying SUGRA models which present a rather special property of factorizable Kähler invariant function. This condition, however, seems not to be typical in EFT from string compactifications, therefore are quite restrictive. Motivated by the possibility of having stable Minkowski solutions, independently of having or not properly integrated the heavy fields, Blanco-Pillado *et. al.* constructed a model realizing Minkowski SUSY vacua with stabilized moduli [37], and general conditions for the stability of meta-stable vacua were worked out by Gómez-Reino and Scrucca in [38, 39].

The main aim of the present thesis is to determine the conditions under which a freezing of fields *à la* KKLT is a reliable approximation for a rather generic class of $4D$, $\mathcal{N}=1$, SUGRA theories resembling the flux compactification scenario. Regarding these results we construct an explicit model where stabilization of light moduli is realized in a Minkowski vacuum.

The structure of the report is as follows: chapter 2 is devoted to a quick review of the fundamental tools and concepts we will be dealing with in the rest of the study. This chapter also serve as a further clarification of the SUGRA EFT framework we glanced above; chapter 3 present the results of ref.[2] studying the conditions for global and local SUSY theories under which freezing of moduli fields H is a reliable description in systems with no gauge interactions and without matter fields. The class of theories we study are described by a superpotential of the form,

$$W = W_0(H) + \epsilon W_1(H, L)\,, \tag{1.3}$$

with $\epsilon \ll 1$, and L are the light multiplets described by the EFT after the integration of the H multiplets. We allow a generic Kähler potential, with the only assumption that the eigenvalues of the Kähler metric are parametrically larger than ϵ. We find that for this generic case the H multiplets can be reliable frozen as far as they sit in a approximate SUSY point in the H-directions, and a hierarchy of $\mathcal{O}(\epsilon)$ between the masses of the H and L fields is realized. This last condition in the SUGRA case further translates to a condition on the VEV of the superpotential $\langle W_0 \rangle \sim \mathcal{O}(\epsilon)$. The condition of a hierarchy in the masses is relaxed once one restricts Kähler potentials of the factorizable form $K = K_H(H) + K_L(L)$, as we briefly show in general and with a particular relevant instance of this sort, namely the LARGE volume compactifications [40, 41, 42, 43] arising from flux compactifications of type-IIB superstrings. The procedure first follows the standard way of integrating fields out from the component Lagrangian, and then show how the same result is easily obtained by integrating the complete H supermultiplets simultaneously; chapter 4, presenting the results of ref.[3], generalize the study done in chapter 3 by introducing gauge dynamics an matter fields. The schematic form of the superpotential in this more complicated set-up is taken as follows:

$$W = W_0(H) + Y_N(H, M)C^N + \epsilon\Big[W_1(H, M) + \mu_M(H, M)C^M\Big] , \tag{1.4}$$

where M denotes any kind of light field, charged or neutral, with VEV of $\mathcal{O}(1)$, and the C matter fields with VEV $\mathcal{O}(\epsilon)$. W_0, Y_N, W_1 and μ_M, are arbitrary holomorphic functions, constrained only by gauge invariance, we restrict the study to the case $N \geq 3$ and $M \geq 2$. With the assumption of neutral H fields the conclusions are mainly the same found in the previous case, but with the extra comment of pointing out the order on C, in the superpotential and Kähler potential, where the proper integration of the H fields starts to be relevant. Notice that introducing gauge dynamics we have further $\mathcal{O}(1)$ dynamics coming form the D-term potential. We will see that heavy fields getting mass from these dynamics cannot in general be frozen, being the correct procedure to integrate the full broken gauge sector. This chapter is mainly devoted to the case of generic Kähler potential, where a hierarchy in the masses is compulsory. However, a section is added to the results of ref.[3], giving a glance in the situation of the factorizable case and in particular for the LARGE volume scenarios; chapter 5 presents the results of ref.[1] with a detailed study of a particular instance for the second step stabilization in the Two-Step procedure, stabilizing two moduli in a Minkowski space. For this set-up an important rôle is played by a field depended Fayet-Iliopoulos (FI) term, giving rise to a generalized SUGRA Fayet-like SUSY breaking. This two moduli case, contrary to the single moduli one studied by Dudas *et. al.* in [44], allows to get less suppressed gaugino masses compared to the gravitino one. It opens also, by the addition of extra non-perturbative effects, the possibility to render rather natural the value for the parameters in the Lagrangian. The set-up is general enough to be implemented in type-II and Heterotic constructions; chapter 6 resume the conclusion and outlook possible further directions to explore.

Chapter 2

Wandering on the Landscape

This chapter introduces the main tools, ideas and conventions we use in the following. We define the Lagrangian for a generic EFT and the classical integration of fields and, in SUSY theories, of supermultiplets. The $\mathcal{N} = 1$ SUGRA Lagrangian in $4D$ and its generalities is introduced, being the framework for the rest of the work as an EFT from string compactifications. In this context we introduce the moduli fields and the possible mechanism to stabilize them. We review the appearance of field dependent FI-terms in string compactifications which we will use in the last part of the thesis. Finally the KKLT work [24] is presented in some detail as a benchmark to introduce the Two-Step moduli stabilization procedure.

2.1 Effective field theories

String phenomenology, although inspired by a possible microscopic description of fundamental interactions, is for most of the porpoises and scenarios studying only the low energy remnants of the theory, being the ones we can experimentally test in a controllable way. Indeed, the highest energy scale that we can reach in the forthcoming LHC era is far below a suggested string scale, or even a GUT scale, by several orders of magnitude. Therefore, we find ourselves always dealing with, testing, an EFT, as we have been continuously during the last century, like in the case of Fermi theory of weak interactions.

The guiding idea on writing down an effective Lagrangian for a field theory is the fact that at low energies modes with very large masses cannot be excited, however still their effects should be seen in the parameters of the low energy theory. Being the energy the key point in this *rationale* a generic EFT is represented by a Lagrangian written as an expansion in powers of momentum [45],

$$\mathcal{L} = \sum_i \frac{c_i}{\Lambda^{d_i - 4}} \mathcal{O}_i \,, \tag{2.1}$$

where c_i are dimensionless couplings known as Wilson coefficients, Λ is the cut-off scale and $\mathcal{O}_i$ are operators of dimension d^i of the light fields. Then, for energies well below Λ the infinite sum in (2.1) can be consistently truncated to a finite set of operators so to get a testable theory[1].

The Effective Lagrangian is then reliable only up to the scale Λ where new degrees of freedom are expected[2] and one has two points of view to look at it. In a bottom-up perspective the Wilson coefficients are parameters to be fixed by experimental data, meanwhile in the top-down perspective are given in terms of the microscopic parameters after the effects of energies above Λ has been summed up. Clearly this last one is the one followed in string phenomenology.

When summing up the contributions from the higher energies it is convenient to choose a basis in the degrees of freedom, ϕ^m, such that can be splitted between the heavy ones, ϕ^H, with masses larger than Λ and the light ones, ϕ^L. Then in the path integral formulation is clean the definition of the effective action

$$e^{iS_{Eff}(\phi^L)} \sim \int [d\phi^H] e^{iS(\phi^L,\phi^H)} \,, \tag{2.2}$$

such that the ϕ^H modes are *integrated out*. The integral, as usual, is dominated by the classical solution,

$$\left. \frac{\delta S}{\delta \phi^H} \right|_{\phi_0^H} = 0 \,, \tag{2.3}$$

minimizing the action. Thus on has

$$e^{iS_{Eff}(\phi^L)} \sim e^{iS(\phi^L,\phi_0^H)} \int [d(\delta\phi^H)] e^{i \frac{\delta^2 S}{\delta\phi^H \delta\phi^H}(\phi_0^H)\delta\phi^H \delta\phi^H + \dots} \,, \tag{2.4}$$

where we show only the first quantum correction given by the ϕ^H mass terms. At the classical level the effective Lagrangian is then given by

$$\mathcal{L}_{Eff}(\phi^L) = \mathcal{L}(\phi^L, \phi_0^H(\phi^L)) \,. \tag{2.5}$$

[1]Part of this analysis clearly relies on the assumption that the Wilson coefficients have no anomalous small or large values.

[2]In general the degrees of freedom describing the underlying theory can be completely different to the ones in the effective one, as is the case of QCD with baryon and mesons as relevant degrees of freedom at low energies and quarks for higher ones.

with $\phi_0^H(\phi^L)$ the solution to the classical equation of motion,

$$\frac{\partial \mathcal{L}}{\partial \phi^H} = 0\,. \tag{2.6}$$

In case one is dealing with heavy vector bosons related to a gauge symmetry the analysis is still valid, with the extra ingredient of a gauge fixing to get rid of the redundant degrees of freedom [46].

Being the resulting theory only valid at low energies a further commonly done approximation is to restrict the EFT to the two derivative level, i.e. to require in the kinetic terms of the effective Lagrangian only terms with mass dimension four. In other words restrict to slowly varying space-time solutions. This translates to require independence of the solutions of eq.(2.6) on the derivatives. Then, the full e.o.m. (2.6) reduces to only the one from potential part of the Lagrangian,

$$\frac{\partial V}{\partial \phi^H} = 0\,. \tag{2.7}$$

In practice, the identification of the physical heavy and light fields is typically not straightforward as the Lagrangian is not written in term of the canonically normalized mass eigenstates. It turns out, however, that a precise identification of the physical heavy modes is not necessary as far as the fields that are integrated out be the main components of them. Indeed, the integration of such fields will affect the parameters of the low energy EFT Lagrangian precisely in a way to get the correct physical quantities at low energies. In some cases, as the ones we will be dealing with, from the form of the scalar potential, or its second derivative matrix, it is rather easy to directly identify the main components of the non-canonical normalized heavy directions, say ϕ^H, with $H = 1, \ldots, n_H$. One might argue that this is not enough as the physical states, obtained by the canonical normalization, in general mix the ϕ^H with the previously though light modes. However, as we will see in chapter 3, there is a particular choice for the canonical basis, $\hat{\phi}^m$, where a subset of them $\hat{\phi}^H$, with $H = 1, \ldots, n_H$, do not have components in the non-canonical light directions. This means, that if the metric is regular enough not to introduce further hierarchies, the masses of the $\hat{\phi}^H$ are of the same order of the ϕ^H ones and their mixings with the light physical modes are of the same order as well, therefore, the canonical normalization is irrelevant for the purposes of calculating the low energy EFT for the light physical fields, and in practice can be avoided. Notice, that this consideration, plus the restriction of a two derivative low energy effective Lagrangian, implies that we can consistently forget completely the kinetic terms for the fields and work only a the level of the potential, so that the analysis is considerably simplified.

2.1.1 Effective SUSY theories

At the two derivative level a Lagrangian of the $\mathcal{N} = 1$ SUSY theory of chiral and vector multiplets is completely fixed by: the superpotential, $W(\phi)$, the Kähler potential, $K(\phi, \bar{\phi}, V^A)$, and the gauge kinetic functions, $f_{AB}(\phi)$. The superpotential and gauge kinetic functions are holomorphic functions of the chiral multiplets, ϕ, and the Kähler potential a real function of the chiral and vector multiplets, V^A [3]. Then a SUSY invariant Lagrangian is written as an integral over the Berenzin coordinates θ,

$$\mathcal{L} = \int d^4\theta K + \int d^2\theta \left(W + \frac{1}{4} f_{AB} \mathcal{W}^{\alpha A} \mathcal{W}_\alpha^B \right) + h.c.\,. \tag{2.8}$$

where $\mathcal{W}_\alpha^A$ is the gauge field strength chiral multiplet, $\mathcal{W}_\alpha = -\frac{1}{4}\overline{DD}\left(e^V D_\alpha e^V\right)$, with D_α the SUSY covariant derivative [7]. The e.o.m. in this case can be written in a compact form by taking the variation over full supermultiplets [47]. Writing the D-term density, i.e. the first term in the Lagrangian, as a F-term one using the SUSY covariant derivatives, $\int d^2\bar{\theta} K = -\frac{1}{4}\overline{D}^2 K$, the e.o.m. for a heavy chiral multiplet ϕ^H reads,

$$\partial_H W - \frac{1}{4}\overline{D}^2 \partial_H K + \frac{1}{4}\partial_H f_{AB} \mathcal{W}^{\alpha A} \mathcal{W}_\alpha^B = 0\,. \tag{2.9}$$

As we did in the non-SUSY case we would like to truncate the theory at the two derivative level. This, however, should be done in a consistent way with the SUSY algebra. Indeed, the SUSY transformations mix derivatives with fermion modes and auxiliary fields. It turns out that this consistent truncation in the chiral equation (2.9) reduces to neglect all terms with SUSY covariant derivatives, so to get an approximate chiral e.o.m. [48],

$$\partial_H W = 0\,. \tag{2.10}$$

This will give an EFT exact at leading order in[4] ∂_μ/M_H, $\psi^i/M_H^{3/2}$, $\lambda^i/M_H^{3/2}$, F^i/M_H^2 and D^A/M_H^2, where M_H is the SUSY mass of the heavy chiral multiplets, given by $M_H \sim \partial_H \partial_H W$. Indeed, the solution to the correct full e.o.m. have deviations from the truncated one of $\mathcal{O}(D^2\phi^i/M_H, D^2 V^A/M_H)$, therefore, in order this to be a good approximation, besides the usual low energy condition $\partial_\mu/M_H \ll 1$, the SUSY breaking scale should be well below the mass scale of the heavy states, i.e. $F^i/M_H^2 \ll 1$ and $D^A/M_H^2 \ll 1$ [49]. For a heavy vector multiplet, V^H, a similar analysis shows that at

[3]In global SUSY the superpotential and the Kähler potential are gauge invariant functions, the gauge kinetic function instead can be charged, as we will see later on more carefully.

[4]The presence of the vector multiplet makes the discussion a bit more subtle and this statement is not strictly correct as we will show more precisely in chapter 4. Still the outcome turns out to be the same so we neglect this subtlety here.

the two derivative level the e.o.m. takes the form [50],

$$\partial_H K = 0\,, \tag{2.11}$$

where again consistency requires $F^i/M_H^2 \ll 1$ and $D^A/M_H^2 \ll 1$ with M_H the SUSY mass of the vector multiplet $M_H^2 = \partial_H \partial_H K$ [49].

These constrains on the auxiliary fields are somehow expected, since we are regarding the effective theory as approximately SUSY when relating the truncation of the derivatives with a truncation in the power of the auxiliary fields and fermion bilinears. Moreover, the mass splitting between the components of the heavy multiplets should be smaller than m_H in order to ensure that none of them remain in the low energy spectrum. This, of course, is the case only if the auxiliary fields of the integrated fields are suppressed. Since at leading order, for generic Kähler potentials, the auxiliary fields of the integrated fields are linear combinations of the light field auxiliary fields, as can be shown by solving the approximate chiral equation, this requirements then translate into the conditions found above.

2.2 $\mathcal{N} = 1$ SUGRA in $4D$

As mentioned in the introduction, out of the infinite number of fields one obtains in the quantization of the strings one is interested only in the massless sector. More precisely the string excitation are characterized by a tower of masses whose lowest value is of order the string scale, $M_{St}^2 \sim 1/\alpha'$. Taking the $10D$ Einstein-Hilbert action, one gets by neglecting such states, and performing a naive compactification over the six dimensional compact manifold Y, one finds

$$S = \frac{g_s^{-2}}{(2\pi)^7 \alpha'^4} \int_{\mathcal{M}_4 \times Y} d^{10}X \sqrt{-g_{10}} R_{10} \sim \frac{g_s^{-2} Vol(Y)_s}{\alpha'} \int_{\mathcal{M}_4} d^4x \sqrt{-g_4} R_4\,, \tag{2.12}$$

where by $Vol(Y)_s$ we denote the volume of the compact manifold in string units, $\ell \sim \sqrt{\alpha'}$. We find, therefore, the following relation with the $4D$ Planck scale,

$$M_{St}^2 \sim \frac{g_s^2}{Vol(Y)_s} M_P^2\,. \tag{2.13}$$

Thus, for a moderate string coupling and volume of the compact space, the string scale is expected to be not so far from the Planck scale[5]. Then is justified to integrate such states out to get an effective description of states lighter than M_{St}. Here, however,

[5]There is still an attractive scenario known as LARGE volume compactification [40, 41, 42, 43] where the volume of the compact space is made anomalously large, such that the string scale can reach values of order TeV with possibilities of being tested at LHC [51].

we do not have all the information of the full action of the theory and the precise procedure described in sec.(2.1) is not possible [52]. Still supersymmetry and gauge symmetry are restrictive enough to allow the general construction of the leading terms, for small curvatures and weak coupling, of such EFT. These are the type-I, type-II and Heterotic SUGRA theories in ten dimensions [13]. Notice that we are throwing away very important information in the context of quantum gravity, which is concerned to the full stringy nature of the fields. Therefore, these SUGRA theories should be regarded valid only for energies well bellow the Planck scale, where gravitational quantum effects are irrelevant.

In the way down to four dimensions by compactifying these $10D$ SUGRA theories we find further heavy states, the Kaluza-Klein modes, with masses of order,

$$m_{KK} \sim \frac{M_{St}}{Vol(Y)_s^{1/6}}\,, \tag{2.14}$$

no so far from the string scale and we might integrated them out as well. The problem again is that in general we don't have the full information to perform a proper integration and still if we had the huge number of field make the task rather impossible. As a matter of fact, even to obtain the full theory involving only the massless states is not a trivial task! However, the fact that all these states are heavy is very robust and quite independent of the dynamics of the massless fields, at least for the field region of interest where the theory is not decompactified, so we can tackle the problem from a bottom-up approach and write down an effective action for the massless states in the aim of section (2.1) [52].

Stated in another way, the theory that we write down is one insensible to: the string nature of the fields, the extra dimensions and quantum gravity effects. Our theory, then, is valid at energies well below M_{St}, M_{KK} and M_P. Notice that implicitly this has behind the assumption of small curvature and weak coupling, where our perturbative approach remains valid. Since we are regarding the compactification in a CY manifold, or orbifold with point group a subgroup of $SU(3)$, the theory is supersymmetric, i.e. a SUGRA theory in $4D$. We restrict our attention to the $\mathcal{N} = 1$ case being the one of phenomenological relevance.

A blind bottom-up approach to the effective action is not a very nice way of proceeding for a candidate to fundamental theory, since we are loosing completely the tightness and rigidity of string theory by allowing arbitrary unknown couplings. Fortunately we can do better in first approximation by arguing a consistent truncation, where the heavy fields are not properly integrated out but simply set to zero, of a well known $4D$ action like the one obtained from toroidal compactification. The resulting Lagrangian, then, is a proper description of the theory for the light fields where the couplings are known from first principles [27] (see also refs.[53] for generalizations and a closer look at possible issues in the procedure.). The theory still is expected to have corrections suppressed by M_{St}, M_{KK} and M_P, which by an abuse of notation we will denote in the following by the same scale M_P.

Being now clear what is going to be the framework on which we work, i.e. $\mathcal{N} = 1$ SUGRA in $4D$, let us explore some of its general features.

2.2.1 Only chiral multiplets

Let us start by considering the case without gauge symmetries, i.e. no vector supermultiplets. Then, as in the global case, at the two derivative level, the theory is completely determined by two functions of the chiral multiplets: the holomorphic superpotential, $W(\phi)$, and the Kähler potential, $K(\phi, \bar{\phi})$. The simplest way to derive the expression in components of the Lagrangian is using the superconformal formalism, extending the $\mathcal{N} = 1$ SUSY algebra by adding the superconformal generators [54]. The system is then extremely constrained by the symmetries that the Lagrangian is easily determined. The procedure introduces new degrees of freedom collected in the conformal gravity multiplet with components, besides the graviton and gravitino, two vector auxiliary fields. Also a compensator chiral multiplet, Φ, is added. Fixing the scalar and spinor components of the compensator and one of the vector auxiliary fields one recovers the symmetries of ordinary SUGRA [55]. The Lagrangian, then, can be written in a way very much resembling the one in global SUSY as an integration over rigid super coordinates,

$$\mathcal{L} = \int d\theta^4 \left(-3e^{-K/3}\Phi\bar{\Phi}\right) + \int d\theta^2 W\Phi^3 + h.c.\,. \tag{2.15}$$

The Berenzin integrals, however, are now deformed by extra-terms with dependencies on the components of the gravity multiplet [54, 56]. At the component level we will mainly interested in the scalar part, where these deformations are not relevant, so the calculation is straightforward and easily written as in the global case. With conventions for the metric signature $(+,-,-,-,-)$ and for the chiral fields $\phi = (\phi, \psi, -F^{\phi})$ [6], then it takes the form

$$\mathcal{L} = K_{i\bar{j}}\partial^{\mu}\phi^i\partial_{\mu}\bar{\phi}^{\bar{j}} - V\,, \tag{2.16}$$

where we regarded the scalar component of the compensator, Φ_0, as a non-dynamical field choosing the gauge $\Phi_0 = M_P e^{K/6}$, with $M_P = M_{Planck}/\sqrt{8\pi} \sim 2.4 \times 10^{-18} GeV$ the reduced Planck mass, so to obtain the canonical Einstein term in the action [55]. The

[6] For simplicity of notation, here and throughout the thesis, we use the same notation for the chiral multiplets and its lowest components, being clear from the context to which one we are referring to.

labels i and j, then, run over the remaining chiral fields which now on are expressed in M_P units. In terms of a re-scaled auxiliary field for the compensator $F^\Phi = M_P e^{K/6} U$, the scalar potential has the following form,

$$\begin{aligned} V &= -\partial_i K F^i \bar{U} - \partial_{\bar{j}} K \overline{F}^{\bar{j}} U - \left(\partial_{i\bar{j}} K - \frac{1}{3M_P^2} \partial_i K \partial_{\bar{j}} K \right) F^i \overline{F}^{\bar{j}} + 3M_P^2 U \bar{U} \\ &\quad + e^{\frac{K}{2M_P^2}} \left(F^i \partial_i W + 3UW + h.c. \right) . \end{aligned} \tag{2.17}$$

Eliminating U,

$$U = \frac{1}{M_P^2} \left(\frac{1}{3} \partial_i K F^i - e^{\frac{K}{2M_P^2}} \overline{W} \right) , \tag{2.18}$$

one has for the auxiliary fields the following expression,

$$\overline{F}^{\bar{i}} = e^{\frac{K}{2M_P^2}} K^{\bar{i}j} \left(\partial_j W + \frac{1}{M_P^2} \partial_j K \, W \right) , \tag{2.19}$$

where the matrix $K^{\bar{i}j}$ is the inverse of the metric $\partial_i \partial_{\bar{j}} K$. The scalar potential for the physical fields then takes the form,

$$V = e^{\frac{K}{M_P^2}} \left(K^{\bar{i}j} D_i W \overline{D}_{\bar{j}} \overline{W} - \frac{3|W|^2}{M_P^2} \right) , \tag{2.20}$$

where we have defined the SUGRA covariant derivative operator $D_i \equiv \partial_i + \partial_i K / M_P^2$. The first thing one notices from the scalar potential is that contrary to the global SUSY case the vacuum energy can be negative. In particular, for SUSY vacua, i.e. $F^i \sim K^{i\bar{j}} D_{\bar{j}} \overline{W} = 0$ [7], the cosmological constant is $\langle V \rangle = -3 \langle e^{\frac{K}{M_P^2}} |W|^2 / M_P^2 \rangle$. It can be shown that the mass term for the gravitino is given by

$$m_{3/2} = e^{\frac{K}{2M_P^2}} |W| / M_P^2 , \tag{2.21}$$

therefore in a SUSY vacuum $\langle V \rangle = -3 m_{3/2}^2 M_P^2$. For phenomenology we are interested in non-SUSY vacua, so we may require $\langle F^i \rangle \sim M_S^2 \neq 0$, with M_S the scale of SUSY breaking, and $\langle V \rangle \sim M_S^4 - 3 m_{3/2}^2 M_P^2$. Notice that in case $M_S^2 \ll m_{3/2} M_P$ still there is no hope to up-lift the cosmological constant, $\langle V \rangle \gtrsim -3 m_{3/2}^2 M_P^2$. Therefore in order to match, or at least get hopefully closer, to the observed vacuum energy we should find solutions such that the expectation value for the classical scalar potential (2.20) vanish, at least up to an accuracy of $\mathcal{O}(M_S^4 / 16\pi^2)$ where quantum corrections start to be relevant and some further considerations are needed to explain the smallness of the cosmological constant.

[7] Like in the global case the breaking of SUSY is parametrized by the VEV of the auxiliary field, as can be understood from the transformation rule for the spinor components, $\delta_\xi \psi \sim -\xi F$.

Before introducing the vector fields let us point out some further properties of the SUGRA Lagrangian. Notice that the Lagrangian as is written in eq.(2.15) is invariant under the following transformations:

$$K \to K + \kappa + \bar{\kappa} \ , \quad W \to e^{-\kappa} W \ , \quad \Phi \to e^{\kappa/3} \Phi \, , \tag{2.22}$$

where κ is a holomorphic function of the chiral fields. Since Φ in not a physical degree of freedom this Kähler invariance is not a true symmetry of the theory, however, it states that the theory is determined by one function only, rather than two as stated above. This function is the invariant combination of the Kähler potential and the superpotential,

$$G = K + M_P^2 \ln \left(\frac{|W|^2}{M_P^6} \right) \, , \tag{2.23}$$

known as generalized Kähler potential or Kähler invariant function. Indeed, whenever, the superpotential is not zero is possible to work in this Kähler gauge, that turns to be useful in many cases, so to write the SUGRA Lagrangian as,

$$\mathcal{L} = \int d\theta^4 \left(-3 e^{-G/3} \Phi \bar{\Phi} \right) + \int d\theta^2 \Phi^3 + h.c. \, . \tag{2.24}$$

The scalar potential then reads,

$$V = -\partial_i G F^i \bar{U} - \partial_{\bar{j}} G \overline{F}^{\bar{j}} U - \left(\partial_i \partial_{\bar{j}} G - \frac{1}{3M_P^2} \partial_i G \partial_{\bar{j}} G \right) F^i F^{\bar{j}} + 3M_P^2 U \bar{U} + 3M_P^3 e^{\frac{G}{2M_P^2}} (U + \bar{U}) \, , \tag{2.25}$$

where the gauge fixing $\Phi_0 = M_P e^{G/2}$, $F^\Phi = M_P e^{G/2} U$ has been implemented in order to get a canonical normalized Einstein-Hilbert action [55]. Notice that if we solve for the auxiliary fields, the expressions are different from the one computed above eqs.(2.18) and (2.19),

$$U = \frac{1}{3M_P^2} \partial_i G F^i - M_P e^{\frac{G}{2M_P^2}} \, , \tag{2.26}$$

$$F^i = M_P e^{\frac{G}{2M_P^2}} G^{i\bar{j}} \partial_{\bar{j}} G \, . \tag{2.27}$$

This is in fact expected since the previous formulation was not invariant under the Kähler transformations, but then, since the VEV of the auxiliary fields are the order parameter of SUSY breaking one may worry about this point. However, due to the fact that physical quantities are given only through Kähler invariant combinations, at the end this mismatch turns out to be non-physical. For example in order to have a correct guess of the SUSY breaking scale and the order of the soft-terms one should use the canonical normalized F-terms, $F^i = |K_{i\bar{i}} F^i \overline{F}^{\bar{i}}|^{1/2}$ no sum over the indices, which exactly coincides in both approaches. The case of anomaly mediation is more delicate due to the fact that has to do with the compensator and gravity multiplets. It turns out that only in the first gauge fixing we choose the vector auxiliary components of the gravity multiplet are completely gauged away, so that the anomaly mediation has only to do with the auxiliary field U. In the second Kähler gauge, instead, one has to be careful of on keeping track on the contribution from the gravity multiplet [8]. In this sense the first gauge is usually preferred.

As a proof of this equivalence between the two descriptions one can show that the scalar potential, which now takes the form,

$$V = M_P^2 e^{\frac{G}{M_P^2}} \left(G^{i\bar{j}} \partial_i G \partial_{\bar{j}} G - 3M_P^2 \right) , \tag{2.28}$$

matches with the one given in eq.(2.20).

2.2.2 Vector multiplets

Vector boson are introduced in the usual way by gauging symmetries, now understood as isometries of the scalar field manifold. This manifold is Kähler with metric $\partial_i \partial_{\bar{j}} G$, then the isometries of the metric consistent with SUSY are generated by holomorphic analytic Killing vectors,

$$X_A = X_A^i \frac{\partial}{\partial \phi^i} \;, \quad \overline{X}_A = \overline{X}_A^{\bar{i}} \frac{\partial}{\partial \bar{\phi}^{\bar{i}}} \;, \tag{2.29}$$

with A running over the dimensions of the isometry group $\mathcal{G}$. A symmetry transformation with infinitesimal real constant parameter λ^A is implemented by the operator $\delta = \lambda^A (X_A + \overline{X}_A)$, then a chiral multiplet transforms as $\delta \phi^i = \lambda^A X_A^i (\phi^j)$. The invariance of the theory under such a transformation translates into the invariance of the generalized Kähler potential,

$$\delta G = X_A^i \partial_i G + \overline{X}_A^{\bar{i}} \partial_{\bar{i}} G = 0 \,. \tag{2.30}$$

Gauging this isometry introduces V^A vector supermultiplets and promotes the transformation parameter to a chiral multiplets $\lambda^A \to \Lambda^A$ so that the transformation operator is now given by $\delta = \Lambda^A X_A + \bar{\Lambda}^A \overline{X}_A - i(\Lambda^A - \bar{\Lambda}^A)\partial_A$, and the invariance condition $\delta G = 0$ reads now,

$$\partial_A G = -i X_A^i \partial_i G = i \overline{X}_A^{\bar{i}} \partial_{\bar{i}} G \,. \tag{2.31}$$

We need also the gauge kinetic function $f_{AB}(\phi^i)$ which is a holomorphic function of the chiral multiplets. It can be charged, transforming non-linearly with charge Q, $\delta f_{AB} =$

[8] I would like to thank K. W. Choi for pointing out this to me.

$i\Lambda^C Q_{CAB}$, and its charge is related to the cancellation of anomalies, as we will see for a particular case in the next section. Then besides the terms we wrote in eq.(2.15), where a dependency on V^A is understood in the Kähler potential, we have a further term with exactly the same form of the global SUSY case,

$$\mathcal{L}_{gau-kin} = \frac{1}{4}\int d\theta^2 f_{AB}(\phi^i)\mathcal{W}^{A\alpha}\mathcal{W}^B_\alpha + h.c.\,, \tag{2.32}$$

where $\mathcal{W}^A_\alpha$ is the gauge fields strength chiral multiplet, $\mathcal{W}_\alpha = -\frac{1}{4}\overline{D}\bar{D}\left(e^V D_\alpha e^V\right)$, with D_α the SUSY covariant derivative, and again the F-term density is deformed by terms dependent on the gravity multiplet. For the scalar potential the situation now is very much like in the global SUSY case, with the only difference that in the global case the Kähler potential and the superpotential are invariant each one apart, here, instead, is their combination in G. The new contribution to the scalar potential in the Wess-Zumino gauge can be written as [57]

$$V_D = \frac{Re(f^{AB})}{2} D_A D_B\,, \tag{2.33}$$

with f^{AB} the inverse of the gauge kinetic function, and

$$D_A = -\partial_A G = iX^i_A\partial_i G = -i\overline{X}^{\bar{i}}_A\partial_{\bar{i}}G\,. \tag{2.34}$$

The mass term for the vector boson is found by expanding the kinetic term for the chiral fields which now are in terms of gauge covariant derivatives, $\mathcal{D}_\mu\phi^i = \partial_\mu\phi^i + iX^i_A V^A_\mu$,

$$\partial_i\partial_{\bar{j}}G\,\mathcal{D}_\mu\phi\mathcal{D}^{\dagger\,\mu}\bar{\phi}^{\bar{j}} \sim \partial_i\partial_{\bar{j}}G\,X^i_A\overline{X}^{\bar{j}}_B V^{\mu A}V^B_\mu\,. \tag{2.35}$$

Thus for the non-canonical normalized vector fields the mass matrix is given by

$$M^2_{AB} = 2\partial_i\partial_{\bar{j}}G\,X^i_A\overline{X}^{\bar{j}}_B\,. \tag{2.36}$$

2.2.3 SUGRA effective theories

In the same spirit we did for the global SUSY we would like to have some e.o.m. consistent with SUGRA at the two derivative level. There have been some confusion in the literature trying to address the integration of full SUSY multiplets in SUGRA theories, this in part due to the complexity on writing the Lagrangian in a close form while adding further degrees of freedom. Indeed, the lowest component we get from the chiral equation we got in the global case, eq.(2.10), is the F-flatness condition for the heavy field direction, so naively one is tempted to generalize the e.o.m. to the one whose lowest component be the F-flatness condition in SUGRA, from eq.(2.19) this is,

$$\partial_H W + \frac{1}{M_P^2}\partial_H K W = 0\,. \tag{2.37}$$

However, notice that this equation cannot be promoted to the supermultiplet level since it is a non-chiral equation, and when trying to solve for the components of the chiral multiplet one finds more equations than variables. At the scalar component level still eq.(2.37) might be the correct one in certain cases as we will see later in chapter 3, but for generic cases the correct chiral e.o.m. consistent for a SUGRA EFT at the two derivative level was recently worked out in [49]. Let us roughly go through the arguments in [49] for the case there are no vector multiplets since is here that the real issue is.

Starting from the Lagrangian (2.15) we can proceed as we did in the global SUSY case[9] by writing the D-density par as a F one, so the exact chiral equation takes the form,

$$\partial_H W - \frac{1}{4}\overline{D}^2(\partial_H K e^{-K/3}\bar{\Phi})\Phi^{-2} = 0\,. \tag{2.38}$$

A consistent SUSY truncation at the two derivative level of the EFT again reduces to neglect the covariant derivative part, and nicely enough the approximate chiral equation exactly coincides with the one of the global SUSY case (2.10). Again the EFT extracted from the solution of the approximate chiral equation is exact only at leading order in the derivatives, fermion bilinear and auxiliary fields. Here, moreover, the auxiliary field of the compensator multiplet has to be taken into account, and the theory is exact only at leading order in U/M_H. From the on-shell expression for U, eq.(2.18), using the condition $F^i/M_H \ll 1$ one finds a constrain in the superpotential $e^{K/2M_P}W/(M_P^2 M_H) \ll 1$. All these conditions again translates to the requirement that the SUSY breaking scale be well below the mass scale of the heavy multiplets. The introduction of the vector multiplets and their integration does not brings further constrains and different e.o.m. than the ones found in section (2.1.1) for the global SUSY case.

2.3 Moduli stabilization

After the easy job of getting rid of the heavy unnecessary fields by truncating the theory obtained through the compactification, one faces the moment to get the hands dirty trying to get some phenomenological significance of all the remaining stuff. In fact, contrary to the ten dimensional case where we have only five possibilities the theory here is quite far from being unique, and is here that the string phenomenology headache

[9] The terms in the F and D densities dependent on the gravity components are uniquely determined so no relevant for the discussion.

begins. In particular among the huge number of effective $4D$ theories we know that only a very small set can be candidate to be extension of the Standard Model [15]. The exclusion criteria are an inter-related set of phenomenological conditions to be realized in the predicted particle physics and cosmology. Tackling each of these subjects apart is already a hard task, and we are going to concentrate mainly in realizing the following items:

1. Low energy SUSY breaking.
2. No exotic light particles.
3. Experimental values for the couplings.
4. Cosmological issues but in particular realizing an almost vanishing cosmological constant.

Interestingly enough all these are faces of the same token, namely the stabilization of moduli fields. Moduli fields parametrize deformations of the compactification manifold which, at tree level, do not change the $4D$ effective energy and therefore correspond to massless scalars in four dimensions [58]. Being string theory such a rigid theory all couplings in four dimensions are given by VEV's of field operators, therefore, an explanation for their values reduce to the one for the VEV for the moduli.

It is clear moreover that moduli, coupled only through gravitational interactions to the visible fields, are obvious candidates for a hidden sector of SUSY breaking. Therefore, the VEV of their auxiliary fields might play an important rôle in the realization of the MSSM and extensions. This scenario, known as SUGRA or Moduli Mediation, contrary to other mechanism of mediation, like Gauge Mediation, is always present. As we saw in section (2.2), in SUGRA theories breaking of SUSY is a compulsory condition to reproduce the observed almost vanishing value for the cosmological constant [21] while getting stabilized moduli with a non-very small gravitino mass.

Depending on the particular details of the compactification the set of moduli is different. Still some of them, the closed string moduli, are rather independent of the set-up and are in general present:

1. The dilaton $s \sim e^{-\phi}$, which is related to the string coupling $g_s^2 \sim e^{\phi}$.
2. Metric moduli: these corresponds to the fields that parametrize the space of CY manifolds. In other words, variations on the metric that leave the CY manifold properties invariant, in particular the Ricci flatness condition,

$$R_{mn}(g + \delta g) = 0 \,, \tag{2.39}$$

with two independent solutions:

(a) Complex Structure moduli, u, related to the pure holomorphic deformations $\delta g = \delta g_{mn}$ are in one-to-one correspondence with the $(2,1)$-forms on the CY. These characterize to the shape of the compact manifold.

(b) Kähler moduli, t, related to the mixed deformations $\delta g = \delta g_{m\bar{n}}$ are in one-to-one correspondence with the $(1,1)$-forms on the CY. These characterize the size of the compact manifold.

An easy way to understand the massless nature of some of these moduli is by realizing the presence of further moduli with axion-like behavior, coming from the compactification of p-form potentials C_p. Indeed, the gauge transformation, $C_p \to C_p + dA_{p-1}$, remains in four dimension reflected as shifts symmetries for these zero modes and in particular forbids mass terms for them. Other consequence of such a symmetry is that these fields cannot appear in the superpotential. One can show that the dilaton and Kähler moduli are always superpartners of such axionic fields, therefore, by holomorphicity of the superpotential these cannot appear neither. A striking consequence of this result is the fact that the superpotential does not suffer from perturbative corrections, since the dilaton and Kähler moduli parametrize the string and world sheet perturbative expansions [59, 60].

At this stage what one finds is a moduli space of SUSY vacua which, besides of being from the beginning not of phenomenological interest, have not explained yet a dynamical compactification of the extra dimensions. Indeed, the size of the compact manifold can be equally infinity. That these vacua be SUSY is not the only phenomenological problem. Moduli fields couple to matter through gravitational interactions, therefore if massless can mediate long range forces that clearly are not observed. But even in case these be massive possible decays may generate entropy that spoils features like big-bang nucleosynthesis, or if stable affect the relict energy density we observe. Evading this issues, known as the "moduli problem", translates into a lower bound on their masses $m_{mod} \geq \mathcal{O}(10)TeV$ [22].

