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Aspects Of the Early Universe Inflation & Baryogenesis

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Introduction

At first sight, Cosmology and Particle Physics seems to be completely unrelated branches of Physics. Cosmology is the study of the physical properties of our Universe seen as a whole of phenomena in space and time, while particle Physics deals with the smallest constituents of matter and their interactions. However and despite the fact that these two disciplines probe two different faces of our world: the biggest and smallest, they can have contact points. The early Universe constitutes the arena for such an interaction. Indeed our Universe's evolution depends crucially on its fundamental constituents and interactions.

As a consequence, the last few years witnessed an intense research activity in the area at the interface of particle Physics and Cosmology, commonly called *Astroparticle Physics*. This revival of interest is mainly motivated by the wealth of data available from astrophysical observations on one hand, and by the lack of particle Physics experimental data. Indeed, particle Physics models accounting for Physics beyond the SM (such as neutrino masses) involve energies much beyond the ones reachable by current terrestrial experiments. Thus, Cosmology is believed to provide the only testing ground for these particle Physics models. This thesis is concerned of various aspect of this connection, namely: Inflationary Cosmology and baryogenesis.

Inflation is by now a well-accepted and robust paradigm able to explain the structure of our Universe. Despite this fact, a unified theory is still lacking. In [1], G. Tasinato and I have been studying the role and effect singlet tadpoles in F-term inflation scenarios. Generally the inflaton is required to be a SM singlet, to keep its potential flat enough for successful inflation. Singlets generically have tadpole diagrams that would cancel out in the limit of exact SUSY. However since during the inflationary era SUSY is effectively broken, there will be a non vanishing contribution of the tadpole, that modify significantly the inflaton potential. As a consequence, the standard F-term inflation scenario is modified. Our main result is that the presence of singlets in a particle Physics models, besides providing natural candidates for the inflaton field, can have interesting cosmological consequences.

Another particularly appealing link between particle Physics and Cosmology appears when one considers the generation of the Baryon Asymmetry of the Universe (BAU). The generation of the BAU represents one of the most prominent puzzles of not only Cosmology but also particle Physics. Any particle Physics model able to give an answer to this question must contain three fundamental ingredients (The three Sakharov conditions). One particularly appealing scenario is *baryogenesis via leptogenesis*. This last exploits the fact that the standard model contains non perturbative objects called *sphalerons* able to wash-out any $B+L$ asymmetry while preserving the orthogonal combination $B-L$. Therefore it is no more necessary to produce a baryon asymmetry, it is enough to have some lepton number that will be reprocessed into a baryon excess, thanks to sphalerons. The initial lepton asymmetry can be produced in many ways. For instance, it can be produced by the out-of-equilibrium decay of see-saw right-handed neutrinos (Fukugita-Yanagida) or by the formation of a scalar condensate carrying non zero lepton number along some MSSM flat direction (Affleck-Dine). The attraction of this scenario resides in its minimality; one just needs right-handed neutrinos that explains elegantly the smallness of neutrino masses wrt to other lepton and quarks, and sphalerons that are already present in the standard model. As a consequence, the scenario gives constraints on the neutrino parameters (masses and mixing).

Successful leptogenesis (FY) requires the presence of enough number of right handed neutrinos. These can either be created via thermal scattering in the thermal bath (thermal creation) or through resonance effects (preheating). A typical problem plaguing thermal leptogenesis scenarios is the tension between the gravitino overproduction bound and the condition for thermal production of right-handed neutrinos. Indeed, the former forces to have lower reheat temperatures $T_{\mathrm{RH}} \lesssim 10^9$ GeV, while the later pushes to higher reheat temperatures $T_{\mathrm{RH}} \gtrsim M_N \gtrsim 10^{10}$ GeV. In [3], motivated by the above, I have been studying leptogenesis at low scale i.e. when the right-handed neutrino masses are of $O(\mathrm{TeV})$, so they can be produced thermally at low reheat temperatures. I focused on two specific leptogenesis scenarios: the Fukugita-Yanagida scenario of out-of-equilibrium decay of right-handed (s)neutrinos and the Affleck-Dine scenario (leptogenesis via the MSSM LH_u flat direction). I found that for the first mechanism (FY), one can achieve a sufficient amount of lepton number provided right-handed neutrinos are degenerate. For the second mechanism (AD), I found that successful BAU is achieved if CP violation coming from SUSY breaking A-terms is maximal.

In leptogenesis scenarios, the amount of the asymmetry depends crucially on the number density and production mechanism of right-handed neutrinos. If the (s)neutrinos are generated thermally, in supersymmetric models there is limited parameter space leading to enough baryons. For this reason, several alternative mechanisms have been proposed. In [4], S. Davidson, M. Peloso, L. Sorbo and I discussed the nonperturbative production of sneutrino quanta by a direct coupling to the inflaton. This production dominates over the corresponding creation of neutrinos, and it can easily (i.e. even for a rather small inflaton-sneutrino coupling) lead to a sufficient baryon asymmetry. We then study the amplification of MSSM degrees of freedom, via their coupling to the sneutrinos, during the rescattering phase which follows the nonperturbative production. This process, which mainly influences the (MSSM) $D-$flat directions, is very efficient as long as the sneutrinos quanta are in the relativistic regime. The rapid amplification of the light degrees of freedom may potentially lead to a gravitino problem. We estimated the gravitino production by means of a perturbative calculation, discussing the regime in which we expect it to be reliable.

Chapter 1

Standard Cosmology: Overview

1.1 Observational facts about our Universe

Much of what we know about our Universe comes from light emitted from distant objects. Progress in spectroscopy, atomic and nuclear Physics has been crucial in probing it at different wave lengths. During the last decade, our understanding of the Universe changed dramatically. Many of its unexpected properties have been discovered as the technological means used to probe it began to be more and more accurate. In particular, various experiments probing the cosmic microwave background radiation (Boomerang, Maxima, Dasi, COBE, WMAP) made available a new wealth of data, that opened a new era in Cosmology: The precision era. On the theoretical front, significant progress have been done in modeling our Universe's properties. As the cosmological parameters are being measured with increasing accuracy, the task of theorists became harder. It switched from explaining a behavior (like expansion, acceleration,...) into explaining the precise value of the parameters, starting from first principles. This constitutes one of the most embarrassing points of contact between Cosmology and Particle Physics. Despite all this, there exist some good point of contact between these two (Big bang nucleosynthesis for e.g.). In this section, we will summarize the state-of-affairs of our Universe from the observational stand point, enumerating its most known properties. Then we will give an overview of Standard Big Bang Cosmology and its shortcomings. Finally, we will give a brief survey of slow-roll inflation.

1.1.1 The expansion

The basic feature of our Universe is that it is expanding. This was established long ago in 1929 by Edwin Hubble, who found a correlation between the recession velocity $\vec{v}$ and distance $\vec{r}$ of a sample of nearby galaxies. This correlation take the form of the commonly known Hubble law

$$\vec{v} = H\,\vec{r}\,, \tag{1.1}$$

where H is the Hubble parameter, describes the expansion rate. The most recent and accurate value for the Hubble constant is [133]

$$H_0 = 100\,h\ \mathrm{km\ sec}^{-1}\mathrm{Mpc}^{-1}, \qquad h = 0.72 \pm 0.02 \pm 0.07. \tag{1.2}$$

As a consequence, light emitted by distant objects (supernovae, galaxies ...) is red shifted *i.e.* its wave length λ is stretched by the expansion. To quantify this phenomenon, we define the cosmological red-shift z as

$$1 + z \equiv \frac{\lambda_{\mathrm{obs}}}{\lambda_{\mathrm{emit}}}. \tag{1.3}$$

In an (adiabatically) expanding FRW Universe with scale factor $a(t)$ (See Sect. 1.2), Eqt.(1.3) can be written as

$$1 + z = \frac{a(t_{\mathrm{obs}})}{a(t_{\mathrm{emit}})} = \frac{T_{\mathrm{emit}}}{T_{\mathrm{obs}}}. \tag{1.4}$$

So the more objects are distant, bigger is their red shift. Since the scale factor is an increasing function of time, objects with high red shifts probe early moments of the evolution of our Universe.

1.1.2 The acceleration

Recent measurements showed, contrary to the expectation, that our Universe is undergoing an accelerating expansion. Indeed, gravity is expected to take over the expansion at a certain (late) time. This observation, confirmed independently by two groups [131, 132], is based on considerations made on high red shifts supernovae. Said more precisely, these analyzes concluded that high red shifts supernovae appear fainter than expected for a slowing down (decelerating) expansion. These observations made theorists think that the Universe is now dominated by a mysterious substance with strange properties (repellent interactions and non clustering). There exist two candidates for such a substance: *dark energy* and *quintessence.*

1.1.3 The Cosmic Microwave Background Radiation

The cosmic microwave background (CMBR) that we observe today represents an instantaneous snap-shot of our Universe when it was 300,000 years old. It was produced when photons decoupled from the other components of the hot plasma. Starting from the surface of last scattering, CMBR photons streamed almost freely, and arrived to us unaltered. They are characterized by a black body spectrum of temperature $T \simeq 2.73$

K with fluctuations of one part of 10^5. Thus the studying the CMBR, one can extract a huge amount of informations about our Universe at its childhood. The CMBR tells us two things. The first is that our Universe is extremely homogeneous and isotropic on large scales, say at distances bigger that 100 Mpc. Of course, the distribution of matter on smaller scales is lumpy, as structures starting from clusters, superclusters, galaxies, the solar system, stars and planet are known to exist. The second thing is that at the time of decoupling, at smaller (angular) scales, there existed density perturbations, which grew to provide the seed for structure formation. Any model of the early Universe Physics should accommodate these two seemingly contradictory facts.

1.1.4 The cosmic mass energy budget

Combining data coming from various measurements (For a review see [135]), we are able to reconstruct the cosmic mass energy budget *i.e.* the detailed balance of the universe components that are: matter which includes ordinary matter (photons, baryons and neutrinos) and dark matter, and possibly other exotic stuff. It turns out, as one can see from Fig (1.1.4) that ordinary matter only constitutes a small fraction of the energy density of the universe and the major part of it is in the form of *dark energy.* Because the different components of the mass/energy budget evolve differently, the composition changes with time. For example, at very early times, photons and other relativistic particles were the dominant component; from 10,000 years until a few billion years ago, matter was the dominant component, and in the

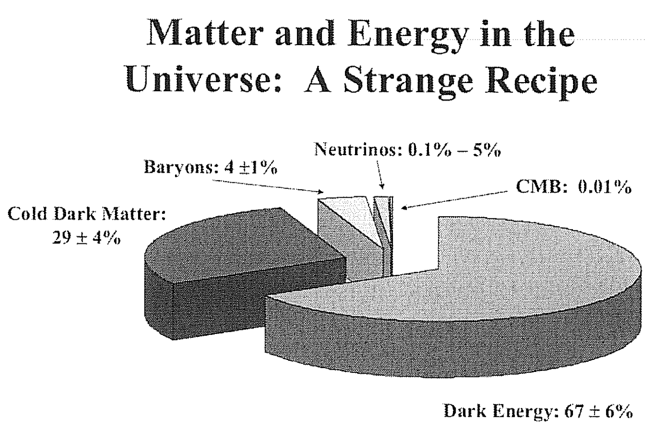


Figure 1.1: The cosmic matter/energy budget (cosmic pie). (From [134])

future dark energy will be the dominant component.

1.2 The Friedmann-Robertson-Walker Universe

As we have seen in the previous section, astronomical observations indicate that our Universe is highly homogeneous and isotropic. Therefore, it can be described by a homogeneous and isotropic metric. Standard Cosmology is based on the FRW metric which is given by

$$ds^2 = dt^2 - a(t)^2 \left\{ \frac{dr^2}{1-kr^2} + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right) \right\} \tag{1.5}$$

where $a(t)$ is the scale factor, t is the cosmic time. The coordinate system (t, r, θ, ϕ) is called co-moving coordinates. The FRW metric describes a homogeneous, isotropic Universe, with a constant curvature $k = 0, \pm 1$. The Einstein equation, including a cosmological constant Λ

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G_N T_{\mu\nu} + \Lambda g_{\mu\nu} \ , \tag{1.6}$$

can be cast in the following form [1]

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G_N}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3}, \tag{1.7}$$

$$\frac{d}{dt}\left(\rho a^3 \right) = -p \frac{d}{dt}\left(a^3 \right) . \tag{1.8}$$

where the energy-momentum tensor was taken as (with the same symmetry of the metric) $T_{\mu\nu} = \mathrm{diag}(\rho, p, p, p)$, where ρ and p represent the energy density and the pressure respectively. These equations are at the basis of Standard big bang model. If $\rho(a)$ is known, one can solve for the scale factor $a(t)$. More generally, for a fluid with equation of state $p = \omega\rho$ we find that

$$\rho \sim a^{-3(1+\omega)}. \tag{1.9}$$

If $\omega \neq -1$, and neglecting the curvature contribution and cosmological constant, we can solve for the scale factor giving

$$a(t) \sim t^{\frac{2}{3(1+\omega)}} \tag{1.10}$$

For example, at early times when the Universe is taught to be radiation dominated (RD) so that $\rho \sim a^{-4}$, then

$$a(t) \sim t^{1/2}. \tag{1.11}$$

[1] Dots refer to derivative with respect to time.

Similarly, for matter dominated (MD) Universe, where $\rho \sim a^{-3}$ one finds

$$a(t) \sim t^{2/3} \tag{1.12}$$

The transition between radiation dominated to matter dominated occurs when $\rho_{\rm rad} = \rho_{\rm matter}$, *i.e.* $T \simeq$ few 10^3 K.

For a Universe dominated by a cosmological constant ($\omega = -1$), $\rho \sim \Lambda$ and

$$a \sim \exp \sqrt{\Lambda/3}\, t. \tag{1.13}$$

In the absence of a cosmological constant, one can define a critical energy density ρ_c given by

$$\rho_c \equiv \frac{3H^2}{8\pi G_N}. \tag{1.14}$$

The Friedmann equation can be written as

$$\Omega - 1 = \frac{k}{a^2H^2}, \qquad \Omega \equiv \frac{\rho}{\rho_c}. \tag{1.15}$$

From the above equation, we see clearly how the Universe energy content determines its geometry. If the energy density is greater (smaller) that ρ_c then the universe is positively (negatively). In the special case where the energy density is exactly the critical one, the spatial geometry is flat. Let's note that in this case, that $k = 0$ is an unstable solution, in the sense that if the energy density decrease or increases by a small amount, the universe will goes inevitably into a phase of $k = \pm 1$ *i.e.* expand forever or recollapses. We will be back to this issue when talking about the flatness problem.

1.3 Problems of standard Cosmology

Despite the success of standard Cosmology in explaining the large-scale isotropy and homogeneity, it fails in giving answers to three basic questions.

1.3.1 The horizon problem

As we have seen in Sect. (1.1.3), the CMBR seen in the sky have a uniform temperature with tiny fluctuations of one part in 10^5. The horizon problem lies in the fact that regions in our universe that could not talk to each other in the past have the same temperature. To illustrate the problem, let us estimate the horizon sizes between the recombination time up to now. The CMBR photons we observe today have decoupled since recombination at $T_{\rm dec} \sim 4000$ K. At that time, the horizon volume was simply $V_{\rm dec} \propto t_{\rm dec}^3$, where $t_{\rm dec}$ is the age of the Universe at $T_{\rm dec}$. Our present horizon volume is $V_0 \propto t_0^3$, where t_0 is the age of the Universe right now. Then $t_{\rm dec} = t_0(T_0/T_{\rm dec})$, where $T_0 \simeq 2.7$ K is the temperature of the CMBR. We can now compare between $V_{\rm dec}$ and V_0. The ratio

$$\frac{V_{\rm dec}}{V_0} = \left(\frac{T_{\rm dec}}{T_0}\right)^{\frac{3}{2}} \sim 5 \times 10^4 \qquad (1.16)$$

shows clearly that there was approximately 10^5 causally disconnected regions at the recombination that now appear to have the same temperature to a high accuracy.

1.3.2 Unwanted relics: The monopole problem

The standard model of strong and electroweak interactions describes successfully all (almost) particle phenomena in our world. At very early times, when the temperature was close to the Planck mass, particle interactions are believed to be described by a Grand Unified Theory (GUT) (For a review see [136]). As the expansion goes, the temperature decrease will trigger a phase transition, where the GUT gauge group is broken down to the standard model one. This breaking will, by the Kibble mechanism [137], generate topological defects: cosmic strings, magnetic monopoles and domain walls, depending on topological considerations. Magnetic monopoles along with proton decay constitutes the most striking imprints of GUTs. Our Universe, as observed today, seem to be free of such relics otherwise they would affect drastically its evolution. This is the essence of the monopole problem. One can estimate how many monopole have been created during the thermal phase transition. According to Kibble, there can be at least one monopole per horizon volume. The horizon size ℓ_H scales as H^{-1}. Furthermore, in a radiation dominated era, the Hubble rate goes as T^2/M_P, where M_P stands for the reduced Planck scale, namely $M_P = \sqrt{8\pi G_N} \simeq 2.4 \times 10^{18}$ GeV. Assuming that the phase transition occurs at $T_C \simeq M_G \simeq 10^{15}$ GeV, one obtains

$$\frac{n_M}{s} \sim \left(\frac{M_G}{M_P}\right)^3 \sim 10^{-9}. \qquad (1.17)$$

The overall mass density of th Universe can be used to place a constraint on the density of monopoles. For $m_M \sim M_G/\alpha_{\rm GUT} \sim 10^{16}$ GeV and $\Omega_M h^2 \lesssim 1$ we get

$$\frac{n_M}{s} \lesssim 10^{-25}. \qquad (1.18)$$

From the above, we see that the estimate (1.17) overweights the bound (1.18) by several orders of magnitude. Clearly there is a monopole problem.

1.3.3 The flatness problem

Observations reveals that the value of Ω is very close to one, in other words that our universe is spatially flat. However, as we mentioned in Sect(1.2), the $k = 0$ point is unstable. To see this, we rewrite (1.15) as

$$\Omega - 1 = \frac{k}{H^2 a^2} \propto k\, t^{\frac{2(1+3\omega)}{3(1+\omega)}} \tag{1.19}$$

where ω= 0, (1/3) for MD (RD). Eqt (1.19) tells us that, since $|\Omega - 1|$ is an increasing function of time, if the present universe is so close to be flat, it have to be more flat at early times. So it must be incredibly fine tuned. The question is how come the energy density in our Universe is so close to the critical one? This is the flatness problem. We will see in Sect(1.4) how inflation will be able to address this problem.

1.4 Inflationary Cosmology

All the problems discussed above can be resolved if the Universe underwent a period where the scale factor grew exponentially (inflated) and very fast [1]. During this inflationary era, the scale factor expands at a rate greater that velocity of light, and so it will put in contact regions that were causally disconnected in the past, giving thus a clear answer to the horizon problem. Similarly, the monopole relic density is diluted by the expansion. Finally, as the universe radius grows, it will become more and more flat. Obviously, inflation does not change the global geometry of the space, it will just make it look more flat locally. We will quantify these statements later on.

The simplest way to make the Universe expand is to let a cosmological constant dominate it. However, as can be seen from (1.13), such a Λ-dominated universe would expand forever at a constant rate. So inflation never ends furthermore there will be an unacceptably high cosmological constant. The most economical way to circumvent this is to consider a scalar field whose potential energy serves as an effective cosmological constant that drives the expansion. This scalar field φ, coined *inflaton* must evolve slowly from a non-vanishing value of its potential down to a global minimum where the potential vanishes.

1.4.1 The slow-roll paradigm

Let us consider a scalar field ϕ (inflaton) in a FRW space with action given by

$$S = \int d^4x\, \sqrt{-g}\, \mathcal{L} = \int d^4x \sqrt{-g} \left[\frac{1}{2}\partial_\mu \phi\, \partial^\mu \phi + V(\phi)\right] \tag{1.20}$$

Neglecting the spatial gradient of the inflaton, the motion of the inflaton will be governed by the equation

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0, \tag{1.21}$$

obtained from minimizing the action (1.20) with respect to ϕ. The non-vanishing components of the energy-momentum tensor, representing the inflaton pressure and energy density, can be readily computed

$$T_{00} = \rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi), \tag{1.22}$$

$$T_{ii} = p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi). \tag{1.23}$$

In order to obtain expansion, the kinetic energy of the inflaton must be negligible wrt its potential energy. Similarly, one expect that the second time derivative of the inflaton in (1.21) to be negligible too. These requirement translates into the following slow-roll conditions

$$\text{(Slow-roll) Inflation} \Rightarrow \begin{cases} \dot{\phi}^2 \ll V \\ 3H\ddot{\phi} \ll V' \end{cases} \Rightarrow \begin{cases} \epsilon \equiv \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1 \\ \eta \equiv M_P^2\left(\frac{V''}{V}\right) \ll 1 \end{cases} \tag{1.24}$$

where ϵ and η quantify the motion of the inflaton. They are known as the slow-roll parameters. The slow-roll paradigm makes the analysis of inflationary dynamics straightforward. One picks a potential $V(\phi)$, then compute the slow-roll parameters, and look for the corresponding values of the inflaton field, namely ϕ_i and ϕ_f. The initial value is the one where the slow-roll conditions are satisfied, while the final one corresponds to its break-down. The number of e-folds, which characterize the amount of inflation can be easily calculated in the slow-roll approximation, it is

$$N(\phi_i \to \phi_f) \equiv \log\left[\frac{a(t_f)}{a(t_i)}\right] = \int_{t_i}^{t_f} H\, dt \tag{1.25}$$

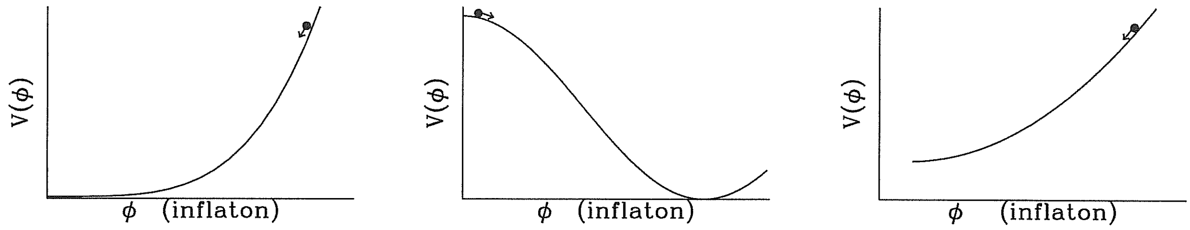
To solve the cosmological problems, N must be bigger that 60.

1.4.2 Inflation and density perturbations

A complete description of the Universe should include a description of deviations from homogeneity, at least in a statistical way. Originally, inflation has been devised to address the three standard cosmological problems (See Sect(1.3)). It was only afterwards that it was realized that inflation could generate density perturbations as well, which is very important for the formation of structures we see. The mechanism for structure formation is very simple; it is based on the attractive property of the gravitational force, that amplifies the primordial perturbations created by the inflaton. This affirmed inflation as a robust paradigm, even if til now a standard model is still lacking.

In the following, we give a brief survey of the theory of cosmological perturbations. More details can be found in excellent reviews (See for e.g [127, 128]). In our slow-roll analysis above, we considered the inflaton as a classical field. In general, we can write the inflaton field as

$$\phi(t) = \phi_0(t) + \delta\phi(\mathbf{x}, t), \tag{1.26}$$



Large field models Small fields models Hybrid models

Figure 1.2: Classification of Inflationary potentials [140].

where $\phi_0(t)$ is the classical value, and $\delta\phi(\mathbf{x},\, t)$ is its quantum fluctuation. This is completely justified since its quantum fluctuation is negligible wrt to the classical value. Inflation generates perturbations through the amplification of quantum fluctuations, which are stretched to astrophysical scales by the rapid expansion. The simplest models generate two types, density perturbations which come from fluctuations in the scalar field and its corresponding scalar metric perturbation, and gravitational waves which are tensor metric fluctuations. The former experiences gravitational instability and leads to structure formation, while the latter can influence the cosmic microwave background anisotropies. Using the slow-roll approximation, one can compute the spectral indices of such perturbations, which are of two kinds: scalar and tensor. They are defined as

$$n_S \simeq 1 - 6\epsilon + 2\eta \quad ; \quad n_T \simeq -2\epsilon \,. \tag{1.27}$$

From the above equations, it is clear that slow-roll inflation generically predicts a flat spectrum *i.e.* $n_s \simeq 1$ and practically no gravitational waves. These quantities (and other) constitute smoking guns of inflationary models. As the precision in cosmological data is improved, it will be possible to pin down the exact form of the inflationary potential. To this end, a classification scheme have to be devised to distinguish the different types of models.

1.4.3 Models of inflation

In Cosmology, the word *model* deserves a different meaning from its counterpart in particle Physics. While in particle Physics, it means a minimal set of choices that stem from first principles (gauge invariance for e.g.) and consistency requirement (renormalization for e.g.), in inflationary Cosmology it just means *inflaton potential*. Much work has been devoted to the construction of inflation models motivated by particle Physics consideration (See for e.g. [4, 9, 8]). This generally proved to be a quite difficult task.

To classify inflationary models, one can begin by counting the fields entering the potential. In its first and simplest version, inflation have been attributed to the dynamics of a single scalar field. These models are called *single-field models.* However, nothing forbids the potential to contain more that one inflaton. *Hybrid models* where inflation is due to the dynamics of a coupled system of two or more fields have been widely studied. They constitute the prototype of particle Physics motivated models. Restricting to single field models, one can refine the classification according to the shape of the potential [140]. Figures 1.4.3 show schematically the different patterns of the potentials (See [140] for details). Large fields models have potentials of the chaotic type ($V(\phi) = m^2\phi^2$ or $\lambda\phi^4$) where the inflaton starts displaced from its minimum $\phi \gg M_P$, probably due to quantum gravity effects, rolls towards the origin. On the contrary, in small fields models, the inflaton rolls from an unstable local minimum located at the origin to a stable one. This kind of potential arises typically in models with spontaneously broken symmetries. On the other hand, hybrid models possess a (false vacuum) minimum with non zero potential. They arise typically in supersymmetry and supergravity models (See Appendix C for a brief review).

Chapter 2

Universal Singlets, Supergravity and Inflation

In supersymmetric theories, the occurrence of universal singlets is a delicate issue, because they usually induce tadpoles that destabilize the hierarchy. We study the effects of these tadpoles in supersymmetric hybrid inflation models. The resulting scenario is generically modified, but it is still possible to achieve inflation in a natural way. It is argued that singlets, despite the problems associated with their presence, can lead to interesting cosmological consequences.

2.1 Introduction

In Particle Physics, the introduction of singlet fields has been invoked in many models to solve various problems. This is done for instance in the Standard Model to give masses to neutrinos with the see-saw mechanism, or in the so called NMSSM for other purposes. In other cases, their presence is actually unavoidable, like in theories that require compactification from higher dimensions. However, it has been pointed out that the presence of these fields induces generically new quadratic divergences at one (or more) loop(s), in particular tadpoles (terms linear in the singlet) that destabilize dramatically the hierarchy [13, 15, 16]. Some efforts have been done to show how to 'tame' these divergences in supergravity, exploiting them to solve some notorious problems [18, 19].

Also in Cosmology, singlets have been shown to be very useful. For example, it was pointed out that singlets can be useful to provide a strong first order phase transition essential for a successful baryogenesis in the NMSSM [23]. In some inflationary models their presence, even if less stressed, is required. However, the tadpole contributions have never been taken into account in the cosmological context. Due to their particular properties, singlets are sensitive to the Planck scale physics. Since Cosmology is the study of the early stages of the universe (just after the Planck era), it is perfectly legitimate to ask whether their presence lead to some consequences. In this chapter, we will consider the modifications required by the presence of these tadpoles in the hybrid inflationary scenario.

By now, it is well established that the inflationary paradigm provides a successful and elegant solution to three essential questions of standard Cosmology: the horizon, the flatness and the monopole problem[1, 2]. It is also widely hoped that successful inflationary models could emerge naturally from pure Particle Physics considerations [3, 4], in the sense that any consistent particle model may have a built-in sector that ensures inflation. Supersymmetric hybrid inflation models appear to be the most promising to achieve this task. Such models (and their extensions) have been constructed and studied extensively [5]. Typically, they are based on superpotentials of the form $W_{\text{inflation}} = \kappa S(\Phi\bar{\Phi} - \mu^2)$, where κ is a dimensionless coupling constant, S is a singlet superfield and Φ, $\bar{\Phi}$ are superfields that are conjugate under some non trivial representation of a group G. At a certain time, inflation is dominated by the F-term of the singlet field ($V_0 = \mu^4$), and this explains the presence of the linear term in the previous superpotential. Usually Φ and $\bar{\Phi}$ are taken to be the Higgs fields that break the GUT gauge symmetry so that $\mu \sim M_{\text{GUT}}$. The resulting scalar potential is the prototype of hybrid inflation [6] except for the mass term for S, which is essential to drive the inflaton to its minimum. Such a slope can however be generated, independently from soft breaking mass terms, by the one loop corrections to the scalar potential along the inflationary trajectory [8]. This model succeed in reproducing the correct values of density perturbation and the spectral index at the price of a small coupling constant ($\kappa \sim 10^{-3} - 10^{-4}$). The generic problem of inflationary models is the stability of the potential. In other words: how to keep the inflaton potential flat enough to achieve successful inflation? Generally, without D-term contribution, supergravity gives new terms to the effective potential of the inflaton that usually destroy the flatness of the potential. However, it is argued that these corrections can be brought under control via a judicious choice of the Kähler potential and the superpotential [11, 5]. Models of inflation in which D-term contributions are considered have been studied [7], showing that it is possible to evade the problems associated with supergravity corrections (See however [24]).

