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Strongly coupled scenarios of electro-weak symmetry breaking

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Chapter 1

Introduction

Physics is a matter of experiments and measurements with the objective of improving our understanding of nature by successive approximations: these approximations are called effective theories. Effective field theories are descriptions valid in the limited range of distances controlled by present and past experiments. An understanding of long range scales can help the researcher in getting some insight on the underlying substructure and in predicting some properties of the small scale physics. In this process, the physicist is guided by several principles whose validity is ultimately dictated by experimental evidences. In the past we have been able to verify the reliability of our principles, and at present we are quite confident of their power. Among the most important guiding principles are the concept of symmetry and naturalness.

The standard model of particle physics is our best description of subatomic processes to date, and it represents a triumph of the symmetry concept. Yet, no one can doubt the standard model is just an approximate description: the nature of neutrino physics and CP violation are not clarified, no candidate for dark matter is present, etc. Nevertheless, its predictions are in remarkable agreement with experiments, suggesting that the new physics is far from being relevant at the scales of interest. The new structure should manifest itself at some much shorter distances.

This experimental evidence is at the heart of the naturalness problem of the model, because there seems to be no physical justification to explain the hierarchy between the masses of the standard model particles and the cut-off scale of the model, estimated to be around the mass scale of the new physics states. A natural electro-weak symmetry breaking sector (EWSB) should be able to split the weak scale from the new physics scale. For this reason, the fundamental Higgs doublet idea as part of the standard model is not really convincing: a non-minimal EWSB sector is required to solve this puzzle.

These theoretical prejudices point towards the detection of new physics around the TeV scale, at a much larger distance than experimentally expected. The main goal of the large hadron collider (LHC) in Geneva will be to uncover the crucial ingredients of the EWSB sector.

What do we expect to see at the LHC? The LHC may see a light CP even scalar only, it may see several new particles, or it may see no new states at all.

The first possibility is to only detect a light scalar with properties similar to the fundamental Higgs boson, the other states being too heavy or too broad to be directly observed. If this scenario is actually realized in nature, then we will need to resolve the couplings of the light scalar with high precision in order to discriminate between the possible candidates. This will not be an easy task for the LHC, but hopefully it will be an accessible goal for a linear collider. If no clear departures from the standard model Higgs couplings are observed, then it will be necessary to critically review our understanding of the naturalness concept.

The second possibility is by far the most interesting from the point of view of both theoretical and experimental research. This is the case in which several new particles below the few TeV range are detected. A detailed spectroscopy would be necessary to understand the fundamental structure of the theory.

In the following we will have much more to say about these two scenarios, but for the moment let me mention that there is in principle a third possibility we cannot ignore, which is the one in which no new particles are observed. Any accelerator has a limited discovery potential, and we cannot neglect the frustrating possibility that the new ingredients required to mediate the EWSB may turn out to be so well hidden not to be seen by our artificial eyes. Though apparently disappointing, this scenario may hide new and interesting physics just around the corner. We must be clever enough and search for the right signals, something may have been missed. A natural place to look at is the scattering of longitudinally polarized W's. Any EWSB sector has to intervene in that process. However, the accuracy with which such an event will be observed at the LHC is practically very small.

From a purely phenomenological perspective it is reasonable to ask if a model-independent study of the new physics is a viable strategy. Following this idea we can consider the standard model in the absence of the Higgs field by assuming that the EWSB sector has been integrated out leaving a low energy effective description of the light degrees of freedom. These light degree of freedom include the Nambu-Goldstone bosons of the $SU(2) \times U(1) \to U(1)$ symmetry breaking pattern. This theory is the so called electro-weak chiral lagrangian and it is discussed in **Chapter 2**.

The UV completion of such a model is formally generic. Yet, the scenario of a weak EWSB dynamics is not very well captured by such a description. Indeed, the description is valid up to the energy scale of the new degrees of freedom, which are by definition very light in a weak dynamics. The conventional perturbative chiral approach says that, at most, this scale is of the order $\Lambda \sim 4\pi v/\sqrt{N}$, where $v \sim 250$ GeV is the electro-weak vacuum and N is the number of Goldstone modes ($N = 3$ in our case). In the energy range $E < \Lambda$ the theory is perturbatively tractable and predictive, as it can be written in terms of a few number of phenomenological parameters. A measurement of some of them may reveal potentially crucial features of the UV dynamics.

The chiral lagrangian approach has been used with great success in the study of the low energy dynamics of QCD, namely the theory of pions. In this respect the observed spectrum of the standard model and the pion physics is completely analogous: the dynamical modes responsible for chiral symmetry breaking are not directly visible, their only detectable effects are encoded into a number of parameters. The scattering of pions have been observed with some precision, and the effective parameters have been measured with reasonable accuracy. The resulting picture can be naively understood in terms of the so called vector meson dominance: there exists an interpolating theory between the UV description and the pion physics in which the relevant dynamical degrees of freedom are the pions and a vector multiplet, the rho meson. A deeper look would reveal that such a multiplet is not sufficient to explain all of the long range effects: subleading corrections are induced by other resonances.

The ultimate reason for the phenomenon of vector meson dominance is that QCD has no wide separation between the string tension and the confinement scale: the energy regime at which the theory of partons becomes strongly coupled is of the same order as the mass scale of the hadrons. Right above that scale the theory is very well described by a weakly coupled QCD.

The QCD analogy suggests that an accurate measurement of the electro-weak chiral lagrangian parameters may provide us with a powerful piece of information even in the unfortunate scenario in which no new states will be directly observed. Unfortunately, simulations show that the precision of the LHC is not adequate and it will not be able to resolve the electro-weak chiral lagrangian parameters. The nature of the EWSB sector will be uncovered at the LHC only if we will be able to directly detect the new states.

A vast literature has been published on the physics of the EWSB sector, and at present there exists a wide spectrum of possible candidates. These can be broadly classified as strongly and weakly coupled theories. There is no compelling reason to prefer a scheme with respect to the other, experiments will tell which path nature has chosen.

The fundamental Higgs doublet idea is by far the most economic possibility. Despite this, no direct detection of its physical excitation has ever been observed. Its phenomenological success is encoded in the smallness of the so called electro-weak precision tests, and any alternative candidate should be good enough to account for this.

A phenomenologically appealing and natural scenario goes under the name of composite Higgs or little Higgs models. In this class of models some new (possibly strong) dynamics induces the appearance of a light Higgs below some scale of the order of a TeV, while the other states are generally heavier. A natural way of realizing this idea is to assume that the Higgs field arises as a pseudo Goldstone mode of some broken approximate global symmetry.

The broken symmetry can be either an internal symmetry or a spacetime symmetry. A scenario invoking broken spacetime symmetries is based on the breaking of the conformal group down to the Poincarè subgroup. In this model the EWSB physics is not specified, and for practical purposes it may be taken to be realized nonlinearly below the scale ~ 1 TeV. The symmetry breaking pattern generates a single Nambu-Goldstone boson, the so called dilaton. As a matter of fact, the leading couplings of the dilaton to the standard model fields are formally equivalent (up to a rescaling) to those of the physical Higgs, namely the electromagnetic singlet physical Higgs. Yet, the standard model fundamental Higgs itself is an approximate dilaton!

Because many models potentially testable at the LHC involve approximately scale invariant theories (including the Randall-Sundrum scenarios, unparticles, walking technicolor, etc.), it is worth analyzing their low energy spectrum with care. The scenario of a light dilaton will be discussed in some details in **Chapter 3** from both an effective 4D perspective and by using the gauge/gravity dual description.

In a strongly coupled scenario as the ones outlined above, a CP even scalar may be the only visible state at the LHC. If this is the case, a discrimination between a composite Higgs or little Higgs scenario and the dilaton scenario becomes really problematic. It turns out that there are a few signals that distinguish them. For example, an enhancement of the Higgs decay into gluons and photons as compared to a generic suppression of all the remaining total rates would be a clear feature of the dilaton. But realistic scenarios are generically more subtle and not so characterizing. A crucial difference between the composite Higgs doublet hypothesis and the dilaton scenario is that the latter is not directly connected with the EWSB sector, and this generally has implication in scattering events of higgses and longitudinal vector bosons at large momentum.

Weakly coupled scenarios necessarily predict new states not too far from the physical Higgs, and hopefully we will be able to tell whether the EWSB sector is controlled by a weak or a strong dynamics by observing them. This may not be a clear signal, though. There are many extensions of the standard model that predict the existence of new weakly coupled vectors above the weak scale. These extensions can either follow from perturbative dynamics, like in the case of exotic Z's and Kaluza-Klein excitations, or from a non-perturbative dynamics.

A generic implication of strongly coupled EWSB sectors is in fact the production of towers of resonances of increasing masses, the lightest of which are expected to be vectorial as well as scalar ones. The phenomenology of the resonances changes significantly depending on the number of fundamental constituents of the strong sector, N. At small N the resonances are strongly coupled among themselves, but weakly coupled to the fermionic currents. This regime is apparently preferred by electro-weak data, although the resonances are very broad and not easy to detect. However, since no description of the dynamics is known one is forced to rescale the QCD predictions, thus concluding that the model is phenomenologically ruled out. This is clearly not a definite answer, because it only applies to QCD-like theories. At large N the theory can be described in terms of weakly coupled resonances which couple quite strongly to the external currents thus generating the well know conflict with the EW precision tests. The characterizing signature of these models is the spectacular resonant processes in Drell-Yan events, with sharp resonant peaks.

It therefore seems that weakly and strongly coupled theories may lead to very similar predictions at low energies. This remark poses potentially serious problems when trying to use the data in disentangling the two classes of candidates. The electro-weak symmetry breaking sector can be unambiguously revealed only if some peculiar pattern manifests itself. Identifying such characterizing signatures is an essential achievement.

Let us focus on scenarios of strong dynamics, then. The idea that the new physics admits a perturbative description up to the Planck scale is too strong and phenomenologically unjustified a statement to be blindly accepted.

Our understanding of the strong dynamics is intimately linked to our knowledge of QCD. The theory of the strong interactions is an asymptotically free theory that confines in the IR. At low momenta, the partons coupling constant exceeds a critical value and induces the breaking of part of the global symmetries of the model, the chiral symmetries. The latter phenomenon is quite a generic property of confining theories, although examples are known where confinement does not imply chiral SB.

In the absence of a Higgs particle, the SM has an induced EWSB driven by the non-zero chiral condensate of QCD. However, being a non-perturbative effect of QCD, the mass scale that characterizes this breaking is the few hundred MeV, far below the observed vector masses. In order to explain this hierarchy one is tempted to introduce a new confining dynamics with a non-perturbative scale of the order of a few hundred GeV. This idea goes under the name of technicolor. If new partons exist charged under this force, and if the chiral symmetry of the partons include the standard model symmetry group, then a generic consequence of the strong dynamics is to generate the correct mass for both W and Z bosons in a way that naturally solves the hierarchy problem.

One of the main concerns about such a strong dynamics is connected with the S-parameter. This parameter measures the amount of violation of the chiral symmetry; apparently, in a strong dynamics such a breaking is too strong, and this is expressed by a large S-parameter, in contrast with experimental expectations. This cannot rule out the idea of a strong dynamics, at most we can conclude that the minimal version of technicolor is disfavored by data, a new and improved version is required. A non-minimal technicolor model is also necessary in order to accommodate the observed standard model fermion mass pattern. A way to achieve this is to charge the standard model fermions under a new force, generally called extended technicolor, that communicates the EWSB to the standard model fermions. There are a variety of patterns in which this can happen. I will not review them here, it suffices to say that most of them predict an intermediate theory written in terms of technicolor and standard model fields, in which there appear unavoidable four fermions operators which are severely constrained by flavor physics.

Quantum field theory seems to offer a possible way out to both the flavor as well as the S-parameter problems. If the theory enters an approximate IR fixed point at a very high scale Λ_{ETC} TeV then one should be able to suppress the FCNC effects with Λ_{ETC} still generating a mass for the light generations of the right order of magnitude. This is the idea behind walking technicolor. As a matter of fact, IR fixed points are present in a wide class of supersymmetric as well as non-supersymmetric strongly coupled models. Therefore, the walking technicolor idea seems quite a realistic possibility.

The improvement in theories with approximate IR fixed points follows from the observation that, if the approximate IR fixed point is strong, the anomalous dimension of the techniquark bilinear can become of order -1 near the critical value at which the chiral symmetry gets broken. As a consequence, the operator that couples to the standard model fermion bilinears and generates a mass for them is effectively a dimension 2 operator (rather than dimension 3 operator, as expected by a semiclassical analysis). The resulting fermion mass operator becomes more relevant than the FCNC four standard model fermions operators and the flavor problem alluded before is alleviated in a way compatible with fermion mass generation, at least for the first two generations, in a quite natural way. Yet, a complete theory of flavor in these models is still lacking.

In a walking dynamics one expects a suppression of the S-parameter, as well, as a consequence of the decrease of convergence in the Weinberg sum rules (of which the S-parameter represents the zeroth order). This property may be seen as the effect of the screening of the precision measurements induced by a light dilaton mode. While no completely satisfactory computation of the S-parameter and of the fermion mass spectrum can be done analytically in these theories, theoretical estimates suggest that non-generic scenarios of strong EWSB cannot be excluded. After all, the apparent versatility of weakly coupled compared to strongly coupled dynamics should be more properly seen as an artifact of our inability in solving the latter! Model building would receive a significant improvement if we were able to control the non-perturbative effects of arbitrary quantum field theories.

Strongly coupled four dimensional theories are not generally solvable analytically, and an indirect alternative must be pursued. A viable way is to resort to some formal theoretical constructions, as that provided by the celebrated gauge/gravity correspondence. Tractable duals of many strong dynamical systems have been found along these lines. Using such a technique we are able to capture features of large N 4D theories at large 't Hooft coupling using extra dimensional field theories at weak coupling.

The regime of validity of the weakly coupled gravitational description is limited to the region in which the dual 4D theory is strongly coupled. The energy range that we expect to be mimicked by the gravity theory is thus in between the first resonance mass and the scale at which the theory ceases to be strong. This can be recognized as a characteristic property of a class of electro-weak symmetry breaking sectors – the so called walking technicolor models – rather than of QCD-like systems. Consequently, these theoretical achievements can be used to analyze the phenomenology of approximately conformal strong dynamics.

A first attempt in this direction is provided by a class of five-dimensional models constructed on a slice of anti de Sitter geometry, the Randall-Sundrum class of models. Realistic dynamics are however only approximately conformal. In order to describe them, the geometry which governs the five-dimensional gravitational picture should depart from anti de Sitter.

In **Chapter 4** the phenomenological implications of these deformations is analyzed. The outcome is that the AdS/CFT correspondence correctly responds to mild deformations of the AdS geometry. It will be shown that the lightest spin-1 resonances are very sensitive to the departure from conformal invariance. If the strong dynamics departs from the conformal regime at an energy scale slightly larger than the confining scale (not to spoil the stability of the hierarchy), the couplings of the lightest resonance to the standard model fermions receive a suppression with respect to those of the heavier excitations, making its effect comparable to that of the second (or even third) one in the low energy regime.

The phenomenological implication can be seen in Drell-Yan processes at sufficiently high transfered momentum: the second (third) resonance can in principle contribute as much as the first, modulo PDFs suppressions of the event. No direct implication on the S-parameter is found, however. The latter is the result of the effect of the full tower of resonances, and the suppression of the first contribution does not affect significantly the overall sum. This is in sharp contrast to what we expect from QCD, where the S-parameter (L_{10} in a more conventional notation) is dominated by the first excited state, the rho meson. In a conformal dynamics the whole tower of resonances contribute, this being a reformulation of the previously mentioned decrease of convergence of the Weinberg sum rules.

Using the gauge/gravity duality we appreciate the problems of strongly coupled theories with the S-parameter as the consequence of tree level vectorial contributions, which are very large compared to the experimentally allowed shift. The fit can be ameliorated by suppressing both the axial and vector current to current correlators that enter in the definition of S. This is done in most of the UV completions of the standard model, for example by decoupling the new dynamics. In this case the new states must live at a scale of several TeV or must be too weakly coupled to the standard model currents, and are therefore outside the resolution of the LHC. Another possibility is to decrease the difference $\langle J_A J_A \rangle - \langle J_V J_V \rangle$ with no substantial suppression of the current to current correlators. A walking dynamics naturally encodes the latter effect. We already observed that in the walking technicolor class the chiral condensate is a more relevant operator than in a generic theory. Hence, the heavy resonances are expected to capture less information about the symmetry breaking, and approximately align to the vectorial excitations.

An even stronger suppression of the S-parameter may be achieved if confinement and chiral symmetry breaking occur at different scales, Λ and Λ_χ respectively. For fixed vector boson masses, $S \propto m_W^2/\Lambda_\chi^2$ and really sets a bound on the chiral symmetry breaking scale, since by definition in a confining theory with no chiral symmetry breaking the S-parameter vanishes. Hence, if the chiral condensate forms at a larger energy than the confining scale, the masses of the vectorial-axial excitations, $M_i \propto \Lambda$, get less constrained by data. The relation $\Lambda < \Lambda_\chi$ is a natural expectation in confining gauge theories, since the decoupling of fermions induced by the condensate triggers the running towards a stronger coupling, and thus confinement.

Because slight deformations of both chiral and conformal symmetry breaking lead to important observable implications, the description of these essential elements should be properly treated. In view of a deeper understanding of the formal apparatus of the gauge/gravity correspondence, a phenomenological study of the impact of these deformations on the LHC physics is needed. In particular, the conformal symmetry violations and the presence of an intermediate scale at which chiral symmetry breaking takes place have some implications on the flavor physics and on the strong $W_L W_L$ scattering, in addition to the already mentioned precision measurements.

In realistic models the mass of the lightest resonance, usually of the order of $\sim 2-3$ TeV, may be decreased around 1.5 TeV by splitting Λ and Λ_χ by a factor slightly different from 1. In this case, rather than observing at most a couple of states up to 3-5 TeV (the cut-off in appropriate processes, like Drell-Yan processes, that will be observed), the LHC will potentially be able to produce several vectorial states. Such an observation would represent an indisputable evidence for the existence of a new strong dynamics.

The present thesis is based on the following papers:

[1] M. Fabbrichesi, A. Tonero and L. Vecchi,
"Gauge boson scattering at the LHC without a light Higgs boson," *Frascati 2006, Monte Carlo's, physics and simulations at the LHC. Part 1*, 162-185

[2] M. Fabbrichesi and L. Vecchi,
"Possible experimental signatures at the LHC of strongly interacting electro-weak symmetry breaking," Phys. Rev. D **76**, 056002 (2007)

[3] L. Vecchi,
"Causal vs. analytic constraints on anomalous quartic gauge couplings,"
JHEP **0711**, 054 (2007)

[4] M. Fabbrichesi, M. Piai, and L. Vecchi,
"Dynamical electro-weak symmetry breaking from deformed AdS: vector mesons and effective couplings," Phys. Rev. D **78**, 045009 (2008)

as well as on a not yet submitted letter. In addition, the following complementary works are reported in the Appendix A and B:

[5] A. A. Andrianov and L. Vecchi,
"On the stability of thick brane worlds non-minimally coupled to gravity,"
Phys. Rev. D **77**, 044035 (2008)

[6] L. Vecchi,
"Massive states as the relevant deformations of gravitating branes,"
Phys. Rev. D **78**, 085029 (2008).

Chapter 2

Higgsless theories

A common prediction of weakly coupled models like the standard model (SM) and minimal supersymmetric extensions, as well as strongly coupled composite models of the Higgs boson, is that the breaking of the electro-weak (EW) symmetry is due to a light—that is, with a mass around a few hundred GeV—Higgs boson.

What happens if the LHC will not discover any light Higgs boson? Most likely, this would mean that the EW symmetry must be broken by a new and strongly interacting sector. In this scenario, it becomes particularly relevant to analyze the physics of massive gauge boson scattering—WW, WZ, and ZZ—because it is here that the strongly interacting sector should manifest itself most directly. This statement can be understood by invoking the equivalence theorem, which associates the Green's functions of external on-shell vectors in longitudinal polarization to the same correlator with external Goldstone bosons. Very heuristically, the gauge fixing imposes a constraint $\partial_\mu \pi = gvW_\mu$ and, since the longitudinal polarization looks like $\epsilon^\mu_{(L)} \to p^\mu/m_W$ in the relativistic limit, we find the relation $W_\mu \epsilon^\mu_{(L)} \to \pi$ for $E \gg m_W$.

Longitudinally polarized gauge boson scattering in this regime looks similar in many ways to $\pi\pi$ scattering in QCD and similar techniques can be used. The natural language is that of the non-linear realization of the electro-weak theory [84].

Consider the case in which the LHC will not find any new particle propagating under an energy scale Λ around 2 TeV. By new we mean those particles, including the scalar Higgs boson, not directly observed yet. Since $\Lambda \gg m_W$, the physics of gauge boson scattering is well described by the SM with the addition of the effective lagrangian containing all the possible operators for the Goldstone bosons (GB)—π^a, with $a = 1, 2, 3$—associated to the $SU(2)_L \times U(1)_Y \to U(1)_{em}$ symmetry breaking. The GB are written as an $SU(2)$ matrix

$$U = \exp\left(i\pi^a\sigma^a/v\right) , \tag{2.1}$$

where σ^a are the Pauli matrices and $v = 246$ GeV is the electro-weak vacuum. The GB couple to the EW gauge and fermion fields in an $SU(2)_L \times U(1)_Y$ invariant way. As usual, under a local $SU(2)_L \times U(1)_Y$ transformation $U \to LUR^\dagger$, with L and R an $SU(2)_L$ and $U(1)_Y$ transformation respectively. The EW precision tests require an approximate $SU(2)_C$ custodial symmetry to be preserved and therefore we assume $R \subset SU(2)_R$.

The most general lagrangian respecting the above symmetries, together with C and P invariance, and up to dimension 4 operators is given in the references in [1] of which we mostly follow the notation:

$$\begin{aligned}\mathcal{L} &= \frac{v^2}{4}\mathrm{Tr}\left[(D_\mu U)^\dagger (D^\mu U)\right] + \frac{1}{4}a_0 g^2 v^2 [Tr(TV_\mu)]^2 + \frac{1}{2}a_1 g g' B_{\mu\nu} Tr(TW^{\mu\nu}) \\ &+ \frac{1}{2} i a_2 g' B_{\mu\nu} Tr(T\,[V^\mu, V^\nu]) + i a_3 g Tr(W_{\mu\nu}[V^\mu, V^\nu]) \\ &+ a_4 [Tr(V_\mu V_\nu)]^2 + a_5 [Tr(V_\mu V^\mu)]^2 + a_6 Tr(V_\mu V_\nu) Tr(TV^\mu) Tr(TV^\nu) \\ &+ a_7 Tr(V_\mu V^\mu) Tr(TV_\nu) Tr(TV^\nu) + \frac{1}{4} a_8 g^2 [Tr(TW_{\mu\nu})]^2 \\ &+ \frac{1}{2} i a_9 Tr(TW_{\mu\nu}) Tr(T[V^\mu, V^\nu]) + \frac{1}{2} a_{10} [Tr(TV_\mu) Tr(TV_\nu)]^2 \,. \end{aligned} \tag{2.2}$$

In (2.2), $V_\mu = (D_\mu U)U^\dagger$, $T = U\sigma^3 U^\dagger$ and

$$D_\mu U = \partial_\mu U + i\frac{\sigma^k}{2} W_\mu^k U - i g' U \frac{\sigma^3}{2} B_\mu \,, \tag{2.3}$$

with $W_{\mu\nu} = \sigma^k W^k_{\mu\nu}/2 = \partial_\mu W_\nu - \partial_\nu W_\mu + ig[W_\mu, W_\nu]$ is expressed in matrix notation.

The conventional procedure to treat this theory is to view it as an effective field theory with a finite cutoff scale Λ. Naturalness arguments lead to $\Lambda < 4\pi v$, but we will see that this bound can be made stronger by requiring unitarity of the underlying theory. Any correlator is then expanded in powers of the dimensionful coupling $1/v$ and thus represents expansion in powers of the external momentum. Since the coupling is a dimensionful quantity, we know that an infinite number of counterterms are needed to remove the singularities and make sensible UV-insensitive predictions. In this sense the action is said to be non-renormalizable and its predictive power is recovered at any order in the expansion in powers of the momentum. The success of this approach is tight to the natural implementation of the low energy theorems of the current algebra.

This lagrangian, as any other effective theory, contains arbitrary coefficients, in this case called a_i, which have to be fixed by experiments or by matching the theory with a UV completion. The coefficients a_2, a_3, a_9 and $a_4, a_5, a_6, a_7, a_{10}$ contribute at tree level to the gauge boson scattering and represent anomalous triple and quartic gauge couplings respectively. They are not directly bounded by experiments. On the other hand, the coefficients a_0, a_1 and a_8 in (2.2) are related to the electro-weak precision measurements parameters S, T and U [87] and therefore directly constrained by LEP precision measurements.[1]

[1] The authors of [3] defined the complete set of EW parameters up to $O(p^6)$ which includes—in addition to S, T and U—W and Y. These latter come from $O(p^6)$ terms and can be neglected in the present discussion.

2.1 Phenomenological constraints

The EW precision measurements test processes in which oblique corrections play a dominant role with respect to the vertex corrections. This is why we can safely neglect the fermion sector (in our approximate treatment) and why the parameters S, T, U, W and Y represent such a stringent phenomenological set of constraints for any new sector to be a candidate for EW symmetry breaking (EWSB). The good agreement between experiments and a single fundamental Higgs boson is encoded in the very small size of the above EW precision tests parameters. The idea of a fundamental Higgs boson is perhaps the most appealing because of its extreme economy but it is not the only possibility and what we do here is to consider some strongly interacting new physics whose role is providing masses for the gauge bosons in place of the Higgs boson.

To express the precision tests constraints in terms of bounds for the coefficients of the low-energy lagrangian in eq. (2.2) we have to take into account that the parameters S, T and U are defined as deviations from the SM predictions evaluated at a reference value for the Higgs and top quark masses. Since we are interested in substituting the SM Higgs sector, we keep separated the contribution to S of the Higgs boson and write

$$S_H + S = S_{EWSB}\,, \tag{2.4}$$

and analog equations for T and U. The contributions coming from the SM particles, including the GB, are not relevant because they appear on both sides of the equation. S_H is given by diagrams containing at least one SM Higgs boson propagator while S_{EWSB} represents the contribution of the new symmetry breaking sector, except for contributions with GB loops only. We find that, in the chiral lagrangian (2.2) notation,

$$\begin{aligned} S_{EWSB} &= -16\pi a_1 \\ \alpha_{em} T_{EWSB} &= 2g^2 a_0 \\ U_{EWSB} &= -16\pi a_8 \end{aligned} \tag{2.5}$$

The coefficients a_0, a_1 and a_8 typically have a scale dependence (and the same is true for S_H, T_H and U_H) because they renormalize the UV divergences of the GB loops which yields a renormalization scale independent S, T and U. One expects by dimensional analysis that $U \sim (m_Z^2/\Lambda^2)T \ll T$ and therefore U is typically ignored. The relationships (2.5) have been used in [4] to study the possible values of the effective lagrangian coefficients in the presence of SM Higgs boson with a mass larger than the EW precision measurements limits.

Using the results of the analysis presented in [3], taking as reference values $m_H = 115$ GeV, $m_t = 178$ GeV and summing the 1-loop Higgs contributions, we obtain:

$$\begin{aligned} S_{EWSB} &= -0.05 \pm 0.15 \\ \alpha_{em} T_{EWSB} &= (0.3 \pm 0.9) \times 10^{-3} \end{aligned} \tag{2.6}$$

at the scale $\mu = m_Z$. We shall use these results to set constraints to the coefficients of the effective lagrangian (2.2).

The smallness of the parameter T can be understood as a consequence of an approximate symmetry of the underlying theory under which the matrix U carries the adjoint representation. In fact, if we require a global $SU(2)_L \times SU(2)_R \to SU(2)_C$ pattern the $T = U\sigma^3 U^\dagger$ operator would not be present in the non-gauged chiral lagrangian. The gauge interactions break explicitly this symmetry through $SU(2)_R \supset U(1)_Y$ (and consequently by $SU(2)_C \supset U(1)_{em}$) thus producing a non-vanishing T parameter as a small loop effect proportional to g'^2. Moreover, any new EWSB sector must eventually be coupled with some new physics responsible for the fermions masses generation and thus requiring a breaking of the $SU(2)_C$. Due to this approximate symmetry we expect the couplings $a_{0,2,6,7,8,9,10}$ to be subdominant with respect to the custodial preserving ones.

Most of the strongly coupled theories have large and positive S_{EWSB} and the assumption that this sector respects an exact custodial symmetry is in general in contrast with smaller values of the S parameter. In fact, a small deviation from the point $T_{EWSB} = 0$ can lead to a negative correction of the same order in the S parameter. Using the effective lagrangian formalism and going to the unitary gauge we find

$$\begin{aligned} S_{EWSB} &= \frac{4}{\alpha_{em}} \left(s_W^2 \Delta_Z - c_W^2 \Delta_A \right) \\ U_{EWSB} &= -\frac{8 s_W^2}{\alpha_{em}} \left(\Delta_Z + \Delta_A \right) \end{aligned} \tag{2.7}$$

where the $\Delta_{A,Z}$ are the shifts in the photon and Z^0 kinetic terms due to new physics—once the shifts in the W propagators have been rescaled to write its kinetic term in the canonical way [5]. If a new theory has $\Delta = \Delta^0 + \hat{\Delta}$ with Δ^0 a custodial symmetric term and $\hat{\Delta}$ small custodial-symmetry breaking term satisfying $s_W^2 \hat{\Delta}_Z - c_W^2 \hat{\Delta}_A = -\varepsilon \alpha_{em}$ then $S_{EWSB} = S^0 - 4\varepsilon$ and $U_{EWSB} = O(\varepsilon)$. This result agrees with the experiments: a large and positive S can only be consistent with data if T is greater than zero.

Bearing the above arguments in mind, we can, in first approximation, consider the custodial symmetry to be exact and therefore discuss only those terms in the lagrangian (B.1) that are invariant under this symmetry. Gauge boson scattering is then dominated by only two coefficients: a_4 and a_5.

Bounds on the coefficients a_4 and a_5 can be obtained by including their effect (at the one-loop level) into low-energy and Z physics precision measurements. They are referred as indirect bounds since they only come in at the loop level. As expected, these bounds turn out to be rather weak [80] :

$$\begin{aligned} -320 \times 10^{-3} \leq a_4 \leq 85 \times 10^{-3} \\ -810 \times 10^{-3} \leq a_5 \leq 210 \times 10^{-3} \end{aligned} \tag{2.8}$$

at 99% C.L. and for $\Lambda = 2$ TeV. Comparable bounds were previously found in the papers in ref. [12]. As before, slightly stronger bounds can be found by a combined analysis. Notice that the $SU(2)_C$ preserving triple gauge coupling a_3 has not been considered in the computations leading to the previous limits. Once its contribution is taken into account, the LHC sensitivity and the indirect bounds presented here are slightly modified although the ranges shown are not changed drastically.

In addition, even though the LHC will explore these terms directly, its sensitivity is not as good as we would like it to be and an important range of values will remain unexplored. Let us consider the capability of the LHC of exploring the coefficients a_4 and a_5 of the effective lagrangian (2.2). This has been discussed most recently in [80] by comparing cross sections with and without the operator controlled by the corresponding coefficient. They consider scattering of W^+W^-, $W^\pm Z$ and ZZ ($W^\pm W^\pm$ gives somewhat weaker bounds) and report limits (at 99% CL) that we take here to be

$$\begin{aligned} -7.7 \times 10^{-3} \leq a_4 \leq 15 \times 10^{-3} \\ -12 \times 10^{-3} \leq a_5 \leq 10 \times 10^{-3}\,. \end{aligned} \tag{2.9}$$

The above limits are obtained considering as non-vanishing only one coefficient at the time. It is also possible to include both coefficients together and obtain a combined (and slightly smaller) limit. We want to be conservative and therefore use (2.9). Comparable limits were previously found in the papers of ref. [10]. To put these results in perspective, limits roughly one order of magnitude better can be achieved by a linear collider [91].

2.2 Perturbative unitarity bound

Being interested in the EW symmetry breaking sector, we will mostly deal with longitudinally polarized vector bosons scattering because it is in these processes that the new physics plays a dominant role. We can therefore make use of the equivalence theorem (ET) wherein the longitudinal W bosons are replaced by the Goldstone bosons [6]. This approximation is valid up to orders m_W^2/s, where s is the center of mass (CM) energy, and therefore—by also including the assumptions underlaying the effective lagrangian approach—we require our scattering amplitudes to exist in a range of energies such as $m_W^2 \ll s \ll \Lambda^2$.

Assuming exact $SU(2)_C$ and crossing symmetry, and at leading order in the SM gauge couplings ($g = g' = 0$), the elastic scattering of gauge bosons is described by a single amplitude $A(s,t,u)$:

$$\mathcal{M}(ij \to kl) = \delta_{ij}\delta_{kl}A(s,t,u) + \delta_{ik}\delta_{jl}A(t,s,u) + \delta_{il}\delta_{jk}A(u,t,s). \tag{2.10}$$

The function A is symmetric under the exchange of its last two arguments.

Up to $O(p^4)$, and by means of the lagrangian (2.2) we obtain [88]

$$\begin{aligned} A(s,t,u) &= \frac{s}{v^2} \\ &+ \frac{4}{v^4}\left[2a_5(\mu)s^2 + a_4(\mu)(t^2+u^2) + \frac{1}{(4\pi)^2}\frac{10s^2+13(t^2+u^2)}{72}\right] \\ &- \frac{1}{96\pi^2 v^4}\left[t(s+2t)\log(\frac{-t}{\mu^2}) + u(s+2u)\log(\frac{-u}{\mu^2}) + 3s^2\log(\frac{-s}{\mu^2})\right] \end{aligned} \tag{2.11}$$

where s,t,u are the usual Mandelstam variables satisfying $s+t+u=0$ which in the CM frame and for any $1+2 \to 1'+2'$ process can be expressed as a function of s and the scattering angle θ as $t=-s(1-\cos\theta)/2$ and $u=-s(1+\cos\theta)/2$.

The couplings $a_{4,5}(\mu)$ appearing in (2.11) are the effective coefficients renormalized using the minimal subtraction scheme and they differ by an additive finite constant from those introduced in [88]. In the latter non-standard renormalization, the numarator of the one loop term in the first bracket of (2.11) is shifted from $10s^2+13(t^2+u^2)$ to $4s^2+7(t^2+u^2)$.

The beta functions for these coefficients are found to be

$$\mu\frac{da_{4,5}}{d\mu} = \frac{b_{4,5}}{(4\pi)^2} \tag{2.12}$$

where $b_4 = 1/6$ and $b_5 = 1/12$. The naturalness argument that sets the cutoff scale at $\Lambda < 4\pi v$ can be reformulated by stating that a_i must be of the order of their beta functions, therefore at the percent level. We will show in the following sections that this range of the parameters is also predicted by more model-dependent analysis compatible with the phenomenological constraints.

The GB carry a conserved isospin $SU(2)_C$ charge $I=1$ and we can express the total amplitude $\mathcal{M}$ as a sum of a singlet $A_0(s,t,u)$, a symmetric $A_2(s,t,u)$, and antisymmetric two index $A_1(s,t,u)$ representations. It is useful for a later discussion to determine the form of these amplitudes for the case of a coset $O(N)/O(N-1)$, the case under study being the $N=4$ case. From a simple computation one gets:

$$\begin{aligned} A_0(s,t,u) &= (N-1)A(s,t,u) + A(t,s,u) + A(u,t,s) \\ A_1(s,t,u) &= A(t,s,u) - A(u,t,s) \\ A_2(s,t,u) &= A(t,s,u) + A(u,t,s)\,. \end{aligned} \tag{2.13}$$

From the above results, we obtain the amplitudes for the scattering of the physical longitudinally polarized gauge bosons as follows:

$$\begin{aligned} A(W^+W^- \to W^+W^-) &= \frac{1}{3}A_0 + \frac{1}{2}A_1 + \frac{1}{6}A_2 \\ A(W^+W^- \to ZZ) &= \frac{1}{3}A_0 - \frac{1}{3}A_2 \end{aligned}$$

$$
\begin{aligned}
A(ZZ \to ZZ) &= \frac{1}{3}A_0 + \frac{2}{3}A_2 \\
A(WZ \to WZ) &= \frac{1}{2}A_1 + \frac{1}{2}A_2 \\
A(W^\pm W^\pm \to W^\pm W^\pm) &= A_2 \,. \qquad (2.14)
\end{aligned}
$$

It is useful to re-express the scattering amplitudes in terms of partial waves of definite angular momentum J and isospin I associated to the custodial $SU(2)_C$ group. These partial waves are denoted t_{IJ} and are defined, in terms of the amplitude A_I of (2.13), as

$$
t_{IJ} = \frac{1}{64\,\pi}\int_{-1}^{1} d(\cos\theta)\, P_J(\cos\theta)\, A_I(s,t,u)\ . \qquad (2.15)
$$

Explicitly we find:

$$
\begin{aligned}
t_{00}^{(2)} &= \frac{s}{16\,\pi v^2}, \\
t_{00}^{(4)} &= \frac{s^2}{64\,\pi v^4}\left[\frac{16(11a_5+7a_4)}{3} + \frac{101/9 - 50\log(s/\mu^2)/9 + 4\,i\,\pi}{16\,\pi^2}\right], \\
t_{11}^{(2)} &= \frac{s}{96\,\pi v^2}, \\
t_{11}^{(4)} &= \frac{s^2}{96\,\pi v^4}\left[4(a_4-2a_5) + \frac{1}{16\,\pi^2}\left(\frac{1}{9}+\frac{i\,\pi}{6}\right)\right], \\
t_{20}^{(2)} &= \frac{-s}{32\,\pi v^2}, \\
t_{20}^{(4)} &= \frac{s^2}{64\,\pi v^4}\left[\frac{32(a_5+2a_4)}{3} + \frac{273/54 - 20\log(s/\mu^2)/9 + i\,\pi}{16\,\pi^2}\right], \qquad (2.16)
\end{aligned}
$$

where the superscript refers to the corresponding power of momenta.