Given its importance, generating non-trivial dynamics for the moduli, and finding appealing solutions for the potential generated, is one of the most intense areas of research in string phenomenology. The early attempts in generating moduli potentials where mostly concentrated in understanding possible non-perturbative dynamics, such as gaugino condensation and world sheet instantons, which break the sifts symmetries mentioned above [61] and then are potential sources of moduli dependency in the potential [62, 63, 64]. Furthermore, being suppressed can naturally solve the hierarchy problem dynamically as first proposed by Witten in ref.[65]. These lead to SUGRA models, so-called racetrack models [66], where the combination of various non-perturbative sectors compete realizing the stabilization of some set of the bulk moduli and simultaneously give a dynamical explanation for a low SUSY breaking scale [63, 67]. In the case of gaugino condensation the moduli dependent superpotential is understood from the fact that moduli control the gauge couplings, g^2_{YM}, or in general the gauge kinetic function (see eq.(2.32)). Therefore, if the hidden sector contains asymptotically free theories, as is usually the case, a gaugino condensate can be realized, $\langle\lambda\lambda\rangle = \Lambda^3_{np}$, at the corresponding non perturbative scale $\Lambda_{np} \sim M_P \exp\left(-\frac{8\pi^2}{b_0 g^2_{YM}}\right)$ scale, with b_0 the beta-function coefficient, $\beta(g_{YM}) = -b_0 g^3_{YM}/16\pi^2$, and g_{YM} the gauge coupling at M_P scale. This generates an effective superpotential of the form [68, 69, 48],

$$W_{ADS} \sim \Lambda^3_{np} \sim e^{-\frac{24\pi^2 f_{YM}(\phi^i)}{b_0}}\,, \tag{2.40}$$

where we used holomorphicity of the superpotential and the relation $g^{-2}_{YM} = Re(f)$, with f the holomorphic kinetic function. A similar result would come from field theory instantons. Stringy instantons are generated by the wrapping of stringy objects around topological non-trivial cycles of the internal space. For example a string world-sheet can wrap around a 2-cycle Γ, leading to a instanton correction to the action of the form [70],

$$W_{inst} \sim e^{-\frac{\mathcal{A}(\Gamma)}{2\pi\alpha'}}\,, \tag{2.41}$$

with $\mathcal{A}$ the area of the wrapped cycle which depends on the metric moduli. p-branes wrapping p-cycles lead to a similar effect [71].

During the last decade the developing of compactification with background fluxes, flux compactifications [19], leads a considerable progress in the understanding and generation of moduli potentials. In this case it is the energy of the fluxes threading the cycles of the manifold what gives rise to a tree level moduli dynamics. From a $4D$ point of view the fluxes induce a superpotential that for the type-IIB string compactifications takes the form [72],

$$W_{flux} = \int_Y \Omega_3 \wedge G_3\,, \tag{2.42}$$

where Ω_3 is the holomorphic $(3,0)$-form on the compact manifold[10], Y, and $G_3 = F_3 + iS\,H_3$ the combined three-flux, with F_3 and H_3 the RR and NSNS field strength and S the dilaton chiral multiplet. In this way for type-IIB one can stabilize all complex structure, present inside Ω_3, and the dilaton moduli. Notice that in this case the use of "moduli stabilization" is a bit misleading since in fact fields which acquire masses

[10]The back reaction due to turning-on the fluxes is such that the compact manifold is not longer a CY. The most stable case is type-IIB compactifications, where in adiabatic approximation the manifold continues to be a CY manifold up to a warping [23].

trough this mechanism are not moduli in the proper sense being its potential generated at tree level. Still, one refers in this terms to this way of proceed since in general the introduction of the fluxes is performed adiabatically around an unperturbed moduli space of a CY, where much more control is possible.

Notice that the flux dynamics are naturally large. Indeed, due to the Dirac quantization the fluxes can only take discrete values. A first consequence of this is that provides large masses to the corresponding stabilized moduli which naively may be comparable with the Kaluza–Klein scale, being both an effect of the compactification. However, the precise scale for the flux induced masses turns out to be,

$$m_{flux}^2 \sim e^{K/M_P^2} |W_{flux}|^2 / M_P^2 \sim \frac{M_P^2}{Vol(Y)_s^2}\,, \tag{2.43}$$

where we have used a known result $K \sim -2\ln\left(Vol(Y)_s\right)$. This should be compared with $m_{KK}^2 \sim M_P^2/Vol(Y)_s^{4/3}$, where we have used eq.(2.13) and eq.(2.14). Therefore, for rather large volume of the compact space $m_{flux} \ll m_{KK}$ and our SUGRA EFT approach is still justified.

Still, the fact that the flux induced superpotential be of order M_P^3 is a worrisome point for phenomenology, since in general will induce a SUSY breaking at M_P scale, unless a fine tuning is implemented. We will expand further on this point in the forthcoming chapters.

2.4 Fayet-Iliopoulos terms in string theory

On the last part of the thesis we will work an explicit set up where moduli fields are stabilized in a nearly Minkowski space with the help of the a D-term potential with a field dependent FI-term. In this section we show how these kind of contribution appear in general, and then in two explicit frameworks, Heterotic and type-IIB Orientifold compactifications.

Let us first point out that the invariance condition $\delta G = 0$ allows scenarios where the superpotential is not invariant, i.e., $iX_A^i \partial_i W = \xi_A W$, with ξ_A a real constant. This R-symmetry when gauged generates a D-term of the form

$$D_A = iX^i \partial_i K + M_P^2 \xi_A\,, \tag{2.44}$$

where a FI-term, ξ_A, has been induced. Recently, however, it has been found that such terms are inconsistent with SUGRA theories [73], since it requires the theory to be invariant under an exact global symmetry, therefore, here on we forget this option. Still, however, there is the possibility to have field dependent terms that for some purposes look like genuine constant FI-terms.

As already mentioned some of the moduli present shift symmetries. Denoting by S such modulus the symmetry acts like,

$$S \to S + i\delta \,, \tag{2.45}$$

with delta a constant real parameter, i.e. S is realized in a non-linear representation of the isometry group with $X^S_A = i\delta$. This symmetry implies that the chiral multiplet can appear only through the combination $S + \bar{S}$, then Kähler potential is necessarily of the form,

$$K(S, \bar{S}) = K(S + \bar{S}) \,, \tag{2.46}$$

and the superpotential is S-independent. If we now gauge the symmetry as shown in the section (2.2.2) gauge invariance of the Kähler potential implies then

$$K(S, \bar{S}, V) = K(S + \bar{S} + \delta\, V) \,, \tag{2.47}$$

and the D-term generated has the following form,

$$D = iX^j_A \partial_j K + iX^S_A \partial_S K = iX^j_A \partial_j K - \delta\, K' \,, \tag{2.48}$$

where j runs over the chiral fields but S and the prime stands for derivatives under S. Notice, then, that the term generated by the charged S behaves like a FI-term, field dependent though, for the remaining fields. Is important to stress that being field dependent the dynamics involved are far richer and can lead to very different results. In particular these ones lead inevitably to a spontaneous breaking of the gauge symmetry, as can be easily seen from the mass term for the vector field (2.36),

$$M^2_V \sim 2K''\delta^2 \,. \tag{2.49}$$

Further important consequences that does not allow to treat them as genuine FI will be addressed in chapter 4.

Let us see more precisely how these terms appear in the specific examples of Heterotic and type-IIB Orientifold compactifications with magnetized D-branes.

2.4.1 Heterotic string

A naive calculation of the triangular anomalies in Heterotic compactifications shows a non-vanishing result. However, being string theory a self-consistent theory these should be somehow cancelled. The immediate candidate to do the job is the imaginary component of dilaton multiplet defined by

$$S = s + i\sigma\,, \tag{2.50}$$

where s is related to the string coupling, $s = g_s^{-2} = e^{-\phi}$, and σ is defined from the zero mode, $B_{\mu\nu}$, of the B_2 2-form, through a $4D$ duality transformation,

$$H_{\lambda\mu\nu}\epsilon^{\lambda\mu\nu\rho} = e^{\phi}\partial^{\rho}\sigma\,. \tag{2.51}$$

Indeed, the gauge kinetic function at three level is given by the dilaton $f = S$, and a coupling $\sigma F\tilde{F}$ is realized. A coupling $\sigma R\tilde{R}$ is realized as well, so a shift in σ seems to do the job. In order to see that indeed is the shift symmetry what has been gauged and that this is the way the anomaly is cancelled, let us compactify the Green-Schwarz term of the ten dimensional action [11], $B \wedge F \wedge F \wedge F$, taking two of the fields strengths, F_2, to lie completely in the internal space [74], then a term of the form

$$B \wedge F \sim A^{\mu}\partial_{\mu}\sigma\,, \tag{2.52}$$

comes out, showing the covariantization of the kinetic term for σ, $\mathcal{L} \sim (\partial_\mu\sigma + \delta A_\mu)^2$, as becoming charged, then a generalized Green-Schwarz mechanism has been implemented.

Nicely enough a shift in σ if enough for cancelling all the possible anomalies as the anomaly coefficients for all of them exactly coincide [75, 76],

$$\delta = \frac{M_P}{192\pi^2} TrQ\,, \tag{2.53}$$

where Q refers to the charge operator of the pseudo-anomalous $U(1)$ gauge symmetry. At the end we have a theory with a Kähler potential for the dilaton S [27],

$$K = -M_P^2 \ln\left(\frac{S + \bar{S} + \delta V}{M_P}\right) \tag{2.54}$$

inducing a field dependent FI

$$D \sim \frac{\delta}{S + \bar{S}}\,. \tag{2.55}$$

Notice that what we are finding is a loop correcting to the Kähler potential. Indeed, although the superpotential can be shown to be exact at tree level in perturbation theory there is nothing to prevent the Kähler potential to get both string and world sheet perturbative corrections.

2.4.2 $D3/D7$ type-IIB orientifolds

In this case the relevant field is the Kähler modulus, $T^i = t^i + i\tau^i$, with t^i defined as the Einstein frame volume of a 4-cycle Σ^i, and

$$\tau^i = \int_{\Sigma^i} C_4 \,, \tag{2.56}$$

the component of the RR 4-form, C_4, along Σ^i. For a $D7$-brane wrapping a 4-cycle Σ^i the Dirac-Born-Infeld action in the string frame is[11],

$$S_{DBI} = -\mu_7 \int_{\mathcal{M}_4\times\Sigma^i} d^8\xi e^{-\phi}\sqrt{det(\iota^* g + 2\pi\alpha' F)} \sim \int_{\mathcal{M}_4\times\Sigma^i} d^8\xi\sqrt{-g}e^{-\phi}F_{\mu\nu}F^{\mu\nu} \,, \tag{2.57}$$

where μ_7 is the brane tension and $\iota^* g$ denote the pullback of the ten dimensional metric on $\mathcal{M}_4 \times \Sigma^i$. The last expression shows the gauge kinetic term obtained by expanding the DBI-action, and where one identifies the real part of the modulus T_i, i.e. the volume of the 4-cycle, as the gauge couplings. Indeed, its imaginary part has the necessary coupling of the gauge kinetic function found from the Chern-Simons part of the action,

$$S_{CS} = -\mu_7 \int_{\mathcal{M}_4\times\Sigma^i} \sum_p \iota^* C_p \wedge e^{2\pi\alpha' F} \sim \int_{\mathcal{M}_4\times\Sigma^i} C \wedge F \wedge F \,. \tag{2.58}$$

In order to see how the modulus gets charged let us expand the 4-from as $C_4 = C_2^i \wedge \omega_i + \ldots$ and extract from the CS action the following term

$$S_{CS} \sim Q_i^\sigma \int_{\mathcal{M}_4} C_2^i \wedge F \sim Q_i^\sigma A^\mu \partial_\mu \tau_i \,, \tag{2.59}$$

where $Q_i^\sigma \equiv \int_{\Sigma^i} \iota^*\omega_i \wedge F$ is then the charge of τ, and we have used the fact that C_2^i is Hodge dual to τ_i. Notice that is not immediate that there is a non vanishing charge. Indeed in order Q_i^σ not to vanish Σ^i should have self intersections over a 2-cycle on which the world-volume flux, F, is non-trivial [77].

For the Kähler moduli the Kähler potential does not have in general a simple explicit expression as was the case of the dilaton, however, it can be written at tree level as

$$K(T + \bar{T}) = -2M_P^2 \ln(\mathcal{V}) \,, \tag{2.60}$$

with $\mathcal{V}$ the volume of the CY which can be viewed as a implicit function of the Kähler moduli. The in the D-term a field dependent FI-term is induced of the form,

$$D \sim Q_i^\sigma \frac{\partial_{T_i}\mathcal{V}}{\mathcal{V}} \,. \tag{2.61}$$

[11] Here and in the following we omit terms involving the NSNS B_2 field, being irrelevant for our discussion.

2.5 The KKLT scenario

The discussion at the end of section 2.3 shows that there are enough mechanism in order to stabilize all closed string moduli. This was first exploited in the context of type-IIB compactifications by Kachru, Kallosh, Linde and Trivedi in [24], by combining tree-level flux induced dynamics with non-perturbative ones. Their approach, known sometimes as "Two-Step Stabilization", starts by observing that from the flux dynamics the complex structure and dilaton are fixed with a generic mass of order M_P. Therefore, one can analyze the stabilization of the Kähler moduli using the EFT where these heavy fields have been integrated out. However, they do not properly integrated out the heavy modes as explained in section (2.1), but regards, instead, these moduli as simply frozen out at some SUSY point of the flux superpotential, so that the resulting effective description for a single Kähler modulus was given by,

$$W = W_0 + A(S_0, U_0)e^{-aT/M_P} \,. \tag{2.62}$$

Where S_0 and U_0 are the frozen values for the heavy moduli, $W_0 = W_{flux}(S_0, U_0)$, and the last part is the non-perturbative part whose coefficients may depend on the heavy moduli, with a a parameter of order one[12]. The Kähler potential also has a constant term plus the one dependent on the Kähler moduli,

$$K = K_{SC}(S_0, U_0) - 3M_P^2 \ln\left(\frac{T + \bar{T}}{M_P}\right) \,. \tag{2.63}$$

A factorization of this sort is only a tree-level result, as both α' and g_s corrections may affect this form, however, one may argue these effects to be irrelevant for some range on the values of the moduli, like large volume, controlling the perturbative expansion. This system has a SUSY vacuum dictated by

$$D_T W = \partial_T W + \frac{1}{M_P^2}\partial_T K W = 0, \tag{2.64}$$

leading to the following equation,

$$-aAe^{-aT/M_P} - \frac{3M_P}{(T + \bar{T})}(W_0 + Ae^{-aT/M_P}) = 0 \,. \tag{2.65}$$

The solutions stabilizes the Kähler moduli at $T \sim \frac{M_P}{a} \ln\left(\frac{W_0}{A}\right)$, with canonical normalized mass $m_T^2 \sim e^{K/M_P}(T + \bar{T})|\partial_T \partial_T W|^2 \sim \frac{W_0^2}{M_P^2} \ln\left(\frac{W_0}{A}\right)$. The space is then an AdS with

[12] More precisely a depends on the range of the gauge group generating the non-perturbative dynamics. However this is irrelevant for the present discussion and we simply regard $N \sim 2 \div 10$.

cosmological constant

$$V = -3e^{\frac{K}{M_P^2}}\frac{|W|^2}{M_P^2} \sim -\frac{3M_P}{(T+\bar{T})^3}|W_0|^2 \,. \tag{2.66}$$

In order to have control in the perturbative correction mentioned above, one has to realize rather large values for T. Since A is expected to be of $\mathcal{O}(M_P^3)$, then W_0 should be tuned to a small value, something a bit weird since W_{flux} is expected to be also of $\mathcal{O}(M_P^3)$. Notice that the solution being a competing effect between W_0 and the non-perturbative superpotential leads to $Ae^{-aT/M_P} \sim W_0$. Since W_{np} is the first order term from a series of non-perturbative effects further suppressed by e^{-aT/M_P}, better it to be very small in order to trust this contribution alone, so we end again in requiring a tuning for a small W_0. We will see in chapter 3 that this condition is also necessary for the freezing of the complex and dilaton fields to be a reliable approach. In the original paper KKLT assume a value of $W_0 \sim 10^{-5}$ arguing the possibility of tuning by the presence of several fluxes.

In order to break SUSY and at the same time uplift the vacuum KKLT propose the introduction of $\overline{D3}$-branes living in the bottom of a warped throat. The potential generated by the $\overline{D}3$-branes has the form,

$$V_{\overline{D3}} \sim \frac{a}{(T+\bar{T})^2} \,, \tag{2.67}$$

with a a constant dependent of the string coupling, the $\overline{D3}$-brane tension and the warp factor. Since the warping generates a red-shift it is possible to fine tune the cosmological constant to lie near Minkowski, i.e. $a \sim |W_0|/T$ parametrically, without destroying the vacuum. Such explicit breaking of SUSY due to $\overline{D3}$-brane can be argued to be the effect of non-linearized SUSY after an spontaneous breaking in the $\overline{D3}$-brane, i.e. a goldstino living in the $\overline{D3}$-brane [78], so that some quantitative studies of interesting physical quantities, such as soft parameters [79], can be performed. However, still it is not clear this be the case so this turns out to be the weakest point in the KKLT construction. After the work of [24], indeed, many works have appeared where the explicit SUSY breaking is replaced by spontaneous F or D-term breaking of different kinds [80, 81, 38, 30, 82, 83, 44, 33, 84, 1].

One of the first proposals to spontaneously break SUSY started by noticing that the D-term contribution to the scalar potential (2.33), contrary to the F-term part, is positive definite, so one may wonder if it can be used to get a vanishing cosmological constant from an original SUSY vacuum [85]. In particular the FI-term we computed in section (2.4.2), with the holomorphic gauge kinetic function $f \sim T$, generates a scalar potential of the form,

$$V_D \sim \frac{(Q^\sigma)^2}{(T+\bar{T})^3}\,, \tag{2.68}$$

that in principle could play the same role of the $\overline{D}3$-brane in the up-lifting but know realized as a D-term breaking.

Unfortunately, working with the definition of G eq.(2.23), the expression for D_A eq.(2.34), one finds,

$$D_A = iX^i_A \frac{D_i W}{W}\,. \tag{2.69}$$

Thus contrary to the flat case the D-term SUSY breaking is linked to the F-term one, i.e. one cannot get pure D-term breaking, as soon after pointed out by Choi *et. al.* in [79], and this scenario cannot be realized. We will see, furthermore, that although it seems that still the D-term contribution to the scalar potential, being naturally of order M_P^4, can easily overcome the F-term part, dynamically the VEV for the D-term is suppressed and at the vacuum its almost negligible. This shows how difficult in general is to realize stabilized heavy moduli in a nearly Minkowski space.

One of the strongest assumptions in the KKLT approach, and all the following realizations, is that the complex structure and dilaton decoupled effectively from the Kähler moduli dynamics, so that a naive freezing of their VEV was enough to account a proper integration of these modes. In the following two chapters we will explore in full detail this procedure in a rather generic class of SUGRA theories, and workout the condition under which this is a reliable approximation.

Chapter 3

Two-Step Moduli Stabilization: pure Moduli Case

At the end of last chapter we saw the explicit instance of the KKLT scenario where all moduli where stabilized. These results strongly relay on the assumption that the stabilization can be performed in two steps, stabilizing the complex structure and dilaton moduli first using the flux induced dynamics, and then, in a second step stabilize the Kähler moduli using non-perturbative effects in a sort of EFT where the dilaton and complex structure are frozen rather than integrated out. The approach is at first glance consistent given the strengths of the dynamics involved. Indeed, the non-perturbative dynamics are expected to be suppressed compared with the flux induced ones. But already at this point the approach is not completely save since strong dynamics not always imply a large inertia, i.e. masses, as was pointed out in [25] by Choi *et. al.*, and some direction may still remain flat. Going beyond this first observation things start to be less clear as the proper integration of the fields seems complicated enough to generate a proper EFT completely different to the simple one obtained by freezing the fields to some fixed value. As a matter of fact there are not only criticisms and warnings on the procedure [28, 86, 87] but also explicit claims that the proper procedure might change completely the dynamics of the light fields [29]. Still, due to the huge multiplicity of fields this Two-Step stabilization, using the simplified EFT, seems to be the only practical way we have to tackle semirealistic string compactifications. In this chapter we carefully explore this issue, finding the conditions for the reliability of the procedure, in the case where there are only moduli fields.

The study, although rather general, is restricted to a class of $4D$ $\mathcal{N}=1$ SUSY theories that reflects the flux compactification scenarios, described by a superpotential of the form[1],

$$W(H,L) = W_0(H) + \epsilon\, W_1(H,L)\,, \tag{3.1}$$

with $\epsilon \ll 1$. The Kähler potential, however, is left arbitrary with the only assumption that the eigenvalues of the associated Kähler metric are parametrically larger than ϵ. We regard the H fields as stabilized by W_0 with an $\mathcal{O}(1)$ non-canonical normalized mass. As we will see, the assumption we do on the metric allows to unequivocally identify the canonical normalized heavy fields with the H fields, justifying our schematic notation of H for the heavy and L for the light fields.

We proceed in an expansion in ϵ, first using the orthodox approach dealing with ordinary fields, restricted to the scalar components, and then using the compact way of integrating full chiral multiplets simultaneously, finding in both approaches agreement in concluding that the simplified version is a reliable description at leading order in ϵ, as far as the scalar components of the H multiplets satisfy the leading F-flatness conditions. In global SUSY this can be immediately promoted to a chiral equation $\partial_H W_0 = 0$ to be satisfied by the fixed chiral fields H. For SUGRA theories two cases are realized: i) for generic Kähler potentials the results for the global case continue to hold with a further restriction on the VEV of the superpotential, namely $W_0 \sim \mathcal{O}(\epsilon)$. These, can be also translated as the conditions of an $\mathcal{O}(\epsilon)$ mass hierarchy between the two sets of fields; ii) for special Kähler potentials realizing and approximate factorization, i.e. $K = K_H(H)+K_L(L)+\mathcal{O}(\epsilon)$, W_0 can arbitrary values at the vacuum and the leading F-flatness conditions, to be satisfied by the scalar componets H_0, are given by $\partial_H W_0 + \partial_H K_H W_0 = 0$.

Notice that is only in the first case that one can speak about integrating out heavy fields. Indeed, not only the gravitino but all fields get SUGRA contributions to their masses that are of order $e^{K/2}|W|$, so only in this case the remaining L will have a hierarchy in the masses of $\mathcal{O}(\epsilon)$.

Interestingly enough, in both cases what is found is that the contribution for SUSY breaking from the frozen field is always suppressed by ϵ compared with the ones of the L-fields. This is an important property no only for the reliability of the simple model but also for the fact we are regarding the simple EFT of the L fields as a SUSY theory.

3.1 Non-Supersymmetric σ-Model

Before entering into the analysis of the case we are interested in, let us study a non-SUSY bosonic σ-model whose results being general provide some simplification in the analysis of the SUSY case.

[1]From here on we work for simplicity in natural units, setting $M_P = 1$.

Let us consider a system of n_H+n_L interacting real scalar fields H^i and L^α, $i=1,\dots,n_H$, $\alpha=1,\dots,n_L$, with Lagrangian density

$$\mathcal{L}=\frac{1}{2}g_{MN}(\phi^M)\partial\phi^M\partial\phi^N-V(\phi^M)\,, \tag{3.2}$$

the general form of potential is chosen such that naturally presents a hierarchy in the masses and therefore a reliable freezing is expected,

$$V(\phi^M)=V_0(H^i)+\epsilon\,V_1(H^i,L^\alpha)\,, \tag{3.3}$$

where $M=1,\dots,n_H,1+n_H,\dots,n_H+n_L$, $\phi^M=(H^i,L^\alpha)$ and $\epsilon\ll 1$. Writing explicitly ϵ we parametrize the magnitude of the second part of the potential. Indeed the analysis not necessarily implies a small coupling between the H and L, but rather that we explore the system in a field region where the second part of the potential and all its derivatives are suppressed compared to the ones of first one. The splitting between the fields H^i and L^α is dictated by V_0, namely we call H^i (the "heavy fields") the ones appearing in V_0, in this way we can define more precise the small parameter as the ratio of masses squared $\epsilon\simeq\frac{m_L^2}{m_H^2}$.

We assume that at a given vacuum $\langle\phi^M\rangle$, the metric g_{MN} is non-singular and $\partial_i\partial_jV_0$ is positive definite with all eigenvalues parametrically larger than ϵ. Under these assumptions, at leading order in ϵ, the simple effective low-energy Lagrangian associated to $\mathcal{L}$ is

$$\mathcal{L}_{sim}=\frac{1}{2}g_{\alpha\beta}(L^\alpha,H_0^i)\partial L^\alpha\partial L^\beta-\Big[V_0(H_0^i)+\epsilon\,V_1(H_0^i,L^\alpha)\Big]\,, \tag{3.4}$$

where H_0^i are the leading order VEV's for H^i, independent of L^α, satisfying $\partial_iV_0(H_0^j)=0$. We want to show that $\mathcal{L}_{sim}$ provides the correct effective Lagrangian for arbitrary kinetic mixing terms at leading order in an expansion in ϵ.

Let us look, in an expansion in ϵ, $\phi^M=\phi_0^M+\epsilon\phi_1^M\dots$, for (space-time independent) vacua of $\mathcal{L}$ by studying the extrema of the potential V in the original, non-canonically normalized, field basis given by H^i and L^α. The leading order solutions, H_0^i, for the H^i are determined at $\mathcal{O}(\epsilon^0)$ from the equations $\partial_iV=\partial_iV_0=0$. At this order, L^α are undetermined, since $\partial_\alpha V_0$ trivially vanishes. At $\mathcal{O}(\epsilon)$, we get the leading VEV's L_0^α for the light fields from $\partial_\alpha V_1(H_0^i)=0$ and the first corrections H_1^i to the VEV's of the heavy fields from the linear equations $\partial_i\partial_jV_0(H_0^k)H_1^j+\partial_iV_1(H_0^k,L_0^\alpha)=0$.

With the approximate vacuum (H_0^i,L_0^α) determined, we can diagonalize the metric in order to identify the canonically normalized field fluctuations $\hat\phi_c$ starting from the field fluctuations $\hat\phi=\phi-\langle\phi\rangle$ [2]. At leading order the matrix to diagonalize is $g_{MN,0}=$

[2] We thank A. Romanino for essentially providing us the argument that follows.

$g_{MN}(H_0^i, L_0^\alpha)$ and whose eigenvalues we assume to be parametrically larger than ϵ. Nicely enough, a positive definite real symmetric matrix can always be written as the product of a lower triangular matrix times its transpose[3]. We then write $g_0 = (T^{-1})^t T^{-1}$, so that $\hat{\phi} = T\hat{\phi}_c$, where

$$T = \begin{pmatrix} (T_H)^i_j & 0 \\ (T_{HL})^\alpha_i & (T_L)^\alpha_\beta \end{pmatrix} \tag{3.5}$$

and $\hat{\phi} = (\hat{H}^i, \hat{L}^\alpha)^t$. In the canonical basis, the Lagrangian (3.2) reads

$$\begin{aligned}\mathcal{L} = \quad & \left[\frac{1}{2} + \mathcal{O}(\epsilon)\right]\left[(\partial \hat{H}^i_c)^2 + (\partial \hat{L}^\alpha_c)^2\right] + \dots \\ & + \Big[V_0(H^i_0 + \epsilon H^i_1 + (T_H\hat{H}_c)^i) + \epsilon V_1(H^i_0 + \epsilon H^i_1 + (T_H\hat{H}_c)^i, L^\alpha_0 + (T\hat{\phi}_c)^\alpha) + \mathcal{O}(\epsilon^2)\Big],\end{aligned} \tag{3.6}$$

where the ellipsis in eq.(3.7) stands for all the higher order terms arising from the expansion of the metric in quantum fluctuations, the specific form of which are not needed.

The next step would be to diagonalize the mass matrix of the heavy fields, but it will not be explicitly needed. Indeed, we see from the term V_0 in eq.(3.7) that the fields $\hat{H}^i_c$ have all a leading mass term of $\mathcal{O}(\epsilon^0)$, the $n_H \times n_H$ mass matrix being of the form $M^2 = T_H^t M_0^2 T_H$, where $(M_0^2)_{ij} = \partial_i \partial_j V_0(H_0^i)$. Since by assumption M_0^2 is positive definite, so it is M^2. Due to the form of the potential in eq.(3.7), integrating out the fluctuations $\hat{H}^i_c$ at quadratic level will only affect the effective theory at $\mathcal{O}(\epsilon^2)$, so we can simply set $\hat{H}^i_c = 0$ if we want a reliable Lagrangian up to $\mathcal{O}(\epsilon)$. We can go back to non-canonically normalized fields $\hat{L}^\alpha = (T_L^{-1}\hat{L}_c)^\alpha$. Since $T_L^{-1} = (T^{-1})^\alpha_\beta$, the kinetic mixing matrix reads now $(T_L^{-1})^t (T_L)^{-1} = g_{\alpha\beta,0}$ and all the terms in the ellipsis in eq.(3.7), when $\hat{H}^i_c = \hat{H}^i = 0$, simply reproduce the full field-dependent metric $g_{\alpha\beta}(L^\alpha, H_0^i)$ appearing in eq.(3.4). The resulting full effective Lagrangian becomes

$$\mathcal{L}_{full} = \left[\frac{1}{2} g_{\alpha\beta}(L^\alpha, H^i_0) + \mathcal{O}(\epsilon)\right] \partial L^\alpha \partial L^\beta - \Big[V_0(H^i_0 + \epsilon H^i_1) + \epsilon V_1(H^i_0, L^\alpha) + \mathcal{O}(\epsilon^2)\Big]$$

and hence, modulo irrelevant constant terms, at leading order in ϵ we get the desired result

$$\mathcal{L}_{sim} = \mathcal{L}_{full}\,. \tag{3.7}$$

An important lesson we learn from this exercise, is that, thanks to the Cholesky decomposition to diagonalize the metric, the heavy mass eigenstates are uniquely determined by the potential independently of the kinetic mixing. Indeed, the canonical basis we construct is such that the canonical heavy states are only combinations of the H^i modes

[3] This procedure, called Cholesky decomposition, is unique if we require the diagonal entries of T to be strictly positive. See e.g. [88].

and being the eigenvalues of the metric of $\mathcal{O}(1)$ the coefficients in the linear combination for the canonical fields $\hat{L}^i$ cannot induce large enhancements so to generated large masses. Therefore, for matters of the analysis we are interested in, we can forget about the kinetic terms, and canonical normalization of the fields, and work with the original modes as they appear in the potential.

3.2 Global SUSY σ-Model

The results found in the previous section, being of general validity, apply to SUSY theories as well. However, the supersymmetric structure of the theory allows us to introduce the suppression parameter in a sightly different way, namely directly in the superpotential leading to a different definition of it in term of the masses $\epsilon \simeq \frac{m_L}{m_H}$, as we will see soon. As mentioned in the introduction we study a SUSY theory specified by a Kähler potential $K(\phi, \bar{\phi})$, taken to be generic but regular, and a superpotential, taken as in eq.(3.1):

$$W(H^i, L^\alpha) = W_0(H^i) + \epsilon W_1(H^i, L^\alpha)\,, \tag{3.8}$$

using the same conventions as before, but considering that now H^i and L^α are complex (super)fields.

3.2.1 Component approach

Let us first treat this SUSY theory as an ordinary one by forgetting the relation about the multiplet components and work at the level of the scalar potential. This approach being the common one for any field theory may be clearly clean as procedure, however, as we will see later on, keeping in mind the supermultiplet nature of the fields allows a more straightforward way of getting the same results.

Once the auxiliary fields are integrated out the scalar potential of the theory is given by

$$V = g^{\bar{M}N}\overline{F}_{\bar{M}}F_N\,, \tag{3.9}$$

with $F_M = \partial_M W$ and $g^{\bar{M}N}$ the inverse matrix of $g_{M\bar{N}} = \partial_M \partial_{\bar{N}} K$. The potential V is a sum of three terms when expanded in ϵ: $V = V_0 + \epsilon V_1 + \epsilon^2 V_2$, with

$$\begin{aligned}
V_0 &= g^{\bar{j}i}\overline{F}_{\bar{j},0}F_{i,0}\,,\\
V_1 &= g^{\bar{j}i}\overline{F}_{\bar{j},1}F_{i,0} + g^{\bar{\alpha}i}\overline{F}_{\bar{\alpha},1}F_{i,0} + c.c.\,,\\
V_2 &= g^{\bar{j}i}\overline{F}_{\bar{j},1}F_{i,1} + g^{\bar{\beta}\alpha}\overline{F}_{\bar{\beta},1}F_{\alpha,1} + (g^{\bar{\alpha}i}\overline{F}_{\bar{\alpha},1}F_{i,1} + c.c.)\,,
\end{aligned} \tag{3.10}$$

where $F_M = F_{M,0} + \epsilon F_{M,1}$ and

$$\begin{aligned} F_{i,0} &= \partial_i W_0\,, \qquad F_{\alpha,0} = 0\,, \\ F_{i,1} &= \partial_i W_1\,, \qquad F_{\alpha,1} = \partial_\alpha W_1\,. \end{aligned} \tag{3.11}$$

We assume that at the vacuum the metric $g_{M\bar{N}}$ is positive definite and that $\partial_i\partial_j W_0$ is non-degenerate with eigenvalues parametrically larger than ϵ. We now establish that if the heavy fields sit at a SUSY vacuum at leading order in ϵ, the bosonic low energy effective theory of the light fields L^α is described by the simple SUSY effective theory with $K_{sim}(L^\alpha, \bar{L}^\alpha) = K(H_0^i, L^\alpha, \bar{H}_0^i, \bar{L}^\alpha)$, $W_{sim} = W_0(H_0^i) + \epsilon W_1(H_0^i, L^\alpha)$:

$$V_{sim} = \tilde{g}^{\bar{\alpha}\alpha}\tilde{F}_\alpha \overline{\tilde{F}}_{\bar{\alpha}}, \tag{3.12}$$

with $\tilde{g}_{\alpha\bar{\alpha}} = \partial_\alpha\partial_{\bar{\alpha}} K_{sim}$ and $\tilde{F}_\alpha = \partial_\alpha W_{sim}$.

Like in the non-SUSY case we start by finding the vacuum in an expansion in ϵ:

$$\langle \phi^M \rangle = \phi_0^M + \epsilon\phi_1^M + \epsilon^2\phi_2^M + \dots\,. \tag{3.13}$$

The equations of motion (e.o.m.) up to $\mathcal{O}(\epsilon^2)$ read

$$(\partial_M V)_0 = \partial_M V_0 = 0\,, \tag{3.14}$$

$$(\partial_M V)_1 = \partial_M V_1 + (\partial_M\partial_N V_0)\phi_1^N + (\partial_M\partial_{\bar{N}} V_0)\bar{\phi}_1^{\bar{N}} = 0\,, \tag{3.15}$$

$$\begin{aligned} (\partial_M V)_2 = {}& \partial_M V_2 + (\partial_M\partial_N V_1)\phi_1^N + (\partial_M\partial_{\bar{N}} V_1)\bar{\phi}_1^{\bar{N}} \\ &+ (\partial_M\partial_N V_0)\phi_2^N + (\partial_M\partial_{\bar{N}} V_0)\bar{\phi}_2^{\bar{N}} \\ &+ \frac{1}{2}(\partial_M\partial_N\partial_P V_0)\phi_1^N\phi_1^P + \frac{1}{2}(\partial_M\partial_{\bar{N}}\partial_{\bar{P}} V_0)\bar{\phi}_1^{\bar{N}}\bar{\phi}_1^{\bar{P}} + (\partial_M\partial_N\partial_{\bar{P}} V_0)\phi_1^N\phi_1^{\bar{P}} = 0, \end{aligned} \tag{3.16}$$

where all quantities in eqs.(3.14), (3.15) and (3.16) are evaluated at $\phi^M = \phi_0^M$. At $\mathcal{O}(\epsilon^0)$, the equations $(\partial_k V)_0 = 0$ can generally admit both SUSY and non-SUSY solutions. Interestingly enough only the first ones admit a decoupling between the H and the L sectors. Indeed, the SUSY solutions,

$$F_{i,0}(H_0^i) = \overline{F}_{\bar{i},0}(\bar{H}_0^i) = 0\,. \tag{3.17}$$

are completely independent of the L^α and furthermore the e.o.m. $(\partial_\alpha V)_0 = 0$ are identically satisfied when $H^i = H_0^i$, so that L^α are not determined at this order. The non-SUSY solutions instead, in general do not satisfy none of these two conditions due to the presence of the inverse metric in V_0 mixing both sectors[4].

[4]Notice that decoupling for the non-SUSY solutions would be obtained in case the Kähler potential satisfies a nearly factorizable condition, $K = K_H(H) + K_L(L) + \epsilon K_{mix}(H, L)$, which effectively leads us to exactly the non-SUSY σ-model we studied in section (3.1).

Nicely enough, around the SUSY solutions the e.o.m. of the light fields at $\mathcal{O}(\epsilon)$, $(\partial_\alpha V)_1 = 0$, are automatically satisfied as well, since

$$\partial_\alpha \partial_M V_0 = \partial_\alpha \partial_{\bar{M}} V_0 = \partial_\alpha V_1 = 0\,, \tag{3.18}$$

when evaluated at $H = H_0^i$, as can easily be checked using the explicit expressions in eq.(3.10). The displacement of the heavy field VEV's at $\mathcal{O}(\epsilon)$ is calculated by taking $M = j$ in eq.(3.15). We get

$$H_1^i = -(K^{-1})^i_{\bar{j}} \overline{F}_1^{\bar{j}}\,, \tag{3.19}$$

with $K_j^{\bar{i}} = g^{\bar{i}k}\partial_k\partial_j W_0$ and $F_1^i = \overline{F}_{\bar{M},1} g^{\bar{M}i}$, evaluated again at $H^i = H_0^i$. At this point it is clear that for consistency of our procedure that all eigenvalues of the matrix $\partial_i\partial_j W_0$ should be of $\mathcal{O}(1)$, in other words all H^i fields should acquire large masses from W_0.

At $\mathcal{O}(\epsilon^2)$ we finally get non-trivial e.o.m. for L^α as well as the 2^{nd} order displacement of the heavy fields H_2^i, whose explicit form will not be needed. Both H_2^i and L_0^α arise at $\mathcal{O}(\epsilon^2)$, but the e.o.m. of L^α do not depend on H_2^i, as can be seen from eqs.(3.16) and (3.18). By plugging eq.(3.19) in eq.(3.16) and after some algebra, we could establish that the e.o.m. that determine L_0^α in the full theory are the same as the one obtained in the simple theory where H^i are frozen to their leading values H_0^i. The best and more instructive way to proceed, however, is by finding the leading power in ϵ of the F terms and their derivatives, evaluated at the shifted vacuum $H_0^i + \epsilon H_1^i$. We get

$$\begin{aligned} F_i &= \mathcal{O}(\epsilon), \qquad F_\alpha = \mathcal{O}(\epsilon), \qquad F^i = \mathcal{O}(\epsilon^2), \qquad F^\alpha = \mathcal{O}(\epsilon)\,, \\ \partial_j F_i &= \mathcal{O}(1), \quad\; \partial_i F_\alpha = \mathcal{O}(\epsilon), \quad \partial_\beta F_i = \mathcal{O}(\epsilon), \quad \partial_\beta F_\alpha = \mathcal{O}(\epsilon)\,. \end{aligned} \tag{3.20}$$

Interestingly enough, although in general the backreaction of the light fields on the heavy ones induce F_i–terms of $\mathcal{O}(\epsilon)$, the upper components F^i, directly related to the VEV of the auxiliary fields, vanish at this order since

$$\overline{F}^{\bar{i}}(H_0^i + \epsilon H_1^i) = \left[g^{\bar{i}M} F_{M,1}(H_0^i) + g^{\bar{i}j}\partial_k F_{j,0} H_1^k\right]\epsilon + O(\epsilon^2) \tag{3.21}$$

and the first two terms in eq.(3.21) exactly cancel, due to eq.(3.19). This implies that at linear order the F_i and F_α terms are related as follows:

$$F_i = -\tilde{g}_{i\bar{j}} g^{\bar{j}\alpha} F_\alpha + \mathcal{O}(\epsilon^2)\,, \tag{3.22}$$

with $\tilde{g}_{i\bar{j}}$ the inverse metric of $g^{\bar{j}i}$, not to be confused with $g_{i\bar{j}}$. Using eq.(3.21) and the relation (3.22), after some straightforward algebra one finds the desired identification

$$\partial_\alpha V = \partial_\alpha V_{sim} + \mathcal{O}(\epsilon^3), \tag{3.23}$$

which implies that the location of the vacuum is reliably computed in the simple theory. In order to establish eq.(3.23), it is very useful to use the matrix identity

$$\tilde{g}^{\bar{\alpha}\alpha} = g^{\bar{\alpha}\alpha} - g^{\bar{\alpha}i}\tilde{g}_{i\bar{j}}g^{\bar{j}\alpha}\,, \tag{3.24}$$

where $\tilde{g}^{\bar{\alpha}\alpha}$ is the inverse of $g_{\alpha\bar{\alpha}}$, appearing in V_{sim}, not to be confused with $g^{\bar{\alpha}\alpha}$. Thanks to eq.(3.24), in particular, it is easy to show that $F^\alpha = \tilde{F}^\alpha$ at leading order in ϵ.

