Flavor Changing Neutral Current Solutions to the Solar Neutrino Problem

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Abstract

We analyse how neutrino flavor changing neutral current interactions can play a relevant role to solve the solar neutrino problem since they can produce a sizeable neutrino transition magnetic moment or lead to matter-enhanced neutrino oscillations. Such oscillations can take place even if neutrinos have zero mass and there is no mixing in the vacuum. We discuss, in particular, the effects of neutrino flavor changing neutral current interactions arising in the minimal supersymmetric standard model when R-parity is not imposed.

À Norma

(Esteja sempre por perto)

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Contents

Introduction

The most successful model of the elementary particle physics and the corresponding one for the solar physics are not compatible with each other. In fact, the so-called standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ model of elementary particles affirms that neutrinos created in the Sun, in the way described by the standard solar model (SSM) [1], should arrive at the Earth surface without undergoing any substantial alteration in their path from the central parts of the Sun to the surface of the Earth. In particular, the number of solar neutrinos to be detected at Earth is expected to be approximately equal to just the number of neutrinos predicted by the standard solar model to be created in the Sun with momentum directed to the Earth and with energy exceeding the threshold value for the relevant detector.

Nevertheless, over a period of two decades the Homestake detector [2] has recorded a systematic deficit in the number of solar neutrino captures in ^{37}Cl compared with those expectations coming from the standard solar model. The observed suppression ratio R =(count-rate)/(SSM prediction) averaged over time is

$$R_{Cl} = 0.27 \pm 0.04 .$$

This deficit has been confirmed by measurements with Kamiokande-II $\nu - e$ scattering detector [3], which finds

$$R_{\rm KII} = 0.46 \pm 0.08 .$$

This unexpected discrepancy has been known as the solar neutrino problem and its understanding represents one of the biggest challenges to particle physicists and astrophysicists.

Obviously one can think of two main possibilities to understand the observed solar neutrino deficit. Either the standard solar model has some problem and gives an overestimated total number of neutrinos produced in the Sun with energy exceeding the threshold value for the available detectors. Or the standard $SU(3)_C \times SU(2)_L \times U(1)_{Y}$ model with its minimal fermionic content is not the ultimate truth in the elementary particle world and we must look for some more complete extension of this model.

In fact, the two existing proposed solutions to the solar neutrino problem that have received by far the greatest attention adopt this last possibility and indirectly suggest some kind of extension of the standard $SU(3)_C \times SU(2)_L \times U(1)_{Y}$ model since both of them postulate the existence of physical processes not allowed by the standard model. One of these solutions [4][5][6] is based on the neutrino oscillations hypothesis [7], which in the more conventional case requires that neutrinos be massive, and the other one [8] assumes that electron neutrino has a diagonal or transitional magnetic moment. The absence of right-handed neutrinos (ν_R) as well as leptonic flavor changing neutral currents in the standard model of elementary particles is the direct responsible for the vanishing of neutrino magnetic moment and for the impossibility of finding neutrino oscillations (deriving from the neutrino masslessness) in this model.

In the present thesis we will discuss how one of the most appealing extension of the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ model, the minimal supersymmetric standard model [9] can play a relevant role to produce solutions to the solar neutrino problem when the imposition of R-parity conservation is relaxed. These solutions are a consequence of the appearance of individual leptonic number (L_e, L_μ and L_τ) violation in this model.

Results

The presence of R-parity violating interactions in supersymmetric models can produce neutrino magnetic moments of order $\mu_\nu \simeq 10^{-13}\mu_B$ [10]. In this version of the theory, the main constraint on μ_ν is set by the upper bound on the neutrino masses ($m_{\nu_e} \lesssim 10eV$ from laboratory experiments, $m_{\nu_\mu}, m_{\nu_\tau} \lesssim 100eV$ from cosmology). Although a value of $\mu_\nu \simeq 10^{-13}\mu_B$ is not close to the present laboratory bounds ($\mu_\nu \lesssim 10^{-10} - 10^{-9}\mu_B$ for the electron and muon neutrinos), it is far larger than the value we would obtain in the standard model adding a right-handed neutrino to its particle content as a $SU(2)_L$ singlet, indeed this "standard model value" [11] of the neutrino magnetic moment is found to be $\mu_\nu/\mu_B \lesssim (10^{-19} - 10^{-18})(m_{\nu_e}/eV)$. If we implement suitable symmetries in the Lagrangian, which implies departing from the minimal supersymmetric standard model, although still remaining with minimal particle content, an order of magnitude can be gained, leading to $\mu_\nu \lesssim 10^{-12}\mu_B$ [10]. This upper bound is obtained by applying the same naturalness criterion as before. In this case, the main constraint comes from the electron mass (or the d-quark mass) rather than the neutrino mass itself.

Taking into account the relevant phenomenological constraints we show also that the neutrino flavor changing neutral current interactions in matter can cause a resonant transition of the solar electron neutrinos into, e.g., tau neutrinos with a probability compatible with the qualitative features of the solar neutrino observations. We will find that such transitions exist even if neutrinos have zero mass and there is no mixing in the vacuum [12]. These transitions can take place in the core of the Sun and can be of adiabatic type. The corresponding transition probability does not depend on the solar neutrino momentum.

We analyse also the possibility that neutrinos do have a mass, but the effects of the existence of vacuum neutrino mixing on the solar neutrino transitions are negligible, the transitions of interest being induced by neutrino flavor changing neutral current interactions with the nucleons in the Sun [12][13] *). The solution of the solar neutrino problem one obtains in this case is practically equivalent to the "conventional MSW" nonadiabatic solution.

How this thesis is organized

In the first chapter of this work we give an introduction to the minimal supersymmetric standard model and we emphasize the possibility of violating R-parity. Special attention will be given to the constraints on the coupling constants associated with R-parity breaking appearing from the severe bounds of flavor changing neutral currents in the presently available experimental data. For experts in these supersymmetric models we recommend just the reading of the section 1.c, where these bounds are discussed. In the Chapter 2, a discussion concerning the non-vanishing neutrino magnetic moment appearing in these models is presented. Finally, in the Chapter 3, we introduce the interesting possibility of having matter-enhanced neutrino oscillation induced by neutrino flavor changing neutral current interactions in matter.

*) This possibility was discussed also in reference [14].

1

MSSM: the option of breaking R-parity

Supersymmetric models [9] are very appealing. Besides placing bosons and fermions in aesthetically attractive common irreducible multiplets, their low energy limit presents also the remarkable possibility of solving the naturalness problem of the gauge hierarchy which are usually present in models containing elementary scalars [15]. In this chapter we will introduce the supersymmetric version of the $SU(3)_C \times SU(2)_L \times U(1)_Y$ standard model, including explicit R-parity breaking. We will be particularly interested in a remarkable feature of such theory: in the absence of any particular symmetry (R-parity, for instance) flavor changing neutral current interactions of the neutrinos appear at tree-level. We will pay special attention to the existing constraints on the coupling-constants related to interactions leading to flavor changing neutral currents.

Specifically, we will discuss the minimal supersymmetric standard model (MSSM) added by terms in the superpotential which break R-parity. By "minimal" we mean the low energy limit of spontaneously broken $N = 1$ supergravity theories [16] which supersymmetrize the standard model [17], promoting the "standard" fields to superfields, and present the following two features: *i*) all scalar kinematic terms are canonical [9] and *ii*) R-parity is implemented so that no baryon and/or lepton number violating terms appear in the superpotential. In this way, since we are relaxing the condition *ii*), we are leaving the idea of "minimality".

1.a. The Supersymmetric Lagrangian

According to Haag-Lopuszanski-Sohnius theorem [18], which generalizes the Coleman-Mandula result [19] to anticommuting operators, the most general algebra of the S-matrix consistent with relativistic quantum field theory is given by the so-called super-Poincaré algebra, where in additional to usual Poincaré algebra, some relations of commutation and anticommutation among the energy-momentum operator P_μ, the six Lorentz generators $M_{\mu\nu} = -M_{\nu\mu}$ and some new anticommuting two-component spinorial operators Q_α ($\alpha = 1, 2$), the supersymmetry generators transforming as $(1/2, 0)$

under Lorentz, appear (see reference [20] for details).

We shall restrict our analyses of building realist supersymmetric models to the case where the number N of supersymmetry generators Q_α is equal to one since, in this case, we have the possibility of describing chiral fermions *). Also, we consider a very elegant and useful technique to deal with supersymmetry, namely, the superfield formalism introduced by Salam and Strathdee [21]. Here the representation of supersymmetry on the superspace is considered. Elements of the $N = 1$ superspace are labeled by the coordinates $Z^A = (x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})$, where θ^α and $\bar{\theta}^{\dot{\alpha}}$ are respectively left- and right-handed anticommuting Weyl spinors.

Superfields are just functions of superspace coordinates, defined in such a way that a supersymmetry transformation of a superfield consists of a total derivative in superspace. Then, the integral over superspace of a scalar function of superfields is certainly invariant under supersymmetric transformations. This is the way to write supersymmetric Lagrangians using the superspace technique.

Before writing the general form of a global $N = 1$ supersymmetric invariant Lagrangian, it is useful to consider particular superfields which define irreducible supersymmetric representations. Let us consider scalar superfields which transform as scalar fields under Poincaré group. A chiral superfield

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*) It is possible to verify that if $N \geq 2$ right- and left-handed fermions lie in the same representation of the supersymmetry group. Many problems arise from this, e.g., $W^\pm$ couples equally to right- and left-handed fermions inducing parity conservation in contradiction with the experimental data.

$\Phi(Z)$ satisfies the further constraint

$$\bar{D}_{\dot{\alpha}}\Phi(Z) = 0, \tag{1.1}$$

while, for an antichiral superfield:

$$D_{\alpha}\Phi(Z) = 0, \tag{1.2}$$

where $\bar{D}_{\dot{\alpha}}$ and D_{α} are covariant derivatives which, in a convenient representation, take the form:

$$D_{\alpha} = \frac{\partial}{\partial\theta^{\alpha}} + i\sigma^{\mu}_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_{\mu} \tag{1.3}$$

$$\bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i\theta^{\alpha}\sigma^{\mu}_{\alpha\dot{\alpha}}\partial_{\mu}, \tag{1.4}$$

where the σ^{μ} are related to the Pauli matrices σ^{i} (i=1,2,3) in the following way:

$$\sigma^{\mu} = (1, \sigma^{i}). \tag{1.5}$$

Moreover, one defines real superfields imposing the following constraint

$$V(Z) = V^{\dagger}(Z). \tag{1.6}$$

Each superfield contains a multiplet of fields which provides a supersymmetric representation. These component fields can be recovered by expanding the superfield in powers of θ and $\bar{\theta}$, using the anticommuting properties of the fermionic coordinates. The general content of a chiral superfield is given by the complex fields $A(x)$ and $F(x)$ and the Weyl spinor $\psi_{\alpha}(x)$. The field $F(x)$

is an auxiliary field that does not propagate. The real superfield V(Z) contains the vector field $v_\mu(x)$, the Weyl spinor $\lambda_\alpha(x)$ and the auxiliary scalar field $D(x)$, in addition to one real and one complex scalar and one Weyl spinor, which can be gauged away, in the case of massless supermultiplet. A massive real superfield describes one real scalar, one Dirac fermion and a spin 1 field.

At this point we are able to write the general form of the $N = 1$ global supersymmetric invariant Lagrangian. It is just the integral over the superspace of a scalar function of superfields. Obviously, since chiral superfields are defined by (1.1), the integration over $d^2\theta$ (and not over $d^2\bar{\theta}$) is sufficient to construct a supersymmetric invariant Lagrangian when we are dealing with this type of superfields. In this way:

$$L = \int d^4x d^2\theta d^2\bar{\theta}\mathcal{L}_D + \Big(\int d^4x d^2\theta\mathcal{L}_F + h.c.\Big) \tag{1.7}$$

where $\mathcal{L}_D$ and $\mathcal{L}_F$ are a real and a chiral combination of superfields, respectively.

1.b. Building models: low-energy supergravity

In the early attempts of constructing supersymmetric extensions of the standard model, it was soon realized that the superpartners of the known particles could not be identified with any other particle already present in the standard model. This can be easily understood since, e.g., the fermionic partners of the gauge bosons are in a real gauge representation and this is not the case for quarks and leptons. If the Higgs particle was the superpartner of the leptons, the vacuum expectation value of the sneutrinos would break the lepton number, moreover, the sneutrino is not able to give mass to both up and down quarks. Even more, if one introduces only one Higgs doublet superfield, masses for both up and down quarks will not be generated; furthermore, the fermionic partner of the usual Higgs doublet renders the particle content of the model anomalous. Two superfields doublets are sufficient to cancel anomalies and to provide masses to all quarks and leptons. Therefore, a realistic $N = 1$ supersymmetrization of the standard model should at least contain one new particle for each known particle and a further non-standard Higgs superfield. The particle content of the minimal supersymmetric standard model is given in Table 1.1.

The set of fields presented in Table 1.1 defines the minimal particle content of a candidate for a realistic supersymmetric model. The list of superfields is, however, not complete for a basic reason that has to do with the supersymmetry-breaking problem. Supersymmetry cannot be an exact symmetry of Nature, as is evidenced by the fact that the masses of the ordinary particles and those of their superpartners are not the same. The supersymmetry-breaking problem has many similarities with the breaking of a standard bosonic gauge symmetry, since, in fact, supersymmetry, when extended to include gravity [22], is itself a local symmetry. In fact, local

Table 1.1: Minimal supersymmetric standard model particle content.

Superfields	Component Fields	$3_C \times 2_L \times 1_Y$ quantum numbers	Name
Matter Fields			
Q	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$(3, 2, \frac{1}{6})$	quark
	$\begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix}$		squark
u^c	u_L^c	$(3, 1, -\frac{2}{3})$	R−up
	$\tilde{u}_L^c$		squark
d^c	d_L^c	$(3, 1, \frac{1}{3})$	R−down
	$\tilde{d}_L^c$		squark
L	$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	$(1, 2 - \frac{1}{2})$	lepton
	$\begin{pmatrix} \tilde{\nu}_L \\ \tilde{e}_L \end{pmatrix}$		slepton
e^c	e_L^c	$(1, 1, 1)$	R − lepton
	$\tilde{e}_L^c$		slepton
Gauge Fields			
G	$g_i (i = 1, .., 8)$	$(8, 1, 0)$	gluon
	$\tilde{g}_i$		gluino
V	$\begin{pmatrix} W^{\pm} \\ W^3 \end{pmatrix}$	$(1, 3, 0)$	W
	$\begin{pmatrix} \tilde{W}^{\pm} \\ \tilde{W}^3 \end{pmatrix}$		W − ino
B	B	$(1, 1, 0)$	B
	$\tilde{B}$		B − ino

Table 1.1: Supersymmetric standard model particle content (cont').

Superfields	Component Fields	$3_C \times 2_L \times 1_Y$ quantum numbers	Name
Higgs Fields			
H_1	$\begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix}$	$(1, 2, \frac{1}{2})$	Higgs
	$\begin{pmatrix} \tilde{\phi}_1^+ \\ \tilde{\phi}_1^0 \end{pmatrix}$		Higgsino
H_2	$\begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}$	$(1, 2, -\frac{1}{2})$	Higgs
	$\begin{pmatrix} \tilde{\phi}_2^+ \\ \tilde{\phi}_2^0 \end{pmatrix}$		Higgsino

supersymmetry is also called supergravity.