The contributions from $J \geq 2$ starts at order p^4, while the $I = 1$ channel is related to an odd spin field due to the Pauli exclusion principle. The $(I = 2, J = 0)$ channel has a dominant minus sign which, from a semiclassical perspective, indicates that this channel is repulsive and we should not expect any resonance with these quantum numbers.

The effective lagrangian (2.2) and gauge boson scattering were extensively discussed in [8].

The amplitudes (2.11) (or, equivalently (2.16)) show that, for $s \gg m_W^2$, the elastic scattering of two longitudinal polarized gauge bosons is observed with a probability that increases with the CM energy s. We expect that for sufficiently large energies the quantum mechanical interpretation of the S-matrix will be lost. This fact can be restated more formally in terms of the partial waves defined in eq. (2.16). The unitarity condition for physical values of the CM energy $s < \Lambda^2$ can be written as

$$
\mathrm{Im}\, t_{IJ}(s) \geq \mid t_{IJ}(s) \mid^2, \qquad (2.17)
$$

where the equality applies under the inelastic production threshold. Since the latter starts at order p^6 we can reformulate our unitarity requirement up to order p^4 terms as $\mathrm{Im}\, t^{(4)}_{IJ}(s) =| t^{(2)}_{IJ}(s)\ |^2$. We can now find a more stringent bound on the cutoff scale by assuming that in the perturbative region $s < \Lambda^2$ the theory does not violate the unitarity bound. A necessary condition to satisfy is that $|Re(t_{IJ})| < 1/2$, which at leading order and for $I = 0$ yelds $s < 8\pi v^2 = \Lambda^2 \sim (1.3\ \mathrm{TeV})^2$. This bound holds if the perturbative expansion adopted is effective, and it is more correctly referred to as perturbative unitarity cutoff. In this sense the constraint holds irrespective of the value of the a_i and is even lower when loops are included. We explicitly show the unitarity bound thus obtained as a dashed line in the plots presented below in Figs. 2.3,2.4 and Figs. 2.5,2.6.

2.3 Axiomatic bounds

The general structure of an effective lagrangian is dictated by the interplay between quantum mechanics, Poincaré invariance, and internal symmetries. Its coefficients are not constrained by the symmetries and must be determined by experiments. Unitarity usually sets an upper bound on the energy scale below which a perturbative effective approach is reliable.

We can interpret the standard model (SM) as an effective theory extending its lagrangian to include new non-renormalizable operators with unknown coefficients. Some of them enter the scattering amplitudes of longitudinally polarized vector bosons. These are called anomalous quartic gauge couplings since they measure the deviation from the SM predictions. These coefficients are necessarily connected with the not yet observed Higgs sector. In the case the Higgs boson is not a fundamental state, or even no Higgs boson will be observed, they provide important informations on the nature of the electro-weak symmetry breaking sector. Whereas there are no significant experimental bounds on them at the moment [80], theoretical arguments can reduce significantly their allowed range and can serve as a guide for future experiments.

We briefly review an analytical tool which has been used in the context of the chiral lagrangian of QCD to constrain some effective coefficients.

Consider a multiplet of scalar particles, which to be definite we call pions π^a, having mass m. Assume they are lighter than any other quanta and that they have appropriate quantum numbers to forbid the transition $2\pi \to \pi$. The other states can be general unstable quanta of complex masses M much greater than $2m$. The S-matrix element for a general transition $2\pi \to 2\pi$ is a Lorentz scalar function of the Mandelstam variables s, t, u and of the mass m^2.

We study the amplitude for the elastic scattering $\pi^a\pi^b \to \pi^a\pi^b$ and assume it can be analytically continued to the complex variables s, t. We denote this analytical function by $F(s, t)$ and require that its domain of analyticity be dictated entirely by the optical theorem and the crossing symmetry. More precisely, we assume that the singularities come from simple poles in the correspondence of the physical masses of the quantum states which can be produced in the reaction, and branch cuts in the real axis starting at the threshold of multi-particle production.

Since no mass-less particle exchange is included in $F(s,t)$, the analytical amplitude satisfies a twice subtracted dispersion relation for a variety of complex t [82]. For any non singular complex point s,t we can write:

$$\frac{1}{2}\frac{d^2F(s,t)}{ds^2}+P=\int_{4m^2}^{\infty}\frac{dx}{\pi}\left\{\frac{ImF(x+i\varepsilon,t)}{(x-s)^3}+\frac{ImF_u(x+i\varepsilon,t)}{(x-u)^3}\right\} \tag{2.18}$$

where we defined $u=4m^2-s-t$ and used the crossing symmetry to write the amplitude in the u-channel as $F_u(x,t)=F(4m^2-x-t,t)$.

The P on the left hand side of (2.18) denotes the second derivative of the residues. By the analyticity assumption this term comes entirely from the complex simple poles produced by the exchange of unstable states. In our discussion the pole term can be neglected since its contribution turns out not to be relevant .

In the case of forward scattering ($t=0$) the imaginary part $ImF(x,0)$ is proportional to the total cross section of the transition $2\pi\rightarrow$'everything' and is therefore non negative. The crossing symmetry leads to a similar result for the u-channel. We conclude that $F''(s,0)$ is a strictly positive function for any real center of mass energy s in the range $0\leq s\leq 4m^2$.

The analyticity assumption can be used to generalize the domain of positivity of the imaginary part of the amplitude. This can be seen by expanding $ImF(x+i\varepsilon,t)$ in partial waves in the physical region and observing that, due to the optical theorem and the properties of the Legendre polynomials, any derivative with respect to t at the point $x\geq 4m^2$, $t=0$ is non negative. The Taylor series of $ImF(x+i\varepsilon,t)$ for $t\geq 0$ is therefore greater than zero. Since an analog result holds for the u-channel, we conclude that the second derivative $F''(s,t)$ is strictly positive (and analytical) for any real kinematical invariant belonging to the triangle $\Delta=\left\{s,t,u|\,0\leq s,t,u\leq 4m^2\right\}$.

In QCD, the scattering of pions at a scale comparable with their masses is very well described by the chiral lagrangian. The 4 pion operators produce order s^2 corrections to the scattering amplitude and eq. (2.18) implies positive bounds on some combination of their coefficients (see [83], for example).

Similar bounds may be obtained for the SM. The anomalous quartic gauge couplings enter the scattering amplitude of two longitudinally polarized gauge bosons at order s^2. We expect that the method outlined in the previous section may be used to bound these coefficients.

There exists, however, a fundamental difference from the QCD case. The assumptions made to derive the relation (2.18) are the analytic, Lorentz and crossing symmetric nature together with the asymptotic behavior of the amplitude $F(s,t)$. A sufficient condition for the latter hypothesis to hold is that no massless particle exchange contribute to F (Froissart bound). In the electroweak case this latter assumption is not natural because of the presence of the electromagnetic interactions.

Although we may consider only amplitudes with no single photon exchange (like $W^{\pm}Z^0 \to W^{\pm}Z^0$ for example), there is still an operative difficulty due to the fact that the amplitude F is generally dominated by the SM graphs at low energy scales. These latter give rise to positive contributions to $F(s,t)$, since the SM is well defined even for vanishing coefficients, and one is lead to conclude that eq. (2.18) implies that the effective operators involved cannot produce a "too large and negative" contribution to the amplitude $F(s,t)$ and that, as a consequence, no significant bound can be derived in the gauged theory. Notice that this is also true in the absence of a light Higgs boson as far as the CM energy is of the order of the Z^0 mass.

One way to overcome these apparent complications is considering amplitudes with no single photon exchange and evaluating them at a high scale $s \gg m_Z^2$ with the equivalence theorem (ET). In this case one has to prove the positivity of the second derivative of the amplitude is guaranteed in the energy regime in which the approach is defined [85].

Another way, which we decide to follow, is working in the global limit. The crucial observation in order to justify this assumption is that in the matching between the effective lagrangian and the UV theory the transverse gauge bosons contribute, because of their weak coupling, in a subdominant way to the effective coefficients of our interest. An accurate estimate of them, and the respective bounds, can therefore be obtained neglecting completely the gauge structure and studying the coefficients of the global theory.

Using this conceptually different (though operationally equivalent) perspective we can study any two by two elastic scattering amplitude and generalize the analysis of [85] to non-forward scattering.

2.3.1 Derivation of the analytical bounds

We first specialize to the case there appears no Higgs-like boson under a cut off Λ.

In this context the basic tool is a non linearly realized effective lagrangian for the breaking pattern $SU(2) \times U(1) \to U(1)$.

Assuming $m_Z^2 \ll \Lambda^2$ and working at energies comparable with the Z^0 mass, the most general lagrangian respecting the above symmetries and up to $O(s^2)$ is given in the previous section.

We stress that in this idealized scenario the π^a are exact Goldstone bosons. To avoid any complication with the asymptotic behavior of the amplitude we can introduce by hand a π^a mass and proceed as in QCD. This mass is actually the consequence of an explicit symmetry breaking term in the UV theory. Being interested in constraining the underlying symmetric theory we are forced to take $m^2 \ll m_Z^2, s$. The bounds we derive differ from the QCD ones for this very reason.

Although no mass gap is present in this context, an approximate positive constraint for

$F''(s,t)$ can be derived. This we do by noticing that a general dispersion relation like (2.18) can be used to bound the anomalous quartic couplings only if the $O(s^3)$ contribution to $F(s,t)$ is negligible. In this regime the second derivative $F''(s,t)$ is dominantly s independent and, for a small non vanishing imaginary part for s, the dispersion relation can be approximated as:

$$\frac{1}{2}\frac{d^2F(s,t)}{ds^2} \simeq \int_0^\infty \frac{dx}{\pi}\left\{\frac{ImF(x+i\varepsilon,t)}{x^3}+\frac{ImF_u(x+i\varepsilon,t)}{x^3}\right\}\left(1+O\left(\frac{s,t}{\Lambda^2}\right)\right) \tag{2.19}$$

where the limit $m^2/s \to 0$ was assumed and the resonant pole term has been neglected. Eq. (2.19) shows that, as far as $O(s^3)$ are negligible compared to $O(s^2)$, the second derivative of the amplitude is strictly positive.

Before evaluating the bounds we notice that the smallness of the EW precision tests T parameter [87] is conveniently achieved by assuming the existence of an approximate global $SU(2)_C$ custodial symmetry under which the Goldstone boson matrix transforms as the adjoint representation. The dominant coefficients associated to anomalous quartic gauge operators are a_4 and a_5 and any $\pi^a\pi^b \to \pi^c\pi^d$ scattering amplitude can be written in terms of the function $A(s,t,u)$. The relevant processes turn out to be:

$$\begin{aligned} A(\pi^0\pi^0 \longrightarrow \pi^0\pi^0) &= A(s,t,u)+A(t,s,u)+A(u,t,s) \\ A(\pi^\pm\pi^0 \longrightarrow \pi^\pm\pi^0) &= A(t,s,u). \end{aligned} \tag{2.20}$$

We can now derive (2.20) twice with respect to s and evaluate the result at $s+i\varepsilon, t$, where $0 < s,t \ll \Lambda^2$. It is convenient to choose a different representation for the kinematical invariants in order to eliminate the logarithms in the final result. We define a scale $w = \sqrt{s(s+t)} = \sqrt{-su} > s$ and obtain:

$$\begin{aligned} a_4(w)+a_5(w) &> -\frac{1}{16}\frac{1}{(4\pi)^2} \\ a_4(w) &> \frac{1}{12}\frac{1}{(4\pi)^2}\left(-\frac{7}{6}+\frac{1}{8}\left(\frac{w}{s}+\frac{s}{w}\right)^2\right). \end{aligned} \tag{2.21}$$

For $t=0$ we have $a_4+a_5 \gtrsim -0.40\times 10^{-3}$ and $a_4 \gtrsim -0.35\times 10^{-3}$ at an arbitrary scale $w = s \ll \Lambda^2$. This result coincides with the one obtained in [85], as expected.

In the case of non-forward scattering, the bound on $\alpha_4(w)$ cannot get arbitrarily large (large w or, equivalently, large t) because at some unknown scale, much smaller than Λ^2, the $O(s^3)$ corrections become relevant in the determination of the amplitude and the bound would not apply. Without a detailed knowledge of the perturbative expansion in the weak coupling s/Λ^2, (that is, of the full theory!) we cannot realistically tell which is the strongest bound derived by this analysis.

What we can certainly do is to compare (2.21) with the well known constraints on the corresponding parameters $l_1 = 4a_5$ and $l_2 = 4a_4$ of QCD. Strong bounds on these coefficients have been evaluated in the triangle Δ [89]. We may interpret our analysis as a study of the axiomatic constraints on the two pion amplitudes in the complementary region $m^2 \ll s \ll \Lambda^2$. Using the notation introduced in [88] we translate (2.21) into $2\bar{l}_1 + 4\bar{l}_4 \gtrsim 3$ and $\bar{l}_2 \gtrsim 0.3$. These constraints are compatible with the experimental observations [90] but are less stringent than those obtained in [89].

We conclude that our analysis does not lead to an improvement of the bounds on $\bar{l}_{1,2}$. If the chiral symmetry is exact, on the other hand, eqs. (2.21) represent stringent bounds on the anomalous quartic couplings implied by the assumptions of analyticity, crossing symmetry, unitarity and Lorentz invariance of the S-matrix.

Eq. (B.37) is not rigorous if a light state enters the processes under consideration and therefore (2.21) are not valid if a Higgs-like scalar propagates under the cutoff. In the next paragraph we discuss an approach which works in this context as well, provided the chiral symmetry is exact.

2.3.2 Causal bounds

Given a general solution of the equations of motion derived from (2.2) we can study the oscillations around it. Consistency with Special Relativity requires the oscillations to propagate sub-luminally. This request may be expressed as a constraint on the same coefficients which enter the elastic scattering of two Goldstone bosons because the dynamics of the oscillation on the background can be interpreted as a scattering process on a macroscopic ‘object‘. If the background has a constant gradient, the presence of super-luminal propagations sum up and can in principle become manifest in the low energy regime [81].

A constant gradient solutions admitted by the lagrangian (2.2) is defined by $\pi_0 = \sigma C_\mu x^\mu$, where σ is a generic isospin direction and the constant vector C_μ is fine-tuned in order to satisfy $C^2 \ll v^4$. The quadratic lagrangian for the oscillations $\delta\pi = \pi - \pi_0$ around the background have the general form:

$$\mathcal{L} = \delta\pi \left(p^2 + \frac{\alpha}{v^4}\,(Cp)^2\right) \delta\pi, \tag{2.22}$$

with $\alpha = a_4, a_4 + a_5$. In the evaluation of (2.22) we neglected $O(Cx/v)$ terms. We can imagine in fact the non trivial background to be switched on in a finite space-time domain so that the latter approximation is seen as a consequence of the fine-tuning of the parameter C_μ.

A perturbative study of the interacting field $\delta\pi$ is in principle possible for energies under a certain scale (to be definite we call this scale the cut-off of the effective theory). By assumption, this cut off is arbitrarily close to Λ as C^2/v^4 goes to zero and, having this fact in mind, we simply denote it as Λ.

A necessary condition for such a perturbative study to make any sense is that the quadratic lagrangian be well defined. This is the case for (2.22) only if $\alpha \geq 0$. In fact, the field $\delta\pi$ has velocity $dE/dp = E/p$ (where $p^\mu = (E, \bar{p})$ and $|\bar{p}| = p$) and for $\alpha < 0$ its quanta propagate super-luminally.

It is important to notice that the presence of super-luminal modes is not the consequence of a bad choice of the vacuum. The quadratic hamiltonian is stable in any vacuum (parametrized by C_μ) if α is 'sufficiently small' but generally leads to violations of the causality principle of Special Relativity when $\alpha < 0$. In the latter hypothesis then different inertial frames may not agree on the physical observations and, for example, the quadratic hamiltonian may appear unbounded from below to a general Lorentzian frame boosted with a sufficiently high velocity.

We finally interpret the constraint $\alpha \geq 0$ as a causal bound.

The effective coefficients α which appear in the perturbative analysis are actually the renormalized couplings so that the above bound can be extended to all energy scales $w < \Lambda^2$, where the perturbative study is assumed to be meaningful, after taking into account the running effect:

$$\begin{aligned} a_4(w) + a_5(w) &\geq \frac{1}{8}\frac{1}{(4\pi)^2}\log\left(\frac{\Lambda^2}{w}\right) \\ a_4(w) &\geq \frac{1}{12}\frac{1}{(4\pi)^2}\log\left(\frac{\Lambda^2}{w}\right). \end{aligned} \tag{2.23}$$

The result cannot be redone for the QCD case because the above chosen background π_0 does not solve the equations of motion when $m \neq 0$.

This approach may be applied even to scenarios in which a scalar Higgs, composite or fundamental, can propagate under the cut off. In this latter case the causal constraints read $a_4 \geq 0$ and $a_4 + a_5 \geq 0$ but now the coefficients do not have any 1-loop scale dependence because the theory has no extra-SM divergences at order s^2. Therefore, the possibility $a_4 = a_5 = 0$ can not and must not be excluded (This is the SM case). The analytical bounds, which would imply a strict inequality, do not apply as already noticed.

In conclusion, the causal one relies on the absence of superluminal propagations. The analytical one relies on the assumption of analyticity, crossing and Lorentz symmetry together with a good behavior at infinity of the scattering amplitude $F(s,t)$. The latter method works in the context of a strongly coupled theory with no Higgs propagating at low energy only. In this scenario (2.21) can be compared to (2.23). We see that the bound on $a_4 + a_5$ is clearly dominated by the causal result and that this is also the case for a_4 if, roughly, the ratio $(w/s)^2$ does not exceed $16\log(\Lambda/\sqrt{w})$. We cannot tell if the analytical bound still apply up to this scale

More importantly, if the fermionic effects are considered separately from $a_{4,5}$, a realistic estimate of the constraints should take the fermions couplings to the Goldstone bosons into account. It is easy to see that the one loop effect induced by the SM fermions gives rise to a positive contribution to the second derivative of the amplitude. This of course lowers the analytical bounds while the causal argument remains valid and (2.23) is not altered.

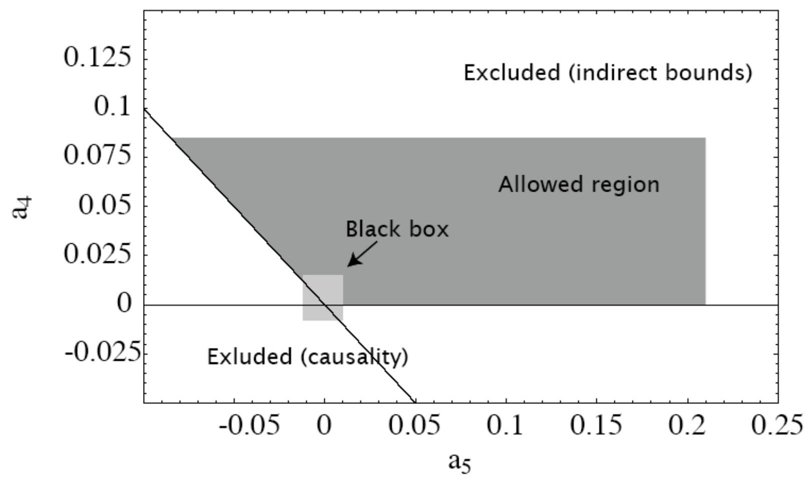

Figure 2.1: The region of allowed values in the a_4-a_5 plane (in gray) as provided by combining indirect bounds and causality constraints. Also depicted, the region below which LHC will not able to resolve the coefficients (Black box).

The bound (2.23) for the higgsless scenario, together with the constraint $a_4 \geq 0$ and $a_4 + a_5 \geq 0$ for the light Higgs-like scenario provide the most stringent and reliable bounds on the effective coefficients $a_{4,5}$.

In order to have a rough estimate of (2.23) we assume $\Lambda \sim 1$ TeV and get $a_4 + a_5 \gtrsim 3.8\times10^{-3}$, $a_4 \gtrsim 2.5\times10^{-3}$ at the Z^0 pole. These values lie inside the very wide experimental bounds $-0.1 \lesssim a_{4,5} \lesssim 0.1$. Eqs. (2.23) significantly reduce the allowed range.

A direct measurement of the anomalous gauge couplings turns out to be of fundamental importance in order to have some insight on the actual nature of the electroweak breaking sector [92]. LHC may improve the bounds [80] by an order of magnitude but the linear collider seems far more appropriate to resolve the coefficients [91]. The measurement of a negative value of a_4 and $a_4 + a_5$ at the next linear collider would therefore signal a breaking of causality, irrespective of the presence of a light scalar state like the Higgs boson. This seems a rather unlikely possibility because it would require too drastic a modification of our physical understanding. A more conservative point of view consists in interpreting the bounds (2.23) as theoretical constraints on the full theory.

Notice that the constraints in eq. (2.23) remove a quite sizable region (most of the negative values, in fact) of values of the parameters a_4 and a_5 allowed by the indirect bounds alone. Fig. 2.1 summarizes the allowed values in the a_4-a_5 plane and compare it with LHC sensitivity.

2.4 Model dependent bounds

Given the results in Fig. 2.1, we can ask ourselves how likely are the different values for the two coefficients a_4 and a_5 among those within the allowed region. Without further assumptions, they are all equally possible and no definite prediction is possible about what we are going to see at the LHC.

In order to gain further information, we would like to find relationships between these two coefficients and between them and those of which the experimental bounds are known. In order to accomplish this, we have to introduce some more specific assumptions about the ultraviolet (UV) physics beyond the cut off of the effective lagrangian. We do it in the spirit of using as much as we know in order to guess what is most likely to be found.

Our strategy is therefore to use our prejudices—that is, model-dependent relationships among the coefficients of the effective lagrangian—plus general constraints coming from causality and analyticity of the amplitudes to see what values the relevant coefficients of the effective electro-weak lagrangian can assume without violating any of the current bounds.

We are aware that in many models the relations among the coefficients we utilize can be made weaker and therefore our bounds will not apply. Nevertheless we find it useful to be as conservative as possible and explore—given what we know from electro-weak precision measurements and taking the models at their face values—what can be said about gauge boson scattering if electro-weak symmetry is broken by a strongly interacting sector. Within this framework, we find that the crucial coefficients are bound to be smaller than the expected sensitivity of the LHC and therefore they will be probably not be detected directly.

As a first step, simple relations for a_4 and a_5 are found by means of assuming that their values are dominated by the integration of particles with masses larger than the cut off. Loops affect are generally not negligible in strongly coupled theories, but we have phenomenological evidences from QCD that even in these cases this approach is reliable[2]. We content ourself with this observation and assume that the leading contribution on the parameters a_i comes from the tree level integration of resonances of arbitrary spin. As an example, consider the tree level integration of an isosinglet massive spin-2 field of mass M. The minimal coupling of the spin-2 field ($I = 0$) to the non-linear sigma model is

$$-\frac{1}{2}g_{\mu\nu}\Delta^{\mu\nu;\sigma\rho}g_{\sigma\rho} + \frac{a}{\Lambda}g_{\mu\nu}\partial^{\mu}U\partial^{\nu}U. \tag{2.24}$$

The propagator P satisfies $\Delta P = \delta$ and reads

$$P^{\mu\nu;\sigma\rho} = -\frac{\frac{1}{2}\left(g_{\mu\sigma}g_{\nu\rho} + g_{\mu\rho}g_{\nu\sigma}\right) - \frac{1}{3}g_{\mu\nu}g_{\sigma\rho}}{p^2 - M^2}. \tag{2.25}$$

[2]One can imagine that the UV degrees of freedom are substituted by a number of resonances of arbitrary spin in the IR regime. The latter can be thought as weakly coupled theories in the large number of UV degrees of freedom

From this we can integrate out the tensor and obtain:

$$\frac{a^2}{2\Lambda^2}\partial_\mu U\partial_\nu U P^{\mu\nu;\sigma\rho}\partial_\sigma U\partial_\rho U = \frac{a^2}{2\Lambda^2 M^2}\left((\partial_\mu U\partial_\nu U)^2 - \frac{1}{3}(\partial_\mu U\partial^\mu U)^2\right) \tag{2.26}$$

that is $\alpha_4 > 0, \alpha_4 + 3\alpha_5 = 0$. In a similar way one can show that an isoscalar scalar gives $\alpha_4 = 0, \alpha_5 > 0$ while an isovector vector gives $\alpha_4 > 0, \alpha_4 + \alpha_5 = 0$. In general, a necessary requirement from stability is therefore $\alpha_4, \alpha_4 + \alpha_5 \geq 0$, in agreement with the previous analysis. Scalar $I = 2$ particles also give $a_4 = -3a_5 > 0$, while spin-2 $I = 2$ tensor give $a_5 > 0$ and $a_4 = 0$.

This exercise provides us with some insight into the possible and most likely values for the coefficients. In particular we can see the characteristic relations between a_4 and a_5 depending on the different quantum numbers of the resonance being integrated. Moreover, one can easily verify that the tree level integration of an arbitrary massive state is consistent with the causality constraints – evaluated at the cut-off scale in the spirit of the Wilson approach – obtained above provided the massive fields are not tachionic.

A further step consists in assuming a specific UV completion beyond the cut off of the effective lagrangian in eq. (2.2). Two scenarios which can be studied with the effective lagrangian approach are a strongly interacting model of a QCD-like theory, a strongly coupled theory at large N, and the strongly coupled regime of a model like the SM Higgs sector in which the Higgs boson is heavier than the cut off or too broad to be seen. For each of these scenarios it is possible to derive more restrictive relationships among the coefficients of the EW lagrangian and in particular we can relate parameters like a_0 and a_1 to a_4 and a_5. These new relationships make possible to use EW precision measurements to constrain the possible values of the coefficients a_4 and a_5.

Modeling a confining dynamics

This scenario is based on a new $SU(N)$ gauge theory coupled to new fermions charged under the fundamental representation. By analogy with QCD these particles are invariant under a flavor chiral symmetry containing the gauged $SU(2)_L \times U(1)_Y$ as a subgroup. Let us consider the case in which no other GB except the 3 unphysical ones are present and therefore the chiral group has to be $SU(2)_L \times SU(2)_R$, with $U(1)_Y \subset SU(2)_R$. The new strong dynamics leads directly to EWSB through the breaking of the axial current under the confining scale around $4\pi v$ and to the appearance of an unbroken $SU(2)_{L+R} = SU(2)_C$ custodial symmetry. Following these assumptions, there are no bounds on the new sector from the parameter T and the relevant constraints come from the S parameter only.[3]

At energies under the confining scale, the strong dynamics can be described in terms of the hadronic states. As a preliminary model, consider the idealized scenario in which

[3]We are not concerned here with the fermion masses and therefore we can bypass most of the problems plaguing technicolor models.

the gluonic degrees of freedom have been integrated leaving an action for the NGB and constituent quarks. In this case we find that a_4 and a_5 are finite at leading order in N, and (by transforming the result of [16] for QCD) read $a_4 = -2a_5 = -a_1/2$, which provide us with the link between gauge boson scattering and EW precision measurements—the coefficient a_1 being directly related to the parameter S as indicated in eq. (2.5).

In a more refined approach, the non-perturbative effects have been integrated out giving rise to a gluon condensate. The result, again at leading order in $1/N$, becomes [17]:

$$\begin{aligned} a_4 &= \frac{N}{12(4\pi)^2} \\ a_5 &= -\left(\frac{1}{2} + \frac{6}{5}\langle G^2\rangle\right) a_4 \,, \end{aligned} \tag{2.27}$$

where $\langle G^2\rangle$ is an average over gauge field fluctuations. The latter is a positive and order 1 free parameter that encodes the dominant soft gauge condensate contribution which there is no reason to consider as a negligible quantity. Without these corrections the result is equivalent to those obtained considering the effect of a heavier fourth family, as seen above. Causality requires $\frac{6}{5}\langle G^2\rangle \leq \frac{1}{2}$ and therefore we will consider values of $\langle G^2\rangle$ ranging between $0 < \langle G^2\rangle < 5/12$.

The S parameter gives stringent constraints on N:

$$S_{EWSB} = \frac{N}{6\pi}\left(1 + \frac{6}{5}\langle G^2\rangle\right) \tag{2.28}$$

which is slightly increased by the strong dynamics with respect to the perturbative estimate, in good agreement with the non-perturbative analysis given in [87]. From the bounds on S_{EWSB}, we have $N < 4$ (2σ) and N < 7 (3σ) respectively. The relevant bounds on a_4 is then obtained via a_1 and yields

$$0 < a_4 < \frac{S_{EWSB}}{32\pi} \,. \tag{2.29}$$

We are going to use the bounds given in eq. (2.27) and eq. (2.29). Notice that the coefficients a_i are scale independent at the leading order in the $1/N$ expansion.

Taking a_1 at the central value of S_{EWSB} gives $a_4 < 0$, which is outside the causality bounds. This is just a reformulation in the language of effective lagrangians of the known disagreement with EW precision measurements of most models of strongly interacting EW symmetry breaking.

We expect vector and scalar resonances to be the lightest states in analogy with QCD. The high spin or high $SU(2)_C$ representations considered earlier are typically bound states of more than two fermions and therefore more energetic. Their large masses make their contribution to the a_i coefficients subdominant.

The relations (2.23) and (2.27) satisfied by the model imply that $-a_4 < a_5 < -a_4/2$, an indication that scalar resonances give contributions comparable with the vectorial ones in the large-N limit (see the discussion at the beginning of this section). If vectors had been the only relevant states, the relation would have been $a_4 = -a_5$. It is useful to pause and compare this result with that in low-energy QCD with 3 flavors.

Whereas in the EW case we find that the large-N result indicates the importance of having low-mass scalar states, the chiral lagrangian of low-energy QCD has the corresponding parameters L_1 and L_2 saturated by the vector states alone. This vector meson dominance is supported by the experimental data and in agreement with the large-N analysis, which in the case of the group $SU(3)$ is different from that of the EW group $SU(2) \times U(1)$. Even though the scalars have little impact on the effective lagrangian parameters of low-energy QCD, they turn out to be relevant to fit the data at energies larger than the ρ mass where the very wide σ resonance appearing in the amplitudes is necessary. One may ask if something similar applies to the EWSB sector, it being described by a similar low-energy action. The answer is positive and it ultimately follows from the fact that the vectors posses 3 polarizations: in order to ensure perturbative unitarity, the high energy contribution of the NGB must be compensated by scalar degrees of freedom. With vectors only the restoration is partial, as we will see in the next paragraph.

The larger dark triangle in Fig. 2.2 shows the allowed values for the coefficients a_4 and a_5 as given by eq. (2.27) and eq. (2.29). The gray background is drawn according to the causality constrain which is assumed scale independent to be consistent with the leading large-N result.

Large N models

Five-dimensional higgsless models [61] have been proposed to solve the naturalness problem of the SM. They describe a gauge theory in a 5D space-time that produces an appropriate tower of massive vectors through the choice of particular boundary conditions. Our world is supposed to be realized on a 4 dimensional brane (3-brane). The lightest Kaluza-Klein modes are interpreted as the physical $W^{\pm}$ and Z^0 while those starting at a mass scale Λ represent a new weakly coupled sector.

The physics of these models is very well understood in terms of a deconstruction of the extra dimension. In this language the tower of spin-1 fields is interpreted as the insertion of an infinite number of hidden local symmetries and can be seen as a modeling of a large N dynamics in which the spin-1 fields dominate the IR physics. For the moment we maintain a purely 5-dimensional perspective, we will have to say more about these scenarios and the dual interpretation in a following chapter.

The scale of unitarity violation is automatically raised to energies larger than 1.3 TeV because the term in the amplitude linearly increasing with the CM energy s is not present in these models. That this is potentially possible can be seen by looking at the contribution of a single vector to the tree-level fundamental amplitude:

$$A(s,t,u) = \frac{s}{v^2} - \frac{3M_V^2 s}{\hat{g}^2 v^4} + \frac{M_V^4}{\hat{g}^2 v^4}\left(\frac{u-s}{t-M_V^2} + \frac{t-s}{u-M_V^2}\right) \tag{2.30}$$

with $\hat{g}$ (not to be interpreted as a gauge coupling) and M_V^2 representing the only two parameters entering up to order p^4. The limit $s \ll M_V^2$ corresponds to integrate the vector out and gives the low energy theorem with the previously mentioned $a_4 = -a_5 = 1/(4\hat{g}^2)$. The condition $M_V^2 = \hat{g}^2 v^2/3$ erases the linear term but cannot modify the divergent behavior of the forward and backward scattering channels. In fact we still find the asymptotic form $t_{00}(s) \simeq \hat{g}^2/(36\pi)\log(s/M_V^2)$ which has to be roughly less than one half to preserve unitarity. This shows why models with only vector resonances cannot move the UV cut off too far from the vector mass scale, as opposed to what happens in the case of scalar particles.

These 5D models fear no better than technicolor when confronted by EW precision measurements. There exists an order 1 mixing among the interpolating gauge bosons – those residing on the SM brane, and thus coupling to SM currents – and the physical heavy vectors which contribute a tree level $W_\mu^3 - B_\nu$ exchange and consequently a $S_{EWSB} \propto 1/(gg')$. In 5D notation and for the simplest case of a flat metric, $S_{EWSB} = O(1)/g^2 \simeq R/g_{(5)}^2$, in agreement with [22]. This result can be ameliorated by the introduction of a warped 5D geometry, or boundary terms or even by a de-localization of the matter fields [23].

These models present the relation $a_4 = -a_5$ which is characteristic of all models with vector resonances only. This line in the $a_4 - a_5$ plane of Fig. 2.2 lies on the causality bound and coincides with the QCD-like scenario in which the strong dynamical effect $\langle G^2\rangle$ is maximal.

The coefficient a_4 is related to a_1. We find that

$$a_4 = -\frac{1}{10}a_1\,, \tag{2.31}$$

and therefore,

$$a_4 = -a_5 = \frac{\pi^2}{120}\frac{v^2}{M_1^2} = \frac{S_{EWSB}}{160\pi}\,. \tag{2.32}$$

The constraints on S of eq. (A.26) lead to $M_1 > 2.5$ TeV which implies a violation of unitarity, and consequently the need of a UV completion for the 5D theory, at the scale $\sim M_1^2$.

The parameters a_4 and a_5 are—as in the other scenarios considered—too small to be directly detected at the LHC. The large mass M_1 of the first vector state makes it hard for the LHC to find it.

In case of a warped fifth dimension these relations are slightly changed but the tension existing between the unitarity bound (which requires a small M_1^2 to raise the cut off above 1.3 TeV) and the S parameter (which requires a large M_1^2) remains a characteristic feature of these models.

Heavy-Higgs scenario

This scenario is a bit more contrived than the previous ones and a few preliminary words are in order.

In the perturbative regime, a scalar Higgs-like particle violates unitarity for masses of the order of 1200 GeV [18]. Moreover, the mass of the Higgs is proportional to its self coupling and from a naive estimate we expect the perturbation theory to break down at $\lambda \sim 4\pi$, that is $m_H \sim 1300$ GeV. What actually happens in the case of a non-perturbative coupling is not known. Problems connected with triviality are not rigorous in non-perturbative theories and therefore the hypothesis of a heavy Higgs cannot be ruled out by this argument.

In order to have a more qualitative understanding of the non-perturbative regime we review the analysis of the large N approximation for the ϕ^4 theory. The formalism introduced here is useful in the study of some non-perturbative features of the non-linear model, as well.

Consider the action:

$$\mathcal{L}_{L\sigma M} = \frac{1}{2}(\partial_\mu U)^2 - \frac{\alpha}{2}\left(U^2 - f^2\right) + \frac{\alpha^2}{2\lambda} + HU, \tag{2.33}$$

where U is a N-plet of scalars and H an external current (the indeces are suppressed). The integration of α is trivial and leads to an equivalent action

$$\mathcal{L}_{L\sigma M} = \frac{1}{2}(\partial_\mu U)^2 + \frac{\lambda f^2}{2}U^2 - \frac{\lambda}{4}U^4 - \frac{\lambda}{4}f^4 + HU, \tag{2.34}$$

which represents the well known linear sigma model (LσM). As the bare coupling is sent to infinity one recovers the non-linear model.

We can now study the ground state of the model. Integrating in the $N-1$ variables U_i and allowing the possibility that the fields U – we conventionally choose it to point in the direction N – and α develop constant vevs

$$\langle U_N \rangle \equiv v; \quad \langle \alpha \rangle \equiv m^2,$$

we find

$$f^2 + \frac{m^2}{\lambda} - N\int \frac{d^dp}{(2\pi)^d}\frac{i}{p^2 - m^2} - v^2 = 0; \quad m^2 v = 0 \tag{2.35}$$

The solutions can be either $v \neq 0$ or $m^2 \neq 0$. In the case $m^2 > 0$ the U propagator contains a mass term m^2 (we see that $m^2 < 0$ is pathologic): the symmetry $O(N)$ is unbroken. In this case it is mandatory that $\lambda > 0$ because of stability arguments. If $m^2 = 0$ then (bearing miracolous cancellations between the bouble diagram and the bare mass f^2) the symmetry is spontaneously broken and necessarily $f^2 \geq 0$. The fluctuations of the field U_N

is the Higgs field. The latter can decay into an α excitation, and subsequently to NGbs, and becomes unstable. The low energy excitations are $N-1$ massless NGBs, as expected by perturbation theory.