The leading order equivalence of the naive low-energy effective theory with the full one proceeds along the same lines of the general non-SUSY case discussed before, now using the generalization of the Cholesky decomposition for hermitian matrices to diagonalize the metric $g_0 = (T^{-1})^\dagger T^{-1}$. However, as we learnt in section 3.1 this procedure does not introduce any important information given our assumption on the metric, therefore we can proceed in terms of the non-canonical basis which also allows more clarity in the procedure.

In order to integrate out the H^i modes we solve, in an expansion up to the gaussian level, the e.o.m. for the fluctuations,

$$V(H,L) = V(\langle H\rangle, L) + \partial_I V(\langle H\rangle, L)\hat{H}^I + \frac{1}{2}\partial_I\partial_J V(\langle H\rangle, L)\hat{H}^I\hat{H}^J + \mathcal{O}(H^3)\,, \tag{3.25}$$

where for simplicity we have collected the holomorphic and antiholomorphic indices in a single notation, $I = i, \bar{i}$. The solution for $\hat{H}^I$ reads then,

$$\hat{H}^I = -V^{IJ}V_J\,, \tag{3.26}$$

where $V_I = \partial_I V$, and V^{IJ} is the inverse matrix of $V_{IJ} = \partial_I\partial_J V$. Therefore, the full effective scalar potential, reads

$$V_{full} = V(\langle H\rangle, L) + V_{int}(\langle H\rangle, L), \tag{3.27}$$

where

$$V = F^M F_M\,, \tag{3.28}$$

is the microscopical scalar potential, and

$$V_{int} = -\frac{1}{2}V_I V^{IJ} V_J|_{H=\langle H\rangle}\,, \tag{3.29}$$

is the potential term induced by a gaussian integration of the heavy moduli. Corrections to eq.(3.29), due to cubic or higher terms in the heavy field fluctuations are negligible.

Given the form of eq.(3.29), the knowledge of the potential at $\mathcal{O}(\epsilon^2)$ requires to compute the $\mathcal{O}(\epsilon)$ terms in $\partial_I V$ and the $\mathcal{O}(1)$ in V^{IJ}, the latter arising from the inverse of $V_{i\bar{j}}$.

One gets then

$$\begin{aligned} \partial_i\partial_{\bar{j}}V|_0 &= \partial_{\bar{j}}\overline{F}_{\bar{k},0}\, g^{\bar{k}l}\, \partial_i F_{l,0}\,, && (3.30)\\ \partial_i V|_1 &= \overline{F}_{\bar{M}} g^{\bar{M}l} \partial_i F_{l,0}\,, && (3.31)\end{aligned}$$

where $F_{i,0} = \partial_i W_0$. Using eqs.(3.30) and (3.31), we find

$$V_{int} = -\overline{F}_{\bar{M}} g^{\bar{M}i} \tilde{g}_{i\bar{j}} g^{\bar{j}N} F_N\,, \tag{3.32}$$

where $\tilde{g}_{i\bar{j}}$ is the inverse of $g^{\bar{j}i}$, not to be confused with $g_{i\bar{j}}$. The full effective scalar potential reads then

$$\begin{aligned} V_{full} &= g^{\bar{M}N}\overline{F}_{\bar{M}}F_N - \overline{F}_{\bar{M}} g^{\bar{M}i}\tilde{g}_{i\bar{j}} g^{\bar{j}N}F_N + \mathcal{O}(\epsilon^3)\,,\\ &= \epsilon^2 \tilde{g}^{\bar{\alpha}\alpha} F_\alpha \overline{F}_{\bar{\alpha}} + \mathcal{O}(\epsilon^3)\,,\\ &= V_{sim} + \mathcal{O}(\epsilon^3)\,, && (3.33)\end{aligned}$$

where $\tilde{g}^{\bar{\alpha}\alpha}$ is the inverse of $g_{\alpha\bar{\alpha}}$, the metric appearing in the simple model where the heavy fields are frozen, and the identity (3.24) between the metrics has been used. Being the potential (3.33) of $\mathcal{O}(\epsilon^2)$, it is enough to keep the leading terms H_0^i for the position of the VEV's in the heavy directions, obtaining finally the equivalence at leading order between the two actions[5].

The way we are proceeding might be confusing as we are showing that only once the scalar fields are integrated out one finds the correct matching between the full and simple descriptions. Indeed if one tries naively to freeze the H^i modes in (3.25) the matching does not proceed. The point is that in the simple description we are freezing full supermultiplets, instead, working with the scalar potential for the scalar components we have properly integrated out the auxiliary components so we are forced to do as well for the scalar components. Alternatively, once the vacuum is located, one can go to the full scalar lagrangian where the auxiliary fields have not been integrated out yet,

$$V_{aux} = -g_{M\bar{M}} F^M \overline{F}^{\bar{M}} + F^M \partial_M W + \bar{F}^{\bar{M}} \partial_{\bar{M}} \overline{W}\,. \tag{3.35}$$

Using the results we found above and keeping only up $\mathcal{O}(\epsilon^2)$ terms one gets

$$V_{aux} = -g_{\alpha\bar{\alpha}} F^\alpha \overline{F}^{\bar{\alpha}} + F^\alpha \partial_\alpha W + \bar{F}^{\bar{\alpha}} \partial_{\bar{\alpha}} \overline{W} + \mathcal{O}(\epsilon^3)\,, \tag{3.36}$$

[5]Notice that alternatively we could regard the VEV of H^i as a function of the L^i using (3.19) as a field identity, since the VEV of the L^α is not determined by the e.o.m. at this order. In this case eq.(3.22) holds also as field identity so that $V_{int} \sim \mathcal{O}(\epsilon^4)$, and with (3.24) one immediately gets

$$V_{full} = \epsilon^2 \tilde{g}^{\bar{\alpha}\alpha} \overline{F}_{\bar{\alpha},1} F_{\alpha,1} = V_{sim} + \mathcal{O}(\epsilon^3)\,. \tag{3.34}$$

which exactly coincides at leading order with the component lagrangian one obtains from the simple superpotential and Kähler potential

$$V_{aux} = V_{sim,\,aux} + \mathcal{O}(\epsilon^3)\,, \tag{3.37}$$

without the need of integrating out. Essentially what we learn from this example is that we have to treat all the components of the supermultiplets in the same footing as far as the low energy EFT is nearly SUSY, implying that the freezing should be performed at the superpotential and Kähler potential and not at the scalar potential after the integration of the auxiliary components[6].

3.2.2 Manifestly SUSY approach

Since we are dealing with a SUSY theory a much more clever approach is to take the advantage of having the supermultiplet formalism and integrate simultaneously full heavy multiplets. This, moreover, by definition takes into account the issues pointed out at the end of section (3.2.1). The way to proceed was reviewed in section (2.1.1) where under some constrains on the auxiliary components of the light multiplets we show that the correct chiral e.o.m. to obtain the EFT at the two derivate level is given by

$$\partial_i W = 0\,. \tag{3.38}$$

There is also pointed out that the EFT one gets by using this approximate e.o.m. is exact only at leading order in the fermion bilinears and auxiliary fields, in particular up to quadratic order in the auxiliary fields. Given the from of the superpotential we are studying it is always warranted that $F^\alpha \sim \mathcal{O}(\epsilon)$ around the solutions of eq.(3.38), so that the possible corrections are indeed irrelevant for our purposes of computing the low energy EFT at leading order in ϵ, and we can safely stick to the result from eq.(3.38). On the other hand, it is clear that in case one wants to study the non-SUSY solutions we point out for the leading scalar potential eq.(3.14), which do not allow decoupling, one should use the full chiral equation and not the approximation eq.(3.38).

Let us, then, solve eq.(3.38) in an expansion in ϵ:

$$H^i = H^i_0 + \epsilon H^i_1(L) + \mathcal{O}(\epsilon^2) \tag{3.39}$$

[6] To a similar conclusion arrive the authors of [53] studying the consistency of truncating the Kaluza–Klein modes in the EFT after compactification. However, they did not point out the fact that the underlying reason was the consistent treatment of the supermultiplets.

where H_0^i are defined by $\partial_i W_0(H_0) = 0$. The effective Kähler and superpotential read

$$W_{full} = W_{sim} + \epsilon^2\Big(\frac{1}{2}\partial_i\partial_j W_0 H_1^i H_1^j + \partial_i W_1 H_1^i\Big) + \mathcal{O}(\epsilon^3)\,, \quad (3.40)$$

$$K_{full} = K_{sim} + \epsilon\Big(\partial_i K_{sim} H_1^i + \partial_{\bar{i}} K_{sim} \bar{H}_1^{\bar{i}}\Big) + \mathcal{O}(\epsilon^2)\,, \quad (3.41)$$

where $W_{sim} = W(H_0)$ and $K_{sim} = K(H_0, \bar{H}_0)$. The leading shift H_1^i equals

$$H_1^i = -W_0^{ij}\partial_j W_1, \quad (3.42)$$

with $W_0^{ij} \sim \mathcal{O}(1)$, the inverse of $W_{0,ij} \equiv \partial_i\partial_j W_0$. In the same way that in the previous approach in order our expansive approach be consistent we should require that all the eigenvalues of $\partial_i\partial_j W_0$ be $\mathcal{O}(1)$. The solution gives then,

$$W_{full} = W_{sim} - \frac{1}{2}\epsilon^2 \partial_i W_1 W_0^{ij} \partial_j W_1 + \mathcal{O}(\epsilon^3)\,. \quad (3.43)$$

$$K_{full} = K_{sim} - \epsilon\Big(\partial_i K_{sim} W_0^{ij}\partial_j W_1 + \partial_{\bar{i}} K_{sim} \overline{W}_0^{\bar{i}\bar{j}} \partial_{\bar{j}} \overline{W}_1\Big) + \mathcal{O}(\epsilon^2)\,. \quad (3.44)$$

However, we do not need this explicit form to realize that the EFT exactly coincide at leading order in ϵ with the simple one. For the scalar potential, in particular, we recover the matching (3.33) and (3.37). Notice that working at the supermultiplet level is not only far more straightforward but also carries more information than working at the scalar potential level, as now also the information about the fermionic sector is precisely specified. One important result we can recover from the chiral equation (3.38) is the suppression of the SUSY breaking contribution from the H^i sector, encoded in the expectation value of the auxiliary fields, as can be seen from its θ^2 component,

$$F^i = -W_0^{ij}\partial_j\partial_\alpha W_1 F^\alpha + \mathcal{O}(\epsilon^2) \quad (3.45)$$

$$\sim \mathcal{O}(\epsilon) F^\alpha + \mathcal{O}(\epsilon^3)\,. \quad (3.46)$$

3.3 Supergravity

Let us now turn to SUGRA, the case we are mainly interested in. It turns out that the analysis closely follows the lines of the flat SUSY case considered in the previous section, with one extra important requirement, which is a consequence of the universal nature of the gravitational interactions. As before we first proceed in the orthodox way of integrating out fields from the scalar potential, allowing us to identify two possible scenarios, and then turn to the simultaneous integration of full off-shell supermultiplets to match the results.

3.3.1 Scalar components level

The superpotential is taken as in eq.(3.8) and all the assumptions of the flat case continue to hold here. The SUGRA scalar potential was derived in section (2.2.1). Here we recall it in the two ways derived there,

$$V = e^K \left(g^{\bar{M}N} \overline{F}_{\bar{M}} F_N - 3|W|^2 \right) = e^G \left(g^{\bar{M}N} G_{\bar{M}} G_N - 3 \right) , \tag{3.47}$$

where, $F_M = D_M W = \partial_M W + (\partial_M K)W$, $G = K + \ln |W|^2$ and its derivatives $G_M = \partial_M G = F_M/W$. As remarked above, gravity makes less transparent the decoupling of the heavy fields from the light ones. This is best seen if we expand the scalar potential in ϵ and analyze the leading term V_0 which reads

$$V_0 = e^K \left(g^{\bar{M}N} \overline{F}_{0,\bar{M}} F_{0,N} - 3|W_0|^2 \right) , \tag{3.48}$$

where

$$F_{0,i} = \partial_i W_0 + (\partial_i K) W_0 \, , \qquad F_{0,\alpha} = (\partial_\alpha K) W_0 \, . \tag{3.49}$$

It is immediately clear from eqs.(3.48) and (3.49) that already at $\mathcal{O}(\epsilon^0)$ and even for SUSY solutions, $F_{0,i} = 0$, in general there is no decoupling between the fields H^i and L^α, as was the case for flat space. However, decoupling at this level, i.e. rendering $F_{0,i}$ independent of L^α, is easily obtained in any of the following cases: i) the VEV of the superpotential satisfies $\langle W_0 \rangle \sim \mathcal{O}(\epsilon)$, and ii) K is factorizable at leading order, i.e. all its mixed derivatives are suppressed, e.g. $\partial_i \partial_{\bar{\alpha}} K \sim \mathcal{O}(\epsilon)$[7]. We now separately discuss the two situations.

3.3.1.1 Small $\langle W_0 \rangle$

When $\langle W_0 \rangle \sim \mathcal{O}(\epsilon)$, the expansion of the G_M's is taken as follows:

$$G_i = G_{i,-1} + G_{i,0} \, , \qquad G_\alpha = G_{\alpha,0} \, , \tag{3.50}$$

where we count the powers of ϵ by taking into account the presence of W in the denominator of the G factors. In this way terms of $\mathcal{O}(\epsilon^{-1})$ appear, but no poles in ϵ arise, being e^G of $\mathcal{O}(\epsilon^2)$. In the particular "Kähler gauge" in which we defined the theory, we

[7] Interestingly enough this is exactly the condition we found for decoupling around the non-SUSY solution of the leading scalar potential in the flat case. However, as we will see, here it does not immediately implies a mass hierarchy between the two sectors.

have

$$\begin{aligned} G_{i,-1} &= \frac{\partial_i W_0}{W}\,, \quad G_{i,0} = \partial_i K + \frac{\partial_i W_1}{W}\,, \\ G_{\alpha,0} &= \partial_\alpha K + \frac{\partial_\alpha W_1}{W}\,. \end{aligned} \tag{3.51}$$

As in the previous flat space analysis, let us first show that the location of the vacuum is reliably computed in the simple theory, namely that $\partial_\alpha V_{sim} = \partial_\alpha V + \mathcal{O}(\epsilon^3)$, where V_{sim} is the simple effective potential obtained by setting $H^i = H^i_0$ in K and W. The full e.o.m. for the fields can be written as

$$\partial_M V = G_M V + e^G (G^P \nabla_M G_P + G_M) = 0\,. \tag{3.52}$$

In eq.(3.52), $G^P = G_{\bar P}\, g^{\bar P P}$, $\nabla_M G_P = \partial_M G_P - \Gamma^Q_{MP} G_Q$ is the Kähler covariant derivative and $\Gamma^Q_{MP} = (\partial_M g_{P\bar Q}) g^{\bar Q Q}$ are the holomorphic components of the affine connection. The expansion of $\partial_M G_P$ gives

$$\begin{aligned} \partial_j G_i &= (\partial_j G_i)_{-2} + (\partial_j G_i)_{-1} + (\partial_j G_i)_0\,, \quad \partial_\beta G_i = (\partial_\beta G_i)_{-1} + (\partial_\beta G_i)_0\,, \\ \partial_j G_\alpha &= (\partial_j G_\alpha)_{-1} + (\partial_j G_\alpha)_0\,, \qquad\qquad\qquad \partial_\beta G_\alpha = (\partial_\beta G_\alpha)_0\,. \end{aligned} \tag{3.53}$$

For simplicity, we do not write the explicit forms of the derivatives of the G's in terms of K and W, being straightforward to derive these expressions. At $\mathcal{O}(\epsilon^0)$, $(\partial_i V)_0 = 0$ is satisfied by $(\partial_j G_i)_{-2} = -(G_i)_{-1}(G_j)_{-1} = 0$, i.e. $\partial_i W_0 = 0$, which fix H^i_0. The equations $(\partial_\beta V)_0 = (\partial_\beta V)_1 = 0$ automatically vanish for $H^i = H^i_0$. The equations $(\partial_j V)_1 = 0$ give the linear order displacement of the heavy fields (the analogue of the flat space formula (3.19):

$$H^i_1 = -(\hat K^{-1})^i_{\bar j} (\overline{G}^{\bar j})_0\,, \tag{3.54}$$

with $\hat K^{\bar i}_j = g^{\bar i k}(\partial_k G_j)_{-1}$ and $(\overline{G}^{\bar j})_0 = (G_{\bar M})_0 g^{\bar M i}$, evaluated at $H^i = H^i_0$.

At the shifted vacuum $H^i_0 + \epsilon H^i_1$ we have

$$\begin{aligned} G_i &= \mathcal{O}(1), \qquad G_\alpha = \mathcal{O}(1), \qquad G^i = \mathcal{O}(\epsilon), \quad G^\alpha = \mathcal{O}(1)\,, \\ \nabla_\beta G_i &= \mathcal{O}(1), \quad \nabla_\beta G_\alpha = \mathcal{O}(1), \qquad V = \mathcal{O}(\epsilon^2)\,. \end{aligned} \tag{3.55}$$

Using eq.(3.24), we also have

$$\begin{aligned} G^\alpha &= G_{\bar\alpha} \tilde g^{\bar\alpha\alpha} + \mathcal{O}(\epsilon)\,, \\ \nabla_\beta G_\alpha &= \partial_\beta G_\alpha - \tilde g^{\bar\gamma\gamma} \partial_\beta g_{\alpha\bar\gamma} G_\gamma = (\tilde\nabla_\beta G_\alpha) + \mathcal{O}(\epsilon)\,, \end{aligned} \tag{3.56}$$

where $\tilde\nabla$ is the covariant derivative constructed in the subspace parametrized by the scalar fields L^α only, namely the one entering in V_{sim}. Finally, using eqs.(3.55) and (3.56), it follows that $\partial_\beta V = \partial_\beta V_{sim} + \mathcal{O}(\epsilon^3)$, where

$$V_{sim} = e^G \Big[\tilde{g}^{\bar{\alpha}\alpha} \overline{G}_{\bar{\alpha}} G_\alpha - 3 \Big] , \tag{3.57}$$

with heavy moduli frozen at H^i_0.

The equivalence of the simple low-energy effective theory with the full one proceeds along the same lines of the flat space SUSY case discussed before, with small changes in the expressions. Again, the knowledge of the potential (3.27) at $\mathcal{O}(\epsilon^2)$ requires to compute the $\mathcal{O}(\epsilon)$ terms in $\partial_I V$ and the $\mathcal{O}(1)$ in V^{IJ}. One gets,

$$\partial_i \partial_{\bar{j}} V|_0 \;=\; e^K \partial_{\bar{j}} \overline{F}_{\bar{k},0}\, g^{\bar{k}l}\, \partial_i F_{l,0} \,, \tag{3.58}$$

$$\partial_i V|_1 \;=\; e^K \overline{F}_{\bar{M}} g^{\bar{M}l} \partial_i F_{l,0} \,, \tag{3.59}$$

where $F_{i,0} = \partial_i W_0$. Using eqs.(3.58) and (3.59), we find

$$V_{int} = -e^K \overline{F}_{\bar{M}} g^{\bar{M}i} \tilde{g}_{i\bar{j}} g^{\bar{j}N} F_N \,, \tag{3.60}$$

where $\tilde{g}_{i\bar{j}}$ is the inverse of $g^{\bar{j}i}$, not to be confused with $g_{i\bar{j}}$. The full effective scalar potential reads then

$$\begin{aligned} V_{full} &= e^K \Big(g^{\bar{M}N} \overline{F}_{\bar{M}} F_N - 3|W|^2 \Big) - e^K \overline{F}_{\bar{M}} g^{\bar{M}i} \tilde{g}_{i\bar{j}} g^{\bar{j}N} F_N + \mathcal{O}(\epsilon^3) \,, \\ &= \epsilon^2 e^K \Big(\tilde{g}^{\bar{\alpha}\alpha} F_\alpha \overline{F}_{\bar{\alpha}} - 3|W|^2 \Big) + \mathcal{O}(\epsilon^3) \,, \\ &= V_{sim} + \mathcal{O}(\epsilon^3) \,, \end{aligned} \tag{3.61}$$

where as in the global SUSY case we used the identity (3.24) between the metrics, and the fact that for the knowledge of the potential (3.61) up to $\mathcal{O}(\epsilon^2)$, it is enough to keep the leading terms H^i_0 for the position of the VEV's in the heavy directions, obtaining once more equivalence at leading order between the two actions.

We can also proceed from the full component potential eq.(2.17) and using the information we have of the vacuum, with the on-shell expression for the auxiliary fields, $F^\alpha \sim \mathcal{O}(\epsilon)$, $F^i \sim \mathcal{O}(\epsilon^2)$ and $U \sim \mathcal{O}(\epsilon)$, keep up to $\mathcal{O}(\epsilon^2)$ terms and find direct matching at leading order with the full component lagrangian obtained from the simple superpotential and Kähler potential without the need of integrating out the scalar components. In fact the discussion at the end of section (3.2.1) holds verbatim here in SUGRA.

3.3.1.2 Almost factorizable Kähler potential

A decoupling between the H and L fields is possible also for generic $\langle W_0 \rangle$, provided that K is almost factorizable, namely

$$K(\phi, \bar{\phi}) = K_H(H^i, \bar{H}^{\bar{i}}) + K_L(L^\alpha, \bar{L}^{\bar{\alpha}}) + \epsilon K_{mix}(\phi^M, \bar{\phi}^{\bar{M}})\,. \tag{3.62}$$

With K as in eq.(3.62), the SUSY equations $F_{0,i} = 0$ do not depend on L^α and lead to solutions of the e.o.m. $(\partial_i V)_0 = 0$ on a generic non-SUSY vacuum with $F_{0,\alpha} \neq 0$. It turns out that in this case the situation can be sightly generalized. Indeed, at $\mathcal{O}(\epsilon^0)$ the system satisfy the general factorizable condition discussed in [36] for the generalized Kähler potential,

$$G_0(H, L) = G_H(H) + G_L(L)\,, \tag{3.63}$$

which in terms of the Kähler potential and superpotential reads $K = K_H(H^i, \bar{H}^i) + K_L(L^\alpha, \bar{L}^\alpha)$ and $W = W_H(H^i)W_L(L^\alpha)$. The outcome of such a system has been widely studied in series of papers by Achucarro *et. al.* [33, 35, 34]. It is clear now that the SUSY solutions, i.e. $G_{0,i} = G_{H,i} = 0$, are independent of the L^α fields as $\partial_\alpha \partial_j G \equiv 0$. The fact that these are indeed solution of the e.o.m. at $\mathcal{O}(\epsilon^0)$ can be easily check from the corresponding scalar potential which takes the form,

$$V_0 = e^{G_H + G_L}\left(G_H^{i\bar{j}} \partial_i G_H \partial_{\bar{j}} G_H + G_L^{\alpha\bar{\beta}} \partial_\alpha G_L \partial_{\bar{\beta}} G_L - 3\right)\,, \tag{3.64}$$

with $G_H^{i\bar{j}}$ ($G_L^{\alpha\bar{\beta}}$ resp.) the inverse of $\partial_i \partial_{\bar{j}} G_H$ ($\partial_\alpha \partial_{\bar{\beta}} G_L$ resp.). Then the e.o.m. for the H^i fields at this order read,

$$\partial_i V_0 \;=\; V_0 \partial_i G_H + e^{G_H + G_L} \partial_i \left(G_H^{j\bar{j}} \partial_j G_H \partial_{\bar{j}} G_H\right)\,. \tag{3.65}$$

The situation, in fact, is more rigid since that all the mixed derivatives $\partial_i \partial_{\alpha_1} \ldots \partial_{\alpha_n} V_0$ vanish as can be seen from eq.(3.64). As a result, $\partial_H V$ is at most of $\mathcal{O}(\epsilon)$, $\partial_H^2 V$ is of $\mathcal{O}(1)$, implying that $\hat{H}^i$, given in (3.26), is $\mathcal{O}(\epsilon)$, and the corrections due to the integration of the $\hat{H}^i$ start only at $\mathcal{O}(\epsilon^2)$. Therefore, not only the leading bosonic effective Lagrangian at $\mathcal{O}(\epsilon^0)$ is hence reliably determined by just freezing the heavy fields to their VEV's H_0^i, but also the next to leading potential at $\mathcal{O}(\epsilon)$.

Notice that contrary to the previous case the $\mathcal{O}(\epsilon^0)$ e.o.m. for the L^α fields are not automatically satisfied therefore the leading order VEV L_0^α is determined from the begining[8]. This is related to a crucial difference between this scenario and the one where

[8]Notice that for realistic scenarios, where the superpotential is of the form (3.1), the dynamics at this level are solely given by the dependency on the Kähler potential. In this case quite intricate dependencies, arising possibly from α', loop and non-perturbative effects to the Kähler potential, will be in general required in order not to get trivial solutions as run away directions.

we require a small VEV for W_0, given by the fact that here the distinction of the fields by heavy and light is misleading since in general all fields, starting from the gravitino, will acquire a mass squared of order $e^K|W_0|^2$ due to usual SUGRA contributions. This fact seems to be in tension with our argument of neglecting the canonical normalization of the fields, since the heavy states were uniquely determined by the H^i fields. It turns out that here the situation is even simpler since not only the metric factorizes at leading order, by definition, but also the mass matrix, hence the canonical normalized mass eigenstates start to mix only at $\mathcal{O}(\epsilon)$, and the discussion remains unchanged.

An interesting possibility, and probably the most relevant in realistic scenarios (see footnote (8) in this chapter), is that the SUGRA EFT model is of the no-scale type, with

$$G_0^\alpha G_{\alpha,0} = g_L^{\bar\alpha\alpha}\partial_{\bar\alpha}K_L\partial_\alpha K_L = 3\,, \tag{3.66}$$

with g^L the inverse of the Kähler metric $\partial_\alpha\partial_{\bar\alpha}K_L$. When eq.(3.66) is valid, eq.(3.64) simplifies to

$$V_0 = e^{G_0}G_0^iG_{i,0}\,, \tag{3.67}$$

and $\partial_\alpha V_0$ automatically vanish for $G_{i,0} = 0$. The situation is now very similar to the non-SUSY case discussed in section 3.1. The fields L^α are generally fixed by the $\mathcal{O}(\epsilon)$ e.o.m. $\partial_\alpha V_1 = 0$, where V_1 is the next scalar potential term, whose explicit form will not be needed. The expansion in fluctuations of V_0 is as before, with $\hat{H} \sim \mathcal{O}(\epsilon)$. The leading bosonic effective Lagrangian arises at $\mathcal{O}(\epsilon)$ and is determined by just freezing the heavy fields to their VEV's H_0^i. As we will see in section (3.3.3), the LARGE volume models of [40, 41] belong to this class of models.

Let us do an important remark at this stage. As said in the beginning of this section the factorizability condition is a constrain on the form of the Kähler potential and superpotential to be realized in the field region around the solution of the e.o.m.. For the Kähler potential this form is given in eq.(3.62) and for the superpotential in general this can be stated like,

$$W = W_H(H)W_L(L) + \epsilon W_{mix}(H,L)\,, \tag{3.68}$$

which for the usual case, $W_L = 1$, reduces to the form in eq.(3.8). Then, it is easy to see that in scenarios like the KKLT one, where at the vacuum the non-perturbative part of the superpotential is comparable to the VEV of W_0 (see section (2.5) and chapter 5), an approximate factorizable form can only be realized if the VEV for W_0 is tuned, as in this case ϵ can be defined as $\epsilon \simeq \langle W_0/\partial^2_{HH}W_0\rangle$. We conclude, thus, that for for such cases the tuning is compulsory even if one can argue for a near factorizable from for the Kähler potential.

3.3.2 Manifestly SUSY approach

In the same spirit of the global case we can enjoy the fact that the fields are arranged in supermultiplets in order to integrated them out simultaneously. It turns out that the first scenario we realized in section (3.3.1) where the VEV of the superpotential is require to be suppressed, $\langle W \rangle \sim \mathcal{O}(\epsilon)$, fulfill all the conditions we describe in section (2.2.3) where we discuss the procedure to follow in order to get a consistent SUGRA EFT at the two derivative level. Indeed, given this constrain on the VEV of the super potential we have that $F^\alpha \sim U \sim \mathcal{O}(\epsilon)$, so that all possible corrections are negligible for our purposes, in particular generate $\mathcal{O}(\epsilon^3)$ corrections in the scalar potential. Therefore, we can safely use the approximate chiral equation we derive in sec.(3.3.1), implying that the analysis is exactly the same we did for the flat case in sec.(3.2.2) with the very same conclusions.

A different story happens in the case we leave W_0 arbitrary. Indeed in this case since we are dealing with fields that are not really heavy even the two derivative approximation in the kinetic terms seems to be misleading. It seems, therefore, that we have to keep further terms coming from the second term in eq.(2.38) and we cannot neglect the supercovariant derivates contribution the the full chiral e.o.m. . Instead of trying to figure out from the full chiral equation (2.38) if is possible to obtain another approximative expression for this particular case, let us follow an analogous procedure to solve for all the components of the chiral multiplet (for simplicity we restrict the analysis to the scalar and auxiliary components).

Starting from the full potential (2.25) we derive at the end of sec.(2.2.1), adding the kinetic terms we have,

$$\begin{aligned} \mathcal{L} \;=\; & G_{M\bar{N}}\partial_\mu\phi^M\partial_\mu\bar{\phi}^{\bar{N}} + G_M F^M\bar{U} + G_{\bar{M}}\overline{F}^{\bar{M}}U \\ & + \left(G_{M\bar{M}} + \frac{1}{3}G_M G_{\bar{M}}\right) F^M F^{\bar{M}} - 3U\bar{U} - 3e^{\frac{G}{2}}(U+\bar{U})\,, \end{aligned} \tag{3.69}$$

where the subindices indicate, as usual, derivatives. We choose to work in the Kähler gauge where only the Kähler invariant function appears since the factorizability and Kähler invariance are completely implicit on it. Let us take the e.o.m. for the F^i and the scalar component H^i.

$$3e^{G/2} + 3\bar{U} - G_{\bar{i}}\overline{F}^{\bar{i}} = 0\,, \tag{3.70}$$

$$\begin{aligned} & - G_{iN}F^N\bar{U} - G_{i\bar{M}}\overline{F}^{\bar{M}}U - \left(G_{iM\bar{N}} - \frac{1}{3}(G_{iM}G_{\bar{N}} + G_M G_{i\bar{N}})\right) F^M\overline{F}^{\bar{N}} \\ & + \frac{3}{2}e^{G/2}(U+\bar{U})G_i - G_{i\bar{M}}\Box\bar{\phi}^{\bar{M}} + G_{i\bar{M}\bar{N}}\partial^\mu\bar{\phi}^{\bar{M}}\partial_\mu\bar{\phi}^{\bar{N}} = 0\,. \end{aligned} \tag{3.71}$$

The first equation with the on-shell expression for U (2.26) leads to the on-shell expression for the auxiliary fields (2.27). From the second expression regarding the almost factorizable nature of G and keeping only the leading terms in ϵ,

$$\begin{aligned} &- G_{ij}F^j\bar{U} - G_{i\bar{j}}\overline{F}^{\bar{j}}U - G_{ij\bar{k}}F^j\overline{F}^{\bar{k}} + \frac{1}{3}G_{ij}G_{\bar{N}}F^j\overline{F}^{\bar{N}} + \frac{1}{3}G_M G_{i\bar{j}}F^M\overline{F}^{\bar{j}} \\ &\qquad + \frac{3}{2}e^{G/2}(U+\bar{U})G_i - G_{i\bar{j}}\Box\bar{H}^{\bar{j}} + G_{i\bar{j}\bar{k}}\partial^\mu\bar{H}^{\bar{j}}\partial_\mu\bar{H}^{\bar{k}} \;=\; \mathcal{O}(\epsilon)\,. \end{aligned} \tag{3.72}$$

The first consequence of the factorizability is that the higher derivative corrections in the kinetic terms are actually suppressed, since the derivative dependence on the L^α appear only in the $\mathcal{O}(\epsilon)$ terms. Therefore, a two derivative approximation in this sector is still possible. Looking for slowly varying solutions and using the on-shell expression for the U field and eq.(3.70), one finds

$$G_{ij}F^j\bar{U} + \frac{1}{2}(U+3\bar{U})G_{i\bar{j}}\overline{F}^{\bar{j}} - G_{ij\bar{k}}F^j\overline{F}^{\bar{k}} + \frac{1}{3}G_{ij}G_{\bar{N}}F^j\overline{F}^{\bar{N}} + \frac{1}{3}G_M G_{i\bar{j}}F^M\overline{F}^{\bar{j}} = \mathcal{O}(\epsilon)\,. \tag{3.73}$$

These, have as solutions $F^i \sim \mathcal{O}(\epsilon)$, which imply from eq.(3.70), the scalar components to be solution for

$$\partial_i G = \frac{1}{W}(\partial_i W + \partial_i K W) = \mathcal{O}(\epsilon)\,. \tag{3.74}$$

Indeed these are the F-flatness conditions we found at leading order in the previous section. However, let us stress that this last equation cannot be promoted to the superfield level in order to solve for the full chiral multiplet H^i as it is not a chiral equation, so one would face more equations than fields to be solved [49]. Plugging back these solutions in (3.69) keeping only up to $\mathcal{O}(\epsilon)$ terms, one gets

$$\begin{aligned} \mathcal{L} \;=\;& G_{\alpha\bar{\beta}}\partial_\mu L^\alpha\partial_\mu\bar{L}^{\bar{\alpha}} + G_\alpha F^\alpha\bar{U} + G_{\bar{\alpha}}\overline{F}^{\bar{\alpha}}U \\ &+ \left(G_{\alpha\bar{\beta}} + \frac{1}{3}G_\alpha G_{\bar{\beta}}\right)F^\alpha F^{\bar{\beta}} - 3U\bar{U} - 3e^{\frac{G}{2}}(U+\bar{U}) + \mathcal{O}(\epsilon^2)\,, \end{aligned} \tag{3.75}$$

where we have kept some $\mathcal{O}(\epsilon^2)$ terms in order not to be forced to split the G function and overload the notation. Notice that the only place where the VEV of H^i appear at $\mathcal{O}(\epsilon^0)$ is the one in the exponential factor of the $(U+\bar{U})$ term. However, expanding around the leading solution, $\partial_i G = 0$, the corrections will appear only at the $\mathcal{O}(\epsilon^2)$, so we can safely keep only the leading VEV for the H^i, as well in the rest of the terms where it appears since these are at most $\mathcal{O}(\epsilon)$. In this way we find again the matching at $\mathcal{O}(\epsilon)$ between the two descriptions.

3.3.3 The LARGE volume scenario

Although the almost factorizable case seems quite appealing, allowing the integration of fields with same mass scale as the L fields, its explicit realization from a string compactification does not seems much realistic. Indeed, although at first order the Kähler potential turns out to be completely factorizable between the different set of moduli, α' corrections lead to mixings. For the case of type-IIB orientifold compactifications on CY it takes the form [89],

$$K = -2\ln\left[\mathcal{V}(T^i) + \xi(S+\bar{S})^{3/2}\right] + \mathcal{K}(S+\bar{S}, U^j, \bar{U}^j)\,, \tag{3.76}$$

where $\mathcal{V}$ is the CY volume, which depends on the Kähler moduli T^i. S and U^i are the dilaton and complex structure moduli. Notice, however, that for points in the moduli space that realize a huge volume this Kähler potential takes the form of almost factorizability discussed in section (3.3.1.2). It turns out that type-IIB orientifold compactifications on CY indeed admit vacua where the internal volume is exponentially large, resulting on so called LARGE volume models [40, 41], so let us do a close-up to this particular scenario (see also ref.[35]).

A generic study of this kind of vacua is more complicated than for the KKLT like ones, since the form of the Kähler potential for the Kähler moduli plays a dramatic rôle in the stabilization [90], rising possible different scaling behaviors for different models. There is, however, a class of models that are expected to encode most of the important features of this scenario, namely the “swiss-cheese” manifold compactifications. The simplest of such compactifications is realized in the manifold described by a hyper-surface in the projective space $\mathbf{CP}^4_{[1,1,1,6,9]}$, with $h^{1,1}=2$ and $h^{2,1}=272$ Kähler and complex structure moduli, respectively. In order to be able to treat this system, we keep only one complex structure modulus, denoted by U in the following. As will be clear, our conclusions do not really depend on such drastic simplification. The Kähler potential (3.76) then takes the form

$$K = -2\ln\left[\mathcal{V}(T,t) + \xi(S+\bar{S})^{3/2}\right] - \ln(S+\bar{S}) - \ln(U+\bar{U})\,, \tag{3.77}$$

with the volume of the CY given by [91]:

$$\mathcal{V}(T,t) = \frac{1}{9\sqrt{2}}\left(T_r^{3/2} - t_r^{3/2}\right)\,. \tag{3.78}$$

In eq.(3.78) we generally denoted by $X_r = (X+\bar{X})/2$ the real part of a complex field X. Consistent LARGE volume vacua are located at point satisfying $T_r \gg 1$ and $t_r \gtrsim 1$ [90], and we can choose as expansion parameter $\epsilon \equiv 1/\mathcal{V} \sim T_r^{-3/2}$. The superpotential is the sum of a flux superpotential $W_0 = W_0(S,U)$ and a non–perturbative term $W_1 =$

$W_1(S, U, t)$.[9] The stabilization to large volume requires the non–perturbative part to scale like the inverse of the volume, i.e. $W_1 \sim \epsilon$, hence the superpotential has again the form of eq.(3.8):

$$W = W_0(S, U) + \epsilon W_1(S, U, t)\,. \tag{3.79}$$

The nearly factorization of eq.(3.77) is made evident by redefining $T \to \epsilon^{-2/3} T$,

$$K = K_H(S, U) + K_L(T, t) + \epsilon K_{mix}(S, T, t)\,, \tag{3.80}$$

with $K_H = -\ln(4S_r U_r)$ and $K_L = -2\ln\mathcal{V}$, therefore, from the results of section (3.3.1.2) no fine-tuning on W_0 is required. This is neither required from low energy SUSY, i.e. small gravitino mass, as the hierarchy is now given by the exponential factor in the Kähler potential, $m_{3/2}^2 = e^K|W|^2 \sim |W_0|^2/\mathcal{V}^2$, a property that makes this kind of vacua very appealing.