As we have seen, many characteristics of supersymmetric models have been largely used in building inflation models. In fact, the singlet nature of the inflaton is a crucial feature, since it protects it from acquiring a too large mass, that will ruin inflation. However, the particular properties of singlets have not been explored yet in inflation, and this is the main purpose of this chapter, at least in a specific example. We will see, in a particular model, that the presence of singlet fields provide a Particle Physics realization of a specific version of hybrid inflation, the so called mutated hybrid inflation [12]. The chapter is organized as follows. In Section 2.2, we briefly review the properties of singlets in supergravity. In Section 2.3, we will focus our discussion on the case of the superpotential of supersymmetric hybrid inflation, showing that the presence of tadpoles generically changes the scalar potential that drives inflation. In Section 2.4, without analyzing in full details the consequences of these modifications, we notice that, in a certain regime, the modified scalar potential can provide a realization of the mutated hybrid inflation scenario. Section 2.4 is devoted to the study of the stability of the potential under one-loop and supergravity corrections. Finally, in Section 2.6, we give our conclusions.

2.2 Universal Singlets in Supergravity

In Particle Physics models, universal singlets are fields that do not transform under any gauge symmetry of the Lagrangian. Therefore, roughly speaking, in non supersymmetric models containing a scalar singlet field s, nothing will forbid the appearance in the Lagrangian of terms such as $a\Lambda^3 s + b\Lambda^2 s^2 + c\Lambda s^3 + \text{h.c.}$ with a, b, $c \sim \mathcal{O}(1)$. Moreover, the natural value for Λ is M_P[1], so singlets will get masses and vev's of $\mathcal{O}(M_P)$. If not coupled to light fields, they will decouple from the low energy theory. Instead, if they are coupled to light fields, they will communicate to them their large vev, destabilizing dramatically the hierarchy.

One could think that invoking supersymmetry will ameliorate the things, but the situation remains the same also in SUSY models [13]. Indeed, it has been shown that, if a supergravity model contains singlets, they can destabilize the mass hierarchy, introducing new quadratic contributions coming from tadpoles [15, 16]. These new quadratic terms have been used to communicate supersymmetry breaking in a particular way [17], to generate the GUT scale [18] and to solve the μ-problem [19] (See also [20] for an early attempt).

For concreteness, let us consider a supersymmetric model with a visible sector containing an universal singlet superfield $S = s + \theta^2 F_s$, and a hidden sector, whose fields are denoted generically with $\Sigma = \sigma + \theta^2 F_\Sigma$, responsible for supersymmetry breaking. Following [17], tadpoles arise due to terms like

$$\delta K = \left[1 + \frac{c}{M_P}(S + S^\dagger)\right] \Sigma\Sigma^\dagger \tag{2.1}$$

in the Kähler potential. The higher order term, proportional to c, is allowed by all the gauge symmetries, and it is generically present in the Kähler potential just because S is a universal singlet.

[1]Throughout the paper, M_P stands for the reduced Planck scale, namely $M_P = M_{\rm Planck}/\sqrt{8\pi} \simeq 2.4 \times 10^{18}$ GeV.

The low-energy Lagrangian contains the following D-term contribution [17] [2]

$$\mathcal{L}_D = \int d^2\theta d^2\bar{\theta} \ e^{K/M_P^2} K, \tag{2.2}$$

where K here is the Kähler potential written in terms of superfields. After integrating out the hidden fields, the effective potential coming from the tadpole is given by [17, 18]

$$\Delta V_{\text{tadpole}} = \gamma \frac{M_f^4}{M_P}(s + s^\dagger) + (\alpha\beta F_s M_f^2 + \text{h.c.}) \tag{2.3}$$

where α is a parameter (See [18, 19]) related to the SUSY breaking in the hidden sector, and β and γ are loop factors that are less than one. The mass M_f stands for the scale of breaking of supersymmetry in the hidden sector, i.e. $\lambda F_\Sigma\rangle = M_f^2$. The loop factors and α will be an essential ingredient for our discussion[3]. They are related to c, to the number of hidden fields and to the detailed structure of the Kähler potential; their typical value is in the range $\mathcal{O}(1-10^{-4})$. In the usual gravity mediated supersymmetry breaking models, one arranges for $M_f^2 \sim m_{3/2} M_P$, where the "gravitino mass" is chosen $m_{3/2} \lesssim O(\text{TeV})$, to solve the hierarchy problem.

The full scalar potential will include, in addition to standard terms, the tadpole contribution (cf. Eq. (2.3)). In terms of auxiliary field F_S it reads [18]

$$V_{F_S} = (\beta\alpha M_f^2 F_S + \text{h.c.}) - |F_S|^2 - \left(F_S \frac{\partial W}{\partial S} + \text{h.c.}\right). \tag{2.4}$$

Since the auxiliary fields F_s are non dynamical, they can be eliminated using their equation of motion [4]

$$F_s^\dagger = -\frac{\partial W}{\partial S} + \alpha\beta M_f^2. \tag{2.5}$$

At this point, to continue the discussion, we must consider a specific form of the superpotential. In the next section, we will consider the typical superpotential for supersymmetric hybrid inflation.

2.3 The model

Within the model of the previous section, let us plug in the superpotential of supersymmetric hybrid inflation i.e.

$$W_{\text{inflation}} = \kappa S(\Phi\bar{\Phi} - \mu^2). \tag{2.6}$$

[2]The expression (2.2) comes from a full supergravity computation, see [15, 16] for more details.

[3]The values of α, β and γ are model-dependent. We consider them as free parameters in their respective allowed range.

[4]Notice the presence of the extra piece in the F-term of s, which is due to the tadpole; the effect of the tadpole is to shift the vev of F_S by the amount $\alpha\beta M_f^2$.

κ is a dimensionless coupling constant, S is the singlet chiral superfield, while Φ and $\bar{\Phi}$ are chiral superfields, belonging to the visible sector, that are conjugate under a non trivial representation of some group G. One can always impose an appropriate R-symmetry [5] such that the superpotential (2.6) is the most general renormalizable one. We do not specify the form of the superpotential for the hidden sector.

At tree level, the scalar potential is readily computed. It is

$$V(\varphi, \bar{\varphi}, s) = \kappa^2 |\varphi\bar{\varphi} - \mu^2|^2 + \kappa^2 |s|^2 (|\varphi|^2 + |\bar{\varphi}|^2) + D\text{−terms}. \tag{2.7}$$

where s, φ and $\bar{\varphi}$ are the scalar components of S, Φ and $\bar{\Phi}$. We will restrict ourselves to the D-flat direction $|\varphi| = |\bar{\varphi}|$. Minimizing the potential, one finds that there are two sets of minima. The first is the supersymmetric one, it is located at $|\varphi| = \mu$ and $s = 0$. The second one breaks SUSY, for $s > s_c = \mu$ and $|\varphi| = 0$. Inflation in this scenario proceeds by assuming chaotic initial conditions for the fields s and ϕ. That is, the inflaton field s rolls from $s \gg s_c$ towards the true minimum ($s = 0$), while the "auxiliary" field φ is held at the origin. The universe undergoes an exponential expansion phase (inflation) since its energy density is then dominated by the false vacuum one ($V = \kappa^2\mu^4$). But this will not last forever; as soon as s reaches the critical value s_c, all the fields rapidly adjust to their SUSY vacuum values restoring supersymmetry, and inflation finishes.
Let us include the tadpole contributions to the scalar potential. Using Eqts. (2.4) and (2.5), one obtains the scalar potential as a function of the two fields φ and s

$$V = \alpha^2\beta^2 M_f^4 + \gamma\frac{M_f^4}{M_P}(s+s^\dagger) + 2\kappa^2|s|^2|\varphi|^2 - 2\kappa\alpha\beta M_f^2(|\varphi|^2 - \mu^2) + \kappa^2(|\varphi|^2 - \mu^2)^2. \tag{2.8}$$

Clearly, due to the presence of the linear term in s, the minimum for s is no more at the origin, but it is now given by

$$s = -\frac{\gamma M_f^4}{2\kappa^2 M_P |\varphi|^2}. \tag{2.9}$$

The supersymmetric minimum is recovered when $\gamma = 0$. This corresponds to choose c exactly zero in the expression of the Kähler potential (2.1). However, a priori, we have no obvious reason to enforce it to this value.

The result is that the values of s and $|\varphi|$ are now correlated, and while s rolls down along the inflationary trajectory, φ moves away from the origin. The usual scenario for hybrid inflation is modified, but the new characteristics of the model can still be used in an inflationary context. For simplicity, we will set the scale μ to zero in the scalar potential. The scale M_f, in our case, can take any value below the Planck scale ($M_f \leq M_P$), since we do not aim to provide a phenomenologically acceptable scenario

[5] These symmetries are global, they are likely to be broken by gravitational interactions, so at the end S will not carry any quantum number.

for supersymmetry breaking. We imagine that this is achieved by some other sector of the model.

The resulting potential, with μ set to zero, looks similar to another realization of hybrid inflation, the mutated hybrid inflation. Indeed, some years ago, Stewart proposed a new version of hybrid inflation based on a potential of the form [12]

$$V(\phi,\psi) = V_0\left(1-\frac{\psi}{M}\right)+\frac{\lambda}{2}\psi^2\phi^2. \tag{2.10}$$

The inflationary trajectory is obtained by minimizing on ψ. Along this trajectory, both ψ and φ roll. The potential, as a function of φ, reads

$$V = V_0\left(1-\frac{V_0}{2\lambda M^2\phi^2}\right). \tag{2.11}$$

Stewart argued that such a potential can arise from an effective superpotential due to non perturbative effects such as gaugino condensation. In the next two sections, we will see that the addition of singlet tadpoles will provide a new particle physics motivation to this model.

2.4 Inflating with tadpoles

Let us proceed to analyze our potential. Minimizing with respect to s, we end with the scalar potential for the inflaton field φ

$$V = M_f^4\alpha^2\beta^2\left(1-\frac{\gamma^2 M_f^4}{2\kappa^2\alpha^2\beta^2|\varphi|^2M_P^2}\right)+\kappa^2|\varphi|^4-\kappa\alpha\beta M_f^2(\varphi^2+\varphi^{\dagger 2}) \tag{2.12}$$

The potential (2.12) looks very similar to the one of mutated hybrid inflation, except for the two last terms. In order to ignore them we must impose

$$\xi \ll \left(\frac{\beta\alpha}{\kappa}\right)^{1/2}, \tag{2.13}$$

where we have defined $\varphi = \xi M_f$. Furthermore their first and second derivatives must also be negligible with respect to the derivatives of the first term that is supposed to drive inflation. These requirements translate into the following condition

$$\xi \ll \left(\frac{\gamma M_f}{\kappa^2 M_P}\right)^{1/3}. \tag{2.14}$$

To satisfy the slow roll conditions

$$\varepsilon = \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 \ll 1 \text{ and } |\eta| = M_P^2\left|\frac{V''}{V}\right| \ll 1, \tag{2.15}$$

we must have

$$\xi \gg \left(\frac{\gamma}{\beta\alpha\kappa}\right)^{1/2}. \tag{2.16}$$

The number of e-folds is given by

$$N = \frac{1}{M_P^2}\int \mathrm{d}\varphi \frac{V}{V'} \simeq \frac{1}{4}\xi^4 \left(\frac{\beta\alpha\kappa}{\gamma}\right)^2. \tag{2.17}$$

The COBE density perturbation normalization corresponds to

$$\frac{V^{3/2}}{{M_P}^3 V'} \simeq 2\sqrt{2}N^{3/4}\left(\frac{\kappa}{\gamma}\right)^{1/2}(\alpha\beta)^{3/2}\left(\frac{M_f}{M_P}\right) = 6\times 10^{-4}, \tag{2.18}$$

and for $N \approx 60$, we obtain the following expression for M_f

$$M_f \simeq 10^{-5}\left(\frac{\gamma}{\kappa}\right)^{1/2}\frac{M_P}{(\alpha\beta)^{3/2}}. \tag{2.19}$$

As in the usual mutated hybrid inflation [12], the spectral index of density perturbations is given by

$$n_S \simeq 1 - 6\varepsilon + 2\eta \simeq 1 - \frac{3}{2N}. \tag{2.20}$$

For $N \simeq 60$, it gives $n \simeq 0.975$.

Combining Eqts. (2.19), (2.14) and (2.16) one ends with

$$\kappa \ll 10^{-5}. \tag{2.21}$$

This constraint is not surprising. In fact the smallness of the coupling constant κ is a typical prediction of hybrid inflation models [5].

Eventually, combining Eqts.(2.17) and (2.19), we obtain

$$M_f \simeq 10^{-5}\frac{\xi}{\beta\alpha}M_P. \tag{2.22}$$

To achieve inflation, the parameters of the model must obey various constraints. However, it is possible to fulfill them in a natural way. As an example, we take α and β to their maximal value i.e. α, $\beta \sim 1$: this choice allows to avoid fine tuning for the other parameters. Taking $\kappa \sim 10^{-6}$, one can consider the loop factor γ in the allowed range $\gamma \sim 10^{-1} - 10^{-4}$. Consequently, the range for ξ is $10 \ll \xi \ll 10^3$. We get a scale of SUSY breaking of the order $M_f \simeq 10^{14} - 10^{17}$ GeV, and the lower one ($M_f \simeq 10^{14}$ GeV) is the typical scale for a model of mutated hybrid inflation.

Usually, inflation finishes when the slow roll conditions are no more valid. This happens generally before the inflaton reaches the true minimum. There the inflaton begins oscillating coherently reheating the universe. Also in our model, the inflation ends when the slow roll conditions, represented by formula (2.16), break down. Actually, the inflaton field energy lies between the two scales given by equations (2.16) and (2.14): this means that nor the inflaton φ nor the singlet s reach the true minimum of the scalar potential at the end of inflation.

2.5 Stability of the potential

The tree level scalar potential usually receives corrections due to loop effects and to supergravity contributions. Such corrections, in our case [6], are dangerous because they can destabilize the inflationary trajectory. The one-loop corrections, as in usual superymmetric theories, depend on the mass splitting between the members of the supermultiplet, induced by the supersymmetry breaking. More precisely, the Coleman-Weinberg one-loop effective potential [21] shows that these corrections are proportional to the fourth power of the mass splitting. In our case, it is easy to see that this quantity, being proportional to the tiny coupling constant κ (See Eq. (2.21), is small enough to render these corrections negligible during the inflationary era.

Unfortunately, the situation with supergravity corrections is much more delicate. Although tadpole contributions, which are an essential ingredient for our model, come from a D-term, our scenario is actually an F-term inflationary one. Consequently the scalar potential receives the usual supergravity corrections to F-terms.

As clearly explained in [11], these corrections are generically non negligible [7], and one should expect new contributions to the scalar potential in Eq. (2.8), proportional to $M_f^4(|\varphi|^2 + |s|^2)/M_P^2$. In our case, due to the fact that the scale M_f is so large, these corrections are potentially important. Hopefully, other contributions, in a more refined version of our model, would cancel or keep under control such dangerous terms. However we will not consider this issue since it is out of the scope of the paper (See [25, 26, 27] for interesting ideas in this direction).

2.6 Conclusions

The presence of singlets in supergravity is a problematic issue, because they usually destabilize the hierarchy. Only in the past few years, it has been realized that their properties can provide interesting phenomenological models in Particle Physics [22]. Singlet fields, in the past, have also been used in Cosmology. For example, it was pointed out in [23] that singlets can be useful to provide a strong first order phase transition essential for a successful baryogenesis in the NMSSM, and moreover they are extensively used in inflationary models.

In this chapter, we have shown that these fields can have other cosmological applications, and in a supergravity framework. Indeed, we have shown that due to the presence of the tadpole contributions, the usual hybrid inflation scenario is

[6] In some models, these corrections are actively used to drive inflation (see as an example [14]), but we will not consider this possibility.

[7] Unless some fine tuning in the Kähler potential is made either by choosing the arbitrary Kähler couplings to be very small [5] or by choosing a specific form of the Kähler potential (and the superpotential), that can be ascribed for example to superstrings constructions [11].

generically modified. We point out that it is possible to use singlet tadpoles in a simple way to provide a new realization for a different scenario of hybrid inflation: the so called mutated hybrid inflation. In this framework, we have shown that it is possible to obtain an inflationary regime for a natural choice of the parameters.

There is no doubt, despite the unavoidable problems associated to their presence, that singlets tadpoles can lead to interesting cosmological implications.

Chapter 3

Baryogenesis through Leptogenesis

We know from the early days of P.A.M. Dirac that for any particle of mass m with definite quantum numbers (electric charge, baryon number), there exists a corresponding anti-particle with the same mass m and opposite quantum numbers. On the other hand, our Universe, at least on observable scales, seems to be made of matter, more precisely of baryons. One could think that this does not necessarily precludes a Universe where islands of antimatter could exist as well. In this case, huge burst of gamma rays, due to the annihilation of matter against anti-matter, would have been observed. This possibility has been considered in [129] where on general grounds a matter-antimatter Universe was empirically excluded.

This is not the only clue for a baryon asymmetric Universe. The computation of light elements abundances [107] shows a complete agreement with observation provided a small baryon excess have to be present before the nucleosynthesis epoch. The required value is quantified by the baryon to photon ratio η_B given by [59]

$$\eta_B \equiv \frac{n_B}{n_\gamma} \simeq (2.6 - 6.3) \times 10^{-11} \tag{3.1}$$

In addition, recent results from the WMAP experiment give [139]

$$\eta_B = (6.5\ ^{+0.4}_{-0.3}) \times 10^{-10}, \tag{3.2}$$

in excellent agreement with the BBN constrain (3.1). Explaining this tiny value represents a challenge. There exists many mechanisms of baryogenesis [66] that can account for this tiny value.

3.1 Baryogenesis: Basics

As we have learned in Sect. 1.4, inflation dilutes any particle relic density, setting the initial conditions for the big bang. Actually, the Universe must go through another phase called *reheating*, where the energy stored in the inflaton field is converted into radiation. Baryogenesis is the process of generation of the baryon asymmetry starting from the matter-antimatter symmetric Universe resulting immediately after the reheating process. In 1976, A. Sakharov identified the three essential conditions that any baryogenesis scenario have to satisfy in order to accomplish its task [34].

3.1.1 Baryon number violation

Although baryon number is classically conserved in the standard model, it is no more the case at the quantum level (See Sect(3.2) for some details). Electroweak baryogenesis (For a review see e.g. [126]) and leptogenesis are two examples of scenarios exploiting this observation. Baryon number is also generically violated (at the renormalizable level) in GUT, as quarks and leptons are part of the same multiplets. Baryogenesis scenarios based on the decay of heavy GUT particles have been considered long ago, however they have at least two drawbacks. The first is that baryon number violating interactions are also responsible of proton decay, so they are tightly constrained. Secondly, the mass of the decaying heavy particles is so high that producing such a particle in the thermal bath, would also produce a bunch of other harmful relics like the gravitino for example (See sect.). A possible solution to this problem have been considered in [86], where the heavy particle are produced at lower temperatures via preheating.

3.1.2 C and CP violation

C and CP violation ensures that the rate of the reactions producing baryons is different from the one producing anti baryons, *i.e.*

$$\Gamma(... \to B + ...) \neq \Gamma(... \to \bar{B} + ...) \tag{3.3}$$

This ingredient is typically present in all models and particularly in the SM. Parity is broken in the SM by the distinction made between right-handed and left handed fermions, while CP violation is contained in the CKM matrix, though not in a sufficient amount. A larger amount of CP violation can be obtained in the supersymmetric version of the SM. In scenarios based on the delayed decay of heavy particles, C and CP violation occurs at one loop as a consequence of the the interference of tree level and one loop diagrams.

3.1.3 Departure from thermal equilibrium

In order that the B, C and CP violating reactions to be efficient, they must occur in a state of out of equilibrium. If it were not the case, the inverse reaction would recombinate its products, averaging the yet produced baryon number to zero. Namely

$$\Gamma(X \to B + Y) \neq \Gamma(B + Y \to X). \tag{3.4}$$

In an expanding Universe, this is such provided the rate of particle interactions given by $\Gamma_{\text{int}} \simeq \langle n\sigma v\rangle$, where n is the number density, σ is the interaction cross section, and v is the velocity of particles, is faster than the Hubble rate.

3.2 Baryon number violation in the SM: Sphalerons

One of the great successes of the standard model of strong and electroweak interactions is its ability to explain why baryon and lepton number (B and L) are conserved. Indeed, perturbatively (and at the renormalizable level), the SM has 4 conserved quantum numbers: L_e, L_μ, L_τ -leptonic numbers and B the baryon number. These quantum numbers happen to be conserved accidentally. Their conservation can be tracked to the conservation of the baryonic and leptonic current defined as

$$J^B_\mu = \frac{1}{3} \sum_{\substack{\text{colors}\\ \text{generations}}} \left(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d\right) \tag{3.5}$$

$$J^L_\mu = \sum_{\ell} \left(\bar{\ell}\gamma_\mu \ell + \bar{\nu}_\ell \gamma_\mu \nu_\ell\right) \tag{3.6}$$

However, due to the quantum anomaly

$$\partial_\mu J^\mu_B = \partial_\mu J^\mu_L = \frac{n_f}{32\pi^2}\text{Tr}\left(F_{\mu\nu}\tilde{F}^{\mu\nu}\right), \tag{3.7}$$

where n_f is the number of families, baryon and lepton numbers are violated. From the above equation, it is clear that the combination $B - L$ is still conserved, while $B + L$ is not. Integrating the above equation and discarding the surface term, we obtain

$$N_B = N_L \propto n_f N_{\text{CS}}, \tag{3.8}$$

where N_{CS} is a Chern-Simmons topological charge, that labels the vacuum. Figure (3.1) shows schematically how this configuration looks like. The system can pass from one vacuum to the other by tunneling. Due to non perturbative nature of this configuration (called *sphaleron*), the rate at which B and L are violated at zero temperature is very small. However at non zero temperature, this rate can be bigger. One can compute such a rate, it is given by [60]

$$\Gamma \sim \begin{cases} \exp\left(\frac{-4\pi}{\alpha_W}\right) \sim 10^{-160}, & T = 0 \\ \\ (\alpha_W T)^4 \left(\frac{m_{sph}}{T}\right)^7 \exp\left(-\frac{m_{\text{sph}}}{T}\right), & T < T_C \\ \\ \alpha_W^5 T^4 & T > T_C \end{cases} \tag{3.9}$$

where $m_{\text{sph}} \sim m_W/\alpha_W$ is the sphaleron mass and T_C is the critical temperature at which the electroweak phase transition takes place. Sphaleron interactions are in

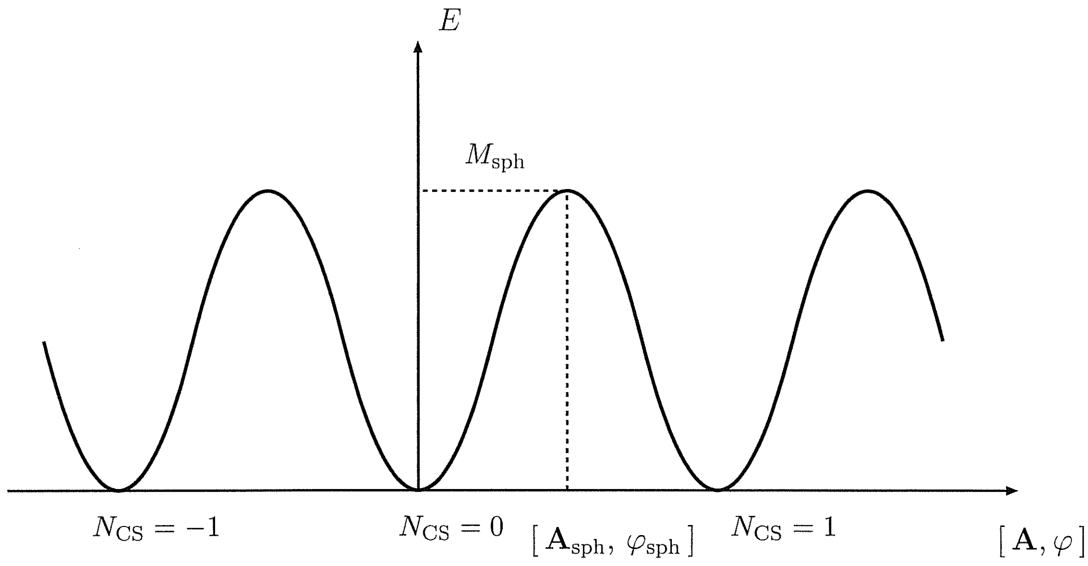


Figure 3.1: A Schematic behavior of the energy dependence on the configuration of the gauge and Higgs fields $[\,\mathbf{A}(x), \varphi(x)\,]$. The minima correspond to topologically distinct vacua with different baryon Chern-Simmons number ($N_{\rm CS}$)

.

equilibrium at temperatures ranging from 10^{12} GeV down to the electroweak scale. As a result, any baryon number produced in this range of temperature is inevitably erased. This is one of the main drawbacks of GUT baryogenesis.

Since the SM describes perfectly the fundamental interactions at low energies, the sphaleron configuration is inherent of any model of baryogenesis. Owing to its minimality, electroweak baryogenesis [126] is one of the most popular models for generating the baryon asymmetry of the Universe. The necessary ingredients are built-in: the sphalerons provide the baryon number violating interactions, the C and CP violation are naturally present in the SM (and even more in the supersymmetric version), and finally the out-of-equilibrium condition is satisfied provided the electroweak phase transition is of the strongly first order. Unfortunately electroweak baryogenesis requires the lightest Higgs particle to be unacceptably light (See [130] for an update). In the next section, we will be interested in an alternative baryogenesis scenario: *leptogenesis*, where lepton number is converted into a baryon asymmetry through sphalerons.

3.3 See-Saw Phenomenology and Leptogenesis

In this section, we introduce our notation for the SUSY see-saw and outline its low-energy implications. The aim is to make contact between realistic see-saw models, and the one generation toy models in which we will study the sneutrino production. We discuss the lepton asymmetry that can be produced in (s)neutrino decay, which implies a lower bound on the mass of the lightest r.h. (s)neutrino. Then, we briefly review different mechanisms for r.h. neutrino production, namely thermal and non thermal. The terms neutrino and sneutrino will be used interchangingly in discussing thermal production, which is similar for bosons and fermions. Concerning the non-thermal case, instead, different results are obtained for the two species, and in section 5.3 we review the ones for the neutrinos. Nonthermal production of sneutrinos is instead discussed in the next chapter.

Let us consider the Minimal Supersymmetric Standard Model (MSSM) extended with three r.h. neutrino superfields N_i (sometimes called the minimal supersymmetric see-saw model). The relevant couplings of the r.h. neutrinos are given by the superpotential [1]

$$W_N = h_{ji}\, L_i \cdot H_u\, N_j + \frac{1}{2}\, M_k\, N_k^2 \,, \tag{3.10}$$

where L_i and H are the lepton and the Higgs doublets, respectively, and h is a 3×3 complex Yukawa matrix. We will neglect the phases in our analysis of N production, because CP violation is not required for this process. We work in the r.h. neutrino mass basis, where the mass matrix M is diagonal, and we disregard the possibility of nearly degenerate r.h. (s)neutrinos [90, 91, 65] (i.e. we assume that the difference of neutrino masses is of order their mass).