We focus on the broken phase for obvious reasons and evaluate the scattering amplitude for two NGBs. All interactions are mediated by the α fields, and its propagator is thus an essential ingredient. The α 1PI is obtained as a sum of a bouble and a mixing term with the Higgs, which gives:

$$\begin{aligned} -i\Sigma &= (-i)^2 \frac{N}{2} \int \frac{d^d p}{(2\pi)^d} \frac{i}{p^2} \frac{i}{(p-q)^2} + (-iv)^2 \frac{i}{q^2} \\ &= -i \left[-\frac{N}{2} J(d, q^2, 0) + \frac{v^2}{q^2} \right]. \end{aligned} \tag{2.36}$$

By a cut-off regularization we find $J = \log\left(\frac{-\Lambda^2}{s}\right)/(4\pi)^2$. The full propagator is found in the usual fashion by contracting with the bare propagator $i\lambda$:

$$D(q^2) = i\lambda \sum_{n=0}^{\infty} (\lambda\Sigma)^n = \frac{i\lambda}{1 - \lambda\left[-\frac{N}{2}J(d, q^2, 0) + \frac{v^2}{q^2}\right]}. \tag{2.37}$$

The divergence in J can be eliminated by reabsorbing it into the bare coupling λ and defining the renormalized coupling $\bar{\lambda}$:

$$\frac{1}{\lambda} + \frac{N}{32\pi^2} \log\left(\frac{-\Lambda^2}{s}\right) \equiv \frac{1}{\bar{\lambda}} + \frac{N}{32\pi^2} \log\left(\frac{-\mu^2}{s}\right)$$

or

$$\bar{\lambda} = \frac{\lambda}{1 + \frac{N\lambda}{32\pi^2} \log\left(\frac{\Lambda^2}{\mu^2}\right)}. \tag{2.38}$$

From the latter we see that for any possible (positive) bare coupling the renormalized coupling goes to zero as the cut-off is removed. This is the celebrated triviality of the LσM. The beta function is found to be

$$\mu \frac{d\bar{\lambda}}{d\mu} = \frac{N}{(4\pi)^2} \bar{\lambda}^2 \tag{2.39}$$

and coincides with the 1-loop result.

We are now in a position to evaluate the physical Higgs mass by looking at the poles of (2.37). Because of the scale independence of the amplitude, we conveniently choose the renormalization scale μ as the modulus of the (complex) pole $\sqrt{s} = \bar{m} - i\Gamma/2 = \mu e^{-i\theta}$, that is

$$\bar{m} = \mu \cos 2\theta; \qquad \Gamma = 2\bar{m} \tan 2\theta. \tag{2.40}$$

One of the conditions for the vanishing of the denominator in (2.37) is

$$\tan 2\theta = \frac{N\bar{\lambda}}{32\pi^2}(2\theta - \pi) \tag{2.41}$$

and expresses the fact that the ratio $\Gamma/\bar{m}$ is an increasing function of the coupling $\bar{\lambda}N$. A detailed analysis shows that the mass $\bar{m}$ is limited to a region below around 1 TeV, while the width increases more rapidly than the perturbative estimate. By comparing a two-loop analysis and a next to leading calculation in $1/N$, one concludes that the loop expansion provides a remarkably careful prediction [126]. For completeness we mention that the propagator has also a tachionic pole at $|p^2| > \Lambda^2$, which is therefore unphysical (using dimensional regularization this is not apparent).

Our study reveals that a residual Higgs field is present for any value of the coupling in the hundred of GeV region. However, because of the large decay width into NGBs (this is replaced by a decay into gauge bosons in the SM, but as far as the Higgs mass exceeds $2m_W$ the result applies. We assume this is the case), the pole is most likely invisible to the detectors and the phenomenology resembles that of a higgsless theory. As long as we intend such a broad Higgs boson only as a modeling of the UV completion of the EW effective lagrangian, we can safely study this scenario by using the perturbative expansion and assuming a heavy Higgs mass.

The effective lagrangian parameters in the case of a heavy Higgs can be computed by retaining only the leading logarithmic terms to yield $a_4 = -a_1$ and $a_4 = 2a_5$, which contains the link between gauge boson scattering and the coefficient a_1 we need. A more complete computation [19] gives

$$\begin{aligned} a_4(m_Z) &= -\frac{1}{12}\frac{1}{(4\pi)^2}\left(\frac{17}{6} - \log\frac{m_H^2}{m_Z^2}\right) \\ a_5(m_Z) &= \frac{v^2}{8m_H^2} - \frac{1}{24}\frac{1}{(4\pi)^2}\left(\frac{79}{3} - \frac{27\pi}{2\sqrt{3}} - \log\frac{m_H^2}{m_Z^2}\right) \end{aligned} \tag{2.42}$$

and

$$S_{EWSB} = \frac{1}{12\pi}\left(\log\frac{m_H^2}{m_Z^2} - \frac{5}{6}\right). \tag{2.43}$$

The causality constrain (2.23) applied to the above coefficients implies a bound on the possible values of the cutoff compared to the integrated mass, $m_H/\Lambda > 1$. Putting these equations together, we obtain:

$$a_4 = \frac{1}{16\pi}\left(S_{EWSB} - \frac{1}{6\pi}\right)$$
$$a_4 = 2a_5 - \frac{v^2}{4m_H^2} + \frac{1}{12}\frac{1}{(4\pi)^2}\left(\frac{141}{6} - \frac{27\pi}{2\sqrt{3}}\right) \tag{2.44}$$

As before in the large-N scenario, the central value of S_{EWSB} yields a value of a_4 outside the causality bounds.

The perturbative results should be compared to the large N estimate. The amplitude for the NGB scattering is given at leading order in $1/N$ by $A = iD$, with D given by (2.37). In terms of the renormalized coupling we have:

$$\begin{aligned} A &= \frac{\frac{s}{v^2}}{1 - \frac{s}{v^2}\left[\frac{1}{\bar{\lambda}} + \frac{N}{32\pi^2}\log\left(\frac{-\mu^2}{s}\right)\right]} \qquad (2.45) \\ &= \frac{s}{v^2} + \frac{s^2}{v^4}\left[\frac{1}{\bar{\lambda}} + \frac{N}{32\pi^2}\log\left(\frac{-\mu^2}{s}\right)\right] + \ldots \end{aligned}$$

At this point we can collect these results with those of the previous section and conclude that in both scenarios under study, the limits on the coefficients a_4 and a_5 are well below LHC sensitivity (compare Fig. 2.1 and Fig. 2.2). If this is the case, the LHC will probably not be able to resolve the value of these coefficients because they are too small to be seen. It goes without saying that this can only be a provisional conclusion in as much as in many models the relations among the coefficients we utilize can be made weaker by a variety of modifications which make the models more sophisticated. Accordingly, our bounds will not apply and the LHC may indeed measure a_4 or a_5 and we will then know that the UV physics is not described by the simple models we have considered.

2.4.1 Experimental signatures: resonances

Even though the values of the coefficients may be too small for the LHC, the perturbative unitarity of the amplitudes is going to be violated at a scale around 1.3 TeV unless higher order contributions are included. Following the well-established tradition of unitarization in the strong interactions, we would like to consider the Padé approximation [25].

This procedure is carried out in the language of the partial waves introduced in (2.16). The Padé coefficient $t^{[r,r]}$ is defined as

$$t^{[r,r]} = \frac{n_0 + n_2 + \ldots + n_{2r}}{d_0 + d_2 + \ldots + d_{2r}} = t^{(2)} + \ldots + t^{(2r)}, \tag{2.46}$$

where t_i is the order i partial wave and the n_i, d_i's are determined by an order by order matching procedure. The coefficient is defined such to satisfy identically the elastic unitarity

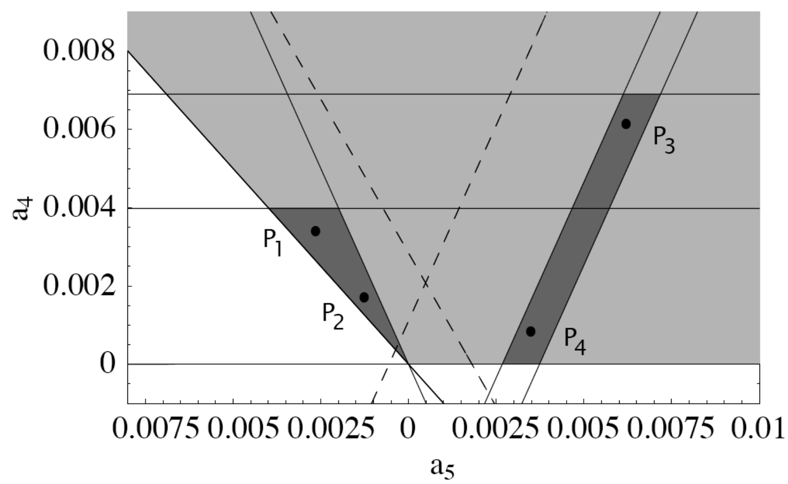

Figure 2.2: Model-dependent bounds for the coefficients. Horizontal lines mark the bounds from EW precision tests for the QCD-like scenario (lower line) and heavy-Higgs scenario (higher line). Four representative points are indicated: P_1 and P_2 for the former and P_3 and P_4 for the heavy Higgs. The two oblique dashed lines represent, respectively, the region of vector resonances (left side of dashed line with positive angular coefficient) and of scalar resonances (right side of dashed line with negative angular coefficient). Notice that the range of this figure is all within the black box of Fig. 2.1.

relation

$$Im[t^{[r,r]}] = |t^{[r,r]}|^2.$$

At leading non-trivial order we find that

$$t^{[1,1]} = \frac{t^{(2)}_{IJ}}{1 - t^{(4)}_{IJ}/t^{(2)}_{IJ}} = t_{IJ}(s) + O(s^3)\,. \qquad (2.47)$$

Equation (2.47) coincides with the result of the so called inverse amplitude method (IAM), which is an alternative approach to the unitarization procedure which makes use of dispersion theory.

The Padé, IAM, approximation has given remarkable results describing meson interactions, having a symmetry breaking pattern almost identical to our present case. To show that its predictions actually capture some non-perturbative effect we compare the singlet partial wave for a large N chiral lagrangian computation, for which a leading order amplitude can be derived analytically, to the leading order Padé approximation obtained above.

In the large N limit one finds

$$A^{(2)} = \frac{s}{v^2} \qquad A^{(2)} = \left(\frac{s}{v^2}\right)^2 \frac{N}{2} J. \tag{2.48}$$

Because at large N $t_{00} = NA_0$ we get the Padé coefficient

$$t^{[1,1]} = \frac{t_{00}^{(2)}}{1 - 16\pi J t_{00}^{(2)}}. \tag{2.49}$$

Remarkably, this is the same solution one finds at leading order in $1/N$! The latter can be easily deduced by taking the limit $\lambda \to \infty$ (bare coupling) in the amplitude derived in the previous section from the large N linear sigma model.

Having justified the physical interest of the above unitarization scheme, we would like now to apply equation (2.47) to the perturbative chiral lagrangian amplitudes in order to link the possible resonances predicted by the Padé approximation to the anomalous quartic gauge couplings a_4 and a_5.

By substituting the expressions (2.16) into (eq. (2.47)) we find resonant poles in the symmetric and antisymmetric channels. The corresponding masses and widths of the first resonances are:

$$m_S^2 = \frac{4v^2}{\frac{16}{3}\left[11a_5(\mu) + 7a_4(\mu)\right] + \frac{1}{16\pi^2}\left[\frac{101-50\log(m_S^2/\mu^2)}{9}\right]}\,, \qquad \Gamma_S = \frac{m_S^3}{16\pi v^2}\,, \tag{2.50}$$

for scalar resonances, and

$$m_V^2 = \frac{v^2}{4\left[a_4(\mu) - 2a_5(\mu)\right] + \frac{1}{16\pi^2}\frac{1}{9}}\,, \qquad \Gamma_V = \frac{m_V^3}{96\pi v^2}\,, \tag{2.51}$$

for vector resonances.

A few words of caution about the IAM approach are in order.

The IAM derivation of (eq. (2.47)) (which we do not review here) makes it clear that the resonances obtained represent the lightest massive states we encounter (above the Z pole) in each channel (it represents the first pole in the complex s-plane, and thus determines the radius of convergence of the chiral expansion. These resonances are not necessarily the only massive states produced by the non-perturbative sector, but those with higher masses are expected to give subdominant contribution.

Since we neglect $O(s^3)$ terms, the regime $s \sim m_{res}^2$ is not actually trustable. The larger the resonance peak, the larger the error: we expect the IAM prediction to give more accurate results in the case of very sharp resonances. This is the reason behind the success of the IAM for the vector resonances in QCD as opposed to the more problematic very broad

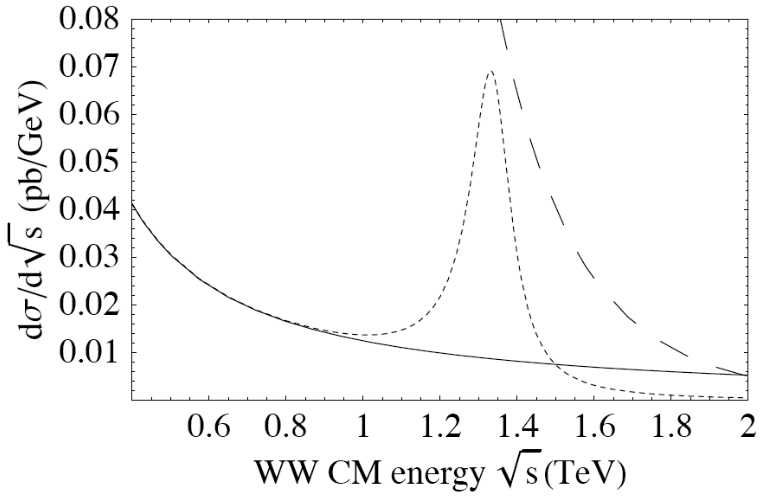

Figure 2.3: Parton-level cross sections for WW scattering. The continuous line is the result of the effective lagrangian. The long-dashed line is the limit after which unitarity is lost. The dashed line with a peak is the amplitude in presence of a vector resonance in the QCD-like scenario. The figure corresponds to the representative point P_1 discussed in the text.

scalar σ. Nevertheless, we consider the IAM result a remarkable prediction, given the very small amount of information needed.

Onother way to check the physical reliability of this method consists in separating the $a_{4,5}$ plane into three areas depending on the predicted lowest laying resonances being a vector, a scalar or even both of them. This partition follows the coefficients patterns one expects by studying the tree level values for a_4 and a_5 as given in section 2.4. It is represented in Fig. 2.2 by the two oblique and dashed lines which mark the limit where Γ/M is less or more than $1/4$ for the case of scalar (oblique line with negative angular coefficient) and vector (oblique line with positive angular coefficient) resonances.

A naive estimate—based on integrating out massive states like in the vector meson dominance of QCD—shows that for resonance masses M between the range of hundreds GeV and a few TeV we should expect the a_i coefficients to range from 10^{-2} to 10^{-3}, which agrees with the IAM formula.

Gauge boson scattering and the presence of resonances have previously been discussed in a number of papers [26, 27].

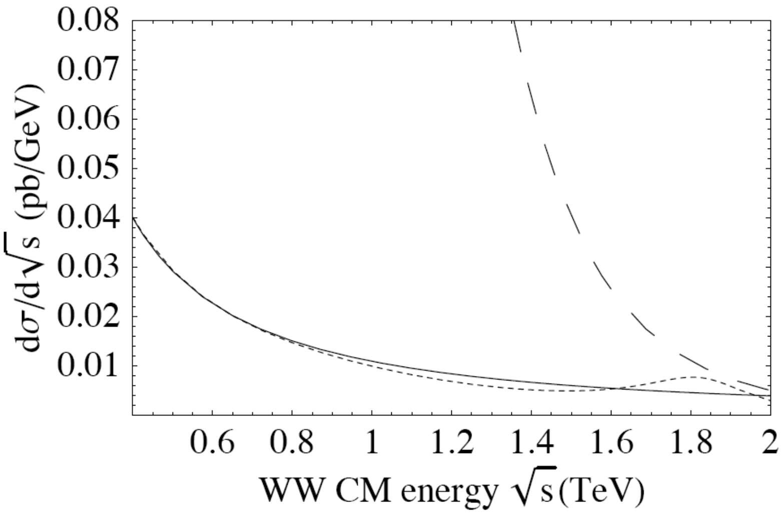

Figure 2.4: Parton-level cross sections for WW scattering. The continuous line is the result of the effective lagrangian. The long-dashed line is the limit after which unitarity is lost. The dashed line with a peak is the amplitude in presence of a vector resonance in the QCD-like scenario. The figure corresponds to the representative point P_2 discussed in the text.

2.4.2 Parton-level cross sections

Our plan is to choose two representative points for each of the considered scenarios in the allowed $a_4 - a_5$ region and then find the first resonances appearing in the $W_L W_L$ elastic scattering using the IAM approximations. The points are shown in Fig. 2.2. We take

$$P_1 : \begin{cases} a_4 &= 3.5 \times 10^{-3} \\ a_5 &= -2.5 \times 10^{-3} \end{cases} \quad \text{and} \quad P_2 : \begin{cases} a_4 &= 1.7 \times 10^{-3} \\ a_5 &= -1.3 \times 10^{-3} \end{cases} \tag{2.52}$$

for the QCD-like scenario and

$$P_3 : \begin{cases} a_4 &= 5.7 \times 10^{-3} \\ a_5 &= 6.0 \times 10^{-3} \end{cases} \quad \text{and} \quad P_4 : \begin{cases} a_4 &= 3.5 \times 10^{-3} \\ a_5 &= 0.7 \times 10^{-3} \end{cases} \tag{2.53}$$

for the heavy-Higgs scenario.

The first pair corresponds to having vector resonances at

$$\begin{cases} m_V &= 1340 \text{ GeV} \\ \Gamma_V &= 128 \text{ GeV} \end{cases} \quad \text{and} \quad \begin{cases} m_V &= 1870 \text{ GeV} \\ \Gamma_V &= 346 \text{ GeV} \end{cases} \tag{2.54}$$

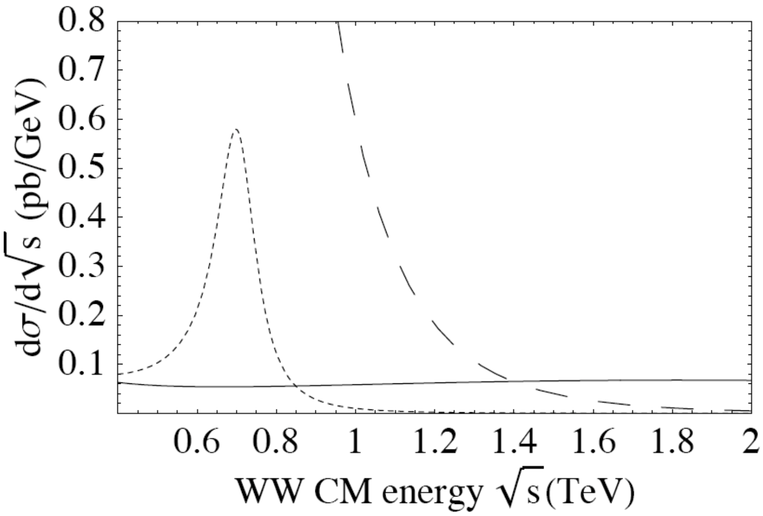

Figure 2.5: Parton-level cross sections for WW scattering. The continuous line is the result of the effective lagrangian. The long-dashed line is the limit after which unitarity is lost. The dashed line with a peak is the amplitude in presence of a scalar resonance in the heavy-Higgs scenario. The figure corresponds to the representative point P_3 discussed in the text.

together with heavier (2 TeV) and very broad scalar states, while the second pair to scalar resonances at

$$\left\{\begin{array}{lcl} m_S & = & 712 \text{ GeV} \\ \Gamma_S & = & 78 \text{ GeV} \end{array}\right. \quad \text{and} \quad \left\{\begin{array}{lcl} m_S & = & 1250 \text{ GeV} \\ \Gamma_S & = & 237 \text{ GeV} \end{array}\right. \tag{2.55}$$

These points are representative of the possible values and span the allowed region. The resonances become heavier, and therefore less visible at the LHC, for smaller values of the coefficients. Accordingly, whereas points P_1 and P_3 give what we may call an ideal scenario, the other two show a situation that will be difficult to discriminate at the LHC.

We can now consider the physical process $pp \to W_L W_L jj + X$ and plot its differential cross section in the WW CM energy $\sqrt{s}$ for the values of the coefficients a_4 and a_5 we have identified. To simplify, we will use the effective W approximation [29].

Once the amplitude $A(s,t,u)$ is given, the differential cross-section for the factorized WW process is

$$\frac{d\sigma_{WW}}{d\cos\theta} = \frac{|\mathcal{M}(s,t,u)|^2}{32\pi\, s}. \tag{2.56}$$

while the differential cross section for the considered physical transition $pp \to W_L W_L jj + X$

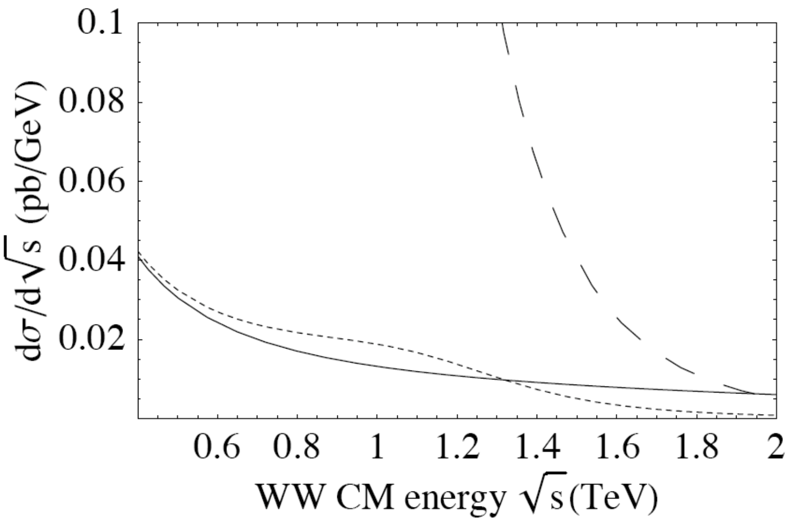

Figure 2.6: Parton-level cross sections for WW scattering. The continuous line is the result of the effective lagrangian. The long-dashed line is the limit after which unitarity is lost. The dashed line with a peak is the amplitude in presence of a scalar resonance in the heavy-Higgs scenario. The figure corresponds to the representative point P_4 discussed in the text. Notice that this plot has rescaled vertical axis with respect to Fig 2.5 because of the smallness of the resonant peak.

reads:

$$\frac{d\sigma}{ds} = \sum_{i,j} \int_{s/s_{pp}}^{1} \int_{s/(x_1 s_{pp})}^{1} \frac{dx_1\, dx_2}{x_1 x_2 s_{pp}} f_i(x_1, s)\, f_j(x_2, s) \frac{dL_{WW}}{d\tau} \int_{-1}^{1} \frac{d\sigma_{WW}}{d\cos\theta} d\cos\theta \tag{2.57}$$

where $\sqrt{s_{pp}}$ is the CM energy which we take to be 14 TeV, as appropriate for the LHC, and

$$\frac{dL_{WW}}{d\tau} \approx \left(\frac{\alpha}{4\pi \sin^2\theta_W}\right)^2 \frac{1}{\tau} \left[(1+\tau)\ln(1/\tau) - 2(1-\tau)\right] \tag{2.58}$$

where $\tau = s/(x_1 x_2 s_{pp})$. For the structure functions f_j we use those of ref. [30].

The high-energy regime will be very much suppressed by the partition functions so that the resonances found by (2.50) and (2.51) turn out to be the only phenomenologically interesting ones. Because of this, we can safely make use of the approximation (2.47) in the whole range from 400 GeV to 2 TeV and thus we take $A(s,t,u)$ to be given by the IAM unitarization of (2.14).

Figures 2.3, 2.4 and 2.5, 2.6 give the cross section for the QCD-like and heavy-Higgs scenario, respectively. The scalar resonance corresponding to P_3 is particularly high and

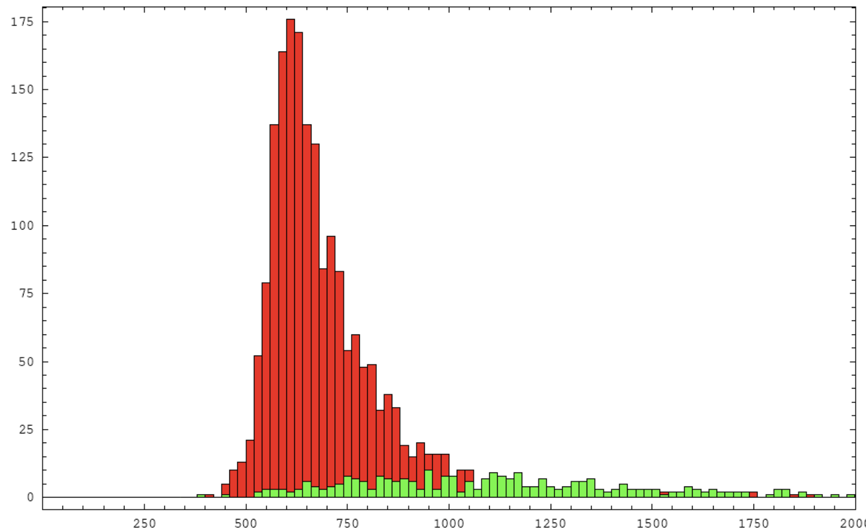

Figure 2.7: Resonant events ($a_4 = 5.7 \times 10^{-3}$; $a_5 = 6.0 \times 10^{-3}$) and ($a_4 = a_5 = 0$) as a function of the invariant mass in a simple data simulation (10000 events).

narrow and a very good candidate for detection. For a LHC luminosity of 100 fb^{-1}, it would yield 10^6 events after one year. If it exists, it will appear as what we would have called the Higgs boson even though it is not a fundamental state and its mass is much heavier than that expected for the SM Higgs boson.

The actual signal at the LHC requires that the parton-level cross sections derived here be included in a Montecarlo simulation (of the bremsstrahlung of the initial partons, QCD showers as well as of the final hadronization) and compared with the expected background and the physics of the detector. In the papers of ref. [27, 28] it has been argued that resonances in the range here considered can be effectively identified at the LHC. Similar signals have also been analyzed in [31].

A reconstruction of one of the resonances discussed is shown in Fig. 2.7. We have used a modified subroutine [28] in Pythia [32] and then PGS [33]. The background has been cut according to the following requirements:

1. events with one lepton, missing energy and at least 3 jets;
2. p_t lepton and missing energy greater than 40 GeV;
3. $|\eta|$ of lepton less than 2.5;
4. p_t of the hardest jet greater than 150 GeV.

Chapter 3

Conformal symmetry and EW theory

In this chapter I will discuss scenarios of EWSB in which conformal strong dynamics plays a crucial role.

There are at least two motivations to consider conformal field theories (CFT). The first one is computational. CFTs are highly constrained and a lot of information can be deduced on pure symmetry grounds. The second motivation is related to naturalness. A CFT is built of operators with exact (quantum) dimension 4, so that no hierarchy problem can arise in such a framework. In fact, no mass scale is even present! Since we know at least two fundamental scales of nature, the weak and the Planck scales, the CFT can only be approximate. Nevertheless, if a nearly conformal sector dominates the energy range in between these two scales, the hierarchy problem would be at least stabilized.

Several models of nearly conformal new physics have been proposed by now. These include walking technicolor, conformal technicolor, Randall-Sundrum scenarios, and unparticles. We will focus on the phenomenological implications of a class of models in which the (approximate) conformal group is spontaneously broken.

3.1 From the extra dimension and back

In this section (and most extensively in the next chapter) we would like to elaborate on the appealing possibility of describing a certain class of strongly coupled 4D models in terms of extra dimensional theories. A complete review of the reasoning that lead to this conclusion would need an entire volume, hence we decide to simply comment on some of the general features.

The idea that an extra dimension can provide an approximate description of a large N confined theory has a phenomenological origin in the concept of hidden local symmetries. A confining theory with large number of fundamental constituents is expected to generate an infinite number of weakly coupled resonances, the lightest of which are naturally scalars and vectors. The hidden local symmetry idea instructs the physicist on how to introduce a vector field in a systematic and minimal way. One assumes that in the low energy effective theory, typically characterized by a broken global symmetry pattern $SU(N_L)\times SU(N_L)\rightarrow SU(N_V)$, an additional spontaneously broken $SU(N_f)$ gauge group is present. It turns out that the approach is phenomenologically viable, in particular one finds that the ρ meson in QCD is described by the vector field of the broken $SU(N_f)$ at a remarkable level of accuracy. It is now clear that if our aim is to describe an infinite number of resonances we should introduce an infinite number of hidden local symmetries. The outcome is understood via the concept of dimensional deconstruction: the infinite set of symmetries can be effectively described in terms of an extra dimensional theory with an $SU(N_f)$ gauge symmetry in the bulk, the Kaluza Klein modes being the resonances.

The phenomenological relation between the extra dimension and large N theories which we illustrated above can be made formal in terms of the celebrated AdS/CFT correspondence. The latter was originally conjectured between a full string theory, namely Type IIB on $AdS_5\times S_5$ and $\mathcal{N}=4$ SYM in $d=4$ dimensions, but at present a large part of the physical community believes that similar correspondences can be found between $non-AdS$ and $non-CFT$ theories. The conjectures has now turned into a more general Gauge/Gravity correspondence. We will show later on an explicit study of the reliability of the correspondence in a non-conformal example.

The Randall-Sundrum model is the simplest phenomenological realization of the above ideas. Consider a $5-$dimensional world described by the 4D coordinates x^μ and y. The fifth coordinate is assumed to be an orbifolded segment bounded by two branes at the points $y=y_\pm$. The action is $\int d^4x\mathcal{L}$, with:

$$\begin{aligned}\mathcal{L} &= 2\int_{y_+}^{y_-} dy\sqrt{-g}\left[-M^3R-V\right]+\sqrt{-g_+}\left[-V_+\right]+\sqrt{-g_-}\left[-V_-\right] \\ &+ \ \dots\end{aligned} \tag{3.1}$$

where the dots stand for a Gibbons-Hawking term and possible additional terms, and $g_{+,-}$ denotes the determinant of the brane induced metric at the points $y=y_\pm$ respectively. The spacetime ranges from $y_+\leq y\leq y_-$ and posses a Z_2 reflection symmetry around the two branes. The factor of 2 in front of the integral over the extra dimension accounts for the orbifold symmetry with fixed points $y=y_\pm$.

We are interested in finding a natural background solution with four dimensional Poincare invariance. The most general line element compatible with this requirement is

$$ds^2=e^{2A}\eta_{\mu\nu}dx^\mu dx^\nu-dy^2, \tag{3.2}$$

with A a function of y. The equations of motion for a line element of the form (3.9) reduce to the following independent equations:

$$A'^2 = -\frac{V}{12M^3} \qquad A'' = -\frac{V_+}{6M^3}\delta(y-y_+) - \frac{V_-}{6M^3}\delta(y-y_-). \tag{3.3}$$

Our background solution reads $A = -k|y|$, provided the cosmological constants are fine tuned to satisfy $V = -12k^2M^3$, $V_+ = -V_- = 12kM^3$. We will assume $k > 0$, that means that the brane at $y = y_-(y_+)$ must have negative (positive) tension. All the mass scales are naturally of the order the 5D Planck scale M. In such a class of models the 4D Planck mass is related to the 5D one as

$$M_P^2 = 2\int_{y_+}^{y_*} dy\, e^{2A} = \frac{M^3}{k}\left(e^{-2ky_+} - e^{-2ky_-}\right). \tag{3.4}$$

The background geometry describes a slice of the AdS_5 space $A = -ky$, for $-\infty < y < +\infty$. Details of the spectrum of generically warped 5D models are given in the Appendix A.

The effect of the RS background on particle physics is illustrated by considering a scalar field Φ localized on the negative tension brane:

$$\begin{aligned} \mathcal{L}_{scalar} &= \sqrt{-g_-}\left[\partial_\mu\Phi\partial_\nu\Phi g_-^{\mu\nu} - M^2\Phi^2\right] \\ &= e^{-4ky_-}\left[(\partial\Phi)^2 e^{2ky_-} - M^2\Phi^2\right] \\ &= (\partial\bar{\Phi})^2 - (Me^{-ky_-})^2\bar{\Phi}^2 \end{aligned} \tag{3.5}$$

where we canonically normalized the field by performing a rescaling $\Phi = \bar{\Phi}e^{ky_-}$. We see that the 4D scalar mass, of order Planck from the 5D perspective, turns out to be suppressed by an exponential factor. This result is easily understood in terms of the gravitational redshift. Indeed, given an arbitrary 4D process, different observers localized at different points in the extra dimension would conclude that the scale of the process is different: $(p^\mu e^{ky})^2$ is the invariant scale.

Elaborating on these remarkable observations Randall and Sundrum proposed a solution to the Hierarchy problem of the SM: they put the SM fields on the negative tension brane and fixed $k(y_- - y_+) \sim 30$ such that the relation $Me^{-ky_-} \sim$ TeV for $M \sim M_{Pl}$ (the conventional choice y_+ has been made without loss of generality). In this way, a natural Higgs mass of order the Planck scale translates into the EW scale. The model is not completely satisfactory, though. In fact the redshft applies to any mass scale of the model. In particular this means that possible higher dimensional operators (FCNC) are suppressed by a scale of order $\sim$ TeV: the theory under consideration contains additional degrees of freedom not far from the weak scale.

A more realistic version of the model requires the SM fermions, at least the lightest ones (which are mainly subjected to EW and flavor precision measurements), to be spread into the bulk, while leaving the Higgs boson localized in the IR brane in order to solve the hierarchy problem. Because of that we are forced to place the gauge symmetry of the SM (plus, eventually, some additional generators) in the bulk, as well. The resulting picture has been studied in extreme detail in the last years and it is found to be a compelling scenario for the physics beyond the SM.

All of the above results, including all the phenomenological predictions, can be understood using the language of the Gauge/Gravity correspondence. The AdS background is associated to an approximate conformal symmetry of the dual 4D theory at large N. The region $y \sim y_+$ is mapped to the UV region, while the regime $y \sim y_-$ is the IR, where all 4D distances are enlarged. The CFT is not exact. The UV brane y_+ acts as a UV regulator in that it sets a finite 4D Planck mass (which otherwise would diverge for pure AdS) and therefore represents an explicit breaking of the conformal symmetry. The IR brane represents a spontaneos breaking of the CFT; it introduces a mass scale in the model and makes it a phenomenologically appealing scenario.

3.1.1 Resonances

A detailed analysis of the spectrum in general brane models see Appendix A,B. Here we focus on some properties of the AdS space-time.

To understand the role of the IR we decouple the UV brane by sending the cutoff to infinity $y_+ \to -\infty$, and study the effect of a negative tension brane on the fields propagating in the bulk. The motion of a test body subject to the gravitational field (3.9) is governed by the geodesic equation

$$\begin{aligned} \frac{d^2x^\mu}{d\theta^2} &= -2A'\frac{dy}{d\theta}\frac{dx^\mu}{d\theta} \\ \frac{d^2y}{d\theta^2} &= -A'e^{2A}\left(\frac{dx^\mu}{d\theta}\right)^2. \end{aligned} \tag{3.6}$$

The first equation is easily solved as $dx^\mu = v^\mu e^{-2A} d\theta$, with v^μ arbitrary constants. Identifying θ with a time variable, the integration constant v^μ can be interpreted as a 4D velocity. This identification can be made more explicit by rewriting the equation for $y(\theta)$ as a one dimensional problem of classical mechanics with hamiltonian

$$E = \left(\frac{dy}{d\theta}\right)^2 - v^2 e^{-2A}. \tag{3.7}$$

The potential energy defined by the warp factor attracts test particles around its saddle points $A' = 0$ if these are local minimum of A, i.e. $A'' \geq 0$. One can recognize the latter as the condition for a negative tension defect.

Since ordinary bodies are attracted by the negative tension domain, the 5D profiles describing them are expected be peaked on it. This is at the heart of the rich Kaluza-Klein phenomenology which characterizes the original work of Randall and Sundrum [133], where the standard model was placed on a negative tension brane.

Let us illustrate this point by considering a 5D scalar Ψ on the background $A = k|y|$ (we chose $y_- = 0$ for simplicity). The eigenvalue equation for the 4D modes Ψ_m reads

$$-\Psi_n'' - 4A'\Psi_n' + m_5^2\Psi_n = M_n^2 e^{-2A}\Psi_m. \tag{3.8}$$

The 4D mass term becomes irrelevant in the asymptotic region $k|y| \gg 1$ indicating that the spectrum is discrete with mass scale determined by the curvature scale,[1] $M_n \sim nk$. This is tantamount to say that the wavefunctions Ψ_m are integrable functions of the variable y.

Notice that 4D radiation does not feel any potential at all ($v^2 = 0$ in (3.7)), and the perturbations are not normalizable. This is what happens to the 4D graviton, for which a UV cutoff is required in order to recover renormalizability.

We can now interpret the localization effect discussed above in terms of the gauge/gravity correspondence: the presence of a negative tension brane leads to the appearence of a mass gap irrespective of the noncompact dimension (irrespective of the absence of an UV cut-off). We identify the IR scale as the confining scale of a possibly dual 4D theory which has a CFT behavior in the far UV [106]. The confinement is not linear, as in the case of QCD. In fact the spectrum follows the pattern $M_n^2 \propto n^2$ characteristic of a hard wall, rather than the pattern $M_n^2 \propto n$ as in a linearly confining theory. We do not have time to discuss this issue here, suffices to say that linearly confining theories are realizable in an extra dimensional context.

Let us analyze the hard wall scenario from now on. That the IR breaking of the CFT is spontaneous can be shown in different ways. Here we focus on the spectrum and notice that, whenever an IR is present, the theory has a massless scalar mode which couples conformally to gravity and to the 4D physics on the brane. This is the expected NGB of the CFT breaking: the dilaton.