It is straightforward to expand G_M in powers of ϵ. We get

$$\begin{aligned} G_S &= G_{S,0} + \mathcal{O}(\epsilon)\,, & G_U &= G_{U,0} + \mathcal{O}(\epsilon)\,, \\ G_T &= \epsilon^{2/3} G_{T,2/3} + \mathcal{O}(\epsilon^{5/3})\,, & G_t &= \epsilon\, G_{t,1} + \mathcal{O}(\epsilon^2)\,, \end{aligned}$$

where, as before, $G_M = \partial_M G$, G the Kähler invariant function, and

$$\begin{aligned} G_{S,0} &= -\frac{1}{2S_r} + \frac{\partial_S W_0}{W_0}\,, & G_{U,0} &= -\frac{1}{2U_r} + \frac{\partial_U W_0}{W_0}\,, \\ G_{T,2/3} &= -\frac{3}{2T_r}\,, & G_{t,1} &= \frac{3\sqrt{t_r}}{2T_r^{3/2}} + \frac{\partial_t W_1}{W_0}\,. \end{aligned}$$

The expansion of the scalar potential V is as follows: $V = \epsilon^2 V_2 + \epsilon^3 V_3 + \mathcal{O}(\epsilon^4)$, where

$$V_2 = e_2^G\Big[g_0^{\bar U U} G_{\bar U,0} G_{U,0} + g_0^{\bar S S}\overline{G}_{\bar S,0} G_{S,0}\Big]\,, \tag{3.81}$$

$$e^G = \epsilon^2 e_2^G + \mathcal{O}(\epsilon^3), \quad e_2^G = \frac{81|W_0|^2}{2S_r U_r T_r^3}\,. \tag{3.82}$$

It is important to notice that, aside from the overall T–dependence appearing in e_2^G, V_2 depends on S and U only. A possible non-trivial Kähler moduli dependence of $\mathcal{O}(\epsilon^2)$ in V_2 exactly cancels the -3 term in V_2, due to the approximate no-scale structure of the Kähler potential, for which [92]

$$\sum_{i,j=T,t} g^{\bar j i}\partial_i K \partial_{\bar j} K = 3 + \mathcal{O}(\xi\epsilon)\,. \tag{3.83}$$

[9] Since T is very large, possible non–perturbative terms of the form $\exp(-aT)$ are totally negligible.

An effective decoupling between S, U and the Kähler moduli appears at this order, so that the leading e.o.m. for S and U admit the SUSY solutions

$$G_{S,0}(S_0,U_0) = G_{U,0}(S_0,U_0) = 0\,. \tag{3.84}$$

Exactly as we found for the generic approximate factorizable case, around the solutions (3.84), all the terms linear in the heavy field fluctuations $\hat{H} = \hat{S}, \hat{U}$ of the form $c_n \hat{H}\hat{L}^n$, where $\hat{L} = \hat{T}, \hat{t}$, $n \geq 1$, vanish. These terms can only arise from V_3, so that schematically we have $V \sim \mathcal{O}(\epsilon^2)\hat{H}^2 + \mathcal{O}(\epsilon^3)c_n\hat{H}\hat{L}^n$, implying that $\hat{H} \sim \mathcal{O}(\epsilon)$. Hence integrating out $\hat{H}$ can only result in effective couplings of $\mathcal{O}(\epsilon^4)$ or higher and hence we can effectively fix $\hat{H} = 0$. Computing the full leading effective potential V_3 for T and t. We find

$$V_3 = \frac{27\Big[81\xi_0|w_0|^2 + 4\sqrt{2t_r}T_r^3|\partial_t w_1|^2 - 3t_r T_r^{3/2}(\bar{w}_0\partial_t w_1 + c.c.)\Big]}{T_r^{9/2}U_{r,0}S_{r,0}}\,, \tag{3.85}$$

where $\xi_0 \equiv \xi S_{r,0}^{3/2}$, $w_0 = W_0(S_0,U_0)$ and $w_1(t) = W_1(S_0,U_0,t)$. It is easy to check that V_3 precisely coincides with the simple scalar potential constructed from $W_{sim} = W(S_0,U_0)$ and $K_{sim} = K(S_0,U_0)$, where the dilaton and the complex structure modulus are frozen at their SUSY values S_0, U_0 [40]. The scaling in ϵ of the mass spectrum is easily computed. The complex structure modulus, the dilaton and the gravitino mass arise from V_2 and hence

$$m_{3/2} \sim m_S \sim m_U \sim \epsilon\,. \tag{3.86}$$

The Kähler structure moduli mass matrix arises from V_3. Given the structure of the kinetic metric, it is simple to see that

$$m_T \sim \epsilon^{3/2}\,, \quad m_t \sim \epsilon\,. \tag{3.87}$$

The scalings (3.86) and (3.87) are in agreement with the one reported in the table 1 of [41]. We see in this explicit example how although the absence of a hierarchy in the mass scales of the H and L fields, the model is effectively decoupled and the simple naive EFT, where one freezes S and U, is reliable.

Let us remark that the system as is so far lacks from an uplifting sector. Indeed, although clearly a non-SUSY vacuum, the cosmological constant turns out to be still an AdS with deepness of $\mathcal{O}(\epsilon^3)$ [40], so the model should necessarily be extended, as we will do at the end of next chapter.

Chapter 4

Two-Step Moduli Stabilization: Matter and Gauge Interactions

In the previous chapter we got an important general result, proving the reliability of the Two-Step moduli stabilization procedure by matching the full actions in both descriptions at leading order in ϵ, for a general class of SUSY models characterized by a superpotential that mimics the flux compactification scenarios. For a generic Kähler potential with arbitrary mixings the requirements found can be summarized as the condition of ensuring a mass hierarchy between the two sector. This condition is instead relaxed if the Kähler potential realizes a nearly factorizable form. The analysis there was restricted to the pure moduli case, namely, no gauge interactions and no $\mathcal{O}(1)$ couplings involving the light fields.

The aim of the present chapter is to generalize the previous study by adding vector multiplets, V^A, and matter-like chiral multiplets with $\mathcal{O}(1)$ field dependent couplings. Models of this sort necessarily imply various scales, associated to different symmetry breakings, SUSY breaking, gauge symmetry breaking, etc. In order to keep our analysis as simple as possible, and yet capture the essential features, we will assume the presence of just two kinds of light charged fields, characterized by having VEV parametrically larger than ϵ and of $\mathcal{O}(\epsilon)$. And denote them by Z and C respectively. Furthermore, in order to ensure light masses for the matter fields[1] we only consider Yukawa and higher order couplings, however, we briefly discuss how the inclusion of messenger-like fields can affect the analysis for the visible sector. The remaining light moduli are denoted by M.

[1]When the gauge symmetry is broken, a combination of the fields L actually get a heavy mass. Nevertheless, in order to distinguish them from the fields H appearing in eq.(4.1) below, with an abuse of language, we will keep calling them light.

The schematic form of the superpotential in this generalized set-up is taken as follows:

$$W = W_0(H) + Y_N(H,M,Z)C^N + \epsilon\Big[W_1(H,M,Z) + \mu_M(H,M,Z)C^M\Big]\,, \tag{4.1}$$

where W_0, Y_N, W_1 and μ_M ($N \geq 3, M \geq 2$), are arbitrary holomorphic functions, constrained only by gauge invariance. The Kähler potential is arbitrary, with the only assumptions that admits a Taylor expansion in the charged fields C and that, as in the previous case, all the eigenvalues of the associated Kähler metric are parametrically larger than ϵ. Similarly, the gauge kinetic functions f are taken to be arbitrary, but regular, moduli dependent holomorphic functions.

Under these consideration there is no difference between the charged field with $\mathcal{O}(1)$ VEV and the moduli, since no assumption is required in the form of the superpotential and Kähler potential, e.g. polynomial in Z. In fact, the moduli can be charged as well, via non-linear realization, as explained in section (2.4). Therefore we will not make distinction between these two kinds of fields denoting them simply by M. In the same spirit, the H^i fields can be matter-like multiplets, however, we will mostly concentrate to the case in which they are all moduli-like fields.

First of all we notice that, contrary to the H fields, massive gauge fields do not generally admit a freezing. More precisely, while superpotentials of the form (4.1) include a very wide class of known superpotentials arising from string compactifications, the class of Kähler potentials which would allow a freezing of the vector fields is quite limited and not very interesting, unless the vector field is heavy and decoupled, in which case one can trivially set it to zero. Hence we will not insist in freezing massive vector fields, but rather we will only show how the freezing of the heavy chiral fields is (not) affected in presence of heavy vector fields, the latter being always properly integrated out.

On the other hand, the scalar condition $F_{H,0} = \partial_H W_0 = 0$ in general does not fix all the VEV of the H fields, since gauge invariance constrains the form of W_0. Secondly, only gauge invariant combinations of the heavy moduli can be frozen, being only in this case that freezing is a well-defined gauge invariant statement. The orthogonal combinations will not appear in W_0, but possibly in other terms of the superpotential (4.1), in combination with light fields and/or in D-terms. Hence, these remaining gauge invariant combinations will typically be relevant in the low-energy dynamics and should properly be included among the light fields. More precisely, one can neglect non-neutral heavy moduli, assuming a chosen gauge-fixing where they are gauged away, but then one has to carefully take into account the dynamics of the associated massive vector-super field. In the context of SUSY breaking, the possible D-term SUSY breaking contributions hidden in the massive vector super field are generally non-negligible. We emphasize this point because, although already present in the literature in various contexts [50, 36], it has been overlooked in some string constructions, where gravity and moduli are neglected altogether. The impossibility of naively neglecting non-neutral moduli has nothing to do with gravity but is purely dictated by gauge invariance, so it remains also in the global limit with gravity decoupled.

Along the lines of the previous chapter, we compare the "full" effective theory obtained by classically integrating out the heavy fields to the "simple" one obtained by just "freezing" them out. We mainly use the manifestly SUSY approach introduced in section (2.1.1) and (2.2.3), which demonstrates itself to be clearer and simpler in the previous cases, with few necessary comments coming from the orthodox way of integrating the scalar components. We find that in the charged field range $|C| \lesssim \mathcal{O}(\epsilon)$, for any values of the M fields where the superpotential W has the form (4.1), the simple theory is a reliable effective field theory. More precisely W_{sim}, K_{sim} and f_{sim} differ, from W_{full}, K_{full} and f_{full}, only by operators which are sub-leading in an EFT sense and correct the coefficients of the already existing couplings by a small amount. The leading C-dependent part of the scalar potential is identical in both theories. On the other hand, cubic terms of the schematic form C^3 in K and C^6 terms in W (and higher) are not naively reliable in the simple theory, so care has to be taken in working with it anytime higher order operators are considered. These results exactly hold in the SUGRA extension upon the constrain $W \sim \mathcal{O}(\epsilon)$ is satisfied at the vacuum.

The way we proceed, namely using the approximate chiral equation to integrate chiral multiplets, does not allow to tackle directly the case with $\langle W_0 \rangle \sim \mathcal{O}(1)$, as was already pointed out in section (3.3.2), so we mainly concentrate to the $\langle W \rangle \sim \mathcal{O}(\epsilon)$ situation. At the end of the chapter, however, we do a short analysis in the orthodox way of integrating out fields from the Lagrangian for the case $\langle W_0 \rangle \sim \mathcal{O}(1)$, with again an explicit example in the LARGE volume scenario.

In this chapter we keep track of the masses for the light fields, m_L, and the ones of the heavy vector, m_V, and chiral fields, m_H, but as before we set the cut-off of the microscopic theory Λ to be the reduced Planck mass M_p and use units in which $\Lambda = M_p = 1$. In order not to introduce some additional hierarchy of scales, we assume that $m_H/M_p \gg \epsilon$ and that $m_V \sim m_H$, recalling that $\epsilon \approx m_L/m_H$.

4.1 O(1) Yukawa couplings

Before turning our attention to full models with gauge interactions and vector multiplets, it is useful to work in absence of the latter, in the limit of vanishing gauge couplings.

As explained in the beginning of the chapter the introduction of the Yukawa couplings forces us to distinguish, among the light fields L, between the fields with $\mathcal{O}(m_H)$ and $\mathcal{O}(\epsilon m_H)$ VEV's, denoting them by M^μ ($\mu = 1, \ldots, n_M$) and C^α ($\alpha = 1, \ldots, n_C$) respectively. We use calligraphic letters $\mathcal{A}, \mathcal{B}, \ldots$, to collectively denote all the light field indices: $\mathcal{A} = (\alpha, \mu)$ and $M, N, \ldots$ to collect all fields indices, heavy and light, $M = (i, \mathcal{A}) = (i, \alpha, \mu)$. Finally, we denote by $L^{\mathcal{A}} = (C^\alpha, M^\mu)$ and by $\phi^M = (H^i, C^\alpha, M^\mu)$ the set of all light and of all light+heavy fields, respectively. The superpotential (4.1) written in a more precise form is as follows:

$$W = W_0(H^i) + \eta\, \widetilde{W}_0(H^i, M^\mu, C^\alpha) + \epsilon\, W_1(H^i, M^\mu, C^\alpha)\,. \tag{4.2}$$

with $\widetilde{W}_0$ and W_1 gauge-invariant polynomials in the fields C^α, with field-dependent couplings:

$$\begin{aligned} \widetilde{W}_0 &= Y_{3,\alpha\beta\gamma}(H^i, M^\mu) C^\alpha C^\beta C^\gamma + \mathcal{O}(C^4)\,, \\ W_1 &= \widetilde{W}_1(H^i, M^\mu) + \mu_{2,\alpha\beta}(H^i, M^\mu) C^\alpha C^\beta + \mathcal{O}(C^3)\,. \end{aligned} \tag{4.3}$$

The requirement that W_0 give a supersymmetric mass of $\mathcal{O}(m_H)$ to the heavy fields and that the fields L have a mass of $\mathcal{O}(m_L)$ fix $W_0, \widetilde{W}_1$ and $\mu_{2,\alpha\beta}$ to be of $\mathcal{O}(m_H)$. The parameter η is a dummy variable which will be useful in what follows, but that eventually will be taken to be equal to 1. The Kähler potential is of the form

$$K = K_0 + K_{1,\alpha\bar{\beta}} C^\alpha \bar{C}^{\bar{\beta}} + (K_{2,\alpha\beta} C^\alpha C^\beta + c.c) + \mathcal{O}(C^3)\,, \tag{4.4}$$

with K_0, K_1 and K_2 arbitrary functions of H^i, M^μ and their complex conjugates, constrained only by gauge invariance.

Our aim is to compare the theory defined above by W and K (the full theory) with the simple effective one where the H^i are frozen at their leading VEV's H_0^i. As before, we solve the approximate chiral e.o.m. $\partial_H W =$ perturbatively in ϵ and, at each order in ϵ, further expand in η:

$$H^i = H_0^i + \eta \delta H_0^i(L) + \mathcal{O}(\eta^2) + \epsilon \Big[H_1^i(L) + \mathcal{O}(\eta) \Big] + \ldots \tag{4.5}$$

where, as usual, H_0^i are defined by $\partial_i W_0(H_0) = 0$ and we regard all eigenvalues of the matrix $\partial_i \partial_j W_0(H_0)$ to be of $\mathcal{O}(m_H)$. The effective superpotential and Kähler potential read

$$\begin{aligned} W_{full} &= W_{sim} + \eta^2\Big(\frac{1}{2}\partial_i\partial_j W_0 \delta H_0^i \delta H_0^j + \partial_i \widetilde{W}_0 \delta H_0^i\Big) + \epsilon\eta\Big(\partial_i\partial_j W_0 \delta H_0^i H_1^j + \partial_i \widetilde{W}_0 H_1^i \\ &\quad + \partial_i W_1 \delta H_0^i\Big) + \epsilon^2\Big(\frac{1}{2}\partial_i\partial_j W_0 H_1^i H_1^j + \partial_i W_1 H_1^i\Big) + \mathcal{O}(\eta^3, \eta^2\epsilon, \eta\epsilon^2, \epsilon^3)\,, \qquad (4.6) \\ K_{full} &= K_{sim} + \eta\Big[\partial_i K_{sim} \delta H_0^i + \partial_{\bar{i}} K_{sim} \delta \bar{H}_0^{\bar{i}}\Big] + \mathcal{O}(\epsilon, \eta^2)\,. \qquad (4.7) \end{aligned}$$

The leading shift δH_0^i is

$$\delta H_0^i = -W_0^{ij}\partial_j \widetilde{W}_0\,, \qquad (4.8)$$

with $W_0^{ij} \sim 1/m_H$ and H_1^i as in eq.(3.42). By plugging back eqs.(3.42) and (4.8) in eqs.(4.6) and (4.7), one easily finds

$$\begin{aligned} W_{full} &= W_{sim} - \frac{1}{2}\eta^2 \partial_i \widetilde{W}_0 W_0^{ij} \partial_j \widetilde{W}_0 - \epsilon\eta \partial_i \widetilde{W}_0 W_0^{ij} \partial_j W_1 - \frac{1}{2}\epsilon^2 \partial_i W_1 W_0^{ij} \partial_j W_1 + \\ &\quad \mathcal{O}(\eta^3, \eta^2\epsilon, \eta\epsilon^2, \epsilon^3)\,, \qquad (4.9) \\ K_{full} &= K_{sim} - \eta\Big[\partial_i K_{sim} W_0^{ij} \partial_j \widetilde{W}_0 + \partial_{\bar{i}} K_{sim} \overline{W}_0^{\bar{i}\bar{j}} \partial_{\bar{j}} \overline{\widetilde{W}_0}\Big] + \mathcal{O}(\epsilon, \eta^2)\,. \qquad (4.10) \end{aligned}$$

Let us now see the structure of the induced operators, not present in the simple model, and their possible relevance, recalling that the meaningful region we can explore in the light charged field directions is defined to be $|C|/m_H \lesssim \mathcal{O}(\epsilon)$, otherwise large masses would be generated. We will assume that $H^i \sim \mathcal{O}(m_H)$ to simplify the following scaling analysis. The leading C-dependent terms of W_{full} and K_{full} (which are in W_{sim} and K_{sim}) are of $\mathcal{O}(\epsilon^3 m_H^3)$ and $\mathcal{O}(\epsilon^2 m_H^2)$ respectively, and it is easily shown that correspondingly the leading C-dependent terms $V(C)$ of the scalar potential are of $\mathcal{O}(\epsilon^4 m_H^4)$. Notice that it is crucial to take $\exp(K/2) W_0 = \mathcal{O}(\epsilon m_H)$ in SUGRA, otherwise terms of $\mathcal{O}(\epsilon^3 m_H^4)$ would appear in the scalar potential, invalidating the equivalence of the simple and full theories. It is straightforward to see that all the induced couplings appearing in W_{full} are at most of $\mathcal{O}(\epsilon^4 m_H^3)$ and those appearing in K_{full} of $\mathcal{O}(\epsilon^3 m_H^2)$, so that

$$V(C)_{full} = V(C)_{sim} + \mathcal{O}(\epsilon^5 m_H^5)\,. \qquad (4.11)$$

Eq.(4.11) implies that the e.o.m. of the light fields C^α are the same in both approaches up to $\mathcal{O}(\epsilon^3)$, which is the first non-trivial order for these fields, being by assumption $\langle C^\alpha \rangle \sim \mathcal{O}(\epsilon m_H)$. Of course, as far as the C–independent $\mathcal{O}(\epsilon^2)$ scalar potential is concerned, the results of chapter 3 still applies, implying, in particular, that the leading e.o.m. of the M fields are identical in the full and simple theories. Notice that the $\mathcal{O}(\epsilon^5 m_H^5)$ terms in eq.(4.11) arises only from $\mathcal{O}(\epsilon)$ corrections to coefficients of operators present in $V(C)_{sim}$ and not from the new higher derivative operators induced by the heavy field integration. The latter, as we will see below, are sub-dominant, being at most of $\mathcal{O}(\epsilon^7 m_H^6)$.

The structure of the higher dimensional operators which are generated by the heavy field integration is easily seen from W_{full} and K_{full}. The term proportional to η^2 in W_{full} gives rise to new induced couplings between the charged fields of the form $Y_{N_l} Y_{N_m} C^{N_l+N_m}$. Their coefficients scale as $1/m_H$, which is higher than the natural scale $\mathcal{O}(1)$ for such operators. It is easy to see that the coefficients of all the higher dimensional operators induced by the terms in W_{full} proportional to $\epsilon\eta$ and ϵ^2 are smaller than their natural values. Similarly, the terms proportional to η in the Kähler potential K_{full} give rise to C^N higher dimensional operators with coefficients of $\mathcal{O}(1/m_H)$, higher than their $\mathcal{O}(1)$ natural values, implying that holomorphic cubic terms in C are not reliable in the simple effective Kähler potential. One can also compute the structure of the lowest dimensional induced operators appearing in the scalar potential $V_{full}(C)$, but possibly not present in $V_{sim}(C)$. We get

$$\begin{aligned}\delta V(C) \sim \quad & \frac{Y_{N_l} Y_{N_m} Y_{N_n}}{m_H} C^{N_l+N_m+N_n-2} + \frac{Y_{N_l} Y_{N_m} \mu_{M_n} \epsilon}{m_H} C^{N_l+N_m+M_n-2} \\ & + \frac{Y_{N_l} \mu_{M_m} \mu_{M_n} \epsilon^2}{m_H} C^{N_l+M_m+M_n-2} + \dots . \end{aligned} \tag{4.12}$$

where we have schematically denoted by Y_{N_l} and μ_{M_m} couplings and their derivatives with respect to H and M. In eq.(4.12) we omitted generic M dependent coefficients and we have not distinguished between fields and their complex conjugates. The ellipsis contains terms which are of the same or higher order in ϵ. The lowest dimensional operators appearing in $\delta V(C)$ are of order C^5 but with suppressed couplings. The only operators with coefficients higher than their natural values are those appearing in the first term of eq.(4.12), of $\mathcal{O}(C^7)$.

Let us point out, however, that a more proper point of view on the simple theory as an EFT should regard it as a theory valid only up to the mass scale of the heavy fields H rather than up to the cut-off scale, so that higher order operators in the simple model are naturally suppressed by m_H and not by one, i.e. the cut-off scale. Therefore, the corrections we are finding for the coupling $C^{N_l+N_m}$ in the superpotential, and $C^{N_l+N_m+N_n-2}$ in the scalar potential, should be compared with the natural coupling in the simple model correctly scaled now by $\mathcal{O}(m_H^{3-N_l-N_m})$ and $\mathcal{O}(m_H^{6-N_l-N_m-N_n})$ respectively, which indeed are larger than the one induced by the integration as far as $m_H \leq 1$, as it is in fact the case. Similar discussion follows with the induced couplings in the Kähler potential.

4.1.1 Comments on the component approach

Probably the simplest way to recover the previous result using the orthodox way of integrating fields from the scalar potential as we did in chapter 3, is by realizing that in the limit in which Y_N are of $\mathcal{O}(\epsilon)$, the superpotential (4.2) is effectively of the form (3.1) and the results of chapter 3 apply. In particular these imply $\delta V(C) = \mathcal{O}(\epsilon^3)$, explaining why the minimum sum of powers of Y_N and ϵ in eq.(4.12) is three. The exercise of computing the effective scalar potential shows besides the terms reported in (4.12) further terms, all of them again scaling as $\mathcal{O}(\epsilon^3)$ in the limit $Y_N \to \epsilon$. Indeed, it is straightforward to explicitly check that cubic or higher order terms in the light auxiliary fields (including the compensator in SUGRA) can be also of the same order of the ones obtained above, so is expected, in the sense explained in section (2.2.3), that these further terms appear. Luckily enough these are at most of the same order and moreover, of the very same structure! We choose, however, not to show the procedure here as is still rather complicated compared to the one done supersymmetrically, and at the same time not elucidating at all. Instead, let us try to show how the terms we found appear from the integration of the heavy modes, and how further terms, with the characteristics we just claimed indeed do appear.

The Yukawa term in the superpotential induce terms in the potential of the form

$$\int d\theta^2 Y_N(H,M) C^N \Phi^3 \sim e^{K/2} \partial_i Y_N C^{N-1} \psi^i \psi^C \,, \tag{4.13}$$

where Φ is the compensator multiplet whose scalar component we have fixed to $e^{K/3}$ (see sec.(2.2.1)), and ψ^i (ψ^C resp.) is the spinor component of H^i (C resp.). From these couplings the integration of the heavy spinor ψ^i induces a term in the effective Lagrangian of the form,

$$\sim \frac{1}{m_H} \partial_i Y_{N_l} \partial_j Y_{N_m} C^{N_l+N_m-2} \psi^C \psi^C \,. \tag{4.14}$$

which can be easily understood as coming from the term $\partial_i Y_{N_l} \partial_j Y_{N_m} C^{N_l+N_m}/m_H$ in the effective superpotential we found by the manifestly SUSY approach. A coupling like,

$$\int d\theta^4 (-3e^{-K/3} \Phi \bar{\Phi}) \sim \partial_i \partial_\mu \partial_{\bar{\mu}} \partial_{\bar{\nu}} K_0 \psi^i \psi^\mu \bar{\psi}^{\bar{\mu}} \bar{\psi}^{\bar{\nu}} \,, \tag{4.15}$$

combined with the one in eq.(4.13) generates the following effective coupling,

$$\sim \frac{1}{m_H} \partial_i Y_N C^{N-1} \bar{\psi}^{\bar{\mu}} \psi^\mu \bar{\psi}^{\bar{\nu}} \psi^C \,, \tag{4.16}$$

which can be identified with the leading induced couplings of the form C^N/m_H we found in the effective Kähler potential. Of course these are not all the possibilities but from these examples is clear how is possible to express them in a manifestly SUSY way by inferring the corresponding term ins the Kähler potential or superpotential.

Let us see an example of an extra term we cannot see using the approximate chiral equation. In the scalar potential we find terms of the form,

$$\int d\theta^2 Y_N(H,M) C^N \Phi^3 \sim e^{K/2} F^C Y_N C^{N-1} \sim e^K \left(\partial_{\bar{i}} \overline{W}_0 C + \bar{Y}_{N_m} \bar{C}^{N_m-1}\right) Y_{N_l} C^{N_l-1} , \tag{4.17}$$

where we have used the on-shell expression for F^C eq.(2.19) and, in order to express explicitly the power of C, we used the fact that $K^{\alpha\bar{i}} \sim C^\alpha$ and $K^{\alpha\bar{\beta}} \sim 1$. So that the couplings $\partial_{\bar{j}\bar{i}} \overline{W}_0 Y_N \bar{H}^{\bar{i}} C^N \sim m_H H^i C^N$ and $\partial_i K H^i C^{N_l-1} \bar{C}^{N_m-1}$ are realized, where we have used $\partial_{ji} W_0 \sim m_H$. These, when the H^i are integrated out, since the scalar field propagator goes like m_H^{-2}, induce the following couplings,

$$\sim \frac{1}{m_H} Y_{N_l} Y_{N_m} \bar{Y}_{N_n} C^{N_l+N_m-1} \bar{C}^{N_n-1} . \tag{4.18}$$

Notice that although these have the same structure of the couplings we show in eq.(4.12) the Yukawa couplings here can be constant, as far as there is Kähler mixing between H^i and C^α. Indeed, from the approximate chiral equation we used in the manifestly SUSY approach, it is clear that there these couplings necessarily require the Yukawas to depend in the H^i fields.

There are other kind of induced couplings, like the ones coming by combining the first term in eq.(4.17) with itself, that are not suppressed by the heavy fields mass,

$$\sim Y_{N_l} \bar{Y}_{M_m} C^{N_l-1} \bar{C}^{N_m-1} . \tag{4.19}$$

However, the coupling $\partial_{\bar{j}\bar{i}} \overline{W}_0 Y_N \bar{H}^{\bar{i}} C^N$ actually arises from all the terms $F^M \partial_M W$ in the potential, and the sum of the coefficients of the induced coupling leads exactly to the one obtains from the simple model. Indeed, as mentioned above, in the limit $Y_N \to \epsilon$ these couplings are $\mathcal{O}(\epsilon^2)$ therefore from the results of chapter 3 such a coupling should match a leading order with the one of the simple model.

4.2 Chiral and vector multiplets

We introduce in this section the vector multiplets by switching on the gauge couplings. We assume that at the vacuum a gauge group $\mathcal{G}$ is spontaneously broken to a subgroup $\mathcal{H}$ at a scale parametrically larger than ϵ. The gauge group $\mathcal{H}$ might be further broken to a subgroup, but only at scales of $\mathcal{O}(\epsilon m_H)$. As introduced in section (2.2.2) we denote by X_A^M and $\overline{X}_A^{\bar{M}}$ the holomorphic and anti-holomorphic Killing vectors generating the (gauged) isometry group $\mathcal{G}$, defined as $\delta\phi^M = \lambda^A X_A^M$, $\delta\bar{\phi}^{\bar{M}} = \lambda^A \overline{X}_A^{\bar{M}}$, with λ^A

infinitesimal real parameters. The corresponding D-terms are

$$D_A = iX_A^M G_M = -i\overline{X}_A^{\bar{M}} G_{\bar{M}}\,, \tag{4.20}$$

where $G = K + \log|W|^2$ and $G_M = \partial_M G$ are the Kähler invariant function and its derivatives[2]. We denote by capital Latin letters $A, B, \ldots = 1, \ldots, \mathrm{adj}(\mathcal{G})$, the gauge group indices, not to be confused with the light field indices $\mathcal{A}, \mathcal{B}, \ldots$ introduced before. For simplicity of presentation, we take the holomorphic gauge kinetic functions f_{AB} diagonal in the gauge indices, so that

$$f_{AB} = \delta_{AB} f_A(H^i, M^\mu, Z^{\hat{\alpha}}, C^\alpha) \tag{4.21}$$

are generic holomorphic functions and $\mathrm{Re}\, f_A = 1/g_A^2$, with g_A the coupling constants of $\mathcal{G}$.

As already stated in the beginning of the chapter, contrary to the pure F-term case, the condition $\partial_i W_0 = 0$ may not fix all the fields H^i. Indeed, gauge invariance relates the derivatives of W_0 by

$$X_A^i \partial_i W_0 = 0\,, \tag{4.22}$$

so that in general they are not linearly independent and some of the fields may remain unfixed. Thus, if the X_A^i are not all vanishing, one can always choose a basis in which some of the fields do not appear at all in W_0. It is very simple to explicitly construct such a basis for the relevant case where the H^i are moduli shifting under a $U(1)$ gauge symmetry. If $\delta H^i = i\xi^i\Lambda$, with Λ the chiral super-field associated to the $U(1)$ transformation, one can always choose to parametrize W_0 in terms of, say, $M^1 \equiv H^1$ (with $\xi^1 \neq 0$) and the $n_H - 1$ gauge invariant operators $H_{GI}^i = \xi^1 H^i - \xi^i H^1$, with $i > 1$. One can invert this relation and use the fields H_{GI}^i in W_0. In this field basis, the only non-vanishing isometry component is X^1. From eq.(4.22) we immediately see that W_0 is independent of M^1, depending only on the $n_H - 1$ gauge invariant fields H_{GI}^i. Of course the same argument can be repeated for each $U(1)$ generator independently. Hence there is no well defined meaning in freezing M^1 being necessarily a flat direction of W_0. The situation in fact stronger since the concept of freezing a charged fields is not even a meaningful gauge invariant statement, since $\delta M^1 \neq 0$ and thus M^1 will necessarily enter in W (if any) in a gauge invariant combination with light fields. One might, instead, get rid of M^1 by gauging it away, namely choosing a gauge (which is not the Wess-Zumino gauge) where it is a constant. The dynamics of M^1, however, do not disappear being encoded in the $U(1)$ vector multiplet, more precisely in the longitudinal component of

[2]As explained in section (2.4) we do not consider constant FI-terms as they are incompatible with a quantum description of SUGRA, as it is the case for string theory.

the gauge field and the lowest auxiliary component, which in the new gauge are dynamical. If the vector field is sufficiently massive and is integrated out, the effects of M^1 will eventually appear in new contributions to the Kähler potential of the remaining light fields. A direct relevant consequence of the above result is the impossibility of naively neglect moduli fields responsible for field-dependent Fayet-Iliopoulos terms introduced in section (2.4), and/or forgetting the implicit gauge-fixing taken behind this choice, as sometimes done in the literature in local string constructions, where all moduli dynamics is neglected altogether. Clearly, this obstruction has nothing to do with gravity and is purely dictated by gauge invariance, therefore holds in the global SUSY case as well. This observation actually dates back to [50] and was more generally expressed in [36]. In conclusion, for all proper heavy fields we must impose[3]

$$X_A^i = 0\,. \tag{4.23}$$

In addition to eq.(4.23), for simplicity we also assume that the remaining isometry components do not depend on H^i, $X_A^{\mathcal{A}} = X_A^{\mathcal{A}}(L^{\mathcal{B}})$.

Notice that the vector multiplets do not enter in the approximate chiral e.o.m. for the heavy fields, eq.(2.10). Therefore, the inclusion of gauge dynamics does not affect the results of section (4.1). However, a claim made in obtaining this equation in section (2.1.1) was not fully true (see footnote (4) in chapter 2), namely that neglecting the supercovariant derivatives in the e.o.m. lead to and exact theory at the two derivative level even in presence of vector multiplets. It turns out , in fact, that even at the two derivative level terms with covariant derivatives in the Kähler potential may now appear, coming from products of the auxiliary fields of the form DF or D^2. However, being the auxiliary component D the $\theta^2\bar{\theta}^2$ component of the vector multiplet, these terms in K arises only from induced terms with at least two covariant derivatives, and by dimensional analysis necessarily further suppressed by m_H. They are, then, in the low energy EFT necessarily suppressed by ϵ or ϵ^2 with respect to the F^2 generated terms and can thus be neglected. We stress the fact that is only in the low energy EFT, where possible heavy vector fields have been already integrated out, that these terms are always negligible, being clear that before integrating out the vector multiplets the coefficients for such terms can be $\mathcal{O}(m_V/m_H)$, i.e. non-suppressed.

The conclusion of the previous discussion is that we can safely use the solutions we found, eqs.(3.42) and (4.8), which lead to the full effective superpotential and Kähler potential, shown in eqs.(4.9) and (4.10), and now also to the effective holomorphic gauge

[3] Of course there is the possibility of vector like representations that acquire heavy masses and vanishing VEV avoiding this obstruction, since for them $\langle X_A^i \rangle = 0$, like in the case for charged Kaluza–Klein fields. We do not consider such a case here being, though, clear that a proper integration induces only non-renormalizable couplings suppressed by their mass.

kinetic functions f_{AB},

$$f_{AB,full} = f_{AB,sim} + \partial_i f_{AB} \delta H_0^i + \mathcal{O}(\epsilon, \eta^2)\,. \tag{4.24}$$

Eq.(4.24) implies that all C–independent terms and C–dependent ones up to C^{N-1} (included) entering in the f_{AB} are reliable at $\mathcal{O}(1)$ in ϵ.

Let us analyze the effective action first in the unbroken gauge symmetry case, namely all M fields neutral. The effective Kähler potential (4.10) gives rise to corrections to the D-terms with schematic form

$$\begin{aligned} D_{A,full} - D_{A,sim} &= iX_A^\alpha \partial_\alpha \Big[\partial_i K_{sim}(\eta \delta H_0^i + \epsilon H_1^i) + h.c.\Big] + \mathcal{O}(\epsilon^2, \eta^2, \epsilon\eta) \\ &= \frac{Y_{N_i}}{m_H} C^{N_i+2} + \epsilon C^2 + \frac{\epsilon \mu_{M_i}}{m_H} C^{M_i+2} + \dots\,, \end{aligned} \tag{4.25}$$

where we have set $\eta = 1$ in the second row of eq.(4.25) and where, for concreteness, we have counted the powers of C (including their complex conjugates) by taking linear realizations of the gauge group (X_A^α proportional to C^α). Gauge invariance of $\widetilde{W}_0$ and W_1 has been used, constraining $X_A^\alpha \partial_\alpha \delta H_0^i$ and $X_A^\alpha \partial_\alpha H_1^i$ to vanish.

We can easily extract from eq.(4.25) the lower dimensional operators generated by the heavy field integration, appearing in the D–term scalar potential $V_{D,full}$ but not present in $V_{D,sim}$. They are of the form $g^2 Y_{N_i} C^{N_i+4}/m_H$ and $\epsilon \mu_{M_i} g^2 C^{M_i+4}/m_H$. Both are at most of $\mathcal{O}(\epsilon^7 m_H^6)$ and hence irrelevant. As already mentioned, in presence of vector multiplets, quadratic terms in the auxiliary fields of the form DF and D^2 are missed. It is useful to explicitly see how this discrepancy arises in our class of models. By construction, all the effective operators one obtains from the effective D-term scalar potential using the approximate chiral e.o.m where covariant derivatives are neglected, give rise to only operators proportional to g^2, as above. On the other hand, it is obvious that in presence of a Kähler mixing term between the charged fields and the heavy fields, higher order operators with coefficients proportional to $(g^2)^2/m_H^2$ should be expected. By explicitly computing the scalar potential in components, indeed, we find

$$\delta V_D \sim \frac{g^2 Y_{N_i}}{m_H} C^{N_i+4} + \frac{\epsilon g^2 \mu_{M_i}}{m_H} C^{M_i+4} + \frac{(g^2)^2}{m_H^2} C^8\,. \tag{4.26}$$

The first two terms are exactly those found in the manifestly SUSY case, whereas the latter arises from the undetected D^2 terms. As advertised before, this term is negligible being of $\mathcal{O}(\epsilon^8 m_H^6)$.

In presence of charged M fields the gauge group $\mathcal{G}$ is spontaneously broken to a subgroup $\mathcal{H}$, and we will distinguish between the two set of generators by a splitting in the gauge index $A = (a, \hat{a})$, $\hat{a} \in \mathcal{G}/\mathcal{H}$, $a \in \mathcal{H}$. The analysis of the low energy EFT in this case is more involved due to the presence of extra heavy stuff, the vector bosons $A_{\hat{a}}$, associated with the broken generators, which acquire masses through the Higgs mechanism, and the real scalar fields superpartners of the would-be Goldstone bosons, which acquire a heavy mass from the D-term dynamics. These fields combine in a massive vector multiplet of $\mathcal{N}=1$ SUSY, and their masses squared are splitted by the SUSY breaking scale encoded in the VEV of the auxiliary field $D^{\hat{a}}$. Such fields cannot be frozen to their VEV, as can easily understood by the fact that the superpartner of the would-be Goldstone is stabilized by the D-term dynamics which has $\mathcal{O}(1)$ dependencies on the light fields VEV, so a proper integration is necessary.

Since the mass splitting inside the massive vector multiplet is dictated by the VEV of D, a manifestly SUSY integration of a vector super field requires that $\langle D\rangle/m_V^2 \ll 1$, with m_V the gauge field mass. In our case this condition is always satisfied, as shown in Appendix A, being $\langle D^{\hat{a}}\rangle = \mathcal{O}(\epsilon^2)$. As shown in section (2.1.1) a vector super field is supersymmetrically integrated out, neglecting covariant derivative terms coming from the holomorphic gauge field action, by setting [50]

$$\partial_{V^{\hat{a}}} K = 0\,, \tag{4.27}$$

$V^{\hat{a}}$ being the vector-superfields associated to the broken generators. For typical charged field Kähler potentials, eq.(4.27) does not admit a simple constant solution, meaning, again, that in general is hopeless to freeze the heavy vector multiplet.