In analogy with the Higgs sector, one needs a (super-Higgs) [23] sector which spontaneously breaks supersymmetry. This contains the Goldstone fermion, which the gravitino (the spin 3/2 superpartner of the spin 2 graviton) eats to become massive. Furthermore one needs some coupling between this (super-Higgs) sector and the standard superparticle sector to transmit the breaking of supersymmetry.

At this point, i.e., in the specification of this coupling, the different models may depart from each other. The models that have received by far the greatest attention are those in which these two sectors communicate only through gravitational couplings (suppressed by powers of $1/M_P$, where M_P is the Planck scale) [17]. The super-Higgs sector is usually called the "hidden sector", since it is coupled to ordinary (super-) matter only through a very weak interaction, the gravity.

One wants to extract the low-energy physics of these models, that is, those effects that are not suppressed by the gravitational coupling itself. These are most simply obtained by considering [24] the limit of infinity Planck scale (M_P), holding fixed the gravitino mass. The result of this limiting procedure is an effective low-energy supergravity theory which is just the global supersymmetric Lagrangian of the ordinary (super-) matter fields plus certain soft supersymmetry-breaking terms

$$L = L_{\text{susy}} + L_{\text{soft}}. \tag{1.8}$$

Among the light particles one will also have, apart from the graviton itself, the gravitino and some debris of the hidden sector, which are, however, decoupled from ordinary matter in the $M_P \to \infty$ limit.

L_{susy} in equation (1.8) is specified by the standard gauge group other than a gauge invariant function $f(\Phi)$ of chiral superfields entering in $\mathcal{L}_F$ (see equation (1.7)), usually called superpotential. $f(\Phi)$ is constrained by renormalizability to be a polynomial in chiral fields of order three at most. The superpotential $f(\Phi)$ of the minimal supersymmetric standard model can be divided in two parts. The first one has a close analogy with the standard Yukawa potential, where the usual fields are promoted to the superfields of Table 1.1:

$$f = h_U Q H_1 u^c + h_D Q H_2 d^c + h_L L H_2 e^c + \mu H_1 H_2 \tag{1.9}$$

Q, u^c, d^c,... are scalar superfields, and so they possess scalar and fermionic components with the same quantum numbers. h_U, h_D and h_L are the corresponding Yukawa coupling constants and μ is a mass coupling between the two Higgses.

Nevertheless, completely "new" renormalizable couplings, with no analogous in the Standard Model, can be also written. Such terms violate lepton (L) number:

$$f^{\Delta L \neq 0} = \frac{1}{2}\lambda_{ijk}[L_i, L_j]e^c_k + \lambda'_{ijk} L_i Q_j d^c_k \tag{1.10}$$

and baryon (B) number:

$$f^{\Delta B \neq 0} = \lambda''_{ijk} u^c_i d^c_j d^c_k. \tag{1.11}$$

First of all, we would like to emphasize that these L- and B-violating terms arise in a complete natural way in the minimal supersymmetric standard model, since, contrarily to the standard model case, renormalizability does not prevent the appearance of such interactions. Nevertheless, it must be noticed that some combinations of these couplings can generate too fast tree-level proton decay and are phenomenologically unacceptable. An appealing way of eliminating unwanted interactions in the Lagrangian is the imposition of a suitable symmetry violated by these interactions. The so-called R-parity [25] is by far the most invoked such symmetry. The R-parity of a particle with baryon number B, lepton number L and spin S, is a multiplicative number associated to this particle defined in the following way:

$$(-1)^{2S+3B+L}. \tag{1.12}$$

In fact, from this definition it is easy to infer that R-parity is $(+1)$ for ordinary standard particles (including Higgses) and (-1) for their supersymmetric partners. Consequently, couplings appearing in (1.10) and (1.11) are characterized by the breaking of the R-parity and do not exist when this symmetry is respected.

As we will see later, the presence of some (not all) of the R-parity violating terms appearing in (1.10) and (1.11) can be phenomenologically viable as can be seen also from the rich literature on the R-parity breaking supersymmetric models [25].

Finally we will conclude this section saying that the set of supersymmetric soft breaking terms takes a quite simple aspect in the case of minimal $N = 1$ supersymmetric theories. It is given by:

$$\begin{aligned} L_{\text{soft}} &= m^2 \sum_i |\phi_i|^2 + M \sum_a \bar{\lambda}^a \lambda^a + \\ &+ [Am(h_U \tilde{Q} H_1 \tilde{u}^c + h_D \tilde{Q} H_2 \tilde{d}^c + h_L \tilde{L} H_2 \tilde{e}^c) + Bm\mu H_1 H_2 + h.c.]. \end{aligned} \tag{1.13}$$

The supergravity induced terms (1.13) contain a common mass m for all scalars (ϕ_i) and a common mass M for all fermionic partners of gauge fields, the gauginos (λ_a are two-components gaugino Weyl spinors). Supergravity is broken in such a way that one obtains m and M of order of Fermi scale. Universal mass terms for the scalars are also necessary to avoid large flavor changing neutral currents at low energies. A and B are two dimensionless coefficients which parametrize the arbitrary choice of the hidden sector. These parameters are taken to be real to avoid the occurrence of new sources of, possibly too large, CP violation.

1.c. Constraints on R-parity breaking interactions

As we have just seen, even in supersymmetric models with the minimal field content, baryon and lepton number conservation are not automatically ensured by gauge invariance as occurs in the standard $SU(3) \times SU(2) \times U(1)$ model. Furthermore, the presence of baryon and lepton-violating terms in low-energy supergravity models are related with the violation of a discrete symmetry called R-parity: the most general (R-parity violating) superpotential contains interactions which break baryon and lepton-number. Since R-parity $(+1)$ is associated with ordinary particles and (-1) with their superpartners, the immediate phenomenological consequence of this R-parity violation is the absence of any mechanism which prevents the lightest supersymmetric particle from decaying and an excess of missing energy in colliders is not any more a signature for supersymmetry.

In the main part of this work we shall be very interested in some specific phenomenological consequences of those interactions which violate R-parity and consequently baryon and/or lepton-number. In spite of that, present experimental evidence is absolutely consistent with the conservation of baryon number and three separate lepton numbers, the electron number L_e, the muon number L_μ and the tau number L_τ. Also, the conversion of a quark of one flavor (d, u, s, c, b, t) into a quark of another flavor is forbidden in strong and electromagnetic interactions. In the standard model, the weak interactions violate this conservation law through interactions of the charged $W^\pm$ gauge boson with quarks in a manner described by the Cabibbo-Kobayashi-Maskawa mixing matrix [26] in such a way that no flavor changing neutral current occurs at tree-level and its appearance at the 1-loop level through W-bosons and quarks exchange is strongly suppressed by the so-called GIM-mechanism [27] *). In this way, we could expect strong constraints on R-

*) Flavor changing neutral current processes represent one of the biggest challenges of any extension of the standard model which predicts new physics at energies not much higher than the Fermi mass scale. In particular, it was realized [28] that even in the (R-parity conserving) minimal supersymmetric standard model, many sources of flavor changing neutral currents, other than the mere supersymmetrization of the one-loop standard model contributions with the quark and W in the internal lines replaced by their superpartners, arise. Particularly important are those processes where a quark interacts with a squark of different flavor mediated by a gluino (for more details of such processes see, e.g., [29] and references therein).

parity violating interactions arising from available data.

First of all, we can notice that if couplings that violate lepton- and baryon-number appearing respectively in equations (1.10) and (1.11) are simultaneously non-vanishing, proton-decay mediated by a superpartner of the right-handed d-quark (the $\tilde{d}_L^c$ squark) arises at tree-level and it is found to be too fast. That is why we will concentrate in this work on models which present just one type of interactions which break R-parity: either those lepton-violating interactions or those baryon-violating ones. A rich literature shows that indeed both possibilities are phenomenologically viable [25]. In particular, we will deal with terms appearing in (1.10) which violate lepton but not baryon number and neglect terms appearing in (1.11).

Some of the expected stringent constraints on the couplings which appear in (1.10) were discussed in reference [30]. Its authors consider the limits on these R-parity breaking interactions arising from processes at low energy which receive some contribution from these interactions when just one of those couplings (λ_{ijk} or λ'_{ijk}) appearing in (1.10) is not vanishing. Their results are displayed in the Table 1.2 that we took directly from reference [30]. We can notice that, roughly speaking, the bounds on the relevant couplings range between 10^{-2} and 10^{-1} for couplings involving the first two generations, whilst they can be less stringent when the third generation is involved. Most of the limits shown in Table 1.2 depend on the assumed values of the masses of the relevant supersymmetric partners of the charged leptons and quarks for which experimental lower bounds lying typically in the interval $(50 \div 120)$ GeV exist. The limits quoted above correspond to values of the slepton and squark masses equal to 100 GeV.

Nevertheless, it was noticed in reference [10] that the simultaneous presence of a pair of those couplings generally leads to limits stronger than the product of the individual limits quoted in Table 1.2. We give below some examples of such situation.

(i) The astonishing experimental upper bound on the branching ratio of the $\mu \to 3e$ process, $\mathrm{BR}(\mu \to 3e) < 1.0 \times 10^{-12}$ [31], gives strong limits on the product $\lambda_{122}\lambda_{121}$ entering the one-loop contribution to this process:

$$\lambda_{122}\lambda_{121} \lesssim 10^{-6}\Big(\frac{m_0}{100\,\mathrm{GeV}}\Big)^2. \tag{1.14}$$

(ii) From the one-loop $\mu \to e\gamma$ transition which contribution to the associated branching ratio must be roughly smaller than the experimental bound $\mathrm{BR}(\mu \to e\gamma) < 4.9 \times 10^{-11}$ [31], we get:

$$\lambda_{233}\lambda_{133} \lesssim 10^{-5}\Big(\frac{m_0}{100\,\mathrm{GeV}}\Big)^2. \tag{1.15}$$

(iii) In the same way [10], no limit exists on the product $\lambda_{123}\lambda_{232}$ of the couplings entering the one-loop diagram contributing to the non-diagonal element $m_{\nu_e\nu_\mu}$ of the neutrino mass matrix stronger than the limit obtained requiring that this same contribution be less than 10 eV:

$$\lambda_{123}\lambda_{232} \lesssim 4 \times 10^{-4}\frac{m_0}{\tilde{m}}\frac{m_0}{100\,\mathrm{GeV}}. \tag{1.16}$$

The parameters m_0 and $\tilde{m}$ appearing in equations (1.14)-(1.16) are a common sfermion mass and a typical supersymmetric mass, respectively. Both these masses can be taken to be approximately equal to 100 GeV.

ijk	$\lambda_{ijk} <$
121	$0.10^{(f)X}\ 0.04^{(a)}\ 0.29^{(e)}$
122	$0.10^{(f)X}\ 0.04^{(a)}\ 0.34^{(e)}$
123	$0.04^{(a)}\ 0.34^{(e)}\ 0.24^{(f)}$
131	$0.10^{(c)}\ 0.24^{(f)}$
132	$0.10^{(f)X}\ 0.10^{(c)}$
133	$0.10^{(c)}\ 0.24^{(f)}$
231	$0.09^{(d)X}\ 0.10^{(f)X}\ 0.26^{(e)}\ 0.12^{(c)}\ 0.24^{(f)}$
232	$0.09^{(d)X}\ 0.12^{(c)}$
233	$0.09^{(d)X}\ 0.12^{(c)}$

ijk	$\lambda'_{ijk} <$
111	$0.03^{(a)X}0.05^{(b)}0.26^{(g)}0.30^{(g)}$
112	$0.03^{(a)X}0.05^{(b)}0.30^{(g)}$
113	$0.03^{(a)X}0.05^{(b)}0.26^{(f)}0.30^{(g)}$
211	$0.22^{(h)X}0.09^{(b)}0.11^{(h)}$
212	$0.09^{(b)}0.11^{(h)}$
213	$0.09^{(b)}0.11^{(h)}$
121	$0.26^{(g)}0.45^{(f)}$
122	$0.45^{(f)}$
123	$0.26^{(f)}0.45^{(f)}$
133	$0.26^{(f)}$
221	$0.22^{(h)X}$
231	$0.22^{(h)X}$
131	$0.26^{(g)}$

Table 1.2: The 1σ limits on the R-breaking couplings λ and λ' of equation (1.10) in units of $(m_{\tilde{f}}/100\,\mathrm{GeV})$, where $m_{\tilde{f}}$ is the appropriate sfermion mass, from: (a) charged-current universality; (b) $\Gamma(\pi \rightarrow e\nu)/\Gamma(\pi \rightarrow \mu\nu)$; (c) $\Gamma(\tau \rightarrow e\nu\bar{\nu})/\Gamma(\tau \rightarrow \mu\nu\bar{\nu})$; (d) $\Gamma(\tau \rightarrow \mu\nu\bar{\nu})/\Gamma((\mu \rightarrow e\nu\bar{\nu})$; (e) $\nu_\mu e$ scattering; (f) forward-backward asymmetry in e^+e^- collisions; (g) atomic parity violation and eD asymmetry; (h) ν_μ deep-inelastic scattering. The superscript X denotes that the corresponding number is at the 2σ level, and is excluded at 1σ level.

(iv) It is possible also to obtain some similar bounds on the products of couplings λ'_{ijk} coming from the existing upper limit on the branching ratio of the lepton charge nonconserving decays like $\tau \to \rho^0 + e$ of the τ-lepton. Other L_e and L_τ nonconserving τ decays, like $\tau^\pm \to e^\pm + \gamma$, $\tau^\pm \to e^\pm + e^+ + e^-$, etc. , are also possible due to (1.10), but the leading contribution in the $\tau^\pm \to \rho^0 + e^\pm$ decay amplitude corresponds to tree diagrams with exchange of virtual $\tilde{t}_L$-squark leading to strong limits. The experimental bound $BR(\tau^- \to \rho^0 + e^-) < 4 \cdot 10^{-5}$ [31] implies [12]:

$$|\lambda'_{131}\lambda'_{331}| < 1.8 \times 10^{-2}(\frac{m(\tilde{t})}{100\ GeV})^2, \tag{1.17}$$

where $m(\tilde{t})$ is the effective stop mass $^{*)}$. A similar, though slightly weaker, bound is obtained considering the decay $\tau^\pm \to \pi^0 + e^\pm$.

There are some bounds on λ_{ijk} and λ'_{ijk} entering equations (1.10) which are by far more stringent than those ones presented above. They come from our understanding of the observed matter-antimatter asymmetry in the universe. The possibility of generating the cosmological baryon asymmetry via non-equilibrium B-, C- and CP-violating interactions in the early universe

$^{*)}$ In the case of the $\tilde{t}_{L,R}$-squarks, given that the $\tilde{t}_L - \tilde{t}_R$ mixing term in the $\tilde{t}_{L,R}$ mass matrix is proportional to the t-quark mass, there might exist a relatively light mass eigenstate $\tilde{t}_1$ together with maximal mixing ($\theta_{\tilde{t}} \simeq \pi/4$). In equation (1.17) $m(\tilde{t})$ would then denote the quantity $m(\tilde{t}_1)\sqrt{2}$. In the general case $m^{-2}(\tilde{t}) = m^{-2}(\tilde{t}_1)\cos^2\theta_{\tilde{t}} + m^{-2}(\tilde{t}_2)sin^2\theta_{\tilde{t}}$, where $\tilde{t}_2$ is the heavier mass eigenstate of the $\tilde{t}_L - \tilde{t}_R$ mass matrix.