3.1.2 The radion

With the introduction of dynamical gravity the interbrane distance fluctuates. The associated quantum is called radion and will be described in some detail in this section, see also Appendix A and B. Because our classical solution does not fix the interbrane distance, the radion is allowed to acquire any background value, i.e. it describes a flat direction. This

[1] The 4D mass parameters measured by a local observer at $y = 0$ are redshifted with respect to those measured by an observer at $y \neq 0$. Because of our choice of normalization ($A(0) = 0$), the quantities described above, in particular the estimate $M_n \propto k$, must be interpreted as IR quantities. We can make contact with the results obtained in a compact Randall-Sundrum scenario by normalizing the warp factor as $A(y_*) = 0$ thus rescaling all the 4D masses $m \to m_{UV} e^{-ky_*}$ and obtaining, for example, the more familiar expression $M_n^2 \sim (ke^{-k(y_+ - y_-)}$. As a check of the procedure we can set $A = k(|y| - y_*)$ in eq. (3.8) and observe that our estimate $M_n \sim k$ traslates in $(M_n e^{ky_*})^2 \sim k^2$, which is the expression found in [133].

implies that the radion has no potential and is therefore massless. This may not be true at next to leading order in the large N expansion. Similarly, that is certainly no more true at the quantum level because any dynamical mode propagating in the bulk induces a Casimir force between the branes, and thus introduces a nontrivial dependence of the vacuum on the interbrane distance.

In order to identify the wavefunction of the radion it suffices to work at infinitesimal level in the perturbations. However, it is convenient to keep the treatment as general as possible. It is always possible to choose a gauge in which the line element is

$$ds^2 = W^2 \eta_{\mu\nu} dx^\mu dx^\nu - Y^2 dy^2 \tag{3.9}$$

where, without loss of generality, I assume $Y > 0$. The remaining gravitational degrees of freedom are spin-1 and spin-2 fields and describe dynamical and nondynamical components of the gravitons sector. I will not discuss the latter here, it suffices to say that the spectrum is composed of a tower of massive states and a massless mode following from the preserved local 4D diffeomorphism invariance and describing the fluctuations of the 4D background $\eta_{\mu\nu}$. The scalars W, Y are functions of the 5D coordinates, and their infinitesimal fluctuations (F_W, F_Y) around the vacuum are defined as

$$W^2 = e^{-2ky}(1 + F_W + \ldots), \quad Y^2 = 1 - F_Y + \ldots$$

The vanishing of the 4D cosmological constant imposes a relation between these two fields, as we now show.

A useful expression can be written for arbitrary 4D metric $\hat{g}_{\mu\nu}(x)$:

$$-\sqrt{g}R = \sqrt{-\hat{g}}\left[-6\hat{g}^{\mu\nu}\partial_\mu (WY)\partial_\nu W - W^2 Y \hat{R} + 12\frac{W^2}{Y}W'^2 + (\ldots)' + \partial_\mu B^\mu\right]. \tag{3.10}$$

The last term is a 4D boundary action and can be neglected as usual. The total derivative in y, on the other hand, cancels with the addition of the Hawking-Gibbons term. The second term contains no derivatives of the fields W, Y and represents a potential term which must vanish once the contribution of the cosmological constants are included. The latter condition implies that the two scalar functions W, Y are not independent. Given the cosmological constants $V = -kV_+ = kV_-$, and no additional sources for gravity (GW), a sufficient condition for the absence of the potential is

$$W' = -kWY. \tag{3.11}$$

The constraint (3.11) will play the major role on our discussions to follow.

For completeness we show the cancellation explicitly

$$\begin{aligned} \frac{\mathcal{L}_{pot}}{\sqrt{-\hat{g}}} &= 2\int_{y_+}^{y_-} dy \left[12k^2M^3 - V\right] W^4 Y - V_+ W_+^4 - V_- W_-^4 \\ &= -4V\int_{y_+}^{y_-} dy W^4 Y + kV\left[W^4\right]_{y_+}^{y_-}, \end{aligned} \tag{3.12}$$

where from now on $[f]_{y_+}^{y_-} \equiv f(y_-) - f(y_+)$.

The linearized equations for the perturbations are found by varying the EOM. In our set up the EOM read $R_{AB} = -\Lambda g_{AB}/3$, where Λ contains both the bulk and the brane tensions. The variation of the $\mu\nu$ part leads to the nontrivial condition

$$-\frac{1}{2}\frac{\partial_\mu\partial_\nu Y^2}{Y^2} - \frac{\partial_\mu\partial_\nu W^2}{W^2} + \eta_{\mu\nu}(\ldots) = 0. \tag{3.13}$$

The cancellation of the $\eta_{\mu\nu}$ term is ensured by the vanishing of the 4D cosmological constant and the condition that $F_W, F_Y \propto Q$, where Q is a 4D scalar satisfying the Klein-Gordon equation. From the cancellation of the $\partial_\mu\partial_\nu$ term it follows the nontrivial constrain $F_Y = 2F_W$.

Imposing finally (3.11) we have $F_W' = kF_Y$ and eventually conclude that $F_Y = Qe^{-2A}$, where $Q(x)$ is a massless 4D field.

We can now deduce a metric valid at any order in the perturbation Q. This can be made in a consistent way by satisfying the constrain (3.11) at the nonlinear level. The result reads

$$ds^2 = e^{2A+Qe^{-2A}}\hat{g}_{\mu\nu}dx^\mu dx^\nu - (1 - Qe^{-2A})^2 dy^2, \tag{3.14}$$

where $A = -ky$, and coincides with the proposal of Rubakov et al. A formally equivalent form holds in the presence of a nontrivial warping. In this case A' is no more a constant and the relation (3.11) gets modified.

It is not difficult to see that thinking of the extra dimensional coordinate as a degree of freedom can be made a perfectly physical statement by a simple change of reference frame. For this purpose it is convenient to define a convenient coordinate w such that $kdw = k(1-F)dy = d(ky - Qe^{2ky}/2)$. Under the change of coordinates the metric (3.14) translates in

$$ds^2 = e^{-2kw}\eta_{\mu\nu}dx^\mu dx^\nu - dw^2 + O(\partial Q/k).$$

The $\partial Q/k$ terms appear in the g_{5A} components[2]. The point here is that the variable w, with $2kw = 2ky - Qe^{2ky}$, actually represents a dynamical mode. The variable w determines the (x,y)-dependent proper length of the extra dimension ($w = \int ds = \int dy(1-F)$) and its classical vacuum is y. Notice that in terms of w the branes are bent (x-dependence) and placed at $w_\pm$, with $2kw_\pm = 2ky_\pm - Qe^{2ky_\pm}$.

A typical bulk field of classical dimension would couple to the physical coordinate w in a scale invariant way, up to derivative couplings, and the field w can be interpreted as a dilaton of the 4D theory.

[2]Thanks to the fundamental constrain (3.11) the result generalizes to any metric of the form (3.9) with W function of y only and $kw = -\log W$.

Let us focus on the gravity part and write down the 4D lagrangian for a generic line element (3.9) and 4D gravity. Substituting the metric in (3.1), and using (3.11), the 4D lagrangian becomes

$$\mathcal{L}_{eff} = \sqrt{-\hat{g}}\frac{M^3}{k}\left[6(\partial W)^2 + W^2\hat{R}\right]_{y_+}^{y_-}. \tag{3.15}$$

The effective action for the field Q is straightforwardly obtained from the previous expression by substituting the appropriate W. This effective action has been used by Rattazzi and Zaffaroni to spell out the dual interpretation of the physics of the IR brane [101]. These authors studied the effective theory (3.15) in the absence of the UV brane regulator ($y_+ \to -\infty$) and argued that the presence of the IR brane in the AdS background can be interpreted as a spontaneous breaking of the conformal symmetry of the strongly coupled 4D dual theory. In this language the corresponding NGB, the dilaton, must be identified with W_-.

In the Appendix C a detailed study of the effective approach to spontaneously broken CFTs is given. From those considerations one sees that, from a purely classical perspective, the dilaton can be identified as the conformal excitation of the metric. We thus understand that the above 5D picture agrees with these expectations provided the anomalous dimensions of the KK excitations can be neglected. At leading order in a large N expansion, this is certainly true. In the following section we study in great detail the physics of the dilaton (or equivalently of the radion in a generalized RS scenario).

3.2 Spontaneously broken CFT

We would like to better understand the physics of the dilaton from a purely 4D perspective. I will elucidate some general properties of spontaneously broken scale invariant theories by making use of an example of classical field theory. The general treatment is given in Appendix C.

Consider the following classical lagrangian for two scalar fields Φ and χ:

$$\frac{1}{2}(\partial\Phi)^2 + \frac{1}{2}(\partial\chi)^2 - \frac{g^2}{2}(\Phi\chi)^2. \tag{3.16}$$

The action is invariant under discrete Z_2 symmetries and conformal invariance.

The action is manifestly invariant under scale transformtions

$$\Phi(x) \to \Phi'(x) = e^{\lambda}\Phi(e^{\lambda}x)$$

and similarly for χ, where λ is arbitrary. We can construct an appropriate stress tenor (obtained as described in the Appendix) of the form:

$$\begin{aligned}\Theta_{\mu\nu} &= \partial_\mu\Phi\partial_\nu\Phi + \partial_\mu\chi\partial_\nu\chi - \frac{1}{2}\eta_{\mu\nu}((\partial\Phi)^2 + (\partial\chi)^2 - g^2(\Phi\chi)^2) \\ &+ \frac{1}{6}(\eta_{\mu\nu}\partial^2 - \partial_\mu\partial_\nu)(\Phi^2 + \chi^2).\end{aligned} \tag{3.17}$$

On the solution the stress tensor is traceless and conserved, as usual, and we find a conserved current $S^\mu = x_\nu \Theta^{\mu\nu}$. Whenever such a stress tensor can be constructed, additional 4 currents K^μ are found to be conserved. These define the special conformal transformtions.

A scale invariant theory is a theory with no mass scales (in particular no running of the couplings). Therefore it may seem impossible to generate a nonvanishing background which triggers the symmetry breaking. It is worth to realize that such a pattern is really possible in the presence of flat directions. The vacuum structure of our model is easily derived and is parametrized by $\Phi\chi = 0$. Without loss of generality we take the vacuum solution to be $\Phi = 0$ and $\chi = f$, where f is completely arbitrary. Re-expressing the lagrangian in terms of the rescaled fields $\bar{\Phi} = \Phi$ and $\bar{\chi} = \chi - f$, we find:

$$\frac{1}{2}(\partial\bar{\Phi})^2 + \frac{1}{2}(\partial\bar{\chi})^2 - \frac{m^2_{CFT}}{2}\bar{\Phi}^2\left(1 + \frac{\bar{\chi}}{f}\right)^2, \tag{3.18}$$

which describes a scalar $\bar{\Phi}$ of mass $m^2_{CFT} = (gf)^2$ interacting with a massless scalar $\bar{\chi}$.

The massless scalar χ is the dilaton, the Goldstone mode of the CFT breaking, and has nonderivative couplings governed by its vacuum f. On our solution we find $\Theta^\mu_\mu = m^2_{CFT}\bar{\Phi}^2(1 + \bar{\chi}/f)$ and

$$-f\partial^2\bar{\chi} = \Theta^\mu_\mu.$$

This expression can be rewritten as the conservation of the total current $S^\mu + f\partial^\mu\bar{\chi}$: in the presence of spontaneous breaking the NGB restores the symmetry, locally.

At the quantum level the coupling g is expected to run, thus breaking explicitly the classical scale invariance. However, one may suspect that appropriate field theories may reproduce the invoked mechanism quantum mechanically. An explicit realization has been found in a 3-dimensional $O(N)$ model at leading order in $1/N$ [157].

A phenomenologically viable scenario should account for an explicit breaking of the CFT. Appropriate field theories are expected to reproduce the invoked mechanism in a quantum mechanical context. Generic strong dynamics are not natural candidates. An heuristic picture is that the conformal breaking is governed by the running of the coupling; in the IR the coupling is strong, as well as a typical anomalous dimension, and there is no small parameter suppressing the dilaton mass compared to the dynamically generated scale. This reasoning may change if the small coupling is taken to be $1/N$. Explicit examples of the latter are provided by the class of theories considered in [158] and theories duals to RS.

One can give a qualitative argument in favor of the existence of a light dilaton mode in confining large N gauge theories, at least in the supersymmetric limit[3]. The reason is that in supersymmetry the axial current and the scale current are part of the same multiplet. At leading order in $1/N$ the anomalous axial symmetry is exact and in a non-chiral vacuum it generates an approximate GB (the η' in QCD). As far as the supersymmetry is preserved at long distances we should expect a CP even pGB to be generated, as well.

[3] I thank Sergio Cecotti for pointing out this observation.

3.3 SM embedding into the CFT

Consider a theory in which a non-compact continuous symmetry G is spontaneously broken by the vacuum expectation value of a number of operators, and assume that the overall SB scale is f. A general result of the symmetry algebra is that any operator $\mathcal{O}$ transforming under some representation of G, which is involved in the breaking, has a non-trivial overlap with the GB of the broken symmetry. The amount of mixing is measured in units of $\langle\mathcal{O}\rangle/f$.

A phenomenologically intriguing implication can be obtained if the Higgs sector of the SM is charged under a generator of G. Under this assumption the excitations of the Higgs sector mix with S, the GB of the broken symmetry. If the Higgs vev is the only source of symmetry breaking the mixing is maximal and the fields coincide.

We can make contact with the conventional rules of non-linear realizations by writing a general operator $\mathcal{O}$ transforming under the representation D_Δ of the symmetry group G as

$$\mathcal{O} = D_\Delta(e^S)\langle\mathcal{O}\rangle + \dots \tag{3.19}$$

where the dots stand for the massive excitations of the field, and D_Δ depends on the broken generators via S. It is now evident that the mixing generally sums up into the representation D and has no effect in the case of compact symmetries. This is a reformulation of the well known property of shift symmetry invariance of the GB. This is not generally true for non-compact space-time symmetries, as the lagrangian itself need not be invariant under the symmetry transformation.

We discuss the possibility that the UV completion of the SM contains a conformal sector spontaneously broken at the scale $\sim f$ down to the Poincare group. The EWSB is then triggered at $v < f$. We further assume that the fields responsible for EWSB are too massive or too broad to be detected, so that the details of EWSB are not relevant. Below, roughly, $\Lambda \sim 4\pi v$ the only excitable states are postulated to be the SM fields plus the GB of the conformal symmetry, the dilaton.

The most predictive scenario constructed out of this idea is the one in which the SM fields are embedded in the scale invariant theory. In this case the couplings of the dilaton to the spin-1, spin-1/2, and the NGBs $SU(2)$ matrix U are governed by the CFT symmetry. At leading order these are (the dilaton is assumed to be canonically normalized)

$$\begin{aligned}
\mathcal{L}_{embed} &= v^2|DU|^2\left(\frac{\chi}{f}\right)^2 + m\bar{\psi}U\psi\left(\frac{\chi}{f}\right) - \frac{1}{4g^2}F_{\mu\nu}^2 + \dots \\
&= \frac{1}{2}m_A^2A^2\left(1+\frac{\bar{\chi}}{f}\right)^2 + m\bar{\psi}\psi\left(1+\frac{\bar{\chi}}{f}\right) + \frac{\beta}{2g}F_{\mu\nu}^2\frac{\bar{\chi}}{f} + \dots
\end{aligned} \tag{3.20}$$

Notice that in the second equality the vectors have been canonically normalized. The phenomenology of (3.20) has been studied by the authors of [161].

The scaling dimension Δ of the SM fields have been taken to coincide with the classical one. Generally, the $\Delta > 4$ operators are either less relevant for our purposes or constrained by EWP data. An exception appears to be the coupling to the unbroken gauge bosons mediated by the scale anomaly, because no such a coupling occurs at the "renormalizable" level.

The model (3.20) has a dual interpretation in terms of the RS1 scenario with a heavy Higgs, where the SM fields were placed on the IR brane. The physics of the RS1 model has been extensively studied in the literature and it is known to suffer both a naturalness and a phenomenological problem. The first is due to the presence of a fine tuning between the 5D cosmological constant and the brane tension required to generate the RS geometry. The second issue is related to the unsatisfactory suppression of dangerous higher dimensional operators. In fact, as the hierarchy is fixed so that the higgs sector is naturally at the weak scale, the non-renormalizable interactions on the IR brane are suppressed by the TeV scale.

From our 4D perspective we can rephrase these apparently model-dependent properties as general consequences of the set up. In the absence of an explicit breaking of the CFT, the dilaton is an exact GB and parametrizes a flat direction. Its effective lagrangian must have no potential, because the only CFT invariant candidate, $V = a\chi^4$, would imply an unbroken vacuum $\chi = 0$. Therefore, the integration of the SM particles must exactly compensate the contribution of the heavy composites that we did not take into account in (3.20). Because by dimensional analysis $a_{SM} \sim g_\rho^2/(16\pi^2)(m_{SM}/f)^2$, we expect the neglected composites to be not too far from the weak scale even if the CFT states $\sim g_\rho f$ are parametrically heavier.

Although this naturalness argument is not definite, we would like to discuss a broader class of theories in which the SM fields act as external sources with respect to the CFT, somewhat in analogy with what is done for the SILH. In the gauge/gravity language this amounts to introduce SM sources on the UV brane. This discussion is the subject of the next section.

3.4 SM breaking of the CFT

A phenomenologically acceptable realization of the dilaton scenario requires the CFT to be explicitly broken [4]. Such a breaking also modifies the couplings to the SM fields.

In the following we discuss the phenomenology of a class of models in which the SM itself represents the source of explicit breaking. In this scenario the dilaton gets a 1-loop potential of the form

$$V(\chi) \sim \frac{1}{16\pi^2}(m_{SM}m_\rho)^2 F(\chi/f) \tag{3.21}$$

[4] In principle one may accomodate an exact GB in a viable framework provided f is very large. In that case the model would look like a higgsless theory. We do not consider this possibility further.

where $m_\rho = g_\rho f$ is the scale of new excitations, m_{SM} is a typical SM mass, and F is an arbitrary function. The dilaton mass would be of order

$$m_\chi^2 \sim \frac{g_\rho^2}{16\pi^2}\left(\frac{v}{f}\right)^2 m_t^2 < \left(\frac{v}{f}\right)^2 m_t^2. \tag{3.22}$$

The full potential for χ is $V_{CFT} + \epsilon V_{SM}$, where the first induces $\chi = f$ and the second is suppressed with respect to the first by at least a factor $\epsilon = (v/f)^2$. Because the top quark is naturally coupled more strongly we expect a new phase transition to occur. The true vacuum is found at $\chi = f' = f(1 + O(\epsilon))$, and can differ from f by order 1 corrections only if $v \sim f$ in the first place. The dilaton interactions are suppressed by the new scale f'.

If the SM induced CFT breaking is strong, that is $v \sim f$, then the possibility $f' < v$ cannot be excluded. In this case the dilaton couplings to the SM fields would be enhanced with respect to the standard set up, though our guiding symmetric principles would start vacillating.

3.4.1 Higgs-dilaton mixing

From the general rules illustrated above we argue that the Higgs sector is necessarily mixed to the dilaton. To show this explicitly we focus on a simplified theory in which the Higgs sector is described by an interpolating Higgs doublet. For simplicity, we further assume that $\Delta(\mathcal{O}_1\mathcal{O}_2) = \Delta(\mathcal{O}_1) + \Delta(\mathcal{O}_2)$, which is rigorously true at leading order in large N and SUSY theories with R-symmetry. In this case the most general action for the Higgs is constructed out of the covariant derivative

$$\left(\partial_\mu + iA_\mu - \Delta\frac{\partial_\mu\chi}{\chi}\right)H, \tag{3.23}$$

and, neglecting sources of explicit symmetry breaking ($m_\chi^2 \ll m_h^2$ in our case), the potential

$$V(\chi, H) = \chi^4 \hat{V}\left(\frac{H^\dagger H}{\chi^{2\Delta}}\right) \tag{3.24}$$

The kinetic term induces a mixing between the Higgs after EWSB which agrees with the one found in RS from the non-minimal coupling $\mathcal{R}H^\dagger H/6$. Deviations from the conformal factor 1/6 account for the breaking of special conformal symmetries, and it amounts to including additional $\alpha H^\dagger D_\mu H \partial^\mu\chi/\chi + h.c. + \beta H^\dagger H(\partial\chi)^2/\chi^2$ operators in our 4D language.

The system can be easily diagonalized by defining the zero scaling dimension field $H \to H(f/\chi)^\Delta$. After EWSB a generic Higgs sector mediates a coupling of the dilaton to the SM via the substitution

$$h \to v\left(1 + \Delta\frac{\bar{\chi}}{f} + \frac{1}{2}\Delta(\Delta-1)\frac{\bar{\chi}^2}{f^2} + \ldots\right) \tag{3.25}$$

In principle, a large anomalous dimension for the interpolating Higgs may compensate the suppression v/f. However, the dimension of the Higgs cannot be arbitrary large if we require our model to be self consistent. For example, assuming that the top Yukawa coupling becomes strongly coupled at the NDA scale $\sim 4\pi v$ requires $\Delta < 2$.

Under the strong assumption that the SM feels the CFT dynamics only via the Higgs, the dilaton couplings to the SM fields formally coincide with those of a fundamental Higgs boson, modulo universality violations induced by the anomalous dimension $\Delta - 1$ of the Higgs sector.

3.4.2 FCNC effects

Flavor changing neutral currents are severely suppressed in the SM. We now consider the constrains applying to possible flavor violating couplings of the dilaton.

The dilaton does not mediate FCNC effects at leading order in the CFT breaking parameters. This is the case if the SM fermions are embedded into the CFT, see (3.20). This result also generalizes to the case in which the SM fermions are not embedded, provided they couple (linearly or bilinearly) to operators with a flavor universal representation under the CFT. Under this assumption, and after EWSB, the most general Yukawa coupling would in fact be written as

$$F_{ij} v \, \bar{\psi}_i \psi_j \left(\frac{\chi}{f} \right)^{\Delta} = m_{ij} \bar{\psi}_i \psi_j \left(1 + \Delta \frac{\bar{\chi}}{f} + \dots \right) \tag{3.26}$$

where Δ is an arbitrary number, i, j are flavor indeces, and F_{ij} is a dimensionless scale invariant function, which we can identify with the Yukawa matrix.

If the above assumption does not apply, the radion can have flavor violating Yukawas proportional to the CFT breaking parameters. These emerge in the low energy dynamics from a non-scale invariant function $F_{ij} = F_{ij}^{(0)} + F_{ij}^{(1)} \bar{\chi}/f + \dots$ of the dilaton, with $F_{ij}^{(a)}$'s naturally of the same order.

We would like now to show how these results arise by looking at specific UV realizations. One can identify two distinct ways to generate the SM flavor structure. The first contains the minimal flavor violation (MFV) class and is based on the coupling

$$\mathcal{L}_1 = y_{ij,ab} \bar{\psi}_i \psi_j \mathcal{O}_{ab} \tag{3.27}$$

where the CFT operators $\mathcal{O}_{ab}$ have generally different scaling dimensions Δ_{ab}. After EWSB the low energy EFT is obtained by integrating out the energetic fluctuations of the CFT and reads

$$\begin{aligned} \mathcal{L}_1 &= y_{ij,ab} \, \bar{\psi} \psi \langle \mathcal{O}_{ab} \rangle \left(\frac{\chi}{f} \right)^{\Delta_{ab}} + \frac{y^2}{m_\rho^2} \bar{\psi} \psi \bar{\psi} \psi + \dots \\ &= m_{ij} \psi_i \psi_j \left(1 + \Delta_{ij} \frac{\bar{\chi}}{f} + \dots \right) + \frac{y^2}{m_\rho^2} \bar{\psi} \psi \bar{\psi} \psi + \dots \end{aligned} \tag{3.28}$$

The coefficients of the four fermion operator is a symbolic representation for the sum $y^2/\Lambda^2 = y_{ij,ab} D_{abcd} y_{kl,cd}$, where the D matrix generates from the propagator of the operator $\mathcal{O}_{ab}$. Notice that for a universal dimension $\Delta_{ab} = \Delta$ the mixing matrix becomes flavor blind, $\Delta_{ij} = \Delta$

The second class approaches the flavor problem by employing a seesaw type mechanism. This is the class usually implemented in the RS scenarios with SM fermions in the bulk. The bare lagrangian now reads:

$$\mathcal{L}_2 = \lambda_L^{ia} \psi_L^i \mathcal{O}_R^a + \lambda_R^{jb} \psi_R^j \mathcal{O}_L^b \tag{3.29}$$

and leads to an EFT of the form

$$\begin{aligned}
\mathcal{L}_2 &= \lambda_L^{ia} \lambda_R^{jb} v^{ab} \psi_L^i \psi_R^j \left(\frac{\chi}{f}\right)^{\Delta_R^a + \Delta_L^b - 4} + \frac{\lambda^4}{m_\rho^2} \bar{\psi}\psi\bar{\psi}\psi + \dots \\
&= \lambda_L^{ia} \lambda_R^{jb} v^{ab} \psi_L^i \psi_R^j \left(1 + (\Delta_R^a + \Delta_L^b - 4)\frac{\bar{\chi}}{f} + \dots\right) + \frac{\lambda^4}{m_\rho^2} \bar{\psi}\psi\bar{\psi}\psi + \dots \\
&= m_{ij} \psi_i \psi_j \left(1 + \Delta_{ij}\frac{\bar{\chi}}{f} + \dots\right) + \frac{\lambda^4}{m_\rho^2} \bar{\psi}\psi\bar{\psi}\psi + \dots
\end{aligned} \tag{3.30}$$

The appearance of the power of the dilaton follows from the expansion of the operator $\int d^4y \mathcal{O}_R^a(x)\mathcal{O}_L^b(y)$ in the process of CFT integration. Again, if the dimensions $\Delta_{L,R}$ were not to depend on the family label a, b, the radion coupling would be flavor diagonal $\Delta_{ij} = \Delta_R + \Delta_L - 4$.

In summery, the dominant FCNC effects are described by the following lagrangian:

$$\bar{\psi}_i \psi_j \left[m_i \delta^{ij} \left(1 + b\frac{\bar{\chi}}{f}\right) + m b^{ij} \frac{\bar{\chi}}{f} \right] + C_{ijkl} \bar{\psi}_i \psi_j \bar{\psi}_k \psi_l, \tag{3.31}$$

where the factor m in front of the flavor violating term reminds us that we should expect a power of the Yukawa coupling. The flavor mixing term b^{ij} is severely constrained by FCNC bounds. Tree level exchanges of the dilaton lead to 4 fermions interaction with coefficients $C \sim (mb^{ij}/f)^2/m_\chi^2$. The strongest bound for a generic LR mixing and CP violating operator comes from the $K\bar{K}$ mixing. The UTFit analysis reports the bound $Im(C) < (10^5 \text{ TeV})^{-2}$ (we are neglecting subleading logarithmic running effects), which translates to

$$|b^{ij}|\frac{v}{f} < 10^{-2} \left(\frac{m_\chi}{100\, GeV}\right). \tag{3.32}$$

In the intriguing regime $v \sim f$ the b_{ij} effects should be neglected compared to the genuinely flavor diagonal b. More generally, the coupling becomes potentially interesting from a phenomenological perspective for relatively large dilaton masses. In the latter case, however, the dominant decay mode is expected to be into massive vectors, as for a fundamental Higgs, and branching ratios into fermions become much less accessible. One can estimate flavor violating events to be down by an order $BR(\chi \to t\bar{c}) \sim 10^{-3}$ in the optimistic case $v \sim f$.

The 4-fermions contact terms in (3.31) can be generated by the tree level integration of heavy composites. As the above discussion makes it clear, their coefficients are of the order $C \sim y^2/m_\rho^2$, where $y \sim m_\psi/v$. The bounds on these coefficients can be obtained from the bounds on the dilaton couplings by noting the correspondence $m_\rho \sim f m_\chi/(|b_{ij}|v) > 10$ TeV. Roughly, we should require a lower bound $f \sim$ TeV for a maximally strong dynamics ($g_\rho \sim 4\pi$). From these qualitative estimates we see that the ratio $v/f \sim$ few$\times 0.1$ in generic models, while $v \sim f$ (or even $v > f$) seems to be compatible with FCNC observations only if the MFV is at work, and in that case $b_{ij} = 0$ by definition.

3.5 Phenomenology of a light scalar

Suppose the LHC detects a light CP even scalar and no other states: can we tell whether this particle is really the physical excitation of a Higgs doublet or a dilaton?

A fundamental difference between the dilaton scenario and a light Higgs doublet model like the strongly interacting light Higgs (SILH) considered in [162], is that the former has generally a low impact in the unitarization of the W elastic scattering. Hence, at energies at most of order $4\pi v$ we expect the Higgs excitations to emerge, while new physics is delayed at a much higher scale in the SILH scenario. Under the assumption that these heavy composites are not directly observed, the above question can be technically rephrased by asking whether the $O(4)$ structure of the SILH scenario is visible or not at energies below or at most of order $\sim 4\pi v$. Let us focus on this point.

If we were able to isolate the strong dynamics from the explicit breaking, the GB sectors of the two theories would look radically different. On the one hand, the SILH scenario, as the fundamental Higgs of the SM, possesses an $O(4)$ symmetry. Events with an odd number of Goldstones π (the would be longitudinal vector bosons) or Higgses h (the would be physical vacuum excitation) are forbidden. On the other hand, no such symmetry is present in the dilaton model, in which the strong dynamics triggers $\bar{\chi} \to 2\pi$ events.

In the broken electro-weak phase the light states of the two strong sectors behave analogously. Once an explicit breaking source is added to the SILH scenario (the SM in [162]), the $O(4)$ invariance is violated. At leading order in a derivative expansion p^2/f^2, the $O(4)$ violating processes are mediated by the Higgs potential and the operator [162]

$$\frac{c_H}{f^2}\partial_\mu \left(H^\dagger H\right) \partial^\mu \left(H^\dagger H\right). \tag{3.33}$$

with $H^\dagger H = (v+h)^2 + \bar{\pi}^2$. The relative amplitude can be estimated to be $\mathcal{A}(h \to 2\pi) = 2\lambda v + c_H v p^2/f^2$, and similarly for $2h \to h$ (notice that the amplitudes vanish as $v \to 0$). For completeness we mention that, thanks to the equivalence theorem, the elastic scattering of longitudinally polarized vector bosons behaves as $\mathcal{A}(2\pi \to 2\pi) \sim c_H p^2/f^2$ at large momenta $p^2 \gg m_W^2$.

The dilaton self-couplings are, in the absence of explicit breaking of the CFT, derivatively induced:

$$\frac{(\partial\chi)^2}{2} + \frac{c_1}{(4\pi)^4}\frac{(\partial^2\chi)^2}{\chi^2} + \frac{c_2}{(4\pi)^4}\frac{(\partial\chi)^4}{\chi^4} + \dots, \tag{3.34}$$

where the coefficients c_i have been estimated using naive dimensional analysis to be of order 1. The second term in (3.34) may be discarded when working at $O(p^4)$. Both $2\bar{\chi} \to 2\bar{\chi}, \bar{\chi}$ events start at $O(p^4)$ and are substantially suppressed in the regime of validity of our effective description. At small energies these events are therefore dominated by the potential interactions induced by the explicit CFT deformations. The induced potential have been studied in some details in [161]. A $\sim$ 500 GeV linear collider, like the ILC, can test the triple dilaton coupling with an accuracy up to 10%, thus providing useful pieces of information about the source of explicit breaking. At scales bigger than f, the strong dynamics modifies the dilaton vertices. This observation becomes crucial in the regime $v \sim f$.

For what concerns GB scattering, the dilaton model behaves as a higgless scenario as the CFT is decoupled; hence we find $\mathcal{A}(2\pi \to 2\pi) \sim p^2/v^2(1 - v/f)^2$.

Reactions involving both dilaton and Goldstone bosons (see (3.20)) scale as $\mathcal{A}(\bar{\chi} \to 2\pi) = p^2/f$. At the characteristic energy $p^2 = m_\chi^2$ we recover the SM result, provided $m_\chi^2 = 2\lambda f^2 = 2\lambda v^2$. At energy scales $p^2 \gg v^2$, the observation of off-shell $O(4)$ violating processes would be a distinctive signature of the dilaton model. These may be potentially tested as excesses in $pp \to jjV_LV_L \to jjX$ events at large invariant mass M_X^2. However, the existence of the required energy range in a way compatible with our effective description demands for a hierarchical relation $v \ll f$. In the dilaton model, this hierarchy would be responsible for strong departures from the Higgs couplings allowing an easy identification anyway.

We now turn to more model dependent features characterizing the dilaton scenario. Let us consider the most general lagrangian for a CP even scalar S. In the unitary gauge this reads as follows:

$$\begin{aligned}\mathcal{L}_{eff} &= \frac{1}{2}m_A^2 A^2\left(1 + 2a_1\frac{S}{v} + a_2^2\frac{S^2}{v^2}\right) \\ &+ \bar{\psi}_i\psi_j\left[m_i\delta^{ij}\left(1 + b\frac{S}{v}\right) + mb^{ij}\frac{S}{v}\right] \\ &+ c\frac{g_{SM}^2}{16\pi^2}F_{\mu\nu}^2\frac{S}{v} + \dots,\end{aligned} \tag{3.35}$$

where we are assuming an approximate custodial symmetry of the strong dynamics. The coefficients a_i, b, and c are $O(1)$. The SM Higgs boson is just a particular case with $a_1 =$

$a_2 = b = 1$ and $b_{ij} = c = 0$, while in the SILH model we have [162]

$$a_1 = a_2 = \frac{1}{\sqrt{1 + c_H \frac{v^2}{f^2}}}; \quad b = \frac{1 + c_y \frac{v^2}{f^2}}{\sqrt{1 + c_H \frac{v^2}{f^2}}}; \quad b_{ij} = 0; \quad c = 0. \tag{3.36}$$

The dilaton model in which the SM fields are embedded into the CFT corresponds to

$$a_1 = a_2 = \frac{v}{f}; \quad b = \frac{v}{f}; \quad b_{ij} = 0; \quad c = \frac{8\pi^2 \beta_{SM}}{g_{SM}^3} \frac{v}{f}, \tag{3.37}$$

while order one deviations from this pattern are expected if the CFT is generally broken, in particular a non-zero b_{ij} may be allowed.

A discussion of the flavor violating b_{ij} has been given in the previous section. For generic flavor models, the FCNC bounds prefer $v < f$. Because the factor v/f is universal in the dilaton scenario, suppressions of the dilaton total rates compared to those of a Higgs would be clear and visible signals of a generic dilaton model (for $(v/f)^2 > 0.1$ the LEP bounds already exclude the mass range 90 GeV $< m_\chi <$ 110 GeV). Yet, branching ratios are essentially unaffected.

If the SM represents an explicit breaking of the CFT, the dilaton does not couple to the unbroken gauge group directly via the trace anomaly. The coupling is mediated by a mixing between composites and SM vectors of order g_{SM}/g_ρ, and by the integration of charged heavy CFT composites. The resulting operator has been included in (3.35) with a model dependent $c = O(v/f)$ factor. Such a contribution can be in principle sizable if the CFT sector has a large number of degrees of freedom at the scale $\sim g_\rho f$, and may dominate over the top loop (which scales as $c = O(1)$). A large c in the dilaton scenario may represent the most accessible signal, especially in the ambiguous regime $v \sim f$.

This discussion does not generalize to the case of a SILH, where the couplings $hgg, h\gamma\gamma$ arise from an operator $H^\dagger H F_{\mu\nu}^2$ which violates the Goldstone shift symmetry and is therefore expected to come along with an additional suppression g_{SM}^2/g_ρ^2, which is of the order of a loop factor for a strong dynamics. In the SILH model the $h \to gg, \gamma\gamma$ processes are dominated by the top loop and we can safely assume $c = 0$ if the new dynamics is strong.

In summery, a discrimination between a dilaton and a Higgs field, either fundamental or composite, is possible as the result of combined fits and sufficiently high precision. The somewhat unnatural scenario in which the SM is part of the (approximate) CFT is characterized by the same branching ratios as a fundamental Higgs but universally suppressed total decays with respect to it. If no suppression from the Higgs couplings is observed, then $v \simeq f$ and a strong enhancement in the $\chi \to 2g, 2\gamma$ events is expected. If the SM breaks explicitly the CFT, the situation is a bit more subtle. Again, if no significant deviations from the Higgs couplings are observed we are in the regime $v \simeq f$, but in this case deviations in decays into massless gauge bosons may not be visible. However, signals from the $O(4)$

violating off-shell processes described above, as well as strong departures from the dilaton self couplings would be clear features of a dilaton.

Chapter 4

EWSB from deformed AdS

In recent years, based on the ideas of the AdS/CFT correspondence [55], and on the pioneering work of Randall and Sundrum [56, 57], many models have been investigated which exhibit in the low energy region the basic properties expected in a walking theory, while being calculable. Examples now exist of models that are compatible (within the errors) with the precision data and can be discovered at the LHC. The literature on the subject is already extensive [58, 108, 60, 61, 62].

Most of these models assume a conformal behavior of the strongly coupled sector in the energy region spanning few orders of magnitude above the electro-weak scale and the existence of a weakly-coupled effective field theory description of the low-energy dynamics of the resonances. The construction of the effective field theory is derived by writing a weakly coupled extra-dimension model with a non-trivial gravity background, and by using the dictionary of the AdS/CFT correspondence to relate back to four dimensions. A generic phenomenological feature of all these models is that, unless a clever mechanism arranging for non-trivial (often fine-tuned) cancellations is implemented, a quite severe lower bound on the mass $M_1 > 2.5$–3 TeV of the lightest spin-1 resonance (techni-rho) results, in particular from the bounds on the electro-weak parameter $\hat{S}$ [51, 52]. This result, together with the assumption that the effective field theory be weakly coupled (and hence calculable), gives rise to a spectacular signature (a sharp resonance peak) at the LHC [64]. Unfortunately, it is very difficult to distinguish it from the signature of a generic, weakly-coupled extension of the standard model with an extended gauge group, predicting a new massive Z' gauge boson.