Integrating out a vector super field implies a choice of gauge fixing. The physical gauge where one gets rid of the eaten Goldstone bosons and their superpartners is the superfield version of the unitary gauge. On the other hand, it is practically easier to work in a gauge where an arbitrary chiral charged field $\widetilde{M}$ with a non-vanishing component in the would-be Goldstone directions, i.e. $\langle \widetilde{X}_{\hat{a}}\rangle \neq 0$, is frozen at its VEV $\widetilde{M}_0$. This can be done for each broken generator so that $\dim \mathcal{G}/\mathcal{H}$ light chiral multiplets (or combinations thereof) are gauged away from the theory. Let us denote by $L^{\mathcal{A}'}$ the remaining directions, with $\mathcal{A}' = 1,\ldots,n_L - \dim\mathcal{G}/\mathcal{H}$, and by $V_0^{\hat{a}}$ the solution to eq.(4.27). Plugging back in the Lagrangian $V^{\hat{a}} = V_0^{\hat{a}}$, we get the SUSY effective theory with heavy vector fields integrated out. As far as the chiral fields are concerned, the correction terms one obtains from the holomorphic gauge kinetic terms are negligible, being suppressed by four covariant derivatives with respect to the corrections terms coming from the effective Kähler potential $K' = K(V^{\hat{a}} = V_0^{\hat{a}})$.[4] The gauge fixing should also be plugged in W, giving rise to a superpotential W', which is a function of the H^i and $L^{\mathcal{A}'}$. The effective

[4] A simple explicit expression for $V_0^{\hat{a}}$ and K' is obtained at the Gaussian level. In this case one gets,

$$V_0^{\hat{a}} = -\widetilde{m}^{-2}_{\hat{a}\hat{b}} K_{\hat{b}}/2\,, \tag{4.28}$$

D'_a term now reads

$$D'_a = iX_a^{\mathcal{A}'}G'_{\mathcal{A}'} = -i\overline{X}_a^{\bar{\mathcal{A}}'}G'_{\bar{\mathcal{A}}'}\,, \tag{4.30}$$

with $X_a^{\mathcal{A}'}$ the components of the pulled back isometry vectors on the Kähler invariant function G', defined by the gauge fixing. Since the latter is linear in the fields, the pull-back is trivial and $X_a^{\alpha'}$ are nothing else than the original isometry vector components along the non-gauged away directions $L^{\mathcal{A}'}$.
After having integrated out the massive vector fields and their scalar partners, we get an intermediate effective theory given by the Kähler potential K', isometry Killing vectors $X_a^{\mathcal{A}'}$, superpotential W' and gauge kinetic functions f_a, with $\mathrm{Re}\, f_a = 1/g_a^2$. No field with non-vanishing VEV and charged under $\mathcal{H}$ appears in this theory. All the charged M have been either gauged away or appear as gauge singlet combinations in the intermediate theory, effectively behaving as new gauge invariant fields. By expanding K' in powers of the charged fields we get a Kähler potential of the form (4.4) (where K'_0, K'_1 and K'_2, in turn, can be expanded in powers of the heavy vector mass) and we are effectively back to the case discussed before of unbroken gauge group, comparing the resulting theory after integrating out the H with the simple one defined by $K'_{sim} = K'(H_0)$, $W'_{sim} = W'(H_0)$, $f_{a,sim} = f_a(H_0)$.

Usually one is interested in the simple model where the vector multiplets have not been integrated out, as wee will see in an explicit example in chapter 5, so one might be interested in comparing the low energy EFT of this simple theory with the one of the full integration we just preformed. Nicely enough, this is exactly what is done in last step above, as the K'_{sim} and W'_{sim} exactly coincide with the Kähler and superpotential obtained by integrating out the heavy vector fields from the simple theory, when the same gauge-fixing taken in the full theory is used. This closes the proof of the reliability of the simple description even in presence of gauge dynamics.

4.3 Comments on including messenger fields

As mentioned in the beginning of the chapter, the study with only cubic, and higher, $\mathcal{O}(1)$ couplings for the matter fields in the superpotential is rather restrictive, as in principle there might be heavy fields relevant for the physics of the visible sector. A first example of this sort of fields are the messenger fields of gauge mediation scenarios [93], so let us study this case in some detail. This kind of fields will appear in the original

where $\widetilde{m}^2_{\hat{a}\hat{b}} = 2\langle g_{M\bar{N}}X_{\hat{a}}^M\overline{X}_{\hat{b}}^{\bar{N}}\rangle$ is the non-canonically normalized mass matrix for the gauge fields and $K_{\hat{a}} = \partial_{V^{\hat{a}}}K|_{V=0}$. Plugged back in K, it gives

$$K' = K(V^{\hat{a}} = 0) - K_{\hat{a}}\widetilde{m}^{-2}_{\hat{a}\hat{b}}K_{\hat{b}}/4\,. \tag{4.29}$$

superpotential with a $\mathcal{O}(1)$ mass term and no direct coupling with the visible matter, so it is enough to add the term

$$W \sim m_\ell \ell \tilde{\ell}\,, \tag{4.31}$$

with m_ℓ a field dependent mass parameter, and ℓ-$\tilde{\ell}$ a vector-like pair of messenger fields. Notice that the mass term, in order the messenger to be kept in the low energy EFT, should be smaller than m_H. Their Kähler potential is like for the the other charged fields eq.(4.4), with the only constrain of allowing only non-renormalizable mixing with the visible sector. Nicely enough the analysis performed in section (4.1) is general enough that we can directly read the new induced coupling in presence of these fields. Regarding the pure messenger sector we read the following leading induced couplings in the superpotential and Kähler potential,

$$W_{full} \sim \frac{m_\ell^2}{m_H}(\ell\tilde{\ell})^2\,, \tag{4.32}$$

$$K_{full} \sim \frac{m_\ell}{m_H}\ell\tilde{\ell}\,. \tag{4.33}$$

Like before, the meaningfull region in field space we can explore is bounded by $|\ell| \lesssim \mathcal{O}(\epsilon m_H)$, otherwise the vector bosons of the Standard Model would acquire large masses. In such a case the leading terms for the messengers in the superpotential and Kähler potentail, given by the terms in W_{sim} and K_{sim} are of $\mathcal{O}(m_\ell m_H^2 \epsilon^2)$ and $\mathcal{O}(m_H^2\epsilon^2)$ respectively, meanwhile the induced couplings on each one scale like $\mathcal{O}(m_\ell^2 m_H^3 \epsilon^4)$ and $\mathcal{O}(m_\ell m_H \epsilon^2)$ respectively. The induced coupling in the superpotential is clearly suppressed, instead, the one for the Kähler potential is less obvious. In this case a sightly suppression is taken part due to the constrain $m_\ell \ll m_H$ so that the induced coupling still can be neglected. However, more important is the fact that this coupling does not enters in the leading terms of the metric, which are the most relevant for the e.o.m. and the masses of the messengers, that is information that can affect the visible sector. Indeed, in the same way we did with the visible sector, one can check that the leading corrections to the simple scalar potentail of the pure messenger sector are again solely due to the shift in the VEV of H^i and scale like $\mathcal{O}(m_\ell^2 m_H^2 \epsilon^3)$, while the leading term in the simple scalar potential scale like $\mathcal{O}(m_\ell^2 m_H^2 \epsilon^2)$, then the e.o.m. are not affected at leading order. The terms due to the new induced coupling, as in the previous case, are further suppressed with the leading terms scaling like $\mathcal{O}(m_\ell^2 m_H^3 \epsilon^4)$. s Having checked the reliability of the simple description in the pure messenger sector, let us turn to the possible effects due to new induced coupling on the visible sector. From the results of section (4.1) we read the following induced direct couplings between the messenger and the visible matter:

$$W_{full} \sim \frac{1}{m_H} m_\ell Y \ell\tilde{\ell} C^3 + \frac{\epsilon}{m_H} m_\ell \mu \ell\tilde{\ell} C^2\,. \tag{4.34}$$

Notice that in these couplings a further small suppression appear due to the condition $m_\ell \ll m_H$, so that the induced couplings are smaller than the natural ones with $\mathcal{O}(1)$ coefficients. As before these couplings in the Lagrangian are completely irrelevant, as can be checked by noticing that with a mixing in the Kähler potential of the form $K \sim (|\ell|^2 + |\tilde{\ell}|^2)|C|^2$ the induced coupling from the messenger point of view is for all effects exactly like a mass term of $\mathcal{O}(m_\ell m_H^2 \epsilon^3)$, and from the visible sector point of view is for all effects like a Yukawa coupling of $\mathcal{O}(\epsilon^2 m_\ell / m_H)$ and a mass term of $\mathcal{O}(\epsilon^3)$.

Before trying to figure out more in detail the effects of such coupling on the soft terms for the visible sector, let us point out some terms potentially present from moduli mediation and the scaling for them. From the F-term density in the Lagrangian, we have the following contributions to the A-term, $B\mu$-term and soft masses,

$$\int d\theta^2 (YC^3 + \epsilon\mu C^2)\Phi^3 \sim e^{K/2}\partial_\nu Y F^\nu C^3 + \epsilon e^{K/2}\partial_\nu \mu F^\nu C^2 + e^K \epsilon^2 |\mu|^2 |C|^2 \,, \tag{4.35}$$

where as usual the compensator have been fixed to $\Phi = e^{K/6}$ and used $F^C \sim \epsilon\bar{\mu}\bar{C}$. We see, then, that the A-terms are $\mathcal{O}(\epsilon)$, meanwhile the mass terms are $\mathcal{O}(\epsilon^2)$. The D-term density of the Lagrangian also contributes to the mass terms, using eq.(4.4) we have (neglecting the explicit μ term in the original superpotential which leads to the same conclusions),

$$\int d\theta^4 (-3e^{-K/3})|\Phi|^2 \sim \partial_\nu \partial_{\bar{\nu}} K_1 |F^\nu|^2 |C|^2 + \partial_\nu \partial_{\bar{\nu}} K_2 |F^\nu|^2 C^2 \,, \tag{4.36}$$

so again we find an $\mathcal{O}(\epsilon^2)$ for these terms. Although we have pointed out only few contributions to the non-canonically normalized soft terms it turns out that the complete set of contributions lead to the same order in ϵ [94, 93].

To extract information on the soft terms from the couplings (4.34), we integrate out the messengers, which necessarily is a one-loop procedure. As usual the superpotential is not affected being protected by non-renormalization properties [93], instead, in the Kähler potential a mixing between the mass operator of the messenger and the operators in eq.(4.34) is induced, generating a direct coupling between the spurion field, X, inside $m_\ell \sim X$, and the visible sector. More precisely, up to loop and $\mathcal{O}(1)$ coefficients, the following terms are generated in the Kähler potential (see e.g. ref.[93] pg. 88),

$$K \sim \frac{|m_\ell|^2}{m_H}(YC^3 + \epsilon\mu C^2)\ln(X\bar{X}) \,. \tag{4.37}$$

Plugging these in the D-term density of the Lagrangian, we read the following contributions to A-term, $B\mu$-term and soft-mases,

$$\mathcal{L} \sim \frac{m_\ell^2}{m_H}\left|\frac{F^X}{m_\ell}\right|^2 \left(YC^3 + \epsilon\mu C^2\right) + \epsilon^2 \frac{|m_\ell|^2}{m_H}\frac{F_X}{m_\ell}|\mu|^2|C|^2 \,. \tag{4.38}$$

where we have used $F^C \sim \epsilon\bar{\mu}\bar{C}$ and regarded $m_\ell \sim X$. We see, then, that all of them are suppressed by $\mathcal{O}(\epsilon)$ terms, and further powers of m_ℓ/m_H, compared to the standard contributions we figured out above. Therefore the contributions from such coupling, and the couplings itself, can be consistently neglected and the analysis done in the previous sections for the visible sector is unaffected by adding the messengers.

4.4 Generalized nearly factorizable models

As already pointed out in section (3.3.3), the explicit realization from string theory of the approximate factorizable case is more restrictive since imposes further conditions on the form of the Kähler potential. This situation is even clearer once matter, not necessarily all in the visible sector, is added. Indeed, being the kinetic functions a function of both, the complex structure and the Kähler moduli, the factorizability, as we stated it, is completely lost upon some of these fields can acquire $\mathcal{O}(1)$ VEV. Still a generalization of the nearly factorizable case can be constructed inspired in the LARGE volume scenarios we studied in section (3.3.3). It follows from this discussion that in this case a real distinction exits between the moduli and the matter, therefore we will distinguish them denoting the matter, in general, by Q, so that $L = \{M^\mu, Q^\alpha\}$ and $\phi^M = \{H^i, M^\mu, Q^\alpha\}$. Consider now a system described by a Kähler invariant function of the form[5],

$$G = G_H(H^i, \bar{H}^{\bar{i}}) + G_M(M^\mu, \bar{M}^{\bar{\mu}}) + \epsilon G_{mix}(\phi^M, \bar{\phi}^{\bar{M}})\,, \tag{4.39}$$

which in terms of the Kähler potential and superpotential can be realized with $K = K_H(H) + K_M(M) + \epsilon K_{mix}(\phi^M)$ and $W = W_H(H)W_M(M) + \epsilon W_1(\phi^M)$, so that the fields Q only appear in the suppressed parts. The structure of inverse metric is then of the following form,

$$(K_{M\bar{N}})^{-1} \sim \begin{pmatrix} \mathcal{O}(1) + \mathcal{O}(\epsilon)\tilde{g}(Q) & \mathcal{O}(1)\tilde{h}(Q) \\ \mathcal{O}(1)\overline{\tilde{h}(Q)} & \mathcal{O}(\epsilon^{-1}) \end{pmatrix}, \tag{4.40}$$

[5]Let us for the moment deal only with matter fields with $\mathcal{O}(\epsilon)$ dependency in the superpotential so that can develop $\mathcal{O}(1)$ VEV's, being these the problematic ones for moduli stabilization issues.

where the first block refers to the moduli directions, and $\tilde{g}$ and $\tilde{h}$ are moduli dependent functions of Q. The scalar potential, therefore, can be written as

$$V = V_0(\tilde{\phi}^{\tilde{M}}, \bar{\tilde{\phi}}^{\bar{\tilde{M}}}) + \epsilon V_1(\phi^M, \bar{\phi}^{\bar{M}}) + \mathcal{O}(\epsilon^2)\,, \tag{4.41}$$

where we denoted by $\tilde{\phi}$ the set of all moduli, $\tilde{M} = \{i, \mu\}$. Notice that in the leading part no contribution from the Q sector is introduced, i.e. is exactly the same leading potential studied before in section (3.3.3). We have, then, that the analysis done before for the integration of the H modes follows verbatim, namely $\hat{H} \sim \mathcal{O}(\epsilon)$ and the corrections to the simple model start at $\mathcal{O}(\epsilon^2)$, i.e, the leading and sub-leading potentials for the moduli M are reliable, and for the for the matter Q only its leading one. The canonical normalization and mass eigenstates mixings are again no an issue due to the form of the metric and the hierarchy between the potentials fixing moduli and the Q. Notice, moreover, that indeed there is still a suppression for the expectation value of the auxiliary fields of the H multiplets, with canonical normalized F-terms ($F_c^M = |K_{M\bar{M}}\overline{F}^{\bar{M}} F^M|^{1/2}$ no sum),

$$F_c^\mu \sim \mathcal{O}(1)\,, \quad F_c^Q \sim \mathcal{O}(\epsilon^{1/2})\,, \quad F_c^i \sim \mathcal{O}(\epsilon),. \tag{4.42}$$

This, therefore, presents a generalization of the factorizable model proposed in [36], which allows a more natural introduction of matter-like fields in the game. The introduction of gauge dynamics then is straightforward as far as we keep the H fields neutral and the gauge kinetic function to have only suppressed dependency on H. In this case, is easy to check that,

$$\partial_i V_D \sim \mathcal{O}(\epsilon)\,, \tag{4.43}$$

so that the previous discussion is not affected. Adding gauge dynamics at this stage is, however, far from being realistic, as the dynamics are all of $\mathcal{O}(1)$ so that SUSY is broken at the Plank scale!. As we saw in section (3.3.3) with the LARGE volume scenario this set-up can be connected to realistic models, so let us now turn to study this case in particular.

4.4.1 LARGE volume scenario with matter

The LARGE volume scenario present in fact the generalized form for the Kähler potential introduced in eq.(4.39), with a generic Kähler potential for the matter fields of the form,

$$K \sim \frac{Z(U, \bar{U}, t, \bar{t})}{\mathcal{V}^\alpha} |Q|^2\,, \tag{4.44}$$

where Q is a generic charged field and for simplicity we have omitted possible index contractions and we have allowed a further dependency on the small Kähler modulus independent of the implicit one in the volume. The modular weight α is expected to be a positive number, making also reliable the expansion done in powers of the matter fields, so that a suppression on such terms is indeed realized. As pointed out at the end of section (3.3.3) the introduction of extra stuff developing non-vanishing VEV is compulsory for these scenarios, as the moduli alone are stabilized in an AdS space which should be up-lifted somehow [83].

Let us see that the situation here is very much as the explained for the generalized factorizable case, studying the simplest model which already contains the novel features, namely a single Q without any dependency on the superpotential, so that the superpotential continue to have the form we used in section (3.3.3). Forgetting for the moment possible D-term dynamics, one can compute the scaling of the leading terms depending on Q,

$$V \sim \frac{|Q|^2}{\mathcal{V}^{2+\alpha}}\Big(\big(D_U W|_{Q=0} + W_0(S,U)\big)\overline{W}_0 + h.c.\Big)\,, \tag{4.45}$$

where $\mathcal{O}(1)$ coefficients are omitted (recalling that for this case our expansion parameter is defined as $\epsilon \sim \mathcal{V}^{-1}$). The Q independent part continues to be like in section (3.3.3), with a leading potential scaling as $\mathcal{O}(\epsilon^2)$ fixing the dilaton and complex structure and the one scaling as $\mathcal{O}(\epsilon^3)$ generating dynamics for the Kähler potential. Being $\alpha > 0$ the new contributions to the potential does not destabilize the SUSY solutions for the S and U, which then do not affect the equation of motion of Q, as their contribution come from the two first terms in eq.(4.45). This is exactly what happens for the Kähler moduli in the previous case. Indeed, the first non-vanishing contributions scale like $|Q|^4\mathcal{O}(\epsilon^{2+2\alpha})$, as can be easily seen form the terms $e^K|D_UW|^2$ and $e^K K^{\bar{C}U} D_U W \bar{D}_{\bar{C}}\overline{W} + h.c.$ knowing $K^{\bar{C}U} \sim \bar{C}$. Notice, though, that these terms absent in the case U is regarded as frozen, i.e. the simple model, can start to affect the V_3 potential, eq.(3.85), for the Kähler moduli if $\langle Q\rangle \sim \mathcal{O}(1)$. This imposes a constrain in the modular weight α, namely $\alpha > 1/2$. In fact, this constrain can be found from the analysis of the induced scalar potential once the complex structure moduli are integrated out. Doing the same analysis we did for the pure moduli case on sees that now the fluctuations $\hat{H}$ scale like $\hat{H} \sim \mathcal{O}(\epsilon) + |Q|^2\mathcal{O}(\epsilon^\alpha)$, therefore, the induced scalar potential, scales like, $V_{ind} \sim \mathcal{O}(\epsilon^4) + |Q|^2\mathcal{O}(\epsilon^{3+\alpha}) + |Q|^4\mathcal{O}(\epsilon^{2+2\alpha})$, so that the leading scalar potential for Q is not affected, but if Q has $\mathcal{O}(1)$ VEV, the last term scales as the leading scalar potential for the Kähler moduli unless $\alpha > 1/2$. The model, however, as is so far leads to vanishing VEV of Q, and depending on the value of α it can be stable or a tachionic direction, so the analysis done is misleading as it regards $Q \sim \mathcal{O}(1)$.

A more interesting and realistic situation happens by adding the D-term potential. The explicit system we now study is a stack of branes wrapping the exponentially large cycle, whose size is characterized by the modulus T, and turn-on some world-volume fluxes.

The modulus T then gets charged under a $U(1)$ symmetry as explained in section (2.4.2) (this model was worked out in detail for the simple case by Cremades *et. al.* in ref.[83]). Taking Q as an open string mode of the stack of branes, with a corresponding charge q_Q, the D-term potential takes the schematic form,

$$V_D \sim \frac{1}{T+\bar{T}}\left(\frac{\delta}{\mathcal{V}^{2/3}} + \frac{q_Q Z(U,\bar{U},t,\bar{t})|Q|^2}{\mathcal{V}^{\alpha}}\right)^2 , \tag{4.46}$$

where we have regarded the gauge kinetic function as $f_X \sim T$. Let us see that in this, now complete, system the simple description is always a reliable approximation. The D-term potential scales as $\mathcal{O}(\epsilon^{2/3})(\mathcal{O}(\epsilon^{2/3})+\mathcal{O}(\epsilon^{\alpha})|Q|^2)^2$, and its derivative with respect to U scales like $\mathcal{O}(\epsilon^{2/3+\alpha})|Q|^2(\mathcal{O}(\epsilon^{2/3})+\mathcal{O}(\epsilon^{\alpha})|Q|^2)$ so that seems possible a correction to the $\mathcal{O}(\epsilon^2)$ e.o.m. for the complex structure from the F-term potential. It turns out that this is not the case as two possibilities can happen. That inside the D-term there are no cancellations, so that Q is likely to be stabilized at vanishing VEV, and no effects for the e.o.m. of U arise[6]. Another possibility is that a non-vanishing VEV of Q cancels the D-term. Let us explore this second case more carefully. The full e.o.m for Q, including the leading terms from the F-term potential is schematically given by [83],

$$\frac{|w_0|^2 Q}{\mathcal{V}^{2+\alpha}} + \frac{1}{T+\bar{T}}\left(\frac{\delta}{\mathcal{V}^{2/3}} + \frac{q_Q Z(U,\bar{U},t,\bar{t})|Q|^2}{\mathcal{V}^{\alpha}}\right)\frac{q_Q Z(U,\bar{U},t,\bar{t})Q}{\mathcal{V}^{\alpha}} = 0 , \tag{4.47}$$

so that the non-vanishing solution we are looking for is given by

$$|Q|^2 \sim \frac{\delta}{q_Q}\mathcal{V}^{\alpha-2/3} . \tag{4.48}$$

Therefore, the contribution from the D-term potential to the e.o.m. of U scales as $\epsilon^{8/3}$ and its leading SUSY solutions are preserved.

Plugging back the solution for Q in the D-term and F-term parts we get the respective scalings $\mathcal{O}(\epsilon^{-10/3})$ and $(\delta/q_Q)\mathcal{O}(\epsilon^{-8/3})$. This last one is naturally larger than the V_3 potential for the Kähler moduli, so in order not to spoil the stabilization of the moduli we show in section (3.3.3) and potentially help with the uplifting of the AdS vacuum a tune of order $\mathcal{O}(\epsilon^{1/3})$ is needed[7]. Let us consider, then, that this is the case and take $\delta \sim \mathcal{O}(\epsilon^{1/3})$. Notice, that the terms in the F-term potential we found above not present in the simple model, scaling as $|Q|^4\mathcal{O}(\epsilon^{2+2\alpha})$, scale now as $\delta^2\mathcal{O}(\epsilon^{10/3})$, so are harmless for the Kähler moduli potential.

[6] This case still may destabilize the Kähler moduli, and a tuning in the Green-Schwarz coefficient δ of order ϵ should be argued.

[7] In ref.[83] the tuning is regarded in the δ Green-Schwarz coefficient, however, for volumes larger than 10^5 in string units, such a tuning is rather unnatural even in presence of warpings [95].

The derivatives for the complex structure have as leading terms,

$$\partial_U V \sim \mathcal{O}(\epsilon^3) + \mathcal{O}(\epsilon^{2+\alpha})|Q|^2 + \mathcal{O}(\epsilon^{2/3+\alpha})|Q|^2 D\,, \tag{4.49}$$

where D is the functional form of the D-term. Then the induced potential due to the integration has the scaling properties,

$$V_{ind} \sim \mathcal{O}(\epsilon^{\alpha})\Big(\mathcal{O}(\epsilon^3) + \mathcal{O}(\epsilon^{5/3})D\Big)|Q|^2 + \mathcal{O}(\epsilon^{2\alpha})\Big(\mathcal{O}(\epsilon^2) + \mathcal{O}(\epsilon^{2/3})D + \mathcal{O}(\epsilon^{-2/3})D^2\Big)|Q|^4\,, \tag{4.50}$$

where we have neglected the Q-independent part since we know already is completely irrelevant. Plugging the computed VEV for Q,

$$V_{ind} \sim \delta\left(\mathcal{O}(\epsilon^{11/3}) + \mathcal{O}(\epsilon^{7/3})D\right) + \delta^2\left(\mathcal{O}(\epsilon^{10/3}) + \mathcal{O}(\epsilon^2)D + \mathcal{O}(\epsilon^{2/3})D^2\right)\,. \tag{4.51}$$

With the first terms in each parenthesis we recover the terms we read before directly from the F-term potential. Once we regard the cancellation of the D-term due to the VEV of Q, $D \sim \mathcal{O}(\epsilon^{4/3})$ (notice also that $\partial_T D \sim \mathcal{O}(\epsilon^{4/3})$ as well), all these terms scale as $\mathcal{O}(\epsilon^{10/3})$ or larger powers, even before requiring $\delta \sim \mathcal{O}(\epsilon^{1/3})$, then all these terms are irrelevant for the simple model up to $\mathcal{O}(\epsilon^3)$ included and the stabilization for the Kähler moduli in the simple model is reliable. Remarkably enough this result is independent of the value for the modular weight of Q.

4.4.1.1 Induced couplings

Now that we have seen that at the level of stabilization of the moduli things are save, at least for the particular model studied, let us see the possible induced couplings generated in the visible sector by the integration of the heavy fields in a more general framework. The scaling of the kinetic function for visible matter is known to be $\alpha = 2/3$ [96] and let us not to consider Kähler mixings with matter-like fields developing $\mathcal{O}(1)$ VEV. We proceed as in section (4.1.1) drawing the corrections we obtain integrating out he components of the complex structure. The larger trilinear coupling between the complex structure and the visible sector due to the Kähler mixing comes from the term in the potential[8],

$$\int d^4\theta(-3e^{-K/3}\Phi\bar{\Phi}) \sim \epsilon^{4/3}\partial_U Z\, F^T U \bar{\psi}^{\bar{Q}}\psi^Q\,, \tag{4.52}$$

where we have used $\partial_T \mathcal{V} \sim (\epsilon^{1/3})$, and the usual normalization for the compensator Φ. With the canonical normalized $F_c^T = |K_{T\bar{T}}F^T\overline{F}^{\bar{T}}|^{1/2} \sim \mathcal{O}(\epsilon)$, as can be easily check

[8]Exactly the same conclusion can be raised if instead of T in the coupling we take t, the small Kähler modulus.

from $F^T \sim e^{K/2} K^{\bar{T}T} K_T W_0 \sim \mathcal{O}(\epsilon^{1/3})$, the coupling scales as $\mathcal{O}(\epsilon^{7/3})$ [9]. Integrating out the complex scalar field U leads to a coupling

$$\frac{\epsilon^{14/3}}{m_U^2} |\bar{\psi}^{\bar{Q}} \psi^Q|^2 \sim \epsilon^{8/3} |\bar{\psi}^{\bar{Q}} \psi^Q|^2 \,, \tag{4.53}$$

where we have used the fact that $m_U^2 \sim \mathcal{O}(\epsilon^2)$. A term in the Kähler potential of the form $|Q|^4/\mathcal{V}^\beta$, generates also such a coupling scaling as $\epsilon^\beta |\bar{\psi}^{\bar{Q}} \psi^Q|^2$. Thus as far as $\beta < 8/3$ the corrections to such term in the Kähler potential are irrelevant and the coupling in the simple description is reliable.

To introduce the $\mathcal{O}(1)$ Yukawa we can use the results of section (4.1.1), and directly see the scalings of the induced couplings,

$$\frac{e^K}{m_H} \partial_i Y_{N_l} \partial_j Y_{N_m} C^{N_l+N_m-2} \psi^C \psi^C \sim \mathcal{O}(\epsilon) \,, \tag{4.54}$$

scaling exactly as the ones generated by a term $\mathcal{O}(1) C^{N_n+N_m}$ in the superpotential so this term is, like in the KKLT-like case, no reliable in the simple description. The coupling coming from the term $\partial_i \partial_\mu \partial_{\bar{\mu}} \partial_{\bar{\nu}} K_0 \psi^i \psi^\mu \bar{\psi}^{\bar{\mu}} \bar{\psi}^{\bar{\nu}}$ in the Lagrangian may carry further scalings from the derivatives on the volume. Let us work the case where all spinors ψ^μ belong to the small modulus t multiplet, which is the most dangerous case since all derivatives can come from Z in eq.(4.44), so that the coupling simply scales as $\mathcal{O}(\epsilon^{2/3})$, and the induced coupling scales as

$$\mathcal{O}(\epsilon^{2/3}) \frac{e^{K/2}}{m_H} \partial_i Y_N C^{N-1} \bar{\psi}^{\bar{\mu}} \psi^\mu \bar{\psi}^{\bar{\nu}} \psi^C \sim \mathcal{O}(\epsilon^{2/3}) \,. \tag{4.55}$$

This coupling is generated from a term in the Kähler potential of the form $\mathcal{O}(\epsilon^\beta) C^N$. If the only dependency on t comes from the volume then the respective coupling scales as $\mathcal{O}(\epsilon^{\beta+3})$, and the induced coupling will be always grater than the original one unless one allows large negative values for β. If the coupling instead carries further t dependencies apart from those in the volume the coupling scales simple as $\mathcal{O}(\epsilon^\beta)$ and $\beta < 2/3$ ensures the reliability of the couplings in the simple model. Notice that the analysis of contributions to the Kähler potential is rather vague as we lack of a precise definition of naturalness, being the relevant suppression parameter the volume, which in principle can appear with any (likely negative) power. From the term

$$\int d\theta^2 Y_N(U) C^N \Phi^3 \sim e^{K/2} F^C Y_N C^{N-1} \sim e^K \partial_{\bar{i}} \overline{W}_0 Y_{N_l} C^{N_l} \,, \tag{4.56}$$

[9] More properly one should take the canonical normalized coupling in order to do a proper estimate. However, since the kinetic function for the complex structure scales as $\mathcal{O}(1)$ this is irrelevant once we compare coupling with the same power of L fields.

using again $K^{C\bar{U}} \sim C$. The coupling, then, scales like $e^K \partial_{\bar{U}\bar{U}} \overline{W}_0 Y_N \bar{U} C^N \sim \mathcal{O}(\epsilon^2)\bar{U} C^N$. Combined with itself leads to the effective coupling,

$$\mathcal{O}(\epsilon^4)\frac{1}{m_U^2} Y_{N_n} \bar{Y}_{N_m} C^{N_n-1} \bar{C}^{N_m-1} \sim \mathcal{O}(\epsilon^2) Y_{N_n} \bar{Y}_{N_m} C^{N_n-1} \bar{C}^{N_m-1} . \tag{4.57}$$

This term is already generated in by the term $Y_N C^N$ in the superpotential, leading to a term scaling as $\mathcal{O}(\epsilon^{4/3})$, using $K^{C\bar{C}} \sim \epsilon^{-2/3}$, therefore the induced terms are suppressed and the original ones appearing in the simple superpotential are reliable.

As said in the beginning, a general analysis of the LARGE volume models seems not as straightforward as in the KKLT-like case, already without the inclusion of the visible sector. The explicit set-up we studied here turns out to be rather save, at least up to the mass level for the moduli and leading couplings in the visible sector. However, being the scalings determined also by the stabilization procedure, this results most probably cannot be blindly generalized to any situation. This simply further motivates the usual more conservative approach [42, 97], where the vacuum is computed in the simple model, but then, for the computation of the soft-terms, knowing the order of the Kähler mixings one estimates the order of the respective F-terms for the complex structure and dilaton. Indeed, one can easily calculate the canonical normalize F-terms using $K^{S\bar{T}} \sim \mathcal{O}(\epsilon^{1/3})$, $K^{U\bar{T}} \sim \mathcal{O}(\epsilon^{\alpha-2/3})|Q|^2$ and $K^{U\bar{Q}} \sim Q$. In our case

$$\begin{aligned} &F_c^T \sim \mathcal{O}(\epsilon) \ , \quad F_c^t \sim \mathcal{O}(\epsilon^{3/2}) \ , \quad F_c^Q \sim \delta^{1/2}\mathcal{O}(\epsilon^{4/3}), \\ &\quad F_c^S \sim \mathcal{O}(\epsilon^2) \ , \quad F_c^U \sim \delta\, \mathcal{O}(\epsilon^{5/3}) \, . \end{aligned} \tag{4.58}$$

With $\delta \sim \mathcal{O}(\epsilon^{1/3})$, necessary to tune the cosmological constant, the sector $Q - U$ is further suppressed, but any way is clear that the S and U contributions are suppressed. So that one can estimate the soft-term contributions from all the sectors instead of integrate them out and take into account the modified theory. This approach at the level of the e.o.m. and masses would lead to the very same results, being this one far more straightforward and simple.

Chapter 5

Moduli Stabilization in Minkowski vacua

The the study of chapters 3 and 4 gives a very powerful result concerning, among others, the stabilization of moduli, telling us that one can consistently forget about the dynamics of many of the moduli, regarded as frozen by the flux induced dynamics. This gives us more freedom on building and playing with toy models of moduli stabilization, which in order to be more or less analytical are restricted to a small set of the moduli. In the present chapter an explicit model is constructed [1] realizing tow moduli stabilization in Minkowski vacua. These two moduli are the light fields of our previous chapters, so we concentrate here only to the second step of the Two-Step moduli stabilization procedure. For completeness in the framework of the previous chapters, Appendix B shows a numerical instance comparing the Two-Step procedure, followed here, and the full one for the simpler case with a single modulus.

A well-known, simple and interesting SUSY breaking mechanism is the Fayet-Iliopoulos model, which is based on a FI-term for a $U(1)_X$ gauge symmetry [98]. Its simplest implementation requires two charged fields, ϕ and χ, with opposite $U(1)_X$ charges q_ϕ and q_χ. The requirement of minimizing the D_X term in the scalar potential induces one of the charged fields, say ϕ, to get a non-vanishing VEV. If the only relevant superpotential term coupling ϕ and χ is linear in χ, a simple effective Polonyi-like superpotential term is induced and SUSY is broken because $F_\chi \neq 0$. In string theory, as explained in section (2.4), field-dependent effective FI-terms generally arise due to a non-linear transformation of some modulus U under a would-be anomalous $U(1)_X$ gauge symmetry. Moduli stabilization and Fayet-like SUSY breaking mechanisms are hence interconnected in string theory and one might wonder if their combined action can efficiently be embedded in a KKLT-set up to provide a spontaneous SUSY breaking mechanism, which also does not need to assume a complete decoupling between moduli stabilization and SUSY breaking, like in [24]. A Fayet-like SUSY breaking mechanism has been shown to successfully give rise to low energy SUSY breaking on a Minkowski/dS vacuum for a KKLT-like SUGRA model, where the FI modulus is identified with the universal Kähler modulus and $q_\phi = -q_\chi$ [44]. The resulting soft mass parameters for the visible sector are realistic, but present a moderate hierarchy between gaugino and scalar masses, unless one complicates the model by introducing additional (messenger-like) fields [44]. The main drawback of the model of [44] is the introduction of an unnaturally small mass term $m\phi\chi$ with $m \sim O(10^{-11} \div 10^{-12})$, in addition to the usual KKLT fine-tuning of assuming a tiny constant superpotential term w_0, which is roughly of the same order as m. A more satisfactory explanation of the smallness of the $\chi\phi$ coupling is necessary, mainly because higher-order terms of the form $c_n(\phi\chi)^n$ with $n > 1$ will generally lead to a restoration of SUSY.

The aim of this chapter is to study in some detail how a Fayet-like SUSY breaking mechanism can be realized in string-derived SUGRA theories. We will study the dynamics of a SUGRA system with two moduli, the FI modulus U and another neutral modulus Z, the two charged fields ϕ and χ, with arbitrary q_χ charge (the $U(1)_X$ charges are normalized so that $q_\phi = -1$) and extra hidden vector-like matter and two/three condensing gauge groups responsible for non-perturbatively generated terms necessary to stabilize U and Z. We specify only the schematic form of the Kähler potential, which is taken quite generic. As far as the superpotential is concerned, we consider both the cases of moduli-independent couplings of the form $Y\chi\phi^{q_\chi}$ and non-perturbatively generated moduli dependent couplings of the form $Y\exp(-\gamma_U U - \gamma_Z Z)\chi\phi^{q_\chi}$, where $\gamma_{U,Z}$ are some unspecified constants. Once the hidden mesons of the condensing gauge groups are integrated out, the superpotential becomes the sum of exponential terms (including a constant term w_0) and of the coupling $\chi\phi^{q_\chi}$. In order to have as much as possible analytic control on this complicated system, we first look for SUSY vacua when χ is decoupled and then we consider its backreaction, which generally gives rise to non-SUSY vacua. In this approximation, the coupling $\chi\phi^{q_\chi}$ is effectively an "up-lifting" term, required to pass from the AdS SUSY vacuum to a Minkowski/dS one with low-energy SUSY breaking. We find that the last step puts quite severe constraints on the parameter space of the superpotential. In particular, we find that the non-perturbatively generated couplings can alleviate the tuning needed on Y from 2 to 6 orders of magnitude, depending on the model considered, but taken alone they do not allow us to have $Y \sim \mathcal{O}(1)$, since the back-reaction induced by the up-lifting term becomes so large that the non-SUSY vacuum either disappears or always remains AdS. Natural values of Y can however be obtained by assuming that $|q_\chi| > q_\phi$ by a few, so that $\langle\phi\rangle^{q_\chi}$ can be responsible for the remaining necessary suppression, though this might be hardly realized in explicit string constructions.

Most of our interest is in the hidden sector dynamics of the theory and hence we will not systematically study how SUSY breaking is mediated to the visible sector or the precise form of the soft parameters, most of which are necessarily model dependent. We just notice that gravity mediation of SUSY breaking is preferred to avoid small moduli masses, linked to the gravitino mass, and that the general pattern of the soft mass terms seem very promising. In particular, the gaugino masses, which in string-inspired models with purely gravity mediated SUSY breaking are often considerably smaller than the non-holomorphic scalar masses, can be naturally made heavier in our set-up, thanks to the presence of two moduli. In presence of the FI modulus only, gauginos can take a mass only by assuming a U-dependent gauge kinetic function for the visible gauge group. Since U transforms under a $U(1)_X$ gauge transformation, anomaly arguments require that some visible matter field has to be charged under $U(1)_X$, which in turn gives rise to heavy non-holomorphic scalar masses, induced by the D_X term [44]. This problem is now simply solved by assuming that the gauge kinetic function for the visible sector depends on Z only. Moreover, when $\gamma_Z \neq 0$, the back-reaction of the up-lifting term on Z will give rise to an enhancement of F_Z, so that eventually gauginos just a few times smaller than the gravitino can be obtained.

The fine-tuning problem to get $Y \ll 1$, by means of the flatness condition, is just a reflection of the other fine-tuning problem which requires $w_0 \ll 1$ to get low-energy supersymmetry breaking. Motivated by the idea of solving this second tuning problem, we have also analyzed the situation in which w_0 vanishes, assuming that some stringy symmetry forbids its appearance. In this case, the moduli stabilization mechanism boils down to a racetrack model [66], where the scale of supersymmetry breaking is dynamically generated. Interestingly enough, the back-reaction of the up-lifting term is milder than before and it is now possible to achieve $Y \sim \mathcal{O}(1)$ with small q_X charges, relying on $\gamma_{Z,U}$ only. Aside from the usual cancellation of the cosmological constant, this model is hence completely natural. When two moduli are considered, similarly to before, one can have a not too hierarchically spectrum of soft masses, although the gauginos are now a bit lighter than the models with $w_0 \neq 0$ considered before.