3.3.1 The CP asymmetry and the bound on M_1

The lepton asymmetry produced in the decay of N_i can be written

$$Y_L \equiv \frac{N_L - N_{\bar{L}}}{s} = \epsilon_i\, \frac{N_{N_i}}{s} \kappa_i, \tag{3.11}$$

where N_{N_i} is the total number density of the ith heavy (s)neutrino species prior to its decay, s is the entropy density at decay [2], κ_i parametrises washout effects due to subsequent lepton number violating interactions, and ϵ_i arises from the CP violation of the N_i decay. It is given by [91]

$$\begin{aligned}
\epsilon_i &\equiv \frac{\sum_j \Gamma(N_i \to \ell_j h_u) - \sum_j \Gamma(N_i \to \bar{\ell}_j \bar{h}_u)}{\sum_j \Gamma(N_i \to \ell_j h_u) + \sum_j \Gamma(N_i \to \bar{\ell}_j \bar{h}_u)} \\
&= -\frac{1}{8\pi} \frac{1}{(YY^\dagger)_{ii}} \sum_{k \neq i} \mathrm{Im}\left[\{(YY^\dagger)_{ik}\}^2\right] \left[F_V\left(\frac{M_k^2}{M_i^2}\right) + F_S\left(\frac{M_k^2}{M_i^2}\right)\right]
\end{aligned} \tag{3.12}$$

where F_V and F_S are the contributions of the vertex and self-energy respectively. They are given by

[1] The superfield N written in eqn. (1) actually denotes an anti-(s)neutrino. However, for brevity

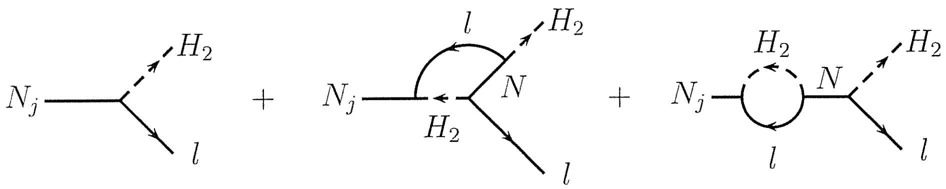


Figure 3.2: *Tree level and one-loop diagrams contributing to heavy neutrino decays.*

$$F_V(x) = \sqrt{x}\ln\left(1+\frac{1}{x}\right), \quad F_S(x) = \frac{2\sqrt{x}}{x-1} \tag{3.13}$$

We suppose for the moment that some number density of N_i is produced in the early Universe, and concentrate on how large an asymmetry can be generated. The asymmetry ϵ_i is determined by the masses and couplings of the r.h. (s)neutrinos, which are given in eqn. (3.10). However, it can be related to, and therefore constrained by, low energy observables.

Considering hierarchical r. h. neutrinos, the CP asymmetry produced in the decay of a r.h. (s)neutrino can conveniently be parameterized as

$$\begin{aligned} \epsilon_i &= \frac{3}{8\,\pi}\frac{M_i\, m_3}{\langle H\rangle^2}\delta_{\rm CP} \\ &\simeq 10^{-6}\left(\frac{M_i}{10^{10}{\rm GeV}}\right)\left(\frac{m_3}{0.05\,{\rm eV}}\right)\delta_{\rm CP}. \end{aligned} \tag{3.14}$$

By using eqs. (3.18) and (3.12), it is possible to show [67, 68, 69] that for the case ϵ_1, $\delta_{\rm CP}$ satisfies the upper bound [3]

$$|\delta_{\rm CP}| \leq 1\ . \tag{3.15}$$

By combining the two last expressions, one finds an upper bound on the parameter ϵ_1 which scales linearly with the r.h. (s)neutrino mass M_1 *i.e.*

$$|\epsilon_1| \lesssim \frac{3}{8\,\pi}\frac{M_i\, m_3}{\langle H\rangle^2}. \tag{3.16}$$

This bound implies the following lower bound on M_1 for leptogenesis to be viable

$$M_1 \gtrsim \eta_B \frac{1-C}{C}\left[\frac{N_{N_i}}{s}\,\frac{3}{8\pi}\frac{m_3}{\langle H\rangle^2}\right]^{-1} \tag{3.17}$$

3.3.2 Low-energy observables

The mass m_3 in equation (3.14) denotes the mass of the heaviest left-handed neutrino. The light neutrino mass matrix is obtained by integrating out the heavy r.h. neutrinos

we will refer to it as a (s)neutrino.

[2]Any subsequent entropy production leads to further dilution of the asymmetry.

[3]We will use the parametrisation (3.14) for all the ϵ_i, $i = 1..3$. It is possible that $\delta_{CP} \leq 1$ for ϵ_2 and ϵ_3 (assuming no cancellations in the formulae), although this has not been shown.

to give the see-saw formula

$$m_\nu = -h^T M^{-1} h \langle H_u^0 \rangle^2 . \tag{3.18}$$

We will assume that the light neutrino masses m_i are hierarchical, so $m_3 \simeq \sqrt{\Delta m^2_{atm}}$ [63].

If h is written in the charged lepton mass eigenstate basis (neutrino flavour basis), then m_ν is diagonalised by the MNS matrix U [92], which can be written $U = V \cdot \text{diag}(e^{-i\phi/2}, e^{-i\phi'/2}, 1)$, with

$$V = \begin{bmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{bmatrix} .$$

In this matrix, $c_{12} = \cos\theta_{12}$, and so on. Atmospheric [63, 93] and solar [94, 95] data imply that θ_{12} and θ_{23} are large, approaching $\pi/4$. θ_{13} is constrained to be $\lesssim 0.1$ by the CHOOZ experiment [96]. In a supersymmetric scenario, there is additional information about h and M available in the slepton mass matrix. The neutrino Yukawa $h^\dagger h$ appear in the renormalization group equations for the soft slepton masses, and thereby induce flavour violating slepton mass terms [97]: $[\tilde{m}_L^2]_{ij}$. In a simple-minded leading log approximation, these off-diagonal mass matrix elements are

$$[\tilde{m}_L^2]_{ij} \simeq \frac{(3m_0^2 + A_0^2)}{8\pi} [V_L]^*_{ki} [V_L]_{kj} h_k^2 \log\left(\frac{M_k}{M_{GUT}}\right) \tag{3.19}$$

where h_i are the eigenvalues of h, m_0^2 and A_0 are soft parameters at the GUT scale, and we introduce a new matrix V_L which diagonalises $h^\dagger h$ in the charged lepton mass eigenstate basis ($V_L h^\dagger h V_L^\dagger$ = diagonal). The branching ratio for $\ell_j \to \ell_i\gamma$ can be roughly estimated as [97]:

$$\text{BR}(\ell_j \to \ell_i \gamma) \propto \frac{\alpha^3}{G_F^2} \frac{|[\tilde{m}_L^2]_{ij}|^2}{\tilde{m}_L^8} \tan^2\beta \tag{3.20}$$

where $\tilde{m}_L^2$ is the slepton mass scale. The experimental bound $BR(\mu \to e\gamma) < 1.2 \times 10^{-11}$ [98] implies $[\tilde{m}_L^2]_{\mu e} \lesssim 10^{-3} \tilde{m}_L^2$, for $\tilde{m}_L \simeq 100$ GeV. This constrains the angles in V_L for given neutrino Yukawas h_i. It can be shown that M and h have the same number of parameters as the weak scale neutrino and slepton mass matrices. Furthermore, in a SUSY scenario with universal soft masses at the GUT scale, M and h can be parametrised with $\tilde{m}_L^2$ and m_ν [99]. The r.h. neutrino masses and Yukawa couplings can be therefore reconstructed (*in principle*, but not in practise [99]) from the weak scale neutrino and sneutrino mass matrices, so that ϵ_i can be expressed in terms of weak scale variables. An analytic approximation for ϵ_1 can be found in [100, 101]:

$$\epsilon_1 \simeq \frac{3h_1^2}{8\pi \sum_j |W_{1j}|^2 m_j^2} \text{Im}\left\{\frac{\sum_k W_{1k}^2 m_k^3}{\sum_n W_{1n}^2 m_n}\right\} , \tag{3.21}$$

where m_i are the light neutrino masses, h_1 is the smallest eigenvalue of h, and $W = V_L U$ is the rotation from the basis where the ν_L masses are diagonal to the basis where $h^\dagger h$ is diagonal. h_1 is in practise unmeasurable; however, if h has a hierarchy similar to the up Yukawa matrix h_u, then $h_1^2 \sim 10^{-8}$, and ϵ_1 will only be large enough if there is some enhancement from the imaginary part. There are two simple limits for the matrix W, which are motivated by model building. The first is $V_L \simeq 1$, and corresponds to an almost diagonal slepton mass matrix (in the charged lepton mass eigenstate basis). This means that the large mixing observed in the MNS matrix U must come from the r.h. sector [102]. The second option is $W \simeq 1$, so $V_L \simeq U^\dagger$. This would arise if the large ν_L mixing is induced in the l.h. sector [103]. In the $V_L = 1$ case, eqn. (3.21) gives [100]:

$$\epsilon_1 \simeq -\frac{3h_1^2}{8\pi D}\,\mathrm{Im}\left\{\frac{m_1^3\; c_{13}^2 c_{12}^2\; e^{i\phi} + m_2^3\; c_{13}^2 s_{12}^2\; e^{i\phi'} + m_3^3\; s_{13}^2\; e^{2i\delta}}{m_1\; c_{13}^2 c_{12}^2\; e^{i\phi} + m_2\; c_{13}^2 s_{12}^2\; e^{i\phi'} + m_3\; s_{13}^2\; e^{2i\delta}}\right\} \tag{3.22}$$

$$\simeq -\frac{3h_1^2}{4\pi}\left\{\left(\frac{m_3}{m_2}\right)^3 2s_{13}^2\sin(2\delta-\phi') - \frac{m_1}{m_2}\sin(\phi-\phi')\right\}\;, \tag{3.23}$$

where $D = m_1^2\; c_{13}^2 c_{12}^2 + m_2^2\; c_{13}^2 s_{12}^2 + m_3^2\; s_{13}^2$, and in the second equation, the solar and atmospheric angles have been taken to be $\pi/4$. If we estimate the phases to be $O(1)$, $h_1 \sim$ the up Yukawa, and $m_3^2/m_2^2 \sim \Delta m_{atm}^2/\Delta m_{sol}^2$, this gives $\epsilon \lesssim 10^{-7}(s_{13}/.1)^2$, where we have scaled the unmeasured angle θ_{13} by its upper bound. [4] This is barely large enough for thermal leptogenesis. However, we remind that h_1 is unknown and it can well be $h_1 > 10^{-4}$. The second case, where $W \simeq 1$, can arise if M and $h^\dagger h$ are almost simultaneously diagonalisable [5]. For small angles in W, the approximation for ϵ can be extracted from (3.22), replacing the angles of the MNS matrix by the angles of W, and setting the cosines $\to 1$. When $W \sim 1$, then $V_L \simeq U^\dagger$, so it is the MNS angles that appear in equation (3.19), and $BR(\mu \to e\gamma) < 1.2 \times 10^{-11}$ [98] implies an upper bound on the CHOOZ angle $\theta_{13} < .02$ (for $\tilde{m}_L = 100$ GeV, $h_3 = 1$)[104]. To conclude, we briefly comment on the parameter κ_i. After the asymmetry is generated in the out-of-equilibrium decay of the r.h. neutrino, lepton number violating interactions which could wash out the asymmetry must be out of equilibrium. This is a fairly straightforward requirement when considering the decay of the lightest r.h. neutrino N_1 [66]; it is more complicated in the case of $N_{2,3}$ decaying at $T \gtrsim M_1$ [105]. The fraction of the asymmetry which survives these interactions is $\kappa_i \leq 1$.

[4] This estimate is not significantly changed if the angles in V_L are small compared to θ_{13}. CHOOZ experiment: $\theta_{13} \lesssim .1$. If $\theta_{13} \ll .1$, but $\theta_{13} < [V_L]_{12}, [V_L]_{13} < .1$, then the formula for ϵ is similar to (3.23), with the replacement $\theta_{13} \to \theta_{L1j}$ and $\delta \to \varphi_{1j}$ (where $[V_L]_{12} = \cos\theta_{L13}\sin\theta_{L12}e^{i\varphi_{12}}$, $[V_L]_{13} = \sin\theta_{L13}e^{i\varphi_{13}}$).

[5] The "almost" is important; if $W = 1$, there is no CP violation, so $\epsilon = 0$.

Chapter 4

Leptogenesis at low scale

A typical problem of the leptogenesis scenario is the mismatch between the maximum reheat temperature implied by gravitino overproduction bound and the minimum temperature required to create thermally the lightest right-handed neutrino. We explore the possibility of baryogenesis via leptogenesis in the presence of low scale mass right-handed neutrino. In such a scenario, right-handed neutrinos are created thermally at low reheat temperatures without relying on non-perturbative production mechanisms. We focus on two specific realizations of the scenario, namely the out-of-equilibrium decay of right-handed neutrinos (Fukugita-Yanagida) and the leptogenesis via the LH_u flat direction (Affleck-Dine). We find that in general, the two scenarios are able to produce the required baryon excess for a reasonable amount of CP violation.

4.1 Introduction

Recent experimental results gave overwhelming evidence that neutrinos have small but non-vanishing masses [28]. In the standard model (SM), neutrinos are exactly massless, hence the explanation of neutrino experiments requires Physics beyond the SM. Furthermore, neutrino masses appear to be very small with respect to the other fermions ones. If neutrinos are Majorana particles, it is possible to accommodate small neutrinos masses in the SM by introducing the lepton number violating effective operator [29] $O_{\rm eff} = \alpha_{ij}\ell_i^T\tau_2\vec{\tau}\ell_j H^T\tau_2\vec{\tau}H/M$, where ℓ_i and H are the lepton and the Higgs doublet respectively. Here M is the scale where "new Physics" is expected to occur, is usually taken as the Planck or the GUT scale. In the former case, the presence of this lepton number violating operator is motivated by the common belief that gravity does not respect any global quantum number [30, 31], or at least this is what happens for example in black holes and wormholes –no hair theorems. In the latter case ($M = M_{\rm GUT}$), the effective operator arises via the see-saw mechanism [32], when integrating-out the heavy right-handed neutrinos (RHNs hereafter).

On the other hand, our Universe appears to be constituted exclusively of baryons. In order not to spoil the Big Bang Nucleosynthesis (BBN) successful predictions of the observed light elements abundances [33], a small baryon excess have to be present. The required value is quantified by the baryon-to-entropy ratio and is given by $Y_B \equiv n_B/s = (7.2 \pm 0.4) \times 10^{-11}$ [59]. To accomplish successfully their task, baryogenesis scenarios [57] have to satisfy three essential conditions [34], namely: *(i.) Baryon number violation*, *(ii.) C and CP violation*, and *(iii.) Departure from thermal equilibrium*. One particularly appealing scenario is the leptogenesis scenario, where lepton number, produced either by the out-of-equilibrium decay of heavy RHN's [35] or by the decay of a scalar condensate carrying non-zero lepton number [36, 37], is reprocessed to a baryon asymmetry via the sphalerons interactions. Given the experimental evidence that lepton number is violated in neutrino oscillation and the fact that proton decay have not been observed yet, the present experimental situation seems to favor this scenario over the other existing baryogenesis scenarios. A generic problem of thermal leptogenesis scenarios is the mismatch between the maximum reheat temperature implied by gravitino overproduction and the minimum temperature required to thermally create heavy RHNs $T_{\rm RH} \gtrsim 10^{10}$ GeV. To reconcile these two facts, non-thermal creation of RHNs in a low reheat temperature plasma were considered. These mechanisms, however, involve non-perturbative dynamics and are in general sensitive to inflation models. Furthermore, they lead to even more stringent bounds on the reheat temperature, due to the non thermal production of moduli and gravitinos [38, 39].

The aim of this chapter is to address this issue in a different perspective. We will consider the situation where the reheat temperature is low (may be as low as the TeV) and we will only consider thermal production of RHNs. This will naturally lead us to consider a class of see-saw models (that we will subsequently call low-scale see-saw models), where RHNs have TeV masses instead of the conventional unification scale. The chapter is organized as follows. In section 4.2, we give our main motivation for the scenario. In section 4.3, we study leptogenesis through the out-of-equilibrium decay of low scale RHNs. In section 4.4, we turn to the Affleck-Dine scenario. Finally, in section 4.5, we summarize our conclusions.

4.2 The gravitino problem vs. thermal leptogenesis

As any unwanted relic, gravitinos represents a potential danger for the thermal history of the Universe. Gravitinos are created predominantly via $2 \to 2$ inelastic scatterings of gluons and gluinos quantas. Their relic density and contribution to the energy density are given by [40]

$$Y_{3/2} = 1.1 \times 10^{-10} \left(\frac{T_{\rm RH}}{10^{10}\ {\rm GeV}} \right) \left(\frac{100\ {\rm GeV}}{m_{3/2}} \right)^2 \left(\frac{m_{\tilde{g}}}{1\ {\rm TeV}} \right)^2 , \tag{4.1}$$

$$\Omega_{3/2} h^2 = 0.21 \left(\frac{T_{\rm RH}}{10^{10}\ {\rm GeV}} \right) \left(\frac{100\ {\rm GeV}}{m_{3/2}} \right) \left(\frac{m_{\tilde{g}}}{1\ {\rm TeV}} \right)^2 , \tag{4.2}$$

where $m_{\tilde{g}}$ denotes the gluino mass. The requirement that, if unstable, their late decay do not disrupt the successful BBN predictions, and if stable, their energy density do not overclose the Universe, put tight constraints on their relic abundance. It has been noted that if $m_{3/2}$ >10 TeV or $m_{3/2} <$ keV, then there is no gravitino problem [41, 42]. These requirements can be relaxed if there is a period of inflation and the constraints apply only on post-inflation abundances. From the expression (4.1), one sees that the gravitino abundance scales linearly with the reheat temperature, therefore the bound on $Y_{3/2}$ translates onto the following bound on the maximum allowed reheat temperature $T_{\rm RH}$ [43]

$$T_{\rm RH} \lesssim (10^6 - 10^9)\ {\rm GeV\ for}\ m_{3/2} = 100\ {\rm GeV} - 1\ {\rm TeV}\,. \tag{4.3}$$

There exists however more stringent bounds on $T_{\rm RH}$ from non-thermal production. For generic supersymmetric inflation models, the bound can be as tight as [39] $T_{\rm RH} \lesssim 10^5 (V^{1/4}/10^{15}$ GeV), where $V^{1/4}$ is the height of the inflationary potential. Let us now see the constraints on the reheat temperature coming from leptogenesis. In the original see-saw model [32] the mass scale of RHNs is typically of $\mathcal{O}(10^{10} - 10^{15})$ GeV. In addition, the bound on the CP parameter [69] for hierarchical RHN's in thermal leptogenesis implies a lower bound on the mass of the lightest RHN $M_{N_1} \gtrsim 10^{10}$ GeV. Consequently, if the thermal leptogenesis scenario is truly *the mechanism* responsible for the the generation of the Baryon Asymmetry of the Universe (BAU), RHNs of this mass have to be produced after inflation. This means a high reheat temperature, at least as high as the mass of the lightest RHN, *i.e.* 10^{10} GeV, potentially conflicting with the gravitino bound discussed above. A possible way out to get around this problem is to produce RHNs non-thermally, that is during an efficient preheating phase [44]. Non-thermal production, however, can lead in some cases to even more stringent bounds on the reheat temperature. Indeed, for typical hybrid inflation models, the upper bound on the reheat temperature can be as low as 1 TeV [38].

From the above discussion, it is clear that any compelling solution to this problem will, in one way or another, involve low reheat temperatures. After all, we dont know the thermal history of our Universe before BBN. All we know experimentally is that $T_{\rm RH} \geq T_{\rm BBN} \sim$ MeV. In this chapter, we will consider a rather exotic solution to this problem, namely the case for leptogenesis when RHNs have a low scale mass. The first benefit of such an approach is that RHNs can be produced thermally with a low reheat temperature $T_{\rm RH} \sim \mathcal{O}$(TeV), avoiding thus the creation of dangerous relics, like heavy GUT monopoles, and more importantly suppressing the creation of gravitinos. On theoretical grounds, nothing forbids the mass of RHNs to be of $\mathcal{O}(\text{TeV})$. In fact this situation is encountered in many cases (See for *e.g.* [45, 46, 47, 48]). This is also a typical situation that arises in models where the fundamental scale (the GUT scale and/or the quantum gravity scale) is of $\mathcal{O}(\text{TeV})$. In this case, the Yukawa couplings of RHNs have to be much smaller to produce phenomenologically acceptable light neutrino masses. Such a fine-tuning is stable under radiative corrections because that Yukawa couplings are self renormalizable and is protected by supersymmetry. There remains the question of how such suppressed Yukawa couplings can arise in a concrete model. This can be achieved for example by the mean of some R-symmetry that forbids the bare Yukawa coupling between the left and the right-handed neutrinos. As a result the leading Yukawa couplings will be suppressed by powers of a heavy scale [45, 46]. The Yukawa suppression can be obtained upon integrating-out some heavy field as well [49].

4.3 Thermal Leptogenesis with TeV scale RHNs

We begin by reviewing the basics of the out-of-equilibrium decay leptogenesis scenario. Consider the Minimal Supersymmetric Standard Model extended by three RHNs, one for each generation. The interactions of the RHNs are given by the following superpotential

$$W_N = Y_{ij} L_i H_u N_j + \frac{1}{2} M_i N_i^2 \tag{4.4}$$

After integrating-out the RHN and electroweak symmetry breaking, the light neutrinos mass matrix will be given by the familiar see-saw formula

$$m_\nu = -Y^T M^{-1} Y \langle H_u \rangle^2 \,. \tag{4.5}$$

In this scenario, the RHNs must decay out-of-equilibrium. A measure of the departure from thermal equilibrium is given by the parameter K defined as

$$K \equiv \left.\frac{\Gamma_N}{2\,H}\right|_{T=M_N} , \tag{4.6}$$

where Γ_N is the decay rate of RHNs and H is the expansion rate of the Universe. The decay is out-of-equilibrium when $K \lesssim 1$. The final baryon asymmetry reprocessed by sphalerons is given by [60]

$$Y_B \equiv \frac{n_B}{s} = \left(\frac{8n_g + 4n_H}{22n_g + 13n_H}\right) \frac{n_L}{s} \,, \tag{4.7}$$

where n_g and n_H counts the number of fermion generations and Higgses respectively.

The lepton asymmetry produced by the CP-violating out-of-equilibrium decay of the RHNs can be computed using

$$\frac{n_L}{s} = \kappa \frac{\epsilon}{g_*}, \tag{4.8}$$

where g_* is the effective degrees of freedom and κ is the dilution factor, computed by integrating the relevant set of Boltzmann equations [50, 51]. The parameter ϵ characterizing CP violation in the RHNs decay, can be defined for each RHN separately as [91]

$$\begin{aligned} \epsilon_i &\equiv \frac{\sum_j \Gamma(N_i \to \ell_j h_u) - \sum_j \Gamma(N_i \to \bar{\ell}_j \bar{h}_u)}{\sum_j \Gamma(N_i \to \ell_j h_u) + \sum_j \Gamma(N_i \to \bar{\ell}_j \bar{h}_u)} \\ &= -\frac{1}{8\pi} \frac{1}{(YY^\dagger)_{ii}} \sum_{k \neq i} \mathrm{Im}\left[\{(YY^\dagger)_{ik}\}^2\right] \left[F_V\left(\frac{M_k^2}{M_i^2}\right) + F_S\left(\frac{M_k^2}{M_i^2}\right)\right] \end{aligned} \tag{4.9}$$

where F_V and F_S are the contributions of the vertex and self-energy respectively. They are given by

$$F_V(x) = \sqrt{x} \ln\left(1 + \frac{1}{x}\right), \quad F_S(x) = \frac{2\sqrt{x}}{x-1} \tag{4.10}$$

Now, applying the above formulae to TeV mass RHNs, one immediately sees that, due to the smallness of the Yukawa couplings, the decay of RHNs is automatically out-of-equilibrium. In addition to the decay processes, there can be other competing processes that might bring the RHNs to thermal equilibrium, depleting any pre-existing lepton number. These processes have to be out-of-equilibrium too, *i.e.* $\Gamma \simeq \langle n\sigma v\rangle \ll H$. The first such process is the $\Delta L = 2$ scattering $\ell h_u \leftrightarrow \bar{\ell}\bar{h}_u$, via both s and t channel. Other competing processes may involve the t-(s)quark, such as $Nt(\bar{b}) \leftrightarrow \ell b(\bar{t})$. It turns out that due to the Yukawa coupling suppression all these processes are out-of-equilibrium. Finally, it has been noted [52] that the process $W^\pm W^\pm \to \ell^\pm \ell^\pm$, mediated by virtual left-handed neutrinos can lead to stringent constraints on their masses. In our case, it leads to a very mild constraint. So far for the out-of-equilibrium conditions, now we concentrate on the CP violation parameter ϵ. As we have seen previously, due to the smallness of the Yukawa couplings, it is very easy to satisfy the out-of-equilibrium condition, however the resulting CP violation parameter ϵ is too small. This is due to the fact that the decay rate Γ and the CP-parameter ϵ are both proportional to the same Yukawa couplings combination. From Eqts (4.9, 4.10), one sees that the two contributions to the CP parameter ϵ are sensible to two completely different patters of RHNs masses. While the vertex contribution F_V is enhanced for large hierarchies, the self-energy contribution F_S is so when RHNs are (quasi-)degenerate [1]. In order to enhance the value of ϵ, one have to

[1] For perturbation theory to hold, the mass splitting $\delta M_{ik} = |M_i - M_k|$ must satisfy $\delta M_{ik} \gg \Gamma$,

exploit the properties of the two functions F_V and F_S. In the next subsection, we will consider the case where RHNs are nearly degenerate [54, 53]. There exist however another possibility, related to the fact that RHNs masses and soft SUSY breaking A-terms are of the same order *i.e.* $\mathcal{O}(\text{TeV})$.

4.3.1 Leptogenesis with quasi-degenerate TeV scale RHNs

Consider a model where two out of the three RHNs are quasi degenerate, that is N_1, N_2 and N_3 have masses M_1, $M_2 \sim \mathcal{O}(\text{TeV}) \ll M_3$ respectively. The mass splitting $\delta M_{12} \equiv |M_2 - M_1| = \delta \cdot M_0$, where $M_0 \sim$ TeV). Due to their suppressed Yukawa's, RHNs will be long-lived enough to eventually dominate the Universe before decaying. The condition for RHNs dominance can be written $\Gamma_N \ll \Gamma_\varphi$, where Γ_φ is the decay rate of the inflaton [67]. While, RHNs can hardly dominate the energy density of the Universe because of Pauli blocking, this can happen more easily for their scalar partners the RH sneutrinos. Moreover, due to quantum de Sitter quantum fluctuations [55] and for $H_{\text{inf}} \gg M_N \sim \mathcal{O}(\text{TeV})$, RH sneutrinos become coherent over super-horizon scales and can be considered as classical fields with the constant value (vev) $\langle \widetilde{N}^2 \rangle = 3H_{\text{inf}}^4/8\pi^2 M_N^2$. Therefore if the RH sneutrinos scalar potential is just given by the mass term, they are likely to dominate quickly the energy density of the Universe. Given the above discussion, one can compute the lepton asymmetry produced during the decay of N_2 using

$$\frac{n_L}{s} = \frac{3}{4}\frac{T_{N_2}}{M_2}\epsilon_2 \,, \tag{4.11}$$

where T_{N_2} is the decay temperature of N_2's computed by equating the energy density of RHNs with the energy density of the Universe when $H \sim \Gamma_2$. Since N_1 and N_2 are quasi-degenerate, we can safely ignore the vertex contribution to the CP parameter ($F_S \gg F_V$). Using Eqts (4.9) and (4.10), we can compute the total CP parameter $\epsilon \simeq \epsilon_1 + \epsilon_2$, giving

$$\epsilon \simeq \frac{1}{8\pi} \sum_{i=1,2} \frac{1}{(YY^\dagger)_{ii}} \,\text{Im}\left[\{(YY^\dagger)_{12}\}^2\right] \frac{1}{\delta} \tag{4.12}$$

A rough estimate of the required degeneracy gives $\delta \sim \mathcal{O}(10^{-6} - 10^{-7})$, and perturbativity is clearly satisfied (See footnote 1 on page 41). Such a degeneracy could be ascribed for example to a flavor symmetry, the parameter δ would then characterize its breaking. In the simplest case, the flavor group G_f is taken as a Z_2 and the RHNs have different parity Z_2 assignments, *i.e.* $N_1 \sim$ odd (even) and $N_2 \sim$ even (odd) under Z_2. The flavor symmetry is broken by the vev of the odd field ψ. Restricting

where Γ is the decay rate of RHNs, otherwise one can no more trust the perturbative calculation based on Eqts (4.9,4.10) and one have to rely on a resummation approach [53]. In the limit of exact degeneracy, the CP parameter vanishes.

to the 12 block, the resulting mass matrix for the RHNs is

$$M_R \sim M_0 \begin{pmatrix} 1 & \delta/2 \\ \delta/2 & 1 \end{pmatrix} \tag{4.13}$$

with $\delta/2 \equiv \langle\psi\rangle/\Lambda$. The diagonalization of the mass matrix yields two quasi-degenerate RHNs with a mass-splitting δM_0.