[32] is vulnerable to the effects of lepton and baryon-number violating interactions after the grand unified epoch since these interactions may erase any cosmic baryon-asymmetry previously existing. Even in the standard model, quantum effects can lead to transitions which violate baryon and lepton number when the temperature is above the Fermi scale [33], however the linear combination $B - L$ is conserved throughout. In this case it is possible to preserve a preexisting baryon-asymmetry if a $B - L$ excess existed at very early time. It is not difficult, in fact, to achieve such excess in most Grand Unification Theories (GUT). However, if there were also other interactions which violate $B - L$, any pre-existing baryon-asymmetry could be eliminated [34]. This is what would occur if the R-parity violating terms in (1.10) led relatively large $B - L$ violating processes. To avoid this problem, λ_{ijk} and λ'_{ijk} couplings should be small enough to keep the $B - L$ violating phenomena they generate slower than the expansion of the universe at any moment before the electroweak phase transition. These argumentations lead to the severe constraints [34]:

$$\lambda_{ijk}\,,\ \lambda'_{ijk} \leq 10^{-8} \tag{1.18}$$

Bounds (1.18) are sufficiently strong to spoil any possibility of observing some effect induced by broken R-parity interactions. Nevertheless, even if there is a widespread consensus that baryon and lepton violating processes become strong enough at temperatures above the Fermi barrier, there exists at least two ways to avoid the constraints (1.18) [35]. The first obvious option would be to regenerate the erased cosmological baryon asymmetry at temperatures below the electroweak phase transition. This proves to be very difficult, though not impossible. A second and more attractive way is the spontaneous breaking of R-parity at a temperature where non perturbative effects are no longer capable of producing baryon and lepton violating fast transitions *). Therefore, we believe that constraints (1.18) can be avoid.

The bounds on the products of couplings constants λ_{ijk} and λ'_{ijk}, shown in equations (1.14)-(1.17), will be particularly useful for our discussion of possible contributions to neutrino magnetic moment coming from these R-parity broken supersymmetric models and matter-enhanced neutrino oscillations induced by flavor changing neutral currents, to be discussed in Chapter 2 and Chapter 3, respectively.

*) For an example of spontaneous breaking of R-parity see, e.g., reference [36].

2

Neutrino Magnetic Moment

A sizeable neutrino magnetic moment has been evoked as a possible way to explain [8] the observed reduction of the solar neutrino flux and its apparent modulation with the solar activity [2]. Such depletion of the flux of neutrinos coming from the Sun could be physically understood in the following way: If the solar neutrino is a Dirac particle, the magnetic field in the convective zone of the Sun will eliminate part of the original left-handed electron neutrinos ν_{e_L} rotating them into right-handed neutrinos ν_{e_R}, which do not feel ordinary weak interactions and escape detection. If the solar neutrino is a Majorana particle, and lepton-number violating interactions exist, one will find magnetic transitions from left-handed electron neutrinos ν_{e_L} to right-handed antineutrinos of different flavor, say, muon or tau antineutrinos ($\nu^c_{\mu R}$ or $\nu^c_{\tau R}$), when crossing the solar convective zone. Even if $\nu^c_{\mu R}$ and $\nu^c_{\tau R}$ are sensitive to weak interactions, they are very badly detected in present experiments for energetic reasons. In order for this depletion to be effective and a good description of the solar neutrino data [2] achieved, the neutrino magnetic moment μ_ν should be of the following size [8]:

$$\mu_\nu \gtrsim 10^{-11}\mu_B\left(10^4\,\text{Gauss}/B_\odot\right), \tag{2.1}$$

where μ_B is the Bohr magneton and $B_\odot$ is the magnetic field in the convective zone of the Sun that can vary in the range 10^3 Gauss $\lesssim B_\odot \lesssim 10^5$ Gauss. Furthermore , in the particular case where the magnetic moment is of (transition) Majorana type, in order to have the solar ν_{e_L} effectively transmuted into a neutrino of different flavor, say, $\nu^c_{i_R}$ $(i = \mu, \tau)$ by the magnetic field, these two energy levels should be highly degenerate. Namely, the following inequality should take place:

$$\Delta m^2_\nu = |m^2_{\nu_e} - m^2_{\nu_i}| < 10^{-7}\text{eV}^2. \tag{2.2}$$

In this chapter we would like to discuss the possible contributions to the neutrino magnetic moment arising in models where neutrino flavor changing neutral current interactions are present. We will follow the idea presented in reference [37] of connecting a sizeable neutrino magnetic moment with R-parity breaking in supersymmetric theories to give a more complete analysis of the problem, and get the naturally attainable values of the neutrino magnetic moments in different versions of the theory. Applying a naturalness criterion, i.e., barring accidental cancellations among possible different contributions to the neutrino or the charged fermion masses, we come to the following conclusions [10]. In the minimal supersymmetric standard model with R-parity broken in the superpotential, one can have neutrino magnetic moments of order $\mu_\nu \simeq 10^{-13}\mu_B$. In this version of the theory, the main constraint on μ_ν is set by the upper bound on the neutrino masses ($m_{\nu_e} \lesssim 10eV$ from laboratory experiments, $m_{\nu_\mu}, m_{\nu_\tau} \lesssim 100eV$ from cosmology). Although a value of $\mu_\nu \simeq 10^{-13}\mu_B$ is not close to the present laboratory bounds ($\mu_\nu \lesssim 10^{-10} - 10^{-9}\mu_B$ for the electron and muon neutrinos), it is far larger than the "standard model value" [11] *) of the neutrino magnetic moment $\mu_\nu/\mu_B \lesssim (10^{-19} - 10^{-18})(m_{\nu_e}/eV)$.

To obtain $\mu_\nu \simeq 10^{-13}\mu_B$ in the minimal supersymmetric standard model does not require any particular symmetric structure of the Lagrangian. If such a symmetry is implemented, which requires departing from the minimal supersymmetric standard model, although still remaining with minimal particle content, an order of magnitude can be gained, leading to $\mu_\nu \lesssim 10^{-12}\mu_B$. This upper bound is obtained by applying the same naturalness criterion as before. In this case, the main constraint comes from the electron mass (or the d-quark mass) rather than the neutrino mass itself.

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*) Since in the standard model there is no right-handed neutrino and leptonic flavor changing neutral currents do not exist as well, the neutrino magnetic moment is identically vanishing. By "standard model value" of the neutrino magnetic moment we mean the value we would obtain in the standard model adding a right-handed neutrino to its particle content as $SU(2)_L$ singlet.

—2.a. Neutrino magnetic moment in the R-parity broken MSSM

The minimal supersymmetric standard model with explicit R-parity breaking via L-violation is described by the superpotential given in equations (1.9) and (1.10). The couplings induced by this superpotential are the ones that give rise to the neutrino magnetic moments, as well as to the neutrino mass. In particular, Figure 2.1 shows all the one loop diagrams, arising from the purely leptonic couplings in (1.10), that contribute to the ν_e-ν_μ magnetic moment $\mu_{\nu_e \nu_\mu}$ after insertion of a photon vertex on any internal line. The diagrams of Figure 2.1 (a),(b) are characterized by the fact that the couplings involved conserve the difference between the electron and muon lepton number $L_e - L_\mu$, whereas the diagrams of Figure 2.1 (c),(d) respect L_τ. Analogous diagrams exist, originating from the λ'_{ijk} couplings in (1.10) involving the quark fields, with the internal leptons (sleptons) replaced by quarks (squarks). In all of these diagrams, that contribute as well to the mass term $m_{\nu_e \nu_\mu}$ if no photon vertex is inserted, an helicity flip on the internal fermion line is necessary. As explicitly indicated, this also requires a mixing of the scalar leptons associated with the different chiralities, described by the scalar Lagrangian term $\Delta_{ij} \tilde{l}_i \tilde{l}^c_j$.

In a general softly broken supersymmetric lagrangian, the mixing square mass matrix Δ_{ij} receives two distinct contributions: one from a supersymmetric coupling, which in the case of minimal supersymmetric standard model can be read from the superpotentional (1.9): $\mu(< H_2 > / <$

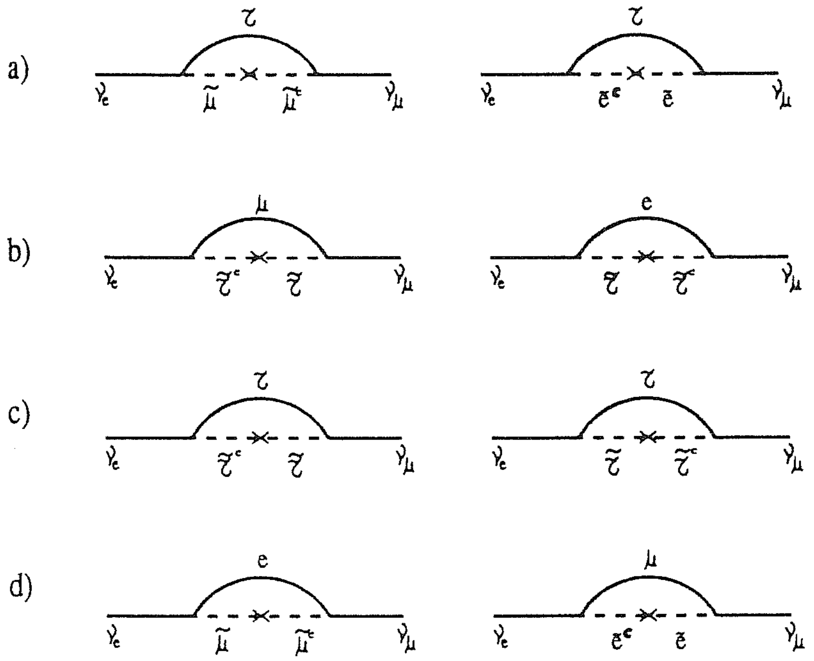

Figure 2.1: Supersymmetric contributions to $m_{\nu_e \nu_\mu}$ and to $\mu_{\nu_e \nu_\mu}$, when a photon line is inserted on each internal line.

$H_1 >) m_i \delta_{ij}$ in the physical basis for the leptons of mass m_i ($<>$ indicates vacuum expectation value), and another term $\Delta_{ij}^{\rm soft}$ coming from the soft supersymmetry-breaking Lagrangian. The minimal supersymmetric standard model is characterized by the property that also $\Delta_{ij}^{\rm soft}$ is diagonal in the physical leptons basis and proportional to the lepton masses. One can therefore write

$$\Delta_{ij}^{\rm minimal} = \tilde{m} m_i \delta_{ij} \qquad (2.3)$$

where $\tilde{m}$ can be obtained from (1.9) and (1.13): $\tilde{m} = \mu(< H_2 > / < H_1 >$

) + Am. Furthermore, we can notice that in the limit of vanishing lepton masses, the sleptons are degenerate at a mass $m_0 = m$ (the splitting between left and right sleptons is irrelevant to the present discussion).

Notice that, in general, many more diagrams can be drawn, contributing to the diagonal neutrino mass terms, whereas the corresponding contributions to the diagonal neutrino magnetic moments vanish by Lorentz symmetry.

Any of the diagrams of Figure 2.1 contribute to $m_{\nu_e \nu_\mu}$ and $\mu_{\nu_e \nu_\mu}$ (after insertion of the photon line) respectively as

$$\delta m_{\nu_e \nu_\mu} \simeq \frac{\lambda_a \lambda_b}{16\pi^2} m_l \frac{\Delta}{m_0^2} \tag{2.4a}$$

$$|\delta \mu_{\nu_e \nu_\mu}| = \mu_B \frac{\lambda_a \lambda_b}{2\pi^2} \frac{m_e m_l}{m_0^2} \left(\log\left(\frac{m_0}{m_l}\right) - 1 \right) \frac{\Delta}{m_0^2} \tag{2.4b}$$

where m_l is the mass of the internal charged lepton, λ_a and λ_b are the appropriate couplings, and Δ is the required mixing term. Of course $m_l/m_0 \ll 1$ and we are also making an expansion in Δ/m_0^2. For any of the individual corresponding contributions in (2.4) we have

$$|\delta \mu_{\nu_e \nu_\mu}| = \mu_B \delta m_{\nu_e \nu_\mu} \frac{8 m_e}{m_0^2} \left(\log\left(\frac{m_0}{m_l}\right) - 1 \right). \tag{2.5}$$

For each single product $\lambda_a \lambda_b$ there are two comparable graphs contributing to $\delta m_{\nu_e \nu_\mu}$ and to $|\delta \mu_{\nu_e \nu_\mu}|$. If we now require that each contribution to $\delta m_{\nu_e \nu_\mu}$ proportional to a fixed product $\lambda_a \lambda_b$ be less than $10 eV$, we get a bound on the corresponding $|\delta \mu_{\nu_e \nu_\mu}|$ with a logarithmic dependence on the masses m_l of the lepton exchanged. Numerically

$$|\delta\mu_{\nu_e\nu_\mu}| \lesssim \mu_B \cdot (0.2\ -\ 0.4) \cdot 10^{-13} \left(\frac{100\mathrm{GeV}}{m_0}\right)^2 , \tag{2.6}$$

where the coefficient depends on which pair of graphs one is considering. From the negative experimental searches of sleptons, m_0 can be as low as 50 GeV, implying

$$|\delta\mu_{\nu_e\nu_\mu}| \leq O(10^{-13}\mu_B). \tag{2.7}$$

This bound can be saturated if and when the bound on the neutrino mass is saturated. In turn, for any given pair of diagrams (with definite lepton-slepton pair exchange), using (2.3) for the scalar mixing, this only depends on the values of the L-violating couplings λ_a, λ_b.

Effects associated with the individual presence of any of these couplings have been analysed in reference [30] and shown in Table 1.2 of Chapter 1. The corresponding bounds obtained are typically of order $|\lambda_{ijk}|, |\lambda'_{ijk}| \lesssim O(10^{-1})$. We have already seen also that the simultaneous presence of a pair of couplings required by the mass or the magnetic moment diagrams generally leads to stronger limits. Rare processes like $\mu \to 3e$ or $\mu \to e\gamma$ give bounds on pairs of λ_{ijk} (see equations (1.14) and (1.15)) stronger than the products of the individual limits [30] which do not allow the saturation of (2.6) by the corresponding diagrams. On the other hand, the limits on the products $\lambda_{123}\lambda_{232}$ of the couplings entering the first ones of the diagrams of Figure 2.1 (a),(b) come from exactly the requirement that this contribution to $m_{\nu_e\nu_\mu}$ must be less than $10eV$, resulting equation (1.16). As a consequence, the related diagrams may saturate the corresponding neutrino mass and may give contributions to the magnetic moments of order $10^{-13}\mu_B$.

We neglected in our discussion the contribution of the second two diagrams of Figure 2.1 (a),(b) since it cannot saturate the bound (2.7). In fact, these diagrams, which in the minimal supersymmetric standard model of equation (2.3) are proportional to the electron mass, are mostly limited by the product $\lambda_{123}\lambda_{131} < 4 \cdot 10^{-3}(m_0/100\text{GeV})^2$ of the corresponding individual bounds listed in Table 1.2. [30]. This ensures that their contribution to $m_{\nu_e\nu_\mu}$ be an order of magnitude smaller than 10 eV.

Analogous considerations can be made for the diagrams involving the quark and squarks exchanges. Some of them may indeed give contributions comparable to the largest terms involving leptons (sleptons) only ($\delta\mu \simeq O(10^{-13}\mu_B)$). However one must be careful about the effects induced by the Cabibbo-Kobayashi-Maskawa mixings which result into the breaking, together with the R-parity breaking couplings, of all the individual lepton numbers.