Indisputable evidence proving that a strongly-coupled sector is responsible for electroweak symmetry breaking would be the discovery of at least the first two spin-1 resonances, hence proving that these new particles are not elementary, but higher energy excitations of a composite object. The major obstacle against this scenario is the unfortunate numerology emerging from the combination of precision data and LHC high-energy discovery reach. If

$M_1 > 2.5$–3 TeV, than it follows that the mass of the second resonance must be $M_2 >$ 5–6 TeV and just beyond the region where LHC data are expected to give convincing evidence [63]. Yet, a pretty mild relaxation of the experimental bounds would be enough to change this situation radically, since $M_1 \sim 1.5$ TeV would imply $M_2 \approx 2.5$–4 TeV, well within reach even at moderate luminosity [64]. It is hence timely, just before LHC starts collecting data, to question how accurate the AdS/CFT description of realistic dynamical electro-weak symmetry breaking is, and whether some of the approximations implied by this description could account for the desired softening of the bounds, without at the same time spoiling the calculability of the effective field theory.

In analogy with [56], the five-dimensional picture usually contains two hard boundaries representing the UV and IR cut-off between which the theory is conformal. This is the weakest link with the idea that electro-weak symmetry breaking be triggered by a non-abelian gauge theory with an approximate IR fixed point. Taken literally, this picture means that, both in the UV and in the IR, conformal symmetry is lost instantaneously, via a sharp transition. As for the UV cut-off, this is not a real problem from the low-energy effective field theory point of view. The details of how an asymptotically-free fundamental theory in the far UV enters a quasi-conformal phase below the UV cut-off, can always be reabsorbed (via holographic renormalization [65, 60]) in the definition of otherwise divergent low-energy parameters of the effective field theory, defined at a given order in the perturbative expansion of the effective field theory itself.

Rather different is the case against using a hard-wall regulator in the IR. There is no sense in which IR effects decouple and can be renormalized away, and hence the low-energy effects we are interested in, when comparing the effective field theory to the experimental data, are inherently sensitive to the choice of the IR regulator. On the one hand, the very validity of the effective field theory description based on the AdS/CFT dictionary requires that the hard-wall cut-off be at least a reasonable leading order approximation (otherwise the effective field theory itself would be strongly coupled, and not admit a controllable expansion). On the other hand, corrections are expected to be present, and estimating their size and understanding their phenomenological consequences is crucial, at the very least in order to know what to expect in experiments such as those at the LHC, which is going to test precisely the energy range close to the IR cut-off.

To be more specific. In the IR, three different phase transitions are taking place: electro-weak symmetry breaking, conformal symmetry breaking and confinement. These cannot define three parametrically separate scales, since they are all triggered by the same physical effect, namely the fact that the underlying (unknown) theory possesses an approximate fixed point in the IR. Hence the RG flow of the underlying dynamics is not going to reach the IR fixed point (which is only approximate), but will drift away from it at low energies, after spending some time (walking) in its proximity. Yet, there is no reason to expect these three effects to arise precisely at the same energy (temperature), and they might define three distinct critical scales (temperatures) that differ by $O(1)$ coefficients.

An illustration of this point can be obtained by considering an $\mathcal{N}=1$ supersymmetric QCD model with N_c colors and N_f fermions. At least at large-N_c, for $3N_c/2 < N_f < 3N_c$, the theory is asymptotically free, but has a fixed point in the IR [66, 67] (for recent progress towards the rigorous construction of the gravity dual see, for instance, [68]). If N_f is not far from the lower bound, so that the theory is strongly coupled at distances larger than a UV cut-off $1/L_0$, then the theory might be approximately described by a large-N_c conformal field theory at strong 't Hooft coupling. Suppose now that at some smaller energy, characterized by a length scale $\bar{L} \gg L_0$, for some reason (for example the existence of a suppressed symmetry-breaking higher-order operator, which acquires a large anomalous dimension in the IR turning it into a relevant deformation) a symmetry-breaking condensate forms, reducing further N_f to a value N_f' closer to or below $3N_c/2$. Symmetry-breaking drives the theory away from the original fixed point, and induces the loss of conformal symmetry. The coupling now runs fast (because the coupling itself was already big and large anomalous dimensions are present), and (depending on N_f') the theory either enters a new conformal phase at stronger coupling or confines. The breaking of the global $SU(N_f)_L \times SU(N_f)_R$, conformal symmetry-breaking and confinement take all place approximately at the same scale. Yet, the energy at which the coupling reaches its upper bound defines a new scale L_1 which might well be some numerical factor away from $\bar{L}$, the scale at which the RG-flow trajectory departed away from the fixed point.

If this is the qualitative behavior of the UV-complete dynamical model that is ultimately responsible for electro-weak symmetry breaking , describing it as a slice of AdS space between two hard walls is a good leading-order approximation. Nevertheless, we may wonder whether a factor of 3 or 4 separating the scales of conformal symmetry-breaking and confinement can be completely ignored, in the light of the phenomenological consequences at the LHC that a mere factor of two might have. In this paper, we study the effect of such a factor. We consider the simplest possible effective field theory description of dynamical electro-weak symmetry breaking as a 5D weakly-coupled system (see also [60]), introduce (besides the UV brane at L_0 and the IR brane at L_1) a new discontinuity at the scale $\bar{L}$, very close to the IR scale L_1, and assume that the background deviates from the AdS case for $\bar{L} < z < L_1$.

As for the origin and description of electro-weak symmetry-breaking, we will treat it as a completely non-dynamical effect localized in the IR, somehow in the spirit of Higgless models. The breaking could take place at L_1 as well as at $\bar{L}$ (or anywhere in between), as suggested by the SQCD example above. We compare the effects on the electro-weak precision parameter $\hat{S}$ in these two cases, as illustrative of two extreme possibilities, without committing ourselves to either of them. The idea that chiral symmetry breaking might, for a generic model, take place at a scale higher than confinement has been in the literature for a while [69], has been supported by lattice evidence in some special case [70], and has recently been discussed also in string-inspired models [71].

A realistic model should also implement a dynamical mechanism generating the mass of the standard model fermions. This can be done either via extended technicolor higher-order interactions between the standard model fermions and the new strong sector [72, 73] (represented in the 5D picture by Yukawa interactions localized at the UV, with the symmetry-breaking vacuum expectation value not localized, but exhibiting a non-trivial power-law profile in the bulk), or via the assumption that standard model fields are themselves (partially) composite, in the spirit of topcolor and related models [74] (which would imply the fermions be allowed to propagate in the bulk of the 5D model). A detailed discussion of how the global family symmetry of the standard model is broken would be required in order to study how the phenomenology of flavor-changing transitions and the physics of the third generation would be affected by the proposed modification of the background. In this paper, we treat the standard model fermions as non-dynamical fields, described by a set of external currents, and do not address the problem of their mass generation. For some recent studies of the flavor problem in the context of warped extra-dimension models, see [75].

4.1 Preliminaries

A non-trivial departure of the dynamics of the spin-1 resonances, with respect to that on pure AdS geometry, may be either due to a modification of the gravity background or to the presence of a non-dynamical background (dilaton). Since we consider an effective field theory where only spin-1 states are dynamical, it is not possible to distinguish between these two effects at this level. We choose to describe the model in terms of a deformation of the gravity background, for simplicity.

Consider the five-dimensional space described by the metric

$$\mathrm{d}s^2 \;=\; a(z)^2\left(\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu \,-\, \mathrm{d}z^2\right)\,, \tag{4.1}$$

where $L_0 < z < L_1$. We will assume that the geometry approaches pure AdS in the UV region, $a(z) \to L/z$ as $z \to L_0$, and departs from it at a scale $z \sim \bar{L}$. In most of the calculations we take $L_0 = L$ for simplicity.

We are interested in describing a model that at low-energy (below $1/L_1$) can be matched to the electro-weak chiral Lagrangian [76]. This requires to introduce a 5-dimensional gauge group which is at least $SU(2)_L \times U(1)_Y$, but may be enlarged to accommodate custodial symmetry. Irrespectively of the details, the model contains a vectorial sector (the neutral part of which consists of the photon and its excitations) and an axial sector (containing the Z boson and its excitations). In this paper we describe only the phenomenology connected with the neutral gauge bosons, hence we dispense with the details of the complete symmetry group. For concreteness, we take the vectorial sector to be described by the pure Yang-Mills $SU(2)$ theory with the following action:

$$\mathcal{S} \;=\; \int \mathrm{d}^4x \int_{L_0}^{L_1} \mathrm{d}z\sqrt{G}\left[G^{MR}G^{NS}\left(-\frac{1}{2}\mathrm{Tr}\, F_{MN}F_{RS}\right)\right. \tag{4.2}$$

$$+2g\delta(z-L_0)G^{MN}\mathrm{Tr}\,J_M A_N\Big]\ ,$$

where $F_{MN} = \partial_M A_N - \partial_N A_M + ig[A_M, A_N]$ is the field strength tensor of $A_M = A^a_M T^a$ with $T^a = \tau^a/2$ the generators of $SU(2)$, where g is the (dimensionful) gauge coupling, and where $J_M = (J_\mu(x), 0)$ is the four-dimensional external current localized on the UV-brane.

Quantization requires to add appropriate gauge-fixing terms, canceling the mixing terms between spin-0 and spin-1 fields, which in unitary gauge implies $A_5(x,z) = 0$.

After Fourier transforming in 4D, $A_\mu(x,z) \equiv \int \frac{\mathrm{d}^4 q}{(2\pi)^2} e^{iqx} A_\mu(q) v(q,z)$, the free bulk equations read

$$\partial_5\left[a(z)\partial_5 v(q,z)\right] \;=\; -q^2 a(z) v(q,z)\,. \tag{4.3}$$

Substituting the solutions in the action, and canceling the boundary terms at $z = L_1$, without breaking the gauge symmetry, requires to impose Neumann boundary conditions:

$$\partial_5 v(q, L_1) \;=\; 0\,. \tag{4.4}$$

This set of equations admits always a constant, massless zero mode.

Finally, the action can be rewritten as a pure boundary term at the UV, from which one can read the vector two-point correlator, that for $L_0 \to L$ is

$$\Sigma_V(q^2) \;=\; g^2 \frac{v(q,L_0)}{\partial_5 v(q,L_0)}\,. \tag{4.5}$$

The latter can be expanded in the vicinity of the poles $q^2 \sim M_i^2$ as

$$\Sigma_V(q^2) \;\sim\; e^2 \frac{R_i}{q^2 - M_i^2}\,, \tag{4.6}$$

where $M_0 = 0$ and M_i $(i = 1, 2, \ldots)$ are the masses of the excited states. The residues $R_0 = 1$ and R_i define the effective couplings to the four-dimensional currents normalized to the coupling e^2 of the massless mode (to be identified with the electro-magnetic coupling of the photon).

The (dimensionful) bulk coupling g controls the perturbative expansion used to extract this correlator. It is not directly related to the effective coupling e of the standard model gauge boson (photon), but rather is related to the strength of the effective interactions among its heavy (composite) excitations. The relation between these two effective couplings depends on how the theory is regularized in the UV, and is not a calculable quantity, because of the divergences in the $L_0 \to 0$ limit. A rigorous treatment requires to introduce appropriate counterterms and treat the ratio $e^2 L/g^2$ as a free parameter. For the purposes of this paper, which primarily require comparing identical UV settings with different IR deformations, we can simplify this procedure by assuming that $L_0 \ll L_1$ be finite and fixed, and express this ratio as a function of the scales and couplings in the model. We discuss later how good the perturbative expansion is by estimating the size of the effective self-coupling of the composite states.

In order to compute $\hat{S}$ one has to introduce also the axial-vector excitations, and a symmetry-breaking mechanism. For the purposes of this paper, we only consider the Higgsless limit, defined by the introduction of a localized, infinitely massive Higgs scalar which assumes a non-trivial symmetry-breaking vacuum expectation value.

The axial-vector modes $v_A(q,z)$ still satisfy Eq (4.3), but their boundary conditions (and the gauge fixing action) are modified. We consider two cases in the following. In the first, symmetry-breaking takes place on the boundary L_1 so that the axial-vector profiles $v_A(q,z)$ obey generalized Neumann boundary condition:

$$\partial_5 v_A(q,L_1) + m v_A(q,L_1) = 0\,. \tag{4.7}$$

The effective symmetry-breaking parameter m has dimension of a mass. In the limit $m \to 0$ one recovers the symmetric case, while for $m \to +\infty$ one recovers the Dirichelet boundary conditions. The mass of the Z boson depends on m is such a way that it vanishes for vanishing m, but is determined by L_1 for arbitrarily large m. In the second case we consider a symmetry-breaking vacuum expectation value localized at a different point $\bar{L} < L_1$ in the fifth dimension. The modifications to be implemented in this case will be discussed in the next sections.

All of this allows to define the axial-vector correlator $\Sigma_A(q^2)$ by replacing in Eq. (4.5) $v_A(q,z)$ and its derivative to $v(q,z)$. After these manipulations, the precision parameter $\hat{S}$ is given by

$$\hat{S} = e^2 \cos^2\theta_W \frac{\mathrm{d}}{\mathrm{d}q^2}\left(\frac{1}{\Sigma_V(q^2)} - \frac{1}{\Sigma_A(q^2)}\right)\Bigg|_{q^2=0}\,, \tag{4.8}$$

where e has been defined before, and corresponds to the electro-magnetic coupling, while θ_W is the effective weak-mixing angle. We recall here that an approximate extrapolation to large Higgs masses yields the experimental limit $\hat{S} < 0.003$ at the 3σ level [52].

4.1.1 Pure AdS background

We summarize here the results of the case in which the background is purely AdS with $a(z) = L/z$, and assume for simplicity that $L_0 = L$. The vector correlator is

$$\Sigma_V^{(0)}(q^2) = \frac{g^2\left(J_0(L_1q)Y_1(Lq) - J_1(Lq)Y_0(L_1q)\right)}{q\left(J_0(L_1q)Y_0(Lq) - J_0(Lq)Y_0(L_1q)\right)}\,. \tag{4.9}$$

In order to discuss the spectrum and couplings, the following approximations can be used:

$$\Sigma_V^{(0)}(q^2) \simeq \frac{g^2 J_0(L_1q)}{Lq^2\left(\frac{\pi}{2}Y_0(L_1q) - J_0(qL_1)\left(\gamma_E + \log\frac{Lq}{2}\right)\right)} \tag{4.10}$$

$$\simeq \frac{g^2}{Lq^2\left(\frac{\pi}{2}\tan(L_1 q - \frac{\pi}{4}) - \left(\gamma_E + \log\frac{Lq}{2}\right)\right)}, \tag{4.11}$$

the first of which is valid for $L \ll L_1$, and the second for $qL_1 > 1$.

From (4.10) one can read the coupling of the zero mode:

$$e^2 = \frac{g^2}{L\log L_1/L}. \tag{4.12}$$

From (4.11) one can look for the poles and the residues R_i. The poles (besides the pole at zero) are in the vicinity of those of $\tan(L_1 q - \pi/4)$:

$$M_i \simeq \frac{\pi}{4L_1}\left((4i-1) - \frac{2}{\gamma_E + \log(4i-1)\pi L/(8L_1)}\right), \tag{4.13}$$

while the residues are approximately given by:

$$R_i \simeq \frac{4\log(L_1/L)}{-2 + \frac{2\pi L_1 M_i}{1+\sin(2L_1 M_i)}}, \tag{4.14}$$

with $i = 1, 2, \ldots$. These approximations are acceptably accurate as long as $L \ll L_1$. A numerical calculation will be performed later on, when discussing the phenomenology for some relevant choice of parameters.

The axial correlator can be computed exactly:

$$\Sigma_A^{(0)}(q^2) = \frac{g^2\left(\left(qJ_0(L_1q) + mJ_1(L_1q)\right)Y_1(Lq) - J_1(Lq)\left(qY_0(L_1q) + mY_1(L_1q)\right)\right)}{q\left(qJ_0(L_1q)Y_0(Lq) + mJ_1(L_1q)Y_0(Lq) - J_0(Lq)\left(qY_0(L_1q) + mY_1(L_1q)\right)\right)}, \tag{4.15}$$

and, for $L_0 \ll L_1$, yields

$$\hat{S} = \frac{\cos^2\theta_W L_1 m(3L_1 m + 8)}{4(L_1 m + 2)^2 \log\left(\frac{L_1}{L}\right)}. \tag{4.16}$$

In the limit $m \to +\infty$ we have

$$\hat{S} = \frac{3\cos^2\theta_W}{4\log L_1/L}. \tag{4.17}$$

Imposing the (3σ-level) experimental limit we find that

$$\frac{g^2}{L} = e^2\log\frac{L_1}{L} = e^2\frac{3\cos^2\theta_W}{4\hat{S}} > 20, \tag{4.18}$$

where $e^2 \simeq 0.1$ is the effective coupling of the electro-magnetic $U(1)_Q$ in the standard model. Since, as discussed later, g^2/L gives a measure of the effective strength of the self interactions between resonances (and the dimensionful coupling g is the expansion parameter in the 5D action) the experimental bounds are satisfied only at the price of loosing calculability, as is the unfortunate case also when trying to build QCD-like technicolor models in 4D, either using the large-N expansion, hidden local symmetry, or deconstruction (see for instance [79]). We do not discuss further this limit.

In the more interesting and realistic case in which $mL_1 \ll 1$, the axial-vector spectrum and couplings are approximately the same as the vectorial sector. In this framework m is just a free parameter, and we treat it as such. With finite $mL_1 \ll 1$, the mass of the lightest axial-vector state is approximately $M_Z^2 \simeq m/(L_1 \log(L_1/L))$, and hence

$$\hat{S} \simeq \frac{\cos^2\theta_W}{2\log L_1/L}\, mL_1 \simeq \frac{\cos^2\theta_W}{2} M_Z^2 L_1^2 \tag{4.19}$$

satisfies the bounds on $\hat{S}$ for $1/L_1 > 1$ TeV, which depending on the value of L_0/L_1 translates into a bound $M_1 > 2.5-4$ TeV. For instance, for $g^2/L < 1/2$ it requires $M_1 > 2.8$ TeV, and consequently $M_2 > 6$ TeV, which is beyond the projected reach of the LHC searches.

4.2 Departure from AdS

We now consider the possibility that conformal invariance be violated at some energy regime above the confinement scale and suppose there exists a hierarchy of scales $L_0 = L < \bar{L} < L_1$ such that the space is the usual AdS for $L_0 < z < \bar{L}$, but departs from it in the IR region $\bar{L} < z < L_1$. Our aim is to model this behavior without affecting the approximate description of confinement provided by the IR brane (different motivations lead the authors of [108] to other parameterizations). The simplest form one can choose in order to achieve this goal is a power-law warp factor

$$a(z) = \begin{cases} \frac{L}{z} & z < \bar{L} \\ \frac{L}{z}\left(\frac{\bar{L}}{z}\right)^{n-1} & z > \bar{L} \end{cases} . \tag{4.20}$$

This parameterization may be viewed as a leading order approximation of a smooth background describing the appearance of some relevant deformation in the conformal field theory before the underlying fundamental theory confines.

We will see later that a power-law avoids generating an explicit mass gap from the bulk equations, so that the quantity $1/L_1$ can still be interpreted as the scale of confinement. Moreover, with our parameterization we can solve the equations exactly and in a very straightforward way, which is in itself a welcome property when modeling an otherwise untreatable dynamical system.

Most of the algebraic manipulations can be performed for generic n. Yet, we discuss explicitly only the $n > 1$ case. A variety of arguments, all ultimately descending from unitarity, suggest that we should limit ourselves to $n \geq 1$. An extra-dimensional argument can be derived along the lines of [77], in which it is shown how the weaker energy condition leads to a c-theorem controlling the behavior of the curvature in crossing a phase transition towards the IR. This is related to the fact that, in the context of strongly-coupled four-dimensional models, in going through a phase transition it is reasonable to expect the effective number of light degrees of freedom to decrease [78]. Hence the effective coupling of the effective field theory description, which is related to the $1/N$ expansion, is expected to increase. We show later in the paper that the effective self-coupling of the heavy resonances is enhanced for $n \geq 1$, in agreement with the four-dimensional intuitive expectation, and that this enhancement is controlled by a power of the ratio of relevant scales, in agreement with naive expectations for a theory with a generic deformation due to a relevant operator. The fact that all of our results agree with the intuitive interpretation gives an indication in support both of the power-law parameterization chosen here and of the $n \geq 1$ restriction.

The solutions to the bulk equations in the IR region $z > \bar{L}$ are of the form

$$v^{IR}(q,z) \;=\; z^{\frac{n+1}{2}}\left(c_1^{IR}(q)J_{\frac{n+1}{2}}(qz) + c_2^{IR}(q)Y_{\frac{n+1}{2}}(qz)\right)\,, \tag{4.21}$$

while in the UV region

$$v^{UV}(q,z) \;=\; z\left(c_1^{UV}(q)J_1(qz) + c_2^{UV}(q)Y_1(qz)\right)\,. \tag{4.22}$$

The bulk profile is obtained by applying the IR boundary conditions to v^{IR}, and then by requiring that the junction of the two solutions be smooth, so that no boundary action localized at $\bar{L}$ is left:

$$\partial_5 v^{IR}(q,L_1) \;=\; 0\,, \tag{4.23}$$
$$v^{IR}(q,\bar{L}) \;=\; v^{UV}(q,\bar{L})\,, \tag{4.24}$$
$$\partial_5 v^{IR}(q,\bar{L}) \;=\; \partial_5 v^{UV}(q,\bar{L})\,. \tag{4.25}$$

The correlator is then obtained from Eq. (4.5) by using v^{UV}. From all of this, one can extract the masses and couplings of the resonances. In particular, the coupling of the zero-mode (photon) is

$$e^2 \;=\; \frac{(n-1)\frac{g^2}{L}}{(n-1)\log(\frac{\bar{L}}{L_0}) + \left(1-\left(\frac{\bar{L}}{L_1}\right)^{n-1}\right)}\,. \tag{4.26}$$

For $n = 1$, or for $\bar{L} = L_1$, one recovers the AdS result (4.12). For $n > 1$ and $\bar{L} < L_1$ this estimate is enhanced (for fixed g^2/L). In order to understand how significant this effect is, one needs to compare this coupling to the effective self-coupling, which is discussed in the next section.

Analytical expressions for the couplings and masses of the vector-like resonances are rather involved. In order to gain a semi-quantitative understanding of how these quantities are modified with respect to the pure AdS case, we discuss the (unrealistic) extreme case in which $\bar{L} = L_0 \ll L_1$. For $qz \gg 1$:

$$J_{\frac{n-1}{2}}(qz) \simeq \sqrt{\frac{2}{\pi q z}} \cos\left(qz - \frac{n\pi}{4}\right) , \tag{4.27}$$

$$Y_{\frac{n-1}{2}}(qz) \simeq \sqrt{\frac{2}{\pi q z}} \sin\left(qz - \frac{n\pi}{4}\right) \tag{4.28}$$

and the masses of i-th resonances, for $n > 1$, are approximately given by the zeros of $J_{\frac{n-1}{2}}(qL_1)$,

$$M_i(n) \simeq \frac{2i-1}{2}\frac{\pi}{L_1} + \frac{n\pi}{4L_1} , \tag{4.29}$$

with $i = 1, 2, \ldots$. This agrees with the pure AdS case ($n = 1$), at least for $L/L_1 \ll 1$, and explicitly shows that the introduction of the non-conformal region $\bar{L} < z < L_1$ affects only modestly the masses of the vectorial excitations. For the more realistic case in which $L_0 \ll \bar{L} < L_1$ a numerical study is necessary and will be presented in section VI. The main features emerging from that analysis are the following. The spectrum of massive modes with masses comparable with the new scale $1/\bar{L}$ is going to be increased by approximately $(n-1)\pi/(4L_1)$ with respect to the AdS case. The spectrum connects back to the pure AdS case for higher excitation number i. As for the residues, the couplings to the currents of the heavy modes are approximately going to be suppressed with a power-law dependence $\approx \left(\frac{\bar{L}}{L_1}\right)^{(n-1)}$ with respect to the AdS case. Again, this suppression applies only to the lightest resonances, those for which the mass is slightly shifted to higher values.

Analytical expressions for both M_Z and $\hat{S}$ can be written in closed form, but from a practical point of view it is convenient to discuss the two limits $mL_1 \ll 1$ and $\bar{m}L_1 \ll 1$. In the phenomenologically relevant region of parameter space the complete expressions are found to be accurately approximated by the following formulas.

If the symmetry breaking takes place at L_1 the IR boundary conditions for the axial-vectors become:

$$\partial_5 v_A^{IR}(q, L_1) + m v_A(q, L_1) = 0 , \tag{4.30}$$

$$v_A^{IR}(q, \bar{L}) = v_A^{UV}(q, \bar{L}) , \tag{4.31}$$

$$\partial_5 v_A^{IR}(q, \bar{L}) = \partial_5 v_A^{UV}(q, \bar{L}) . \tag{4.32}$$

For generic $n > 1$ and in the limit $mL_1 \ll 1$, the mass of Z boson and $\hat{S}$ read

$$M_Z^2 \simeq \left(\frac{\bar{L}}{L_1}\right)^{n-1} \frac{(n-1)m}{L_1\left(1-(\bar{L}/L_1)^{n-1}+(n-1)\log \bar{L}/L\right)}\,, \tag{4.33}$$

$$\hat{S} \simeq \cos^2\theta_W \left(\frac{1}{n+1}+\frac{1}{2}(\bar{L}/L_1)^2-\frac{1}{n+1}(\bar{L}/L_1)^{n+1}\right) L_1^2 M_Z^2\,. \tag{4.34}$$

For small $\bar{L}/L_1$ this approximation would not hold, because of the dependence of M_Z on m and on $\bar{L}/L_1$. We do not admit a parametric separation between $\bar{L}$ and L_1, and hence the approximations are acceptable.

The other extreme possibility we are interested in is the one in which the symmetry-breaking condensate is localized at $\bar{L}$, for which the boundary conditions become

$$\partial_5 v_A^{IR}(q, L_1) = 0\,, \tag{4.35}$$

$$v_A^{IR}(q,\bar{L}) = v_A^{UV}(q,\bar{L})\,, \tag{4.36}$$

$$\partial_5 v_A^{IR}(q,\bar{L}) = \partial_5 v_A^{UV}(q,\bar{L}) + \bar{m} v_A(q,\bar{L})\,. \tag{4.37}$$

For generic n and at leading order in $\bar{m}L_1 \ll 1$ we have:

$$M_Z^2 \simeq \frac{(n-1)\bar{m}}{\bar{L}\left(1-(\bar{L}/L_1)^{n-1}+(n-1)\log \bar{L}/L\right)}\,, \tag{4.38}$$

$$\hat{S} \simeq \frac{\cos^2\theta_W\left(n+1-2\left(\bar{L}/L_1\right)^{n-1}\right)}{2(n-1)}\bar{L}^2 M_Z^2 \tag{4.39}$$

Notice how the dependence of M_Z on $\bar{m}$ is not suppressed by powers of $\bar{L}/L_1$, as in the former case, where m came from a localized term at L_1. This result agrees with the intuitive notion that moving the symmetry-breaking towards the UV enhances its effect for the zero-mode, while suppressing the mass splitting of the heavy resonances. The result is well illustrated by $\hat{S}$, which is proportional to M_Z^2 through the position $\bar{L}$ or L_1 of the symmetry-breaking condensate in the fifth dimension.

4.3 Estimating the strength of the self-interactions

The departure from conformal invariance, explicitly added via a power-law deviation from the AdS background in the IR region, might imply that the dynamics of the effective field theory itself be strongly coupled, as is the case for a QCD-like dynamical model. It has to be understood if the effective field theory treatment still admits a power-counting allowing to use a cut-off L_0 much larger than the electro-weak scale. A fully rigorous treatment of this problem is not possible, because it requires to extend the effective field theory Lagrangian beyond the leading order in $1/N_c$. Yet, a reasonable estimate of the effective coupling can be extracted by looking at the cubic and quartic self-couplings of the resonances, the structure of which (at the leading order) is dictated by 5D gauge-invariance.

Consider first the pure AdS background and define

$$g_\rho^{(i)\,2} \equiv \frac{g^2}{L}\frac{\int_{L_0}^{L_1}\frac{\mathrm{d}z}{z}|v(M_i,z)|^4}{\left(\int_{L_0}^{L_1}\frac{\mathrm{d}z}{z}|v(M_i,z)|^2\right)^2}\,. \tag{4.40}$$

The expansion parameter is related to g_ρ, which we define as the asymptotic limit of the effective self-coupling for large excitation number. As long as $L_0 \ll L_1$ and $M_i L_1 \gg 1$, the bulk profiles of the heavy modes can be approximated by

$$v(M_i,z) \propto \frac{z}{\sqrt{M_i}}J_1(M_i z) \propto \frac{\sqrt{z}}{M_i}\cos\left(M_i z - \frac{3\pi}{4}\right) \tag{4.41}$$

yielding

$$g_\rho^2 \equiv \lim_{i\to+\infty} g_\rho^{(i)\,2} \simeq \frac{3}{4}\frac{g^2}{L}\,. \tag{4.42}$$

For the smallest values of $i = 1, 2$ this is a moderate underestimate. For instance for $i = 1$, from the exact solution one obtains $g_\rho^{(1)\,2} \sim 1.2 g^2/L$. The meaning of this definition of g_ρ is that it gives a reasonable estimate of the strength of the self-coupling of the resonances, and hence of the expansion parameter of the effective field theory (which is related to the large-N_c expansion). As expected, this turns out to be controlled by g^2/L, up to $O(1)$ coefficients. The actual value of g^2 is related (with the treatment of the UV cut-off used here) to the coupling of the zero mode $e^2 = g^2/(L \log L_1/L_0)$, so that $g_\rho^2 \approx e^2 \log(L_1/L_0)$. This yields the relation between strength of the effective coupling and the effective cut-off in the UV, which as expected is logarithmic, ultimately because of conformal symmetry. The requirement that this defines a perturbative coupling g_ρ^2 implies a bound on L_1/L_0. Choosing for instance $L_1 = 100 L_0$ (a value that is not justifiable by applying naive dimensional analysis to the electro-weak chiral Lagrangian), yields $g_\rho^{(i)\,2} \approx 0.3$, which means that the effective field theory admits an acceptable expansion in powers of $g_\rho^2/(4\pi)$ even with large choices of the UV cut-off $1/L_0$.

Generalizing this estimate in presence of the non-trivial background (4.20) is somehow more difficult, largely because of the junction conditions at $\bar{L}$. This can be done numerically, but for the present purposes a semi-quantitative assessment of the size of the effective coupling suffices. We again focus on large values of $M_i L_1$ and modify the definition of the effective couplings to

$$g_\rho^{(i)\,2} \equiv \frac{g^2}{L}\frac{\int_{L_0}^{L_1}\frac{\mathrm{d}z}{z^n}|v(M_i,z)|^4}{\bar{L}^{n-1}\left(\int_{L_0}^{L_1}\frac{\mathrm{d}z}{z^n}|v(M_i,z)|^2\right)^2}\,. \tag{4.43}$$

The specific case we are interested in lies somewhere in between the pure AdS and the pure power-law. In the latter case an acceptable approximation would be:

$$v(M_i, z) \;\propto\; \frac{z^{\frac{n+1}{2}}}{\sqrt{M_i}} J_{\frac{n+1}{2}}(M_i z) \;\propto\; \frac{z^{\frac{n}{2}}}{M_i} \cos\left(M_i z - \frac{(n+2)\pi}{4}\right) . \tag{4.44}$$

The effective coupling receives power-law contributions in $L_1/\bar{L}$, plus terms that are logarithmic in $L_0/\bar{L}$ and hence subleading $O(1)$ corrections. The power-law is the most important effect and, for large choices of $L_1/\bar{L}$ and in the case $n > 1$, we obtain:

$$g_\rho^2 \;\simeq\; \frac{3}{2(n+1)} \frac{g^2}{L} \left(\frac{L_1}{\bar{L}}\right)^{n-1} \tag{4.45}$$

$$\simeq\; \frac{3e^2}{2(n^2-1)} \left(\frac{L_1}{\bar{L}}\right)^{n-1} \tag{4.46}$$

which, as in the pure AdS case, represents a defective approximation by roughly a factor of 2 for the very first resonance. We see that, for g_ρ to be acceptably small as to define an expansion parameter, $L_1/\bar{L}$ cannot be large.

The power-law dependence on $\bar{L}/L_1$ in Eq. (4.45) is expected in a non-conformal effective theory, in presence of relevant operators, in which case there cannot be a substantial scale separation between the UV cut-off and the mass scale L_1 of the effective theory itself. This result agrees with naive dimensional analysis counting. For instance, taking $\bar{L} = L_0$ implies that the model is strongly coupled, unless $(L_1/\bar{L})^{n-1} \ll 4\pi$, which implies a very low cut off, and the impossibility of describing the resonances as weakly coupled.

Notice that this result depends smoothly on $n > 1$. But in trying to extend the analysis to $n < 1$ one immediately faces a problem. For instance, for $L_0 \to \bar{L} \ll L_1$, $n < 1$, and keeping g^2/L fixed, the effective coupling becomes vanishingly small. This behavior would imply that, in the region of the parameters space in which the theory admits an effective approach, the original conformal theory flows into a new phase that is described by a new effective field theory which has effectively a weaker coupling. This violates the intuitive expectations, according to which such a phase transition always drives the theory towards stronger coupling, such that the new effective field theory has always a smaller number of light degrees of freedom, and hence a larger expansion parameter. Though not rigorous, this argument seems to support the hypothesis that only $n > 1$ is an admissible choice.

From the phenomenological point of view, one way to assess how strongly coupled is the first resonance, is to consider γ_1, the first excited mode with the quantum numbers of a photon, and compare its partial width into two standard model fermions f to the partial width into two on-shell W bosons, namely:

$$\frac{\Gamma(\gamma_1 \to f\bar{f})}{\Gamma(\gamma_1 \to W^+W^-)} \;\approx\; \frac{8\alpha}{3} R_1 \frac{48\pi}{g_\rho^2} \simeq \frac{\pi R_1}{g_\rho^2} . \tag{4.47}$$

For a weakly-coupled theory this approximate estimate should be $O(1)$ or bigger. In other words, a rough estimate of the width of the first resonance gives $\Gamma \approx g_\rho^2 M_1/(48\pi)$, and hence the approximation of treating this resonance as infinitely narrow (as expected at large-N_c) makes sense only as long as g_ρ^2 is at most some $O(1)$ number. A more detailed study of this quantities, and the phenomenological consequences relevant at LHC energies, will be discussed in a subsequent study.

4.4 Phenomenological implications

4.4.1 Spectrum and couplings to the currents

We start with a numerical analysis of the spectrum and couplings of the vectorial excited states. We perform the numerical analysis because the results discussed in the previous section for these quantities give only semi-quantitative approximate expressions. Since we always consider values of m and $\bar{m}$ that are small compared to $1/L_1$, the results apply also to the axial-vector modes, irrespectively of the choice of localizing the symmetry-breaking effects at L_1 or at $\bar{L}$.

The masses M_i depend in a complicated way on L_1, $\bar{L}$, L_0, and n. In Figure 4.1 we plot the mass (in units of $1/L_1$) for the first three excited states, as a function of $L_1/\bar{L}$. We compare four choices of the relevant parameters, characterized by $n = 2, 3$ and by the choice of the UV cut-off $L_0 = L_1/20$ and $L_0 = L_1/100$. The masses are very mildly UV sensitive and, as anticipated, slightly larger than in the $n = 1$ case (pure AdS), which is recovered when $\bar{L} = L_1$. The enhancement is proportional to n and it affects the heavier states only for large values of $L_1/\bar{L}$.

The coupling R_i is, in the pure AdS case, a monotonically decreasing function of the excitation number i. In Figures 4.2 and 4.3 we plot the numerical results obtained for this quantity, for the same choices of parameters used for the masses. In going from $L_1/\bar{L} = 1$ (pure AdS) to larger values and/or to large n, a suppression of the coupling is obtained for the lightest state. This suppression is a very big effect, and it becomes relevant at large values of $L_1/\bar{L}$. As a result, for instance in the case $n = 3$, with $L_1/\bar{L} > 4$ the third resonance has the strongest coupling, followed by the second and by the first.

Before concluding this section it is worth to comment on the significance of the numerical results presented. The approach used in this paper has some limitations: the dependence of the physical quantities M_i, R_i, and e on the unphysical UV cut-off $1/L_0$ should be removed by an appropriate renormalization procedure, and the truncation at tree-level of the perturbative expansion introduces a systematic error on the estimates. The main physical information that emerges from the numerical study is that, while the modification of the masses due to the departure from pure AdS background is of a size comparable with the expected systematic error, and hence should not be taken too literally, on the other hand the change by factors of $O(2-4)$ in the coupling to the currents is so large that we

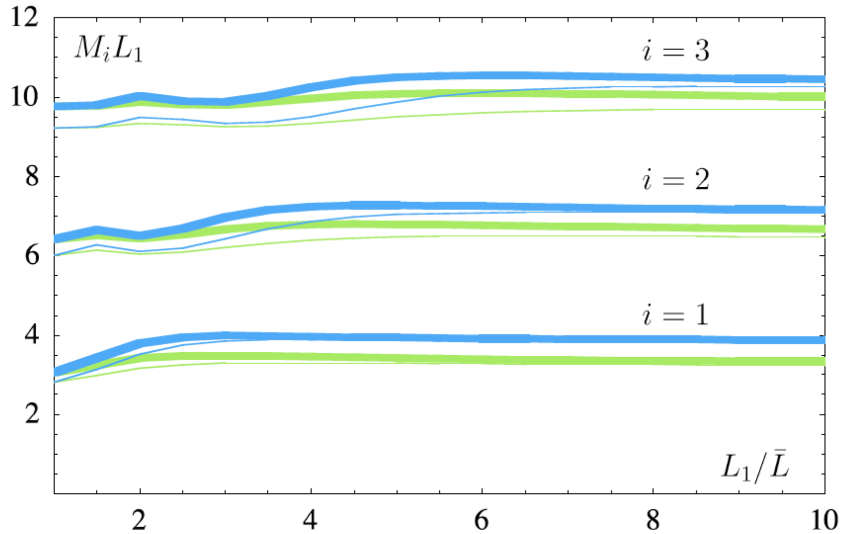

Figure 4.1: Masses M_iL_1 of the first $i = 1, 2, 3$ excited vector modes, as a function of $L_1/\bar{L}$. The four curves are drawn for $n = 2$ (green/light grey) and $n = 3$ (cyan/dark grey), and for $L_0 = L_1/20$ (thick line) and $L_0 = L_1/100$ (thin line).

expect it to be a robust and observable result.