4.3.2 Leptogenesis from soft SUSY breaking A-terms

In the traditional leptogenesis scenario, the contributions of the soft SUSY breaking A-terms to the CP parameter ϵ are usually neglected. Indeed, SUSY breaking will induce the following A-terms

$$\mathcal{L}_{\rm soft} = A_{ij} m_{3/2} \widetilde{L}_i \widetilde{N}_j H_u + \text{h.c.}. \tag{4.14}$$

Let us consider the following vertex diagrams, where in the tri-scalar vertex we put the A-term contribution from Eqt (4.14) instead of the standard SUSY one.

Estimating the contribution of the SUSY soft breaking A-terms to ϵ and comparing it to the standard SUSY one for each of the two considered diagrams, we obtain

$$\frac{\epsilon^{\rm soft}_{(a)}}{\epsilon^{\rm SUSY}} \sim |A| \frac{m_{3/2}}{M_i} \sin\delta_{\rm soft}\,, \tag{4.15}$$

$$\frac{\epsilon^{\rm soft}_{(b)}}{\epsilon^{\rm SUSY}} \sim |A|^4 \left(\frac{m_{3/2}}{M_i}\right)^4 \sin\delta_{\rm soft} \tag{4.16}$$

where $\delta_{\rm soft}$ is an effective soft CP phase. From (4.15), we see that in the conventional leptogenesis scenario, where the mass of the lightest RHN is $M_1 \simeq 10^{10}$ GeV, $\epsilon_{\rm soft}$ is suppressed with respect to $\epsilon^{\rm SUSY}$ at least by a factor of 10^{-7}. However, in our scenario, where $M_i \sim m_{3/2}$, the CP asymmetry parameter $\epsilon_{\rm soft}$ is no more suppressed. It can

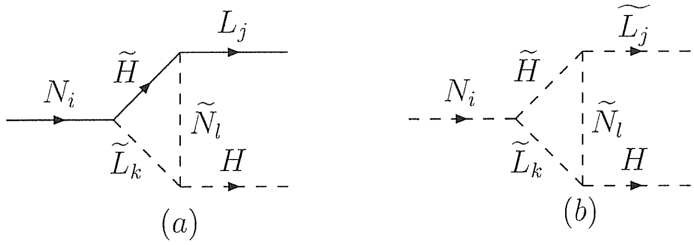


Figure 4.1: SUSY breaking A-term contributions to the CP parameter ϵ.

even dominate the over the SUSY contribution depending on the value of the soft parameters. This means that CP violation may completely originate from the soft SUSY breaking sector, like in the Affleck-Dine case. However, besides enhancing the amount of CP violation, the soft SUSY breaking interactions could bring the RHNs decay at equilibrium, erasing considerably the produced lepton number. A more accurate analysis, requiring the integration of Boltzmann equations, is necessary to reach a firm conclusion.

Finally, it is worth noticing from Eqt. (4.2) that gravitinos could no more constitute a sizable amount of dark matter in our scenario. Indeed, $\Omega_{3/2}\, h^2 = 0.01 - 1$, requires the gravitino to be lighter and/or the gluinos masses to be heavier.

4.4 Affleck-Dine leptogenesis with TeV scale RHNs

Now, we turn to investigate the Affleck-Dine mechanism [36, 37] in the presence of TeV scale RHNs. Consider the LH_u MSSM flat direction given by [2]

$$L_i = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi \\ 0 \end{pmatrix}, \quad H_u = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \varphi \end{pmatrix} \tag{4.17}$$

This flat direction is lifted by the non-renormalizable operator $W_{\rm NR} = \lambda (L\, H_u)^2/M = \lambda \varphi^4/4M$. This operator can be generated via the see-saw mechanism when integrating-out the heavy RHNs. The evolution of the scalar condensate φ in the expanding background is dictated by the classical equation of motion

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{\partial V(\varphi)}{\partial \varphi^*} = 0 \tag{4.18}$$

where $V(\varphi)$ is the full potential, including the soft masses, the Hubble induced masses and the A-terms (both from SUSY breaking and the Hubble induced ones)[3].

$$\begin{aligned} V(\varphi) &= (m_{3/2}^2 - c_H H^2)|\varphi|^2 + a_H H \frac{\varphi^4}{4M} \\ &\quad + a_m m_{3/2} \frac{\varphi^4}{4M} + \text{h.c.} + \frac{|\lambda|^2}{M^2} |\varphi|^6 . \end{aligned} \tag{4.19}$$

The constants c_H and a_H depend on the detailed structure of the Kahler potential. In particular the sign of c_H is crucial for the validity of the AD scenario. We assume throughout the chapter that it is positive ($c_H > 0$). The evolution of the scalar condensate follows three phases. During inflation, when $H \gg m_{3/2}$ the field φ is over-damped and it settles away from the origin at a distance

$$|\varphi_0| \simeq \left(\frac{c_H M^2 H^2}{|\lambda|^2} \right)^{1/4} . \tag{4.20}$$

[2]The factor $\sqrt{2}$ is necessary to have a canonical kinetic term for φ (The Kahler potential is $K = H_u H_u^\dagger + LL^\dagger = \varphi \varphi^\dagger$).

[3]Here, we are simply ignoring thermal effects [56].

From the last equation, one sees that φ is displaced farther as the neutrino Yukawa coupling λ is smaller. That is why L_i in Eqt. (4.17) is usually chosen as the neutrino with the smallest Yukawa coupling, L_1 say. When $H \approx m_{3/2}$, the A-terms enter into play and the condensate begins to oscillate. In general, when taking into account thermal effects, the condensate begins to oscillate when the decreasing expansion rate reaches a certain value denoted $H_{\rm osc}$, determined when the thermal contributions are taken into account [56]. At later times when $H \ll m_{3/2}$, the lepton number is essentially conserved. The evolution of the lepton number, n_L defined as

$$n_L = \frac{i}{2}(\varphi^* \dot{\varphi} - \varphi \dot{\varphi}^*), \tag{4.21}$$

follows the equation

$$\dot{n_L} + 3Hn_L = \mathrm{Im}\left[\varphi \frac{\partial V(\varphi)}{\partial \varphi}\right] \tag{4.22}$$

The generated lepton asymmetry can be approximated by integrating the equation (4.22). This gives

$$n_L \approx \frac{m_{3/2}}{2M}\mathrm{Im}(a_m \varphi^4)t \tag{4.23}$$

In a matter dominated Universe, the expansion rate scales with time as $H = 2/3t$. Plugging this into the last equation, we get

$$\frac{n_L}{s} \approx \frac{1}{12}\left(\frac{T_{\rm RH}}{H_{\rm osc}}\right)\left(\frac{m_{3/2}}{M_*}\right)\left(\frac{M}{M_*}\right)\frac{\delta_{\rm eff}}{|\lambda|^2}, \tag{4.24}$$

where $M_* \equiv M_{\rm Planck}/\sqrt{8\pi} \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck mass and we have dropped constants of $\mathcal{O}(1)$. The effective CP-violating parameter $\delta_{\rm eff}$ is defined as

$$\delta_{\rm eff} \simeq \sin\left(4 \arg \varphi + \arg a_m\right) \tag{4.25}$$

Now, specializing to the low scale see-saw models [46, 45], where Yukawa couplings come-out naturally suppressed as $\lambda \sim |Y^{\rm eff}|^2 \sim m_{3/2}/M_*$, we get

$$\frac{n_L}{s} \approx \frac{1}{12}\left(\frac{T_{\rm RH}}{H_{\rm osc}}\right)\delta_{\rm eff} \tag{4.26}$$

Usually the effective CP-violating parameter is assumed to be maximal *i.e.* $\delta_{\rm eff} \simeq 1$. In our case there is no need to do so, since the reheating temperature can be as low as $m_{3/2}$ so TeV mass RHNs are produced thermally, while the gravitinos are not. Typically, the condensate begins to oscillate at $H_{\rm osc} \gtrsim m_{3/2}$. In the extreme case when $T_{\rm RH} \simeq H_{\rm osc} \simeq m_{3/2}$, only a small amount of CP is sufficient to reproduce the observed value, namely $\delta_{\rm eff} \simeq 10^{-9} - 10^{-10}$. Up to now, we did not specify the transmission mechanism of SUSY breaking. We just assumed that some hidden sector will produce the soft breaking scalar masses and A-terms. In the gravity-mediated scenario the A-terms are known to be of the form $am_{3/2}W$ + h.c.. This means that

$a_m = b_m\lambda$ and $a_H = b_H\lambda$, where b_H, $b_m \sim O(1)$. In this case, the resulting lepton asymmetry is given by

$$\frac{n_L}{s} \approx \frac{1}{12}\left(\frac{T_{\rm RH}}{H_{\rm osc}}\right)\left(\frac{m_{3/2}}{M_*}\right)\delta_{\rm eff} \tag{4.27}$$

where now the CP violation parameter is defined as

$$\delta_{\rm eff} \simeq \sin\left(4\arg\varphi + \arg a_m + \arg\lambda\right) \tag{4.28}$$

For the typical values $H_{\rm osc} \sim m_{3/2}$, $T_{\rm RH} \sim 10^9$ GeV and $\delta_{\rm eff} \sim O(1)$, we obtain the right value for the lepton asymmetry.

4.5 Conclusions

To conclude, motivated by the potential conflict between the gravitino overproduction bound and the high reheat temperature required to produce RHNs thermally, we investigated the baryogenesis through leptogenesis scenario in the presence of low scale RHNs. We have seen that, in such a scenario the Yukawa couplings of RHNs have to be suppressed, in order to give rise to acceptable light neutrino masses. This suppression proved to be useful for many purposes, in particular in satisfying the out-of-equilibrium condition in the FY scenario. Due to this suppression, however, the resulting CP was too small. We used two different mechanisms to enhance the CP parameter: the degeneracy of RHNs and soft A-terms. In the latter case, the necessary CP violation may come entirely from the soft SUSY A-term. We also considered leptogenesis via the LH_u flat direction. We have seen that for generic SUSY breaking scenarios, AD leptogenesis with low scale RHNs is possible, though with reheat temperatures higher than TeV.

Chapter 5

Non thermal leptogenesis and rescatterring

As we have seen in the previous chapters, the observed baryon asymmetry of the Universe can be due to the $B-L$ violating decay of heavy right handed (s)neutrinos. The amount of the asymmetry depends crucially on their number density. If the (s)neutrinos are generated thermally, in supersymmetric models there is limited parameter space leading to enough baryons. For this reason, several alternative mechanisms have been proposed. We discuss the nonperturbative production of sneutrino quanta by a direct coupling to the inflaton. This production dominates over the corresponding creation of neutrinos, and it can easily (i.e. even for a rather small inflaton-sneutrino coupling) lead to a sufficient baryon asymmetry. We then study the amplification of MSSM degrees of freedom, via their coupling to the sneutrinos, during the rescattering phase which follows the nonperturbative production. This process, which mainly influences the (MSSM) $D-$flat directions, is very efficient as long as the sneutrino quanta are in the relativistic regime. The rapid amplification of the light degrees of freedom may potentially lead to a gravitino problem. We estimate the gravitino production by means of a perturbative calculation, discussing the regime in which we expect it to be reliable. The chapter is organised as follow. First, we will review the basics of thermal production and its drawbacks. This will bring us to consider alternative production mechanisms and namely we will consider non thermal production mechanism (preheating). Then, we will see what are the consequences of SUSY on this mechanism, namely the rescattering of the flat directions. At the end we draw our conclusions.

5.1 Introduction and summary

The generation of the Baryon Asymmetry of the Universe (BAU) [57] represents one of the puzzles of Cosmology. Three ingredients are required [58] to achieve this task: baryon number violation, C and CP violation, and departure from thermal equilibrium. The baryon number violation can be challenging to implement, because it must be consistent with the current lower bound on the proton lifetime, $\tau_p \gtrsim 10^{32}$ years [59]. The Standard Model (SM) is a C and CP violating theory, and contains non-perturbative $B + L$ violating interactions (sphalerons) [60]—which are rapid in the early Universe but unable to mediate proton decay. However, it seems difficult to use this baryon number violation to create the asymmetry in the SM [61] and its more popular extensions [62]. An attractive alternative is to generate a lepton asymmetry [35] in some C, CP and lepton number (L)-violating out-of-equilibrium interaction, and then allow the sphalerons to reprocess part of it into a baryon asymmetry. An appealing feature of this scenario is that while neutrino masses are experimentally observed [63] (and are L-violating, if they are Majorana), there is still no evidence for baryon number violation.

The above idea is naturally implemented [35] in the context of the see-saw [32], which is a minimal mechanism for generating neutrino masses much smaller than the ones of the charged leptons. Three right-handed (r.h.) neutrinos N_i are added to the SM particle content, given Yukawa interactions with the lepton and Higgs doublets, and large Majorana masses. This gives the three light neutrinos very small masses, due to their small mixing with the heavy r.h. neutrinos through the Dirac mass. Grand Unified Theories (GUT) and their supersymmetric versions, that constitute natural candidates for the Physics beyond the SM, often contain r.h. neutrinos in their particle content. In this chapter we consider the supersymmetric version of the see-saw mechanism, which is theoretically attractive because it addresses the hierarchy between the Higgs and r.h. neutrino masses.

The r.h. (s)neutrinos of the see-saw can generate the BAU via leptogenesis, in a three steps process [35, 64, 65, 66]. First, some (CP symmetric) number density of (s)neutrinos is created in the early Universe. Then, a lepton asymmetry is generated in their CP violating out-of-equilibrium decay. Finally, the lepton asymmetry is partially reprocessed into a baryon one by the $B + L$ violating interactions, provided it is not washed out by lepton number violating scatterings. In this chapter, we are mostly interested in the first step, although in the next section we will also briefly review the decay and washout processes.

The most straightforward and cosmological model-independent mechanism to generate r.h. (s)neutrinos is via scattering in the thermal bath [66]. However, as discussed in section 3.3, the parameter space available is restricted in supergravity-motivated models. Indeed, unless some enhancement of the CP asymmetry characterizing the r.h. (s)neutrino decay is present (which occurs for example if they are nearly degenerate in mass) the generation of a sufficient lepton number poses a rather strong lower bound on their mass [67, 68, 69] (see also [70]). For thermal production, this translates into a lower bound on the reheating temperature T_{RH} of the thermal bath. On the other hand, if supergravity is assumed, T_{RH} cannot be taken arbitrarily large without leading to an overproduction of gravitinos [71, 72, 73, 74, 75, 40]. The two requirements are compatible only provided a nearly maximal CP asymmetry (again, banning any possible enhancement from mass degeneracy) is present in the r.h. (s)neutrino decay. If this is not the case, alternative mechanisms of production for the r.h. (s)neutrino have to be considered.

As remarked in [76], leptogenesis can be achieved if at least one r.h. sneutrino has a smaller mass than the Hubble parameter, *i.e.* [1] $M_N < H$, during inflation. In this case, quantum fluctuations of this sneutrino component are produced during the inflationary expansion, and amplified to generate a classical condensate. The decay of the condensate eventually generates the required lepton asymmetry. The above requirement $M_N < H$ is not trivially satisfied in a supergravity context, since supergravity corrections typically provide a mass precisely of order H to any scalar field of the model [77]. In this case, however, a suitable choice of the Kähler potential can induce a negative mass term $m^2_{\rm ind} \simeq -H^2$, so that a large expectation value will be generated for the sneutrino component during inflation [77]. This also leads to the formation of a condensate during inflation, and to successful leptogenesis as in the previous case.

Large variances can be produced during inflation if the sneutrinos are not too strongly coupled to the inflaton field ϕ, since this would generate a high effective mass which could fix $\langle \tilde{N} \rangle = 0$ during inflation. However, if one of the r.h. (s)neutrinos is coupled to the inflaton, there is the obvious possibility that a sufficient amount of (s)neutrino quanta is generated when the inflaton decays. Quite remarkably, for a rather wide range of models this decay occurs in a nonperturbative way [78, 79] (this is known in the literature as *preheating* [79]). In models of chaotic inflation [80], this is due to the coherent oscillations of the inflaton field, which can be responsible for a parametric amplification of the bosonic fields to which the inflaton is coupled [2]. It is important to remark that this resonant amplification does not require very high couplings between the inflaton and the produced fields. For a coupling of the form $(g^2/2)\,\phi^2\,N^2$ in the scalar potential, resonant amplification of the field N already occurs for $g^2 \gtrsim 10^{-8}$ [84], if the mass of N is negligible at the end of inflation, and if a massive inflaton is considered. For a massless inflaton ($V(\phi) = \lambda\,\phi^4/4$), an efficient resonance is present also for much smaller values of g (we will show this explicitly in section 5.2), since in this case the resonance is not halted by the expansion of the Universe [85].

[1] We will use N as a shorthand for the superfield, its scalar and fermionic component. We explicitly refer to the particle type when this could cause a confusion.

[2] A nonperturbative inflaton decay also occurs for hybrid inflation [81, 82, 83]. However, we will not discuss this possibility here.

If the produced particle is very massive ($M_N \gtrsim m_\phi$), the effectiveness of the resonance becomes a highly model dependent issue. A potential of the form $V(\phi) + M_N^2 N^2/2 + g^2 \phi^2 N^2/2$, has been considered in the literature mainly to discuss the production of heavy bosons needed for GUT baryogenesis [86]. Working in the Hartree approximation, it has been found [84] that a resonance is effective only provided the coupling g^2 satisfies $g^2 \gtrsim 10^{-7} (M_N/m_\phi)^4$. Taking into account all the other backreaction effects, a stronger lower bound on g has to be expected [79, 84], since the latter typically limits the growth of the fluctuations amplified by the resonance.

Very different bounds can be expected for different potentials. Consider for example $V(\phi) + (M_N + g\,\phi)^2\, N^2/2$. In this case, due to the high initial amplitude of the inflaton oscillations, the total mass of N can vanish at some discrete points even for a coupling as small as $g^2 \sim 10^{-10}\, (M_N/m_\phi)^2$. Whenever $M_N + g\,\phi = 0$, parametric amplification of N occurs. Thus, the lower bound valid for the previous potential is considerably weakened. Although this second choice of the potential may seem *ad hoc*, we note that it is the one which arises in supersymmetric models if both the r.h. sneutrino mass and interaction with the inflaton are encoded in the superpotential, $W(N) \supset M_N N^2 + g\,\phi\, N^2$. We regard this as a very natural possibility.

The idea of a nonperturbative production associated to the vanishing of the total mass has been applied to leptogenesis in [44]. The analysis of [44] focused on the production of r.h. neutrinos, with a mass term of the form $(M_N + g\,\phi)\,\bar{N}N$. From the results of [44], and from the analytical computations of [87], it can be shown that a sufficient lepton asymmetry is generated if the mass of the r.h. neutrinos is higher than about 10^{14} GeV, and if their coupling to the inflaton satisfies $g \gtrsim 0.03$ (we will derive these bounds in section 5.4). Here we note that this high coupling can in principle destabilize through quantum effects the required flatness of the inflaton potential. This, in addition to the strong hierarchy between the r.h. neutrino mass and the electroweak scale, motivates the study of the supersymmetrized version of the mechanism proposed in [44].

One of our aims is to show explicitly that, in the supersymmetrized version of the above model, the nonperturbative production of the r.h. sneutrinos is much more efficient than the one of the neutrinos. Due to supersymmetry, the inflaton couples with the same strength both to the r.h. neutrinos and to the sneutrinos, so that if the former are produced at preheating this will also occur for the latter. However, while production of fermions is limited by Pauli blocking, the production of scalar particles at preheating is characterized by very large occupation numbers. This high production has typically a big impact on the dynamics of the inflaton field. The most immediate backreaction effect is the generation of an effective potential for the zero mode of the inflaton. This effective potential, taken into account in the Hartree approximation [79], is typically comparable with or even dominant over the tree level potential $V(\phi)$. There are however two equally important effects which are beyond the Hartree approximation. The first is due to the scatterings of the produced quanta against the zero mode of the inflaton. This destroys the coherence of the oscillations, thus ending the resonant production characterizing the early stage of preheating [79]. The second is the amplification of all the other fields to which the produced quanta are coupled. This is a very turbulent process, dominated by the nonlinear effects caused by the very high occupation numbers of the fields involved. As a result, all these mutually interacting fields are left with highly excited spectra far from thermal equilibrium [88]. Both these effects are denoted as *rescattering* [84]. [3]

Rescattering strongly affects some of the outcomes of the analytical studies of preheating of bosons, which hardly go beyond the Hartree approximation. For this reason, the results presented in our work are obtained with numerical simulations on the lattice. More precisely, the code "LATTICEEASY" [89], by G. Felder and I. Tkachev, has been used (details are given in section 5.4). Full numerical calculations on the lattice are however rather extensive. We have found that the necessary computing time is reduced in the conformal case, that is with the inflaton potential $V(\phi) = \lambda\,\phi^4/4$, and with a r.h. sneutrino mass which is negligible during the early stages of preheating. For this reason, in our computations we fixed $M_N = 10^{11}$ GeV, which is smaller than the Hubble parameter during inflation, but still high enough to require a nonthermal production of the sneutrinos. The numerical results show a very efficient production of r.h. sneutrinos and inflaton quanta at preheating/rescattering. Even for a coupling inflaton-sneutrino as small as $g^2 \sim$ few $\times\, 10^{-12}$, the produced quanta come to dominate the energy density of the Universe already within about the first 5 e-folds after the end of inflation. In particular, the energy density stored in sneutrinos is typically found to be a fraction of order one of the total energy density, so that a sufficient leptogenesis is easily achieved at their decay.

R.h. sneutrinos are coupled to Higgs fields and left handed (l.h.) leptons through the superpotential term $h\,N\,H\,L \subset W$ (responsible for the Yukawa interaction which provides a Dirac mass to the neutrinos). Thus, one may expect that quanta of the latter fields are amplified by the rescattering of the r.h. sneutrinos produced at preheating. We study this possibility in section 5.5, showing that indeed the amplification occurs for a wide range of values of the coupling h. Part of the analysis follows the detailed discussion on rescattering given in [88], where the numerical code [89] used here was also employed. However, the analysis of [88] is focused on the production of massless particles, while we show that the non vanishing mass of the sneutrinos can have some interesting consequences. More precisely, when the sneutrino quanta become non relativistic (let us denote by $\hat{\eta}$ the time at which this happens) their

[3] In some works, the term *rescattering* refers only to the scatterings of the produced quanta against the zero mode of the inflaton. Here we keep the original meaning given in [84].

rescattering effects become much less efficient. Thus, a strong amplification of the MSSM fields at rescattering can take place only if the coupling h is sufficiently large so that the amplification occurs before $\hat{\eta}$. As a consequence, for massive sneutrinos and for small values of h, the number of MSSM quanta produced at rescattering is an increasing function of h. However, the production is actually *disfavored* when the coupling h becomes too high. This is simply due to energy conservation, since the energy associated to the interaction term between the sneutrino and the MSSM fields cannot be higher than the energy initially present in the sneutrino distribution (equivalently, one can say that, for a too high coupling h, the non vanishing value of the sneutrinos gives a too high effective mass to the MSSM fields, which prevents them from being too strongly amplified). Posing quantitative bounds on the coupling h would require some better (analytical) understanding of the details of rescattering than we presently have. However, the numerical results shown in section 5.5 may give an idea of the expected orders of magnitude.

An important remark is in order. When we speak about the amplification of MSSM fields coupled to the r.h. sneutrinos we have actually in mind amplification of $D-$flat directions (let us generally denote them by X). Indeed, $D-$terms provide a potential term of the form $\Delta V \sim g_G^2 |Y|^4$ for any scalar non flat direction Y. Since g_G is a gauge coupling ($g_G = O\left(10^{-1}\right)$), we expect such terms to prevent a strong amplification of Y, again from energy conservation arguments. Another important issue which emerges when gauge interactions are considered is whether gauge fields themselves are amplified at rescattering. We believe that, at least in the model we are considering, also the amplification of gauge fields will be rather suppressed. The scalar distributions amplified at rescattering break much of the gauge symmetry of the model. This gives the corresponding gauge fields an effective mass in their dispersion relation (analogous to the thermal mass acquired by fields in a thermal bath) of the order $m^2 \sim g_G^2 \langle X^2 \rangle$. As we extensively discuss in the Chapter, in the class of models we are considering the nonthermal distributions formed at rescattering are characterized by a typical momentum several orders of magnitude smaller than this mass scale. For this reason, one can expect that such heavy gauge fields cannot be strongly amplified. [4] In our opinion, an explicit check of these conjectures by means of numerical simulations could be of great interest, especially considering the great importance that gauge fields could have for the thermalization of the scalar distributions.

To conclude, we discuss the production of gravitino quanta from the scalar distributions generated at rescattering. We already mentioned that in order to avoid a thermal overproduction of gravitinos an upper bound has to be set on the reheating

[4] Gauge fields which are not coupled to the fields generated at rescattering will not acquire this high effective mass. However, being uncoupled, they will not be amplified either.

temperature T_{RH} of the thermal bath, $T_{RH} \lesssim$ few $\times\, 10^{10}$ GeV [40]. The requirement of a low reheating temperature can be seen as the demand that the inflaton decays sufficiently late, so that particles in the thermal bath have sufficiently low number densities and energies when they form. If $H \simeq 10^{12}$ GeV at the end of inflation, and if the scale factor a is normalized to one at this time, the generation of the thermal bath cannot occur before $a \simeq 10^7$. Gravitino overproduction is avoided by the fact that in the earlier times most of the energy density of the Universe is still stored in the coherent inflaton oscillations. On the contrary, we have already remarked that preheating/rescattering lead to a quick depletion of the zero mode in the first few e-folds after the end of inflation. [5]

The question whether also the distributions formed at rescattering may lead to a gravitino problem is thus a very natural one, and section 5.6 of the chapter is devoted to some considerations on this regard. [6] To provide at least a partial answer to this question, we distinguish the period during which rescattering is actually effective from the successive longer thermalization era. The computation of the amount of gravitinos produced during the earlier stages of rescattering appears as a very difficult task. The numerical simulations valid in the case of bosonic fields indicate that a perturbative computation (with dominant $2 \to 2$ scatterings taken into account) can hardly reproduce the numerical results, and that probably $N \to 2$ processes ($N > 2$) have also to be taken into account (we discuss this point in more details in section 5.5). It is expected that the same problem will arise also for the computations of the quanta of gravitinos produced by the scalar distributions which are being forming at this stage. The end of rescattering/beginning of the thermalization period is instead characterized by a much slower evolution of the scalar distributions. In particular, the total occupation number of all the scalar fields is (approximatively) conserved, which is interpreted [88] by the fact that $2 \to 2$ processes are now determining the evolution of their distributions. Motivated by this observation, we assume that $2 \to 2$ interactions are also the main source of production for gravitinos from this stage on.

[5]The situation is even more enhanced for hybrid inflationary model, in which the energy density of the zero modes of the scalars gets dissipated within their first oscillation [81, 82, 83].

[6]We acknowledge very useful discussions with Patrick B. Greene and Lev Kofman on this issue.

Mechanism	N Yukawa h	N mass	ϕ-N coupling
Thermal	$10^{-5}\text{eV} < \tilde{m}_1 < 10^{-3}\text{eV}$	$10^9\text{GeV} \lesssim M_1 \lesssim T_{RH}$	irrelevant
Affleck–Dine	$10^{-9}\text{eV} < m_{\nu_1} < 10^{-4}\text{eV}$	$M_i < H_{\text{infl}}$	$\begin{cases} M_i^{\text{eff}} < H_{\text{infl}} \\ \left(M_i^{\text{eff}}\right)^2 < 0 \end{cases}$
Pert.ϕ decay	$\Gamma_{LV} < H(\tau_i)$	$\left\{ \begin{array}{c} M_i < m_\phi/2 \\ M_i > m_\phi/2 \end{array} \right\}$	$\text{Br}(\phi \to N_i N_i) \sim 1$ $\text{Br}(\phi \to N_i^* N_i^*) \sim 1$
N preheating eq. (5.3)	$\Gamma_{LV} < H(\tau_i)$	$M_i \gtrsim 10^{14}\,\text{GeV}$	$g_i \gtrsim 0.03$
$\tilde{N}$ preheating/resc. eq. (5.8)	$\Gamma_{LV} < H(\tau_i)$	$M_i \lesssim g_i\, 10^{17}\,\text{GeV}$	$g_i \gtrsim \sqrt{\lambda}$

Table 5.1: Summary of parameters for which leptogenesis could work, for different r.h. (s)neutrino production mechanisms. In (s)neutrino production mechanisms which do not require the Yukawa coupling (non-thermal mechanisms), the constraint on the Yukawa matrix is that lepton number violating interactions in the thermal soup be out of equilibrium after the r.h. (s)neutrinos decay at τ_i. This also implies $M_i > T(\tau_i)$, and possibly additional constraints on L violating processes mediated by $N_j, j \neq i$. Recall $\tilde{m}_i$ parametrises the N_i decay rate, and is defined after eqn. (5.1). The Affleck–Dine mechanism proceeds through generation of large expectation values either for a small [120] or a tachyonic [77] effective mass of sneutrinos M^{eff} during inflation. The asymmetry made by the perturbative decay of the inflaton can be generated by the on-shell r.h. (s)neutrinos [108], which subsequently decay, or by the decay via off-shell r.h. (s)neutrinos (N_i^*) to Higgses and leptons [109]. The properties of nonperturbative (s)neutrino production analysed in the present chapter is summarized in the last two lines (λ is the inflaton self-coupling). Other scenarios for nonperturbative production after the end of inflation can be envisaged, with model–dependent results.