2.b. Departing from Minimality

We have seen that the minimal supersymmetric standard model may naturally give neutrino magnetic moments which far exceed the so-called

"Standard Model value", without invoking any particular symmetry structure of the Lagrangian. This suggests to ask whether a suitable departure from the minimal supersymmetric standard model may lead to even bigger neutrino magnetic moments. The role of appropriate approximate symmetries of the Lagrangian in connection with the neutrino problem has been recently emphasized [38][39].

Along these lines we require that our Lagrangian, in the limit of vanishing Yukawa couplings, be symmetric under an $SU(2)_H$ horizontal symmetry acting on the first and the second generations. In connection with the neutrino magnetic moment, this same symmetry has already been invoked in a non supersymmetric context in reference [39] and in the supersymmetric case in reference [37]. In our view all of these models have problems with the $e-\mu$ mass splitting, which represents a violation of the $SU(2)_H$-symmetry. The description of the $e-\mu$ mass difference by explicit different Yukawa couplings in a non-supersymmetric context looks unnatural because of the quadratic divergencies induced by loops in asymmetric renormalizable operators. On the other hand, the breaking of the $SU(2)_H$ symmetry by soft terms [37] may account for the $e-\mu$ mass difference generated by radiative corrections only by an unnatural fine tuning.

Here we propose that the $SU(2)_H$ symmetry be respected in the Yukawa-less limit of the supersymmetric Lagrangian, including the soft breaking terms and the L-violating couplings, being just violated by the standard supersymmetric Yukawa couplings for the electron and the muon. For the consistency of the theory, we rely on the absence of quadratic divergencies in supersymmetric theories. Small violations of the $SU(2)_H$ symmetry will also have to be present, for example, in the scalar mixing terms, but they will be controlled by the small Yukawa couplings and they will not be quadratically divergent. Departing from the minimal supersymmetric standard model, we will assume for these mixings terms

$$\Delta_{11} \simeq \Delta_{22} \gg |\Delta_{11} - \Delta_{22}| \simeq \tilde{m} m_\mu. \tag{2.8}$$

Let us now reconsider the diagrams of Figure 2.1. The $SU(2)_H$ symmetry forbids the diagrams 2.1 (c),(d), since the individual couplings entering the diagrams violate the horizontal $SU(2)_H$. On the other hand, the allowed diagrams 2.1 (a),(b) in the exact $SU(2)_H$ limit contribute to the magnetic moment (after the photon insertion), which is an $SU(2)_H$ singlet term, but do not to the mass, which is a triplet. In practical terms, a cancellation takes place in the mass diagrams 2.1 (a),(b) between the electron (selectron) and the muon (smuon) diagram, whereas the related magnetic moment terms add coherently. This allow to evade the neutrino mass bound as the strongest restriction on the magnetic moment. With respect to the previous considerations in the minimal supersymmetric standard model, we offer a symmetry reason for a natural cancellation among different mass terms.

The cancellation is most efficient in the diagrams 2.1 (a), which are the interesting source of the sizeable magnetic moment. Their contribution to $m_{\nu_e \nu_\mu}$ is

$$\delta m_{\nu_e \nu_\mu} = \frac{\lambda_{123}\lambda_{232}}{16\pi^2} m_\tau \frac{\Delta_{22} - \Delta_{11}}{m_0^2} = \frac{\lambda_{123}\lambda_{232}}{16\pi^2} m_\tau m_\mu \frac{\tilde{m}}{m_0^2}, \tag{2.9}$$

which is safe ($\delta m_{\nu_e \nu_\mu} < 10eV$), provided

$$\lambda_{123}\lambda_{232} < 9 \times 10^{-4} \left(\frac{m_0}{\tilde{m}}\right)\left(\frac{m_0}{100\text{GeV}}\right). \tag{2.10}$$

On the other hand, their contribution to $\mu_{\nu_e \nu_\mu}$ is sensitive to $\Delta_{22} \simeq \Delta_{11}$ and not to their difference. The constraint (2.10) is then not enough to bound $\mu_{\nu_e \nu_\mu}$. Rather it is the contribution to the electron mass shown in Figure 2.2 that provides the main restriction. From this diagram, where a neutral gaugino is exchanged, we have

$$\delta m_e = \frac{e^2}{16\pi^2} \sum_\alpha \left(V_{1\alpha}^2 + \frac{1}{\sin^2\theta_W \cos^2\theta_W} V_{2\alpha}^2 \right) F(m_\alpha)\Delta_{11}. \tag{2.11}$$

where θ_W is the weak mixing angle,

$$F(m_\alpha) = \frac{m_\alpha}{m_\alpha^2 - m_0^2}\left[1 + \frac{2m_\alpha^2}{m_\alpha^2 - m_0^2}\log\left(\frac{m_0}{m_\alpha}\right)\right] \tag{2.12}$$

and m_α is the α-th eigenvalue of the neutralino mass matrix, diagonalized by the matrix $V_{\alpha\beta}$ from the usual basis $\chi_N = (\tilde{\gamma}, \tilde{Z}, \tilde{h}_1, \tilde{h}_2)$. We are, of course, using the fact that only the gaugino couplings are relevant in Figure 2.2 and not the couplings to the Higgsinos $\tilde{h}_{1,2}$. From (2.11) and (2.12), requiring $\delta m_e < 0.5$ MeV, $m_{\chi_N} \gtrsim 20$ GeV and $m_0 \gtrsim 50$ GeV, we infer

$$\Delta_{11} \lesssim 100\text{GeV}^2. \tag{2.13}$$

This bound can either be saturated for light gaugino masses m_α's or for heavy ones $m_\alpha \lesssim 1$ TeV.

The same mass insertion as in Figure 2.2 enters in the τ-neutrino mass diagram of Figure 2.3 as well. However, requiring $\delta m_{\nu_\tau} < 100$ eV for consistency with cosmology and using $\Delta_{22} \simeq \Delta_{11} \lesssim 100\text{GeV}^2$ gives $\lambda_{232} \lesssim$

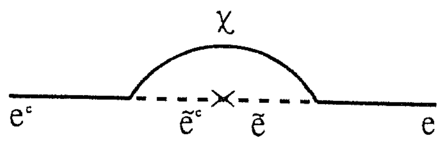

Figure 2.2: Supersymmetric contribution to the electron mass.

$0.05(m_0/100\text{GeV})$, which is only slightly more restrictive than the limit $\lambda_{232} \lesssim 0.09(m_0/100\text{GeV})$ from lepton universality [30] (see Table 1.2) and does not prevent the saturation of the bound (2.10) even taking into account $\lambda_{123} \lesssim 0.04(m_0/100\text{GeV})$ from charged current quark-lepton universality [30] (see again Table 1.2).

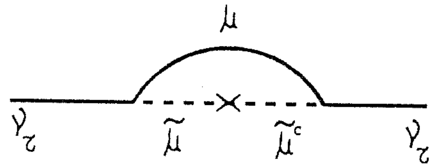

Figure 2.3: Supersymmetric contribution to the tau-neutrino mass.

Putting all together, from the diagrams of Figure 2.1 (a), we find, using (1.16), (2.4b) and (2.13)

$$\delta\mu_{\nu_e \nu_\mu} \lesssim O(10^{-12}\mu_B). \tag{2.14}$$

We think that $\mu_\nu \simeq O(10^{-12} - 10^{-13}\,\mu_B)$ is the largest value one can obtain for the neutrino magnetic moment in a completely natural way in a supersymmetric theory with minimal particle content *).

We have also considered, as in the minimal supersymmetric standard model cases, the possible contribution of the quark-squark diagrams with the $SU(2)_H$-symmetry implemented. In this case a strong constraint comes from the one-loop contribution to the d-quark mass, involving the creation and annihilation of a gluino-sdown pair, analogous to the electron mass diagram of Figure 2.2. Also in view of the bounds on the squarks masses which are stronger than the corresponding ones for the sleptons, the value (2.14) cannot be attained by the quark-squark contributions.

Since we are considering contributions to the transition magnetic moment, in order to obtain a solution of the solar neutrino problem both con-

*) However the authors of reference [40] arrive to some different conclusions. They consider the non-minimal supersymmetric model discussed in Section 2.b. This model takes over the model previously discussed by the same authors [37] with the essential modification of breaking $SU(2)_H$ by the electron and the muon Yukawa couplings rather than by the scalar mass terms. In our opinion, the claim that a neutrino magnetic moment in the range $(10^{-11} - 10^{-10})\mu_B$ can be obtained, underestimates the significance of the bound coming from the electron mass, as we have discussed.

3

MSW effect and FCNC

In the present chapter we will discuss the implications of the existence of flavor changing neutral current (FCNC) interactions of neutrinos with matter leading to matter-enhanced transitions between neutrinos possessing different flavors [12] and the possible solutions of the solar neutrino problem that they imply. We will find that such transitions exist even if neutrinos have zero mass and there is no neutrino mixing in the vacuum [12] (in contrast with the conventional case to be discussed in the next section). We analyse the general features of such transitions using the specific context provided by the supersymmetric theories with broken R-parity discussed in Chapter 1. However, the properties of the neutrino transitions in matter to be discussed in this chapter are general and do not depend on the particular scheme in which such flavor changing neutral current interactions arise.

Taking into account the relevant phenomenological constraints we show that the neutrino flavor changing neutral current interactions can cause a resditions (2.1) and (2.2) must be fulfilled. In our case, the neutrino $\nu_e - \nu_\mu$ components are degenerate since they couple together to form a kind of Dirac neutrino and therefore the condition (2.2) is automatically satisfied. Although the value found for the neutrino magnetic moment is six or seven orders of magnitude larger than the value of μ_ν one would obtain in the Standard Model adding a right-handed neutrino, it is smaller than the value for μ_ν shown in equation (2.1) necessary for the mechanism proposed by Voloshin, Vysotsky and Okun [8] to effectively produce a solution of the solar neutrino problem. Nevertheless, it is important to notice that large uncertainties are associated with the detailed structure of the solar magnetic field. In fact, 10^3 Gauss $\lesssim B_\odot \lesssim 10^5$ Gauss and from equations (2.1) and (2.14) it is possible to conclude that supersymmetric models with broken R-parity may play a relevant role to produce a μ_ν able to explain the observed smaller than expected solar neutrino flux and its apparent anticorrelation with solar activity.

onant transition of the solar electron neutrinos into, e.g., tau neutrinos with a probability compatible with the qualitative features of the solar neutrino observations. The transition in question can take place in the core of the Sun and can be of adiabatic type. The corresponding transition probability does not depend on the solar neutrino momentum.

We analyse also the possibility that neutrinos do have a mass, but the effects of the existence of vacuum neutrino mixing on the solar neutrino transitions are negligible, the transitions of interest being induced by neutrino flavor changing neutral current interactions with the nucleons in the Sun [12][13]. The solution of the solar neutrino problem one obtains in this case is practically equivalent to the "conventional" nonadiabatic solution found in references [41] and [42].

Some phenomenological implications of the particular scheme with neutrino flavor changing neutral current interactions considered are also discussed. The most interesting prediction for lepton physics is that decays in which the τ lepton charge is not conserved, in particular $\tau^{\pm} \rightarrow \rho^{0} + \ell^{\pm}$ and $\tau^{\pm} \rightarrow \pi^{0} + \ell^{\pm}$, should occur with branching ratios close to the existing experimental upper limits.

3.a. MSW effect in the conventional case

The conventional approach involving matter-enhanced neutrino oscillations, the so-called MSW effect [4][5][6] (see also the review articles [43][44]), and the related possible solutions [4][45][46] to solar neutrino problem assumes [7] that the states of the flavor neutrinos ν_ℓ, $\ell = e, \mu, \tau$, produced with definite momentum $\vec{p}$ in vacuum in weak interaction processes are coherent superpositions of the states of neutrinos ν_i, $i = 1, 2, 3$, having the same momentum $\vec{p}$ and definite masses m_i, $i = 1, 2, 3$, some of which are nonzero. It is also implicitly assumed that the weak interaction of the neutrinos is described by the Lagrangian of the standard electroweak theory [47], in other words, that the relevant mechanism of neutrino mass generation does not lead to new nonnegligible interactions of neutrinos with the particles forming the matter. Two conditions have to be fulfilled in this case in order for the transitions (oscillations) between neutrinos possessing different flavors $\nu_\ell \Leftrightarrow \nu_{\ell'}$, $\ell \neq \ell'$, $\ell, \ell' = e, \mu, \tau$, to be possible: at least two of the neutrinos ν_i with definite mass in vacuum must be mass-nondegenerate, and nontrivial neutrino (lepton) mixing in vacuum must exist. In particular, the two neutrino oscillations, say $\nu_e \Leftrightarrow \nu_\mu$, which are characterized in vacuum by two parameters $\Delta m^2 = m_2^2 - m_1^2$ and θ, where m_1 and m_2 are the masses of the corresponding two vacuum mass eigenstate neutrinos (ν_1 and ν_2) and θ is the neutrino mixing angle in vacuum, can take place only provided $\Delta m^2 \neq 0$ and $\sin 2\theta \neq 0$.

Let us first quickly review the "conventional" two generation case where a neutrino state is assumed to be a linear combination of the two flavor states $|\nu_e>$ and $|\nu_\mu>$, for instance:

$$|\nu, t> = C_e(t)|\nu_e> + C_\mu(t)|\nu_\mu> . \tag{3.1}$$

If the neutrinos are massive, then the mass eigenstates need not be identical to the flavor eigenstates, so that the Dirac equations which govern the evolution of the neutrino state are not necessarily diagonal in the flavor basis. This leads to the well known phenomena of vacuum neutrino oscillations. In the presence of matter, the nondiagonal nature of this evolution is further enhanced by coherent forward scattering which can lead to resonant neutrino oscillations. The evolution equations describing this process, up to a term proportional to the identity which contributes just to an overall phase factor to the state $|\nu, t>$, can be written as:

$$\begin{aligned} & i\frac{d}{dt}\begin{pmatrix} C_e \\ C_\mu \end{pmatrix} = \\ & = \begin{pmatrix} 0 & \Delta m^2 \sin 2\theta/4p \\ \Delta m^2 \sin 2\theta/4p & \Delta m^2 \cos 2\theta/2p - \sqrt{2}G_F N_e \end{pmatrix} \begin{pmatrix} C_e \\ C_\mu \end{pmatrix}, \end{aligned} \tag{3.2}$$

where $p = |\vec{p}|$, N_e is the number density of electrons and G_F is the Fermi constant. The constraints $\Delta m^2 > 0$ and $\theta < \pi/4$ are assumed. At a given electron density, N_e, the matter mass eigenstates are

$$\begin{aligned} |\nu_1, N_e> &= \cos\theta_m|\nu_e> - \sin\theta_m|\nu_\mu>, \\ |\nu_2, N_e> &= \sin\theta_m|\nu_e> + \cos\theta_m|\nu_\mu>, \end{aligned} \tag{3.3}$$

where the mixing angle in matter θ_m is given by

$$\sin 2\theta_m = \frac{\sin 2\theta}{\sqrt{(\cos 2\theta - \sqrt{2}G_F N_e 2p/\Delta m^2)^2 + \sin^2 2\theta}}. \tag{3.4}$$

From this equation (3.4) we can define the resonant condition for which $\sin 2\theta_m$ is maximal, i.e., equal to the unity:

$$N_e = N_e^{\rm res} \equiv \frac{\Delta m^2}{2p\sqrt{2}G_F} \cos 2\theta. \tag{3.5}$$

The states appearing in (3.3) evolve in time by the multiplication of a phase factor, if the electron density is a constant. For such a constant density there are three regions of interest:

(i) Well below resonance, $N_e << N_e^{\rm res}$, where the matter mixing angle is $\theta_m \simeq \theta$ and the oscillation length equals to that one in vacuum, $L_{\rm vacuum} = 4\pi p/\Delta m^2$. Typically, this is the region where the solar neutrinos are detected in.