All the above general considerations are supplemented by considering in some detail three specific models: i) an orientifold compactification of type IIB on $\mathbf{CP}^4_{[1,1,1,6,9]}$, where U and Z are identified with the two Kähler moduli of the compactification manifold ($w_0 \neq 0$), ii) an heterotic compactification on a generic CY 3-fold, where U and Z are identified with the dilaton and the universal Kähler modulus, respectively ($w_0 \neq 0$), iii) the model of i) but now assuming $w_0 = 0$. It should be stressed that, in the spirit of our bottom-up approach, none of these models are actually full-fledged string models, but rather they should be seen as inspiring examples to partially fix the arbitrariness in the choice of the Kähler potential, superpotential and gauge kinetic functions we have at the SUGRA level. In all three cases, we have analytically and numerically studied the main properties of the vacuum, including its (meta)stability. The latter is essentially never a serious issue and all the moduli components are always one or two order of magnitudes heavier than the gravitino. In all three cases we have always estimated under which conditions a classical SUGRA analysis is reliable by considering the effect of the universal α' correction to the moduli Kähler potential [99, 89].

5.1 General Two-Moduli model

The SUGRA model we consider consists of two moduli multiplets U and Z, a hidden gauge group of the form $\mathcal{G}_1 \times \mathcal{G}_2 \times U(1)_X$, with $\mathcal{G}_1$ and $\mathcal{G}_2$ non-abelian factors, massless matter charged under $U(1)_X$ and under $\mathcal{G}_1$ or $\mathcal{G}_2$ (but not both) and finally two chiral multiplets ϕ and χ, charged under $U(1)_X$ and singlets under $\mathcal{G}_1 \times \mathcal{G}_2$. For concreteness, we take $\mathcal{G}_i = SU(N_i)$ $(i = 1, 2)$ and consider N_{fA} quarks Q_i and $\widetilde{Q}_i$ in the fundamental and anti-fundamental representations of $\mathcal{G}_i$. This is the field content of our model. We assume that the $U(1)_X$ gauge symmetry is "pseudo"-anomalous, namely that such symmetry is non-linearly realized in one of the two moduli multiplets, U, the latter mediating a generalized Green-Schwarz mechanism [74]. We normalize the $U(1)_X$ charges so that $q_\phi = -1$ and take $q_{Q_i} + q_{\widetilde{Q}_i} > 0$, $q_\chi > 0$, with the same $U(1)_X$ charge for all flavors, for simplicity. The model is finally specified by the Kähler potential, superpotential and gauge kinetic functions. Omitting for simplicity flavor and color indices, the full Kähler potential K_{Tot} and superpotential W_{Tot} are a sum of a visible and hidden sector, $W_{Tot} = W_v + W_h$ and $K_{Tot} = K_v + K_h$, where

$$K_v = \alpha_{iv} Q_{iv}^\dagger e^{2q_{iv}V_X + V_v} Q_{iv} \tag{5.1}$$

represents the Kähler potential of the visible sector, with i running over all visible fields, and we have schematically denoted by Q_{iv} and V_v all the visible chiral fields and vector superfields. For simplicity, we have taken K_v to be diagonal in the visible sector fields. We do not specify the visible superpotential W_v because it will never enter in our considerations.

The Kähler potential for the hidden sector reads[1],

$$
\begin{aligned}
K_h &= K_M + \alpha_\phi \bar\phi e^{-2V_X}\phi + \alpha_\chi \bar\chi e^{2q_X V_X}\chi \\
&\quad + \sum_{i=1,2} \alpha_i \Big(Q_i^\dagger e^{V_i + 2q_{Q_i} V_X} Q_i + \widetilde{Q}_i^\dagger e^{-V_i + 2q_{\widetilde{Q}_i} V_X} \widetilde{Q}_i \Big)\,,
\end{aligned} \tag{5.2}
$$

with the superpotential given by

$$
\begin{aligned}
W_h &= w_0 + Y(U,Z,\phi)\phi^{q_X}\chi + \sum_{i=1,2} c_i(U,Z,\phi)\, Q_i \widetilde{Q}_i\, \phi^{q_{Q_i}+q_{\widetilde{Q}_i}} \\
&\quad + \sum_{i=1,2} \eta_i (N_i - N_{if}) \left(\frac{\Lambda_i(U,Z)^{3N_i - N_{if}}}{\det(Q_i \widetilde{Q}_i)} \right)^{\frac{1}{N_i - N_{if}}}\,.
\end{aligned} \tag{5.3}
$$

The holomorphic gauge kinetic functions in the hidden sector are taken to be

$$
f_i(U,Z) = n_i U + m_i Z + p_i\,; \qquad f_X(U) = n_X U\,. \tag{5.4}
$$

Several comments are in order. The Kähler potentials (5.1) and (5.2) are supposed to be the first terms in an expansion in the matter fields up to quadratic order, K_M, α_{iv}, α_i, α_ϕ and α_χ[2] are generally real functions of $U + \overline{U} + \delta\, V_X$ and $Z + \overline{Z}$. In K_h, V_i and V_X denote the vector superfields associated to the non-abelian groups $\mathcal{G}_i$ and $U(1)_X$, respectively, K_M is the Kähler potential for the U and Z moduli and δ is the Green-Schwarz coefficient. The form of the latter is uniquely fixed by gauge invariance to be

$$
\delta = \frac{(q_{Q_1} + q_{\widetilde{Q}_1})N_{1f}}{4\pi^2 n_1} = \frac{(q_{Q_2} + q_{\widetilde{Q}_2})N_{2f}}{4\pi^2 n_2}\,. \tag{5.5}
$$

In the spirit of the Two-Step moduli stabilization we have allowed in the superpotential (5.3) an arbitrary constant term w_0 that is supposed to be the left-over of all the remaining fields, regarded stabilized by $\mathcal{O}(1)$ dynamics so that can be safely frozen out as shown in chapters 3 and 4. Strictly speaking a constant term in the Kähler potential is also present, however, for non-anomalous large values of the frozen moduli it can be safely neglected being an $\mathcal{O}(1)$ normalization in the physical quantities like the gravitino mass. Consistency of our Two-Step stabilization procedure requires w_0 to be tiny in Planck units. We will later discuss the case in which w_0 exactly vanishes. The hidden Yukawa couplings Y and c_i in W_h are assumed to generally depend on both moduli. Due to the Peccei-Quinn symmetries associated to $\operatorname{Im} U$ and $\operatorname{Im} Z$, the only allowed moduli dependence is exponential. The superpotential (5.3) is manifestly G_i invariant, whereas the $U(1)_X$ invariant is less transparent. Under a $U(1)_X$ super-gauge transformation with parameter Λ, one has $\delta_X V_X = -i(\Lambda - \bar\Lambda)/2$, $\delta_X U = i\delta\Lambda/2$ and $\delta_X \Phi = i q_\Phi \Lambda \Phi$ for

[1] See also [82] where a similar analysis with a single modulus in a KKLT context has been done.

[2] Notice that for simplicity we have taken the same moduli dependent function α_i for the hidden quarks and anti-quarks.

any charged multiplet Φ [3]. Gauge invariance constraints then the couplings Y and c_i to depend on U by means of the gauge invariant combination $\exp(-U)\phi^{\delta/2}$. We then parameterize

$$Y(U,Z,\phi) = Y\phi^{\gamma_U\delta/2}e^{-\gamma_U U-\gamma_Z Z}\,, \qquad c_i(U,Z,\phi) = c_i\phi^{\eta_{i,U}\delta/2}e^{-\eta_{i,U}U-\eta_{i,Z}Z}\,, \tag{5.6}$$

with Y a constant and c_i constant matrices (in flavor space). The phenomenological coefficients $\gamma_{U/Z}$ and $\eta_{i,U/Z}$ are non-vanishing for non-perturbatively generated Yukawa couplings only. The last term in W_h is the non-perturbatively generated superpotential term appearing in $\mathcal{N}=1$ theories for $N_f < N$ [69]. We have found convenient to introduce the factors $\eta_{1,2} = \pm 1$ in eq.(5.3), which will allow us to set to zero the imaginary parts of U and Z. The dynamically generated scales $\Lambda_i(U,Z)$ are field-dependent and follows from the holomorphic gauge kinetic functions (5.4). From eq.(5.4) we have

$$g_i^{-2} = \mathrm{Re}\, f_i(U,Z), \quad g_X^{-2} = \mathrm{Re}\, f_X(U)\,, \tag{5.7}$$

and

$$|\Lambda_i(U,Z)| = e^{-\frac{8\pi^2}{g_i^2(3N_i-N_{if})}}\,. \tag{5.8}$$

The coefficients n_X, n_i, m_i and p_i in eq.(5.4) are model dependent constants, which we keep generic for the moment. Just for simplicity of the analysis, we have assumed that the $U(1)_X$ factor depends only on the U modulus. It is straightforward to check that the non-perturbative superpotential terms in eq.(5.3) are $U(1)_X$ gauge-invariant provided the two equalities in eq.(5.5) are satisfied.

We will mostly be interested in the dynamics of the hidden sector of the theory, assuming that all visible fields vanish. The scalar potential of the theory has the usual SUGRA form introduced in section (2.2) given by $V = V_F + V_D$, with

$$V_F = e^{K_h}\Big(K_h^{I\bar{J}} D_I W_h \overline{D_J W_h} - 3|W_h|^2\Big)\,, \tag{5.9}$$

$$V_D = \sum_{i=1,2}\frac{1}{2\mathrm{Re}\, f_i}D_i^2 + \frac{1}{2\mathrm{Re}\, f_X}D_X^2\,. \tag{5.10}$$

In eq.(5.9), I, J run over the hidden chiral multiplets $Q_i, \widetilde{Q}_i, \phi, \chi, U, Z$, $D_I W_h = \partial_I W_h + (\partial_I K_h)W_h \equiv F_I$ is the Kähler covariant derivative and $K_h^{I\bar{J}}$ is the inverse Kähler metric. In eq.(5.10), D_i and D_X are the D-terms associated to the $\mathcal{G}_A$ and $U(1)_X$ gauge groups,

[3]Notice a sightly change on the normalization of the Vector, V_X, multiplet compared to the one followed in the previous chapters.

whose explicit form is[4]

$$\begin{aligned} D_i^a &= \alpha_i(Q_i^\dagger T^{a_i} Q_i - \widetilde{Q}_i^\dagger T^{a_i} \widetilde{Q}_i)\,, & (5.11)\\ D_X &= \sum_{i=1,2} \alpha_i \Big(q_{Q_i} Q_i^\dagger Q_i + q_{\widetilde{Q}_i} \widetilde{Q}_i^\dagger \widetilde{Q}_i \Big) + \alpha_\chi q_\chi \bar{\chi}\chi - \alpha_\phi \bar{\phi}\phi \\ &\quad -\frac{\delta}{2}\Big[\alpha_i'(Q_i^\dagger Q_i + \widetilde{Q}_i^\dagger \widetilde{Q}_i) + \alpha_\phi' \bar{\phi}\phi + \alpha_\chi' \bar{\chi}\chi + K_M' \Big]\,, & (5.12) \end{aligned}$$

where T^{a_i} in eq.(5.11) are the generators of $SU(N_i)$ and $'$ in eq.(5.12) stands for a derivative with respect to U.

5.1.1 Looking for non-SUSY Minkowski minima

A direct analytical study of the minima of V is a formidable task. However, we will see that it is possible to find non supersymmetric metastable Minkowski minima starting from AdS SUSY vacua when $\chi \ll 1$.

Particularly important for what follows is the $U(1)_X$ D-term. The minimization of D_X induces then a non-vanishing VEV for ϕ (taken real for simplicity):

$$\phi^2_{SUSY} = \frac{-\delta K_M'}{2(\alpha_\phi + \delta\alpha_\phi'/2)}\,. \tag{5.13}$$

Notice that typically $K_M' < 0$, so that the right-hand side in eq.(5.13) is positive and the $U(1)_X$ symmetry is spontaneously broken. From the third term in the superpotential (5.3), we see that ϕ_{SUSY} also induces a mass term for the quarks Q_i and $\widetilde{Q}_i$. Assuming that $\phi_{SUSY} \gg m_{3/2}$, unless the Yukawa couplings $c_i(U, Z, \phi)$ are extremely small, a sufficiently large mass for the quarks Q_i and $\widetilde{Q}_i$ is induced.[5] Under the assumption that at the minimum $W_h \ll 1$, which is obviously required to have a sufficiently light gravitino mass, the quarks can be integrated out by the manifestly SUSY approach introduced in sections (2.1.1) and (2.2.3), and highly exploited in the previous chapters. As well known the space of D-flat vacua for the non-Abelian sectors is parametrized by the meson chiral fields $M_i = Q_i\widetilde{Q}_i$, therefore, integrating out the quarks ensuring D-flatness reduces to integrate out the meson multiplets, i.e. solve the chiral e.o.m. $\partial_{M_i} W = 0$. The solution is given by

$$M_i = \Lambda_i^2 \left(\frac{\Lambda_i}{m_i}\right)^{1-N_{if}/N_i}\,, \tag{5.14}$$

[4]Recall we use the same notation for a chiral multiplet and its lowest scalar component, since it should be clear from the context to what we are referring to.

[5]We assume that the c_i in eq.(5.6) are such that all quarks get a mass when ϕ acquires a VEV.

with $m_i = c_i\phi^{q_i}$, which turns out to be highly suppressed by the dynamical generated scales $\Lambda_i \ll 1$ and the small ratio Λ_i/m_i [6]. The effective superpotential W_{eff} reads

$$W_{eff} = w_0 + f(\phi)e^{-\gamma_Z Z-\gamma_U U}\chi + \sum_{i=1,2} A_i(\phi)e^{-a_i U-b_i Z}\,, \tag{5.15}$$

where

$$\begin{aligned} a_i &\equiv \eta_{i,U}\frac{N_{if}}{N_i} + \frac{8\pi^2 n_i}{N_i}\,, \qquad b_i \equiv \eta_{i,Z}\frac{N_{if}}{N_i} + \frac{8\pi^2 m_i}{N_i}\,, \\ f(\phi) &\equiv Y\phi^{\hat{q}_\chi}\,, \qquad A_i(\phi) \equiv \eta_i N_i e^{-8\pi^2 p_i/N_i}\Big(c_i\phi^{q_i}\Big)^{\frac{N_{if}}{N_i}}, \end{aligned} \tag{5.16}$$

are effective parameters and for simplicity we have defined the effective charges

$$q_i \equiv q_{Q_i} + q_{\tilde{Q}_i} + \eta_{i,U}\delta/2\,, \qquad \hat{q}_\chi \equiv q_\chi + \frac{\gamma_U\delta}{2}\,. \tag{5.17}$$

Due to the highly suppressed size for the solutions M_i we can neglect completely the contribution due to the integration in K_h, so that the resulting effective Kähler potential is simply

$$K_{eff} = K_M + \alpha_\phi\bar{\phi}e^{-2V_X}\phi + \alpha_\chi\bar{\chi}e^{2q_\chi V_X}\chi\,. \tag{5.18}$$

As next step, we look for vacua with $\chi \ll 1$. We expand the scalar potential V_{eff} arising from (5.15), (5.18) and the D_X term in powers of χ, $V_{eff} = \sum_{n,m=0}^\infty V_{n,m}\chi^n\chi^{\dagger m}$, and keep only the leading term $V_0 \equiv V_{0,0}$. It reads

$$V_0 = \frac{1}{2\mathrm{Re}\, f_X}(D_X^{(0)})^2 + e^{K_{eff}^{(0)}}\Bigg[\sum_{i,j=U,Z,\phi} K_{eff}^{(0)i\bar{j}}F_i^{(0)}F_{\bar{j}}^{(0)} - 3|W_{eff}^{(0)}|^2 + V_{up-lift}\Bigg]\,, \tag{5.19}$$

where the F-terms are computed using K_{eff} and W_{eff} and the superscript (0) means that all expressions are evaluated for $\chi = 0$. The first three terms in V_0 correspond to the SUGRA scalar potential that would result from $K_{eff}^{(0)}$, $W_{eff}^{(0)}$ and f_X. The last term

$$V_{up-lift} = \frac{|F_\chi^{(0)}|^2}{\alpha_\chi}\,, \tag{5.20}$$

where $F_\chi^{(0)} = f(\phi)\exp(-\gamma_Z Z - \gamma_U U)$, is effectively a moduli-dependent "up-lifting" term. Let us look for approximate SUSY vacua for U and Z, neglecting for the moment the up-lifting term $V_{up-lift}$, and assuming that at the extremum w_0 is larger than the dynamically generated terms in (5.15). It is easy to solve the system $D_X^{(0)} = F_U^{(0)} = F_Z^{(0)} = F_\phi^{(0)} = 0$. The D_X term trivially vanishes when eq.(5.13) is satisfied, so that ϕ is determined. At a SUSY extremum, gauge invariance implies $F_\phi^{(0)} = -\delta/(2\phi)F_U^{(0)}$, so

[6] Of course, we are here assuming at this stage the existence of a non-runaway minimum for the moduli U, Z.

that we are left to solve the system $F_U^{(0)} = F_Z^{(0)} = 0$. We get

$$U_{SUSY} \simeq \frac{b_2 x_1 - b_1 x_2}{a_1 b_2 - a_2 b_1}\,, \qquad Z_{SUSY} \simeq \frac{a_1 x_2 - a_2 x_1}{a_1 b_2 - a_2 b_1}\,, \tag{5.21}$$

where

$$x_{1,2} = -\ln\left[\pm \frac{w_0}{A_{1,2}(\phi_{SUSY})} \frac{b_{2,1} K_{eff}^{(0)\prime} - a_{2,1} \dot{K}_{eff}^{(0)}}{a_1 b_2 - a_2 b_1}\right], \tag{5.22}$$

and a dot stands for a derivative with respect to Z. By appropriately choosing the signs of η_i appearing in $A_i(\phi)$, see eq.(5.16), we can always set U_{SUSY} and Z_{SUSY} to be real, so that for simplicity of notation in the following we will always assume real fields and real parameters. Since U and Z enter explicitly in the coefficients $x_{1,2}$ above, eqs.(5.21) do not admit explicit analytic solutions. However, the logarithmic dependence on U and Z of $x_{1,2}$ is often mild enough that a good approximate expression for U_{SUSY} and Z_{SUSY} is obtained by taking some educated guess for the moduli in eq.(5.22), compute (5.21), insert the result in (5.22) and compute once again (5.21). The shifts in the fields due to the up-lifting term $V_{up-lift}$ can be found by expanding the extrema conditions $\partial_U V_0 = \partial_Z V_0 = \partial_\phi V_0 = 0$ around the SUSY values (5.21): $U = U_{SUSY} + \Delta U$, $Z = Z_{SUSY} + \Delta Z$, $\phi = \phi_{SUSY} + \Delta\phi$ and keeping the leading term in $V_{up-lift}$ and terms up to linear order in ΔU, ΔZ or $\Delta\phi$ in the remaining terms of the scalar potential. The resulting expressions one gets for the shifts are actually very involved and can be handled only numerically. Some simple approximate formulae can however be derived by using simple scaling arguments to estimate the typical size of the terms entering in the Kähler and superpotential terms, eqs.(5.15) and (5.18). We first notice that eq.(5.21) fixes the sizes of the moduli U and Z at the SUSY point to be inversely proportional to the effective parameters $a_{1,2}$ and $b_{1,2}$. For simplicity, we can take all the $a_{1,2}$ and $b_{1,2}$ parameters that do not vanish to be of the same order of magnitude and denote their common value by a. It then follows that $U \sim Z$ and we can generally denote by X the common modulus VEV. We will use such simplified notation anytime we want to estimate a quantity without giving its explicit expression. Coming back to eq.(5.21), it is clear that aX is approximately a constant, proportional to the $x_{1,2}$ coefficients defined in eq.(5.22). Since $w_0 \ll 1$ is required to have a sufficiently light gravitino mass, this constant is much larger than 1. As a matter of fact, for a wide class of models aX is always in the narrow range $20 \lesssim aX \lesssim 40$, which is essentially the range dictated by the Planck/electroweak scale hierarchy. The parameter $\epsilon \equiv 1/(aX)$ is then small and an expansion in ϵ is possible[7]. The following scaling behaviors are taken at the SUSY

[7] Do not confuse the present ϵ with the one of chapters 3 and 4.

extremum:

$$\partial_X^n K_{eff}^{(0)} \sim \partial_X^n K_M \sim \frac{1}{X^n}\,, \qquad\qquad \partial_X^n \alpha_{\phi,\chi} \sim \frac{\alpha_{\phi,\chi}}{X^n}\,, \quad n>0\,. \qquad (5.23)$$

$$\partial_X W_{eff}^{(0)} \sim (\partial_X K_{eff}^{(0)}) W_{eff}^{(0)} \sim \frac{1}{X} W_{eff}^{(0)}\,, \qquad \partial_X^n W_{eff}^{(0)} \sim a^{n-1}\partial_X W_{eff}^{(0)}\,, \quad n>1\,.$$

The term proportional to $|\phi|^2$ in $K_{eff}^{(0)}$ is sub-leading in ϵ with respect to the purely moduli dependent term K_M. Indeed, $\alpha_\phi |\phi|^2 \lesssim \delta/X$ and, neglecting possible corrections due to $\eta_{iZ/U}$, one has from eqs.(5.5) and (5.16) that $\delta = 2q_i N_{if}/(N_i a_i)$, which typically is less or equal to $1/a$. This explains the first relation in eq.(5.23). We are now ready to estimate the shift of the fields ΔU, ΔZ and $\Delta\phi$ due to the up-lifting term $V_{up-lift}$. In the heavy $U(1)_X$ gauge field approximation $M_X \gg m_{3/2}$, which will always be the case of interest for us, one has (see e.g.[39])

$$\langle D_X \rangle \simeq \frac{2}{M_X^2} e^{K_{eff}^{(0)}} \frac{q_\chi (F_\chi^{(0)})^2}{\alpha_\chi} = \frac{2}{M_X^2} q_\chi e^{K_{eff}^{(0)}} V_{up-lift}\,, \qquad (5.24)$$

where $M_X^2 = 2g_X^2 \alpha_\phi \phi_{SUSY}^2$, so that the D_X term at the minimum is negligibly small. We can use the condition $D_X \simeq 0$ to express $\Delta\phi$ as a function of ΔZ and ΔU. Using eq.(5.13), it is straightforward to see that $\Delta\phi$ scales as

$$\Delta\phi \sim \phi \frac{\Delta X}{X}\,. \qquad (5.25)$$

Next step is to estimate ΔX. Using eqs.(5.13), (5.23), (5.24) and (5.25), it is a simple exercise to see that at leading order in ϵ and up to linear order in ΔX, one has

$$\partial_X V_0 \simeq e^{K_M^{(0)}} K_M^{X\bar{X}} \partial_{\bar{X}} F_{\bar{X}} \partial_X F_X \Delta X + \partial_X (e^{K_M^{(0)}} V_{up-lift}) + e^{K_M^{(0)}} q_\chi V_{up-lift} \frac{\partial_X^2 K_M}{\partial_X K_M} \simeq 0\,, \qquad (5.26)$$

giving

$$\Delta X \sim V_{up-lift} \frac{\partial_X^2 K_M \Big[q_\chi \partial_X^2 K_M + (\partial_X K_M)^2 + \gamma \partial_X K_M \Big]}{\partial_X K_M \Big(\partial_X^2 W_{eff}^{(0)} \Big)^2} \sim \epsilon^2 \frac{V_{up-lift}}{(W_{eff}^{(0)})^2} X (q_\chi + \gamma X)\,, \qquad (5.27)$$

where γ generally denotes γ_U or γ_Z and we have tacitly assumed $q_\chi > 1$ in writing the last relation in eq.(5.27). At a Minkowski minimum, $|V|_{up-lift} \sim |W_{eff}^{(0)}|^2$, so that the fraction in eq.(5.27) is $\mathcal{O}(1)$. If $\gamma_U = \gamma_Z = 0$ and $q_\chi \sim \mathcal{O}(1)$, the relative shifts of the moduli are small: $\Delta X/X \sim \epsilon^2 \sim 10^{-3}$ and certainly the up-lifting term $V_{up-lift}$ does not de-stabilize the system and can be treated as a perturbation, as we did. When γ_U and/or γ_Z are non-vanishing, the up-lifting term $V_{up-lift}$ becomes exponentially sensitive to the values of U, Z. It is then not enough to have $\Delta X/X \ll 1$, but the stronger constraint $\gamma\Delta X \ll 1$ is required, in order to avoid large displacements of $V_{up-lift}$ which can result on the impossibility of finding a Minkowski solution. This results on a bound on the size of γ:

$$\gamma^2 \ll a^2 \,. \tag{5.28}$$

The same constraint $\gamma\Delta X \ll 1$ gives also an upper bound on q_χ, $\epsilon\, q_\chi\gamma/a \ll 1$, which is however quite mild, in light also of eq.(5.28). We do not report the detailed expressions for ΔU and ΔZ in the general case, which are very involved and not illuminating even when expanded in powers of ϵ. Just for concreteness, we report their form for the particular case when $\alpha_\chi = \alpha_\phi = 1$, factorizable K_M, W_{eff} (i.e. $\dot{K}'_M = \dot{W}'_{eff} = 0$) and perturbatively generated Y Yukawa coupling: $\gamma_U = \gamma_Z = 0$. In these approximations, we have

$$\begin{aligned}
\Delta U &\simeq -V_{up-lift}\frac{K''_M[(K'_M)^2 + q_\chi K''_M]}{K'_M(W^{(0)\prime\prime}_{eff})^2}\,, \\
\Delta Z &\simeq -V_{up-lift}\frac{\dot{K}_M\ddot{K}_M}{(\ddot{W}^{(0)}_{eff})^2}\,,
\end{aligned} \tag{5.29}$$

whose scalings are in agreement with the general estimate (5.27). Notice that both ΔZ and ΔU are positive, since $\dot{K}_M$ and K'_M are negative, tending to decrease the up-lifting term. The scaling behaviors of the F-terms at the non-SUSY vacuum are easily found.

$$\begin{aligned}
F^{(0)}_\chi &\sim \alpha_\chi^{1/2} W^{(0)}_{eff}\,, \\
F^{(0)}_X &\sim \epsilon\Big(\gamma + \frac{q_\chi}{X}\Big)\alpha_\chi^{-1/2}F^{(0)}_\chi\,, \\
F^{(0)}_\phi &\sim \frac{\delta}{\phi}\epsilon\Big(\gamma + \frac{q_\chi}{X}\Big)\alpha_\chi^{-1/2}F^{(0)}_\chi\,.
\end{aligned} \tag{5.30}$$

Using eq.(5.30), one can easily estimate $K^{(0)i\bar{j}}_{eff}F^{(0)}_i F^{(0)}_{\bar{j}}$ and $V_{up-lift}$. In agreement with our expectation, $V_{up-lift}$ is the leading term contributing positively to the vacuum energy, justifying the given name of up-lifting term.

Once the approximate vacuum of the leading potential V_0 has been found, given by $U_0 = U_{SUSY} + \Delta U$, $Z_0 = Z_{SUSY} + \Delta Z$, $\phi_0 = \phi_{SUSY} + \Delta\phi$, we turn on χ and verify the validity of our initial assumption $\chi \ll 1$. The easiest way to estimate χ is to use the relation $D_X = iX^i_X F_i/W$ between the D_X term and the F-terms. At the minimum $D_X \simeq 0$, but the F-terms for χ and ϕ contributing to D_X are typically of the same order of magnitude: $\phi F_\phi \sim q_\chi \chi F_\chi$. Using this relation and the scalings (5.30), we get

$$\chi_0 \sim \delta\epsilon\Big(\gamma + \frac{q_\chi}{X}\Big)\frac{1}{q_\chi}\alpha_\chi^{-1/2}\,, \tag{5.31}$$

which proves our initial assumption $\chi \ll 1$. A more accurate estimate of χ_0 might be obtained by considering the next sub-leading potential terms $V_{1,0} = V_{0,1}$, $V_{2,0} = V_{0,2}$

and $V_{1,1}$ obtained by expanding V_{eff} in powers of χ and $\bar{\chi}$. In first approximation, one can freeze U, Z and ϕ at the values U_0, Z_0 and ϕ_0, which extremize V_0, so that χ is determined by a linear equation. The general explicit expression for χ is however very involved and not very interesting, so we will not report it here. For the same reason, we do not report the expressions of the further shifts of U, Z and ϕ induced by the backreaction of χ. They turn out to scale as eqs. (5.25) and (5.27), but are typically smaller in the parameter region we will consider in the following. The location of the vacuum is then slightly shifted but it is not destabilized by the field χ. We can also check how χ changes the values of the F-terms (5.30). One has

$$
\begin{aligned}
F_\chi &= F_\chi^{(0)} + \alpha_\chi \chi (W_{eff}^{(0)} + \chi F_\chi^{(0)}), \\
F_X &= F_X^{(0)} + \chi(\partial_X F_\chi^{(0)} + \partial_X K_{eff} F_\chi^{(0)}), \\
F_\phi &= F_\phi^{(0)} + \chi\Big(\frac{\hat{q}_\chi}{\phi} + \phi\,\alpha_\phi\Big) F_\chi^{(0)}.
\end{aligned}
\tag{5.32}
$$

Using eqs.(5.30) and (5.31), it is straightforward to verify that the effect of χ on F_χ and F_X is negligible, while the second term in F_ϕ is of the same order as $F_\phi^{(0)}$. Hence the scalings (5.30) hold also for the full F-terms F_χ, F_X and F_ϕ, providing a final consistency check of eq.(5.31), which has been derived under this assumption.

Let us now discuss under which conditions the above SUSY breaking mechanism is stable under small deformations. The choice of the superpotential (5.3) was rather ad hoc, since we have considered only linear terms in χ and tacitly assumed that possible higher order terms of the form $Y_n(U,Z,\phi)(\chi\phi^{q_\chi})^n$, with $n>1$, can be neglected. This assumption is actually very strong, since the requirement $|F_\chi^{(0)}| \simeq \alpha_\chi^{1/2}|W_{eff}^{(0)}|$ puts severe constraints on the size of the constant term Y appearing in (5.6). This is particularly clear if one notices that generally $\alpha_\chi \lesssim 1$ and $W_{eff}^{(0)} \simeq w_0 \lesssim \mathcal{O}(10^{-13})$. The more obvious options of assuming a perturbative ($\gamma_{U,Z}=0$) mass term ($q_\chi = 1$) or trilinear coupling ($q_\chi = 2$) leads to a unnaturally small coupling Y. In such a situation, the terms of the form $Y_n(U,Z,\phi)(\chi\phi^{q_\chi})^n$ will lead to a restoration of SUSY and to the destabilization of the non-SUSY vacuum. This is best seen by considering the flat space model with stabilized moduli. In this case, the relevant superpotential term in eq.(5.15) is just the term linear in χ, with the Lagrangian invariant under a $U(1)_R$ symmetry with $R(\chi)=2$, $R(\phi)=0$. An exact R-symmetry is generally necessary to get a SUSY-breaking vacuum [100] and, indeed, in absence of moduli and gravitational dynamics, χ is stabilized at the origin where $U(1)_R$ is unbroken. Any term of higher order in χ will necessarily break $U(1)_R$, leading to the appearance of SUSY vacua. Gravity and moduli explicitly break $U(1)_R$, but if the breaking is small enough their only effect would be to displace a bit χ from the origin, so that $\chi \ll 1$, as predicted by eq.(5.31). If the terms $Y_n(U,Z,\phi)(\chi\phi^{q_\chi})^n$ are all negligibly small, much smaller than χw_0, the SUSY preserving vacua will appear for large values of χ and will not perturb much the (meta)-stable non-SUSY vacuum close to the origin. However, when the terms $Y_n(U,Z,\phi)(\chi\phi^{q_\chi})^n$ become roughly of the same order as χw_0, the SUSY vacua approach the origin and the non-SUSY vacua are destabilized and disappear.

Invoking non-perturbatively generated couplings ($\gamma \neq 0$) alleviate the problem, but it does not solve it, because a natural Y would require $\gamma \sim a$, so that

$$e^{-\gamma U} \sim e^{-a\,U} \sim W_{eff}^{(0)}\,, \tag{5.33}$$

but the constraint (5.28) does not allow such values of γ . A possible way out is to consider higher-order couplings by taking $q_\chi > 2$, so that the effective term $\phi_0^{q_\chi}$ becomes small enough to get not so small values of Y. In this way, we also more effectively suppress the dangerous terms $(\chi\phi^{q_\chi})^n$. Summarizing, the requirement of naturalness and more importantly stability under superpotential deformations with higher powers of χ necessarily require to consider non-renormalizable interactions with $q_\chi > 2$, the precise bound on q_χ depending on $\gamma_{U,Z}$ being zero or not[8].

So far, we have been able to find approximate expressions for the extrema of the scalar potential V_{eff}, but we have still to check whether these vacua are minima or not. At leading order in ϵ and for $\phi_{SUSY}/X \ll 1$, the kinetic mixing of ϕ with the moduli can be neglected and the mass of ϕ is determined by the D-term potential. Its physical mass is

$$m_\phi^2 \simeq 2g_X^2\alpha_\phi\phi_{SUSY}^2\,, \tag{5.34}$$

which is also the mass of the gauge vector boson A_X, as we have seen. Indeed, $\mathrm{Im}\,\phi$ is approximately the would-be Goldstone boson eaten up by A_X after the $U(1)_X$ gauge symmetry breaking. The leading contribution to the χ mass is also easily derived by looking at the terms quadratic in $\chi,\bar{\chi}$. Its physical mass equals

$$m_\chi^2 \simeq \frac{2q_\chi^2}{\alpha_\phi\phi_{SUSY}^2}V_{up-lift}\,. \tag{5.35}$$

From eq.(5.35) we have $m_\chi \gtrsim (q_\chi/\sqrt{\epsilon})\,m_{3/2} \gg m_{3/2}$ and hence the effects of SUSY breaking on m_χ are small, so that m_χ is approximately the mass of both components of the complex field χ. The mass scale of m_ϕ is of order $\sqrt{\epsilon/X}$ and, unless the moduli are very large, it is just one or two orders of magnitude below the Planck scale. The

[8] A cheap way to overcome this problem is to assume that Y is an effective non-perturbatively generated coupling of some other modulus which has been already stabilized. The problem with this approach is the fact that, for example in the type-IIB set up, the fields that generate such non-perturbative Yukawas are Kähler moduli which are not stabilized by fluxes, therefore, seems unavoidable to include their dynamics and stabilization al together at this stage.

moduli masses are more involved and are best described by an effective approach where the massive fields ϕ and A_X are integrated out, approach we will now consider.

5.1.2 Effective description

As we have seen, the cancellation of the $U(1)_X$ D-term given by a non-trivial VEV for ϕ induces large meson masses, as well as large masses for A_X and ϕ itself. One might then not only integrate out the meson fields, as we did, but also A_X and ϕ. As explained in section (4.2) these field in fact build up a massive vector multiplet and we can integrated it out using a manifestly SUSY approach. At the Gaussian level the e.o.m. for the massive vector multiplet $\partial K_{eff}(\phi_0)/\partial V_X = 0$ has solution

$$V_X = -\widetilde{m}_X^{-2} K_{eff,X}/2\,, \tag{5.36}$$

where $\widetilde{m}_X^2$ is the non-canonically normalized mass matrix for the gauge fields and $K_{eff,X} = \partial_{V_X} K|_{V_X=0}$. Plugging back in K_{eff}, it gives

$$\hat{K} = K(V_{\hat{a}} = 0) - \widetilde{m}_X^{-2} K_{eff,X} K_{eff,X}/4\,. \tag{5.37}$$

The physical gauge that get rid of the would-be Goldstone and its superpartners is the super-unitary gauge, which is nothing else that the super-field version of the standard unitary gauge. It is defined as $\langle X_i\rangle \delta\phi^i = 0$, X^i the holomorphic Killing vectors, and in our case, neglecting the χ component, reads

$$\phi = \phi_0 - \frac{1}{2\alpha_\phi \phi_0}\left[(\delta K_M'' + 2\phi_0^2 \alpha_\phi')(U - U_0) + (\delta \dot{K}_M' + 2\phi_0^2 \dot{\alpha}_\phi)(Z - Z_0)\right], \tag{5.38}$$

where ϕ_0, U_0 and Z_0 are the approximate VEV's we have previously found. The would-be Goldstone boson is essentially given by $\mathrm{Im}\,\phi$, being the last two terms in eq.(5.38) suppressed at least by a factor $\sqrt{\epsilon}$. In this way, we can get rid of the ϕ chiral field, substituting eq.(5.38) in both the Kähler potential (5.18) and superpotential (5.15). At leading order, we can neglect the last two terms in eq.(5.38) and just take $\phi = \phi_0$. This in fact is an approximate version of the gauge chosen in chapter 4 to integrate the vector multiplet. Then the solution (5.36) reads approximately as

$$V_X \simeq -\frac{D_X}{2\alpha_\phi \phi_0^2} \simeq -\frac{q_\chi \alpha_\chi}{2\alpha_\phi \phi_0^2}|\chi|^2\,, \tag{5.39}$$

where in the last relation we have completely neglected the U and Z dynamics by taking $D_X \simeq q_\chi \alpha_\chi |\chi|^2$. The approximate version of eq.(5.37) takes the form,

$$\hat{K}_{eff} \simeq \alpha_\chi |\chi|^2 + K_M - \frac{D_X^2}{2\alpha_\phi \phi_0^2} \simeq \alpha_\chi |\chi|^2 + K_M - \frac{q_\chi^2 \alpha_\chi^2}{2\alpha_\phi \phi_0^2} |\chi|^4 \,. \tag{5.40}$$

The superpotential $\hat{W}_{eff}$ trivially follows from (5.15) with $\phi = \phi_0$, so that the field dependent terms $f(\phi)$ and $A(\phi)$ defined in eq.(5.16) become now effective constants $f(\phi_0)$ and $A(\phi_0)$. Notice that $\hat{K}_{eff}$ and $\hat{W}_{eff}$ sensitively depend on ϕ_0, whose precise value cannot be correctly determined without actually using the full underlying model. However, the shifts on ϕ, as computed in the previous subsection, are small enough that at leading order one might safely replace ϕ_0 in all the above formulae (and the one that will follow) by the SUSY value (5.13).