In the thermal case, the gravitino production is dominated by processes having a gravitationally suppressed vertex (from which the gravitino is emitted) and a second vertex characterized by a gauge interaction with one outgoing gaugino. However, we believe that in the present context these interactions will be kinematically forbidden, due to the high effective mass-squared that gauginos acquire from their interaction with the scalar distributions (the argument follows the one already given for gauge fields). Once again we notice that the system is still effectively behaving as a condensate: the number densities of the scalar distributions are set by the quantity $\sqrt{\langle X^2 \rangle}$, which is much higher than the typical momenta of the distributions themselves. This generates a high effective mass for all the particles "strongly" coupled to these scalar fields. A further comparison with the case of a thermal distribution may be useful: in the latter case both the typical momenta and the effective masses are set by the only energy scale present, namely the temperature of the system. As should be clear from the above discussion (see also [88]), the thermalization of the distributions produced at rescattering necessarily proceeds through particle fusion. Only after a sufficiently prolonged stage of thermalization, will the system be sufficiently close to thermodynamical equilibrium so to render processes as the one discussed above kinematically allowed.

In section 5.6 we show that if this class of processes is indeed kinematically suppressed, the production of gravitinos from the distributions formed at rescattering is sufficiently small. However, we remark that this analysis still leaves out the gravitino production which may have occurred at the earlier stages of the rescattering period. Whether this production may be sufficiently strong to overcome the limits from nucleosynthesis remains an open problem.

Let us finally summarize the plan of the chapter. In section 5.2 we discuss leptogenesis with a thermal production of the r.h. (s)neutrinos. Leptogenesis with a nonthermal production of r.h. neutrinos is reviewed in section 5.3. The supersymmetric version of this model is presented in section 5.4, where we study the nonthermal production of sneutrino quanta. Section 5.5 is devoted to the amplification of the MSSM $D-$flat directions due to the rescattering of the r.h. sneutrino quanta. The discussion on the gravitino production is presented in section 5.6, apart from a few technical details which can be found in the appendix A.

5.2 Thermal N_i production

We now consider the case where the lightest r.h. (s)neutrino N_1 is thermally produced after T_{RH} [7]. With hierarchical r.h. neutrino masses, one can typically assume the

[7]We assume in this work that T_{RH} is "large"; for a discussion of baryogenesis in low-T_{RH} models, see *e.g.* [106].

lepton asymmetry to be produced by the decay of the lightest r.h. (s)neutrino N_1. As we shall see, this is a self-consistent assumption, because M_2 and M_3 will turn out to be larger than T_{RH}. To generate a lepton asymmetry, the decay of the N_1 should proceed out of equilibrium. More quantitatively, the ratio of the thermal average of the N_i decay rate and of the Hubble parameter at the temperature $T \simeq M_i$,

$$\frac{\Gamma_{N_i}}{2\,H}|_{T=M_i} \equiv \frac{\tilde{m}_i}{2 \times 10^{-3}\,\mathrm{eV}}\,, \tag{5.1}$$

should be less than unity to have an unambiguously out-of-equilibrium decay. The parameter $\tilde{m}_i$ is defined as $\Gamma_i \langle H\rangle^2/M_i^2$, where $\langle H\rangle$ is the Higgs vev. However, $\tilde{m}_1$ cannot be taken too small if N_1 is produced thermally [66]. Indeed the quantity $\tilde{m}_1$ controls the strength of the interactions of the N_1 with MSSM degrees of freedom, and an efficient thermal production via Yukawa interactions typically requires $\tilde{m}_1 \gtrsim 10^{-5}$ eV. To account for both these effects, $\Gamma_{N_1} \sim H(T = M_1)$ should be taken, so the decay is only barely out of equilibrium, and the final lepton asymmetry has to be computed by integrating the full set of relevant Boltzmann equations [64, 65]. These computations show that a significant portion of the lepton asymmetry is erased by lepton-number violating processes, and that only a fraction $\kappa \lesssim .1$ or less typically survives. Starting from N_1 in thermal equilibrium at $T > M_1$, and collecting all the above informations, the final baryon asymmetry can be estimated to be

$$Y_B \simeq 10^{-10}\left(\frac{M_1}{10^{10}\,\mathrm{GeV}}\right)\left(\frac{m_3}{0.05\,\mathrm{eV}}\right)\left(\frac{\kappa}{0.1}\right)\delta_{\mathrm{CP}}\,. \tag{5.2}$$

This expression has to be compared with the baryon asymmetry required by Big Bang Nucleosynthesis $Y_B \equiv N_B/s \sim (1.7 - 8) \times 10^{-11}$ [107, 59]. As we have anticipated, we see that thermal leptogenesis can be a viable mechanism if the mass of the r.h. neutrino N_1 is sufficiently high. From eqn. (5.2) we find the lower bound $M_1 \gtrsim (10^9 - 10^{10})$ GeV, although it is fair to say that higher values are required if the bound (3.15) is not saturated. If N_1 is generated from the thermal bath, a reheating temperature greater than M_1 is required. In the most favorable case, the required value is only marginally compatible with the bound imposed by gravitino overproduction from the thermal bath [40]. To overcome the potential conflict between the gravitino bound and the requirement of a reheating temperature high enough for leptogenesis, the possibility of producing right-handed neutrinos *non thermally* has been envisaged [44]. The following considerations will be focused on this framework.

5.3 Non thermal production of right handed neutrinos

Many alternatives to thermal (s)neutrino production have been considered in the literature. We concentrate here on mechanisms that involve a direct coupling of N to the inflaton, although in the next section we will comment on differences and similarities with the Affleck-Dine mechanism. The strength of this interaction, relative to the inflaton coupling to other degrees of freedom, is a free parameter; for appropriate values, a lepton asymmetry of the correct magnitude can be produced. The number density of r.h. (s)neutrinos will also depend on the evolution of the inflaton between the end of inflation and reheating. If the inflaton decays perturbatively, right-handed neutrinos with masses less than half the inflaton mass could be produced in the decay [108]. For heavier r.h. neutrinos, one can also envisage the possibility that a sufficient leptogenesis is generated in processes in which they mediate a perturbative inflaton decay [109]. In both cases, the final lepton asymmetry will be proportional to the branching ratio of the inflaton into (either on- or off-shell) neutrinos. A branching ratio of order one is typically required. Right-handed neutrinos with masses greater than that of the inflaton can be produced at preheating, if their interaction with the inflaton is strong enough. The production of heavy fermions (sneutrinos are discussed in the next section) in an expanding Universe was first discussed in ref. [44] (fermionic production in the conformal case was first studied in [110]), where a direct Yukawa coupling to the inflaton ϕ was considered, and the simplest chaotic inflationary scenario with a massive inflaton, $V(\phi) = m_\phi^2\,\phi^2/2$, $m_\phi \simeq 10^{13}$ GeV, was assumed. The relevant part of the lagrangian is

$$\mathcal{L}_{N,\phi} = \bar{N}\;(M + g\,\phi)\;N\;, \tag{5.3}$$

where N is any one of the r.h. neutrinos. We assume that only one r.h. neutrino generation plays an important role in the generation of a lepton asymmetry, and therefore we drop the r.h. neutrino generation index for the remainder of this section. The generalization of the following analysis to three generations is straightforward, at least as long as the r.h. neutrino–inflaton coupling matrix g is diagonal in the r.h. neutrino mass basis (otherwise, the formalism of [111] should be used). After the end of inflation, the inflaton condensate ϕ oscillates about the minimum of its potential with amplitude of a fraction of the Planck mass $M_P \simeq 1.22\cdot 10^{19}\,\mathrm{GeV}$. The total effective mass of the fermion $M + g\,\phi\,(t)$ varies non adiabatically in time, and this leads to a (non perturbative) production of quanta of N. In particular, fermion production at preheating occurs whenever the total effective mass crosses zero. As a consequence, fermions with a mass up to

$$M_{\rm max} \simeq 5\left(\frac{q}{10^{10}}\right)^{1/2} \times 10^{17}\,\mathrm{GeV},\; q \equiv \frac{g^2\phi_0^2}{4m_\phi^2} \simeq 3g^2\,10^{10} \tag{5.4}$$

can be produced [44], irrespective of the value of the reheating temperature of the thermal bath which is formed at later times. The abundance of neutrinos produced at preheating has been computed analytically [87], and it is most conveniently given in terms of the ratio

$$\frac{N_N}{\rho_\phi} \simeq \frac{1}{10^{10}\,\mathrm{GeV}} \frac{1.4 \times 10^{-14}\, q}{M_{10}^{1/2}} \left[\log\left(1.7 \times 10^3\, \frac{q^{1/2}}{M_{10}}\right)\right]^{3/2}, \tag{5.5}$$

where we have defined $M_{10} = M/\left(10^{10}\,\mathrm{GeV}\right)$. The above formula is valid as long as the backreaction of the produced neutrinos on inflaton dynamics is negligible, as it turns out to be the case as long as $q \lesssim 10^8$ [44, 87]. For larger values of q, the effectiveness of preheating increases (by a factor up to about 1.5), and the above equation gives just a lower bound on N_N. In what follows, we will conservatively assume $q < 10^8$. [8] For a massive inflaton, the ratio (5.5) is constant until the inflaton condensate decays. If neutrinos decay before reheating has completed, the resulting baryon asymmetry reads

$$\begin{aligned} Y_B &= \frac{8}{23} Y_{B-L} = \frac{8}{23}\left(-\epsilon\, N_N\, \frac{3}{4}\, \frac{T_{\mathrm{RH}}}{\rho_\phi}\right) \\ &= 4 \cdot 10^{-14}\, \frac{q}{10^8}\, M_{10}^{1/2}\, \frac{T_{\mathrm{RH}}}{10^9\,\mathrm{GeV}}\, \frac{m_{\nu_3}}{0.05\,\mathrm{eV}}\, \delta_{CP} \left[\mathrm{Log}\left(1.66 \cdot 10^3\, \frac{q^{1/2}}{M_{10}}\right)\right]^{3/2} \end{aligned} \tag{5.6}$$

The ratio of N_N to the entropy is constant after reheating, unless it decreased due to some subsequent entropy production [9]. For a given value of the parameter q, the baryon asymmetry (18) is maximized when the mass M has its largest possible value (16). For $T_{\mathrm{RH}} \simeq 10^9$ GeV, $m_{\nu_3} = 0.05$ eV and $\delta_{CP} = 1$, imposing the condition (16) on eqn. (18) it is possible to see that the generation of the observed baryon asymmetry requires g larger than about 0.03. In particular, if we assume $q \simeq 10^8$, then the observed baryon asymmetry can be obtained for a mass of the right–handed neutrino of the order of 10^{14}–10^{15} GeV. Production of r.h. neutrinos at preheating can generate a large enough lepton asymmetry from neutrinos with $M_N > T_{RH}$, so constraints on T_{RH} do not translate into bounds on M_N. We notice however that the large inflaton-neutrino coupling required ($g \gtrsim 0.03$) can in principle modify through quantum effects the small mass or self coupling parameters characterizing the inflaton potential. This constitutes a further motivation for considering supersymmetric models, as we do in the remaining of this work. We will see that the production of the scalar partners of the r.h. neutrinos can significantly affect some of the above considerations.

[8] One should also consider the perturbative decay of the inflaton quanta. Comparing eqn. (5.5) with the number of r.h. neutrinos produced perturbatively in one inflaton oscillation (i.e. the typical timescale for preheating), one can however see that the perturbative production is subdominant when kinematically allowed.

[9] For instance, this is the case if the r.h. neutrinos lifetime is long enough for them to come to dominate the energy density in the Universe, after reheating has completed.

5.4 Supersymmetric see-saw and nonthermal production of right handed sneutrinos

As we have seen in the previous section, preheating can have very important consequences for leptogenesis through the production of r.h. neutrinos [44]. In supersymmetric extensions of the see-saw model, the production of the supersymmetric partners of the neutrinos is even more important. Due to supersymmetry, the inflaton couples with the same strength both to the r.h. neutrinos and to the sneutrinos, so that if the former are produced at preheating this will also occur for the latter. However, while production of fermions is limited by Pauli blocking, the production of scalar particles at preheating is characterized by very large occupation numbers. As a consequence, the production of r.h. sneutrinos can be expected to be more significant than the one of neutrinos, as the numerical results presented below confirm. Production of particles at preheating gives very model dependent results; nevertheless, some general features can be outlined, and the whole process can be roughly divided into three separate stages. The first of them is characterized by a very quick amplification of the fields directly coupled to the inflaton (and of the inflaton field itself, in the case of a sufficiently strong self-interaction) to exponentially large occupation numbers [79]. Very rapidly, the system reaches a stage in which the backreaction of the produced quanta, customarily denoted as rescattering [84], plays a dominant role. In the case of parametric resonance, the scatterings of the quanta against the zero mode of the inflaton destroy the coherence of the oscillations, thus ending the resonant production characterizing the early stage of preheating [79]. An equally important backreaction effect is the amplification of all the other fields to which the produced quanta are coupled. This is a very turbulent process, dominated by the nonlinear effects caused by the very high occupation numbers of the fields involved. As a result, all these mutually interacting fields are left with highly excited spectra far from thermal equilibrium. The latter is actually achieved on a much longer timescale, through an adiabatic (slow) evolution of the spectra, which characterizes the third and final stage of the reheating process. [10] The first stage of preheating is well understood. Particle production is computed in a semi-classical approximation (for a rather general formalism in the case of several coupled fields see [111]), and analytical solutions have been obtained in a broad class of models [78, 79, 117, 85, 110, 87]. Analytical approximations break down when nonlinear processes become dominant. However, the high occupation numbers of the scalar fields involved allow a classical

[10] The thermalization of this system is a very interesting issue, which however we do not discuss in this work - see [88] for a more detailed study. Since in this case thermalization proceeds via particle fusion, an important role may be played by three or five point vertices, which shorten perturbative estimates of the thermalisation timescale [112] (other recent discussions on thermalization can be found in [113, 114, 115, 116]).

Inflaton vev at the end of inflation	$\phi_0 \simeq M_P/3$
Inflaton self-coupling (eqn. (5.7))	$\lambda \simeq 9 \times 10^{-14}$
Neutrino-inflaton coupling (eqn. (5.7))	$\tilde{g} = g^2/\lambda$
Neutrino Yukawa coupling (eqn. (5.15))	$\tilde{h} = h^2/\lambda \ \ (\leftrightarrow \tilde{m} = \frac{h^2 \langle H_u \rangle^2}{M})$
X number density (eqn. (5.10))	$N_{c,X} = [\text{comov. num. den.}]/[\sqrt{\lambda}\phi_0]^3$
X "mass" (eqn. (5.9))	$m^2_{\text{eff},X} = \left(a^2 \langle \frac{\partial^2 V}{\partial \phi_i^2} \rangle - \frac{a''}{a} \right) /[\sqrt{\lambda}\phi_0]^2$

Table 5.2: Translation table between quantities in plots and superpotential parameters (we give the eqn. where the parameter is defined). Recall that η is the conformal time coordinate, $\eta = 0$ at the end of inflation, and subsequently the scale factor is $a(\eta)/a(0) \simeq \eta/2 + 1$.

study of the system. Indeed, in the limit of high occupation numbers quantum uncertainties become negligible, and quantum probabilities show a classical (deterministic) evolution [118]. The latter can be better computed by means of lattice simulations in position space [84, 89, 83], where all the effects of backreaction and rescattering are (automatically) taken into account. A detailed discussion of rescattering and of the approach to thermal equilibrium has been given in [88], where the code "LATTICEEASY" [89], by G. Felder and I. Tkachev, has been used. The numerical results presented in this paper are also obtained with this code. For numerical convenience, we consider a chaotic inflationary scenario with a quartic potential for the inflaton. More specifically, we focus on the superpotential [11]

$$W(\Phi, N) = \frac{\sqrt{\lambda}}{3} \Phi^3 + \frac{1}{2} \left(\sqrt{2}\, g\, \Phi + M \right) N^2 \,. \tag{5.7}$$

The second term of W reproduces the lagrangian (5.3) for the r.h. neutrinos. We denote the scalar components of the inflaton and of the r.h. neutrinos multiplets with ϕ and N, respectively. To simplify the numerical computations, the imaginary components of the scalar fields will be neglected. Therefore, after canonical normalization,

[11] To embed the system in a supergravity context while preserving a flat potential for the inflaton field, one may impose [119] a definite parity for the Kähler potential $\mathcal{K} = \mathcal{K}(\Phi - \Phi^*)$. Doing so, the inflaton is identified with the real direction of the scalar component of Φ, and supergravity corrections can be neglected.

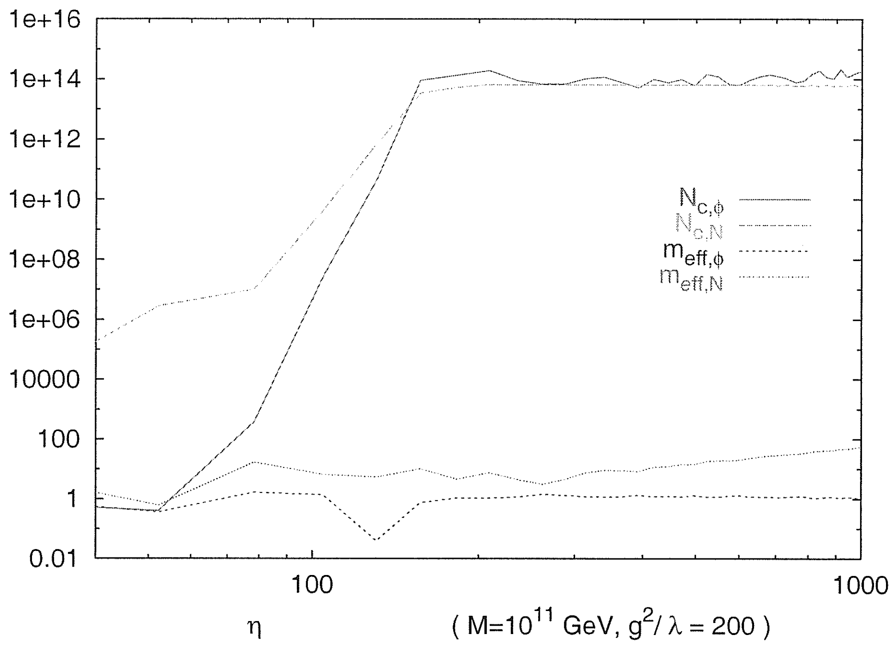


Figure 5.1: Time evolution of the comoving number density and of the effective masses of the inflaton ϕ and of the r.h. sneutrino N. See table 5.2 for notation.

$\phi \rightarrow \phi/\sqrt{2}$, $N \rightarrow N/\sqrt{2}$, we consider the scalar potential [12]

$$V_{\rm scalar} = \frac{\lambda}{4}\,\phi^4 + \frac{1}{2}\,(g\,\phi + M)^2\,N^2\;. \tag{5.8}$$

The size of the temperature fluctuations of the Cosmic Microwave Background sets $\lambda \simeq 9 \times 10^{-14}$, while the neutrino mass M as well as the coupling g to the inflaton are model dependent parameters. The case $M = 0$ is analyzed in detail in [88]. In figure 5.1 we show the time evolution of the comoving number density and of the comoving effective mass of the two scalars ϕ and N, for the particular choice of the parameters $M = 10^{11}$GeV, $\tilde{g} \equiv g^2/\lambda = 200$. The effective mass is defined as

$$m^2_{{\rm eff},\phi_i} \equiv a^2 \langle \frac{\partial^2 V}{\partial \phi_i^2} \rangle - \frac{a''}{a}\;, \tag{5.9}$$

where a is the scale factor of the Universe, normalized to one at the end of inflation, prime indicates derivative with respect to the conformal time η, while $\langle \ldots \rangle$ denotes

[12]We neglect the term quartic in N, subdominant with respect to the mass term $M^2\,N^2$ during most of the preheating/rescattering period, as well as the mixed term $\propto \sqrt{\lambda}\,g\,\phi^2\,N^2$, negligible with respect to the mixed term present in eqn. (5.8) for $g^2 \gg \lambda$. One may also be worried that, if the right–handed sneutrino is charged under some gauge group (as it generally happens in grand unified models) with a gauge coupling g_G not much smaller than one, the corresponding $D-$term $\propto g_G^2\,|N|^4 \subset V$ could prevent the amplification of N at preheating-rescattering. However, at least as long as $\langle N \rangle$ is smaller than the scale at which the gauge symmetry is broken, this term gets actually compensated by a shift of the (much heavier) field responsible for the breaking of the symmetry, and it is thus absent from the effective potential for N [120, 76].

average over the sites of the lattice. The term a''/a appears in eqn. (5.9) because we are considering minimally (rather than conformally) coupled scalars, and it vanishes in a radiation–dominated background. The comoving number density is defined as the integral over momentum of the "occupation number"

$$\begin{aligned} n_k(\eta) &\equiv \frac{1}{2}\left(\omega_k\,|f_k|^2 + \frac{1}{\omega_k}\,|f_k'|^2\right)\,, \\ \omega_k^2 &\equiv k^2 + m_{\rm eff}^2\,, \end{aligned} \tag{5.10}$$

where f_k denotes the Fourier transform (to be evaluated on the lattice) of the rescaled field $a\,\phi$. By definition [89], the quanta stored in the oscillating inflaton condensate do not contribute to $N_{c,\phi}$ in figure 5.1. The three quantities $m_{\rm eff}$, $N_{\rm c}$, and η are all shown in units of $\sqrt{\lambda}\,\phi_0 \simeq 1.25\cdot 10^{12}\,$GeV, with $\phi_0 \simeq 0.342\,M_P$ denoting the value of ϕ at the end of inflation, to the appropriate power. All the numerical results presented in this work are obtained with a two dimensional lattice of size $L = 20\left(\sqrt{\lambda}\,\phi_0\right)^{-1}$ and with $N = 1024^2$ sites (see [89] for details). Figure 5.1 exhibits the features that we have outlined at the beginning of this section, namely a quick stage of exponential growth of the occupation numbers followed by a period in which the occupation numbers are nearly constant. During the first stages of the process, the results presented reproduce very well the ones obtained in [88] for $M = 0$, since the "bare" mass of the r.h. neutrinos is initially negligible. However, the presence of a non vanishing bare mass affects the subsequent evolution of the system. Indeed, when the value of the fluctuations in the sneutrino field becomes comparable with the amplitude of inflaton oscillations, the second term in eqn. (5.8) shifts the minimum of the effective potential of the inflaton, giving it an effective mass that is roughly constant in comoving units. A stronger effect is related to the fact that the sneutrino itself is massive. In rescaled units and for the present choice of the parameters, we have

$$m_{\rm eff,N}{}^2 \simeq \left\langle\left(0.08\,a + 14.1\,\frac{a\,\phi}{\phi_0}\right)^2\right\rangle\,. \tag{5.11}$$

Numerical results show that $\langle\phi\rangle \sim \phi_0/a$ (as one could also see by inspecting the potential (5.8) in Hartree approximation), so that the part in the above equation that depends on ϕ remains of order one. It follows that the r.h. neutrino mass M dominates the effective mass (5.11) for $\eta \geq \hat{\eta} \sim 350$, as clearly indicated by the growth of $m_{\rm eff,N}$ visible in figure 5.1 for $\eta > \hat{\eta}$. Production of sneutrinos at preheating in the present model is strictly related to the production of fermions we have analysed in section 2.3 . In particular, production occurs whenever the effective neutrino mass (5.11) crosses zero. Numerical results show that preheating is terminated by rescattering effects when the scale factor a is of the order of $a_{\rm resc} \simeq 100$. As a consequence, sneutrinos with a bare mass up to $g\,\phi_0/a_{\rm resc} \simeq g\cdot 10^{17}\,$GeV will be efficiently produced at preheating

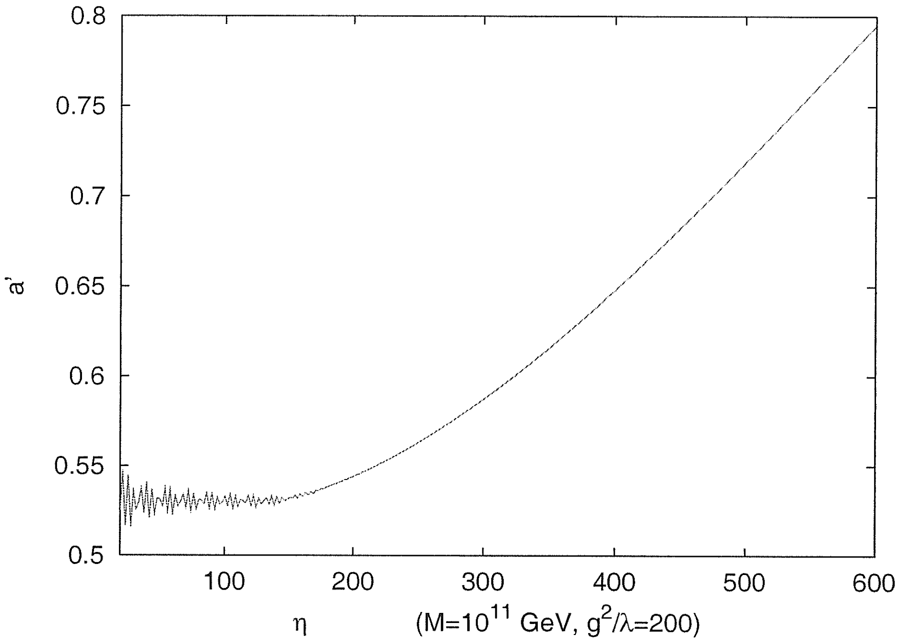


Figure 5.2: Time evolution of the derivative of the scale factor with respect to conformal time. A constant value indicates radiation domination, while a linear growth indicates matter domination.

and will constitute a sizable fraction of the background energy [13]. Numerical results show that after the onset of rescattering, the energy density gets roughly equiparated between the quanta of the two species. As a consequence, in general large couplings correspond to a large interaction energy, and therefore to a smaller number density during the rescattering/thermalization stage [84, 88]. In particular, for this reason a large quartic self–coupling $g_{\rm N}^2\ |N|^4$ for the sneutrino would prevent it from getting large occupation numbers, since energy conservation would impose $\langle N^2\rangle \propto \sqrt{\lambda/g_{\rm N}^2}$. Numerical results also indicate that soon after the beginning of rescattering most of the r.h. neutrino quanta have a momentum of the order $k_* \sim 15\,\sqrt{\lambda}\,\phi_0$. [14] Thus, most of the r.h. neutrinos become non relativistic at a time not much greater than $\hat{\eta}$. From this time on, the energy density of the system redshifts as the energy density of matter. The transition between the two stages of matter and radiation domination is clearly visible in figure 5.2, where we show the time evolution of the derivative of the scale factor with respect to conformal time. As long as the neutrino mass is negligible,

[13]Non-adiabatic production of sneutrinos can occur for bare masses as large as $g\,\phi_0/4 \sim g \cdot 10^{18}$ GeV, but the efficiency of the process will be much lower, because redshift effects will terminate the resonance before rescattering sets in.