(ii) At resonance, $N_e = N_e^{\rm res}$, where the matter mixing angle is $\theta_m = \pi/4$ and the oscillation length is $L_{\rm res} = L_{\rm vacuum}/\sin\theta$, which for small vacuum mixing angle can be many times the vacuum oscillation length.

(iii) Far above resonance, $N_e >> N_e^{\rm res}$, where the matter mixing angle $\theta_m \sim \pi/2$, and the oscillation length $L_{\rm matter} = 2\pi/[(\Delta m^2/2p \cos 2\theta - \sqrt{2}G_F N_e)^2 + (\Delta m^2/2p \sin 2\theta)^2]^{1/2}$ is much smaller than the vacuum oscillation length $L_{\rm vacuum}$.

For the situation of current interest, the solar neutrino case, matter density varies along the neutrino path. Neutrinos are typically produced above resonance, pass through resonance and are detected in vacuum. According to the standard solar model (SSM) [1], the number densities of electrons (protons) and neutrons have spherically symmetric distributions in the Sun, they decrease monotonically, approximately exponentially, in the radial direction from the center to the surface of the Sun. Thus, one has along the path of the solar neutrinos moving radially in the Sun:

$$N_e(t) = N_e(t_0)e^{-(t-t_0)/r_0}, \tag{3.6}$$

where $N_e(t_0)$ is the electron number density in the point of neutrino production and r_0 is a constant - the "scale height". Equation (3.6) provides a rather accurate description of the change of N_e along the neutrino path in the Sun, predicted by the standard solar model, for $r_0 \simeq 0.095 R_\odot$, $R_\odot = 6.96 \cdot 10^5$ km being the solar radius.

Furthermore, solar electron neutrino can undergo two different types of transitions in the Sun [4]. If the electron number density changes sufficiently slowly along the neutrino path, the transition is adiabatic, otherwise it is called nonadiabatic. The condition which determines the type of change of the electron number density $N_e(t)$ in a given point (reached by the neutrino at time t) of the neutrino trajectory can be written in terms of the adiabaticity parameter [4]

$$4n(t) = \sqrt{2} G_F \frac{(N_e^{\rm res})^2}{|dN_e/dt|} \mathrm{tg}^2 2\theta \left(1 + \frac{[1 - N_e(t)/N_e^{\rm res}]^2}{\mathrm{tg}^2 2\theta}\right)^{3/2}. \tag{3.7}$$

If we have along the whole neutrino trajectory

$$4n(t) >> 1 \tag{3.8}$$

the transition is of adiabatic type. In the case where

$$4n(t) \lesssim 1, \tag{3.9}$$

in some points of the neutrino trajectory, we have nonadiabatic transitions. In the case of the Sun ($N_e(t)$ is a smooth monotonically decreasing function of $(t - t_0)$) the quantity $4n(t)$ takes its minimal value at the resonance. Therefore, a given transition will be adiabatic or nonadabatic if the inequality (3.8) or (3.9), respectively, is satisfied at the resonance point [4][48][43]

$$4n_0 >> 1 \longrightarrow \text{adiabatic transition}, \tag{3.10}$$

$$4n_0 \lesssim 1 \longrightarrow \text{nonadiabatic transition}, \tag{3.11}$$

where

$$4n_0 = 4n(t)|_{N_e(t)=N_e^{\text{res}}} \;. \tag{3.12}$$

Criterion (3.10) (or (3.11)) is applicable only for transitions of neutrinos which pass through the resonance point. From equations (3.7) and (3.12) we obtain

$$4n_0(t) = \sqrt{2}G_F \frac{(N_e^{\text{res}})^2}{|dN_e/dt|_{\text{res}}} \text{tg}^2 2\theta. \tag{3.13}$$

The probability that solar electron neutrinos will not transform into neutrinos of different type on their way from the central region to the surface of the Sun and further to the Earth surface $P_{\odot}(\nu_e \to \nu_e; t, t_0)$ (t_0 is the time of neutrino production, $t \geq t_0$) can be represented as a sum of two terms

$$P_{\odot}(\nu_e \to \nu_e; t, t_0) = \bar{P}_{\odot}(\nu_e \to \nu_e; t, t_0) + P_{\odot}^{\rm osc}(\nu_e \to \nu_e; t, t_0) \tag{3.14}$$

where $\bar{P}_{\odot}(\nu_e \to \nu_e; t, t_0)$ is the average probability to find a solar neutrino with momentum $\vec{p}$ at the Earth surface and $P_{\odot}^{\rm osc}(\nu_e \to \nu_e; t, t_0)$ is an oscillating term.

If the neutrino transitions in the Sun are adiabatic (i.e., $N_e(t)$ changes sufficiently slowly and equation (3.10) is fulfilled) [4][43]

$$\bar{P}_{\odot}(\nu_e \to \nu_e; t, t_0) = \bar{P}_{A}(\nu_e \to \nu_e; t, t_0) = \frac{1}{2} + \frac{1}{2}\cos 2\theta_m(t_0)\cos 2\theta \tag{3.15}$$

and

$$P_{\odot}^{\rm osc}(\nu_e \to \nu_e; t, t_0) = P_{A}^{\rm osc}(\nu_e \to \nu_e; t, t_0) = \frac{1}{2}\sin 2\theta_m(t_0)\sin 2\theta \times \\ \times \cos\left\{\frac{\Delta m^2}{2p}\int_{t_0}^{t}\left[\left(1 - \frac{N_e(t')}{N_e^{\rm res}}\right)^2 \cos^2 2\theta + \sin^2 2\theta\right]^{1/2} dt'\right\}. \tag{3.16}$$

While, in the general case, it was shown in references [49], [50] and [51] that for fixed $\sin^2 2\theta$ and $N_e(t_0)$ the two terms appearing in (3.14), the average probability and the oscillating term, are given, respectively, by

$$\bar{P}_{\odot}(\nu_e \to \nu_e; t, t_0) = \bar{P}_{\odot}^{\rm exp}(\nu_e \to \nu_e; t, t_0) \tag{3.17}$$

and

$$P_{\odot}^{\rm osc}(\nu_e \to \nu_e; t, t_0) = P_{NA}^{\rm osc}(\nu_e \to \nu_e; t, t_0) \tag{3.18}$$

for any value of $p/\Delta m^2$ if the value of $\sin^2 2\theta$ is not smaller than few times 10^{-3} *). As we will see later, small values of $\sin^2 2\theta$ ($\sin^2 2\theta < 4 \times 10^{-3}$) are disfavoured by the solar neutrino data. That is why we will use equations (3.17) and (3.18) as a good approximation for the probability $P_{\odot}(\nu_e \to \nu; t, t_0)$ **). In these equations, we have

$$\bar{P}_{\odot}^{\exp}(\nu_e \to \nu_e; t, t_0) = \frac{1}{2} + \left(\frac{1}{2} - P'\right) \cos 2\theta_m(t_0) \cos 2\theta \tag{3.19}$$

and

$$P_{NA}^{\text{osc}}(\nu_e \to \nu_e; t, t_0) = P_A^{\text{osc}}(\nu_e \to \nu_e; t, t_0) + \sum_{i=1}^{3} P_i^{\text{osc}}(\nu_e \to \nu_e; t, t_0), \tag{3.20}$$

where

$$P' = \frac{\exp(-2\pi r_0 \sin^2 \theta \Delta m^2/2p) - \exp(-2\pi r_0 \Delta m^2/2p)}{1 - \exp(-2\pi r_0 \Delta m^2/2p)} \tag{3.21}$$

is the analog of the Landau-Zener probability [52], for the case of exponentially varying density, of matter eigenstate conversion $\nu_1 \to \nu_2$ in the reso-

*) This lower bound for $\sin^2 2\theta$ quoted above is inversely proportional to the values of N_e and r_0 and corresponds to $N_e(t_0) = 20\, N_A\, \text{cm}^{-3}$ and $r_0 = 0.1\, R_{\odot}$.

**) Note, however, that for the case when $\sin^2 2\theta < 4\times10^{-3}$, many subtleties arise, as can be seen in references [49], [50] and [51].

nance crossing (the level-crossing probability) *) and the oscillation terms in equation (3.20) can be written as [51]

$$P_1^{\rm osc}(\nu_e \to \nu_e; t_0, t) = -\sqrt{P'(1-P')}\sin 2\theta \cos 2\theta_m(t_0)\cos(\Phi_{12}+\Phi_{22}) \quad (3.22)$$

$$P_2^{\rm osc}(\nu_e \to \nu_e; t_0, t) = -\sqrt{P'(1-P')}\cos 2\theta \sin 2\theta_m(t_0)\cos(\Phi_{12}-\Phi_{22}) \quad (3.23)$$

$$P_3^{\rm osc}(\nu_e \to \nu_e; t_0, t) = -\frac{1}{2}P'\sin 2\theta \cos 2\theta_m(t_0)\left(\cos 2\Phi_{12} + \cos 2\Phi_{22}\right) \quad (3.24)$$

The explicit form of Φ_{12} and Φ_{22} can be found in reference [51] and a definition of similar functions will be given later in section 3.c.

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*) The old result of Landau and Zener [52] for the probability of a transition at $t = +\infty$ between two levels of a molecule induced by an effective interaction switched on at $t_0 = -\infty$ and changing linearly with time was used in reference [53] in the context of the problem of neutrino transitions in matter to calculate the average electron-neutrino survival probability in the case of linearly varying $N_e(t)$ and infinite initial and final densities. The probability of level-crossing in this case is found to be $P = \exp(-2\pi n_0)$. A more realistic scenario (using finite initial and final densities) in the case of linearly varying matter density was discussed in reference [54] and the same expression for P was derived from exact solutions (using asymptotic series expansions) and the conditions of applicability of the Landau-Zener (i.e., linear density approximation) result for the description of the matter-enhanced solar neutrino transitions were also derived.

We would like to conclude this presentation of the MSW effect in the "conventional" case emphasizing that recent analyses [41][42][55] have shown that when interpreted in terms of transitions (oscillations) of the solar neutrinos (ν_e) into neutrinos of one different type (ν_μ, for instance), the deficit of solar neutrinos detected in the Homestake and Kamiokande II experiments can be explained either by the nonadiabatic solution of solar matter-enhanced transitions of the solar neutrinos [44][45][41][42], characterized by:

$$\begin{aligned} sin^2 2\theta &\geq 4 \times 10^{-3}, \\ \Delta m^2 sin^2 2\theta &= (3.2 \pm 1.0) \times 10^{-8}\,\mathrm{eV}^2, \end{aligned} \tag{3.25}$$

or by existence of "long" wavelength two-neutrino oscillations of the solar neutrino ν_e not affected by the solar matter (vacuum solution) [42][55]:

$$\begin{aligned} sin^2 2\theta &\gtrsim 0.7, \\ \Delta m^2 &\simeq (0.5 \div 2.5) \times 10^{-10}\,\mathrm{eV}^2. \end{aligned} \tag{3.26}$$

3.b. MSW with massless neutrinos

As is quite well known, flavor changing neutral current interactions of neutrinos, which are the basic ingredient of the mechanism of neutrino transitions we shall analyse, do not appear in the standard theory of electroweak interactions [47]. Such flavor changing neutral current interactions arise at higher orders of perturbation theory (at one or higher loop level) in most of its extensions with massive neutrinos (see, e.g., reference [43]). The supersymmetric theories with R-parity breaking represent a remarkable exception since in their framework the neutrino flavor changing neutral current interactions can occur at tree level [25], even in the minimal version of these theories, as it can be induced directly from equation (1.10). Such interactions can arise effectively at tree level as a result of the exchange of virtual sleptons or virtual squarks. Obviously, at least two of the coupling constants λ_{ijk} (e.g., λ_{121} and λ_{231}) or λ'_{ijk} (e.g., λ'_{131} and λ'_{331}) must be nonzero in order that the interaction terms of interest to appear in the effective Lagrangian of the theory. As can be shown, the constraints on the relevant couplings λ_{ijk} following from the data on the flavor changing lepton decays ($\mu^{\pm} \to e^{\pm} + \gamma$, $\mu^{\pm} \to e^{\pm}e^{+}e^{-}$, $\tau^{\pm} \to e^{\pm}e^{+}e^{-}$, etc.) make, however, even the coherent effects of the neutrino flavor changing neutral current interactions, generated by the purely leptonic λ-type interactions in equation (1.10), on the propagation of the neutrinos in matter negligible. This is not the case when the neutrino flavor changing neutral current interactions are due to the λ'-type R-parity nonconserving couplings in equation (1.10) and they involve the neutrinos $\overset{(-)}{\nu}_e$ and $\overset{(-)}{\nu}_\tau$ or $\overset{(-)}{\nu}_\mu$ and $\overset{(-)}{\nu}_\tau$.

To be more specific and to simplify the discussion, let us assume that only the coupling constants λ'_{131} and λ'_{331} in equation (1.10) are different from zero and the fields entering in the corresponding terms in equation

(1.10) have definite mass *). One has in this case for the supersymmetric R-parity nonconserving interaction Lagrangian:

$$
\begin{aligned}
\mathcal{L}_R(x) = \lambda'_{131}\{&\tilde{\nu}_{eL}(x)\bar{d}_R(x)b_L(x) + \tilde{b}_L(x)\bar{d}_R(x)\nu_{eL}(x)+ \\
&+ \tilde{d}_R(x)\bar{\nu^c}_{eR}(x)b_L(x) - \tilde{e}_L\bar{d}_R(x)t_L(x)- \\
&- \tilde{t}_L(x)\bar{d}_R(x)e_L(x) - \tilde{d}_R(x)\bar{e^c}_R(x)t_L(x)\}+ \\
&+ \lambda'_{331}\{\tilde{\nu}_{\tau L}(x)\bar{d}_R(x)b_L(x) + \tilde{b}_L(x)\bar{d}_R(x)\nu_{\tau L}(x)+ \\
&+ \tilde{d}_R(x)\bar{\nu^c}_{\tau R}(x)b_L(x) - \tilde{\tau}_L(x)\bar{d}_R(x)t_L(x)- \\
&- \tilde{t}_L(x)\bar{d}_R(x)\tau_L(x) - \tilde{d}_R(x)\bar{\tau^c}_R(x)t_L(x)\} + h.c.,
\end{aligned}
\tag{3.27}
$$

where $\overset{(\sim)}{\nu}_{eL}(x), ..., \overset{(\sim)}{b}_L(x)$ are the fields of the (scalar) electron neutrino,..., LH bottom (scalar) quark, $\nu^c_{eR}(x) = C(\bar{\nu}_{eL})^T(x)$, $e^c_R(x) = C\bar{e}^T_L(x)$, etc., C being the charge conjugation matrix. We shall assume also in what follows that the coupling constants λ'_{131} and λ'_{331} are real and that apart from $\mathcal{L}_R(x)$ (equation (3.27)) the Lagrangian of the theory coincides with the Lagrangian of the minimal supersymmetric extension of the standard theory, introduced in Chapter 1.