4.4.2 Self-couplings and symmetry-breaking

We want the 5D action to define a reasonable effective field theory treatment of the strong dynamics and of the resulting electro-weak symmetry breaking effects, with a well-behaved perturbative expansion. We implement this requirement by imposing the bound $g_\rho^2 < 1/2$ (a reference value that we fix in such a way that for the choices of parameters discussed here the ratio of partial width estimated in Eq. (4.47) is > 1), where g_ρ has been defined in the body of the previous section. In the pure AdS case we require that $L_1/L_0 < 200$, which means that the model is very modestly sensitive to the position of the UV cut-off and, unless extreme choices of $L_0 \ll L_1$ are used, we can neglect the effect of L_0 in driving the effective coupling strong. We can therefore impose the bound directly on the modification due to the new non-conformal energy regime:

$$\left(\frac{L_1}{\bar{L}}\right)^{n-1} \quad < \quad \frac{(n^2-1)}{3e^2}\,. \tag{4.48}$$

For small values of $n \simeq 1$, the bound is not relevant, unless very large values of L_1/L_0

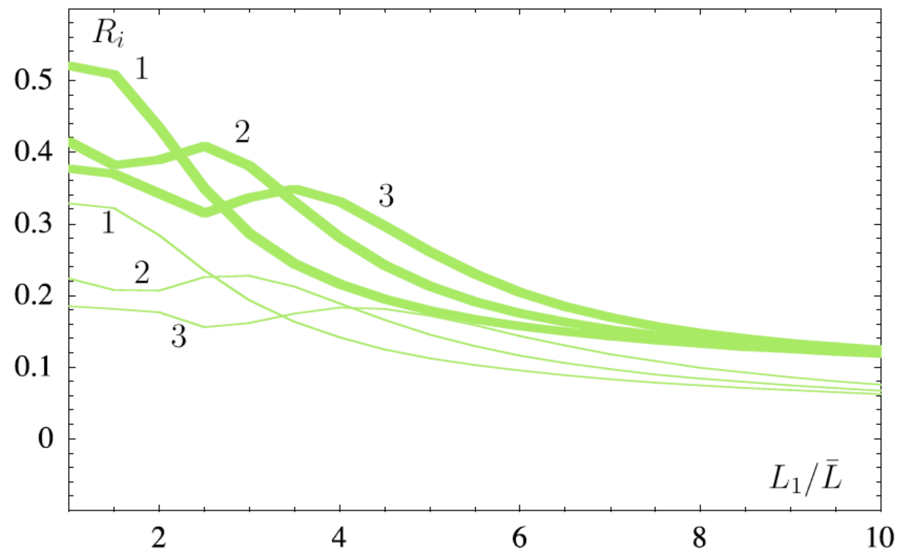

Figure 4.2: Relative coupling R_i to the currents of the first $i = 1, 2, 3$ excited vector modes, as a function of $L_1/\bar{L}$. The curves are drawn for $n = 2$, and for $L_0 = L_1/20$ (thick line) and $L_0 = L_1/100$ (thin line).

are used. We do not discuss further this case. For $n > 3$ the bound is very restrictive, and only $L_1/\bar{L} \sim O(1)$ is allowed. This confirms the intuitive notion that if large power-law deviations are allowed over a large energy window, the model is strongly coupled and does not admit a perturbative and controllable effective field theory expansion. For $n = 2 - 3$, values of $L_1/\bar{L} \sim 3 - 8$ are compatible with the requirement that the effective field theory be weakly coupled, and offer an interesting possibility from the phenomenological point of view. We focus on this possibility.

The effects of symmetry breaking are encoded in the estimate of $\hat{S}$. This is the quantity that ultimately sets a bound on L_1, and hence on the mass of the excited resonances. If the symmetry-breaking effects are localized at L_1, the analytical expression derived in Eq. (4.34) shows that, for all practical purposes, the bounds are the same as those obtained in the pure AdS case, $L_1 < 1$ TeV^{-1}. This is the case because the only sizable suppression factors are the $1/(n+1)$ and the $\bar{L}/L_1$ terms, but at large values of n only $\bar{L}/L_1 \sim 1$ is allowed.

Let us discuss the case in which symmetry-breaking takes place at $\bar{L}$. In order to assess how sizable the reduction in the experimental bounds is, we require that $\hat{S} < 0.003$, and calculate the minimum value of $1/L_1$ which is compatible with this bound, using the expression in Eq. (4.39). We show the result in Figure 4.4 assuming various values of

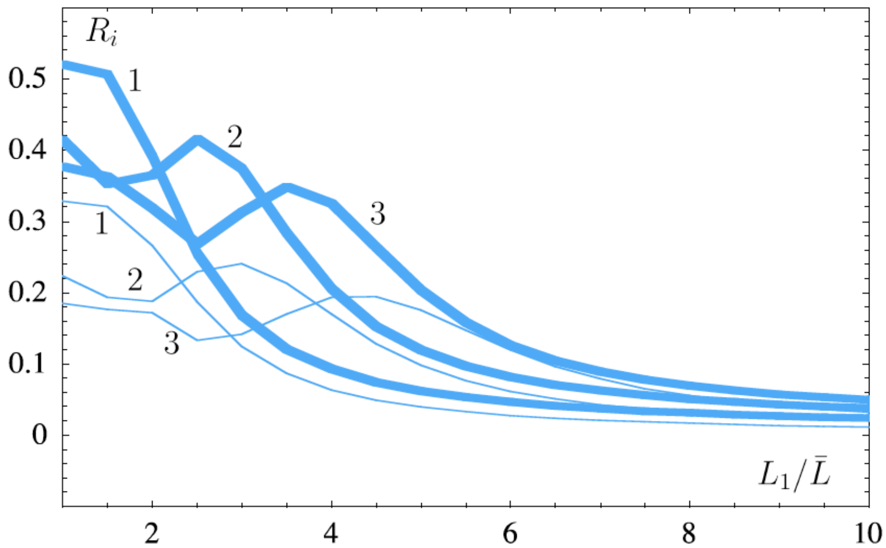

Figure 4.3: Relative coupling R_i to the currents of the first $i = 1, 2, 3$ excited vector modes, as a function of $L_1/\bar{L}$. The curves are drawn for $n = 3$, and for $L_0 = L_1/20$ (thick line) and $L_0 = L_1/100$ (thin line).

$L_1/\bar{L}$. We plot, as a function of n, the lower bound for $\pi/(M_Z L_1)$ starting from the pure AdS case, but without exceeding the (n-dependent) bound in Eq. (4.48). For the reasons already stressed in section VI-A, the identification $M_1 \simeq \pi/L_1$, although not a strict equality, provides a reasonable estimate up to boundary effects, model-dependent shifts, and systematic errors (see also Figure 4.1). We decided to plot this quantity for convenience, since L_1, rather than the masses of the vector excitations, enters the explicit formulae for $\hat{S}$.

In the pure AdS case ($L_1/\bar{L} = 1$), the lower bound in Figure 4.4 implies (using the experimental value of M_Z) $M_1 > 3$ TeV, and $M_2 >$ 6-7 TeV. Going to larger values of $L_1/\bar{L}$ allows for a very significant reduction of such bounds, even when this ratio is small enough to be compatible with the requirement that the effective coupling g_ρ^2 be smaller than 1/2. As a result, the value of the scale $1/L_1$ can be greatly reduced. Values such as $M_1 \sim 1.5$ TeV, $M_2 \sim 3$ TeV and $M_3 \sim 4.5$ TeV are not excluded experimentally.

A detailed calculation of the coupling to the currents and of the partial widths is necessary in order to draw firm quantitative conclusions, but these preliminary estimates indicate that the first three resonances have $R_i \sim 0.15 - 0.35$, while $g_\rho^{(i)\,2} < 0.5$. These resonances should have a sizable branching fraction in standard-model fermions, and a sizable pro-

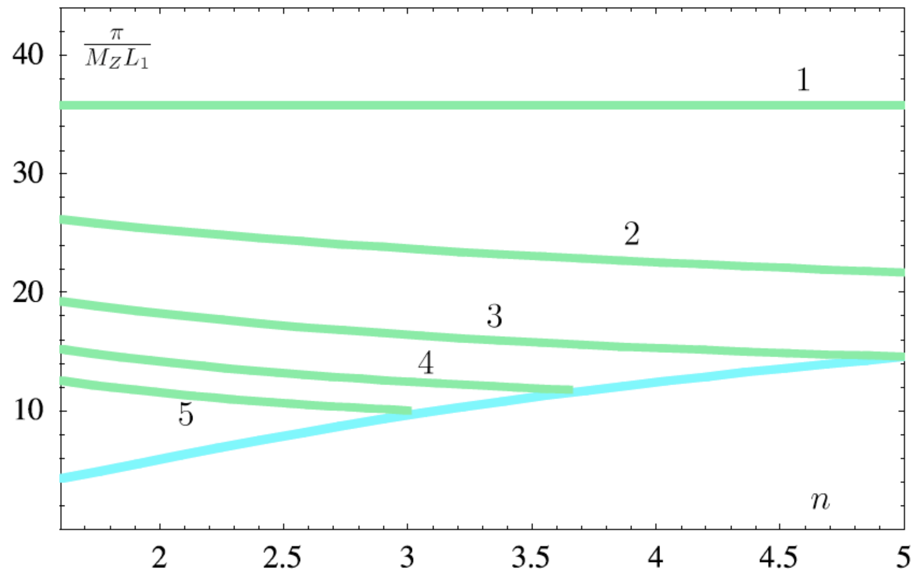

Figure 4.4: Lower bound on $\pi/(M_Z L_1)$ as a function of n in the case in which symmetry-breaking takes place at $\bar{L}$. The green curves (light grey) are obtained using $L_1/\bar{L} = 1, 2, 3, 4, 5$ The cyan curve (dark grey) is obtained by using the limiting value of $L_1/\bar{L}$ such that g_ρ be perturbative. We interrupt the (green) curves obtained at constant $L_1/\bar{L}$ at the value of n for which Eq. (4.48) would not be satisfied, which is at the intersection with the cyan curve.

duction cross-section in Drell-Yan processes. In particular, for this range of masses and couplings, LHC has a good chance of detecting all of these states even at moderate integrated luminosity, by combining data on $\mu^+\mu^-$ and e^+e^- final states.

4.5 Discussion

The starting point for the construction of an effective field theory description of dynamical electro-weak symmetry breaking is the assumption that some fundamental, possibly asymptotically free, field theory, defined in the far UV, flows towards an (approximate) strongly-coupled fixed-point in the IR. Accordingly, there is a regime at intermediate-to-low energies in which the (walking) theory can be described by a weakly-coupled five-dimensional model, in the spirit of the AdS/CFT correspondence. The presence of a deformation away from the AdS metric—in the form of some operator that becomes relevant and dominates the dynamics at long distances—drives the model away from the fixed point (inducing the loss of conformal behavior), produces non-trivial condensates (which trigger spontaneous electro-weak symmetry breaking), and ultimately leads the theory towards confinement (and hence introducing a mass gap in the spectrum of bound states).

This paper proposes a toy-model that allows for a quantitative study of the effects that such a relevant deformation might have on the low-energy observable quantities, in the regime at and below the LHC relevant energies. The basic idea is to parameterize the effects of such a deformation in terms of a power-law departure from the AdS background over a limited energy window just above the scale of confinement. This treatment proves to be useful thanks to its intrinsic simplicity and the lack of any more systematic (calculable) approach. It has its limitations as well. Hence we summarize and critically analyze our results, in order to draw some important model-independent conclusion and in order to highlight the areas where more work, and possibly some guidance from the experimental data to come, are necessary.

First of all, the type of modification of the background we propose has a very modest effect on the spectrum of composite resonances. The properties of such spectrum are still determined by the presence of a hard-wall in the IR, that acts both as a regulator and as a physical scale determining the mass gaps and spacings. It is inappropriate to believe that this model can describe accurately more than a handful of resonances, and one should be very careful when talking about resonances with large excitation number i. Yet, the model-independent message here is quite clear, and very important. While the spectrum is substantially independent of the possible presence, and type, of deformation that is driving the theory away from the fixed point in the IR, the effective couplings of the resonances, both to other resonances and to the standard model fermions, are very sensible to the departure from conformality that this deformation is introducing.

The calculation of the coupling to the currents and the estimate of the self-couplings show a large departure from the expectations based on the pure AdS case, in presence of the same regulators in the IR and in the UV. The coupling to the currents is suppressed, and the suppression in not a universal effect, but rather it is different for different resonances. The self couplings are enhanced with respect to the pure AdS case, following the four-dimensional intuition. This poses some important limitation on how long it is admissible to assume that it will take for the theory to flow from the region in proximity of the IR fixed point, where it is walking, to the new phase transition at which confinement takes place. It is very encouraging that our estimates indicate that this regime, though limited, might be long enough to allow for very sizable $O(2\text{-}4)$ effects to result, without spoiling the calculability of the effective field theory that the AdS/CFT language is supposed to provide.

The deformation responsible for the loss of conformal symmetry might or might not be related with electro-weak symmetry breaking. If not, then electro-weak symmetry breaking is triggered at the same scale as confinement, as is the case for QCD. In this case this model allows us to say that we do not expect any significant modification of the precision electro-weak parameters and of the coefficients of the electro-weak chiral Lagrangian with respect to the results obtained in the pure AdS background. In this case, the couplings of the excited states are the only observable quantities carrying information about the existence of an energy regime above the scale of confinement where the dynamics is not conformal.

At large-N_c or in presence of a complicated fermionic field content in the fundamental theory, the chiral symmetry breaking condensates may form at a temperature larger than the scale of confinement. In this case, the formation of such condensates might itself be the deformation that drives the theory away from the fixed point, and that leads to confinement at some lower scale. The phenomenological consequences of such a scenario are relevant not only for the LHC, but even in analyzing LEP and TeVatron data. Our simple model allows us to show that it is reasonable to expect that in this case the estimates of the coefficients of the chiral Lagrangian (we focused on $\hat{S}$ because best known and most model-independent) might be suppressed by large numerical factors, without entering a strongly coupled regime for the effective field theory, and with a resulting drastic reduction of the experimental bounds on the masses of the lightest new spin-1 states (techni-ρ). This toy-model highlights the fact that, whatever the fundamental theory is in the far UV, if the dynamics contains a mechanism leading to a separation of the scales of chiral symmetry breaking and confinement, then the expectations for $\hat{S}$, and for other precision parameters related with isospin breaking, can be changed drastically. At the LHC, this implies that, without requiring any additional custodial symmetry, nor any fine-tuning, the dynamics itself might be compatible with the detection of the first two or even three excited states, which would provide unmistakable evidence for a strongly-coupled origin of electro-weak symmetry breaking.

The techniques used here, and the choices of parameters we make, are affected by systematic uncertainties. The numerical results we obtain are to be taken as an indication of what is possible, rather than as robust predictions. Yet, part of the results are completely general: for any admissible choice of $L_1/\bar{L}$, of $n > 1$ and of the position in the fifth dimension at which we localize the symmetry-breaking terms, there is always a suppression of the coupling of the vector mesons to the currents, an enhancement of their self-couplings, and a suppression of $\hat{S}$. These are quantitative model-independent results, indicating that for these quantities the pure AdS case yields always a limiting, conservative estimate. And they all point in the direction of making the experimental searches at the LHC easier.

Chapter 5

Conclusions

Strongly coupled electro-weak symmetry breaking scenarios cannot be ruled out. The intrinsic complexity of their dynamical structure is at the root of their versatility, as well as a challenging problem to be solved.

An increasing number of groups have recently renewed their interest on the subject, attacking the calculability problem both by means of lattice simulations and analytical tools. Although identifying theoretically clean, measurable quantities that can help distinguish unambiguously perturbative from non-perturbative scenarios of electro-weak symmetry breaking may not be an easy task, the forthcoming experiments will give us a concrete opportunity to learn more about the mysterious world of the strong dynamics.

After years of mere speculations, particle physics will have fresh new data to work on!

Appendix A

Brane Worlds

Brane worlds [127]–[133] open a way to understand a number of long-standing problems in particle physics such as fermion mass hierarchy and the smallness of cosmological constant (see reviews [134]–[138]). The branes themselves may exhibit their specific excitations - branons which contribute into the discovery potential of high-energy colliders [139].

Theoretically brane worlds could well be created spontaneously if the bulk in extra dimensions is filled not only by gravity but also by primordial [140]-[143] or composite [144] scalar matter self-interacting so that its condensation breaks the translational invariance (in the form of a kink for one extra dimension). Such a configuration is essential also to trigger localization of fermions [127] and possibly of other matter fields [142, 143, 145, 146]. In the absence of gravity the latter localization holds perfectly also for scalar fields in the Goldstone boson sector related to spontaneous breaking of translational invariance. When matter induced gravity affects the geometry in the bulk, the scalar Goldstone mode mix strongly with scalar components of multi-dimensional gravity and can be removed by a gauge choice [147]. The physical scalar zero-mode fluctuation apparently disappears from the particle phenomenology.

The latter mechanism has been analyzed in spontaneously generated brane worlds with minimal gravitational interaction. However if both gravity and scalar fields Φ are induced by more fundamental matter fields then at low energies, from vacuum polarization effects, one recovers also a non-minimal interaction between space-time curvature R and scalar fields $\xi R\Phi^2$. This is a purpose of our work to examine the particle spectrum in interplay of gravity and a bulk scalar field, in the case of the above non-minimal interaction, when a brane world is generated spontaneously. We analyze the perturbative stability against quantum fluctuations, i.e. the absence of tachyons in the spectrum, as well as the phenomenon of (de)localization of light particles on a brane.

One can find that such a term is not in general innocuous. We show that in the case of constant scalar configurations such a term generally causes instability of the scalar field.

Contrary to the previous case, a non minimally coupled scalar matter may be perturbatively stable if its vacuum configuration is non trivial. We prove this in the particular case of brane world scenarios.

It seems to be possible to use the well established perturbative stability of the $\xi = 0$ system [147],[148] and make a conformal transformation to eliminate the ξ term from the action. Doing this, however, we introduce non analytical interactions for the scalar field. Since the theorem for the perturbative equivalence of the on-shell S-matrix is rigorously proved only in the case of analytic change of field variables, we believe that the most trustable way to face the problem is by showing explicitly the positivity of the spectrum.

Appendix A is organized as follows.

In section 1 we introduce our notation and the equations of motion (EOM) for general scalar and gravitational backgrounds. We briefly discuss the effect of the non minimal coupling for the case of constant scalar configurations and introduce the ansatz for the brane configuration.

In section 2 the linear gauge invariance is analyzed. The quadratic action in the oscillations around the background is constructed by invoking the gauge symmetry. As its consequence not all linear equations derived are independent.The decoupling procedure for the quadratic action is finally performed. We show that the gravitational field can be decoupled from the scalar oscillations after a field redefinition. Two different gauges are then chosen to simplify the analysis of the decoupled system, one of which is very convenient to study the spectrum.

In section 3 we discuss the spectrum of the brane scenario. This is composed by a tower of massive spin 2 fields (gravitons) and a tower of spin 0 fields (branons), as expected. We show that both spectra are determined completely by a single function of the warped factor and of the scalar v.e.v. The sign of this function determines the positivity of the kinetic and mass terms for both subsystems. The linearized equations are converted into a Schrödinger form and the localization of wavefunctions is discussed.

A.1 The classical background

We assume the 5D action to have the form:

$$I[g,\Phi] = \int d^5X \; \mathcal{L}(g,\Phi), \tag{A.1}$$

with

$$\mathcal{L} \;=\; \sqrt{|g|}\left\{-M_*^3 R + \partial_A\Phi\partial^A\Phi - V(\Phi) + \xi R\Phi^2\right\}. \tag{A.2}$$

A cosmological constant can be considered as englobed into the scalar potential V.

The 5D coordinates are denoted by $X^A = (x^\mu, z)$, with Greek indices $\mu, \nu, \ldots = 0, 1, 2, 3$ and Latin indices $A, B, \ldots = 0, 1, 2, 3, 5$.

The equations of motion (EOM) are:

$$R_{AB} - \frac{1}{2} g_{AB} R = \frac{1}{M_*^3} T_{AB} \tag{A.3}$$
$$D^2\Phi = -\frac{1}{2}\frac{\partial V}{\partial \Phi} + \xi R \Phi,$$

where $D^2 = D_C D^C$ and D_C is a covariant derivative. The energy momentum tensor reads

$$\begin{aligned} T_{AB} &= \partial_A \Phi \partial_B \Phi - \frac{1}{2} g_{AB} \left(\partial_C \Phi \partial^C \Phi - V(\Phi)\right) \\ &+ \xi \left(R_{AB} - \frac{1}{2} g_{AB} R + g_{AB} D^C D_C - D_A D_B\right) \Phi^2. \end{aligned} \tag{A.4}$$

The presence of a non minimal coupling governed by the parameter ξ may cause destabilization of scalar configurations [149]. This is a general characteristic of constant scalar backgrounds and does not depend on the specific geometry of space-time. To see this we make the trace of the Einstein equation to find the scalar curvature. Substituting it in the EOM for the scalar field we can find it in full generality as:

$$D^2\Phi = -U'_{eff} \tag{A.5}$$

with

$$U'_{eff} = \frac{\frac{1}{2}(-M_*^3 + \xi\Phi^2)V' + \xi\Phi\frac{d}{2-d}V + \xi\Phi\left(1 + \xi\frac{4(d-1)}{2-d}\right)\Phi_{,A}\Phi^{,A}}{-M_*^3 + \xi\Phi^2\left(1 + \xi\frac{4(d-1)}{2-d}\right)}, \tag{A.6}$$

with d being the dimensionality of space-time and V' denoting the derivative with respect to the scalar field. The effective potential U_{eff} determines the stability of a scalar configuration although it does not correspond to its physical energy.

The existence of a global minimum for U_{eff} requires this function to grow ($U'_{eff} \geq 0$) in the limit $\Phi \to +\infty$. For a constant solution

$$U'_{eff} \to \frac{\frac{1}{2}\Phi V' + \frac{d}{2-d}V}{\Phi\left(1 + \xi\frac{4(d-1)}{2-d}\right)}. \tag{A.7}$$

If we assume $V \propto \lambda\Phi^n$ with $\lambda > 0$ and $n > 2d/(d-2)$ then the numerator is always a positive quantity. A necessary condition for having a global minimum is therefore $1 + \xi\frac{4(d-1)}{2-d} > 0$. For $d = 5$ we get the condition $1 - \xi\frac{16}{3} > 0$.

In the case of non trivial scalar configurations the above method cannot be straightforwardly applied since the stability is controlled by a functional.

Let's limit ourselves to the study of background solutions which don't spontaneously break the 4D Poincarè invariance and take $g_{AB} = A^2(z)\eta_{AB}$, with $\eta_{AB} = diag(1, -1, -1, -1, -1)$, and $\Phi = \Phi(z)$. The equations of motion in terms of this ansatz read

$$\frac{1}{2}A^5\left(\frac{\Phi'^2}{A^2} - V(\Phi)\right) + 6AA'^2\left(-M_*^3 + \xi\Phi^2\right) + 4\xi A^2 A'(\Phi^2)' = 0, \tag{A.8}$$

$$-\frac{1}{2}A^5\left(\frac{\Phi'^2}{A^2} + V(\Phi)\right) + 3A^2A''\left(-M_*^3 + \xi\Phi^2\right) + 2\xi A^2 A'(\Phi^2)' + \xi A^3(\Phi^2)'' = 0, \tag{A.9}$$

$$2\left(A^3\Phi'\right)' - A^5\frac{\delta V}{\delta\Phi} + \left(16\xi A^2 A'' + 8\xi AA'^2\right)\Phi = 0 \tag{A.10}$$

where from now on $f' = df/dz$.

Since we have three equations for two unknown functions, one of the above conditions must be redundant. This is a consequence of gauge invariance and can be seen explicitly by subtracting (A.9) from (A.8) to derive an expression for the potential. Differentiating it one can recover (A.10).

We select out the solutions with a definite parity in the fifth direction in respect to $z = 0$, i.e. the potential $V(\Phi)$ is $\mathbf{Z}_2$ symmetric.

A.2 Field excitations

A.2.1 The local invariance

By construction, the action (B.1) is invariant under general diffeomorphisms. Because the space-time variable X is a dummy variable the symmetry can be seen as an invariance of the action under appropriate transformations of the fields. This transformation is the Lie derivative along an arbitrary vector ζ^A defined by the coordinate transformation $X \to \tilde{X} = X + \zeta(X)$. To the first order one finds:

$$\begin{aligned}\tilde{g}_{AB}(X) &= g_{AB}(X) - \zeta^C_{,A}\, g_{CB}(X) - \zeta^C_{,B}\, g_{AC}(X) - g_{AB,C}(X)\, \zeta^C + O(\zeta^2) \\ &= g_{AB}(X) - \zeta_{A;B} - \zeta_{B;A} + O(\zeta^2)\end{aligned} \tag{A.11}$$

where ';' denotes the covariant derivative.

Let's consider the general case of non trivial backgrounds $\bar{g}_{AB}(X)$ and define the fluctuating field $h_{AB}(X)$ as follows:

$$g_{AB}(X) \equiv \bar{g}_{AB}(X) + h_{AB}(X). \tag{A.12}$$

The Lie derivative acting on $h_{AB}(X)$ is highly non linear and maybe expanded in powers of ζ, h. Since the action of our interest is up to quadratic order in the fluctuations, we confine ourselves to the leading order:

$$\tilde{h}_{AB}(X) = h_{AB}(X) - \zeta_{A;B} - \zeta_{B;A} + O(\zeta^2, h\zeta) \tag{A.13}$$

where now the ';' denotes the covariant derivative with respect to the background $\bar{g}_{AB}(X)$.

The same line of reasoning applies to general tensors. We take the fluctuations around the solution of the EOM to be:

$$g_{AB}(X) = A^2(z)\left(\eta_{AB} + h_{AB}(X)\right); \quad \Phi(X) = \Phi(z) + \phi(X). \tag{A.14}$$

Since the 4D symmetry is unbroken we adopt the 4D notation $h_{5\mu} \equiv v_\mu$, $h_{55} \equiv S$. Introducing the notation $\hat{\zeta}_A$, with $\zeta_\mu = A^2\hat{\zeta}_\mu$ and $\zeta_5 = A\hat{\zeta}_5$ for convenience we can express our gauge symmetry as follows

$$\begin{aligned} h_{\mu\nu} &\rightarrow h_{\mu\nu} - \left(\hat{\zeta}_{\mu,\nu} + \hat{\zeta}_{\nu,\mu} - \frac{2A'}{A^2}\eta_{\mu\nu}\,\hat{\zeta}_5\right) \\ v_\mu &\rightarrow v_\mu - \left(\frac{1}{A}\hat{\zeta}_{5,\mu} + \hat{\zeta}'_\mu\right) \\ S &\rightarrow S - \frac{2}{A}\hat{\zeta}'_5 \\ \phi &\rightarrow \phi + \hat{\zeta}_5\frac{\Phi'}{A}. \end{aligned} \tag{A.15}$$

Notice that these transformations are exact up to $O(\zeta^2, h^2, h\zeta)$ terms (where h stands for an arbitrary fluctuation). The symmetry transformations leaving the solution $h_{AB} = \phi = 0$ unchanged are the isometries of the background metric. It is easy to see that they are restricted to the 4D Poincare group. This fact justifies our decomposition of the fields under $SO(1,3)$ representations.

After gauging, the unbroken 4D Poincare group implies the existence of a 4D massless spin 2 field which has to be identified as a graviton. From the above gauge transformations we expect this state to be described by z-independent fluctuations of $h_{\mu\nu}$. This is intuitively understood since the 4D space is flat and this field actually describes the space-time dependent fluctuations of $\eta_{\mu\nu}$.

Since the translations along z are spontaneously broken a Goldstone boson (GB) must appear. We expect only one scalar state because the Lorentz rotations orthogonal to z, though spontaneously broken, don't act independently on the vacuum and therefore don't generate additional massless states [150]. As usual, the GB can be identified as the space-time dependent coordinate $\zeta_5(x) = A\hat{\zeta}_5(x,z)$, where the 4D dependence ensures that its propagation is confined to the unbroken directions. However, due to the gauge nature of the symmetry, the GB is locally gauge equivalent to the zero solution,therefore it is not a physical state and must be rotated away.

We will show in the next section that once the gauge symmetry is completely fixed (the GB has been absorbed) the propagating fields will describe a tower of spin 2 and spin 0 particles.

A.2.2 Quadratic action

We now expand the action in Taylor series up to quadratic order in field fluctuations.

We consider a conformally flat space with D dimensions and write the metric as

$$g_{AB}(X) = A^2(X)\left[\eta_{AB} + h_{AB}(X)\right] \tag{A.16}$$

where the warp factor $A(X)$ is chosen to depend on all coordinates for convenience. The inverse metric and the determinant up to quadratic order in the fluctuations are

$$\begin{aligned} g^{AB} &= A^{-2}\left[\eta^{AB} - h^{AB} + h^{AC}h_C{}^B + \ldots\right]; \\ \sqrt{g} &= A^D\left[1 + \frac{1}{2}h + \frac{1}{8}h^2 - \frac{1}{4}h^{AB}h_{AB} + \ldots\right]. \end{aligned} \tag{A.17}$$

All indexes are raised by the five dimensional flat metric η_{AB}, so that $h = h_{AB}\eta^{AB}$ for example. To avoid misunderstanding, notice that throughout the text we used a four dimensional representation of the gravitational field in terms of $h_{\mu\nu}, v_\mu, S$. Therefore, the 4D trace appearing in the text is actually $h_{\mu\nu}\eta^{\mu\nu}$ and differs from the one in the Appendix because of the S field.

Armed with the above formulas one can prove that the Einstein-Hilbert term $\sqrt{g}R$ can be written, up to quadratic order in the fluctuations, as

$$\begin{aligned} & A^{2-D}\sqrt{g}R \\ = & -2(D-1)\frac{A_{,E}^{,E}}{A} - (D-1)(D-4)\frac{A^{,E}A_{,E}}{A^2} + 2(D-1)\frac{A_{,EF}}{A}h^{EF} \\ + & (D-1)(D-4)\frac{A_{,E}A_{,F}}{A^2}h^{EF} - (D-1)\frac{A_{,E}^{,E}}{A}h + 2(D-1)\frac{A_{,E}}{A}h^{EF}{}_{,F} \\ - & (D-1)\frac{A^{,E}}{A}h_{,E} - \frac{1}{2}(D-1)(D-4)\frac{A^{,E}A_{,E}}{A^2}h + h^{EF}{}_{,EF} - h_{,E}^{,E} \\ - & 2(D-1)\frac{A_{,EF}}{A}h^{EG}h_G{}^F - (D-1)(D-4)\frac{A_{,E}A_{,F}}{A^2}h^{EG}h_G{}^F + (D-1)\frac{A^{,G}}{A}h^{EF}h_{EF,G} \\ - & 2(D-1)\frac{A^{,G}}{A}h^{EF}h_{GE,F} - 2(D-1)\frac{A_{,G}}{A}h^{EG}h^F{}_{E,F} + (D-1)\frac{A_{,F}}{A}h^{EF}h_{,E} \\ + & (D-1)\frac{A_{,EF}}{A}h^{EF}h + \frac{1}{2}(D-1)(D-4)\frac{A_{,E}A_{,F}}{A^2}h^{EF}h + (D-1)\frac{A_{,E}}{A}h^{EF}{}_{,F}h \end{aligned} \tag{A.18}$$

$$
\begin{aligned}
- & \frac{D-1}{2}\frac{A^{,E}}{A}h_{,E}h - \frac{D-1}{4}\frac{A^{,E}_{,E}}{A}h^2 - \frac{1}{8}(D-1)(D-4)\frac{A^{,E}A_{,E}}{A^2}h^2 \\
+ & \frac{D-1}{2}\frac{A^{,G}_{,G}}{A}h^{EF}h_{EF} + \frac{1}{4}(D-1)(D-4)\frac{A^{,G}A_{,G}}{A^2}h^{EF}h_{EF} \\
+ & \frac{3}{4}h_{EF,G}h^{EF,G} - \frac{1}{2}h_{EF,G}h^{GF,E} + h^{EF}{}_{,F}h_{,E} - h^{EG}{}_{,E}h^{F}{}_{G,F} - \frac{1}{4}h_{,E}h^{,E} \\
+ & h^{EF}h_{,EF} + h^{EF}h^{,G}_{EF,G} - 2h_E{}^F h^{EG}{}_{,GF} + \frac{1}{2}h^{EF}{}_{,EF}h - \frac{1}{2}hh^{,E}_{,E}.
\end{aligned}
\tag{A.19}
$$

These formulas were first derived in [153].

As already discussed, the gauge invariance of the action at leading order in ζ also depends on the higher order terms in the fluctuations. At the second order in fluctuations we can get rid of the latter ones by simply imposing the equations of motion.

The transformations (A.15) can help to eliminate gauge degrees of freedom [151] . To simplify the direct calculations as much as possible we adopt the gauge $S = v_\mu = 0$ and explicitly calculate the quadratic lagrangian (A.2) using the formula (A.18) (see also [153]). For a further convenience we write it as a sum of two pieces:

$$
\mathcal{L}_{(2)}(v_\mu = S = 0) = \mathcal{L}_h + \mathcal{L}_\phi, \tag{A.20}
$$

where

$$
\begin{aligned}
& \mathcal{L}_h \\
\equiv\ & A^3\left(-M_*^3 + \xi\Phi^2\right)\left\{-\frac{1}{4}\ h_{\alpha\beta,\nu}h^{\alpha\beta,\nu} - \frac{1}{2}\ h^{\alpha\beta}_{,\beta}h_{,\alpha} + \frac{1}{2}\ h^{\alpha\nu}_{,\alpha}h^{\beta}_{\nu,\beta} + \frac{1}{4}\ h_{,\alpha}h^{,\alpha}\right\} \\
+\ & A^3\left(-M_*^3 + \xi\Phi^2\right)\left\{\frac{1}{4}h'_{\mu\nu}h'^{\mu\nu} - \frac{1}{4}h'^2\right\}
\end{aligned}
\tag{A.21}
$$

and

$$
\begin{aligned}
\mathcal{L}_\phi \ \equiv\ & A^3\phi_{,\mu}\phi^{,\mu} - A^3\phi'^2 - \frac{1}{2}\ A^5\frac{\delta^2 V}{\delta\Phi^2}(\Phi)\phi^2 \\
+\ & \left(A^3\Phi' + 8\xi A^2A'\Phi\right)h'\phi + 2\xi A^3\Phi\left(h^{\mu\nu}_{,\mu\nu} - h^{,\mu}_{,\mu} + h''\right)\phi.
\end{aligned}
\tag{A.22}
$$

The commas denote partial derivatives and all indices are raised up with the Minkowski flat metric. In particular, $h = h_{\mu\nu}\eta^{\mu\nu}$.

We can construct the quadratic action in an arbitrary gauge by invoking gauge invariance and using the iterative procedure we now present. The linear terms in the vector field can be evaluated as follows. After a gauge transformation defined by $\hat{\zeta}_\mu(x,z)$ the fields $h_{\mu\nu}$ and v_μ are changed in such a way that:

$$
\delta I = \int\left\{\frac{\delta I}{\delta h_{\mu\nu}}\left(-\hat{\zeta}_{\mu,\nu} - \hat{\zeta}_{\nu,\mu}\right) + \frac{\delta I}{\delta v_\mu}\left(\hat{\zeta}'_\mu\right) + O(\hat{\zeta}^2, hh\hat{\zeta})\right\} = 0. \tag{A.23}
$$

It is easy to see that this condition is satisfied for any $\hat{\zeta}_\mu$ only if:

$$\begin{aligned}\frac{\delta I}{\delta v^\mu}(v_\mu = S = 0) &= A^3(-M_*^3 + \xi\Phi^2)\left(h_{\mu\nu}^{,\nu} - h_{,\mu}\right)' \\ &+ \left(2A^3\Phi' + 16\xi A^2 A'\Phi\right)\phi_{,\mu} - 4\xi(A^3\Phi\phi_{,\mu})'.\end{aligned} \tag{A.24}$$

An analogous procedure is applicable when we perform a gauge transformation defined by $\hat{\zeta}_5(x,z)$. In this case the variation with respect to S can be derived if we take into account the transformation of ϕ, too. One gets,

$$\begin{aligned}\frac{\delta I}{\delta S}(v_\mu = S = 0) &= -\frac{1}{2}\, A^3\left(-M_*^3 + \xi\Phi^2\right)\left(h_{,\mu\nu}^{\mu\nu} - h_{,\mu}^{,\mu}\right) \\ &- \frac{1}{2}\left(A^3\left(-M_*^3 + \xi\Phi^2\right)\right)' h' \\ &+ (A^3\Phi'\phi)' - 2A^3\Phi'\phi' + 2\xi A^3\Phi\phi_{,\mu}^{,\mu} - 8\xi(A^2 A'\Phi\phi)' \\ &+ (16\xi A^2 A'' + 8\xi A A'^2)\Phi\phi.\end{aligned} \tag{A.25}$$

The quadratic part in S is obtained by requiring that the linear terms in S automatically cancel under a gauge transformation. This is satisfied if and only if the full quadratic action for the S field in the $v_\mu = 0$ gauge is,

$$\mathcal{L}_S \equiv -\frac{1}{4}A^5 V S^2 + \frac{\delta I}{\delta S}(v_\mu = S = 0)S. \tag{A.26}$$

With the inclusion of $\mathcal{L}_S$ to the quadratic action, the first derivative in v_μ receives additional contributions. Repeating the procedure outlined we find,

$$\begin{aligned}\frac{\delta I}{\delta v^\mu}(v_\mu = 0) &= A^3(-M_*^3 + \xi\Phi^2)\left(h_{\mu\nu}^{,\nu} - h_{,\mu}\right)' \\ &+ \left(2A^3\Phi' + 16\xi A^2 A'\Phi\right)\phi_{,\mu} - 4\xi(A^3\Phi\phi_{,\mu})' \\ &- \left[A^3(-M_*^3 + \xi\Phi^2)\right]' S_{,\mu}.\end{aligned} \tag{A.27}$$

From this latter we derive the quadratic action in v_μ:

$$\mathcal{L}_V \equiv \frac{1}{4}A^3\left(-M_*^3 + \xi\Phi^2\right) v_{\mu\nu}v^{\mu\nu} + \frac{\delta I}{\delta v^\mu}(v_\mu = 0)v^\mu, \tag{A.28}$$

where $v_{\mu\nu} = v_{\mu,\nu} - v_{\nu,\mu}$.