[14]The precise value of the typical momentum k_* of the distributions, as well as the time needed for N_c to saturate, are a nontrivial function of $\tilde{g}$, since different values of this parameter lead to different positions (in momentum space) and strengths of the resonance bands [85]. However, the rescattering stage destroys these resonance bands, making the dependence of k_* on $\tilde{g}$ milder.

the energy density of the system redshifts as the one of radiation [85, 88], and the evolution of the scale factor is very well approximated by $a \simeq (1+t)^{1/2} \simeq \eta/2+1$, where we have set $t=\eta=0$ at the end of inflation. Therefore, during the initial stage of radiation domination, a' is constant. In the following matter dominated stage $a \propto \eta^2$, and a' grows linearly with time. To estimate the baryon asymmetry produced from the decay of the r.h. sneutrinos, we need to know the fraction of the entropy of the Universe that is generated in the decay. The baryon asymmetry will be [67]

$$\begin{aligned} Y_B &\simeq -\epsilon \frac{8}{23} \frac{N_N}{s_N} \frac{s_N}{s_{tot}} \simeq \frac{8}{23} \left(-\epsilon \frac{3}{4} \frac{T_N}{M} \right) \frac{s_N}{s_{tot}} \\ &\simeq 0.3 \times 10^{-10} \left(\frac{T_N}{10^6\,\mathrm{GeV}} \right) \left(\frac{m_3}{0.05\,\mathrm{eV}} \right) \frac{s_N}{s_{tot}} \delta_{\mathrm{CP}} , \end{aligned} \tag{5.12}$$

where $s_N \propto g_*(T_N) T_N^3$ is the entropy produced in N decay, and s_{tot} is the total entropy of the Universe. If the r.h. sneutrinos dominate the Universe when they decay, then T_N is the temperature to which the Universe is reheated by the decay of the sneutrinos N, and $s_{tot} \sim s_N$. This condition is satisfied if the inflaton mainly decays only into one r.h. sneutrino family (as it is clearly the case in the one generation model we have studied numerically). However, if the inflaton couples to other scalars (also in particular to the other generations of sneutrinos), these could produce additional entropy. At variance with the case of leptogenesis induced by the decay of right–handed neutrinos, analysed in section 2.3, the baryon asymmetry (5.12) does not depend on the r.h. (s)neutrino mass, that must only satisfy $M > T_N$ in order to prevent thermal regeneration of the r.h. sneutrinos after their decay. This is due the fact that, thanks to Bose statistics, r.h. sneutrinos can get large occupation numbers at preheating (whereas Pauli blocking makes fermion production less efficient), and they can easily represent a substantial fraction of the energy in the Universe. [15] The generalization to the more realistic case of three neutrino families coupled to the inflaton is obtained by considering the following superpotential, in the mass eigenstate basis for the r.h. neutrinos,

$$W = \frac{\sqrt{\lambda}}{3} \Phi^3 + h_{ji}\, L_i \cdot H_u\, N_j + \frac{1}{2} \left(M_j \delta_{jk} + \sqrt{2} g_{jk} \Phi \right) N_k\, N_j . \tag{5.13}$$

This gives a potential for the real components of the scalars

$$V = \frac{\lambda}{4} \phi^4 + \frac{1}{2} \sum_{i,j,k} (g_{ij}\phi + M_i \delta_{ij})(g_{ik}^* \phi + M_i \delta_{ik}) N_j N_k + \ldots \tag{5.14}$$

[15]Notice that for a massless inflaton, the inflaton energy redshifts as radiation, and non-relativistic neutrinos will easily dominate the energy in the Universe. If the inflaton in instead massive, then the energy in sneutrinos would in any case be a fraction of order unity of the background energy, and would start increasing after inflaton decay. As a consequence, the resulting baryon asymmetry would still be given (at most up to factors of order one) by the expression (5.12).

where dots include the terms involving the Yukawa h, which are not relevant for nonthermal N_i generation. We suppose for simplicity that the neutrino–inflaton coupling g_{ij} is diagonal. Energy considerations after rescattering [84, 88] lead to the expectation that the sneutrino family that is most strongly coupled to the inflaton is also the one that will have the smallest number density (clearly, on the assumption that all the sneutrinos are sufficiently coupled to be amplified). This is opposite to the scenario in which sneutrinos are produced by the perturbative decay of ϕ, where N_{N_i} is proportional to the inflaton branching ratio to N_i. However, the presence of the r.h. sneutrino bare masses, as well as the existence of couplings to the Standard Model degrees of freedom (whose effects will be analysed in detail in the next sections), can strongly affect these conclusions. Although the resulting baryon asymmetry (5.12) has the same expression as the one reported in [67], the mechanism that led to a sneutrino dominated Universe is different from the generation of large expectation values considered in ref. [67] or in the Affleck–Dine [120] mechanism. Indeed, the latter is effective if during inflation the sneutrino (or, more generally, the amplified flat direction) has a mass much smaller than the Hubble rate. This requires (besides a sufficiently small bare mass $M \ll H$) that the amplified field does not get a large effective mass through its coupling to the inflaton. It is important to remark that the mechanism we are discussing can provide a sufficient leptogenesis even if the coupling g between the inflaton and the r.h. neutrino multiplets is much smaller than the one needed in non supersymmetric models, i.e. with only the production of the neutrinos taken into account, see eqn. (5.6). Anyhow, even couplings as small as $g^2 \sim 10\,\lambda$ prevent the formation of a large condensate during inflation. Therefore, the two mechanisms can lead to large occupation numbers in complementary regions of the parameter space. Notice that the above discussion applies to every effective mass term that can arise in the potential for a (quasi) flat direction. In particular, it could be interesting to consider the effective mass of the order of the Hubble parameter that is generally induced by supergravity corrections [77], although in this case amplification effects may be weakened by the quick redshift characterizing the nonrenormalizable interactions. If supergravity corrections induce a tachyonic mass $m_{\text{eff}}^2 \simeq -H^2$, a large expectation value will be generated during inflation [77], and the dynamics of preheating will turn out to be rather different from the one considered in the present section. This however requires a suitable nonminimal Kähler potential, and we will not discuss this possibility in this work. Finally, it is worth stressing that both the leptogenesis scenario described in this section and the one considered in [67], although they are related to Affleck–Dine leptogenesis, are somehow different from it for what concerns the fulfillment of the Sakharov CP–violation condition [58]. In the Affleck–Dine scenario, the latter is achieved by the motion of the Affleck–Dine condensate (that requires coherence over many Hubble lengths), while in the mechanism we are analysing, CP–violating sneutrino decays are crucial in the generation of an asymmetry.

5.5 Production of light degrees of freedom at rescattering

The description presented above is further modified by the effects of the coupling of the r.h. neutrino multiplets to the l.h. leptons and Higgses, coming from the superpotential (3.10). Considering for simplicity only one generation, the superpotential (5.7) will be supplemented by

$$\Delta W = h\, N\, L \cdot H \ . \tag{5.15}$$

The resulting scalar potential contains several interaction vertices coming from F–terms. In addition, there are quartic contributions from the D–terms. The presence of the latter in the potential plays an important role for our discussion, since rescattering mostly affects the D–flat directions. To see this, consider the case in which the left–handed selectron and the charged scalar Higgs vanish. The D-terms for the neutral Higgs and sneutrino then take the familiar form of the tree level MSSM Higgs potential, with H_d^0 replaced by $\tilde{\nu}_L$

$$V_D = \frac{g_{\rm SU(2)}^2 + g_Y^2}{8} \left| \left| H^0 \right|^2 - \left| \tilde{\nu}_L \right|^2 \right|^2 \quad . \tag{5.16}$$

The directions that are not D–flat (i.e. the ones for which $|\tilde{\nu}_L| \neq |H^0|$ in the present example) are characterized by a large (gauge) quartic coupling in the scalar potential. Due to this coupling, they cannot be significantly amplified by rescattering effects. On the contrary, D–flat directions have only quartic couplings coming from F–terms as $|\partial W/\partial N|^2$, whose strength h^2 will be typically taken $\ll 1$ in all the cases considered below. Indeed these quartic interactions among the D–flat directions will be neglected in our computation, since they can be relevant only during the thermalization stage, when the variances of these fields have grown to be sufficiently large. The most important F–terms arising from the total superpotential are of the form $\sim h^2\, |N|^2 \left(|\tilde{\nu}_L|^2 + |H^0|^2 \right)$. If we denote by X the D–flat combination $|\tilde{\nu}_L| = |H^0|$, then the relevant coupling of X to the sneutrino field will be simply given by $h^2\, N^2\, X^2$ (as in the previous section, we consider for numerical convenience only real directions). Besides the quartic term $\propto h^2\, X^4$, we will also neglect the interaction term $\propto (g\phi + M)\, N\, X^2 \simeq M\, N\, X^2$, which is responsible for the late time decay of the r.h. sneutrinos (that is, the supersymmetric counterpart of the vertex which gives the decay of the r.h. neutrinos into Higgses and leptons). We thus consider the simplified model characterized by the scalar potential

$$V\left(\phi,\, N,\, X\right) = \frac{\lambda}{4}\, \phi^4 + \frac{1}{2} \left(M + g\, \phi\right)^2\, N^2 + \frac{1}{2}\, h^2\, N^2\, X^2 \ . \tag{5.17}$$

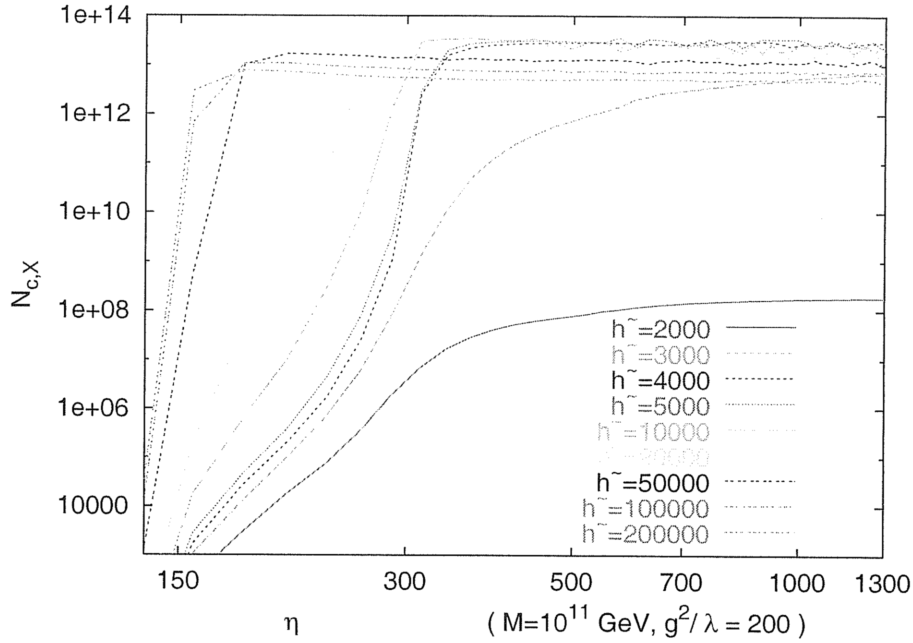


Figure 5.3: Time evolution of the comoving number density of the light quanta X. See table 5.2 for notation.

Neglecting the imaginary directions of the scalar fields, as well as many of the interaction terms, allows a considerable reduction of the computing time needed for the numerical simulations (this is particularly welcome for the extensive computation that we discuss in the next section). The above discussion leads us however to believe that the simplified model should well describe the main features of the preheating and rescattering process for the supersymmetrized see-saw model with a nonperturbative production of the r.h. neutrinos. In figure 5.3 we show the time evolution of the comoving number density of the quanta of X. As in the previous section, we have fixed $M = 10^{11}$ GeV, $\tilde{g} = 200$, while different values of the parameter $\tilde{h} \equiv h^2/\lambda$ are shown. Even if in the simplified model (5.17) the X field is not directly coupled to the inflaton, we see that (for suitable values of the coupling h) it can be highly amplified by the rescattering of r.h. sneutrino quanta. Figure 5.3 shows that the growth of number of X particles is roughly exponential in time. When the effective sneutrino mass is varying non-adiabatically in time and is not negligible with respect to sneutrino typical momenta, the production of X particles cannot be analysed in terms of scatterings of sneutrinos. However, after the end of the parametric resonance period and the onset of rescattering, one can expect that a particle–like picture can give some description of the behavior of the system [88]. In this case, if the dominant contribution to X production process were given by the $2 \to 2$ scattering $NN \to XX$, the rate of growth of N_X should be proportional to h^4. The lattice results appear to present a milder dependence on h, suggesting that the $NN \to XX$ scatterings alone

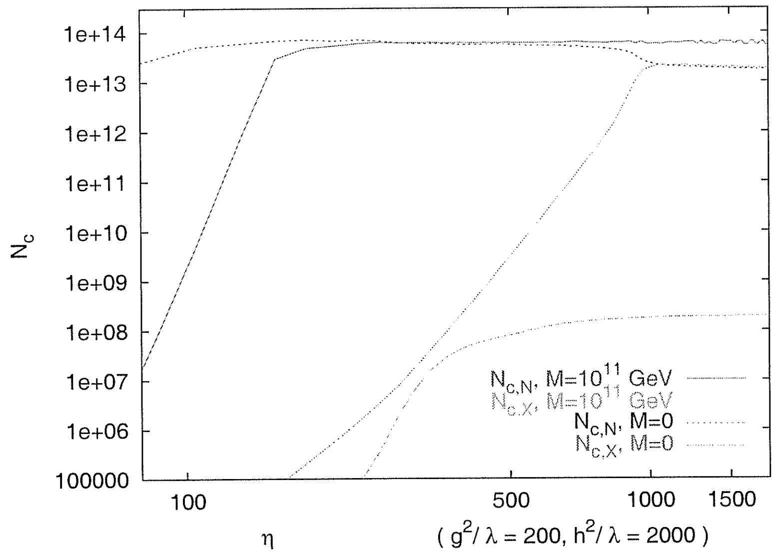


Figure 5.4: Comparison of the growth of the occupation numbers of N and X with and without the r.h. neutrino mass term. Notice that the amplification of X considerably weakens when the r.h. neutrino quanta become non-relativistic, $\eta \sim 350$ .See table 5.2 for notation.

cannot account for the production of X states. In a naive perturbative analysis, the contribution, from $(m+2) \times N \to XX$ processes, to the rate of growth of N_X, is proportional to the $m \times N \to XX$ rate times a factor roughly given by $h^4\, N_N^2/(4\pi p^6)$, where $p \sim 15\sqrt{\lambda}\phi_0/a$ is the typical momentum exchanged. Due to the high density of sneutrinos, the expansion parameter $h^4\, N_N^2/(4\pi p^6)$ is of order unity for the values of h we are considering. Therefore, strongly turbulent processes with many incoming sneutrinos can contribute significantly to the rate of growth of N_X. This is confirmed by the fact that the total number of particles decreases during the stage of generation of the X states, thus showing that particle fusion processes are dominant at this stage [88]. The main features shown in figure 5.3 are shared by the other evolutions with different $\tilde{g}$ that we will consider in the next section, and they can be understood at least qualitatively. As could have been easily guessed, the timescale for the growth of $N_{c,X}$ is a decreasing function of $\tilde{h}$. We also notice from the figure that the amplification of X becomes less efficient as the quanta of N become non relativistic, at $\eta \sim 300$. This can be seen explicitly in figure 5.4, where we show the effect of the r.h. neutrino mass term on the growth of the comoving occupation numbers of the N and X fields. If the two fields are both massless, the rescattering of the quanta of N lifts the X to (practically) the same amplification [88]. We observe that the situation is completely different for the case in which the quanta of N have a sufficiently high mass. Indeed, when the amplification of X from the r.h. neutrino quanta substantially decreases when the latter become non-relativistic. As a consequence of the two

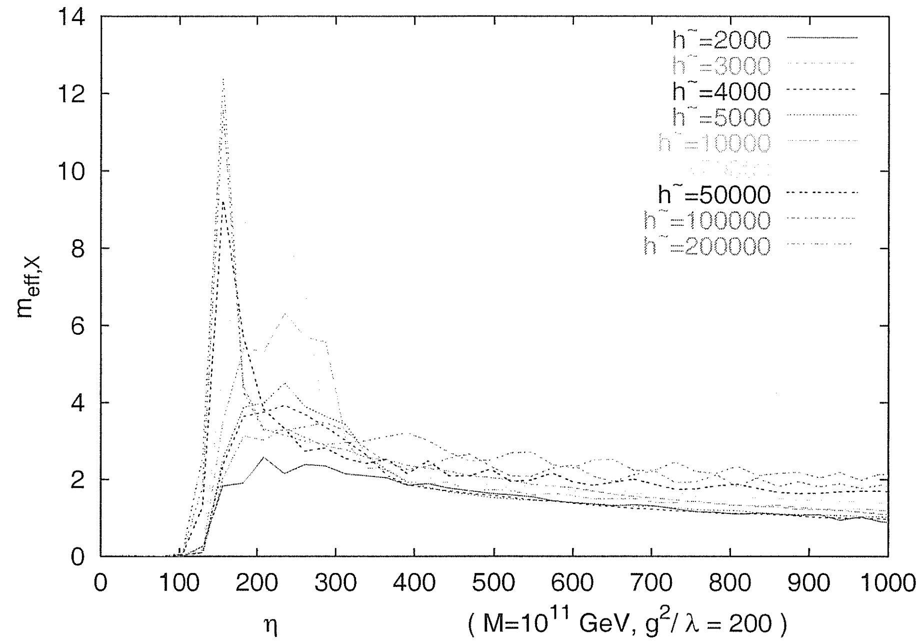


Figure 5.5: Time evolution of the effective mass of the quanta of X . See table 5.2 for notation.

effects mentioned in this paragraph, the asymptotic value of $N_{c,X}$ (at least in the time interval we have considered) decreases at small $\tilde{h}$. Figure 5.3 also shows a decrease of the asymptotic $N_{c,X}$ for high $\tilde{h}$. As discussed in [88], for sufficiently high coupling h the two fields have comparable occupation numbers, $N_{c,N} \simeq N_{c,X}$. Clearly, the higher the coupling is, the sooner this (approximate) equipartition is reached. For a very high h , the potential energy associated to the last term of (5.17) then disfavors the production of the quanta of the two fields, in the same way as a high quartic coupling $\propto N^4$ added to the potential (5.8) would have prevented the amplification of the r.h. neutrinos in the two fields case. In figure 5.5, we show the time evolution of the effective mass of the quanta of X . Comparing it with their distribution in momentum space, one realizes that most of the quanta are always in a relativistic regime.

5.6 Perturbative production of gravitinos

The light quanta X generated at rescattering can in turn be responsible for the production of unwanted relics such as gravitinos. If unstable, gravitinos with a mass of the order of the electroweak scale (which is what we expect in models of gravitationally mediated supersymmetry breaking) disrupt the successful predictions of primordial nucleosynthesis, unless their abundance is below the very stringent bound [74]

$$Y_{3/2} \lesssim 10^{-14} \left(\mathrm{TeV}/m_{3/2}\right) \ . \qquad (5.18)$$

In inflationary theories, several sources of gravitino production have been considered. The most standard of them is the perturbative production from the thermal bath formed at reheating. In this case, the above limit (5.18) translates into the upper bound $T_{\rm rh} \lesssim \text{few} \times 10^{10}$ GeV on the reheating temperature [40]. Other sources of gravitino production can be studied. Since gravitinos arise in supersymmetric models, a very natural "candidate" channel for their production is the decay of the inflaton into its supersymmetric partner (the inflatino) plus a gravitino. This process is however either kinematically forbidden [73] or strongly suppressed [121] by the fact that the difference between the inflaton and the inflatino mass is governed by the supersymmetry breaking scale, which is of the same order of the gravitino mass. The resulting gravitino production is sufficiently small. Recently, the generation of gravitinos at preheating has been extensively discussed, both concerning the relatively easier case of the transverse component [122], and the more delicate issue of the longitudinal component [123, 111, 124]. These studies are focused on the nonperturbative amplification of the gravitino field due to the coherent oscillations of the inflaton, and this mechanism of production is found to be sufficiently limited [111] provided that the inflationary sector of the theory is weakly (e.g. gravitationally) coupled to the one responsible for the present supersymmetry breakdown. In this section we discuss a different possible source of gravitinos, namely a perturbative production from the nonthermal distributions of light MSSM quanta generated at rescattering. A comparison with the standard thermal production may be used as an initial motivation. Concerning the latter, the requirement of a low reheating temperature can be seen as the demand that the inflaton decays sufficiently late, so that particle in the thermal bath have sufficiently low number densities and energies when they form. If $H \simeq 10^{12}$ GeV at the end of inflation, and if the scale factor a is normalized to one at this time, the generation of the thermal bath cannot occur before $a \simeq 10^7$. Gravitino overproduction is avoided by the fact that in the earlier times most of the energy density of the Universe is still stored in the coherent inflaton oscillations. As we have seen in the previous sections, this last assumption is no longer valid if preheating and rescattering effects are important. Indeed, in the model considered above the energy density of the scalar distributions becomes dominant already when the scale factor is of order 100 (the precise value being a function of the parameters of the model). [16] Although this comparison is rather suggestive, it is fair to say that the computation of gravitino production in the context of rescattering is certainly more difficult than the usual thermal production. While in the latter case a perturbative approach can be adopted, and the final result can be readily estimated by computing the rate of $2 \rightarrow 2$ processes with one gravitino in the final state, rescattering is a highly nonlinear phe-

[16] The situation is even more enhanced for hybrid inflationary model, in which the energy density of the zero modes of the scalars gets dissipated within their first oscillation [81, 82, 83].

nomenon. In the bosonic case, we already remarked that naive perturbative estimates poorly reproduce the initial amplification of the fields X. Only towards the end of the rescattering stage the number densities of the amplified fields become sufficiently small so that $2 \to 2$ processes become dominant, as the (approximate) conservation of the total comoving occupation number at relatively late times signals [88]. Unluckily, Pauli blocking forbids fermionic fields to behave classically (in the sense discussed in section 3), and lattice simulations cannot be used. However, we may take the numerical results for bosons as a guideline. Also for the production of fermions more complicated processes than just $2 \to 2$ interactions could be relevant during most of the rescattering stage, while they should be subdominant at sufficiently late time. The latter is presumably set by the same timescale at which rescattering is seen to end in the numerical simulations described in the previous sections. With this in mind, we proceed to an estimate of the amount of gravitinos produced by $2 \to 2$ processes with the fields amplified at preheating/rescattering in the incoming state. We stress once more that this estimate can be reliable only from the beginning of the thermalization stage on, so it should provide a lower bound on the total number of produced gravitinos. It is possible that a higher amount of gravitinos is produced at earlier times, when nonlinear effects cannot be neglected. This computation has been carried out in appendix, where also some details (i.e. concerning the quantization of the system and the choice to focus on the transverse gravitino component) are reported. As for the thermal production, the dominant $2 \to 2$ processes have only one gravitino in the final state, and hence only one gravitationally suppressed vertex. In the thermal case, the dominant processes have a gauge interaction as the second vertex, consider e.g. the process $X\,X \to \tilde{z}\,\psi_{3/2}$ with one higgsino in the propagator. For the nonthermal distributions of scalars that we are considering, however, such processes are kinematically forbidden. This is a crucial point, which poses a significant limit on the estimated production of gravitinos. We can easily understand it using the specific process just mentioned as an example: either the Higgses are not amplified at rescattering (so the above process is irrelevant) or the non-vanishing $\langle H^2\rangle$ provides an effective mass to the zino produced in the interaction. When the light scalar distributions saturate (that is, when the gravitino production can be effective), we find numerically $a\,\sqrt{\langle X^2\rangle} \sim (10^{-2} - 10^{-1})\,\phi_0$. The typical comoving momenta $a\,p$ characterizing the scalar distributions are instead only about one order of magnitude higher than the "inflaton mass" at the end of inflation, $a\,p \sim 15\sqrt{\lambda}\,\phi_0$. As a consequence, the gauginos acquire a mass $m_{\tilde{z}} \sim (10^2 - 10^3)\,p$, which shows that these processes are kinematically forbidden. [17] We notice that, at least for this specific kind of interactions, the system is still effectively behaving as a condensate: the number

[17] Processes with an additional gauge interaction and the gaugino in the propagator are allowed but strongly suppressed. See the appendix for details.

densities of the scalar distributions are set by the quantity $\sqrt{\langle X^2 \rangle}$, which is much higher than the typical momenta of the distributions themselves. This generates a high effective mass for all the particles "strongly" coupled to these scalar fields. Compare this situation with a medium in thermodynamical equilibrium: in this case both the typical momenta and the effective masses are set by the only energy scale present, namely the temperature of the system. As should be clear by the above discussion (see also [88]), the thermalization of the distributions produced at rescattering necessarily proceeds through particle fusion. Only after a sufficiently prolonged stage of thermalization, will the system be sufficiently close to thermodynamical equilibrium so to render processes as the one discussed above kinematically allowed. As we discuss in the appendix, kinematically allowed $2 \to 2$ interactions can be obtained by taking a trilinear interaction coming from the superpotential term (5.15), also responsible for the Dirac mass term for the neutrinos. This can lead to processes of the kind $\tilde{N}_R X \to x\, \psi_{3/2}$, or $X\, X \to N_R\, \psi_{3/2}$. The physical number density of (transverse) gravitinos produced by these processes can be estimated as

$$N_{3/2}(\eta) \sim \frac{h^2}{M_P^2} \left[\frac{N_{c,X}\, N_{c,N}}{a^6} \right] \Bigg|_{\eta=\eta_*} \left[\frac{a(\eta_*)}{a(\eta)} \right]^3 \quad , \quad \eta > \eta_* \; , \qquad (5.19)$$

where η_* is the time (to be determined numerically) at which the expression in the first parenthesis reaches its maximum, while the second parenthesis is a dilution factor due to the expansion of the Universe at later times (see the appendix for details). [18] As we have remarked, this result is subject to the limit (5.18), where $Y_{3/2} \equiv N_{3/2}/s$, and s is the entropy density of the Universe, computed once the dominating thermal bath is formed. For practical use, we find that a more "convenient" bound can be obtained if (5.18) is combined with the result for the baryon asymmetry, eqn. (5.12). For this purpose, consider the ratio

$$\zeta \equiv \frac{Y_{3/2}}{Y_B} = \frac{23}{8\,\epsilon_1} \frac{N_{3/2}(\eta)}{N_N(\eta)} = \frac{23}{8\,\epsilon_1} \frac{N_{3/2}(\eta_*)}{N_N(\eta_*)} \; . \qquad (5.20)$$

The quantity ζ has two main advantages, (i) it is independent of the entropy of the Universe and (ii) it can be computed already at $\eta = \eta_*$, since after this time the two physical number densities $N_{3/2}$ and N_N simply rescale as a^{-3}. While $Y_{3/2}$ must satisfy the upper bound (5.18), the limit $Y_B \gtrsim 10^{-11}$ poses a lower bound on the number density of the sneutrinos, if leptogenesis is assumed to be responsible for the baryon asymmetry of the Universe. Adopting the parameterization (3.14), we then

[18] The time η_* roughly corresponds at the moment at which the distribution of light quanta X starts to saturate. This typically occurs towards the end of the rescattering stage, which guarantees that considering only $2 \to 2$ processes should provide at least an order of magnitude estimate of the gravitino quanta produced at this stage.