The interactions described by the Lagrangian (3.27) can lead to resonant $\overset{(-)}{\nu}_e \rightarrow \overset{(-)}{\nu}_\tau$ transitions in matter [12]. Indeed, if $|\lambda'_{131}| \neq |\lambda'_{331}|$, the $\overset{(-)}{\nu}_e$ and

*) If the quark fields in equation (1.10) are current (flavor) eigenstates, non negligible neutrino masses and mixing would, in general, be generated radiatively. However, we are interested in the present analysis in the case when the neutrino masses and mixing in vacuum are not relevant for the solar neutrino problem.

and $\overset{(-)}{\nu}_\tau$ neutrinos will scatter coherently with different amplitudes not only on the electrons present in matter [5], but also on the d-quarks. Moreover, the process $\overset{(-)}{\nu}_e + d \to \overset{(-)}{\nu}_\tau + d$ is possible at tree level: it is mediated by the exchange of the virtual $\tilde{b}_L$-squark. Correspondingly, the system of evolution equations which describes the $\overset{(-)}{\nu}_e \to \overset{(-)}{\nu}_\tau$ transitions in matter has the form:

$$
\begin{aligned}
i\frac{d}{dt}\begin{pmatrix} \overset{(-)}{A}_e(t,t_0) \\ \overset{(-)}{A}_\tau(t,t_0) \end{pmatrix} &= \\
&= \overset{+}{(-)}\sqrt{2}G_F \begin{pmatrix} 0 & \epsilon(2N_n+N_p) \\ \epsilon(2N_n+N_p) & \epsilon'(2N_n+N_p)-N_e \end{pmatrix} \begin{pmatrix} \overset{(-)}{A}_e(t,t_0) \\ \overset{(-)}{A}_\tau(t,t_0) \end{pmatrix}.
\end{aligned}
\tag{3.28}
$$

Here $\overset{(-)}{A}_e(t,t_0)$ and $\overset{(-)}{A}_\tau(t,t_0)$ are the amplitudes of the probabilities to find the neutrinos $\overset{(-)}{\nu}_e$ and $\overset{(-)}{\nu}_\tau$ at time t if some coherent mixture of $\overset{(-)}{\nu}_e$ and $\overset{(-)}{\nu}_\tau$ was produced in matter at time t_0, N_e, N_p and N_n are the electron, proton and neutron number densities in the point of the neutrino trajectory reached by the neutrinos at time t, $\overset{+}{(-)}\sqrt{2}G_F\epsilon'(2N_n+N_p)$ is the difference between the $\overset{(-)}{\nu}_\tau - d$ and $\overset{(-)}{\nu}_e - d$ forward elastic scattering amplitudes, $\overset{+}{(-)}\sqrt{2}G_F\epsilon(2N_n+N_p)$ is the $\overset{(-)}{\nu}_e + d \to \overset{(-)}{\nu}_\tau + d$ forward scattering amplitude, where

$$\epsilon' = \frac{|\lambda'_{331}|^2 - |\lambda'_{131}|^2}{4m^2(\tilde{b}_L)\sqrt{2}G_F}, \tag{3.29}$$

$$\epsilon = \frac{\lambda'_{331}\cdot\lambda'_{131}}{4m^2(\tilde{b}_L)\sqrt{2}G_F}, \tag{3.30}$$

and $m(\tilde{b}_L)$ is the mass of $\tilde{b}_L$. In all cases of practical interest (propagation of neutrinos in the Sun, in the Earth and in supernovae) the matter is electrically neutral, so $N_p = N_e$. In (3.28) we did not take into account the possible effects of scattering of $\overset{(-)}{\nu}_e$ and $\overset{(-)}{\nu}_\tau$ off neutrinos which can be important for the transitions of the neutrinos emitted by the supernovae (see, e.g., reference [44])

Equation (3.28) implies that the properties of the neutrino (antineutrino) transitions induced solely by the interactions of the neutrinos with the particles forming the matter are very different from the properties of the transitions due to the "conventional" nonzero neutrino masses and nontrivial neutrino mixing [4][43][44]. The neutrino evolution matrix in equation (3.28) and consequently the neutrino transition probabilities of interest, in contrast to the "conventional" transition probabilities, do not depend on the neutrino momentum. It also follows from equation (3.28) that in the case under discussion one has, e.g., for the $\nu_e \to \nu_\tau$ and $\bar{\nu}_e \to \bar{\nu}_\tau$ transition probabilities in matter:

$$P(\nu_e \to \nu_\tau; t, t_0) = P(\bar{\nu}_e \to \bar{\nu}_\tau; t, t_0). \tag{3.31}$$

Equality (3.31) does not hold when the corresponding neutrino and antineutrino transitions in matter are a consequence of the existence of mass-nondegenerate neutrinos and nontrivial neutrino mixing and the effects of matter on the transitions are non negligible (see, e.g., [43] and [44]). Further, the resonance condition for the transitions in matter we are considering has the form:

$$\epsilon'(2N_n + N_e) = N_e. \tag{3.32}$$

where equation (3.32) is supposed to be fulfilled at least in one layer of matter crossed by the neutrinos if N_n or/and N_e vary continuously along the neutrino path (as in the Sun, for example). Unlike the resonance condition in the "conventional" case [6], introduced in equation (3.5), condition (3.32) does not depend on the neutrino momentum and is universal for neutrinos and antineutrinos. Thus, if the $\nu_e \rightarrow \nu_\tau$ transition in matter is resonantly enhanced, so will be the $\bar{\nu}_e \rightarrow \bar{\nu}_\tau$ transition. In contrast, if the $\nu_e \rightarrow \nu_\tau$ and $\bar{\nu}_e \rightarrow \bar{\nu}_\tau$ transitions are due to the "conventional" mechanism and one of the transitions (say, $\nu_e \rightarrow \nu_\tau$) is resonantly enhanced in matter, the other transition ($\bar{\nu}_e \rightarrow \bar{\nu}_\tau$) will be suppressed as the relevant resonance condition can be fulfilled either for the neutrino or for the antineutrino transitions, but not for both types of transitions. We would like to emphasize that the discussed properties of the neutrino transitions in matter caused solely by neutrino interactions are general and do not depend on the particular scheme in which such interactions arise.

Let us check next whether the resonance condition (3.32) can be fulfilled for the solar neutrinos, for the neutrinos crossing the Earth and/or for the supernova neutrinos. We consider first the bounds imposed on the two coupling constants λ'_{131} and λ'_{331} by the existing phenomenology. The data on the parity violation in atoms and on the P-odd asymmetry in the $e - D$ deep inelastic scattering imply [30] (see Table 1.2, Chapter 1):

$$|\lambda'_{131}| < 0.26 \ . \tag{3.33}$$

The most stringent constraint on the product of the two couplings $|\lambda'_{331} \cdot \lambda'_{131}|$ follows from the existing upper limit on the branching ratio (BR) of the lepton charge nonconserving decay $\tau^- \rightarrow \rho^0 + e^-$ of the τ-lepton, shown in equation 1.17. From this inequality, using equation (3.30), one obtains the following constraint on ϵ:

$$|\epsilon| < 2.7 \times 10^{-2} m^2(\tilde{t})/m^2(\tilde{b}_L). \tag{3.34}$$

According to the standard solar model [1], the number densities of electrons (protons) and neutrons have spherically symmetric distributions in the Sun; they decrease monotonically (approximately exponentially) in the radial direction from the center to the surface of the Sun. The ratio of interest $(2N_n + N_e)/N_e$ (see equation (3.32)) is practically constant and equals approximately 1.3 for $0.6R_\odot \lesssim r \lesssim R_\odot$, where r is the distance from the center of the Sun; it takes its maximal value max $[(2N_n + N_e)/N_e] \simeq 2$ at the center of the Sun. Thus, condition (3.32) can be satisfied only for a very small range of values of ϵ', $0.5 \lesssim \epsilon' \lesssim 0.8$, determined by the physics of the Sun. Simple analysis shows that this is a general feature for all schemes in which the solar neutrino transitions are generated only by the interaction of neutrinos with matter. Indeed, the solar neutrinos can scatter only on u- and d- quarks and on electrons in the Sun. For all possible cases of scattering the resonance condition can hold, as it follows from the standard solar model prediction for the relative variation of N_n and N_e (N_p) in the Sun, for intervals of values of the parameters, analogous to ϵ', comparable in magnitude with the one for ϵ' quoted above. In the specific case we consider, the interval of values of ϵ', $0.5 \lesssim \epsilon' \lesssim 0.8$, is equivalent to

$$0.3 \lesssim (|\lambda'_{331}|^2 - |\lambda'_{131}|^2)(\frac{100\ GeV}{m(\tilde{b}_L)})^2 \lesssim 0.5 \tag{3.35}$$

Comparing (3.35) with (3.33) and (1.17) we conclude that, remarkably enough, the existing data on flavor changing neutral currents do not exclude the possibility of a substantial reduction of the solar neutrino flux due to the mechanism discussed.

The neutrino mixing angle in matter $\theta_m(t)$ [5] and the adiabaticity parameter $4n_0$ [1] on which the $\overset{(-)}{\nu}_e \rightarrow \overset{(-)}{\nu}_\tau$ transition probability crucially depends, can be derived from equation (3.28) and are given by the following expressions:

$$tg2\theta_m(t) = \frac{2\epsilon(1 + 2N_n/N_e)}{\epsilon'(1 + 2N_n/N_e) - 1}, \tag{3.36}$$

$$4n_0 = \sqrt{2}G_F\Big(\frac{N_e}{|(\dot{N_n/N_e})|}\Big)_{res} \cdot \frac{2}{\epsilon'}(\frac{\epsilon}{\epsilon'})^2, \tag{3.37}$$

where $(N_e/|(\dot{N_n/N_e})|)_{res}$ is the ratio of the values of N_e and the absolute value of the derivative of the ratio (N_n/N_e) in the resonance layer. Since the resonance condition, equation (3.32), does not depend on the neutrino momentum and the neutrinos are not produced uniformly in the interior of the Sun, the amount of suppression of the different components (^8B, ^7B_e, $p-p$) of the solar neutrino flux is sensitive to the position of the layer of matter in which equation (3.32) holds and to the type of transition [4][43] – adiabatic ($4n_0 >> 1$) or nonadiabatic ($4n_0 \lesssim 1$) – neutrinos undergo. According to the standard solar model [1], approximately 96% of the $p-p$, 97% of the ^7B_e and 98% of the ^8B solar neutrinos are produced in the spherical regions of the Sun defined, respectively, by $r \lesssim 0.22R_\odot$ ($N_e \gtrsim 25.4 N_A cm^{-3}$, N_A being the Avogadro's number), $r \lesssim 0.14R_\odot$ ($N_e \gtrsim 49.5N_A cm^{-3}$), and $r \lesssim 0.10R_\odot$ ($N_e \gtrsim 65.5N_A cm^{-3}$). We will consider first the possibility adiabatic matter-enhanced solar neutrino transitions. In the Sun one has [1] $max\ N_e \simeq 98N_A cm^{-3}$ and $[|(\dot{N_n/N_e})|]^{-1} > R_\odot$, and it is not difficult to check that for $r \lesssim 0.6R_\odot$ ($N_e \gtrsim 0.4N_A cm^{-3}$) the adiabaticity condition, $4n_0 >> 1$, can be satisfied for a rather large range of values of the parameter ϵ compatible with (3.34); for $0.6R_\odot \lesssim r \lesssim R_\odot$, $(\dot{N_n/N_e}) \simeq 0$ and $4n_0 >> 1$ practically for any value of ϵ of physical interest.

The probability $P(\nu_e \to \nu_e; t_E, t_0)$ that the solar ν_e will not transform into ν_τ on their way from the central part of the Sun to the surface of the Earth is given in the case of adiabatic $\nu_e \to \nu_\tau$ transitions by:

$$P(\nu_e \to \nu_e; t_E, t_0) = \frac{1}{2} + \frac{1}{2} cos2\theta_m(t_0) cos2\theta_m(t_s) + \\ + \frac{1}{2} sin2\theta_m(t_0) sin2\theta_m(t_s) cos\{\sqrt{2}G_F \cdot \\ \int_{t_0}^{t_s} N_e[4\epsilon^2(1+2\frac{N_n}{N_e})^2 + (\epsilon'(1+2\frac{N_n}{N_e})-1)^2]^{1/2} dt'\}, \quad (3.38)$$

where $\theta_m(t_0)$ and $\theta_m(t_s)$ are the values of the mixing angle in matter in the point of neutrino production and at the surface of the Sun (reached by the neutrinos at time t_s) and the integration in (3.38) is along the neutrino path in the Sun. The last term in the right-hand side of equation (3.38) is an oscillating term. Taking into account that in the Sun [1] $1.3 \lesssim 1+2N_n/N_e \lesssim 2$, $N_e(t_0) \gtrsim 25 N_A cm^{-3}$ and N_e decreases approximately exponentially with a scale height $r_0 \simeq 0.1 R_\odot$ along the neutrino path, it is not difficult to show that for $0.5 \lesssim \epsilon' \lesssim 0.8$ the oscillating term in $P(\nu_e \rightarrow \nu_e; t_E, t_0)$ is a fast oscillating function of $N_e(t_0)$ – the value of N_e in the point of neutrino production – and the averaging over the region of neutrino production renders it negligibly small. Thus, the suppression of the solar ν_e flux is determined in practice by the sum of the first two terms in the right-hand side of equation (3.38), which represents the average probability $\bar{P}(\nu_e \rightarrow \nu_e; t_E, t_0)$. Further, there are two possibilities. If the resonance condition (3.32) is fulfilled in a layer located outside the region of neutrino production, the transitions of practically all solar ν_e will be resonantly enhanced, but the degree of enhancement will depend somewhat on the location of the point of ν_e production (i.e., on $N_e(t_0)$). Suppose, for instance, that $\epsilon' = 0.72$ (e.g., $\lambda'_{331} = 0.4$, $\lambda'_{131} = 0.05$ and $m(\tilde{b}_L) = 60\ GeV$). In this case the resonance takes place at $r \simeq 0.4 R_\odot$, where $N_e \simeq 3.5 N_A cm^{-3}$ and $(1+2N_n/N_e) \simeq 1.37$. For $\epsilon = 0.07$ the average probability $\bar{P}(\nu_e \rightarrow \nu_e; t_E, t_0)$ takes the values 0.48, 0.42 and 0.40 for neutrinos produced respectively in the layers at $r = 0.1 R_\odot$ ($N_e(t_0) \simeq 66 N_A cm^{-3}$), $r = 0.06 R_\odot$ ($N_e(t_0) \simeq 84 N_A cm^{-3}$) and $r = 0.04 R_\odot$ ($N_e(t_0) \simeq 91 N_A cm^{-3}$). So, the reduction of the flux of the $p-p$ neutrinos will be somewhat less than the reduction of the fluxes of the ^7B_e and ^8B neutrinos. As ϵ decreases $\bar{P}(\nu_e \rightarrow \nu_e; t_E, t_0)$ also decreases and can take values close to zero. For $\epsilon \lesssim 10^{-2}$ the transitions become nonadiabatic , $\bar{P}(\nu_e \rightarrow \nu_e; t_E, t_0)$ begins to rise and if $\epsilon \lesssim 3 \cdot 10^{-3}$ one has $\bar{P}(\nu_e \rightarrow \nu_e; t_E, t_0) \gtrsim 0.8$.