The effective model described by $\hat{K}_{eff}$ and $\hat{W}_{eff}$ is considerably more tractable than the underlying UV model we considered before. In particular, some physical features are more transparent and, in addition, such effective description provides us with an approximate formula for the moduli masses. For instance, it is immediately clear that a vacuum with $\chi_0 \ll 1$ and non-runaway moduli will necessarily break SUSY, since $F_\chi \simeq f \exp(-\gamma_Z Z_0 - \gamma_U U_0) \neq 0$. In fact, as far as χ is concerned, the model is nothing else than a Polonyi model with a deformed Kähler potential. As we already mentioned, in the flat-space limit with decoupled moduli, χ will be stabilized at the origin due to the $|\chi|^4$ term in eq.(5.40). The extrema conditions (5.21) for U and Z are rederived by requiring $\hat{F}_U^{(0)} = \hat{F}_Z^{(0)} = 0$, using the same notation as before. An approximate analytical formula for the moduli mass terms can be derived, once again by keeping the leading term in an expansion in ϵ. One gets

$$m_{i\bar{j}}^2 \simeq e^{K_M} \partial_{\bar{j}} \partial_{\bar{m}} \bar{\hat{W}}_{eff}^{(0)} K_M^{\bar{m}l} \partial_i \partial_l \hat{W}_{eff}^{(0)} \,, \tag{5.41}$$

with the indices running over U and Z. Writing the moduli metric as $g = (T^{-1})^\dagger T^{-1}$, as we did in section (3.1) with the Choleski decomposition, the canonical normalized mass matrix (regarding a small Kähler mixing with χ) is given by

$$\mathcal{M} = T^\dagger m^2 T \,. \tag{5.42}$$

Notice that with an arbitrary vector ϕ,

$$\phi^i \mathcal{M}_{i\bar{j}} \bar{\phi}^{\bar{j}} = \Phi_{\bar{j}} K^{\bar{j}i} \Phi_i \,, \tag{5.43}$$

where $\Phi_i = \partial_i \partial_j \hat{W}_{eff}^{(0)} T_{\bar{j}}^j \bar{\phi}^{\bar{j}}$, i.e. $\mathcal{M}$ is positive definite, so its eigenvalues are all positive, therefore the solution is indeed a minimum. For the particular case of decoupled Kähler and superpotential terms, namely $\dot{\hat{K}}'_M = \dot{\hat{W}}'_{eff} = 0$, both the moduli masses and kinetic terms are already in a diagonal form, and we get

$$m^2_{U,Z} \simeq e^{K_M} \left| \frac{\partial^2_{U,Z} \hat{W}^{(0)}_{eff}}{\partial^2_{U,Z} K_M} \right|^2 \sim \epsilon^{-2} m^2_{3/2} \,. \tag{5.44}$$

The last equality of eq.(5.44) shows the scaling of the moduli masses with respect to the gravitino mass $m_{3/2}$. As can be seen, the moduli are parametrically heavier than the gravitino, which is a cosmologically welcome feature.

An important comment is now in order. One might wonder why we have decided to adopt from the beginning an effective field theory approach for the hidden mesons (and tacitly for all the other moduli possibly responsible for the constant term in the superpotential), integrating them out from the very beginning, and not for ϕ and A_X which are actually even heavier! From a purely effective quantum field theory point of view, this is indeed not justified, the correct procedure being the integration of all the states in the order specified by their mass scales and run the effective parameters down to lower energies. At the classical level we are considering here and when focusing only on the properties of the vacua up to the mass level, however, no real difference occurs and integrating out some state or not is only a matter of simplicity. Contrary to the mesons, which can always be easily integrated out supersymmetrically to a very good approximation, as we did, ϕ and A_X would require more work than what we have shown above to be properly integrated out, because they are more sensible to SUSY breaking effects. Here we are at most able to go to the leading order in ϵ, since the next to leading order would require the knowledge of the VEV at this precision something that is unknown at this stage. More importantly, the approximation of completely neglecting the moduli dynamics in the induced part of the effective Kähler potential and substitute it with just $q_\chi^2 \alpha_\chi^2 |\chi|^4$, as we did in eqs.(5.39) and (5.40), turns out to be in general a quite crude approximation. Both these approximations can be relaxed and we have analytically and numerically checked that the resulting "improved" effective model reproduce pretty well, at a more quantitative level, the main properties of the full theory. Contrary to the naive effective theory we have shown above, however, the improved theory is no less complicated than the full one, so that no real simplification occurs in considering it. More over the full implementation of the gauge fixing would require the knowledge of the numerical values of the gauge fixing equation with a precision of $\mathcal{O}(m^2_{3/2}/m^2_V)$ [9]. On the contrary, the naive model captures all the qualitative features of the full model and, as a matter of fact, it has been crucial to guide us in the analysis of the previous subsection.

[9] For the case of the gauge in which ϕ is fixed to its VEV this is the numerical precision required for $\langle\phi\rangle$.

The analysis of the general two-moduli model performed so far can trivially be reduced to the single modulus (U) case with essentially no effort when $w_0 \neq 0$. We will then not repeat the analysis here, but just point out that all the considerations we made for the two-moduli case apply. The only qualitative difference is that with a single modulus one condensing gauge group is enough.

5.2 Models with $w_0 = 0$

From the previous discussion it is clear that in general one has to face a possible fine-tuning problem to get $Y \ll 1$ in the Yukawa sector for χ, but one might argue that this is, by means of the flatness condition, just a reflection of the other fine-tuning problem required to have a tiny constant superpotential term, $w_0 \ll 1$. In principle, then, we have to face up to three fine-tuning problems, the third being the unavoidable tuning of the cosmological constant. Moreover, up to possible suppressions coming from the $\exp(K)$ term in the scalar potential, w_0 essentially fixes the supersymmetry breaking scale. It would be more desirable, instead, to dynamically generate it. This motivates us to analyze also the case in which w_0 vanishes, assuming that some stringy symmetry forbids its appearance. Let us start by considering a theory with a single modulus (U). Most of the considerations we made for $w_0 \neq 0$ continue to apply for $w_0 = 0$, the main difference being the moduli stabilization mechanism, which now boils down to a racetrack model [66], where the scale of supersymmetry breaking is dynamically generated. The effective Kähler potential is still given by eq.(5.18), with the obvious understanding that K_M, α_ϕ and α_χ do not depend on Z. Similarly, the effective superpotential is as in eq.(5.15), with $\gamma_Z = b_1 = b_2 = w_0 = 0$. The condition of vanishing D_X term still fixes ϕ to the value (5.13). The equation $F_U^{(0)} = 0$, at leading order in ϵ has as solution

$$U_{SUSY} = \frac{1}{a_1 - a_2} \ln \left(-\frac{a_1 A_1(\phi_{SUSY})}{a2 A_2(\phi_{SUSY})} \right) . \tag{5.45}$$

The axionic component of U is always extremized such that the two condensing sectors get opposite signs, therefore for simplicity we take $\eta_1 = -\eta_2 = 1$ and set it to zero. The scaling relations reported in eq.(5.23) are still valid but the very last relation among the derivatives of the superpotential should be reviewed. Indeed, the racetrack models work using the competing effects of the different condensing sectors, as clearly illustrated by eq.(5.45). As a result, there are some cancellations among the condensing scales that hold at the F-term level, but are destroyed once derivatives of F-terms are taken. The result of this is that the scaling behavior of the first derivative of the superpotential still satisfies the relation given by (5.23), but the higher ones are changed to

$$\partial_U^n W_{eff}^{(0)} \sim a^n W_{eff}^{(0)}\,, \quad n > 1\,. \tag{5.46}$$

Eq.(5.26) and the first relation in eq.(5.27) still hold with obvious notation changes, so using eq.(5.46),we now get for the modulus shift:

$$\Delta U \sim \epsilon^4 \frac{V_{up-lift}}{(W_{eff}^{(0)})^2}\, U(q_\chi + \gamma U)\,, \tag{5.47}$$

where we omit the unnecessary subscript U in γ. Comparing eq.(5.47) with eq.(5.27), we notice that the shift of U induced by the up-lifting term is now two or three orders of magnitude smaller than the models with $w_0 \neq 0$, being $\mathcal{O}(\epsilon^4)$ instead of $\mathcal{O}(\epsilon^2)$. The F-terms are also parametrically smaller than before:

$$F_U^{(0)} \sim \epsilon^2 \Big(\gamma + \frac{q_\chi}{U}\Big) \alpha_\chi^{-1/2} F_\chi^{(0)}\,, \qquad F_\phi^{(0)} \sim \frac{\delta}{\phi}\epsilon^2 \Big(\gamma + \frac{q_\chi}{U}\Big) \alpha_\chi^{-1/2} F_\chi^{(0)}\,, \tag{5.48}$$

where $F_\chi^{(0)} \sim \alpha_\chi^{1/2} W_{eff}^{(0)}$. The constraint $\gamma \Delta U \ll 1$ gives now

$$\gamma^2 \ll \frac{a^2}{\epsilon^2}\,. \tag{5.49}$$

We see from eq.(5.49) that values of $\gamma \sim a$ are now allowed, solving the fine-tuning problem in the coupling Y. The shift on the vacuum induced by the backreaction of χ is now comparable or even slightly larger than eq.(5.47). However, it is typically small enough not to destabilize the vacuum. The scaling of χ can still be estimated by the relation $\phi F_\phi \sim q_\chi \chi F_\chi$, giving

$$\chi_0 \sim \delta \epsilon^2 \Big(\gamma + \frac{q_\chi}{X}\Big) \frac{1}{q_\chi} \alpha_\chi^{-1/2}\,, \tag{5.50}$$

which is small, as required. The scalings of the full F terms (5.32) can easily be worked out using eq.(5.50). We find that $F_\chi \sim F_\chi^{(0)}$, $F_X \sim F_X^{(0)}$ and $F_\phi \sim F_\phi^{(0)}$, but the χ-dependent terms in F_X and F_ϕ are non-negligible. The mass of U can be estimated using an effective description, as explained in subsection 2.2. The first relation in eq.(5.44) still holds, but the different scaling (5.46) of the superpotential gives now

$$m_U^2 \sim \epsilon^{-4} m_{3/2}^2\,. \tag{5.51}$$

The above analysis can be extended to the case of two moduli, in which case one has to work with at least three condensing sectors to get viable SUSY solutions. Instead of considering the most general model with three condensing gauge groups, we will now focus on an interesting class of models with decoupled non-perturbative superpotential terms. The effective superpotential W_{RT3} reads now

$$W_{RT3} = f(\phi)e^{-\gamma_Z Z-\gamma_U U}\chi + A_1(\phi)e^{-a_1 U} - A_2(\phi)e^{-a_2 U} + A_3(\phi)e^{-bZ}\,. \tag{5.52}$$

If the condensing scales associated to the gauge groups G_1 and G_2 are much larger than that of G_3, U is approximately stabilized by a racetrack mechanism given by G_1 and G_2 at the value (5.45). With U so stabilized, $W_{RT3}^{(0)}$ (notation as before) boils down to a KKLT-like superpotential which gives the following SUSY extremum for Z (see e.g. [84] for a similar analysis):

$$Z_{SUSY} \simeq -\frac{1}{b}\ln\left[\frac{\dot{K}_M\hat{w}_0}{bA_3}\right], \tag{5.53}$$

where

$$\hat{w}_0 \equiv A_1(\phi_{SUSY})e^{-a_1 U_{SUSY}} - A_2(\phi_{SUSY})e^{-a_2 U_{SUSY}} \tag{5.54}$$

is an effective constant superpotential term. The shifts in the fields induced by the up-lifting term $V_{up-lift}$ can be derived using the by now familiar expansion in $\epsilon \simeq 1/(bZ) \simeq 1/(a_1 U) \simeq 1/(a_2 U) \sim 1/(aU)$. The form of the superpotential and the corresponding different stabilization mechanisms for Z and U do not allow now to consider U and Z together. Indeed, we have now $\partial_U^n W_{RT3}^{(0)} \sim a^n W_{RT3}^{(0)}$, as in eq.(5.46), and $\partial_Z^n W_{RT3}^{(0)} \sim b^{n-1}W_{RT3}^{(0)}/Z$, as in eq.(5.23). The leading terms in the shifts of the fields are as follows:

$$\Delta Z \sim \epsilon^2 q_\chi X + \epsilon^2 X^2\gamma_Z + \epsilon^3 X^2\gamma_U\,, \qquad \Delta U \sim \epsilon^3 q_\chi X + \epsilon^3 X^2\gamma_Z + \epsilon^4 X^2\gamma_U\,, \tag{5.55}$$

where X denotes generically U or Z, assumed to be of the same order of magnitude. The usual bound $\gamma_Z\Delta Z \ll 1$ does not allow for natural values $\gamma_Z \sim a \sim b$, so that we are forced to consider $\gamma_Z = 0$ and $\gamma_U \neq 0$, in which case $\gamma_U \sim a \sim b$ can be taken. The shift of ϕ is given by $\Delta\phi \sim \phi\Delta Z/Z$. The $F^{(0)}$-terms scale as in eq.(5.48), with $F_Z^{(0)} \sim F_U^{(0)}$. The considerations made for χ_0 and the full F-terms in the single modulus case apply also here. The moduli masses depend on the form of the Kähler potential K_M. If K_M is factorizable, then U and Z will have masses as roughly given by eqs.(5.51) and (5.44), respectively. If K_M is not factorizable, then generally the mass of U will be essentially as given by eq.(5.51), whereas Z will be heavier than what predicted by eq.(5.44), depending on the mixing between the two moduli in K_M.

5.3 Soft masses

The natural framework of SUSY breaking mediation in any model with hidden and not sequestered sector, is gravity mediation, with $m_{3/2} \sim \mathcal{O}(\text{TeV})$. A sufficiently heavy gravitino is desirable for cosmological reasons, being the moduli masses proportional to $m_{3/2}$, according to eqs.(5.44) and (5.51)[10]. When the would-be anomalous $U(1)_X$ gauge field is very massive, like the scenario advocated in our paper, one can effectively integrate A_X out and get the effective Kähler potential (5.40), taking care of including possible visible sector contributions to the D_X term, which we will shortly discuss. In this way, all soft parameters can be derived using the standard results of [94] with F-term breaking only. We will not explicitly compute all the resulting soft terms, but rather we will just estimate the size of the gaugino and scalar masses.

Let us start by considering the non-holomorphic soft scalar masses, in which case our considerations will apply to both the models with $w_0 \neq 0$ and $w_0 = 0$. For canonically normalized fields, they read [94]:

$$m_{i\bar{i}}^2 = m_{3/2}^2 - \frac{1}{\alpha_{iv}} F^I F^{\bar{J}} R_{i\bar{i}I\bar{J}}\,. \tag{5.56}$$

In eq.(5.56), $F^I = \exp(\hat{K}/2)\hat{K}^{I\bar{J}} F_{\bar{J}}$, I, J over the hidden sector fields (U, Z and χ) and α_{iv} is the moduli-dependent function appearing in the Kähler potential of the visible sector, eq.(5.1), with i running over the visible sector fields. Two possibilities arise, depending on whether the $U(1)_X$ charges q_{iv} are vanishing or not. When $q_{iv} \neq 0$, $D_X \simeq \alpha_\chi q_\chi |\chi|^2 + \alpha_{iv} q_{iv} |Q^{(iv)}|^2$ and the leading canonically normalized soft mass term reads

$$m_{i\bar{j}}^2 \simeq \delta_{i\bar{j}} m_{3/2}^2 \frac{3 q_\chi q_{iv}}{\alpha_\phi |\phi_0|^2} \gtrsim \delta_{i\bar{j}} m_{3/2}^2 \frac{q_\chi q_{iv}}{\epsilon} \tag{5.57}$$

which arise from the term $F^\chi F^{\bar{\chi}} R_{\chi\bar{\chi}i\bar{j}}$ in eq.(5.56). Using eq.(5.24), eq.(5.57) can be also rewritten in the more conventional form $m^2 = q_{iv} g_X^2 \langle D_X \rangle$.

If $q_{iv} = 0$, $D_X \simeq \alpha_\chi q_\chi |\chi|^2 - \delta/2\alpha'_{iv} q_{iv} |Q^{(iv)}|^2$. The leading term coming from D_X is now of the same order as the universal $m_{3/2}^2$ term appearing in eq.(5.56), so that

$$m_{i\bar{j}}^2 \sim \delta_{i\bar{j}} m_{3/2}^2\,. \tag{5.58}$$

In eq.(5.58) we have not considered the contribution of possible quartic terms in the charged fields of the form $|Q_{iv}|^2 |\chi|^2$ which we have not specified in the Kähler potentials

[10] One might further push $m_{3/2}$ to $\mathcal{O}(10TeV)$ or more, assuming a sequestering of the hidden sector from the visible sector, so that the gravity mediation can be suppressed and anomaly mediation takes over [101]. We will not consider this possibility, which is non-generic.

(5.1) and (5.2). Their contribution can be relevant or even dominant, but it is model-dependent and can easily be derived from eq.(5.56) once these terms are specified. Given eqs.(5.57) and (5.58), the choice $q_{iv} = 0$ is preferred, giving rise to not too heavy scalar masses.

Let us now consider the gauginos. Their canonically normalized masses are

$$m_g = \left| F^I \frac{\partial_I f_v}{2\mathrm{Re}\, f_v} \right| , \tag{5.59}$$

where f_v schematically denotes the holomorphic gauge kinetic functions of the visible gauge group. Let us first discuss the models with $w_0 \neq 0$. In this case, using eq.(5.30), we can easily estimate, for linearly moduli dependent f_v,

$$m_g \sim m_{3/2}\, \epsilon\, X \Big(\gamma + \frac{q_X}{X} \Big) \, . \tag{5.60}$$

We see that $m_g < m_{3/2}$, but on the other hand eq.(5.60) predicts gaugino masses which are typically larger than those found in the original KKLT scenario with $\overline{D}3$-branes. This was already observed in [44] for a model with one modulus and perturbative up-lifting term ($\gamma = 0$). We notice here that when $\gamma \neq 0$ (or $q_X > 1$), eq.(5.60) predicts even larger gaugino masses. In fact, considering that $\epsilon X \gamma \sim \gamma / a$ and the bound (5.28), the gauginos can be made just a few times lighter than $m_{3/2}$, sufficiently heavy to neglect anomaly mediation contributions which become relevant if $m_g \lesssim m_{3/2}/(4\pi^2)$. We believe this is an important welcome feature of models with two moduli. As already argued in [44], in presence of one modulus only, non-vanishing tree-level gaugino masses would require f_v to depend on U. Due to the non-linear transformation of U under $U(1)_X$, anomalous transformations of the action are induced, which must be compensated by $U(1)_X$-G_{vis}^2 anomalies in the fermion spectrum, requiring $q_{iv} \neq 0$ or some other modification, such as the introduction of $U(1)_X$ charged fields, vector-like with respect to G_{vis}, which can also be seen as messenger fields of a high scale gauge mediation. This possibility was studied in some detail for the single modulus case in [102], where among others the stability of the metastable vacuum, once the messengers are introduced, has been check under some mild assumptions. We simply notice that in presence of two moduli, a more economical choice is to assume f_v to depend on the neutral modulus Z only, in which case one can safely take $q_{iv} = 0$. We will see in a specific example in the next section that this choice, together with $\gamma_Z \neq 0$, gives rise to a fully satisfactory scenario for gaugino and scalar mass terms.

The gaugino masses in the models with $w_0 = 0$ sensitively depend on how we choose the exponential term γ in the up-lifting term. As we have seen, a natural up-lifting term requires $\gamma_U \neq 0$ and hence $\gamma_Z = 0$ if we allow the non-perturbatively generated coupling to depend on one modulus only. Furthermore, if we want to avoid introducing additional $U(1)_X$ charged fields, then f_v should depend on Z only. In this case eq.(5.48) gives, for linearly moduli dependent f_v:

$$m_g \sim m_{3/2}\,\epsilon^2\,X\Big(\gamma_U + \frac{q_X}{X}\Big) \sim m_{3/2}\,\epsilon\,, \tag{5.61}$$

where in the last relation the scaling $\gamma_U \sim a$ has been used. The gaugino masses are significantly lighter than the gravitino now, so that anomaly mediated contributions cannot be neglected. Gaugino masses can be increased by allowing γ_Z to be non-vanishing, in which case they scale as in eq.(5.60). If one allows the non-perturbatively generated up-lifting term to depend on both moduli, then γ_U and γ_Z can respectively solve the naturalness problem of the up-lifting coupling and alleviate the modest hierarchy between gaugino and scalar masses.

5.4 Explicit models

The exponential sensitivity of the superpotential (5.15) on the moduli, the not so small value of the expansion parameter $\epsilon \sim 1/30$ and the several other approximations made before do not generally allow for a fully reliable, quantitative analytical study of the theory. Indeed, the main aim of sections (5.1) and (5.2) was to qualitatively characterize the models and to show the existence of metastable Minkowski vacua with low-energy SUSY breaking in a large area in parameter and moduli space, rather than quantitatively study them. The aim of this section is to study at a more quantitative level three specific models, two with $w_0 \neq 0$ and one with $w_0 = 0$. Most of the analysis here is performed numerically, because the exponential nature of the superpotential and the smallness of the D_X term at the minimum require a detailed knowledge of the location of the vacuum, in particular in the moduli directions. In order to appreciate this point, we will report in tables 1, 2 and 3 various quantities of interest computed starting both by the exact numerical vacuum and the approximate analytical one. The latter is found along the lines of subsection 2.1. We start from the SUSY configuration for ϕ, U and Z given by eqs.(5.13) and (5.21) for $w_0 \neq 0$, and eqs.(5.13), (5.45) and (5.53) for $w_0 = 0$. We then expand V_0, taking $V_{up-lift}$ as a perturbation, around the SUSY vacuum, keeping only the linear terms in ΔU, ΔZ and $\Delta\phi$. In this way we get what we denoted by U_0, Z_0 and ϕ_0. We finally compute the VEV of χ as explained below eq.(5.31).

Notice that already the numerical search of exact minima in presence of the D_X-term is not straightforward. The D_X-term is naturally of order one when slightly off-shell, and thus much bigger than the typical values of the F terms, namely one has $V_D \gg V_F$ and all the energy of the system is dominated by V_D, hiding completely the stabilization of the moduli encoded in V_F. On the contrary, in the heavy gauge field approximation, $V_D \ll V_F$ at the minimum, being $\mathcal{O}(F^4)$, see eq.(5.24), which means that a severe fine-tuning takes place in V_D at the minimum. We have been able to circumvent this problem using a linear combination of the equations of motion for ϕ and χ to solve for D_X in terms of F-terms and their derivatives, and replace the result in these. We also found useful, instead of solving the equation of motion for ϕ, to impose that the expression found for D_X to be equal to its expression (5.12) in terms of the fields. The numerical vacua so obtained turns out to be stable and the resulting D_X and F-terms always satisfies the consistency condition (A.17).

5.4.1 Type-IIB model

The first model we consider is based on an orientifold compactification of type IIB string theory on a CY 3-fold obtained as an hyper-surface in $\mathbf{CP}^4$, namely $\mathbf{CP}^4_{[1,1,1,6,9]}$. This CY has $h^{1,1} = 2$ and $h^{2,1} = 272$ Kähler and complex structure moduli, respectively. See [91] for details. In the spirit of the Two-Step moduli stabilization, we assume here that a combination of NSNS and RR fluxes stabilized the dilaton and complex structure moduli supersymmetrically. These, once being frozen out, give rise to a constant superpotential term w_0. We do not specify the detailed string construction which might give rise to the superpotential (5.3). We generally assume that $D7$-branes (and $O7$ planes) must be introduced to generate the non-perturbative superpotential terms in (5.3), as well as the non-linear transformation under $U(1)_X$ of the modulus U, as was shown in section (2.4.2). We will neglect in the following possible open string moduli and consider the dynamics of the two Kähler moduli only, identifying them with the two moduli U and Z. We have now to specify the explicit form of the various terms entering in the Kähler potential (5.2). The purely moduli-dependent function K_M is known. In the usual approximation of neglecting flux effects, it takes the form [91]

$$K_M = -2\ln\mathcal{V}\ , \quad \mathcal{V} = \frac{1}{9\sqrt{2}}\left(\left(\frac{U+\bar{U}}{2}\right)^{3/2} - \left(\frac{Z+\bar{Z}}{2}\right)^{3/2}\right), \tag{5.62}$$

where $\mathcal{V}$ is the volume of the CY manifold. We do not specify the modular functions $\alpha_{1,2}$ for the mesons $M_{1,2}$ since they do not play any role in the limit where the mesons are supersymmetrically integrated out. The modular functions for ϕ and χ, α_ϕ and α_χ in eq.(5.2), are instead relevant but are generally difficult to derive and depend on the underlying string construction. We assume here the following ansatz:

$$\alpha_\phi = \alpha_\chi = \frac{(Z+\bar{Z})}{\mathcal{V}}\,, \tag{5.63}$$

which is simple enough, but not totally trivial. It should be stressed that there is nothing peculiar in the (arbitrary) choice we made in eq.(5.63). Any other choice will be fine as well, provided that α_χ is not too small. Indeed, according to eq.(5.31), $\chi_0 \sim \alpha_\chi^{-1/2}$ and a sufficiently small α_χ can lead to a breakdown of our analysis based on an expansion in χ. We have now to specify the various parameters entering in the hidden superpotential (5.3) and the gauge kinetic functions (5.4). Their choice is somehow arbitrary, but we require that U and Z and the volume of the CY to be sufficiently large to trust the classical SUGRA analysis. We take, for $\eta_1 = -\eta_2 = -1$,[11]

$$\begin{aligned} &N_1 = 40, \quad N_{1f} = 4, \quad q_1 = 1, \; c_1 = 1, \; p_1 = 0, \qquad n_1 = \frac{1}{4\pi}, \; m_1 = 0, \; \eta_{1,U} = \eta_{1,Z} = 0, \\ &N_2 = 25, \quad N_{2f} = 1, \quad q_2 = 0, \; c_2 = 1, \; p_2 = 0, \qquad n_2 = 0, \; m_2 = \frac{1}{4\pi}, \; \eta_{2,U} = \eta_{2,Z} = 0, \\ &w_0 = 9 \times 10^{-14}, \quad Y = 4.2 \times 10^{-5}, \quad q_\chi = 6, \qquad \gamma_Z = \frac{1}{60}, \; \gamma_U = 0, \; n_X = \frac{1}{4\pi}. \end{aligned} \tag{5.64}$$

The exponential moduli dependence in the up-lifting term $V_{up-lift}$ is supposed to arise

	Numerical	Analytical
$\langle U\rangle$	232	230
$\langle Z\rangle$	148	146
$\langle \phi\rangle$	6.2×10^{-2}	6.3×10^{-2}
$\langle \chi\rangle$	2.2×10^{-4}	1.3×10^{-4}
$\langle F_U\rangle$	-4.0×10^{-16}	-2.3×10^{-16}
$\langle F_Z\rangle$	4.2×10^{-16}	2.3×10^{-16}
$\langle F_\phi\rangle$	8.5×10^{-15}	5.1×10^{-15}
$\langle F_\chi\rangle$	2.1×10^{-13}	2.2×10^{-13}
$\langle D_X\rangle$	1.5×10^{-26}	1.6×10^{-27}
$\langle V\rangle$	9.2×10^{-33}	2.2×10^{-32}

	Numerical	Analytical
$m_{3/2}$	1.6	1.6
m_ϕ	7.3×10^{13}	7.4×10^{13}
$m_{\mathrm{Re}(\chi)}$	242	254
$m_{\mathrm{Im}(\chi)}$	241	253
$m_{\mathrm{Im}(\tilde U)}$	140	176
$m_{\mathrm{Re}(\tilde Z)}$	97	156
$m_{\mathrm{Im}(\tilde Z)}$	95	105
$m_{\mathrm{Re}(\tilde U)}$	61	93

TABLE 5.1: VEV's, masses and scales for the IIB model with $w_0 \neq 0$ and parameters given by eq.(5.64). Expectation values are expressed in (reduced) Planck units and masses in TeV units. $\tilde U \sim U+Z$ and $\tilde Z \sim U-Z$ stand for the (approximate) eigenvector mass states. The definitions are slightly different in the numerical and analytical cases due to the diagonalization of the kinetic terms.

from some non-perturbative effect, such as stringy instantons [103]. The supersymmetric vacuum when the χ-sector is turned off is

$$U_{SUSY} = 229\,, \quad Z_{SUSY} = 145\,, \quad \phi_{SUSY} = 6.3 \times 10^{-2}\,. \tag{5.65}$$

We report in table (5.4.1) (left panel) the location of the non-SUSY vacuum, as exactly found numerically and analytically by linearly expanding around the SUSY solution (5.65), as well as the F-terms, D_X and the potential V at the minimum. It can be seen

[11] What actually matters are the values of the phenomenological parameters (5.16), which do not uniquely fix the microscopical ones, as is evident from eq.(5.16). The choice (5.64) is purely illustrative. The same comment also applies to the next two examples below.

that U, Z and ϕ are well reproduced analytically, whereas χ is not, since its VEV is exponentially sensitive to the values of U and Z by means of the non-perturbative terms in W_{eff}. For the same reason F_U, F_Z and F_ϕ are only roughly reproduced. F_χ, instead, is better estimated since it essentially depends on Z only through the mild exponential appearing in the first term in eq.(5.6). The D_X term is also well reproduced because at leading order it is governed by F_χ only, see eq.(5.24). In table (5.4.1) (right panel) the gravitino and all the scalar masses are reported, in TeV units. For simplicity of presentation, we have not written the precise linear combination of mass eigenvectors, but just the main components in field space. As anticipated, there is a hierarchy of scales. Fixing the overall scale such that $m_{3/2} \simeq O(1)$TeV, the field ϕ (and the $U(1)_X$ gauge boson A_X) is ultra-heavy, whereas the moduli and χ have masses $\mathcal{O}(100)TeV$. Like for the F-terms, the masses which do not directly depend on the strong dynamics, namely $m_{3/2}$, whose mass is governed by w_0, and m_ϕ, whose mass is well approximated by eq.(5.34), are well predicted analytically. In agreement with our general observations, $1/(a_1 U) \simeq 1/(b_2 Z) \simeq 1/37 \equiv \epsilon$. It is easy to check that the values of F_ϕ, F_U and F_Z reported in table (5.4.1) agree with the scaling behaviors predicted by eq.(5.30). As observed in subsection 2.1, the stability of the system requires a low value for γ_Z and hence the exponential dependence on Z of the coupling $Y(U,Z,\phi)$ does not help much in getting a not so small Y. This constraints us to choose a rather large value of the $U(1)_X$ charge of χ, $q_\chi = 6$, although such choice might not naturally appear in simple D-brane constructions. We can also compute the universal gaugino masses at the high scale, assuming $f_v = f_2 = m_2 Z$. We get

$$m_g \simeq 380 \text{ GeV}\,, \tag{5.66}$$

which is roughly one quarter the gravitino mass. As explained before, we assume $U(1)_X$ neutral visible matter fields, so that the non-holomorphic soft scalar masses $m \sim m_{3/2} \sim \mathcal{O}(1)$ TeV, instead of $m \simeq g_X\sqrt{D_X q_{iv}} \simeq 70\,\text{TeV}\,\sqrt{q_{iv}}$, valid for $U(1)_X$ charged fields.

We have finally considered the reliability of the SUGRA approximation by considering the α' correction appearing in K_M. For type-IIB orientifolds, this is known to be [89]

$$\mathcal{V} \to \mathcal{V} + \frac{\xi}{2g_s^{3/2}}, \qquad \xi = -\frac{\chi(M)\zeta(3)}{2(2\pi)^3}\,, \tag{5.67}$$

with $\chi(M)$ the Euler characteristic of the CY and $\zeta(3) = \sum_{n=1}^{\infty} 1/n^3 \simeq 1.2$. In writing eq.(5.67), we have frozen the dilaton field S, which is taken non-dynamical and stabilized by fluxes to some value $g_s = \text{Re}\,S$. We have studied how the correction (5.67) roughly affects the model given by the input parameters (5.64), which are kept all fixed with the exception of Y, which is tuned to get an approximately Minkowski vacuum. A sizable correction is expected when $\xi/2g_s^{3/2} \sim \mathcal{V}$. This is indeed the case, although we have numerically checked that the α' correction is non-negligible already when it is $\mathcal{O}(\mathcal{V}/10)$. In our example $\chi(\mathbf{CP}^4_{[1,1,1,6,9]}) = -540$ and for the vacuum shown in table (5.4.1), roughly speaking, $g_s \gtrsim 1/10$ in order to trust the SUGRA analysis and neglect the correction (5.67).

5.4.2 Heterotic model

The second model we consider is inspired by a generic compactification of perturbative heterotic string theory on a CY 3-fold. In such a case, we identify U and Z with the dilaton and the universal Kähler modulus, respectively [12]. Contrary to the IIB case, unfortunately we dot not still have a scenario to stabilize in a controlled way the complex structure moduli in the heterotic string. In addition to that, the presence of a small non vanishing constant superpotential term w_0 is known to be not easily produced. It is well-known that a flux for H induces a constant term in the superpotential [62], but being the H flux quantized, such a constant term is typically of order one in Planck units [104]. In the spirit of our bottom-up SUGRA approach, we do not look for a microscopic explanation for w_0. It might be the left-over term of the F and D terms vanishing conditions for all the extra fields which are supposed to occur in any realistic model, or else the left-over of a flux superpotential of an heterotic string compactified on generalized half-flat manifolds and non-standard embedding, which has been argued to admit quantized fluxes resulting in a small w_0 [105], or the result of approximate R skiametries broken by higher order operators [106]. The classical Kähler potential for the moduli is known to be [27]

$$K_M = -\ln(U+\bar{U}) - 3\ln(Z+\bar{Z})\,. \tag{5.68}$$

The modular functions α_ϕ and α_χ are now functions of Z only and typically expected to be of the form $(Z+\bar{Z})^{-n}$, with $-1 \leq n \leq 0$. We take here $n = 1/3$, so that

$$\alpha_\phi = \alpha_\chi = \frac{1}{(Z+\bar{Z})^{1/3}}\,. \tag{5.69}$$

The parameters entering in the gauge kinetic functions (5.4) and the superpotential (5.3) are quite more constrained with respect to the IIB case. At tree-level $f_i = f_X = U$. A possible Z-dependence can (and generally does) occur only at loop level by means of moduli-dependent threshold corrections. The exponential moduli dependence of the couplings Y and c_i is assumed to arise from world-sheet instantons and hence they depend on Z only. Finally, the size of the hidden gauge groups is bounded. We will

[12]In the more conventional notation introduced in section (2.3) the dilaton and universal Kähler modulus are denoted by S and T, respectively.

look for vacua with $U, Z \gtrsim 1$, which are on the edge of perturbativity, but lie in the phenomenologically most interesting region in moduli space in perturbative heterotic theory. In fact, one should require $\mathrm{Re}\, U \sim 2$ for a successful SUSY GUT model, but since the 10-dimensional string coupling is given by $g_s = \sqrt{Z^3/U}$ [27], perturbativity of the 10d heterotic string also requires $Z^3 < U$ and hence $Z \gtrsim 1$. We then take

$$\begin{aligned}
&N_1 = 5,\ N_{1f} = 4,\ q_1 = 1,\ \ c_1 = \frac{1}{2},\ \ \eta_{1,Z} = 2\pi + \frac{3}{4},\ \eta_{1,U} = 0,\ m_1 = 0,\ n_1 = 1,\ p_1 = 0 \\
&N_2 = 4,\ N_{2f} = 2,\ q_2 = 2,\ \ c_2 = 1,\ \ \eta_{2,Z} = 0,\qquad \eta_{2,U} = 0,\ \ m_2 = 0,\ n_2 = 1,\ p_2 = 0 \\
&w_0 = 3 \times 10^{-15}\ \ ,\ q_X = 10,\ Y = 3.1 \times 10^{-3},\qquad \gamma_Z = 2\pi,\ \gamma_U = 0,\ n_X = 1\,.
\end{aligned} \tag{5.70}$$

The supersymmetric vacuum is at

$$U_{SUSY} = 1.76,\ Z_{SUSY} = 1.19,\ \phi_{SUSY} = 0.14\,. \tag{5.71}$$

In the heterotic case, being the tree-level gauge kinetic functions U-dependent only

	Numerical	Analytical
$\langle U\rangle$	1.78	1.78
$\langle Z\rangle$	1.20	1.20
$\langle \phi\rangle$	0.14	0.14
$\langle \chi\rangle$	-3.2×10^{-4}	-2.0×10^{-4}
$\langle F_U\rangle$	-1.9×10^{-16}	-1.5×10^{-16}
$\langle F_Z\rangle$	-1.4×10^{-15}	-1.0×10^{-15}
$\langle F_\phi\rangle$	-2.1×10^{-17}	-7.2×10^{-18}
$\langle F_\chi\rangle$	4.0×10^{-15}	4.2×10^{-15}
$\langle D_X\rangle$	5.4×10^{-28}	5.9×10^{-28}
$\langle V\rangle$	1.3×10^{-32}	1.7×10^{-32}

	Numerical	Analytical
$m_{3/2}$	1.0	1.0
m_ϕ	3.1×10^{14}	3.1×10^{14}
m_χ	191	200
$m_{\mathrm{Re}(U)}$	98	115
$m_{\mathrm{Im}(U)}$	87	107
$m_{\mathrm{Im}(Z)}$	56	61
$m_{\mathrm{Re}(Z)}$	42	52

Table 5.2: VEV's, masses and scales for the heterotic model with $w_0 \neq 0$ and parameters given by eq.(5.70). Expectation values are expressed in (reduced) Planck units and masses in TeV units.

($m_{1,2} = 0$), one cannot have both $\eta_{1,Z} = \eta_{2,Z} = 0$, since this would lead to the vanishing of the effective parameters $b_{1,2}$ defined in eq.(5.16), which is unacceptable. The particular choice of $\eta_{1,Z}$ in eq.(5.70) is required to fix Z in the small window $1 \lesssim Z \lesssim U$, but of course there are several ways to achieve it in terms of the microscopic parameters, given that what matters are the effective ones defined in eq.(5.16). The asymmetry between U and Z in this heterotic inspired model does not allow to straightforwardly use the general scalings discussed in section 2.1. We do not report the analytical more elaborated analysis which is now required. One can nevertheless check that the general scalings (5.27) and (5.30) still give the rough order of magnitude estimate for the shifts of the fields and the F terms by taking $\epsilon \simeq 1/(a_1 U) \simeq 1/28$. We report in table (5.4.2) (left and right panel) the location of the non-SUSY vacuum, the F-terms, D_X, the potential and the masses for the scalars and the gravitino of the model. As in the IIB model before, χ is not analytically reproduced to a good accuracy, being exponentially sensitive to the values of U and Z. Similarly for F_U, F_Z and F_ϕ. The hierarchy of scales appearing in the gravitino and scalar masses are the same as in the Type IIB model. The large $U(1)_X$ charge of χ, $q_\chi = 10$, allows for a natural explanation of the smallness of the up-lifting term without using tiny values for Y, as was done in (5.64) in the previous example. The tree-level universality of the heterotic holomorphic gauge kinetic functions (for level one Kac-Moody gauge groups, assumed here) fixes $f_v = U$ and hence the universal high scale gaugino masses are calculable and read

$$m_g \simeq 240\ \text{GeV}\,. \tag{5.72}$$

Unfortunately it is not possible now to assume all visible matter fields to be $U(1)_X$ neutral, since $f_v = U$. For the $U(1)_X$ charged fields we get now $m \simeq g_X\sqrt{D_X q_{iv}} \simeq 42\,\text{TeV}\,\sqrt{q_{iv}}$.

The vacuum reported in table (5.4.2) is barely perturbative since the associated 10d string coupling constant $g_s \simeq 1$. One can explicitly see that the situation is similar in the α' expansion, due to the low values of the moduli, by considering again the universal α' correction to the Kähler potential for Z, which now reads [99]

$$(Z+\bar{Z})^3 \to (Z+\bar{Z})^3 + 4\xi, \tag{5.73}$$

with ξ defined as in eq.(5.67). We find that the α' correction (5.73) is generally negligible for $|\xi| \lesssim \mathcal{O}(1/10)$ and are deadly for $|\xi| \gtrsim \mathcal{O}(1)$, with the impossibility of achieving a Minkowski vacuum ($\xi < 0$) or the appearance of tachyons ($\xi > 0$). In the range $\mathcal{O}(1/10) \lesssim |\xi| \lesssim \mathcal{O}(1)$ the qualitative properties of the model are unaffected, but the numerical values reported in table (5.4.2) get corrections of order 100%. Considering that for $|\chi(M)| \sim 10^2$, a typical value for CY manifolds, $|\xi| \sim \mathcal{O}(1)$, it is clear that the phenomenologically interesting region $U, Z \gtrsim 1$ is barely calculable, as we anticipated.