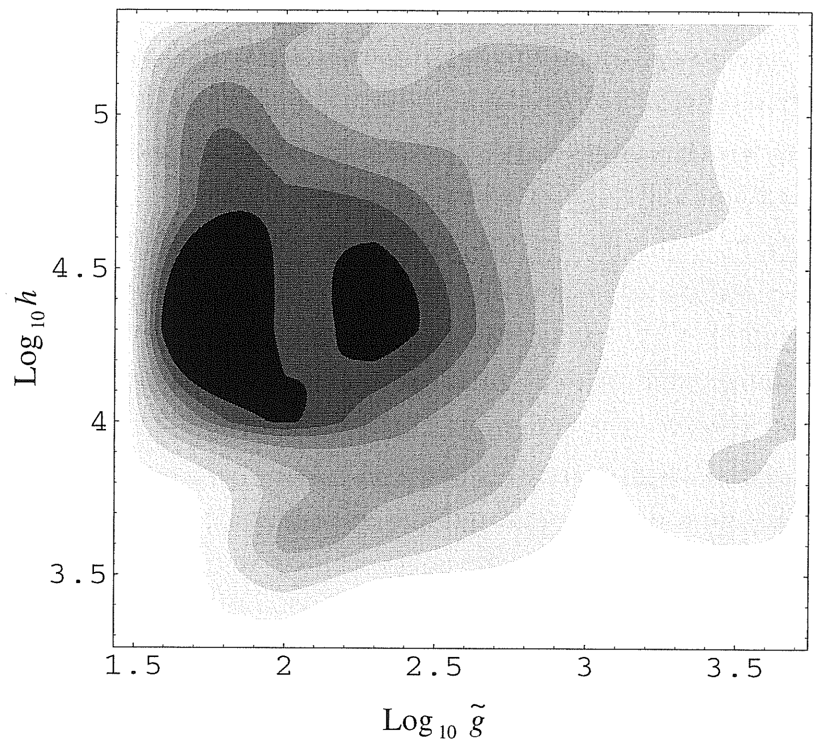


Figure 5.6: Contour plot for the quantity $\tilde{\zeta}/\tilde{h}$, where $\tilde{\zeta}$ is defined in eqn. (5.22). The contour lines range from $\tilde{\zeta}/\tilde{h} = 3 \cdot 10^{-12}$ to $\tilde{\zeta}/\tilde{h} = 1.1 \cdot 10^{-10}$ (darker region).

see that the ratio ζ has to satisfy

$$\zeta \simeq \frac{3 \times 10^6}{\delta_{\rm CP}} \left(\frac{10^{10}\,\text{GeV}}{M}\right) \left(\frac{0.05\,\text{eV}}{m_3}\right) \frac{N_{3/2}\,(\eta_*)}{N_N\,(\eta_*)} < 10^{-3}\ . \tag{5.21}$$

It is important to stress that, unlike the limit (5.18), this bound cannot be ameliorated by an eventual entropy release which may occur between the decay of the r.h. sneutrinos and nucleosynthesis, since both the gravitino and the baryon number densities would be diluted in the same amount. For this reason, we find in the present context the bound (5.21) more significant than the limit (5.18) involving the gravitino abundance alone. We wanted to verify whether the condition (5.21) is respected for the choice $M = 10^{11}\,\text{GeV}$ considered in the previous sections, and for several values of the couplings g and h (defined in the potential (5.17)) in the range $\tilde{g} \in [30, 5000]$, $\tilde{h} \in [2000, 200000]$ (we remind that $\tilde{g} \equiv g^2/\lambda$, and analogously for h). To do so, we have defined

$$\zeta \equiv \frac{1}{\delta_{\rm CP}} \left(\frac{0.05\,\text{eV}}{m_3}\right) \tilde{\zeta}\ , \tag{5.22}$$

and in figure 5.6 we have plotted the quantity $\tilde{\zeta}/\tilde{h}$. In this way, we factor out the explicit dependence of ζ on $\tilde{h}$ coming from the cross section of the dominant $2 \to 2$

processes, see e.g. eqn. (5.19). The qualitative behavior of $\tilde{\zeta}/\tilde{h}$ with h (vertical axis) is easily understood in terms of the arguments used to explain the results of figure 5.3. For relatively low h, the amplification of the X field is weak, and so few gravitinos quanta are produced. As h increases, the amplification becomes stronger, both regarding the final value at which N_X saturates and the rapidity at which the saturation occurs. As a consequence, the number density of produced gravitinos also increases. As h further increases, the rapidity at which X saturates keeps increasing (and so, the time η_* at which most of the gravitino quanta are produced decreases); however, the final value of N_X starts to become smaller, leading to the decrease of $\tilde{\zeta}/\tilde{h}$ that we observe in the figure. For fixed values of h, the final result is also first increasing and then decreasing with g. This behavior is presumably due to the dependence on g of the total number of quanta of both the N and X fields produced at preheating/rescattering (notice that it vanishes both at very small and very high g). However, the interpretation is in this case less clear. Concerning the value of $\tilde{\zeta}$ itself, in the range of coupling considered it reaches the maximal value at $\tilde{g} \simeq 100$, $\tilde{h} \simeq 200000$, where it evaluates to $\tilde{\zeta} \simeq 10^{-5}$. From what we have just said, we expect that $\tilde{\zeta}$ starts decreasing at higher h, although the numerical simulations we have performed show that the decrease starts only at the highest value of h that we have considered. From the definition (5.22), we see that the bound $\zeta < 10^{-3}$ is respected, provided the CP violation encoded in the parameter $\delta_{\rm CP}$ (see eqs. (3.14) and (3.15)) is not too small. However, we remind that our estimates take into account only the gravitino quanta produced from the end of the rescattering stage on, while a higher production at earlier times cannot be excluded.

Conclusions

Despite the tremendous success of spontaneously broken gauge theories, and in particular the standard model of strong and electroweak interactions, in describing low-energy phenomena, there still remains an important unresolved issue. This last manifests itself in two equally serious aspects: experimental and mathematical. The experimental aspect lies in the fact that the Higgs particle has not been detected yet. However, and even if one can alleviate this problem saying that this will be completely resolved (hopefully positively) once the LHC will be operative, the other aspect of the problem is still present. Indeed, the presence of quadratic divergences, which are inherent of theories containing scalar fields, makes the theory mathematically inconsistent. This problem dubbed *hierarchy problem* is the main source of embarrassment in the particle Physics community. Supersymmetry is one of the most popular frameworks to address the hierarchy problem. SUSY brings a plethora of new particles, and as such it brings a whole new set of phenomenology.

On the other hand, the standard big bang model, together with inflation and the theory of cosmological perturbations provides a satisfactory description of our Universe on astronomical scales. In the last two decade, supersymmetry and supergravity have also been used in a cosmological framework, providing attractive solutions to several prominent problems (dark matter, baryogenesis,...). Nonetheless, there are many other instances where the interaction between particle Physics and Cosmology proves to be not so successful (The monopole problem, moduli, gravitinos,...). This fact is somehow expected, since the two theories fundamentally probe different scales. However, if some unified description of all the forces of nature exists, this should not be the case, and the predictions in the two sides must coincide.

In this thesis, we focused on different aspects of the interface between particle Physics and Cosmology. In particular, we were concerned with singlet tadpoles that arise naturally supergravity theories. We showed that they modify significantly the hybrid inflationary potential, and lead to a modified scenario. Then we were interested in baryogenesis, and more precisely in leptogenesis. The attraction of this scenario is that it is able to explain and relate two well-known experimental facts: the baryon asymmetry of the Universe and neutrino masses. In its most simple version (thermal leptogenesis), it suffers from the overabundance of gravitinos. We studied two alternative that provide a solution to the gravitino problem. The first is leptogenesis at low scale, where we found interesting link between the baryon asymmetry and the soft SUSY breaking terms. The second is non thermal leptogenesis.

The merging of particle Physics and Cosmology in a unique and unified framework is for sure one of the most pressing issues in modern theoretical Physics. Perhaps as pressing as finding a unified description of gravity and quantum theory (Quantum gravity). It goes without saying that understanding the Physics pervading in the early Universe would probably shed light on the unified description of the fundamental interactions of nature. We have seen in this thesis some examples of the contact points between these two branches of modern Physics. Given the yet available dazzling array of cosmological data, and the forthcoming ones, we must feel that we are really fortunate to live in such an exciting era for both particle Physics and Cosmology.

Appendix A

Perturbative production of gravitinos

In this appendix we derive eqn. (5.19) of chapter 5. We first estimate the number density of gravitinos produced by the nonthermal distributions formed at rescattering. We remark that at this stage supersymmetry breaking is controlled by the energy density of these distributions. The longitudinal gravitino component (i.e. the goldstino, which is "eaten" in the unitary gauge) is thus provided by a linear combination of the fermionic superpartners of these scalars, and does not coincide with the longitudinal gravitino at late times (at least for the standard case of a present gravitationally mediated supersymmetry breakdown). For this reason, our discussion will be limited to the transverse gravitino component $\psi_{3/2}$. Its mass is given by [125]

$$m_{3/2} = \mathrm{e}^{\mathcal{K}(\phi_i)/M_P^2} \, \frac{|W(\phi_i)|}{M_P^2} \,, \tag{A-1}$$

where, we remind, W and $\mathcal{K}$ are, respectively, the superpotential and the Kähler potential of the model, while $\{\phi_i\}$ denotes the set of scalar fields amplified during preheating and rescattering. To quantize the transverse gravitino, we define a homogeneous mass $m_{3/2}^0$ by replacing in eqn. (A-1) the ($\mathbf{x}-$dependent) values of the fields with their (homogeneous) variances, $\phi\,(t,\,\mathbf{x}) \to \sqrt{\langle\phi^2\rangle\,(t)}$. The difference $\delta m_{3/2} \equiv m_{3/2} - m_{3/2}^0$ will be accounted for in the interaction lagrangian. The main production of gravitinos is expected to occur close to the point at which the number density of light scalars X reaches its maximum, in the same way as most of the thermal production occurs as soon as the thermal bath is generated. From the results of section 4, we observe that the maximum of $N_{c,X}$ is achieved at the end of the rescattering phase. At this moment the thermalization stage begins, during which the variances of the fields show an adiabatic evolution. This allows us a consistent quantization of the gravitino component, since the mass $m_{3/2}^0\,(t)$ is also varying only adiabatically in this period. It is clear that the main concern with the procedure just described is that, contrary to the thermal case, the difference $\delta m_{3/2}$ is now of the same order of $m^0_{3/2}$ itself, at least during the initial part of the thermalization stage. This leads to the problem discussed in section 5, namely to the fact that the perturbative computation of the gravitino production is presumably not as straightforward as in the thermal case, and more complicated processes than just $2 \to 2$ interactions can be expected to be relevant. However, as we have already remarked, the latter should provide at least an order of magnitude estimate for the gravitino produced from the end of the rescattering stage on, and should reasonably lead to a lower bound to the total production. For this reason, we now proceed to a rough estimate of their cross sections. The dominant processes with two gravitinos as outgoing particles have two gravitationally suppressed vertices (i.e. $X\,X \to \psi_{3/2}\,\psi_{3/2}$ with a flat direction fermion x in the propagator; processes coming from the interaction term $\delta m_{3/2}\,\bar{\psi}_{3/2}\,\psi_{3/2}$ are subdominant). Their cross section is of the order $\sigma \sim p^2/M_P^4$, where here and in the following p denotes the typical momentum exchanged in the scattering. As in the thermal case, the distributions of the light quanta are indeed characterized by a typical momentum; while for a thermal distribution $p \sim T$, we now have $p \sim \kappa\sqrt{\lambda}\,\phi_0\,/a\,(t)$, where in the cases shown below κ is a coefficient of order 10 dependent on the specific choice of the parameters. In our estimates we will take $\kappa \sim 15$. [1] Thus, $\sigma \sim 10^{-11}\,M_P^{-2}\,a^{-2}$ for this class of processes. A more efficient production is expected from scatterings with only one gravitationally suppressed vertex, and hence with only one gravitino in the final state. For example, the standard thermal production is mainly due to channels having a gauge interaction as the second vertex, e.g. $H\,H \to \psi_{3/2}\,\tilde{z}$ with an exchanged higgsino. In the present context, however, processes with outgoing gauginos (that we generically denote with $\tilde{g}$) are expected to be kinematically forbidden, since these particles acquire a high effective mass from their interaction with the nonthermal scalar distributions. Indeed, if a scalar field X has a large vev, and an interaction of the form $\sqrt{\alpha}X\psi\tilde{g}$ is present (ψ and $\tilde{g}$ are two component matter fermion and gaugino, we use $\sqrt{\alpha}$ because we have already used g as the inflaton-neutrino coupling) then the gaugino acquires a large Dirac mass $\sim \sqrt{\alpha}\langle X\rangle$ mixing with ψ. We have large variances, rather than a large vev; by analogy with finite temperature, we expect that $\langle X^2\rangle \neq 0$ will generate an effective mass square "$m^2 \sim \alpha\langle X^2\rangle$" in the $\tilde{g}$ and ψ dispersion relations.[2] So for kinematic purposes, we assume that gauginos which couple to the flat direction have masses of order $\sqrt{\alpha\langle X^2\rangle}$. When the light scalar distributions saturate (that is,

[1] The existence of a typical momentum allows the use of the integrated Boltzmann equation to estimate the amount of gravitinos produced. Moreover, since this momentum is much higher than the gravitino mass, the value of the latter does not affect the cross sections for the processes with outgoing transverse gravitinos. This is welcome, since the above (somewhat arbitrary) redefinition $m_{3/2} = m^0_{3/2} + \delta m_{3/2}$ will not affect our estimates.

[2] It is implicit in this discussion that all the fermionic fields are quantized in the same way as done for the gravitinos.

when the gravitino production we are discussing can be effective), we find numerically $\sqrt{\langle X^2 \rangle} \sim (10^{-2} - 10^{-1})\, \phi_0/a$. As a consequence, $m_{\tilde{g}} \sim (10^2 - 10^3)\, p$, and these scatterings are forbidden. One is immediately led to consider processes with an additional $X_i\, \psi_j\, \tilde{g}$ vertex and in which the heavy gaugino is off-shell. Their cross section can be roughly estimated as $\sigma \sim 10^{-2}\, (\alpha/M_P)^2\, (p/m_{\tilde{g}})^2$, which is comparable or smaller than the cross section for the process $X\, X \to \psi_{3/2}\, \psi_{3/2}$ considered above. Finally, there is the possibility that the second vertex comes from the superpotential term (5.15), also responsible for the Dirac mass term for the neutrinos. This can lead to processes of the kind $\tilde{N}_R\, X \to x\, \psi_{3/2}$ or $X\, X \to N_R\, \psi_{3/2}$ (x denoting the fermionic partner of X; all processes have in the propagator the fermionic partner of one of the incoming scalars). The cross sections for these processes are roughly estimated as [3] $\sigma \sim h^2/M_P^2 \simeq 10^{-13}\, \tilde{h}/M_P^2$. Thus, unless of a very small coupling h, the last class of scatterings has the highest cross section and dominates the production of the transverse gravitinos. In particular, processes with one incoming N_R quantum are dominant if $N_{c,X}$ starts to saturate at a smaller value than $N_{c,N}$. Viceversa, scatterings of the kind $X\, X \to N_R\, \psi_{3/2}$ will dominate. Numerical results indicate that the former situation is more often realized. The cases in which the opposite was found are characterized by a relatively high coupling $\tilde{h}$, so that the light degrees of freedom are quickly amplified to values $N_{c,X} \gamma N_{c,N}$. Anyhow, in these cases the cross sections of the two type of processes are clearly of the same order of magnitude. Hence, for brevity of exposition we will only refer to the processes with an incoming r.h. sneutrino, although both the two possibilities have been considered in our estimates. The integrated Boltzmann equation reads

$$\frac{dN_{3/2}}{dt} + 3\, H\, N_{3/2} \simeq \langle \sigma |v| \rangle N_X N_N \quad , \tag{A-2}$$

where the "friction term" due to the expansion of the Universe can be neglected in the estimate of the order of magnitude of gravitinos produced. The whole production time can be then divided in a series of time intervals of duration $H^{-1}\,(t_i)$ each. During each interval, quanta of gravitinos are generated with a density of

$$\delta N_{3/2}^i \sim \frac{h^2}{M_P^2}\, \frac{N_{c,X}\,(\eta_i)}{a\,(\eta_i)^3}\, \frac{N_{c,N}\,(\eta_i)}{a\,(\eta_i)^3}\, H^{-1}\,(\eta_i) \tag{A-3}$$

(notice the presence of the scale factor, since the physical and not the comoving occupation number has to be used in the integrated Boltzmann equation). The function $(N_{c,X}\, N_{c,N}\, H^{-1}\, a^{-6})\,(\eta)$ amounts to zero at the end of inflation, and it reaches a maximum at a time η_*, which can be determined numerically and which roughly

[3] In this estimate it is assumed that the exchanged momentum p is higher or at most comparable with the mass of the r.h. neutrinos. This is certainly true when most of the gravitinos are produced, i.e. when the distributions of light quanta X are about to saturate.

corresponds to the moment at which the comoving number density $N_{c,X}$ starts saturating (this in turns occurs towards the end of the rescattering stage, when $2 \to 2$ processes start dominating). At $\eta > \eta_*$ it then quickly decreases due to the expansion of the Universe. It thus turns out that, as for the thermal production, the gravitino quanta are mostly generated at the time η_*, so that their "late time" physical number density is approximatively given by

$$N_{3/2}(\eta) \sim \frac{h^2}{M_P^2} \frac{N_{c,X}(\eta_*)}{a(\eta_*)^3} \frac{N_{c,N}(\eta_*)}{a(\eta_*)^3} H^{-1}(\eta_*) \left[\frac{a(\eta_*)}{a(\eta)}\right]^3, \qquad \eta > \eta_* \,. \tag{A-4}$$

This is eqn. (5.19) of the main text.

Appendix B

Thermodynamics

In this appendix, we define some useful concepts of equilibrium thermodynamics that will be useful in the rest of the thesis. In thermal equilibrium, the number density n_i, energy density ρ_i and pressure p_i of the set of particles of type i is given by

$$n_i = \frac{g_i}{2\pi^3}\int f_i(\mathbf{p})\mathrm{d}^3p, \tag{A-1}$$

$$\rho_i = \frac{g_i}{2\pi^3}\int E_i f_i(\mathbf{p})\mathrm{d}^3p, \tag{A-2}$$

$$p_i = \frac{g_i}{2\pi^3}\int \frac{\mathbf{p^2}}{3E_i} f_i(\mathbf{p})\mathrm{d}^3p. \tag{A-3}$$

where $f_i(\mathbf{p})$ is the distribution function characterizing a system of particles with momentum between $\mathbf{p}$ to $\mathbf{p}+d\mathbf{p}$. It is given by

$$f_i(\mathbf{p}) = \frac{1}{\exp\left(\frac{E_i-\mu_i}{T}\right)\pm 1}. \tag{A-4}$$

where $E_i = \sqrt{\mathbf{p}^2+m_i^2}$ is the energy of particles of type i given in term of their momenta $\mathbf{p}$ and mass m_i. The plus (minus) sign in the denominator correspond to fermionic (bosonic) particles.

In Table B.1, we show these quantities for the relativistic ($T \gg m_i$) and non-relativistic ($T \ll m_i$) limits. Here, we have assumed $|\mu_i| \ll T$ and no Bose-Einstein condensation ($|\mu_i| < m_i$).

Because the energy density of a non-relativistic particle is exponentially suppressed compared with the relativistic one, the total energy density of the radiation $\rho_{\rm rad}$ is given by the following simple form:

$$\rho_{\rm rad} = \frac{\pi^2}{30} g_*(T) T^4\,, \tag{A-5}$$

where

$$g_*(T) \equiv \sum_{\substack{m_i \ll T \\ i=\text{boson}}} g_i + \frac{7}{8}\sum_{\substack{m_j \ll T \\ j=\text{fermion}}} g_j\,. \tag{A-6}$$

$T \gg m_i$		$T \ll m_i$
fermion	boson	
$n_i = \frac{3}{4} g_i \left(\frac{\zeta(3)}{\pi^2}\right) T^3$	$n_i = g_i \left(\frac{\zeta(3)}{\pi^2}\right) T^3$	$n_i = g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} \exp\left(-\frac{m_i}{T}\right)$
$\rho_i = \frac{7}{8} g_i \left(\frac{\pi^2}{30}\right) T^4$	$\rho_i = g_i \left(\frac{\pi^2}{30}\right) T^4$	$\rho_i = m_i n_i$
$p_i = \frac{1}{3}\rho_i$	$p_i = \frac{1}{3}\rho_i$	$p_i = T n_i \ (\ll \rho_i)$

Table B.1: The number density n_i, energy density ρ_i and pressure p_i of the particle i, which is thermal equilibrium, in the limits of $T \gg m_i$ and $T \ll m_i$. We have assumed $|\mu_i| \ll T$ and $|\mu_i| < m_i$.

If there are particles which have different temperatures from that of the photon T, another factor $(T_i/T)^4$ should be multiplied in the above expression. (For example, at $T \ll$ MeV, neutrinos have temperature $T_\nu = (4/11)^{1/3} T$ for $m_\nu \ll T_\nu$.)

Notice that all leptogenesis mechanisms discussed in this thesis work at temperatures far above the electroweak scale $T \gg 1$ TeV, where all the MSSM particles are expected to be in thermal equilibrium. In this case, we obtain

$$g_* = 228.75 \qquad \text{for} \quad \text{MSSM}\,. \tag{A-7}$$

In the expanding universe, it is convenient to introduce the entropy density s, which is defined by

$$\begin{aligned} s &\equiv \frac{\rho + p}{T} \\ &= \frac{4}{3T}\rho = \frac{2\pi^2}{45} g_*(T) T^3\,. \end{aligned} \tag{A-8}$$

(Again, in the presence of particles with different temperatures, a factor of $(T_i/T)^3$ is multiplied in (A-8). In this case, the g_* in (A-8) becomes slightly different from the g_* in Eqt. (A-5).) Notice that the entropy per co-moving volume sR^3 is conserved as far as no entropy production takes place. Thus it is quite convenient to take the ratio n_X/s when we discuss some number density n_X. For example, if some X-number is conserved, the ratio of the X-number density to the entropy density takes a constant value

$$\frac{n_X}{s} = const\,, \tag{A-9}$$

as long as there is no entropy production, since both n_X and s scales as R^{-3} as the universe expands. As another example, if the X-particle is in thermal equilibrium and relativistic ($T \gg m_X$), the ratio is given by

$$\frac{n_X^{\rm eq}}{s} = \frac{45\zeta(3)}{2\pi^4}\frac{g_X}{g_*(T)} \quad \left(\times\frac{3}{4} \quad \text{for fermion}\right)\,, \tag{A-10}$$

where the temperature (or time) dependence only comes from $g_*(T)$.

Before closing this section, we calculate the relations between the particle number asymmetry $n_i^{(+)} - n_i^{(-)}$ and the particle's chemical potential μ_i, which can be obtained by integrating Eqt. (A-1). In order to calculate the asymmetry, it is necessary to calculate higher order terms than those in Table B.1. The results are given by

$$\begin{aligned} n_i^{(+)} - n_i^{(-)} &= \frac{1}{6} g_i T^3 \left[\left(\frac{\mu_i}{T} \right) + \cdots \right] \qquad \text{for} \quad \text{fermion}\,, \\ n_i^{(+)} - n_i^{(-)} &= \frac{1}{3} g_i T^3 \left[\left(\frac{\mu_i}{T} \right) + \cdots \right] \qquad \text{for} \quad \text{boson}\,, \end{aligned} \tag{A-11}$$

where ellipses denote higher order terms in the expansions of m_i/T and μ_i/T. Here, we have assumed no Bose-Einstein condensation $|\mu_i| < m_i$ for boson, and relativistic limit $m_i \ll T$.

Appendix C

Supersymmetry and Supergravity

In this appendix, we review the main results of supersymmetry and supergravity. More detailed discussion is available in excellent textbooks (See for e.g [138]). We follow the notation of Wess and Bagger through this appendix).

C.1 $N = 1$ Supersymmetry

Supersymmetry (SUSY from now on) is a symmetry relating bosons and fermions. Namely a supersymmetric transformation can be written

$$
\begin{aligned}
Q\,|\text{Boson}\rangle &\sim |\text{Fermion}\rangle, && \text{(A-1)}\\
Q\,|\text{Fermion}\rangle &\sim |\text{Boson}\rangle && \text{(A-2)}
\end{aligned}
$$

From the above equations, it is obvious that the SUSY generator Q is fermionic, it is called the supercharge. It follows that SUSY is a space-time symmetry, extending the Lorentz group. Indeed, it is the maximal extension allowed by virtue of the Coleman-Mandula theorem. The (anti-) commutation relations involving the supercharge can be summarized as follows

$$
\begin{aligned}
\{Q,\bar{Q}\} &= 2\sigma^{\mu}P_{\mu}, && \text{(A-3)}\\
\{Q,Q\} &= \{\bar{Q},\bar{Q}\} = 0, && \text{(A-4)}\\
[P^{\mu},Q] &= [P^{\mu},\bar{Q}] = 0. && \text{(A-5)}
\end{aligned}
$$

Another immediate consequence of these equations is that in order to close the algebra the number of fermionic degrees of freedom have to be equal to the bosonic one; that is $n_B = n_F$. Furthermore, as can be seen from eqn. (A-5) P^2 commutes with Q, and so also the masses of bosons and fermions are equal ($m_B = m_F$). Taking the supersymmetric version of a model involves essentially doubling the spectrum.

C.2 Superspace

For any theory, one can write its corresponding supersymmetric version. The *superspace formalism* simplifies considerably this task. The superspace is obtained by adding four anti-commuting spinor degrees of freedom $\theta^\alpha, \bar{\theta}_{\dot{\alpha}}$ to the space-time coordinates x^μ. As a consequence, any point in the *superspace*, is parametrized by the usual (bosonic) space-time coordinates x^μ, plus the Grassmann (fermionic) coordinates $(\theta, \bar{\theta})$, *i.e.* $X^A \equiv (x^\mu, \theta, \bar{\theta})$. The spinor index is raised and lowered with the ϵ-tensor and $\theta\theta = \theta^\alpha\theta_\alpha = -2\theta^1\theta^2$. Similarly, $\bar{\theta}\bar{\theta} = \bar{\theta}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}} = 2\bar{\theta}_{\dot{1}}\bar{\theta}_{\dot{2}}$. We also have

$$\theta^\alpha\theta^\beta = \frac{1}{2}\epsilon^{\alpha\beta}\theta\theta\,, \qquad \bar{\theta}_{\dot{\alpha}}\bar{\theta}_{\dot{\beta}} = -\frac{1}{2}\epsilon_{\dot{\alpha}\dot{\beta}}\bar{\theta}\bar{\theta}\,, \qquad \theta\sigma^\mu\bar{\theta}\theta\sigma^\nu\bar{\theta} = \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}\eta^{\mu\nu}\,. \tag{A-6}$$

The members of the supermultiplet are gathered in a unique object called *superfield.* In the off-shell formulation, some (non dynamical) auxiliary field is added to the supermultiplet to close the algebra. A superfield is therefore a function on the superspace, say, $F(x, \theta, \bar{\theta})$. Since the θ-coordinates are anti-commuting, the most general $N = 1$ superfield can always be expanded as

$$\begin{aligned} F(x,\theta,\bar{\theta}) &= f(x) + \theta\phi(x) + \bar{\theta}\bar{\chi}(x) + \theta\theta m(x) + \bar{\theta}\bar{\theta}n(x) + \theta\sigma^\mu\bar{\theta}V_\mu(x) \\ &+ \theta\theta\bar{\theta}\bar{\lambda}(x) + \bar{\theta}\bar{\theta}\theta\psi(x) + \theta\theta\bar{\theta}\bar{\theta}d(x)\,. \end{aligned} \tag{A-7}$$

where $f(x)$, $d(x)$, $m(x)$ and $n(x)$ are complex scalar fields, $\phi(x)$ and $\psi(x)$ are left-handed Weyl spinors, $\bar{\chi}(x)$ and $\bar{\lambda}(x)$ are right-handed Weyl spinors, and $A_\mu(x)$ a complex vector field.

C.3 $N = 1$ SUSY representations

Particles transform under supersymmetry in representations containing both bosons and fermions called *supermultiplets.* In the massless case, representations are labelled by the helicity $\lambda = \frac{\mathbf{J}}{\mathbf{p}}$, while in the massive case, they are labelled by the mass and spin (m, J). In its simplest version ($N = 1$), the supermultiplet consists of a field of helicity λ along with its superpartner of helicity $\lambda + 1/2$. There are two useful cases. **The scalar (or chiral) supermultiplet** contains a scalar field $\varphi(x)$ (spin $= 0$) and its fermionic partner $\psi(x)$ (spin $= 1/2$). **The gauge supermultiplet**: contains a gauge field $A^a_\mu(x)$ (spin $= 1$) and its fermionic partner called the gaugino $\lambda^a(x)$ (spin $= 1/2$) , where a is a gauge index.

As one can see from eqn. (A-7), there are a way too much degrees of freedom in $F(x, \theta, \bar{\theta})$. One can reduce them by imposing certain constraints which are preserved by the SUSY transformations. To do so, we introduce the super-covariant derivatives

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha\dot{\alpha}}\,\partial_\mu\,, \qquad \bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i\sigma^\mu_{\alpha\dot{\alpha}}\theta^\alpha\,\partial_\mu\,. \tag{A-8}$$

that commute with Q and $\bar{Q}$. The chiral superfield is obtained imposing the constraint $\bar{D}_{\dot{\alpha}}\Phi = 0$. The chiral superfield can be written as

$$\Phi(x, \theta) = \varphi(x) + \sqrt{2}\theta\psi(x) + \theta\theta F(x). \tag{A-9}$$

Likewise, imposing the reality condition $V^\dagger(x, \theta, \bar{\theta}) = V(x, \theta, \bar{\theta})$, one obtains the gauge superfield

$$V(x, \theta, \bar{\theta}) = -\theta\sigma^\mu\bar{\theta}A_\mu(x) + i\theta\theta\bar{\theta}\bar{\lambda}(x) - i\bar{\theta}\bar{\theta}\theta\lambda(x) + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D(x). \tag{A-10}$$

It is then straightforward to generalize to arbitrary gauge group G, whose elements satisfy the Lie algebra $[T_a, T_b] = it^c_{ab}T^c$, with $\mathrm{Tr}(T_aT_b) = 2k\delta_{ab}$, by writing $V(x, \theta, \bar{\theta}) \equiv T_aV^a a(x, \theta, \bar{\theta})$. One can also construct the field strength superfield

$$W_\alpha = -\frac{1}{4}\bar{D}^2e^{-V}D_\alpha e^V\,, \qquad \bar{W}_{\dot{\alpha}} = -\frac{1}{4}D^2e^V\bar{D}_{\dot{\alpha}}e^{-V}\,, \tag{A-11}$$

where likewise $W_\alpha \equiv T_aW^a_\alpha$. Now, we can write the most general gauge invariant and (global) SUSY Lagrangian for a set of chiral multiplets $\Phi_n(x, \theta)$

$$\mathcal{L} = \sum_n \int \mathrm{d}^4\theta\ \Phi^\dagger_n\mathrm{e}^V\Phi_n + \frac{1}{8kg^2}\int \mathrm{d}^2\theta\ \mathrm{Tr}W^2_\alpha + \int \mathrm{d}^2\theta\ W(\Phi_n) + \mathrm{h.c.} \tag{A-12}$$

where W is a gauge invariant holomorphic function of the superfields Φ_n known as the *superpotential*. Writing the Lagrangian (A-12) in term of the component fields and integrating-out the auxiliary fields using their equation of motion, we obtain the simple expressions for the scalar potential

$$V(\phi_n) = \sum_n |F_n|^2 + \frac{1}{2}\sum_a |D_a|^2\,, \tag{A-13}$$

where the F and $D-$term are given by

$$F_n = \frac{\partial W}{\partial\phi_n}, \qquad D_a = g\sum_{m,n}\phi^\dagger_nT^{mn}_a\phi_m \tag{A-14}$$

Using the above technology, it is now very easy to write the supersymmetric version of any Lagrangian.