Alternatively, the resonance layer can be located in the region of neutrino production and only the solar neutrinos which cross it on the way to the surface of the Sun would undergo a matter-enhanced conversion. If, for example, equation (3.32) is realized in the layer at $r \simeq 0.06R_\odot$ ($\epsilon' = 0.55$), approximately only one quarter of the $p-p$ and one half of the ^7B_e neutrinos will undergo a resonance transition into ν_τ; and $\bar{P}(\nu_e \rightarrow \nu_e; t_E, t_0) \simeq 0.45$ for the neutrinos produced in the layer at $r = 0.04R_\odot$.

The above examples indicate that the mechanism of solar neutrino conversion into ν_τ we have considered leads to values of probability $P(\nu_e \rightarrow \nu_e; t_E, t_0)$ which are qualitatively compatible with the solar neutrino observations. However, only a more comprehensive analysis can show whether this mechanism is indeed compatible with all quantitative aspects of the solar neutrino data.

The neutrino flavor changing neutral current interaction (3.27) can induce $\overset{(-)}{\nu}_e \rightarrow \overset{(-)}{\nu}_\tau$ transitions also in the Earth and in the supernovae, but for different values of the parameter ϵ', smaller than 0.5. Condition (3.32) can be fulfilled for transitions of the neutrinos $\overset{(-)}{\nu}_e$ crossing the Earth for $\epsilon' \sim 1/3$ ($N_n \simeq N_p = N_e$ in the Earth) and for much smaller values of ϵ' in the case of supernova neutrinos (see e.g. [44]). In fact, the data on the neutrino flux emitted by the supernova SN 1987a can possibly be used to exclude some region of the values of the parameters ϵ' and ϵ.

3.c. Neglecting vacuum mixing angle

We shall consider next the theoretical possibility that neutrino flavor changing neutral currents induce effectively relatively large mixing between the flavor neutrinos (say, $\overset{(-)}{\nu}_e$ and $\overset{(-)}{\nu}_\tau$) in matter when the neutrinos possess nonzero masses [12][13]. The mixing generated in matter can actually exceed in some cases the vacuum neutrino mixing. In the Sun, it can be of the magnitude required for the matter-enhanced transitions of the solar neutrinos to be possible when the neutrino mixing in vacuum is too small to generate alone a substantial conversion of the solar neutrinos into neutrinos of a different type. We will see that in the latter case there exists a solution of the solar neutrino problem analogous to the nonadiabatic solution [44][45] found in the "conventional" two-neutrino vacuum mixing case given by equation (3.25) of the section 3.a.

In this section we derive [13] a simple analytic description of the matter-enhanced two-neutrino transitions of the solar neutrinos when the relevant flavor neutrino mixing is generated effectively in matter by flavor changing neutral current interactions. We give also a proof of the existence of solution of the solar neutrino problem in the indicated case.

We shall assume that the neutrino flavor changing neutral current interactions generate a mixing between the $\overset{(-)}{\nu}_e$ and $\overset{(-)}{\nu}_\tau$ neutrinos in matter. We shall neglect the possible effects of existence of $\overset{(-)}{\nu}_e - \overset{(-)}{\nu}_\tau$ mixing in vacuum. Obviously, this approximation can be justified only if the vacuum mixing is much smaller than the mixing induced in matter by the flavor changing neutral current interactions, which will be suppose to be the case. In the limit of negligible vacuum mixing $\overset{(-)}{\nu}_e$ and $\overset{(-)}{\nu}_\tau$ are neutrinos with definite mass in vacuum. We shall denote their masses by m_1 and m_2, respectively, assuming that $m_2 > m_1$. The system of neutrino evolution equations describing the ν_e and ν_τ propagation in matter with varying electron (N_e), proton (N_p) and neutron (N_n) number densities in the case we are interested in can be written in the form [12]:

$$
\begin{aligned}
&i\frac{d}{dt}\begin{pmatrix} A_e(t,t_0) \\ A_\tau(t,t_0) \end{pmatrix} = \\
&= \sqrt{2}G_F N_e(t)\begin{pmatrix} 0 & \epsilon\big(1+2N_n(t)/N_e(t)\big) \\ \epsilon\big(1+2N_n(t)/N_e(t)\big) & N_e^{res}/N_e(t)-1 \end{pmatrix}\begin{pmatrix} A_e(t,t_0) \\ A_\tau(t,t_0) \end{pmatrix}.
\end{aligned}
\tag{3.39}
$$

Here we again indicate $A_e(t,t_0)$ and $A_\tau(t,t_0)$ as being the amplitudes of the probabilities to find neutrinos ν_e and ν_τ, respectively, at time t if a coherent mixture of ν_e and ν_τ states (with moment $\vec{p}$) has been produced at time t_0 ($t \geq t_0$), $N_n(t)$ and $N_e(t)$ are the values of N_n and N_e in the point of neutrino trajectory reached at time t,

$$N_e^{res} = \frac{\Delta m^2}{2p\sqrt{2}G_F}, \tag{3.40}$$

is the resonance electron number density [12] (see also, e.g., references [43][44]), where $\Delta m^2 = m_2^2 - m_1^2$ and $p = |\vec{p}|$, ϵ is a constant proportional to the coupling constant of the effective four-fermion neutrino flavor changing neutral current interaction generating the term $\sqrt{2}G_F\epsilon\big(N_e(t) + 2N_n(t)\big)$ in equation (3.39), and we have made use of the equality $N_e(t) = N_p(t)$ valid for electrically neutral matter. We have assumed in equation (3.39) that neutrinos ν_e can be converted into ν_τ when they scatter coherently on the d-quarks of the nucleons present in matter, i.e., that the flavor changing neutral current process $\nu_e + d \to \nu_\tau + d$ can take place. The term $\sqrt{2}G_F\epsilon(N_e + 2N_n)$ is determined by the forward scattering amplitude of the indicated process and plays the role of a $\nu_e - \nu_\tau$ mixing term in matter. The effective neutrino flavor changing neutral current interaction inducing the reaction $\nu_e + d \to \nu_\tau + d$ can arise at tree level in the supersymmetric theories with R-parity non-conservation [12]. In such theories ϵ is given by equation (3.30) and the limits on the branching ratio of flavor changing decays of the τ-lepton imply the bounds given in equation (3.34). Nevertheless, we would like to emphasize again that our conclusions are general and do not depend on the specific context of the R-parity non-conserving supersymmetric theories.

The neutrino mixing angle in matter $\theta_m(t)$ and the adiabaticity parameter $4n_0$, on which the value of the $\nu_e \to \nu_\tau$ transition probability crucially depends, can be derived from equation (3.39) and are given by the following expressions [12]:

$$tg2\theta_m(t) = \frac{2\epsilon\big(1 + 2N_n(t)/N_e(t)\big)}{N_e^{res}/N_e(t) - 1}, \tag{3.41}$$

$$4n_0 \simeq \sqrt{2}G_F \frac{(N_e^{res})^2}{|(\dot{N}_e)_{res}|}\ \big[2\epsilon\big(1 + 2N_n(t)/N_e(t)\big)_{res}\big]^2, \tag{3.42}$$

where $(\dot{N}_e)_{res}$ and $\big(1 + 2N_n(t)/N_e(t)\big)_{res}$ are the values of of the derivative of $N_e(t)$ and of $\big(1 + 2N_n(t)/N_e(t)\big)$ in the resonance layer (in which $N_e = N_e^{res}$). In deriving equation (3.42) we have assumed that the derivative $\big(N_n(t)\dot{/}N_e(t)\big)$ of the ratio $\big(N_n(t)/N_e(t)\big)$ satisfies the condition $|\big(N_n(t)\dot{/}N_e(t)\big)| << |\big(\dot{N}_e(t)/N_e(t)\big)|$. This condition is fulfilled in the Sun [12], as we have already discussed in section 3.b. Expression (3.42) for $4n_0$ is valid in the case when $N_e(t)$ and $N_n(t)$ vary monotonically along the neutrino path and $N_e(t_0) > N_e^{res}$, $N_e(t_0)$ being the electron number density in the point of neutrino production.

Consider next the transition of the solar ν_e neutrino into ν_τ in the Sun. According to the standard solar model [1], the number densities of electrons roughly change exponentially along the path of the neutrinos moving radially in the Sun, in the way described by equation (3.6).

In contrast to the electron number density the ratio $(N_e + 2N_n)/N_e$ changes very little in the Sun [1]. According to the discussion presented in section 3.b., this ratio is practically constant and equals approximately 1.3 for $0.6R_\odot \lesssim r \lesssim R_\odot$, where r is the distance from the center of the Sun; and it takes its maximal value $max[(N_e + 2N_n)/N_e] \sim 2$ at the center of the Sun. In view of this fact we shall make the following approximation: we shall treat $\big(N_e(t) + 2N_n(t)\big)/N_e(t)$ as a constant which we shall denote by $(1 + 2N_n/N_e)_0$, $1.3 \lesssim (1 + 2N_n/N_e)_0 \lesssim 2$. The possible choice of the values of $(1 + 2N_n/N_e)_0$ ensuring the most accurate description of the $\nu_e \to \nu_\tau$ transitions in the Sun will be discussed latter.

With $\epsilon\big(1 + 2N_n(t)/N_e(t)\big)$ replaced by a constant ϵ_{12},

$$\epsilon_{12} = \epsilon(1 + 2N_n/N_e)_0 \tag{3.43}$$

and $N_e(t)$ given by equation (3.6), the system of neutrino evolution equations (3.39) can be solved exactly [13]. The solutions of (3.39) can be obtained by the method exploited in reference [49] to derive the exact solutions of the system of evolution equations describing two-neutrino oscillations in matter with exponentially varying electron number density in the "conventional" case of existence of vacuum neutrino mixing. As in the vacuum mixing case [49], the solutions of (3.39) of interest are expressed as linear combinations of two linearly independent confluent hypergeometric functions [56]; they take rather simple form when written in terms of the Kummer's functions [56]. We have [13]:

$$\begin{aligned} A(\nu_e \to \nu_e; t, t_0) &= \frac{e^{(Z_0 - Z)a/c}}{W_{12}} \times \\ &\times \left\{ \left[\mathcal{D}_Z \tilde{\Phi}(a, c; Z)\right]_{Z=Z_0} \Phi(a, c; Z) - \left[\mathcal{D}_Z \Phi(a, c; Z)\right]_{Z=Z_0} \tilde{\Phi}(a, c; Z) \right\} \end{aligned} \tag{3.44}$$

$$
\begin{aligned}
A(\nu_e \to \nu_\tau; t, t_0) &= \frac{\epsilon_{12}}{\sqrt{1+4\epsilon_{12}^2}} \frac{1-c}{1+c} \frac{e^{(Z_0-Z)a/c}}{W_{12}} Z\, Z_0 \times \\
&\times \Big\{ \Phi(a+1, 2+c; Z_0)\tilde{\Phi}(a+1, 2+c; Z) - \\
&- \tilde{\Phi}(a+1, 2+c; Z_0)\Phi(a+1, 2+c; Z) \Big\}
\end{aligned} \tag{3.45}
$$

$$
\begin{aligned}
A(\nu_\tau \to \nu_e; t, t_0) &= \frac{\epsilon_{12}}{\sqrt{1+4\epsilon_{12}^2}} \frac{e^{(Z_0-Z)a/c}}{W_{12}} \times \\
&\times \Big\{ \tilde{\Phi}(a, c; Z_0)\Phi(a, c; Z) - \Phi(a, c; Z_0)\tilde{\Phi}(a, c; Z) \Big\}
\end{aligned} \tag{3.46}
$$

$$
\begin{aligned}
A(\nu_\tau \to \nu_\tau; t, t_0) &= \frac{e^{(Z_0-Z)a/c}}{W_{12}} \times \\
&\times \Big\{ \Phi(a, c; Z_0)\mathcal{D}_Z\tilde{\Phi}(a, c; Z) - \tilde{\Phi}(a, c; Z_0)\mathcal{D}_Z\Phi(a, c; Z) \Big\}
\end{aligned} \tag{3.47}
$$

In equations (3.44) - (3.47) $\Phi(a, c; Z)$ $\big(\Phi(a+1, 2+c; Z)\big)$ and $\tilde{\Phi}(a, c; Z) = Z^{1-c}\Phi(a-c+1, 2-c; Z)$ $\big(\tilde{\Phi}(a+1, 2+c; Z) = Z^{-1-c}\Phi(a-c, -c; Z)\big)$, $c \neq 0, \pm 1, \pm 2, \ldots$, are two linearly independent confluent hypergeometric Kummer's functions [56],

$$
\mathcal{D}_Z = -\frac{a}{c} + \frac{d}{dZ}\,, \tag{3.48}
$$

$$
Z_{(0)} = -ir_0\sqrt{2}G_F N_e(t_{(0)})\sqrt{1+4\epsilon_{12}^2}\,, \tag{3.49}
$$

$$
a = -ir_0\frac{\Delta m^2}{2p}\frac{1}{2}\Big(1 - \frac{1}{\sqrt{1+4\epsilon_{12}^2}}\Big)\,, \quad c = -ir_0\frac{\Delta m^2}{2p}\,, \tag{3.50}
$$

and

$$
W_{12} = (1-c)Z_0^{-c}e^{Z_0} \tag{3.51}
$$

is the value of the Wronskian of the two Kummer's functions indicated above at the initial point of the neutrino path $Z = Z_0$.

Few comments are in order. Solutions (3.44) and (3.45) correspond to the initial conditions

$$A_e(t_0, t_0) = 1\,, \;\; A_\tau(t_0, t_0) = 0, \tag{3.52}$$

while solutions (3.46) and (3.47) have been obtained assuming that

$$A_e(t_0, t_0) = 0\,, \;\; A_\tau(t_0, t_0) = 1. \tag{3.53}$$

Thus, $A_e(t, t_0) = A(\nu_e \to \nu_e; t, t_0)$ and $A_\tau(t, t_0) = A(\nu_e \to \nu_\tau; t, t_0)$ if initial conditions are given by equation (3.52), while $A_e(t, t_0) = A(\nu_\tau \to \nu_e; t, t_0)$ and $A_\tau(t, t_0) = A(\nu_\tau \to \nu_\tau; t, t_0)$ if conditions (3.53) are valid. Obviously any other solution of the system (3.39) corresponding to initial conditions different from (3.52) and (3.53) can be expressed as linear combination of the solutions (3.44) - (3.47).

The Kummer's functions entering into expressions (3.44) - (3.47) are characterized by the property [56]:

$$\Phi(a, c; 0) = 1\,, \quad a, c \neq 0, -1, -2, \dots\,. \tag{3.54}$$

One can eliminate the derivatives of the Kummer's functions appearing in equations (3.44) and (3.47) by using the relations [56]:

$$\mathcal{D}_Z \Phi(a, c; Z) = \frac{a}{c}(1 - \frac{a}{c})\frac{1}{(1+c)} Z \Phi(a+1, 2+c; Z), \tag{3.55}$$

$$\mathcal{D}_Z \tilde{\Phi}(a,c;Z) = (1-c)Z\tilde{\Phi}(a+1,2+c;Z). \qquad (3.56)$$

Given the solutions (3.44) - (3.47), one can find the probability $P_{\odot}(\nu_e \to \nu_e; t_S, t_0)$ that solar ν_e will not change into ν_τ on its way from the central part of the Sun, where the solar neutrinos are produced, to the surface of the Sun, t_S being the time at which the neutrino reaches the surface: $P_{\odot}(\nu_e \to \nu_e; t_S, t_0) \simeq P^{\rm exp}(\nu_e \to \nu_e; t_S, t_0)$, where $P^{\rm exp}(\nu_e \to \nu_e; t_S, t_0) = |A(\nu_e \to \nu_e; t_S, t_0)|^2$. The probability $P^{\rm exp}(\nu_e \to \nu_e; t_S, t_0)$ can be calculated, in particular, for any location of the initial point of the neutrino path in the region of neutrino production *). Since for $t \geq t_S$, $N_e(t) = N_n(t) = 0$ and the nondiagonal elements in the neutrino evolution matrix in (3.39) vanish, $\nu_e \to \nu_\tau$ transitions are not possible on the way of solar neutrinos from the surface of the Sun to the surface of the Earth reached at time t_E, and $P^{(\rm exp)}_{\odot}(\nu_e \to \nu_e; t_E, t_0) = P^{(\rm exp)}_{\odot}(\nu_e \to \nu_e; t_S, t_0)$.