The full action to the quadratic order represents finally the sum of (A.22), (B.6), (A.26) and (A.28),

$$\mathcal{L}_{(2)} = \mathcal{L}_h + \mathcal{L}_\phi + \mathcal{L}_S + \mathcal{L}_V. \tag{A.29}$$

This result can be explicitly checked exploiting the formulas in Appendix A, but we stress that its form is completely fixed by gauge invariance. As we have seen, gauge invariance implies that the linearized equations are not all independent. This point will be very useful when solving the coupled mass eigenvalue equations. It is also important to emphasize that the gauge conditions can be imposed already at the lagrangian level if the secondary constraints are also taken into account.

A.3 Decoupling

The physical spectrum can be identified after the system being completely decoupled. It can be achieved by a redefinition of the gravitational field. We will see that the particle spectrum of these models comprise a tower of spin 0 and spin 2 fields.

To unravel completely the quadratic action one has to shift $h_{\mu\nu}$ by a special solution of its equation of motion,

$$\begin{aligned}
&\frac{1}{2}A^3\left(-M_*^3+\xi\Phi^2\right)\left\{\, h^{;\mu}_{\alpha\beta,\mu}-\ h^{;\mu}_{\alpha\mu,\beta}-\ h^{;\mu}_{\beta\mu,\alpha}+h_{,\alpha\beta}+\ \eta_{\alpha\beta}h^{\mu\nu}_{,\mu\nu}-\ \eta_{\alpha\beta}h^{;\mu}_{,\mu}\right\} \qquad \text{(A.30)}\\
&-\frac{1}{2}\left[A^3\left(-M_*^3+\xi\Phi^2\right)\left(h'_{\alpha\beta}-\eta_{\alpha\beta}h'\right)\right]'+\frac{1}{2}\left[A^3\left(-M_*^3+\xi\Phi^2\right)\left(v_{\alpha,\beta}+v_{\beta,\alpha}-2\eta_{\alpha\beta}v^{;\mu}_{\mu}\right)\right]'\\
&-\eta_{\alpha\beta}\left[A^3\Phi'\phi+8\xi A^2A'\Phi\phi\right]'+\ 2\xi A^3\Phi(\phi_{,\alpha\beta}-\eta_{\alpha\beta}\phi^{;\mu}_{,\mu})+2\xi\eta_{\alpha\beta}(A^3\Phi\phi)''\\
&-\frac{1}{2}A^3\left(-M_*^3+\xi\Phi^2\right)\left(S_{,\alpha\beta}-\eta_{\alpha\beta}S^{;\mu}_{,\mu}\right)+\frac{1}{2}\eta_{\alpha\beta}\left\{\left[A^3\left(-M_*^3+\xi\Phi^2\right)\right]'S\right\}'=0.
\end{aligned}$$

This solution can be written in terms of two scalars E, ψ and a vector F_μ,

$$h_{\mu\nu}\to h_{\mu\nu}+F_{\mu,\nu}+F_{\nu,\mu}+E_{,\mu\nu}+\eta_{\mu\nu}\psi \qquad \text{(A.31)}$$

for $F'_\mu=v_\mu$ and:

$$\begin{aligned}
\Xi\psi-\frac{1}{2}\Xi S-\frac{1}{2}(\Xi E')'+2\xi A^3\Phi\phi=0 \qquad \text{(A.32)}\\
3\Xi\psi'+\Xi' S=2A^3\Phi'\phi+16\xi A^2A'\Phi\phi-4\xi(A^3\Phi\phi)',
\end{aligned}$$

where the convenient notation $\Xi=A^3\left(-M_*^3+\xi\Phi^2\right)$ has been introduced. The first(second) condition follows from the off diagonal (diagonal) terms of equation (A.30).

Substituting the redefined tensor field in (A.29) one can rewrite the lagrangian as the sum of a tensorial contribution ($\mathcal{L}_{grav}$) and an action containing the scalar fields S,ϕ,E,ψ and v_μ. Because of the above constraints (A.32) only two scalars out of four are independent and, making use of the gauge freedom defined by $\hat{\zeta}_5$, we conclude that only one of them actually describes a propagating degree of freedom. The vector v_μ is a $\hat{\zeta}_\mu$-gauge variable and can be rotated away.

After the decoupling conditions are imposed we can choose an arbitrary gauge to simplify the analysis of the spectrum. In the light-cone gauge the result should agree with [152]. Here we decide to work with $v_\mu = 0$. This choice leaves a residual freedom parametrized by $\hat{\zeta}_5$ and $\hat{\zeta}_\mu$ with $A\hat{\zeta}'_\mu + \hat{\zeta}_{5,\mu} = 0$, see (A.15).

Now let's examine two simple $\hat{\zeta}_5$-gauge choices. The first one is defined by setting S to be an appropriate function of ϕ which implies $\psi = 0$. In this case the physical branon is given by a ϕ field and the analysis of the constant Φ solutions turns out to be easy. To study the non trivial v.e.v. case (brane solutions) the most convenient choice is $\phi = 0$. In this case the branon field is described by ψ.

General scalar background in $\psi = 0$ gauge

Setting $\psi = 0$ we can interpret the second equation (A.32) as a gauge choice on S. The conditions (A.32) define S and E as non local functions of ϕ. The physical branon turns out to be described by ϕ.

By inserting our shifted tensor field in the quadratic action the linear terms in the physical $h_{\mu\nu}$ are canceled by construction. Moreover, all contributions containing the field E automatically cancel because of the above conditions (A.32). The quadratic action can therefore be written as the sum of the graviton contribution plus a scalar part describing the physical branon:

$$\begin{aligned} &\mathcal{L}_{grav} \\ = \quad & A^3\left(-M_*^3+\xi\Phi^2\right)\left\{-\frac{1}{4}\, h_{\alpha\beta,\nu}h^{\alpha\beta,\nu} - \frac{1}{2}\, h^{\alpha\beta}_{,\beta}h_{,\alpha} + \frac{1}{2}\, h^{\alpha\nu}_{,\alpha}h^{\beta}_{\nu,\beta} + \frac{1}{4}\, h_{,\alpha}h^{,\alpha}\right\} \\ + \quad & A^3\left(-M_*^3+\xi\Phi^2\right)\left\{\frac{1}{4}h'_{\mu\nu}h'^{\mu\nu} - \frac{1}{4}h'^2\right\} \end{aligned} \tag{A.33}$$

and

$$\begin{aligned} &\mathcal{L}_{bran} \\ = \quad & A^3\phi_{,\mu}\phi^{,\mu} - A^3\phi'^2 - \frac{1}{2}\,A^5\frac{\delta^2 V}{\delta\Phi^2}(\Phi)\phi^2 + (8\xi A^2A'' + 4\xi AA'^2)\phi^2 - \frac{1}{4}A^5V(\Phi)S^2 \\ + \quad & \left\{(A^3\Phi'\phi)' - 2A^3\Phi'\phi' + 2\xi A^3\Phi\phi^{;\mu}_{,\mu} - 8\xi(A^2A'\Phi\phi)' + (16\xi A^2A'' + 8\xi AA'^2)\Phi\phi\right\}S. \end{aligned} \tag{A.34}$$

The residual gauge invariance defined by $\hat{\zeta}'_\mu = 0$ will be fixed in the next section.

The gravitational field is now completely decoupled from the scalar degrees of freedom. Expressing S in terms of the field ϕ and integrating by parts we get:

$$\mathcal{L}_{bran} = \tilde{A}^3\left\{\phi_{,\mu}\phi^{,\mu} - \phi'^2 - \tilde{U}\phi^2\right\}. \tag{A.35}$$

The explicit expressions are rather complicated and read:

$$\tilde{A}^3 = A^3 - \frac{2\xi}{\Xi'^2}\left\{\left[A^6(\Phi^2)' + 16\xi A^5 A'\Phi^2\right]\Xi' - 2\xi A^6\Phi^2\Xi''\right\} \tag{A.36}$$

and

$$\begin{aligned}\tilde{A}^3\tilde{U} &= A^3\left[\frac{1}{2}A^2V'' - 8\xi H' - 12\xi H^2\right] \\ &- \frac{A^6}{\Xi'}\left[(1-2\xi)\Phi' + 2\xi H\Phi\right]\left\{A^2V' - \frac{A^5V}{\Xi'}\left[(1-2\xi)\Phi' + 2\xi H\Phi\right] - 24\xi H^2\Phi - 16\xi H\Phi'\right\} \\ &+ \left\{2\xi\frac{A^{11}V}{\Xi'^2}\Phi\left[(1-2\xi)\Phi' + 2\xi H\Phi\right] - \xi\frac{A^8V'}{\Xi'}\Phi\right. \\ &\left.-\frac{A^6}{\Xi'}\left[(1-2\xi)\Phi'^2 - 8\xi^2H^2\Phi^2 + (10\xi - 32\xi^2)H\Phi\Phi'\right]\right\}'.\end{aligned} \tag{A.37}$$

In the above definitions the expression V' denotes the variation with respect to the field Φ. The quantities $H = A'/A$ and $\Xi = A^3\left(-M_*^3 + \xi\Phi^2\right)$ have been used for brevity and the relation $\Xi'' = A^5V$ has also been employed.

Rescaling the field $\phi = \tilde{A}^{-3/2}\Psi$ we obtain the standard form:

$$\mathcal{L}_{bran} = \Psi_{,\mu}\Psi^{,\mu} - \Psi'^2 - U\Psi^2 \tag{A.38}$$

with

$$U = \frac{3}{4}\frac{\tilde{A}'^2}{\tilde{A}^2} + \frac{3}{2}\frac{\tilde{A}''}{\tilde{A}} + \tilde{U}. \tag{A.39}$$

The stability of the configuration is not manifest and will be analyzed later on. For the moment we study two simple limits of the potential: the case $\xi = 0$ and $\Phi' = 0$.

For $\xi = 0$ we have $\hat{A}^3 \to A^3$, $\Xi \to -M_*^3 A^3$ and

$$\tilde{A}^3\tilde{U} \to A^3\left\{\frac{1}{2}A^2V'' + \frac{1}{3HM_*^3}\left[A^2V'\Phi' + \frac{1}{3}\frac{A^2V}{HM_*^3}\Phi'^2\right] + \frac{1}{3A^3}\left[\frac{A^3\Phi'^2}{HM_*^3}\right]'\right\}. \tag{A.40}$$

Making use of the EOM we get:

$$U \to \frac{\Omega''}{\Omega}, \quad \Omega = \frac{A^{3/2}\Phi'}{2H}. \tag{A.41}$$

This result agrees with [147]. In the next section we will derive the generalization of this expression for the case of arbitrary ξ.

Although physically less interesting, the case $\Phi' = 0$ reveals some general feature. We make use of the approximation $\xi\Phi^2 \ll M_*^3$ to simplify the result. It is then straightforward to evaluate the potential:

$$U \to \frac{15}{4}H^2\left(1 - \frac{16}{3}\xi\right) + \frac{1}{2}A^2V'' + O\left(\xi\frac{\Phi^2}{M_*^3}\right). \tag{A.42}$$

A sufficient condition for stability is that Φ be a local minimum of the potential V and $\xi < 3/16$. For ξ sufficiently large we see that the flat geometry $A' = 0$ is favored. This represents the local version of the result obtained in section II.

Non-trivial scalar background in $\phi = 0$ gauge

We now turn to the study of the physical spectrum. A simple check in the EOM reveals two possible solutions for the scalar v.e.v.: a trivial one and a non trivial one. The latter condition in particular requires that $\Phi' = 0$ only in isolated points. The gauge $\phi = 0$ can be satisfied almost everywhere. We now show that the propagating graviton field is described by ψ.

By inserting our shifted tensor field and setting $\phi = 0$, the action simplifies considerably and can be written as the sum of the graviton contribution plus a scalar part:

$$\mathcal{L}_{(2)}(v_\mu = \phi = 0) = \mathcal{L}_h + \mathcal{L}_S = \mathcal{L}_{grav} + \mathcal{L}_{bran} \tag{A.43}$$

where

$$\begin{aligned} \mathcal{L}_{grav} &= \Xi\left\{-\frac{1}{4}\ h_{\alpha\beta,\nu}h^{\alpha\beta,\nu} - \frac{1}{2}\ h^{\alpha\beta}_{,\beta}h_{,\alpha} + \frac{1}{2}\ h^{\alpha\nu}_{,\alpha}h^{\beta}_{\nu,\beta} + \frac{1}{4}\ h_{,\alpha}h^{,\alpha}\right\} \\ &+ \Xi\left\{\frac{1}{4}h'_{\mu\nu}h'^{\mu\nu} - \frac{1}{4}h'^2\right\} \end{aligned} \tag{A.44}$$

and

$$\mathcal{L}_{bran} = \frac{3}{2}\Xi\psi_{,\mu}\psi^{,\mu} - 3\Xi\psi'^2 + \frac{3}{2}\Xi\psi^{,\mu}_{,\mu}S - 2\Xi'\psi'S - \frac{1}{4}\Xi''S^2. \tag{A.45}$$

The first two terms in the latter expression come from the quadratic part in the tensor field ($\mathcal{L}_h$) while the last three from the terms linear in S ($\mathcal{L}_S$). The identity $\Xi'' = A^5V$ has also been used.

Expressing S in terms of the derivative of the field ψ, exploiting (A.32) and integrating by parts in the 4D variables we get:

$$\mathcal{L}_{bran} = \hat{\Omega}^2\left\{\psi_{,\mu}\psi^{,\mu} - \psi'^2\right\} \tag{A.46}$$

where

$$\hat{\Omega}^2 = -3\Xi + \frac{9}{4}\frac{\Xi''\Xi^2}{\Xi'^2} = A^3\left(\frac{3}{2}\frac{\Xi\Phi'}{\Xi'}\right)^2 - 3\Xi\left(\xi A^3\frac{(\Phi^2)'}{\Xi'}\right)^2. \tag{A.47}$$

In the last equality the EOM have been employed.

By studying the linearized equations in the $\phi = 0$ gauge one can actually find a decoupled condition for S and realize that its solution is a divergent function. In our notation its divergence can be traced back to the relation $S \sim \Xi\psi'/\Xi'$ following from (A.32). Since the physical state ψ is delta function normalizable, S will diverge as $1/H \sim z$ for asymptotically constant scalar configurations. The field S cannot be projected out, as it was assumed in [154], because it is not an independent configuration. Therefore the conclusion of [154] on the necessity of having conformal matter on the brane is not well justified.

Introducing the rescaled field $\psi = \hat{\Omega}^{-1}\Psi$ we obtain:

$$\mathcal{L}_{bran} = \Psi_{,\mu}\Psi^{,\mu} - \Psi'^2 - U\Psi^2 \tag{A.48}$$

where

$$U = \frac{\hat{\Omega}''}{\hat{\Omega}} \tag{A.49}$$

is forced by gauge invariance to coincide with (A.39). It is easy to verify it in the case $\xi = 0$ ($\hat{\Omega} \to \Omega$) while for the case $\Phi' = 0$ is not trivial.

Note

In order to better understand the above results we briefly discuss the degrees of freedom involved. It is convenient to decompose the 15 gravitational fields in terms of its traceless-transverse tensor, vectors and scalar components:

$$h_{\mu\nu} = b_{\mu\nu} + f_{\mu,\nu} + f_{\nu,\mu} + E_{,\mu\nu} + \eta_{\mu\nu}\psi \tag{A.50}$$

where $b_{\mu\nu}$ and f_μ satisfy $b_{\mu\nu}^{;\mu} = b = 0 = f_\mu^{;\mu}$. The h_{5A} fields are still denoted as v_μ and S.

Substituting this form in the full quadratic action one can recognize f_μ and E as auxiliary fields. E is in fact a Lagrange multiplier and gives rise to a constraint which is the second equation of (A.32). Gauge invariance requires this latter to be equivalent to the condition $\frac{\delta I}{\delta v^\mu}(v_\nu = 0) = 0$, indicating that the graviphoton appears in the quadratic action only via its kinetic term. The vector f_μ turns out not to be coupled to any field. Its integration does not lead to interesting relations and we will simply ignore it in this discussion.

The above mentioned constraint is of extreme importance because it involves the three scalars ψ, S and ϕ implying that only two of them are independent. Since the field S contains no kinetic term while ψ does, it may be natural to choose ψ and ϕ as the independent variables. The initial 15 + 1 scalar degrees of freedom are now reduced to a traceless and transverse field $b_{\mu\nu}$, a vector v_μ and two scalars ψ, ϕ.

The physical degrees of freedom can be identified after the gauge is completely fixed. For a non compact extra dimension the vector field v_μ can be completely gauged away by an appropriate choice of $\hat{\zeta}_\mu$. The residual gauge symmetry depends on $\hat{\zeta}_5$ and acts on the independent scalar components as

$$\begin{aligned} \psi &\rightarrow \psi + \frac{2A'}{A^2}\hat{\zeta}_5 \\ \phi &\rightarrow \phi + \hat{\zeta}_5\frac{\Phi'}{A}. \end{aligned} \tag{A.51}$$

A gauge dependent combination of ψ and ϕ can then be eaten and the orthogonal combination can be chosen to be the gauge invariant field

$$\Psi \propto \phi - \frac{A\Phi'}{2A'}\psi. \tag{A.52}$$

This is the natural candidate to describe the branon [147]. In the previous subsections we have shown the explicit form of the quadratic action for this field.

A.3.1 Spin 2 fields

Ξ determines the sign of the quadratic Hamiltonian for both branons and gravitons. In order to have a positive definite quadratic energy we require $\Xi < 0$. The opposite sign may indicate a breaking of our semiclassical analysis and it would necessitate a quantum gravity justification. We therefore assume from here on that $\xi\Phi^2 \ll M_*^3$.

Introducing the rescaled field $h_{\mu\nu} = (-\Xi)^{-1/2}\sqrt{2}b_{\mu\nu}$ we obtain:

$$\begin{aligned} \mathcal{L}_{grav} &= \frac{1}{2}\, b_{\alpha\beta,\nu}b^{\alpha\beta,\nu} + b^{\alpha\beta}_{,\beta}b_{,\alpha} - b^{\alpha\nu}_{,\alpha}b^{\beta}_{\nu,\beta} - \frac{1}{2}\, b_{,\alpha}b^{,\alpha} \\ &- \left\{\frac{1}{2}\left(b'_{\mu\nu}b'^{\mu\nu} + Wb_{\mu\nu}b^{\mu\nu}\right) - \frac{1}{2}\left(b'^2 + Wb^2\right)\right\} \end{aligned} \tag{A.53}$$

with

$$W = \frac{1}{2}\frac{\Xi''}{\Xi} - \frac{1}{4}\frac{\Xi'^2}{\Xi^2} = K^2 - K', \quad K = -\frac{1}{2}\frac{\Xi'}{\Xi}. \tag{A.54}$$

The action can be put in standard form if we define

$$b_{\mu\nu}(X) = \sum_m b^{(m)}_{\mu\nu}(x)b_m(z) \tag{A.55}$$

where the wavefunctions $b_m(z)$ satisfy the eigenvalue equation

$$-b_m'' + Wb_m = (-\partial_z + K)(\partial_z + K)b_m = m^2 b_m \qquad (A.56)$$
$$\int dz b_m b_{m'} = \delta_{m,m'},$$

with manifestly positive masses.

The zero mode is:

$$b_0(z) = C_{(1)}\Xi(z)^{1/2} + C_{(2)}\Xi(z)^{1/2} \int^z dz' \frac{1}{\Xi(z')} \qquad (A.57)$$

and it is a physical state if its wave function is normalizable. In the semi-classical approximation $\xi\Phi^2 \ll M_*^3$ we can analyze the convergence by studying the regularity and the asymptotic behavior of A^3. It is easy to see that the integral (A.56) can be convergent only if one of the two constants $C_{(1,2)}$ is zero.

In the above basis the 5D bulk dynamics can be integrated leaving an effective 4D action describing a tower of spin 2 massive states whose quadratic action reads:

$$\begin{aligned} & I_{grav} \qquad (A.58) \\ = & \sum_m \int d^4x \left\{ \left(\frac{1}{2}\, b^{(m)}{}_{\alpha\beta,\nu} b_{(m)}{}^{\alpha\beta,\nu} + \ b_{(m)}{}^{\alpha\beta}_{,\beta} b^{(m)}{}_{,\alpha} - \ b_{(m)}{}^{\alpha\nu}_{,\alpha} b_{(m)}{}^{\beta}_{\nu,\beta} - \frac{1}{2}\, b^{(m)}{}_{,\alpha} b_{(m)}{}^{,\alpha} \right) \right. \\ & \left. - \frac{m^2}{2} \left(b^{(m)}{}_{\mu\nu} b_{(m)}{}^{\mu\nu} - b_{(m)}{}^2 \right) \right\}. \end{aligned}$$

As already discussed, the propagating fields are transverse-traceless as appropriate for massive spin 2 states. The remnant gauge freedom acting on it as $b^0_{\mu\nu} \to b^0_{\mu\nu} - \hat{\zeta}_{\mu,\nu} - \hat{\zeta}_{\nu,\mu}$ with $\zeta'_\mu = 0$ represents the usual gauge symmetry of the 4D graviton field. It is defined by a transverse $\hat{\zeta}_\mu(x)$ satisfying the free scalar field EOM. This is exactly what is needed in order to further reduce the d.o.f. of the on-shell graviton by three units.

A.3.2 Spin 0 fields

The lagrangian describing the spin 0 field is:

$$\mathcal{L}_{bran} = \Psi_{,\mu}\Psi^{,\mu} - \Psi'^2 - U\Psi^2 \qquad (A.59)$$

where

$$U = \frac{\hat{\Omega}''}{\hat{\Omega}} = J^2 - J', \quad J = -\frac{\hat{\Omega}'}{\hat{\Omega}}. \qquad (A.60)$$

This result explicitly shows the positivity of the scalar spectrum.

We expand Ψ in an appropriate basis:

$$\Psi(X) = \sum_m \psi^{(m)}(x)\Psi_m(z) \tag{A.61}$$

with

$$\begin{aligned} -\Psi''_m + U\Psi_m &= (-\partial_z + J)(\partial_z + J)\Psi_m = m^2\Psi_m \\ \int dz \Psi_m \Psi_{m'} &= \delta_{m,m'}. \end{aligned} \tag{A.62}$$

The lightest state is the zero mode:

$$\Psi_0(z) = D_{(1)}\hat{\Omega} + D_{(2)}\hat{\Omega}\int^z dz' \frac{1}{\hat{\Omega}^2}. \tag{A.63}$$

Again, the integration constants $D_{(1)}, D_{(2)}$ cannot be both different from zero. One can easily see from the EOM that the asymptotic form of $\hat{\Omega}$ is the same as that of $A^{3/2}$. We conclude that once the graviton solution has been chosen ($C_{(1)} = 0$ or $C_{(2)} = 0$) the scalar zero mode with a well defined limit at infinity is unique ($D_{(1)} = 0$ or $D_{(2)} = 0$). But, in fact, the two solutions $D_{1,2}$ cannot satisfy simultaneously the convergence at zero and infinity, each of them can satisfy one of the requirements if a smooth limit for $A(z)$ is required. To see this we follow [155] and expand our solution near $z = 0$:

$$\begin{aligned} A &= 1 + az^\alpha + \cdots \\ \Phi &= b + cz^\beta + \cdots \end{aligned} \tag{A.64}$$

where $\beta > 0$ guarantees a smooth $z = 0$ limit for the graviton wavefunction. Imposing the EOM we find $\alpha = 2\beta$ and $\hat{\Omega}^2 \sim 1/z^\alpha$. The mode (A.63) is singular at the origin and must be projected out from the dynamics of the model. The absence of the scalar zero mode was first noticed in [147] (see also [156]).

In order to determine the existence of a mass gap we have to study the asymptotics of $A^3 \sim \hat{\Omega}^2$. We again follow [155] and notice that, for any power law $A \sim z^{-\gamma}$ with $\gamma > 0$, the scalar potential is $U \sim 1/z^2$ and the spectrum turns out to be a continuous starting at zero. For an exponential behavior of A we may have a mass gap. A reasoning to fix the asymptotics of the warp factor may be the requirement of having non singular curvature invariants at any point in the fifth dimension [155]. Assuming this, one has to rule out exponentially varying $A(z)$ and confine the study to power laws. The spectrum which follows is continuous and ranges from zero to infinity.

The massive states behave as a linear combination of regular and irregular Bessel's functions at infinity. Near the origin the potential effectively acts as a repulsive centrifugal force $U \sim \omega^2/z^2$ expelling the wavefunctions of the massive spectrum. They behave like regular Bessel's functions on the brane. This delocalization effect is valid for any value of the 5D Planck scale, namely U does not depend on M_*.

This result poses serious problems concerning the validity of the analysis. In particular, the fact that the scalar perturbations are found to be suppressed at the location of the defect is opposed to what happens in the absence of gravity, where the GB of translational invariance is found to be peaked on it. How is it possible to reconcile our results with the well know results in flat space? How is the long range force mediated by the GB recovered as the Planck mass is sent to infinity? This topic is technically subtle and requires a detail study. This is the subject of the next chapter.

Appendix B

Are brane-excitations really massless?

The spontaneous breaking of continuous symmetries implies the existence of Nambu-Goldstone modes as relevant field coordinates for the low energy dynamics. If the symmetry is gauged a massive representation emerges in place of them due to the Higgs phenomenon. The mass scale generated by this mechanism may represent an important threshold for the IR theory, as the Fermi coupling in the weak interactions.

An analog consideration holds for the case of spontaneous breaking of translational invariance. If the spontaneous breaking refers to a compact coordinate a Higgs phenomenon is expected to involve the graviphotons [130]. A rigorous analysis of the non-linear realization of the space-time symmetries shows that the Nambu-Goldstone boson kinetic term (the Nambu-Goto action) does not provide any mass term for the graviphoton [35], as naively expected by analogy with internal gauge theories. The mass term arises from additional operators that one can build out of the relevant IR variables, and that are essential for the self-consistency of the description since they encode the presence of the symmetry breaking ensuring continuity of the observable as gravity is switched off. In this phenomenological approach the mass of the graviphotons cannot be rigorously connected to other fundamental scales of the theory (such as the brane tension and the 4d Planck mass) and remains a free parameter.

The main purpose of the paper is to analyze the emerging of this gravity-induced scale in an exactly tractable framework. We will study a class of gravitational models in which a scalar field develops a non-trivial background along a single space-like direction. Without loss of generality we consider the 5d lagrangian

$$\mathcal{L} = \sqrt{-g}\left(-M^3 R + g^{AB}\partial_A\Phi\partial_B\Phi - V(\Phi) + \ldots\right) \tag{B.1}$$

where the dots refer to some unspecified theory coupled to the scalar Φ (and gravity).

The 5-dimensional coordinates are denoted as x^μ, y, with $A, B = 0, 1, 2, 3, 5$, Greek indeces referring to the unbroken coordinates, while y denotes the broken spacial direction. The metric signature is "mostly minus".

The y-dependent vacuum expectation value $\langle\Phi\rangle \equiv \Phi_0(y)$ is assumed to trigger the localization of light modes of the above unspecified theory around some point of the y-direction, which we conventionally choose as the origin. This effectively realizes a brane world with a non-trivial dynamics trapped on it. In order to keep our discussion as general as possible we will not write down any explicit function Φ_0, nor any action for the would-be localized fields. We simply stress that natural candidates for Φ_0 are the kink shape proposed in [116] (in this case the dots in (B.1) would stand for fermionic fields), or one of the scalars described in [37] (in this case the dots in (B.1) would stand for gauge fields).

For simplicity the background geometry preserves the $4d$-Poincare symmetry

$$ds^2 = a^2\eta_{\mu\nu}dx^\mu dx^\nu - dy^2, \tag{B.2}$$

where $a = e^A$ and $A = A(y)$. The equations of motion derived from (B.1) reduce to the following independent conditions

$$\begin{aligned} \Phi_0'^2 &= -3M^3A'' \\ V(\Phi_0) &= -3M^3(4A'^2 + A''), \end{aligned} \tag{B.3}$$

where $f' \equiv \partial f/\partial y$. We see that the second equation (B.3) requires $A'' \leq 0$, this being a general consequence of the weak energy condition [96]. Such a constraint implies that no regular solution of the equations of motion is admitted on a circle (we are restricting our analysis to two-derivatives theories). Being interested in the study of translationally invariant theories, we are forced to consider a non-compact extra dimension y.

This, by itself, may not represent a serious problem for our study because the authors of [133] succeeded in making sense of these theories, at least for what concerns the spin-2 sector[1]. Similarly, we will be able to obtain an effective description of the brane fluctuations. Another feature of non-compact extra dimensions is more subtle: the graviphotons are not dynamical fields. This raises up another question: what about the gravity-induced scale if the Higgs mechanism is not canonically realized? An answer to this question is provided by the analysis of [152]. These authors studied in detail the scalar kink background and showed that continuity of physical quantities is ensured by the presence of a massive resonance in the scalar sector.

We will show that this resonance is a frame-dependent state and that the main features of the coupled scalar-gravity system can be captured by a simple model analog to the ones presented in [49]. Under some simplifying assumptions (in this set up the requirement

[1]These theories are expected to provide a sensible effective description of the brane physics even though an apparent inconsistency arises in the evaluation of the self-couplings [40, 133].

$k < w$, where k is the curvature scale and w is the inverse brane width, is not an option), we will be able to extract an explicit expression for the resonant propagator in the longitudinal and axial gauges. In these gauges the resonance couplings to brane localized currents are those of the zero mode of the global theory, except for corrections of order $O(k/w)$. In the realistic limit $k \ll w$, and at energies below the scale w, the resonance represents the only relevant deformation of the brane, very much like the zero mode of the gravity-free model.

In this class of models the resonance mass m_R plays an analog role as the graviphoton mass in compact extra dimensions. For energies much larger than m_R one recovers the Nambu-Goldstone boson dynamics via a generalization of the equivalence theorem. In this regime a non-gravitational description would be a good approximation of the full theory. At scales much smaller, however, the information about the spontaneous brane generation is encoded in non-renormalizable operators suppressed by the scale m_R and thus, for large m_R, it may become more difficult to probe.

In the following we will analyze in detail the physics of the brane fluctuations (the Φ excitations) and their relevance on the brane-localized dynamics, with and without gravity. Even though the class of backgrounds considered here are to some extent special, as we have seen, we believe they can shed light on the effect of the gauging of space-time symmetries in more general scenarios.

B.1 The linearized theory

B.1.1 The scalar sector without gravity

In order to render the paper self-consistent we review some of the basic properties of the non-gravitational theory. Without loss of generality we consider the potential $V = (\delta_\Phi W)^2$, with W an arbitrary function of Φ and a δ_Φ indicating derivative with respect to the field. The system admits degenerate constant vacua which, by convention, have zero energy. These solutions have $\delta_\Phi W = 0$, and consequently $\delta^2_\Phi V = 2(\delta^2_\Phi W)^2 \geq 0$, while non-trivial solutions satisfy $\Phi_0' = \delta_\Phi W$.

We will quantize our theory on the background $\Phi_0(y)$ which, as anticipated in the introduction, is supposed to trigger the formation of a brane at $y = 0$. A prototypical example which will be used as a reference is the kink background

$$\Phi_0 = v \tanh(wy) \tag{B.4}$$

which follows from

$$W = wv\Phi\left(1 - \frac{\Phi^2}{3v^2}\right). \tag{B.5}$$

The spectrum of the field Φ is obtained by studying the linearized equation for the fluctuation $\phi \equiv \Phi - \Phi_0$, which is easily found to be

$$\left(\partial_\mu \partial^\mu - \partial_y^2 + \frac{\Phi_0'''}{\Phi_0'}\right) \phi = 0, \tag{B.6}$$

where because of the y dependence of the background the relation $\delta_\Phi^2 V(\Phi_0) = \Phi_0'''/\Phi_0'$ holds. The eigenmodes $\phi_m(y)$ are solutions of eq. (B.6) provided we replace $\partial_\mu \partial^\mu \to -m^2$. They form a complete set in the space of square integrable functions and can be used to expand in Kaluza-Klein modes the 5d field $\phi(x, y) = \sum_m \phi_m(y) Q_m(x)$ and write an effective 4d lagrangian:

$$\mathcal{L}_{4d} = \sum_m N_m (\partial_\mu Q_m \partial^\mu Q_m - m^2 Q_m^2), \tag{B.7}$$

where $N_m = \int dy\, \phi_m^2$.

The spectrum of the brane fluctuation generally contains a zero mode Q_0 with wave-function $\phi_0 \propto \Phi_0'$ (which we take to be normalizable), possible discrete eigenvalues, and a continuum starting at a threshold defined by the potential $\delta_\Phi^2 V$. The interacting terms are obtained as usual from the convolutions of the 5d profiles and, after the fields Q_m have been properly normalized, they turn out to depend on inverse powers of the normalizations N_m.

An important comment is in order. In the absence of gravity the system is truly translational invariant. A global shift $y \to y + \epsilon$ changes the non-trivial background configuration $\Phi_0(y)$ into a new vacuum solution. Promoting the parameter to a 4d field $\epsilon(x)$ we can identify it with the Nambu-Goldstone mode. Although the Lorentz rotations orthogonal to y are spontaneously broken, as well, they do not act independently on the vacuum [42] so that a single massless mode is predicted. Because of its definition, the Nambu-Goldstone boson cannot be found as a dynamical mode in the linearized approach. At infinitesimal level in the symmetry transformation we see that it coincides with the zero mode $Q_0(x)$, ensuring its mass is exactly zero. But at a non-linear level this is no more true, and all of the Q_m have a non-trivial overlap with it. Hence the fields Q_m are seen to acquire a potential, while the Nambu-Goldstone boson is expected to interact only through derivative couplings. The strength of these couplings follows from the Nambu-Goldstone boson normalization, which also coincides with N_0,

$$\int dy\, \Phi_0'^2 = \int dy\, W' = \Delta W, \tag{B.8}$$

where we defined the topological charge $\Delta W = W(+\infty) - W(-\infty)$. This quantity measures the energy density of the dimensionally reduced system:

$$\rho[\Phi_0] = \int dy \left(\Phi_0'^2 + V\right) = 2\Delta W, \tag{B.9}$$

i.e. the brane tension.

A description of the dimensionally reduced theory can be obtained using an effective approach that makes no reference to the physics responsible for the generation of the defect [43]. This is a very powerful approach if we ignore the dynamics responsible for the brane generation, but it cannot tell much about the relevance of the excitations ignored in the description. Since we have at our disposal an explicit model, an effective description in terms of a Nambu-Goldstone boson is not convenient and we will adopt the language of the linearized theory of the fluctuations. Making contact between these two approaches is not immediate because the Nambu-Goldstone boson is a composite state of the fields Q_m.

We will focus on the brane dynamics at energies much smaller than the characteristic mass of the modes Q_m. This scale also defines the inverse brane width w, so that an observer at momentum $p^2 \ll w^2$ would see the brane as an infinitely thin defect. Consequently we call this regime the thin brane limit. In this regime the mode Q_0 is the only accessible dynamical excitation of the brane, the massive states being integrated out[2]. The low energy dynamics on the brane is thus described by an effective theory of the Q_0 mode coupled (generically through non-derivative operators) to some (unspecified) physics localized at $y = 0$.

Before embarking on the study of the implications of the gauging of the translational symmetry on this effective theory, it is worth to anticipate some tools that will be used in the following sections.

When gravity is switched on, eq. (B.6) gets extremely involved and it will not be easy to extract useful information from it. It is therefore convenient to develop a systematic method to approximate the eigenvalue problem. Let us illustrate how this works in flat space-time. First of all we introduce the following parametrization $\phi_m = \Phi_0' f_m$ so that the eigenvalue equation reads

$$-f_m'' - 2\frac{\Phi_0''}{\Phi_0'} f_m' = m^2 f_m. \tag{B.10}$$

The convenience of this parametrization is that we extracted the zero mode profile and we can now simplify the eigenvalue problem without loosing the main dynamical ingredient. In order to achieve this it is necessary to make some assumptions on the background configuration. We will assume that the defect is exponentially localized as $\Phi_0' \sim e^{-2w|y|}$, i.e. the zero mode is normalizable, and that $\Phi_0'(0)$ is odd, i.e. the zero mode has a sharp peak on the brane. Under our assumptions we can write

$$-f_m'' + 4w\, sign(y) f_m' = m^2 f_m \tag{B.11}$$

from which it follows, except for a normalization, the general solution

$$f_m = e^{2w|y|} \left(\cos(\mu|y|) + \beta_m \sin(\mu|y|)\right) \tag{B.12}$$

[2]Because of the composite nature of the Nambu-Goldstone boson the states Q_m, for $m \neq 0$, cannot decouple in the limit $w \to \infty$. Their integration leads to corrections to the Q_0 potential which are of the same order as the bare couplings.

with $\mu^2 = m^2 - 4w^2$. The boundary condition on the brane $f'(0) = 0$ and normalizability fully determine the spectrum. The simplified problem clearly predicts a zero mode $f_0 = const$, and a continuum $m > 2w$ of delta function normalizable modes[3].