C.4 $N = 1$ Supergravity

If one considers the SUSY parameter as a function of coordinates, one obtains the local version of SUSY, which by construction includes gravity. That's the reason why it is usually called Supergravity (SUGRA). There exists one more useful representation in this case, it is **The gravitational multiplet** containing a tensor field (the graviton) field (spin $= 2$) and its fermionic partner the gravitino (spin $= 3/2$), in addition to other auxiliary fields. The formulation of supergravity is a complicated issue. For the lack of space (and time) we refer to excellent reviews. We will just write the most important formulae, which will be generalizations of the global SUSY case. Before doing so, we need to define two more functions in addition to the superpotential: the Kähler potential and the gauge kinetic function. The Kähler potential K is a real function of the chiral superfields and their complex conjugate. It describes how the scalar components are coupled to each other, according to the relation

$$\mathcal{L}_{\rm kin} = \frac{\partial^2 K}{\partial\phi_n \partial\phi_m^\dagger}(\partial^\mu\phi_n)(\partial_\mu\phi_m^\dagger) \tag{A-15}$$

In the *canonical* case it is just given by $K(\phi_n,\phi_n^\dagger) = \sum_n \phi_n^\dagger\phi_n$. The gauge kinetic function $f_{\alpha\beta}$ determines the kinetic terms of the gauge fields. Any supergravity theory is completely specified by the choice of the three functions W, K and $f|\alpha\beta$. The most general gauge and SUGRA invariant. It can be written as

$$\mathcal{L} = \int d^4\theta\ K(\Phi_n^\dagger e^V,\Phi_n) + \frac{1}{8kg^2}\int d^2\theta\ \mathrm{Tr} f_{\alpha\beta}W_\alpha W_\beta + \int d^2\theta\ W(\Phi_n) + \mathrm{h.c.} \tag{A-16}$$

The scalar potential of $N=1$ supergravity takes the form

$$V_{\rm SUGRA} = e^{\frac{K}{M_P^2}}\left(F_i\left(K^{-1}\right)^i_j F^j - 3\frac{|W|^2}{M_P^2}\right) + \frac{1}{2}\mathrm{Re} f_{ab}^{-1} D_a D_b\,, \tag{A-17}$$

where now the F and $D-$terms are given by

$$F_i \equiv \partial_i W + \frac{K_i W}{M_P}, \qquad D_a = g\sum_{m,n} K_n T_a^{mn}\phi_m \tag{A-18}$$

and the index (subscript) i stands for derivatives with respect to φ_i (φ_i^*).

Bibliography

[1] A. H. Guth, Phys. Rev. D **23** (1981) 347.

[2] A. Linde, **Particle Physics And Inflationary Cosmology**, *Chur, Switzerland: Harwood (1990) 362 p. (Contemporary concepts in physics, 5).*

[3] D. H. Lyth and A. Riotto, Phys. Rept. **314** (1999) 1, hep-ph/9807278.

[4] G. Dvali and A. Riotto, Phys. Lett. B **417** (1998) 20, hep-ph/9706408.

[5] G. Lazarides, hep-ph/0011130; and references therein.

[6] A. Linde, Phys. Rev. D **49** (1994) 748, astro-ph/9307002.

[7] P. Binetruy and G. Dvali, Phys. Lett. B **388** (1996) 241, hep-ph/9606342, E. Halyo, Phys. Lett. B **387** (1996) 43, hep-ph/9606423.

[8] G. Dvali, Q. Shafi and R. Schaefer, Phys. Rev. Lett. **73** (1994) 1886 , hep-ph/9406319.

[9] G. R. Dvali, L. M. Krauss and H. Liu, hep-ph/9707456.

[10] E. J. Copeland, A. R. Liddle, D. H. Lyth, E. D. Stewart and D. Wands, Phys. Rev. D **49** (1994) 6410, astro-ph/9401011.

[11] E. D. Stewart, Phys. Rev. D **51** (1995) 6847, hep-ph/9405389; See also hep-ph/9408302.

[12] E. D. Stewart, Phys. Lett. B **345** (1995) 414 , astro-ph/9407040.

[13] H. P. Nilles, M. Srednicki and D. Wyler, Phys. Lett. B **124** (1983) 337; A. B. Lahanas, Phys. Lett. B **124** (1983) 341.

[14] A. Linde and A. Riotto, Phys. Rev. D **56** (1997) 1841, hep-ph/9703209.

[15] J. Bagger and E. Poppitz, Phys. Rev. Lett. **71** (1993) 2380 , hep-ph/9307317.

[16] J. Bagger, E. Poppitz and L. Randall, Nucl. Phys. B **455** (1995) 59, hep-ph/9505244.

[17] H. P. Nilles and N. Polonsky, Phys. Lett. B **412** (1997) 69, hep-ph/9707249.

[18] C. Kolda and N. Polonsky, Phys. Lett. B **433** (1998) 323 , hep-ph/9805240.

[19] C. Kolda, S. Pokorski and N. Polonsky, Phys. Rev. Lett. **80** (1998) 5263, hep-ph/9803310.

[20] G. F. Giudice and A. Masiero, Phys. Lett. B **206** (1988) 480.

[21] S. Coleman and E. Weinberg, Phys. Rev. D **7** (1973) 1888.

[22] U. Ellwanger, M. Rausch de Traubenberg and C. A. Savoy, Nucl. Phys. B **492** (1997) 21, hep-ph/9611251.

[23] A. T. Davies, C. D. Froggatt and R. G. Moorhouse, Phys. Lett. B **372** (1996) 88, hep-ph/9603388; S. J. Huber and M. G. Schmidt, hep-ph/0003122.

[24] J. R. Espinosa, A. Riotto and G. G. Ross, Nucl. Phys. B **531** (1998) 461, hep-ph/9804214; D. H. Lyth and A. Riotto, Phys. Lett. B **412** (1997) 28, hep-ph/9707273.

[25] M. Bastero-Gil and S. F. King, Nucl. Phys. B **549** (1999) 391, hep-ph/9806477.

[26] J. A. Casas and G. B. Gelmini, Phys. Lett. B **410** (1997) 36, hep-ph/9706439.

[27] M. K. Gaillard, H. Murayama and K. A. Olive, Phys. Lett. B **355** (1995) 71, hep-ph/9504307.

[28] S. Fukuda *et al.* [Super-Kamiokande Collaboration], Phys. Rev. Lett. **85** (2000) 3999;

S. Fukuda *et al.* [SuperKamiokande Collaboration], Phys. Rev. Lett. **86** (2001) 5651;

Q. R. Ahmad *et al.* [SNO Collaboration], Phys. Rev. Lett. **87** (2001) 071301.

[29] S. Weinberg, Summary talk at XXIII Int. Conf. on High Energy Physics, Berkeley, CA, Jul 16-23, 1986, Int. J. Mod. Phys. A **2** (1987) 301.

[30] E. K. Akhmedov, Z. G. Berezhiani and G. Senjanović, Phys. Rev. Lett. **69** (1992) 3013, hep-ph/9205230; E. K. Akhmedov, Z. G. Berezhiani, G. Senjanović and Z. j. Tao, Phys. Rev. D **47** (1993) 3245, hep-ph/9208230.

[31] R. Barbieri and L. J. Hall, Nucl. Phys. B **364** (1991) 27;

R. Barbieri, J. R. Ellis and M. K. Gaillard, Phys. Lett. B **90** (1980) 249.

[32] M. Gell-Mann, P. Ramond and R. Slansky in *Supergravity* (P. van Niewenhuizen and D. Freedman, eds.), (Amsterdam), North Holland, 1979; T. Yanagida in *Workshop on Unified Theory and Baryon number in the Universe* (O. Sawada and A. Sugamoto, eds.), (Japan), KEK 1979; R.N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44**, 912 (1980).

[33] For a review see for *e.g.* K. A. Olive, G. Steigman and T. P. Walker, Phys. Rept. **333** (2000) 389, astro-ph/9905320.

[34] A. D. Sakharov, Pisma Zh. Eksp. Teor. Fiz. **5** (1967) 32 [JETP Lett. **5** (1967) 24].

[35] M. Fukugita and T. Yanagida, Phys. Lett. B **174** (1986) 45; M. A. Luty, Phys. Rev. D **45** (1992) 455.

[36] I. Affleck and M. Dine, Nucl. Phys. B **249** (1985) 361.

[37] M. Dine, L. Randall and S. Thomas, Nucl. Phys. B **458** (1996) 291, hep-ph/9507453.

[38] G. F. Giudice, I. Tkachev and A. Riotto, JHEP **9908** (1999) 009, hep-ph/9907510.

[39] G. F. Giudice, A. Riotto and I. Tkachev, JHEP **9911** (1999) 036, hep-ph/9911302.

[40] M. Bolz, A. Brandenburg and W. Buchmuller, Nucl. Phys. B **606** (2001) 518, hep-ph/0012052.

[41] S. Weinberg, Phys. Rev. Lett. **48** (1982) 1303.

[42] H. Pagels and J. R. Primack, Phys. Rev. Lett. **48** (1982) 223.

[43] J. Ellis, A. Linde, and D. Nanopoulos, Phys. Lett. **B118**, 59 (1982);
D. Nanopoulos, K. Olive, and M. Srednicki, Phys. Lett. **B127**, 30 (1983);
J. Ellis, J. Kim, and D. Nanopoulos, Phys. Lett. **B145**, 181 (1984); For a detailed discussion see also M. Kawasaki and T. Moroi, Prog. Theor. Phys. **93** (1995) 879, hep-ph/9403364.

[44] G. F. Giudice, M. Peloso, A. Riotto and I. Tkachev, JHEP **9908** (1999) 014, hep-ph/9905242.

[45] N. Arkani-Hamed, L. J. Hall, H. Murayama, D. R. Smith and N. Weiner, Phys. Rev. D **64** (2001) 115011, hep-ph/0006312; N. Arkani-Hamed, L. J. Hall, H. Murayama, D. R. Smith and N. Weiner, hep-ph/0007001.

[46] F. Borzumati and Y. Nomura, Phys. Rev. D **64** (2001) 053005, hep-ph/0007018.

[47] E. Dudas and C. A. Savoy, hep-ph/0205264.

[48] S. Nasri and S. Moussa, Mod. Phys. Lett. A **17** (2002) 771, hep-ph/0106107; S. Nasri and M. Trodden, hep-ph/0107215.

[49] C. D. Froggatt and H. B. Nielsen, Nucl. Phys. B **147** (1979) 277.

[50] E. W. Kolb and M. S. Turner, *The Early Universe*, Addison-Wesley (1990).

[51] M. Plumacher, Z. Phys. C **74** (1997) 549, hep-ph/9604229.

[52] U. Sarkar, Phys. Lett. B **390** (1997) 97, hep-ph/9606359.

[53] M. Flanz, E. A. Paschos, U. Sarkar and J. Weiss, Phys. Lett. B **389** (1996) 693, hep-ph/9607310;

A. Pilaftsis, Phys. Rev. D **56** (1997) 5431, hep-ph/9707235.

[54] T. Hambye, Nucl. Phys. B **633** (2002) 171, hep-ph/0111089.

[55] A. D. Linde, Phys. Lett. B **116** (1982) 335.

[56] A. Anisimov and M. Dine, Nucl. Phys. B **619** (2001) 729, hep-ph/0008058; A. Anisimov, hep-ph/0111233.

[57] *see e.g.* W. Buchmuller and S. Fredenhagen, hep-ph/0001098; A. Riotto and M. Trodden, Ann. Rev. Nucl. Part. Sci. **49** (1999) 35; V. A. Rubakov and M. E. Shaposhnikov, Usp. Fiz. Nauk **166** (1996) 493 [Phys. Usp. **39** (1996) 461].

[58] A. D. Sakharov, Pisma Zh. Eksp. Teor. Fiz. **5** (1967) 32 [JETP Lett. **5** (1967) 24].

[59] K. Hagiwara *et al.* [Particle Data Group Collaboration], Phys. Rev. D **66** (2002) 010001.

[60] V. A. Kuzmin, V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B **155** (1985) 36; J. A. Harvey and M. S. Turner, Phys. Rev. D **42** (1990) 3344.

[61] K. Kajantie, M. Laine, K. Rummukainen and M. E. Shaposhnikov, Nucl. Phys. B **466** (1996) 189, hep-lat/9510020.

[62] M. Carena, M. Quiros and C. E. Wagner, Nucl. Phys. B **524** (1998) 3 , hep-ph/9710401. M. Losada, Nucl. Phys. B **537** (1999) 3 hep-ph/9806519. M. Losada, Nucl. Phys. B **569** (2000) 125 hep-ph/9905441.

[63] Y. Fukuda *et al.* [Super-Kamiokande Collaboration], Phys. Rev. Lett. **81** (1998) 1562, Phys. Rev. Lett. **82** (1999) 1810, Phys. Rev. Lett. **82** (1999) 2430.

[64] M. A. Luty, Phys. Rev. D **45** (1992) 455; B. A. Campbell, S. Davidson and K. A. Olive, Nucl. Phys. B **399** (1993) 111 `hep-ph/9302223`; W. Buchmuller and T. Yanagida, Phys. Lett. B **302** (1993) 240; W. Buchmuller and M. Plumacher, Phys. Lett. B **389** (1996) 73 `hep-ph/9608308`; M. Plumacher, Nucl. Phys. B **530** (1998) 207 `hep-ph/9704231`. M. Plumacher, Z. Phys. C **74** (1997) 549 `hep-ph/9604229`.

[65] M. Flanz, E. A. Paschos, U. Sarkar and J. Weiss, Phys. Lett. B **389** (1996) 693 `hep-ph/9607310`; A. Pilaftsis, Phys. Rev. D **56** (1997) 5431 `hep-ph/9707235`;

[66] *see e.g.* W. Buchmüller and M. Plumacher, Int. J. Mod. Phys. A **15** (2000) 5047 `hep-ph/0007176`, *or* W. Buchmuller and M. Plumacher, Phys. Rept. **320** (1999) 329, *and references therein.*

[67] K. Hamaguchi, H. Murayama and T. Yanagida, Phys. Rev. D **65** (2002) 043512 `hep-ph/0109030`.

[68] M. Fujii, K. Hamaguchi and T. Yanagida, Phys. Rev. D **65** (2002) 115012 `hep-ph/0202210`.

[69] S. Davidson and A. Ibarra, Phys. Lett. B **535** (2002) 25 [`hep-ph/0202239`.

[70] W. Buchmuller, P. Di Bari and M. Plumacher, Nucl. Phys. B **643** (2002) 367 [`hep-ph/0205349`. W. Buchmuller, P. Di Bari and M. Plumacher, Phys. Lett. B **547** (2002) 128 [`hep-ph/0209301`.

[71] S. Weinberg, Phys. Rev. Lett. **48** (1982) 1303.

[72] J. R. Ellis, A. D. Linde and D. V. Nanopoulos, Phys. Lett. B **118** (1982) 59.

[73] D. V. Nanopoulos, K. A. Olive and M. Srednicki, Phys. Lett. B **127** (1983) 30; J. R. Ellis, J. S. Hagelin, D. V. Nanopoulos, K. A. Olive and M. Srednicki, Nucl. Phys. B **238** (1984) 453; M. Y. Khlopov and A. D. Linde, Phys. Lett. B **138** (1984) 265.

[74] J. R. Ellis, J. E. Kim and D. V. Nanopoulos, Phys. Lett. B **145** (1984) 181; J. R. Ellis, D. V. Nanopoulos and S. Sarkar, Nucl. Phys. B **259** (1985) 175; R. Juszkiewicz, J. Silk and A. Stebbins, Phys. Lett. B **158** (1985) 463; D. Lindley, Phys. Lett. B **171** (1986) 235; M. Kawasaki and K. Sato, Phys. Lett. B **189** (1987) 23.

[75] M. Kawasaki and T. Moroi, Prog. Theor. Phys. **93** (1995) 879 hep-ph/9403364; J. R. Ellis, D. V. Nanopoulos, K. A. Olive and S. J. Rey, Astropart. Phys. **4** (1996) 371 hep-ph/9505438; S. Sarkar, Rept. Prog. Phys. **59** (1996) 1493 hep-ph/9602260;

[76] H. Murayama and T. Yanagida, Phys. Lett. B **322** (1994) 349 [hep-ph/9310297.

[77] M. Dine, L. Randall and S. Thomas, Phys. Rev. Lett. **75** (1995) 398 hep-ph/9503303. M. Dine, L. Randall and S. Thomas, Nucl. Phys. B **458** (1996) 291 [hep-ph/9507453.

[78] J. H. Traschen and R. H. Brandenberger, Phys. Rev. D **42** (1990) 2491; A. D. Dolgov and D. P. Kirilova, Sov. J. Nucl. Phys. **51** (1990) 172 [Yad. Fiz. **51** (1990) 273];

[79] L. Kofman, A. D. Linde and A. A. Starobinsky, Phys. Rev. Lett. **73** (1994) 3195 hep-th/9405187; L. Kofman, A. D. Linde and A. A. Starobinsky, Phys. Rev. D **56** (1997) 3258 hep-ph/9704452.

[80] A. D. Linde, Phys. Lett. B **129** (1983) 177.

[81] G. N. Felder, J. Garcia-Bellido, P. B. Greene, L. Kofman, A. D. Linde and I. Tkachev, Phys. Rev. Lett. **87** (2001) 011601 hep-ph/0012142; G. N. Felder, L. Kofman and A. D. Linde, Phys. Rev. D **64** (2001) 123517 hep-th/0106179; J. Garcia-Bellido and E. Ruiz Morales, Phys. Lett. B **536** (2002) 193 hep-ph/0109230; J. Garcia-Bellido, M. Garcia Perez and A. Gonzalez-Arroyo, hep-ph/0208228.

[82] T. Asaka, W. Buchmuller and L. Covi, Phys. Lett. B **510** (2001) 271 hep-ph/0104037.

[83] E. J. Copeland, S. Pascoli and A. Rajantie, Phys. Rev. D **65** (2002) 103517 hep-ph/0202031.

[84] S. Y. Khlebnikov and I. I. Tkachev, Phys. Rev. Lett. **77** (1996) 219 hep-ph/9603378, Phys. Lett. B **390** (1997) 80 hep-ph/9608458, Phys. Rev. Lett. **79** (1997) 1607 hep-ph/9610477, Phys. Rev. D **56** (1997) 653 hep-ph/9701423.

[85] P. B. Greene, L. Kofman, A. D. Linde and A. A. Starobinsky, Phys. Rev. D **56** (1997) 6175 hep-ph/9705347.

[86] E. W. Kolb, A. D. Linde and A. Riotto, Phys. Rev. Lett. **77** (1996) 4290 hep-ph/9606260. E. W. Kolb, A. Riotto and I. I. Tkachev, Phys. Lett. B **423** (1998) 348 hep-ph/9801306.

[87] M. Peloso and L. Sorbo, JHEP **0005** (2000) 016 hep-ph/0003045.

[88] G. N. Felder and L. Kofman, Phys. Rev. D **63** (2001) 103503, hep-ph/0011160.

[89] G. N. Felder and I. Tkachev, hep-ph/0011159.

[90] J. Liu and G. Segre, Phys. Rev. D **48** (1993) 4609 hep-ph/9304241. A. Pilaftsis, Nucl. Phys. B **504** (1997) 61 hep-ph/9702393; Int. J. Mod. Phys. A **14** (1999) 1811 hep-ph/9812256. L. Boubekeur, hep-ph/0208003.

[91] L. Covi, E. Roulet and F. Vissani, Phys. Lett. B **384** (1996) 169 hep-ph/9605319.

[92] Z. Maki, M. Nakagawa and S. Sakata, Prog. Theor. Phys. **28** (1962) 870.

[93] S. Hatakeyama *et al.* [Kamiokande Collaboration], Phys. Rev. Lett. **81** (1998) 2016; W. W. Allison *et al.*, Phys. Lett. B **391** (1997) 491; W. W. Allison *et al.* [Soudan-2 Collaboration], Phys. Lett. B **449** (1999) 137.

[94] Q. R. Ahmad *et al.* [SNO Collaboration], Phys. Rev. Lett. **87** (2001) 071301; Q. R. Ahmad *et al.* [SNO Collaboration], nucl-ex/0204008. Q. R. Ahmad *et al.* [SNO Collaboration], nucl-ex/0204009.

[95] B. T. Cleveland *et al.*, Astrophys. J. **496** (1998) 505; Y. Fukuda *et al.* [Kamiokande Collaboration], Phys. Rev. Lett. **77** (1996) 1683; W. Hampel *et al.* [GALLEX Collaboration], Phys. Lett. B **388** (1996) 384; D. N. Abdurashitov *et al.*, Phys. Rev. Lett. **77** (1996) 4708.

[96] M. Apollonio *et al.* [CHOOZ Collaboration], Phys. Lett. B **466** (1999) 415.

[97] F. Borzumati and A. Masiero, Phys. Rev. Lett. **57** (1986) 961.
For more recent analyses, see e.g.: J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, Phys. Rev. D **53** (1996) 2442; S. Lavignac, I. Masina and C. A. Savoy, hep-ph/0106245; J. A. Casas and A. Ibarra, Nucl. Phys. B **618** (2001) 171 hep-ph/0103065.

[98] M. L. Brooks *et al.* [MEGA Collaboration], Phys. Rev. Lett. **83** (1999) 1521, hep-ex/9905013.

[99] S. Davidson and A. Ibarra, JHEP **0109** (2001) 013 hep-ph/0104076.

[100] S. Davidson and A. Ibarra, hep-ph/0206304.

[101] G. C. Branco, R. Gonzalez Felipe, F. R. Joaquim and M. N. Rebelo, Nucl. Phys. B **640** (2002) 202 hep-ph/0202030.

[102] A. Y. Smirnov, Phys. Rev. D **48** (1993) 3264, hep-ph/9304205.

[103] *see e.g.* G. Altarelli and F. Feruglio, Phys. Lett. B **451** (1999) 388 hep-ph/9812475.

[104] J. Sato, K. Tobe and T. Yanagida, Phys. Lett. B **498** (2001) 189 hep-ph/0010348.

[105] R. Barbieri, P. Creminelli, A. Strumia and N. Tetradis, Nucl. Phys. B **575** (2000) 61 hep-ph/9911315.

[106] S. Dimopoulos and L. J. Hall, Phys. Lett. B **196** (1987) 135. K. Benakli and S. Davidson, Phys. Rev. D **60** (1999) 025004 hep-ph/9810280. G. F. Giudice, E. W. Kolb and A. Riotto, Phys. Rev. D **64** (2001) 023508 hep-ph/0005123.

[107] K. A. Olive, G. Steigman and T. P. Walker, Phys. Rept. **333** (2000) 389 astro-ph/9905320.

[108] G. Lazarides and Q. Shafi, Phys. Lett. B **258** (1991) 305. T. Asaka, K. Hamaguchi, M. Kawasaki and T. Yanagida, Phys. Lett. B **464** (1999) 12 hep-ph/9906366; K. Kumekawa, T. Moroi and T. Yanagida, Prog. Theor. Phys. **92** (1994) 437 hep-ph/9405337.

[109] R. Allahverdi and A. Mazumdar, hep-ph/0208268.

[110] P. B. Greene and L. Kofman, Phys. Lett. B **448** (1999) 6 hep-ph/9807339.

[111] H. P. Nilles, M. Peloso and L. Sorbo, Phys. Rev. Lett. **87** (2001) 051302 hep-ph/0102264, JHEP **0104** (2001) 004 hep-th/0103202.

[112] S. Davidson and S. Sarkar, JHEP **0011** (2000) 012 hep-ph/0009078.

[113] J. McDonald, Phys. Rev. D **61** (2000) 083513 hep-ph/9909467.

[114] R. Allahverdi, Phys. Rev. D **62** (2000) 063509 hep-ph/0004035.

[115] G. Mangano, G. Miele, S. Pastor and M. Peloso, Phys. Rev. D **64** (2001) 123509 hep-ph/0102080.

[116] R. Allahverdi and M. Drees, Phys. Rev. D **66**, 063513 (2002) hep-ph/0205246.

[117] Y. Shtanov, J. Traschen and R. H. Brandenberger, Phys. Rev. D **51** (1995) 5438 hep-ph/9407247; J. Baacke, K. Heitmann and C. Patzold, Phys. Rev. D **58** (1998) 125013 hep-ph/9806205; D. Cormier, K. Heitmann and A. Mazumdar, Phys. Rev. D **65** (2002) 083521 hep-ph/0105236.

[118] A. H. Guth and S. Y. Pi, Phys. Rev. D **32** (1985) 1899; D. Polarski and A. A. Starobinsky, Class. Quant. Grav. **13** (1996) 377 gr-qc/9504030.

[119] M. K. Gaillard, H. Murayama and K. A. Olive, Phys. Lett. B **355** (1995) 71 hep-ph/9504307. M. Kawasaki, M. Yamaguchi and T. Yanagida, Phys. Rev. D **63** (2001) 103514 hep-ph/0011104.

[120] I. Affleck and M. Dine, Nucl. Phys. B **249** (1985) 361.

[121] H. P. Nilles, K. A. Olive and M. Peloso, Phys. Lett. B **522** (2001) 304 hep-ph/0107212.

[122] A. L. Maroto and A. Mazumdar, Phys. Rev. Lett. **84** (2000) 1655 hep-ph/9904206;

[123] R. Kallosh, L. Kofman, A. D. Linde and A. Van Proeyen, Phys. Rev. D **61** (2000) 103503 hep-th/9907124; G. F. Giudice, I. Tkachev and A. Riotto, JHEP **9908** (1999) 009 hep-ph/9907510; G. F. Giudice, A. Riotto and I. Tkachev, JHEP **9911** (1999) 036 hep-ph/9911302; R. Kallosh, L. Kofman, A. D. Linde and A. Van Proeyen, Class. Quant. Grav. **17** (2000) 4269, hep-th/0006179.

[124] P. B. Greene, K. Kadota and H. Murayama, hep-ph/0208276.

[125] T. Moroi, Ph.D. thesis, hep-ph/9503210.

[126] M. Trodden, Rev. Mod. Phys. **71** (1999) 1463 hep-ph/9803479.

[127] *Lectures given at ICTP Summer School on Astroparticle Physics and Cosmology, Trieste, Italy, 17 Jun - 5 Jul 2002.*, hep-ph/0210162

[128] A.R. Liddle, D.H. Lyth, *Cosmological inflation and large-scale structure* Cambridge, UK: Univ. Pr. (2000) 400 p.

[129] A. G. Cohen, A. De Rujula and S. L. Glashow, Astrophys. J. **495** (1998) 539 astro-ph/9707087.

[130] M. Carena, M. Quiros, M. Seco and C. E. Wagner, Nucl. Phys. B **650** (2003) 24 hep-ph/0208043.

[131] S. Perlmutter *et al.* [Supernova Cosmology Project Collaboration], Astrophys. J. **517** (1999) 565 astro-ph/9812133.

[132] A. G. Riess *et al.* [Supernova Search Team Collaboration], Astron. J. **116** (1998) 1009 astro-ph/9805201.

[133] W. L. Freedman *et al.*, Astrophys. J. **553** (2001) 47 astro-ph/0012376.

[134] W. L. Freedman and M. S. Turner, arXiv:astro-ph/0308418.

[135] N. A. Bahcall, J. P. Ostriker, S. Perlmutter and P. J. Steinhardt, Science **284**, 1481 (1999) astro-ph/9906463.

[136] P. Langacker, Phys. Rept. **72** (1981) 185.

[137] T. W. Kibble, J. Phys. A **9** (1976) 1387.

[138] J. Wess and J. Bagger, *Supersymmetry And Supergravity*

[139] D. N. Spergel *et al.*, astro-ph/0302209.

[140] E. W. Kolb, hep-ph/9910311.