The expression for $P^{\rm exp}(\nu_e \to \nu_e; t_S, t_0)$ has a relatively simple form for neutrinos produced in the closer (with respect to the Earth) solar hemisphere. Taking into account that $N(t_S) = 0$ and using equation (3.54), we obtain [13]:

$$P^{\rm exp}(\nu_e \to \nu_e; t_S, t_0) = 1 - \frac{\epsilon_{12}^2}{1+4\epsilon_{12}^2}\frac{|Z_0|^2}{|1-c|^2}|\Phi(a-c+1,2-c;Z_0)|^2. \qquad (3.57)$$

In the vacuum limit ($N_e(t_0) = Z_0 = 0$) expression (3.57) gives $P^{\rm exp}(\nu_e \to \nu_e; t_S, t_0) = 1$, as should be expected since the $\nu_e \to \nu_\tau$ transition cannot take place in vacuum.

---

*) For further details see references [57].

On the basis of the result (3.57) we shall obtain next a simple approximate expression for the probability $P^{\exp}(\nu_e \rightarrow \nu_e; t_S, t_0)$. In our derivation we shall follow closely the analysis performed in reference [51]. The indicated analysis together with the results obtained in references [49][50] permitted to derive a complete, simple and very accurate analytic description of the matter-enhanced $\nu_e \rightarrow \nu_{\mu(\tau)}$ transitions (oscillations) of the solar neutrinos in the "conventional" case of existence of neutrino $(\nu_e - \nu_{\mu(\tau)})$ mixing in vacuum.

The first thing to be noticed is that according to the standard solar model [1], solar neutrinos are born in a spherical region in the central part of the Sun ($r \lesssim 0.24 R_\odot$), wherein $25\,\mathrm{cm}^{-3} N_A \lesssim N_e(t_0) \lesssim 100\,\mathrm{cm}^{-3} N_A$, N_A being Avogadro's number, and $r_0 \geq 0.1 R_\odot$. Since [49][51]

$$|Z_0| \simeq 5.2 \times 10^2 \frac{r_0}{0.1 R_\odot} \frac{N_e(t_0)}{20\,\mathrm{cm}^{-3} N_A} \sqrt{1 + 4\epsilon_{12}^2}, \qquad (3.58)$$

in the case of solar neutrinos $|Z_0| >> 1$ and one can use the asymptotic series expansions of the confluent hypergeometric functions (in powers of Z_0^{-1}) [56] to obtain an approximate formula for $P^{\exp}(\nu_e \rightarrow \nu_e; t_S, t_0)$ in terms of elementary functions. It proves convenient [50][51] to express first the function $Z^{1-c}\Phi(a - c + 1, 2 - c; Z)$ appearing in equation (3.57) as a linear combination of two different linear independent confluent hypergeometric functions $\Psi(a, c; Z)$ and $e^Z \Psi(c - a, c; -Z)$ characterized by the property [56]:

$$\lim_{Z \rightarrow 0} \Psi(a, c; Z) = \left[\frac{\Gamma(1-c)}{\Gamma(a - c + 1)}\right] + Z^{1-c}\left[\frac{\Gamma(c-1)}{\Gamma(a)}\right], \qquad (3.59)$$

where $\Gamma(1-c)$ etc. is the gamma function. We get [13]:

$$P^{\exp}(\nu_e \to \nu_e; t_S, t_0) = 1 - \Big\{ P_0' |(-Z_0)^{c-a} \Psi(c-a, c; -Z_0)|^2 \sin^2\theta^0 +$$
$$+ (1 - P_0')|Z_0^a \Psi(a, c; Z_0)|^2 \cos^2\theta^0 - \sqrt{P_0'(1-P_0')} \times$$
$$\times |Z_0^a \Psi(a, c; Z_0)\big[(-Z_0)^{c-a}\Psi(c-a, c; Z_0)\big]^*| \sin 2\theta^0 \cos(\phi_{12} - \phi_{22}) \Big\}. \tag{3.60}$$

In equation (3.60)

$$P_0' = \frac{\exp(-2\pi r_0 \sin^2\theta^0 \Delta m^2/2p) - \exp(-2\pi r_0 \Delta m^2/2p)}{1 - \exp(-2\pi r_0 \Delta m^2/2p)}, \tag{3.61}$$

$$\sin^2\theta^0 = \frac{1}{2}\Big(1 - \frac{1}{\sqrt{1+4\epsilon_{12}^2}}\Big), \quad \mathrm{tg} 2\theta^0 = 2\epsilon_{12} = 2\epsilon(1 + 2N_n/N_e)_0, \tag{3.62}$$

$$\phi_{12} - \phi_{22} = |Z_0| + \phi - \phi_1 - \phi_2 - \frac{r_0 \Delta m^2}{2p} \cos 2\theta^0 \ln|Z_0|, \tag{3.63}$$

where

$$\phi = \arg\Big[Z_0^a \Psi(a, c; Z_0)\big[(-Z_0)^{c-a}\Psi(c-a, c; -Z_0)\big]^*\Big], \tag{3.64}$$

and

$$\phi_1 = \arg\Gamma(-a), \quad \phi_2 = \arg\Gamma[-(a-c)]. \tag{3.65}$$

Obviously, in matter θ^0 plays a role analogous to that of the vacuum mixing angle θ in the "conventional" case. The first two terms in the curly brackets in equation (3.60) represent the average probability $\bar{P}^{\exp}(\nu_e \to \nu_e; t_S, t_0)$, while the last is an oscillating term, $P_{\rm osc}^{\exp}(\nu_e \to \nu_e; t_s, t_0)$.

Using the asymptotic series (i.e., "large" Z_0) expansion of the functions $Z_0^a \Psi(a, c, Z_0)$ and $(-Z_0)^{c-a}\Psi(c-a, c; -Z_0)$ [56] and neglecting terms in the series containing fourth or higher powers of $(Z_0)^{-1}$ we find that under the conditions

$$N_e^{res} < N_e(t_0), \tag{3.66}$$

$$\mathrm{tg}^2 2\theta_m^0(t_0) < 1, \tag{3.67}$$

$$\mathrm{tg}^2 2\theta^0 < 1, \tag{3.68}$$

one has:

$$|Z_0^a \Psi(a, c; Z_0)|^2 = \frac{1 - \cos 2\theta_m^0(t_0)}{2\cos^2\theta^0} - \frac{4\sin^2\theta^0\cos^3 2\theta^0}{|Z_0|^2}\left(\frac{N_e^{res}}{N_e(t_0)}\right)^3, \tag{3.69}$$

$$|(-Z_0)^{c-a}\Psi(c-a, c; -Z_0)|^2 = \frac{1 + \cos 2\theta_m^0(t_0)}{2\sin^2\theta^0} + \frac{4\cos^2\theta^0\cos^3 2\theta^0}{|Z_0|^2}\left(\frac{N_e^{res}}{N_e(t_0)}\right)^3 \tag{3.70}$$

where the terms proportional to $|Z_0|^{-2}$ in equations (3.69) - (3.70) do not exceed approximately 10^{-6} and will be neglect further. The angle $\theta_m^0(t_0)$ entering into equations (3.67), (3.69) and (3.70), is determined by

$$\text{tg}2\theta_m^0(t_0) = \frac{2\epsilon(1+2N_n/N_e)_0}{N_e^{res}/N_e(t_0)-1} = \frac{\text{tg}2\theta^0}{N_e^{res}/N_e(t_0)-1}. \tag{3.71}$$

From equations (3.60), using (3.69) and (3.70), we get for the average probability $\bar{P}^{\text{exp}}(\nu_e \to \nu_e; t_S, t_0)$ and the oscillating term $P_{\text{osc}}^{\text{exp}}(\nu_e \to \nu_e; t_S, t_0)$ in $P^{\text{exp}}(\nu_e \to \nu_e; t_S, t_0)$:

$$\bar{P}^{\text{exp}}(\nu_e \to \nu_e; t_S, t_0) = \frac{1}{2} + (\frac{1}{2} - P_0') \cos 2\theta_m^0(t_0), \tag{3.72}$$

$$P_{\text{osc}}^{\text{exp}}(\nu_e \to \nu_e; t_S, t_0) = \sqrt{P'(1-P')} \sin 2\theta_m^0(t_0) \cos(\phi_{12} - \phi_{22}). \tag{3.73}$$

The quantity P_0' entering into equations (3.72) - (3.73) represents the level-crossing probability (i.e., is an analog of the Landau-Zener probability) in the problem under study.

Two remarks $^{*)}$ concerning the conditions (3.66) - (3.68) under which

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$^{*)}$ Let us note that in contrast to $P_{\text{osc}}^{\text{exp}}(\nu_e \to \nu_e; t_e, t_0)$, equation (3.73), the oscillating term in the probability of the solar neutrino "survival" $P^{\text{exp}}(\nu_e \to \nu_e; t_E, t_0)$ in the "conventional" case exhibits a nontrivial dependence on the Sun-Earth distance [51]. This can lead to a substantial variation (with a period of approximately 10 -100 days) of the fluxes of monochromatic ^7Be and *pep* neutrinos from the Sun [58]. If such variations will be observed, that will rule out the possible mechanism of generation of neutrino mixing in matter, which can lead to matter-enhanced transitions of solar neutrinos discussed in the present article.

the result (3.72) and (3.73) is valid are in order. Inequality (3.68) is automatically fulfilled because of the existing experimental constraints on the possible neutrino flavor changing neutral current interactions (which imply roughly $\mathrm{tg}^2 2\theta^0 < 0.1$). Conditions (3.66) and (3.67) coincide in essence with the conditions one arrives to in the analogous analyses performed [49][51] in the "conventional" vacuum mixing case. Condition (3.66) arises from the requirement that neutrinos pass through a region with resonance density on their way out of the Sun, while condition (3.67) follows from the formal requirements of convergence of series in powers of $\mathrm{tg}^2 2\theta_m(t_0)$ resulting form factors like $\cos 2\theta_m(t_0) = [1 + \mathrm{tg}^2 2\theta_m(t_0)]^{-1/2}$.

From the results of the numerical analyses [51] of the magnitude and the importance of the oscillating terms in the expression for the probability $P^{\mathrm{exp}}(\nu_e \to \nu_e; t, t_0)$ derived in the exponential approximation (3.6) for the $N_e(t)$ in the "conventional" vacuum mixing case discussed in section 3.a., we can conclude that either $|P^{\mathrm{osc}}_{\mathrm{exp}}(\nu_e \to \nu_e; t, t_0)| < 5 \times 10^{-2}$ or $|P^{\mathrm{exp}}_{\mathrm{osc}}(\nu_e \to \nu_e; t_S, t_0)| < 5 \times 10^{-2}$ for all values of interest of the parameters $\Delta m^2/2p$, $N_e(t_0)$ and θ^0. Moreover, $P^{\mathrm{exp}}_{\mathrm{osc}}(\nu_e \to \nu_e; t_S, t_0)$ will be additionally suppressed (most probably, strongly) by the averaging over the dimensions of the region of neutrino production in the Sun. Thus the oscillating term (3.73) in the probability $P^{\mathrm{exp}}(\nu_e \to \nu_e; t_S, t_0)$ can be neglected and with a good precision (not worse than few percent) we have:

$$P^{\mathrm{exp}}(\nu_e \to \nu_e; t_S, t_0) \simeq \bar{P}^{\mathrm{exp}}(\nu_e \to \nu_e; t_S, t_0). \qquad (3.74)$$

On the basis of the results (3.72) and (3.74) we have derived for $P^{\mathrm{exp}}(\nu_e \to$

$\nu_e; t_S, t_0)$ one can obtain a complete analytic description of the solar neutrino transitions of interest (i.e., of $P_{\odot}(\nu_e \rightarrow \nu_e; t_S, t_0)$) much in the same way this was done in section 3.a. [49][50] in the "conventional" vacuum neutrino mixing case *). In order for the analytic description in question to be rather accurate it is necessary to use two different choices of θ^0 in the expressions for $\cos 2\theta^0$ and P'_0, namely, the values of θ^0 in the point of neutrino production in the Sun (i.e., $(1 + 2N_n/N_e)_0 = [1 + 2N_n(t_0)/N_e(t_0)]$) and in the resonance layer [12] (i.e., $(1 + 2N_n/N_e)_0 = [1 + 2N_n(t)/N_e(t)]_{res}$), respectively. With these choices we have: $\cos 2\theta^0_m(t_0) = \cos 2\theta_m(t_0)$,

$$P'_0|_{\theta^0 = \theta'} = P', \qquad (3.75)$$

where

$$\mathrm{tg} 2\theta' = 2\epsilon\big(1 + 2N_n(t)/N_e(t)\big)_{res}, \qquad (3.76)$$

and, consequently,

$$P^{\exp}(\nu_e \rightarrow \nu_e; t_S, t_0) \simeq \bar{P}^{\exp}(\nu_e \rightarrow \nu_e; t_S, t_0) = \frac{1}{2} + (\frac{1}{2} - P') \cos 2\theta_m(t_0). \qquad (3.77)$$

Expression (3.77) essentially coincides with the "conventional" exponential density approximation expression for the average probability of solar neutrino survival in the Sun [49][50] for small values of the neutrino mixing angle in

*) See equations (3.14) - (3.24) where the results of the analytic description of the relevant matter-enhanced transitions (oscillations) of solar neutrinos in the "conventional" case are summarized.

vacuum θ ($\sin 2\theta \lesssim 0.1$) if we identify θ' with θ. This implies that there exists a solution of the solar neutrino problem in the case under consideration which is given approximately by equation (3.25) in which θ is replaced by θ'. It corresponds to nonadiabatic "conventional" transitions shown in equations (3.17) and (3.18) for small vacuum mixing angle θ. It should be noted that for $\sin^2 2\theta' \gtrsim 4 \times 10^{-3}$ one has [50][57] $P_{\odot}(\nu_e \to \nu_e; t_S, t_0) = P^{\exp}(\nu_e \to \nu_e; t_S, t_0)$ and the probability of solar neutrino survival in the Sun is described analytically with a rather high precision just by equation (3.77).

We shall conclude with the following remark. It was assumed in the study presented above that the neutrino mixing is generated effectively in matter by the coherent neutrino flavor changing neutral current scattering on the d-quarks of the nucleons present in matter: $\nu_e + d \to \nu_\tau + d$. As the analysis performed in the previous chapter indicates, similar results can be obtained if neutrino mixing is generated also or only by coherent scattering on the u-quarks: $\nu_e + u \to \nu_\tau + u$.

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