5.4.3 Type-IIB model with $w_0 = 0$

As we have seen, the requirement of a natural Y favors the choice $\gamma_U \neq 0$ for models with $w_0 = 0$. In an heterotic context, where U is the dilaton, we would be forced to invoke exotic non-perturbative couplings. In addition to that, hierarchies would also appear in the mesonic Yukawa couplings c_i. The upper bound on the gauge groups leads to a lower bound on a, $a \gtrsim 15$ and $a_1 - a_2 \sim 3$. Using eq.(5.45), it is easy to see that the phenomenological requirement $U \sim 2$ constraints $A_1/A_2 \sim 10^3$. In light of eq.(5.16), this hierarchy in A_1/A_2 typically induces an even larger hierarchy in the microscopical Yukawa couplings c_i, unless one assumes a hierarchy between them due to, say, world-sheet instantons. For these reasons, as a specific example of model with $w_0 = 0$, we again opt here for a type IIB model on $\mathbf{CP}^4_{[1,1,1,6,9]}$, with U and Z the two Kähler moduli of the CY manifold. No bound on the gauge group arises and the situation seems more favorable. For simplicity we take now trivial Kähler potentials for ϕ and χ, namely

$$\alpha_\chi = \alpha_\phi = 1\,. \tag{5.74}$$

The input parameters are as follows:

$$\begin{array}{lllll} N_1 = 30, \;\; N_{1f} = 2, \;\; q_1 = 2, \;\; c_1 = 1, \;\; p_1 = \frac{18}{50}, & n_1 = \frac{1}{4\pi}, & m_1 = 0, & \eta_{1,U} = \eta_{1,Z} = 0, \\ N_2 = 29, \;\; N_{2f} = 2, \;\; q_2 = 2, \;\; c_2 = 2, \;\; p_2 = 0, & n_2 = \frac{1}{4\pi}, & m_2 = 0, & \eta_{2,U} = \eta_{2,Z} = 0, \\ N_3 = 11, \;\; N_{3f} = 1, \;\; q_3 = 0, \;\; c_2 = 1, \;\; p_3 = 0, & n_3 = 0, & m_3 = \frac{1}{4\pi}, & \eta_{3,U} = \eta_{3,Z} = 0, \\ \qquad Y = \frac{867}{5000}\,, \qquad q_\chi = 2, \;\; n_X = \frac{1}{4\pi}, & \gamma_Z = 0, & \gamma_U = \frac{1}{6}\,. & \end{array} \tag{5.75}$$

The supersymmetric vacuum is at

$$U_{SUSY} = 136, \quad Z_{SUSY} = 63, \quad \phi_{SUSY} = 0.10, \tag{5.76}$$

and the exact non-SUSY vacuum, and its properties, is reported in table(5.4.3). Notice that the model is quite constrained. Given $N_{1,2}$, $N_{1f,2f}$ and $q_{1,2}$, for mesonic Yukawa couplings $c_i \sim \mathcal{O}(1)$, the gauge kinetic functions are essentially fixed by the requirement of low-energy SUSY. A constant threshold correction, appearing in (5.75), can be avoided by allowing a mild tuning between c_1 and c_2. The value of the coupling Y is fixed by the flat condition. As expected from the general arguments of section 4, the gauginos are now light and indeed we get, for $f_v = f_3 = Z/(4\pi)$,

$$m_g \simeq 270 \text{ GeV}\,, \tag{5.77}$$

which is less than one order of magnitude smaller than $m_{3/2}$. We can take $U(1)_X$ neutral visible matter fields, so that the non-holomorphic soft scalar masses $m \simeq m_{3/2}$, instead of $m \simeq g_X\sqrt{D_X q_{iv}} \simeq 80\,\text{TeV}\,\sqrt{q_{iv}}$, valid for $U(1)_X$ charged fields. The analytical and numerical values in this case agree with very good accuracy thanks to the smallness of χ. We have $1/(b_3 Z) \simeq 1/36 \sim 1/(a_{1,2}U) \simeq 1/30$ and the expected scalings for the shifts of the fields and the F terms are satisfied. The value of χ as would be roughly predicted by eq.(5.50) is about one order of magnitude bigger than its actual value, due to accidental numerical factors for which $q_\chi \chi F_\chi \sim \phi F_\phi/10$. Notice how this hybrid model, where the KKLT-like and racetrack stabilizations work together, has all the appealing features

	Numerical	Analytical
$\langle U\rangle$	136.0	136.0
$\langle Z\rangle$	63.3	63.3
$\langle \phi\rangle$	0.101	0.101
$\langle \chi\rangle$	-3.6×10^{-6}	-7.6×10^{-7}
$\langle F_U\rangle$	-2.7×10^{-17}	-2.6×10^{-17}
$\langle F_Z\rangle$	-7.2×10^{-18}	-8.1×10^{-18}
$\langle F_\phi\rangle$	1.5×10^{-16}	1.6×10^{-16}
$\langle F_\chi\rangle$	2.0×10^{-13}	2.0×10^{-13}
$\langle D_X\rangle$	1.2×10^{-26}	1.2×10^{-27}
$\langle V\rangle$	2.4×10^{-32}	2.4×10^{-32}

	Numerical	Analytical
$m_{3/2}$	3.31	3.31
m_ϕ	1.1×10^{14}	1.1×10^{14}
$m_{\mathrm{Im}(\tilde U)}$	6.3×10^{3}	6.3×10^{3}
$m_{\mathrm{Re}(\tilde U)}$	6.3×10^{3}	6.3×10^{3}
$m_{\mathrm{Re}(\tilde Z)}$	229	229
$m_{\mathrm{Im}(\tilde Z)}$	212	212
$m_{\mathrm{Im}(\chi)}$	160	160
$m_{\mathrm{Re}(\chi)}$	160	160

TABLE 5.3: VEV's, masses and scales for the IIB model with $w_0 = 0$ and parameters given by eq.(5.75). Expectation values are expressed in (reduced) Planck units and masses in TeV units. $\tilde U \sim U+Z$ and $\tilde Z \sim U-Z$ stand for the (approximate) eigenvector mass states.

we were looking for. The stabilization via the racetrack mechanism of U allows to get a natural Yukawa coupling Y, and the stabilization of Z after the uplifting generates acceptable gaugino masses and avoid the problem of having too heavy scalar soft masses.

We have numerically checked the reliability of the classical SUGRA analysis by looking at the α' correction (5.67). As expected, the correction becomes sizable for $|\xi|/g_s^{3/2} \gtrsim \mathcal{V}$, i.e, $g_s \sim 1/25$ and is essentially negligible for $|\xi|/g_s^{3/2} \lesssim \mathcal{V}/10$. This results on a mild lower bound on the string coupling: $g_s \gtrsim 1/6$. Contrary to the case with $w_0 \neq 0$ where the possibility of getting a Minkowski vacuum is lost for g_s smaller than the bound, no dramatic consequences appear now, in the sense that the corrections are only quantitative, but the non-SUSY Minkowski vacuum is still there, even for $g_s \sim 1/30$.

Chapter 6

Conclusions and Outlook

In this thesis we have addressed the issue of the Two-Step moduli stabilization in full detail. We found the conditions for the procedure to be reliable around nearly supersymmetric vacua for a generic class of SUSY (global and local) theories whose moduli superpotential is given by[1],

$$W = W_0(H) + \epsilon W_1(H, L)\,, \tag{6.1}$$

with $\epsilon \ll 1$ and the moduli H the supermultiplets to be frozen out. No constraints on the Kähler potential are assumed, provided that it is sufficiently regular. The analysis was performed in full generality first considering only moduli fields and then introducing matter and gauge interactions.

In chapter 3, devoted to the pure moduli case, we found that the simple supersymmetric model, where the H supermultiplets are frozen out at the level of the superpotential and Kähler potential, is a reliable description at leading order in ϵ, as far as the scalar components of the frozen multiplet be solutions of the leading F-flatness conditions, and acquire $\mathcal{O}(1)$ masses from the W_0 dynamics. In the case of SUGRA theories two situations were identified: for a generic Kähler potential a mass hierarchy of $\mathcal{O}(\epsilon)$ is compulsory, turning into a constrain on the VEV of the superpotential, namely $\langle W_0 \rangle \sim \mathcal{O}(\epsilon)$. In this case the leading F-flatness condition take the usual flat form $\partial_H W_0 = 0$, which turns out to be also the chiral equation for the frozen chiral multiplet H_0. The mass hierarchy condition is relaxed for Kähler potentials realizing an approximate factorizable form, $K = K_H(H) + K_L(L) + \epsilon K_{mix}(H, L)$, and W_0 can take arbitrary values at the vacuum. In this case the leading F-flatness condition for the H scalar components are given by $\partial_H W_0 + \partial_H K_H W_0 = 0$. Here, however, although the auxiliary fields F^H are still frozen out at vanishing VEV, there is no chiral equation for which the

[1] By moduli in the L sector we mean any kind of field, also charged and matter like, with $\mathcal{O}(1)$ VEV.

frozen H_0, as chiral multiplet, be solution to. It is clear that in order the EFT theory be described by a superpotential and a Kähler potential, as is the case of the simple SUSY model, the VEV of the auxiliary fields of the H multiplets should be always suppressed compared with the ones of the L ones. Indeed, once we are around the leading SUSY solutions for the H fields this is warranted. For the case of generic Kähler potential the suppression comes from the hierarchy on the masses, $F^H \sim m_L/m_H F^L$, and for the factorizable case by the small mixing between the two sectors. In both cases, then, we have $F^H \sim \mathcal{O}(\epsilon) F^L$.

In chapter 4 the set-up was generalized allowing gauge dynamics and $\mathcal{O}(1)$ Yukawa couplings in the matter sector. We realize, first of all, that freezing of charged H fields, besides the fact that due to gauge invariance the leading F-flatness solution may leave some flat directions, is a gauge non-invariant statement, and freezing of charged fields seems only possible for heavy fields with vanishing VEV, like the Kaluza-Klein resonances. Therefore, the Two-Step stabilization procedure can only be realized for neutral fields, or more general, on gauge invariant combinations of them. This simple observation, which actually has been already pointed out in several places [50, 36], is in fact rather often overlooked in models dealing with FI-terms from string compactifications (see e.g.[107] for an interesting class of F-theory models of this kind). Indeed, as we have seen, in SUGRA only field dependent FI-term can appear, and depend on fields that are charged under the corresponding $U(1)$ sector. These fields, therefore, cannot be regarded as being previously stabilized since the resulting dynamics are potentially completely different. In particular, their SUSY breaking effects are expected to be of the same order of the ones from fields stabilized afterwards.

The conclusions in this generalized set-up are mainly the same as in the pure moduli case. For the case where W_0 is tuned at the vacuum, using an approximate manifestly SUSY approach, we identified the leading corrections in the matter sector due to the presence of the $\mathcal{O}(1)$ couplings, in the form of new induced couplings not reliable in the simple model. More precisely, a term in the superpotential with schematic from $W \sim Y_N C^N$, induces the following couplings:

$$W_{full} \sim \frac{1}{m_H} Y_{N_l} Y_{N_m} C^{N_l+N_m} \,, \tag{6.2}$$

$$K_{full} \sim \frac{1}{m_H} Y_N C^N + h.c. \tag{6.3}$$

Thus, couplings of order C^6, or higher, in the superpotential and holomorphic couplings of order C^3, or higher, in the Kähler potential are not well described in the simple approach. This situation, however, is cured once one realizes that the simple approach cannot be a correct description beyond the m_H scale, so that in using naturalness arguments one should use m_H instead of the cut-off scale $\Lambda \geq m_H$ (indeed, the new cut-off scale is formally m_H). In this case the induced couplings are indeed suppressed compared to the ones with "natural" coefficients.

The assumed form for the moduli superpotential is a quite natural and generic one in the context of low energy SUGRA theories from string compactification, in particular resembles the flux compactifications scenarios. Indeed, the first framework where our results apply are the KKLT-like moduli stabilization models. In this context, the condition we found on the VEV for W_0 was mainly assumed in order to trust the leading perturbative terms in the Kähler potential and the non-perturbative ones in the superpotential, since was under these circumstances that KKLT [24] where able to stabilize the Kähler modulus at sightly large values. Later on, it was stressed that in order to get low energy SUSY breaking this tuning in W_0 should be quite large, of order 10^{-13}. Here, we find again the need of such tuning in order to trust the Two-Step moduli stabilization procedure. We stress, however, that the results presented in this thesis go beyond the e.o.m. and mass level in the effective action, drawing more information than the needed for moduli stabilization matters. In fact, we have been able to check the full low energy EFT action in the case with only moduli fields and find where the first corrections appear once matter is included.

We have treated in a more particular way the LARGE volume scenario of type-IIB orientifold compactifications. This scenario, being in the moduli sector of the factorizable kind mentioned above, gives another explicit realization of decoupling between the two sectors involved in the Two-Step procedure. In this case the analysis is less generic, since the scalings on the volume of the CY can be model dependent. For the explicit model of "swiss-cheese" manifolds we studied such decoupling with only moduli fields in full detail, finding a matching in the description at leading order in $1/\mathcal{V}$.

In order to implement the introduction of matter fields in the nearly factorizable scenario, in section (4.4) we generalized the condition for the Kähler potential allowing for the matter fields to appear only in suppressed terms,

$$K = K_H(H) + K_M(M) + \epsilon K_{mix}(H, M, Q)\,, \tag{6.4}$$

with M the light moduli and Q the matter fields. We saw that this Kähler potential still allows a decoupling between the H and L sectors with an arbitrary value of W_0 at the vacuum. We showed that this set-up is naturally implemented in the LARGE volume scenario, and studied a particular model in this context. The model, although the simplest possible one, has all ingredients that might play an important rôle for moduli stabilization: a FI-term dependent on the large Kähler modulus and a single charged

field Q with no superpotential dynamics, whose VEV is determined through the nearly cancellation of the D-term. We found for this case that the simple description leads again to a reliable action at leading order in $1/\mathcal{V}$, independently of the value for the modular weight of Q. This analysis, being quite particular, cannot be blindly generalized to the full set of models realizing the LARGE volume scenario. In fact, being those an appealing set of vacua for string compactifications, a more generic study on them is one of the first issues this thesis points to as further directions of investigation.
Using a more heuristic approach, but also more model independent, we glanced the induced couplings in the visible sector due to the integration of the heavy fields, where now the possible suppression of the induced couplings comes by inverse powers of the volume. Interestingly enough we found that the superpotential starts to be corrected as in the previous case, eq.(6.2), with coefficients of the order of the natural ones, i.e. $\mathcal{O}(1)$. For the Kähler potential, although the corrections look in general harmless, the analysis is less conclusive, as relays in unknown powers of the volume in the higher order couplings originally presented.

In chapter 5, by invoking a Two-Step procedure, we studied an explicit realization of stabilized two light moduli in Minkowski vacua. The system assumes a pseudo-anomalous $U(1)_X$ gauge symmetry inducing a field dependent FI-term. The last one, together with the presence in the superpotential of a term $\chi\phi^{q_\chi}$ for two charged fields ϕ and χ, forces the system to break SUSY exactly as in the original FI model. Being the FI-term moduli-dependent, the stabilization of the moduli is a crucial (and anyway important) ingredient driving indirectly the breaking. The mechanism can be made stable and relatively natural by invoking a ratio between the $U(1)_X$ charges for ϕ and χ of almost one order of magnitude, or with the help of non-perturbative generated couplings. When the moduli are stabilized without a constant superpotential term w_0, the mechanism is more robust and no bound on the $U(1)_X$ charges arises. The moduli masses are proportional to the scale of SUSY breaking and hence a gravity mediation of SUSY breaking, with a gravitino mass of $\mathcal{O}(\text{TeV})$, is preferred for cosmological reasons. In connection with the bulk of the thesis on the Two-Step stabilization approach, we show in Appendix B how the single modulus case works explicitly, in a numerical comparison between a the One-Step and the Two-Step procedures.

A model stabilizing a single modulus and using the same mechanism to realize Minkowski non-SUSY vacua was first studied in ref.[44]. Generalizing the study to two moduli is not just an academic complication, because in this set-up one modulus necessarily transforms under the $U(1)_X$ gauge symmetry, whereas the second one can be taken neutral. The two-moduli system is then the simplest scenario that describes more realistic situations. Moreover, we have shown how the presence of the second neutral modulus can considerably help in getting sufficiently heavy gaugino without getting at the same time very heavy scalar fields, a property which is not so common in string compactifications.

All our analysis on this system has been based on the search for non-SUSY solutions starting from SUSY ones. It should be stressed that this does not imply that our vacua are small deformations of SUSY ones, since the small parameter ϵ introduced there is fixed by the Planck/weak scale hierarchy and cannot be continuously taken to be zero. On the other hand, we cannot exclude the existence of other interesting non-SUSY vacua which are not detectable with our approach. We think that the above SUSY breaking mechanism, together with the stabilization of moduli by non-perturbative gauge dynamics, is interesting, promising and can be made quite natural, particularly when $w_0 = 0$. The next crucial step, as always when considering bottom-up SUGRA phenomenological models, would be to embed this mechanism in a full string theory set-up.

In looking for such realizations in a full string theory set-up, an important question that is raised, and which can be seen as a continuation to the study of the first chapters of the thesis, is the possibility of neglecting some of the light stuff in a sort of Three-Step procedure. Indeed, despite the huge simplification, one obtains by invoking the Two-Step approach, still one might remain with more moduli than the ones usually regarded in the toy models for moduli stabilization. The first problem one would face is the entanglement between the dynamics of all fields, in general far more complicated than the ones dictated by the superpotential in eq.(6.1). Since these toy models are intended to explain the stabilization of moduli before, or at most meanwhile, SUSY breaking happens, it is clear that the neglected fields should have again suppressed auxiliary fields at leading order. As we have found, this can be obtained by to means: a mass hierarchy and an approximate decoupling between the two sectors. Therefore, one might be tempted to generalize the condition by combining these two, although none of them are as severe as in the cases studied here, so that at the end a strong suppression is still realized. Of course, this is not enough since a decoupling, at least at the level of the e.o.m., on the dynamics of both sector is still to be realized, but already gives hints on a possible approach to be taken. On the other hand, possible effects on the observable sector, in particular on the soft-terms, are expected to impose further constrains on the structure of the models where the procedure can be reliable.

Appendix A

Vacuum in presence of D-terms

In this appendix, along the lines of chapter 3, we show in some more detail how the e.o.m. of the light scalar fields agree at leading order in the full and simple models in presence of vector fields. Since non-trivial e.o.m. for the fields C appear only at $\mathcal{O}(\epsilon^3)$, for simplicity we set them to zero, which is always a solution to their e.o.m., and only study the e.o.m. for the remaining M fields. In order to keep the notation as simple as possible, we omit in this appendix the subscript "full", being understood that any quantity with no specification arises in the full theory.

The new ingredient with respect to the analysis performed in sections (3.2.1) and (3.3.1) is the D-term scalar potential

$$V_D = \frac{1}{2} g_A^2 D_A^2 \,. \tag{A.1}$$

We study the location of the vacuum in both theories in a series expansion in ϵ:

$$\langle \phi^M \rangle = \phi_0^M + \epsilon \phi_1^M + \epsilon^2 \phi_2^M + \dots \,. \tag{A.2}$$

Although the D-term potential does not admit an expansion in ϵ, being governed by generally $\mathcal{O}(1)$ gauge couplings, at the vacuum the D-terms are related to F-terms and hence an expansion in ϵ is still possible.

At $\mathcal{O}(\epsilon^0)$, $\partial_i W_0(H_0) = 0$ solve the F-term e.o.m. for the heavy fields and trivialize the corresponding ones for the light fields: $(\partial_i V_F)_0 = (\partial_{\mathcal{A}} V_F)_0 = 0$. Of course, due to the presence of V_D, this is no longer a sufficient condition but it is still necessary. In this way, the leading order VEV's for the heavy fields are fixed. At $\mathcal{O}(\epsilon^0)$ and $H^i = H_0^i$, the e.o.m. for the light fields are entirely given by the D-term potential:

$$(\partial_{\mathcal{A}} V_D)_0 = \frac{1}{2} (\partial_{\mathcal{A}} g_A^2) D_A^2 + g_A^2 D_A \partial_{\mathcal{A}} D_A = 0 \,, \tag{A.3}$$

evaluated at ϕ_0^M, which admit the simple solution

$$D_A(\phi_0^M) = 0\,. \tag{A.4}$$

When the gauge symmetry is unbroken, eq.(A.4) is a solution to all orders in ϵ. Indeed, from the explicit expression $D_A = iX^i\partial_i K$, it is straightforward to deduce the following general bound,

$$\sqrt{2}g_A D_A \leq \sqrt{g^{\bar{N}N}K_N K_{\bar{N}}}m_{AA}\,, \tag{A.5}$$

with m_{AA} being the diagonal components of the gauge field mass matrix

$$m^2_{AB} = 2g_A\, g_B\, g_{M\bar{N}} X_A^M \overline{X}_B^{\bar{N}}\,. \tag{A.6}$$

For spontaneously broken symmetries, another relation between F and D–terms is valid at the vacuum, of the form $\langle D\rangle \sim \langle F^2\rangle/m_V^2$ (see eq.(A.17) below), where m_V is the typical scale of the heavy vector fields, parametrically larger than ϵ. Requiring the F-terms to be all at most of $\mathcal{O}(\epsilon)$, we conclude that at the vacuum $\langle D\rangle \lesssim \mathcal{O}(\epsilon^2)$. Eq.(A.4) is the only sensible solution to eq.(A.3) for vacua with no $\mathcal{O}(1)$ SUSY breaking. Eq.(A.4) also ensures that at $\mathcal{O}(\epsilon^0)$ the e.o.m. of the heavy fields are automatically satisfied at H_0, since $(\partial_i V_D)_0 = 0$.

At $\mathcal{O}(\epsilon)$, the e.o.m. for the light fields are still given by the D-term potential only, so that

$$(\partial_{\mathcal{A}} V_D)_1 = g_A^2\Big[\partial_N D_A(\phi_0)\phi_1^N + \partial_{\bar{N}} D_A(\phi_0)\phi_1^{\bar{N}}\Big]\partial_{\mathcal{A}} D_A(\phi_0) = 0\,. \tag{A.7}$$

Two possible solutions can be taken. Either $\partial_{\mathcal{A}} D_A(\phi_0) = 0$, which implies $X_A^{\mathcal{A}}(\phi_0^M) = 0$, being $X_A^i = 0$, or $\partial_N D_A(\phi_0)\phi_1^N + \partial_{\bar{N}} D_A(\phi_0)\phi_1^{\bar{N}} = 0$. The two situations correspond respectively to unbroken and broken generators. Indeed, splitting the gauge index $A = (a, \hat{a})$ in eq.(A.6), with $a \in \mathcal{H}$, $\hat{a} \in \mathcal{G}/\mathcal{H}$, we have

$$m^2_{\hat{a}\hat{b}} = \mathcal{O}(m_V^2), \quad m^2_{\hat{a}a} = m^2_{a\hat{a}} = 0, \quad m^2_{ab} = 0\,. \tag{A.8}$$

By taking g_A^2 and the Kähler metric parametrically of order one, eq.(A.8) gives

$$\langle X_a^M\rangle = 0, \quad \langle X_{\hat{a}}^M\rangle = \mathcal{O}(m_V)\,. \tag{A.9}$$

So, we have

$$\begin{aligned} X_{M,a}(\phi_0) = X_a^M(\phi_0) &= 0, \quad a \in \mathcal{H}\,, \\ \partial_N D_{\hat{a}}(\phi_0)\phi_1^N + \partial_{\bar{N}} D_{\hat{a}}(\phi_0)\phi_1^{\bar{N}} &= 0, \quad \hat{a} \in \mathcal{G}/\mathcal{H}\,. \end{aligned} \tag{A.10}$$

Eqs.(A.10) imply that both D_a and $D_{\hat{a}}$ vanish at $\mathcal{O}(\epsilon)$, in agreement with our previous argument that $D_A \leq \mathcal{O}(\epsilon^2)$. When eq.(A.10) is satisfied, the e.o.m. for the heavy fields at $\mathcal{O}(\epsilon)$ are given by V_F only and fix H_1^i as in the pure F-term case studied in chapter 3,

$$H_1^i = -(\hat{K}^{-1})^i_{\bar{j}}(\overline{G}^{\bar{j}})_0\,, \tag{A.11}$$

with $\hat{K}^{\bar{i}}_{j} = g^{\bar{i}k}(\partial_k G_j)_{-1}$ and $(\overline{G}^{\bar{j}})_0 = (\overline{G}_{\bar{M}})_0 g^{\bar{M}i}$,following the notation of section (3.3.1), evaluated at $H^i = H_0^i$. At the shifted vacuum $H_0^i + \epsilon H_1^i$, we have

$$G_i = \mathcal{O}(1)\,,\;\; G_{\mathcal{A}} = \mathcal{O}(1)\,,\;\; G^i = \mathcal{O}(\epsilon)\,,\;\; G^{\mathcal{A}} = \mathcal{O}(1)\,, \tag{A.12}$$

showing that the matching of the F-term part of the e.o.m. of the light fields at $\mathcal{O}(\epsilon^2)$, $(\partial_{\mathcal{A}} V_F)_{full} = (\partial_{\mathcal{A}} V_F)_{sim} + \mathcal{O}(\epsilon^3)$, continues to hold in presence of D-terms. Hence we can just focus on V_D. The term $(\partial_{\mathcal{A}} V_D)_2$ can be written as follows:

$$(\partial_{\mathcal{A}} V_D)_2 = g_{\hat{a}}^2(\phi_0) D_{\hat{a}}(\phi_0 + \epsilon\phi_1 + \epsilon^2\phi_2)\partial_{\mathcal{A}} D_{\hat{a}}(\phi_0)\,, \tag{A.13}$$

where we have explicitly written the order at which the various quantities should in principle be known.

A similar expansion of the e.o.m. can be performed in the simple effective theory, where the D–term reads $V_{D,sim} = g_A^2 (D_A)^2_{sim}/2$, with $(D_A)_{sim} = iX_A^{\mathcal{A}}\partial_{\mathcal{A}} K_{sim}$, and all quantities evaluated at the leading frozen vacuum H_0^i. At $\mathcal{O}(\epsilon^0)$ and $\mathcal{O}(\epsilon)$ we get

$$(D_A)_{sim}(\phi_0) = 0, \quad (X_a^{\mathcal{A}})_{sim}(\phi_0) = 0, \quad \partial_{\mathcal{A}} D_{\hat{a},sim}(\phi_0)\phi^{\mathcal{A}}_{1,sim} + \partial_{\bar{\mathcal{A}}} D_{\hat{a},sim}(\phi_0)\phi^{\bar{\mathcal{A}}}_{1,sim} = 0\,, \tag{A.14}$$

which implies that $D_{sim}(\phi_0) \sim \mathcal{O}(\epsilon^2)$. The form of $(\partial_{\mathcal{A}} V_D)_{sim}$ at $\mathcal{O}(\epsilon^2)$ is the same as eq.(A.13), but written in terms of $(D_A)_{sim}$ and evaluated at H_0^i. Thus, the equivalence of the two descriptions requires that the following non-trivial relation holds:

$$D_{\hat{a}}(\langle\phi^M\rangle) = (D_{\hat{a}})_{sim}(H_0^i, \langle L^{\mathcal{A}}\rangle) + \mathcal{O}(\epsilon^3)\,, \tag{A.15}$$

with $\langle\phi\rangle$ and $\langle L\rangle$ expanded up to $\mathcal{O}(\epsilon^2)$. Luckily enough, we do not need to work out the vacuum up to $\mathcal{O}(\epsilon^2)$, since we can trade $\dim G$ e.o.m. to write $\dim G$ relations between the F and D–terms at the vacuum[1]. Indeed, by taking the combination of the e.o.m. $\mathrm{Im}(X_A^M \partial_M V) = 0$ (the real part being identically vanishing by gauge invariance: $\delta_\lambda V = \lambda^A \mathrm{Re}(X_A^M \partial_M V) = 0$), one easily derive the following equation (see e.g.[108, 39]:

$$q_{AM\bar{M}} F^M \overline{F}^{\bar{M}} - \frac{1}{2} D^B \left[m^2_{AB} + \delta_{AB}(F^M F_M - m^2_{3/2})\right] = 0\,. \tag{A.16}$$

[1]Notice that, modulo global gauge transformations, the vacuum is uniquely determined since the "missing" $\dim G$ e.o.m. are provided by the $\dim G$ D-term constraints (A.4).

where $q_{AM\bar{M}} = \nabla_M \nabla_{\bar{M}} D_A$, $m_{3/2} = \exp(K/2)W$, $F^M = e^{K/2} K^{M\bar{N}}(\partial_{\bar{N}}\overline{W} + \partial_{\bar{N}} K\overline{W})$ and $D_A = Re(f_A)\delta_{AB}D^B/2$ [2]. When $A = a$, the second term in eq.(A.16) vanishes and the equations boil down to a set of constraints for the F-terms at the vacuum, dictated by gauge invariance. When $A = \hat{a}$, instead, we can invert eq.(A.16) to solve for $D^{\hat{a}}$:

$$D^{\hat{a}} = 2m^{-2}_{\hat{a}\hat{b}} q_{\hat{b}M\bar{M}} F^M \overline{F}^{\bar{M}} + \mathcal{O}(\epsilon^4)\,. \tag{A.17}$$

A similar relation occurs in the simple effective theory upon replacing the indices M and $\bar{M}$ with $\mathcal{A}$ and $\bar{\mathcal{A}}$. Given the relation between D^A and D_A and regarding gauge kinetic terms of $\mathcal{O}(1)$ eq.(A.17) tells us that $D_{\hat{a}} \sim \mathcal{O}(\epsilon^2)$, since the F–terms are all at most of $\mathcal{O}(\epsilon)$. In addition, since $F^i(\langle\phi^M\rangle) \sim \mathcal{O}(\epsilon^2)$, we see that eq.(A.15) is easily proved:

$$D^{\hat{a}}(\langle\phi^M\rangle) = 2m^{-2}_{\hat{a}\hat{b}} q_{\hat{b}\mathcal{A}\bar{\mathcal{A}}} F^{\mathcal{A}} \overline{F}^{\bar{\mathcal{A}}}(\phi^M_0) + \mathcal{O}(\epsilon^3) = (D_{sim})^{\hat{a}}(H^i_0, L^{\mathcal{A}}_0) + \mathcal{O}(\epsilon^3)\,. \tag{A.18}$$

We have then established that even in presence of D–terms the location of the vacuum as computed by the simple effective theory is reliable.

[2] If the gauge kinetic functions are not gauge-invariant (e.g. as required by anomaly cancellation), an extra term appears in eq.(A.17). Being of $\mathcal{O}(D^2) \sim \mathcal{O}(\epsilon^4)$, it is completely negligible for our purposes. See e.g. [39] for a more general formula including these terms.

Appendix B

One-Step vs Two-Step Moduli Stabilization: a Numerical Test

In this appendix we apply the results of chapters 3 and 4 to the string-inspired SUGRA toy model studied in chapter 5 for the case with a single modulus. We add to the system two heavy moduli to be frozen out in the simple model. Although the model contains just five complex fields, instead of hundreds as in realistic string models, it is already sufficiently complicated to make an analytical study a formidable task. For this reason we opt here for a numerical analysis. In particular we will show how the vacuum and the scalar mass spectrum as given by the simple effective model is in agreement with the full–fledged analysis. The Kähler and superpotential terms are taken as follows:

$$
\begin{aligned}
K &= -2\ln\Big[(T+\bar{T}-\delta V_X)^{3/2}+\xi(S+\bar{S})^{3/2}\Big]-\ln(U+\bar{U})-\ln(S+\bar{S}) \\
&\quad +\frac{\bar{\phi}\,e^{-2V_X}\phi}{(U+\bar{U})^{n_\phi}}+\frac{\bar{\chi}\,e^{2V_X}\chi}{(U+\bar{U})^{n_\chi}}\,, \qquad \text{(B.1)} \\
W &= aU^2+bU+S(cU^2+dU+e)+mU\phi\chi+\beta U^2\phi^{\alpha\delta/2}e^{-\alpha T}\,. \qquad \text{(B.2)}
\end{aligned}
$$

The heavy fields $H=\{S,U\}$ mimic respectively the dilaton and a complex structure modulus of some IIB flux Calabi-Yau compactification, T represents the overall universal light Kähler modulus, ϕ and χ are two charged fields with $\mathcal{O}(1)$ VEV's and opposite $U(1)$ charge with respect to a $U(1)$ gauge field A_X. The holomorphic gauge kinetic function associated to $U(1)_X$ is taken to be $f_X=T$. The kinetic mixing between heavy and light fields is provided by the universal α' correction to the volume [89], parametrized by ξ in eq.(B.1) and by the complex structure dependence of the kinetic term for ϕ and χ. The expression for the $U(1)_X$ D–term is

$$
D_X = iX^M\partial_M K = \frac{|\chi|^2}{(2U_r)^{n_\chi}}-\frac{|\phi|^2}{(2U_r)^{n_\phi}}+\frac{3\delta T_r^{1/2}}{4(T_r^{3/2}+\xi S_r^{3/2})}\,, \qquad \text{(B.3)}
$$

where the subscript r denotes the real part of the field. The first five terms in the superpotential mimic terms arising from fluxes and should be identified with W_0 in our general analysis (see e.g. eq.(3.53) of [109]). We have introduced non trivial dependencies on the complex structure moduli in the mass term $mU\phi\chi$ (in chapter 5 $Y\phi^{q_\chi}\chi$), responsible for the main SUSY breaking source in the χ direction, and in the non-perturbative generated part $\phi^{\alpha\delta/2}e^{-\alpha T}$ (denoted in chapter 5 as $A(\phi)e^{-aT}$), responsible for the stabilization of T. These are the small superpotential terms defining W_1. The prefactor of the exponential is a function of the complex structure moduli, which for simplicity we have taken to be just quadratic. It should be stressed that the detailed structure of K and W in eqs.(B.1) and (B.2) is pretty arbitrary and not well motivated; it should just be seen as a simple, yet not trivial, string–inspired example where to concretely apply our results.

In choosing the parameters entering in eqs.(B.1) and (B.2) a fine tuning is required in a and b in order to obtain a tiny W_0, ensuring a hierarchy in the masses. We have taken[1]

$$\begin{aligned} a &= -2.55 - 10^{-13}\,, \quad b = 25.5 + 2\cdot 10^{-12}\,, \quad c = 0.25\,, \quad d = -2.45\,, \quad e = -0.5 \\ \alpha &= 1\,, \quad \beta = -0.5\,, \quad \delta = 1\,, \quad m = 2.44\times 10^{-12}\,, \quad n_\chi = 1\,, \quad n_\phi = 0\,, \quad \xi = 0.1 \end{aligned} \tag{B.4}$$

The smallness of m, required to get a dS vacuum with a sufficiently small cosmological constant, justifies the location of the $U\phi\chi$ operator in W_1. The SUSY VEV's of S and U, as given by $\partial_S W_0 = \partial_U W_0$ are precisely $S_0 = U_0 = 10$, with $W_0(S_0, U_0) = 10^{-11}$. In table 1 we report the exact VEV's and F–terms of the fields as numerically computed in the full model and their relative shifts compared to those computed, again numerically, in the simple effective model, with S and U frozen at their values S_0 and U_0.[2] The model above belongs to the general class of models studied in the main text with an ϵ roughly $\mathcal{O}(10^{-13})$. Keeping two significant digits, the physical masses are:

$$\begin{aligned} m_{H_1}^2 &= 1.1\cdot 10^{-2}, \quad m_{H_2}^2 = 1.0\cdot 10^{-2}, \quad m_{\phi_r}^2 = 1.5\cdot 10^{-3}, \\ m_{T_i}^2 &= 4.0\cdot 10^{-27}, \quad m_{T_r}^2 = 3.7\cdot 10^{-27}, \quad m_{\chi_i}^2 = 2.3\cdot 10^{-28}, \quad m_{\chi_r}^2 = 2.3\cdot 10^{-28}. \end{aligned} \tag{B.5}$$

where $H_1 \simeq U+S$, $H_2 \simeq U-S$ and the subscript r and i denote real and imaginary field components, respectively. The masses m_{H1}^2 and m_{H2}^2 refer to both components of the complex scalar fields, being SUSY breaking effects negligible. The imaginary component of ϕ is approximately the Goldstone boson eaten by A_X and hence is exactly massless.

[1] In a realistic string setting, the fluxes are quantized and makes the appearance of terms like a and b in eq.(B.4) rather difficult, if not impossible. We do not care about this possible issue. As explained above, we do not pretend our model to mimic a realistic string set-up in any single detail.

[2] For simplicity of the numerical analysis, we have not integrated out the massive $U(1)_X$ gauge field, as it should be done in both theories. However, this is not going to affect the vacuum or the mass spectrum and hence the results that follow are reliable.

	$\langle X\rangle$	$\Delta\langle X\rangle$	F^X	ΔF^X
S	$10+2\cdot 10^{-13}$	$2\cdot 10^{-14}$	$-9.9\cdot 10^{-28}$	–
U	$10+2\cdot 10^{-13}$	$2\cdot 10^{-14}$	$5.6\cdot 10^{-28}$	–
T	31.4	$1.1\cdot 10^{-15}$	$-3.4\cdot 10^{-15}$	$-1.4\cdot 10^{-16}$
ϕ	0.15	$-1.3\cdot 10^{-15}$	$1.4\cdot 10^{-16}$	$-7.4\cdot 10^{-14}$
χ	0.05	$-7.6\cdot 10^{-14}$	$7.5\cdot 10^{-15}$	$1.5\cdot 10^{-14}$

TABLE B.1: VEV's and $F^N = e^{K/2} g^{\bar M N}\overline{F}_{\bar M}$ terms for the fields and their relative shifts, as derived by a numerical analysis. Here and in the main text $\Delta X \equiv (X_{full} - X_{sim})/X_{full}$. All quantities are in reduced Planck units.

The relative mass shifts are

$$\begin{aligned} \Delta m^2_{\phi_r} &= -4.7\cdot 10^{-15}\,, \qquad \Delta m^2_{T_i} = -2.1\cdot 10^{-14}\,, \quad \Delta m^2_{T_r} = -2.6\cdot 10^{-14}\,, \\ \Delta m^2_{\chi_i} &= 1.4\cdot 10^{-14}\,, \qquad \Delta m^2_{\chi_r} = 1.8\cdot 10^{-14}\,. \end{aligned} \tag{B.6}$$

Finally, we also report the gravitino mass, the D_X–term, the cosmological constant and their relative shifts:

$$\begin{aligned} m^2_{3/2} &= 9.5\cdot 10^{-31}\,, \quad \Delta m^2_{3/2} = -2.7\cdot 10^{-14}\,, \\ D_X &= 3.7\cdot 10^{-27}\,, \quad \Delta D_X = 1.5\cdot 10^{-14}\,, \\ V_0 &= 1.2\cdot 10^{-32}\,, \quad \Delta V_0 = 8.2\cdot 10^{-12}\,. \end{aligned} \tag{B.7}$$

Notice that being the cosmological constant fine-tuned to be "small", namely of order $10^{-2}m^2_{3/2}$, its relative shift is larger. The latter is inversely proportional to the smallness of V_0.

The relative shifts of the various quantities considered are smaller than ϵ due to the fact that the Kähler mixing between the H and L fields coming from (B.1) in the above vacuum are relatively small. This example shows the excellent agreement between the Two-Step and the full moduli stabilization procedure.

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