For later convenience we also define the brane to brane propagator as the ϕ two-point function $G(p^2, y, y')$ in 4d momentum space and for $y = y' = 0$. From its formal definition it follows $G(p^2, y, 0) \propto \phi_p(y)$ for any $y \neq 0$. Since ϕ_m is smooth everywhere and satisfies trivial boundary conditions at the origin, the normalization simply reads $G'(p^2, 0, 0) = 1$. This specifies the solution as

$$G(p^2, y, 0) = \frac{\phi_p(y)}{\phi'_p(0)} \tag{B.13}$$

up to boundary conditions in the asymptotic region $|y| \to \infty$. We can isolate the poles of the discrete spectrum by requiring an asymptotic exponential behavior for the Green's function. This is done by imposing $\beta_p = i$ in the approximate expressions obtained above. The brane to brane correlator is seen to acquires a pole at $p^2 = 0$. Using our parity assumption ($\Phi''_0(0) = 0$) and expanding in p/w while keeping only the dominant contribution we have

$$G(p^2, 0, 0) \sim \frac{4w}{p^2}. \tag{B.14}$$

The pole appears in the real axis and thus corresponds to a physical massless particle. The residue $4w \sim \Phi'^2_0(0)/\int dy\, \Phi'^2_0$ sets the strength of a typical interaction between the zero mode and arbitrary brane-localized currents.

B.1.2 The scalar sector with gravity

Smooth scalar backgrounds triggering brane generation can be found in a gravitational context in terms of a function W [44]. The solutions for (B.3) read $\Phi'_0 = \delta_\Phi W$ and $-3M^3 A' = W$ provided

$$V = \left(\frac{\delta W}{\delta \Phi}\right)^2 - \frac{4}{3}\frac{W^2}{M^3}. \tag{B.15}$$

For example, the kink background is defined by the same W introduced in the previous section. Its backreaction gives rise to a warp factor (assuming $A(y)$ even and choosing the normalization $A(0) = 0$)

$$A = -\frac{2}{9}\frac{v^2}{M^3}\left(\log\cosh(wy) + \frac{1}{4}\tanh(wy)^2\right). \tag{B.16}$$

[3]Under our assumptions, a possible next to higher localized mode would have odd parity and would not be crucial for the effective brane dynamics.

We can interpret this solution as a smooth realization of the Randall-Sundrum geometry. If we explore energies of order $p^2 \ll w^2$, the brane looks like an infinitely thin defect and the warp factor can be approximated as $A = -k|y|$, with

$$k = \frac{2}{9}\frac{v^2}{M^3}w = \frac{\Delta W}{M^3}. \tag{B.17}$$

The second equality holds up to numerical factors and it is completely general. The relation $k \ll w$ is thus forced by the requirement of a sensible semiclassical approach to gravity. The curvature scale k will play an important role in the effective theory.

When translational symmetry is made local a shift of the vacuum along the broken direction no longer identifies a dynamical perturbation. As anticipated in the introduction, no Higgs mechanism is expected since the graviphotons are found to be unphysical perturbations in these backgrounds. The spontaneous breaking will modify the pure scalar sector of the gravitational theory. The analysis of the latter has been carried out in detail in [45, 46] (for a generalization to non-minimal couplings see [100]). Let us review the main conclusions and, for convenience, express the results in the conformal coordinate z, $dy = adz$.

The non-trivial background induces a mixing between the field excitation $\phi = \Phi - \Phi_0$ and the scalar components of the metric. In the longitudinal gauge and at linearized level the perturbed line element reads

$$ds^2 = a^2[(1+F)\eta_{\mu\nu}dx^\mu dx^\nu - (1-S)dz^2]. \tag{B.18}$$

The additional δg_{AB} components include tensor and vector fields which play no role in the diagonalization of the scalar sector. They give rise to a continuous spectrum of massive spin-2 fields, and a normalizable zero mode identified with the graviton. We refer the reader to the literature for details on the spin-2 dynamics, in the following we will discuss the pure spin-0 sector.

The scalar sector is subject to two constraints. In the longitudinal gauge the first requires $S = 2F$, while the second is the formal statement that the fields F and ϕ are not independent fluctuations. For arbitrary gauge choices this reads:

$$a^2(\dot{F} + \dot{A}S) = -\frac{2}{3}\frac{1}{M^3}a^2\dot{\Phi}_0\phi, \tag{B.19}$$

where $\dot{f} \equiv \partial f/\partial z$. We conclude that the system admits a single independent 5d scalar fluctuation.

The fields F and ϕ satisfy two dynamical equations in addition to the constraint (B.19). These are more elegantly expressed in terms of gauge invariant variables [45] as:

$$\begin{aligned}(\mathcal{A}^+\mathcal{A}^- + \partial_\mu\partial^\mu)\mathcal{G} &= 0 \\ (\mathcal{A}^-\mathcal{A}^+ + \partial_\mu\partial^\mu)\mathcal{U} &= 0\end{aligned} \tag{B.20}$$

where

$$\mathcal{A}^{\pm} = \pm\partial_z + \frac{\dot{\mathcal{Z}}}{\mathcal{Z}}, \qquad \mathcal{Z} = a^{3/2}\frac{\dot{\Phi}_0}{\dot{A}}, \tag{B.21}$$

and the diffeomorphism invariant variables are defined in this gauge as

$$\begin{aligned} \mathcal{G} &= a^{3/2}(\phi - \frac{\dot{\Phi}_0}{2\dot{A}}F) \\ \mathcal{U} &= a^{3/2}\frac{F}{\dot{\Phi}_0}. \end{aligned} \tag{B.22}$$

In order to analyze the spectrum we decompose the 5d fields F, ϕ in Kaluza-Klein modes $F(x,z) = \sum_m F_m(z)Q_m(x)$ and $\phi(x,z) = \sum_m \phi_m(z)Q_m(x)$, where the eigenfunctions $F_m(z), \phi_m(z)$ satisfy eqs. (B.19) and (B.20) with $\partial_\mu\partial^\mu \to -m^2$. Because of (B.19) the equations (B.20) are found to be equivalent for $m \neq 0$. More precisely, for any eigenvalue m (including $m = 0$) the eigenfunctions are related by

$$\mathcal{A}^{-}\mathcal{G}_m = -\frac{3M^3}{2}m^2\mathcal{U}_m. \tag{B.23}$$

In order to complete the eigenvalue problem for the modes F_m, ϕ_m and determine the spectrum we need to specify the normalization condition. The 4d kinetic terms for the scalars F,ϕ, and S in the longitudinal gauge is [45, 46, 100]

$$a^3\left(\partial_\mu\phi\partial^\mu\phi - \frac{3}{2}M^3\partial_\mu F(\partial^\mu F - \partial^\mu S)\right). \tag{B.24}$$

Substituting the condition $S = 2F$ we can immediately derive the normalization of the dynamical fields Q_m:

$$\begin{aligned} N_m &= \int dz\, a^3\left[\phi_m^2 + \frac{3}{2}M^3F_m^2\right] \\ &= \frac{3M^3}{2}\int\frac{dy}{a^2}\left[\frac{3}{2}\frac{M^3}{\Phi_0'^2}Y_m'^2 + Y_m^2\right], \end{aligned} \tag{B.25}$$

where in the second equality we changed back to the coordinate y. In the last expression the definition $Y_m \equiv a^2F_m$ and the constraint (B.19), i.e. $Y_m' = -2a^2\Phi_0'\phi_m/3$, have been used.

The authors [46] have proven the hermiticity of the quadratic action in the fluctuations under very general boundary conditions. This ensures that the Kaluza-Klein expansion is meaningful. The 4d dependent coefficients $Q_m(x)$ play the role of the physical 4d fields and satisfy the dispersion relation $(\partial_\mu \partial^\mu + m^2) Q_m = 0$. One can thus write a free lagrangian formally equivalent to (B.7). The crucial difference between these two theories is the spectrum of physical states.

The physical spectrum of the perturbations, including $m = 0$, is specified by the second of the equations (B.20) together with the requirement $N_m < \infty$. Since the normalization condition can be conveniently written in terms of Y_m it is natural to ask for a dynamical equation for this variable. Re-expressing the equation for $\mathcal{U}_m$ in terms of it we find

$$-Y_m'' + 2(A' + \frac{\Phi_0''}{\Phi_0'})Y_m' - 2A''Y = m^2 e^{-2A} Y_m. \tag{B.26}$$

Equations (B.25) and (B.26) define completely the mass eigenvalue problem in the longitudinal gauge [46].

By an inspection of (B.20), and making use of the asymptotic form of the background (B.3), one can conclude that the mass squared are positive and continuous as $m > 0$ [45]. Because of (B.23) we see that the eigenvector of zero mass can be obtained by solving a first order equation $\mathcal{A}^- \mathcal{G} = 0$ (the additional solutions of the system (B.20) do not satisfy the constraint (B.19)). The independent solutions can be derived explicitly and are $\mathcal{G} = 0, \mathcal{Z}$. In terms of the original fields they read

$$\begin{aligned} \phi_0 &= \beta_1 \frac{\Phi_0'}{a^2} - \beta_2 \frac{\Phi_0'}{a^2} \int^y a^2 \\ F_0 &= \beta_1 \frac{2A'}{a^2} + \beta_2 \left(1 - \frac{2A'}{a^2} \int^y a^2\right). \end{aligned} \tag{B.27}$$

The solution $\beta_2 = 0$ resembles the zero mode of the global theory (the global theory admits a second zero mode solution, $\phi_0 = \Phi_0' \int^y dy\, 1/\Phi_0'^2$, which is not reproduced by the gravitational model. This mode is non-normalizable in the non-gravitational model and thus it plays no role in the dynamics), and it is instructive to observe that $N_0(\beta_2 = 0) = \Delta\left(We^{-2A}\right)$. The latter expression has to be compared with the vacuum energy of the gravitational system, i.e. the brane tension,

$$\rho[\Phi_0, A] = 2\Delta\left(We^{4A}\right). \tag{B.28}$$

By self-consistency, the latter must vanish (the effective 4d theory is defined in flat space) so that the solution $\beta_2 = 0$ has a divergent normalization N_0. We stress that the divergence of the integral is entirely due to the mixing with the field F, i.e. the F_m^2 term in the normalization (B.25).

Both the boundary and the divergent nature of N_0 hold for any zero mode solution of the scalar system. To see this explicitly we use equation (B.26) to recast eq. (B.25) in the form

$$N_m = \frac{3M^3}{4} \int dy \left[-\frac{m^2}{A''} \left(\frac{Y_m}{a^2}\right)^2 - \partial_y \left(\frac{Y_m Y_m'}{a^2 A''}\right) \right]. \tag{B.29}$$

By plugging the solutions (B.27) into (B.29) we see that $N_0 \propto e^{-2A(\infty)}$ for any $B_{1,2}$, and therefore diverges. This is tantamount to say that the zero mode decouples from the effective theory.

The absence of a massless spin-0 mode in the effective 4d theory may have a natural interpretation via holography. The background we are studying represents a smooth realization of the Randall-Sundrum geometry and it may be thought of as a dual description of some unknown strongly coupled 4d theory [40]. Under this assumption the IR of the 4d theory should be identified with the large y region, while the UV with the $y \sim 0$ region. Interestingly, the 4d theory is not asymptotically free. This follows from the identification of $1/A'$ with a possible C-function of the strongly coupled theory [96]. Because of the weak energy condition this function increases in the UV, as suggested by 4d intuition. However, regularity of the gravitational background in passing from positive to negative y implies that such a function blows up at $y = 0$ ($A'(0) = 0$), indicating that the number of degrees of freedom of the 4d theory increases as we approach higher and higher energy scales. Although unfamiliar, this behavior does not imply any obvious pathology on the theory and, in particular, the $y \sim 0$ region is well defined on the gravity side. Furthermore, since no conformal symmetry is realized in the UV no exactly massless state is expected.

B.2 The fate of the Goldstone mode

The gravitational theory describes a drastically different spectrum compared to the one of the globally symmetric model. The zero mode predicted by the non-gravitational theory disappears and no brane perturbation is expected to mediate a long range force. Nevertheless, the brane can still be excited at arbitrary small scales since the continuum starts at zero momentum. In this section we will see that this continuum forms a bound state that appears as a resonant mode to a 4d observer residing on the brane.

Resonant states are eigenmodes of the hamiltonian with complex momentum. They were first identified in the context of alpha decay by Gamow by imposing asymptotic outgoing wave behavior on the wavefunctions. An explicit expression of the resonant condition for our system will be presented later on. For the moment let us stress that, by imposing outgoing wave boundary conditions on the wavefunctions F_m, S_m, ϕ_m, one realizes that the only profiles manifesting a resonant behavior are the ϕ_m. Hence, we conclude that the resonance dynamics is frame dependent, since so does the fundamental field Φ. Our strategy will be to derive a resonant condition for ϕ and analyze the physics of the resonance in two gauges widely used in the literature: the longitudinal (see (B.18)) and the axial gauge ($\delta g_{A5} = 0$).

We discuss the resonance physics in the longitudinal gauge, the results in the axial gauge can be obtained analogously. To derive an explicit expression for the dynamical equation for ϕ_m we can differentiate (B.26) and use (B.19). As anticipated in section II this equation can be conveniently written as a function of f, with $\phi = \Phi_0' f$. The expression is rather involved and reads:

$$\left(\frac{f' e^{2A}}{1+\frac{m^2}{2A''}e^{-2A}}\right)' = -2A'' f e^{2A} \tag{B.30}$$

An exact solution of (B.30) cannot be found even for the simplest backgrounds, but we can find an approximate solution by matching two expressions for f_m, namely $f_<$, defined for $0 < |y| < y_*$, and $f_>$, defined for $|y| > y_*$. The distance y_* is of order the brane width $1/w$. For localized defects and $m \neq 0$ the terms $m^2/A'' \gg 1$ as soon as we move away from the brane. Near the brane, on the other hand, the smoothness of the background requires $A' \approx A''' \propto \Phi'' \approx 0$ and leads to a condition for $f_<$. These are:

$$\begin{aligned} -f''_> - f'_>(4A' + 2\frac{\Phi''}{\Phi'}) &= m^2 e^{-2A} f_> \\ -f''_< - 2A'' f_< &= m^2 e^{-2A} f_<. \end{aligned} \tag{B.31}$$

Equations (B.31) can now be exactly solved in the limit $y_* \to 0$, in which the brane appears infinitely thin $A \approx -k|y|$. Assuming exponentially localized scalar defects $\Phi' \sim e^{-2w|y|}$, eqs. (B.31) can be compactly written as

$$-f'' + 4(k+w) sign(y) f' + 4k\delta(y) f = m^2 e^{2k|y|} f. \tag{B.32}$$

Because of the localization of the defect the mixing between ϕ and the scalars F, S reduces to a brane effect and translates into the boundary condition at $y = 0$. The latter corresponds to a localized positive mass term and will play a role in the determination of the resonant condition. Notice that the boundary condition could have been deduced from the would-be massless profile $\phi_0 = \Phi'_0 e^{-2A}$ found in the previous section. As the above derivation makes it clear, however, the simplified eigenvalue problem is a reasonable approximation for massive modes only.

The model described by the equation (B.32) is a particular example of a more general class of theories that will be introduced in the following section. The system (B.32) can be used as a toy model for the description of our non-trivial set up, having the great advantage of being exactly solvable. The solution, up to a normalization, is

$$\begin{aligned} f_m &= e^{\nu k|y|}\left(J_\nu\left(\frac{m}{k}e^{k|y|}\right) + \beta_m Y_\nu\left(\frac{m}{k}e^{k|y|}\right)\right) \\ \nu &= 2\left(1+\frac{w}{k}\right), \end{aligned} \tag{B.33}$$

where J_ν and Y_ν are the Bessel function of order ν of first and second kind respectively. Imposing the boundary condition at the brane and estimating N_m using the normalization (B.25) (for $m \neq 0$ the normalization is dominated by the ϕ integral) we find

$$\beta_m = -\frac{\frac{m}{k}J_{\nu-1}\left(\frac{m}{k}\right) - 2J_\nu\left(\frac{m}{k}\right)}{\frac{m}{k}Y_{\nu-1}\left(\frac{m}{k}\right) - 2Y_\nu\left(\frac{m}{k}\right)} \tag{B.34}$$

$$f_m \propto \sqrt{\frac{m}{k}}\frac{e^{\nu k|y|}}{\sqrt{1+\beta_m^2}}\left(J_\nu\left(\frac{m}{k}e^{k|y|}\right)+\beta_m Y_\nu\left(\frac{m}{k}e^{k|y|}\right)\right)$$

where the proportionality factors are numbers depending on the scale entering Φ_0 (and the brane tension ΔW), which we can simply ignore.

A plot of the spectral density at $y=0$, i.e. $\phi_m^2(0)$, reveals the presence of a continuum starting at zero; this is very suppressed up to an energy of the order $m^2 \sim kw$, where it develops a very sharp peak near the pole of β_m, and finally stabilizes at a scale $m > w$. The peak is the remnant of the delta function localized zero mode density of the global theory which, remarkably, has been shifted to a non-zero mass by gravity. A similar result has been previously found in [152]. The authors of [152] analyzed the kink background in the light-cone gauge and studied the Yukawa potential mediated by the spin-0 system. They showed that the mentioned peak in the spectral density $\phi_m^2(0)$ ensures continuity of the potentials in the limit $M \to \infty$. In order to understand more deeply the nature of this peak we isolate it by demanding for the appearance of resonant modes.

Imposing outgoing wave boundary conditions on (B.33) (setting $\beta_m = i$) we find the explicit form of the resonant state of our model (B.32)

$$\phi_R = \Phi_0' e^{\nu k|y|} H_\nu^{(1)}\left(\frac{m}{k}e^{k|y|}\right), \tag{B.35}$$

where $H_\nu^{(1)} = J_\nu + iY_\nu$ is the Hankel function of the first kind. The boundary condition on the brane fixes completely the eigenvalue at a complex $m = m_R - i\Gamma_R/2$. If the pole is located in the lower half plane, i.e. $\Gamma_R > 0$, an asymptotically outgoing wave manifests the characteristic exponential decay in time (and, typically, an exponential divergence in space) and the quantities m_R and Γ_R can be identified as the mass and the width of the resonance, respectively. The resonant condition for our system finally reads:

$$\frac{m}{k}\frac{H_{\nu-1}^{(1)}\left(\frac{m}{k}\right)}{H_\nu^{(1)}\left(\frac{m}{k}\right)} = 2. \tag{B.36}$$

This condition admits a single solution at a scale $m_R^2 \sim kw$, corresponding to a complex pole in the scalar propagator. A similar procedure applied to the profiles F_m and S_m provides no solution.

An explicit analytical identification of the root $m = m_R - i\Gamma_R/2$ is not possible since, strictly speaking, the Bessel functions cannot be approximated by an expansion in either the small or large argument limit for $m^2 \sim kw$. Nevertheless a numerical investigation shows that for sufficiently large ν the approximation of small argument works quite well.

This approximation allows us to obtain expressions similar to those of [49][4]:

$$x\frac{H^{(1)}_{\nu-1}(x)}{H^{(1)}_{\nu}(x)} \approx x\frac{Y_{\nu-1}}{Y_\nu}\left[1 - ix\frac{J_{\nu-1}}{Y_{\nu-1}}\right], \tag{B.37}$$

where for brevity $x = m/w$, that will be used to estimate the pole in the following. The reliability of the small argument approximation should not come as a surprise, since comparing the small argument limit of (B.36) with the explicit expression for β_m we see that the pole in the Green's function approaches the bump found in the spectral density $\phi_m^2(0)$. The same approximation implies that for $k|y| < 1$ the resonant profile (B.35) acquires approximately the same y dependence of the zero mode $\sim \Phi_0'$, while outside the brane the resonance becomes exponentially divergent because of the positive width.

We can now find an approximate expression for the resonant propagator. Because of the mixing among ϕ, F, and S, the scalars' Green's function is a three by three matrix where each of the entries can be formally written in the canonical form

$$G_{ab}(p^2, y, y') \propto \int dm\, \frac{g_m^{(a)}(y) g_m^{(b)}(y')}{p^2 - m^2},$$

with $g_m^{(a,b)} = \phi_m, F_m, S_m$ (see [152] for an explicit expression of this matrix in the light-cone gauge). The relevant term in our analysis is $G_{\phi\phi}$, the other entries are expected to provide subdominant corrections to the amplitudes as $M \to \infty$. Under the simplifying assumption of an exponentially localized defect, the mixing between the scalar states becomes a pure brane effect and the Green's function $G_{\phi\phi}$ is completely determined by the eigenmode ϕ_p and its boundary conditions. Following an analog procedure as that used in section II we obtain an expression for the brane to brane propagator as:

$$G_{\phi\phi}(p^2, 0, 0) = \frac{1}{\frac{\phi_p'(0)}{\phi_p(0)} - 2k}. \tag{B.38}$$

Setting $\phi_p = \phi_R$ in the above expression and using (B.37) we are able to isolate a pole

$$\begin{aligned} G_{\phi\phi}(p^2, 0, 0) \quad &\sim \quad \frac{1}{p\frac{H^{(1)}_{\nu-1}\left(\frac{p}{k}\right)}{H^{(1)}_{\nu}\left(\frac{p}{k}\right)} - 2k} \\ &\approx \quad \frac{2k(\nu - 1)}{p^2 - m_R^2 + ip\,\Gamma_R(p)} \end{aligned} \tag{B.39}$$

[4]The approximation $x \ll \sqrt{\nu}$ was natural in the framework studied by [49] since they considered a case analog to $w \ll k$, which in our framework is not a reliable limit.

where we have defined:

$$\left(\frac{m_R}{k}\right)^2 \approx 4(\nu-1) \tag{B.40}$$
$$\frac{\Gamma_R(m_R)}{m_R} \approx \frac{\pi/4}{\Gamma(\nu-1)\Gamma(\nu)}\left(\frac{m_R}{2k}\right)^{2(\nu-1)}.$$

As already emphasized, eqs. (B.40) are only approximate, though reliable, expressions of the pole. Eqs. (B.39) and (B.40) represent the main results of the paper.

The resonant propagator should be compared with (B.14). As gravity decouples we have $m_R - i\Gamma_R/2 \to 0$ as $k/w \to 0$, and the resonance exchange mimics the zero mode of the global theory ensuring continuity of the observable as gravity decouples. The ratio $\Gamma_R/m_R \propto (e^2/\nu)^\nu$ is exponentially small and can be neglected in the limit $k \ll w$. The resonance is thus a stable state for all practical purposes and mediates a potential between static brane sources of the form

$$V_R \propto \frac{1}{r}e^{-m_R r}, \tag{B.41}$$

where, in the limit $w/k \gg 1$, $m_R^2 \approx 8wk \ll w^2$. The residue in (B.39) is the $4w$ factor found in (B.14) except for small $\mathrm{O}(k/w)$ corrections. This means that the scale determining the effective coupling of the resonance to brane localized currents is approximately the same as the zero mode of the global theory.

For $k \ll w$ the resonance represents the only relevant brane fluctuation below the cut-off w, as the spectral function $\phi_m^2(0)$ suggests. We can appreciate the negligible impact on the effective theory of the continuum $m \ll m_R$ by estimating the Yukawa potential they mediate on the brane. At 4d distances r large compared to the curvature $kr \gg 1$ these are expected to play a role. Expanding for small arguments our approximate solutions we have $f_m(0) \propto \sqrt{x}xJ_{\nu-1}(x) \propto \sqrt{x}x^\nu$, with $x = m/k$, and the potential behaves as

$$V_{cont} \sim \int dm \frac{\phi_m^2(0)}{r}e^{-mr} \propto \frac{1}{\nu r}\left(\frac{1}{kr}\right)^{2\nu+2} \tag{B.42}$$

for $kr \gg 1$. In the limit $\nu \gg 1$ the effect of the continuum below the resonant peak is highly suppressed. An analog suppression holds for the continuum in the range $m_R < m < w$. (Notice, by the way, that the continuum becomes relevant at $m \sim w$, rather than at $2w$ as in the global theory.)

Having established the dominant role of the resonance in the low energy brane to brane Φ exchanges we conclude that, provided $k \ll w$, the low energy dynamics of the brane can be thought as an effective theory for the resonant mode. Hence the theory acquires a non-trivial dependence on a new scale m_R^2 with respect to the global theory. For $w^2 \gg p^2 \gg m_R^2$ the resonance represents the only dynamical degree of freedom of the brane. In this regime the global description is an approximation of the full theory up to $O(k/w)$ corrections. More interesting is the opposite limit. For $p^2 \ll m_R^2$ no brane excitation can be significantly produced and the effect of the brane fluctuation on the physics localized on the brane is totally encoded in non-renormalizable operators suppressed by the scale m_R.

Let us briefly comment on the generalization to other gauge choices. In the axial gauge ($\delta g_{A5} = 0$) the constraint (B.19) simplifies to $F' = -2\Phi_0'\phi/3$, but the dynamical equation for Y_m can no longer be used to derive a resonant condition for ϕ_m. In fact, in this gauge the diffeomorphism invariant field $\mathcal{U}$ contains the contribution of another scalar component of the metric [45], and thus, although the first equation (B.20) holds in general, it cannot be straightforwardly used to obtain an equation for F, and subsequently ϕ. It is more convenient to derive a third order differential equation for $f = \phi/\Phi_0'$ from the dynamical condition for $\mathcal{G}$ with the help of (B.19). This equation was found in [48], but here it will not be reproduced for brevity. The latter leads to a resonant condition of the same form as (B.36) (with $\nu \to \nu - 1$ and $2 \to 4$), and consistently formulas similar to (B.39), (B.40), and (B.42) can be derived. On the other hand, the same line of reasoning applied to the solution found in [152] using the light-cone gauge reveals the presence of a resonant mode described by an equation similar to (B.36), but with $\nu \to 1$. The solution now has mass and width of comparable magnitude, and both proportional to the curvature scale.

One should not be worried in finding different predictions for the resonant pole because it is an unphysical quantity. The same apparent inconsistency is clearly encountered in the estimate of the potential energy felt by two sources of the scalar ϕ. In the light-cone gauge the authors [152] found that the potential mediated by the continuum of light modes in the regime $kr \gg 1$ is suppressed by $1/(kr)^2$, while in both the longitudinal and the axial gauge the suppression is much more significant (at least $1/(kr)^{2\nu}$) with respect to the long range $1/r$. The invariance of the total amplitude under infinitesimal diffeomorphisms follows from the invariance of the full lagrangian – which includes an interacting term formally written as $\mathcal{L}_{int} \equiv T^{AB}h_{AB} + J\phi$ – while all of the above predictions make reference to the $J\phi$ term only. The discrepancy found in the above computations is of the same order of the gravitons contribution to the full amplitude[5]. Gauge invariance ensures that, once these additional contributions are included, any observer probes the same physics.

B.2.1 An exactly solvable model

The appearance of resonant modes in place of the discrete states of the global theory resembles the situation considered in [49]. In that paper the authors studied the effect of the non-compact Randall-Sundrum warping on chiral fermions localized with the mechanism

[5]Notice that in the light-cone gauge a mixing among the physical gravitons components and the physical spin-0 states is enforced. This mixing suggests that the spin-2 sector has some knowledge of the scale w already at tree level (even in the thin brane limit). In both axial and longitudinal gauges the brane width dependence of the spin-2 lagrangian is traded with a k-dependence in the thin brane limit.

described in [116]. They found that if we supply the fermionic field with a bulk mass, then the massless localized state of the non-gravitational theory disappears and the continuum generates a resonant mode with mass proportional to the small bulk mass and an exponentially suppressed width. We will now show that a similar situation is realized in the spin-0 sector of the gravitational theory considered in the present paper (at least in the gauges studied here).

Consider a bulk scalar field ψ fluctuating in a gravitational background

$$\mathcal{L} = \sqrt{-g}\left(g^{AB}\partial_A\psi\partial_B\psi - m_5^2\psi^2)\right). \tag{B.43}$$

In order to localize light modes on the brane we introduce an attractive delta function mass term.

Let us first discuss the theory in flat space and define $m_5^2 = 4(w^2 - w_b\delta(y))$. The mass eigenvalue problem for the scalar wavefunctions ψ_m can be written as:

$$\begin{aligned} \int dy\, \psi_m(y)\psi_{m'}(y) &= \delta_{m,m'} \\ -\psi_m'' + 4(w^2 - w_b\delta(y))\psi_m &= m^2\psi_m, \end{aligned} \tag{B.44}$$

where w_b is assumed to be a free parameter of order w. The system (B.44) admits a continuum starting at the threshold $2w$ and a single localized mode. The normalizable wavefunction for the latter is $\psi = e^{-\mu|y|}$ and has a mass squared $4w^2 - \mu^2$. The parameter μ is determined by the boundary conditions as $\psi'(0) = -2w_b\psi(0)$, which gives $\mu = 2w_b$. We see that normalizability enforces $w_b \geq 0$ while absence of tachions $w_b \leq w$ (the delta function potential cannot be too attractive).

As gravity is switched on eqs (B.44) receive corrections both in the potential and the kinetic term. For a geometry of the Randall-Sundrum form $ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$, where k sets the curvature scale, we have

$$\begin{aligned} \int dy\, e^{-2k|y|}\psi_m(y)\psi_{m'}(y) &= \delta_{m,m'} \\ -\psi_m'' + 4k\, sign(y)\psi_m' + 4(\bar{w}^2 - \bar{w}_b\delta(y))\psi_m &= m^2 e^{2k|y|}\psi_m \end{aligned} \tag{B.45}$$

where now both $\bar{w}$ and $\bar{w}_b$ may contain curvature corrections of order k/w with respect to w and w_b respectively. Under the assumption $\bar{w}^2 > \bar{w}_b(\bar{w}_b + 2\bar{w})$ the spectrum has no tachionic modes, while the requirement $\bar{w}^2 = \bar{w}_b(\bar{w}_b + 2\bar{w})$ is the necessary condition for the existence of a localized massless mode.

Despite the absence of discrete eigenvalues the model may admit resonant states which behave effectively as particles. For the explicit model (B.45) the resonant condition reads:

$$\frac{m}{k}\frac{H^{(1)}_{\nu-1}\left(\frac{m}{k}\right)}{H^{(1)}_{\nu}\left(\frac{m}{k}\right)} = \left(\nu - 2 - 2\frac{\bar{w}_b}{k}\right) \tag{B.46}$$

$$\nu = 2\sqrt{1+\frac{\bar{w}^2}{k^2}}.$$

Thin resonances exist only if the right hand side of the first equation (B.46) is positive and, not surprisingly, if $\bar{w}_b \neq 0$. The former requirement is the same condition ensuring absence of tachionic excitations. Proceeding as in the previous section we approximately find a solution for $m = m_R - i\Gamma_R/2$, where

$$\begin{aligned} \left(\frac{m_R}{k}\right)^2 &\approx 2(\nu-1)\left(\nu-2-2\frac{\bar{w}_b}{k}\right) \\ \frac{\Gamma_R(m_R)}{m_R} &\approx \frac{\pi/4}{\Gamma(\nu-1)\Gamma(\nu)}\left(\frac{m_R}{2k}\right)^{2(\nu-1)}. \end{aligned} \tag{B.47}$$

The theory analyzed in the text is found to be a particular example of (B.45) with $\bar{w}^2 = w(w+2k)$ and $\bar{w}_b = w-k$ (see eq. (B.32) in the text). Despite the involved expression for the ϕ_m norm obtained in section III, it is easy to verify that the F^2 integral in eq. (B.25) is convergent for any $m \neq 0$ and, thus, the normalization condition for the fluctuations ϕ_m is effectively of the same form as (B.45). As gravity decouples ($k=0$) $\bar{w}, \bar{w}_b \to w$ and the model predicts a localized massless mode[6]. Because of the mixing with gravity, an $O(k/w)$ correction to the mass term is induced and the mode disappears from the spectrum leaving a resonance in its place.

Using this simplified picture one can also deduce the fate of a possibly massive localized mode. An inspection of (B.47) shows that, in the limit $k \ll w$ and for $\bar{w}_b = O(\bar{w})$, a resonance still appears at $m_R \sim w$ but quickly becomes wider as its mass increases. For example the choice $\bar{w}^2 = w(w+2k)$ and $\bar{w}_b = w_b$ leads to the prediction of a broad resonance with mass $m_R^2 \approx 8w^2(1-w_b/w)$, that should be compared with the flat space-time result $4w^2(1-w_b^2/w^2)$. Although this choice seems quite arbitrary, one can verify with technics similar to those used in the bulk of the paper that it provides a good approximation of the next to higher state of the kink background.

[6]The kink background (B.4) reproduces the above potential in the thin brane limit $w \to \infty$. For an observer at a distance $y > 1/w$, where $1/w$ characterizes the thickness of the defect, the background can be simplified by the approximation $\tanh(wy) \simeq sign(y)$. From the identities $sign(y)^2 = 1$ and $\partial_y sign(y) = 2\delta(y)$ we derive $\Phi_0'''/\Phi_0' \simeq -2w^2(2\delta(y)/w - 2)$.

Appendix C

Non-linear realization of broken CFTs

In this Appendix we describe in some details the construction of a field theory of broken scale and conformal invariance. The first section is devoted to a brief review of some properties of the CFTs, the second to the theory of non-linear realization of the broken CFTs, using both a formal perspective and a more pedagogical one.

C.1 Conformal invariance

Many properties of the conformal group can be understood as a consequence of the Weyl invariance:

$$\begin{aligned} g_{\mu\nu}(x) &\rightarrow g'_{\mu\nu}(x) = e^{-2\lambda} g_{\mu\nu}(x) \\ \Phi(x) &\rightarrow \Phi'(x) = e^{\Delta\lambda}\Phi(x), \end{aligned} \tag{C.1}$$

where λ is an arbitrary function. Weyl invariance is not a coordinate transformation. The most general subgroup that can be written as a spacetime transformation is the conformal group. Hence, Weyl invariance in a D dimensional curved spacetime (which is a local symmetry) implies invariance under $SO(2, D)$ conformal invariance in the D dimensional flat spacetime. From now on we focus on $D = 4$.

The conformal symmetry generates the most general group of coordinate transformations $x \rightarrow x' = x + f$ that preserve the causal structure of spacetime:

$$\frac{\partial x'^{\mu}}{\partial x^{\alpha}} \frac{\partial x'^{\nu}}{\partial x^{\beta}} \eta^{\alpha\beta} = |det \frac{\partial x'}{\partial x}|^{1/2} \eta^{\mu\nu}. \tag{C.2}$$

Working at infinitesimal level we find that the transformation $x^\mu \rightarrow x^\mu + f^\mu$ must satisfy

$$\frac{\partial_\mu f_\nu}{2} + \frac{\partial_\nu f_\mu}{2} - \frac{\partial^\alpha f_\alpha \eta_{\mu\nu}}{4} = 0 \tag{C.3}$$

which is solved by $f^\mu = a^\mu + \omega^{\mu\nu} x_\nu + x^2\beta^\mu - 2x\beta x^\mu + \lambda x^\mu$, where $\omega^{\mu\nu} = \omega^{\nu\mu}$ and $\Omega^2 = 1 - 4x\beta + 2\lambda$. The transformations defined by a^μ and $\omega^{\mu\nu}$ form the Poincare group, while λ and β^μ define the dilatations and the so called special conformal transformations, respectively. Integrating the infinitesimal forms we find:

$$\begin{aligned} x^\mu &\rightarrow x'^\mu = e^\lambda x^\mu \\ x^\mu &\rightarrow x'^\mu = \frac{x^\mu + x^2\beta^\mu}{1 + 2x\beta + x^2\beta^2} \quad \left(or \quad \frac{x'^\mu}{x'^2} = \frac{x^\mu}{x^2} + \beta^\mu \right). \end{aligned} \tag{C.4}$$

No proof that scale invariance implies the conformal one exists in $D > 2$, but no non-trivial (and unitary) counterexamples are known in quantum field theories. Yet, finding scale but non-conformal field theories is a trivial exercise at the classical level. In general, a scale invariant theory has a conserved scale current S^μ defined as

$$S^\mu = x^\nu T^\mu_\nu + K^\mu; \qquad T^\mu_\mu + \partial_\mu K^\mu = 0. \tag{C.5}$$

If we are able to find a I such that $\partial_\mu K^\mu = \partial^2 I$ the theory is also conformally invariant. Indeed we can then construct a traceless stress tensor as:

$$\Theta_{\mu\nu} = T_{\mu\nu} - \frac{1}{D-1}\left(\partial_\mu\partial_\nu - \eta_{\mu\nu}\partial^2\right) I. \tag{C.6}$$

This is the case for a certain number of systems. For example, for the classical $D = 4$ lagrangian:

$$(\partial_\mu\phi^i)^2 - \lambda_{ijkl}\phi^i\phi^j\phi^k\phi^l, \tag{C.7}$$

where λ_{ijkl} are totally symmetric fields, we find that $K_\mu = \phi^i\partial_\mu\phi^i$ accomplishes the requirement. One recognizes the traceless stress tensor thus constructed as the one following from coupling the scalars to gravity in a Weyl invariant way. Generally, from the variation $\delta\phi = (\Delta + x^\mu\partial_\mu)\phi$, we have

$$\begin{aligned} S_\mu &= \pi_\mu\delta\phi - x_\mu\mathcal{L} = x^\nu(\pi_\mu\partial_\nu\phi - \eta_{\mu\nu}\mathcal{L}) + \Delta\pi_\mu\phi \\ &= x^\nu T_{\mu\nu} + \Delta\pi_\mu\phi \end{aligned} \tag{C.8}$$

so that in general $K_\mu = \Delta^i\pi^i_\mu\phi^i$.

Once a traceless stress tensor is found, one can construct 4 additional conserved currents as $K^{\mu\nu} = (2x^\mu x^\lambda - \eta^{\mu\lambda}x^2)\Theta^\nu_\lambda$, corresponding to the special conformal transformations. No other conserved currents linear in the stress tensor can be constructed. Indeed, suppose $J^\alpha = f_\nu(x)\Theta^{\alpha\nu}$ is conserved, then using the traceless and symmetric nature of the tensor we have

$$0 = \partial_\mu f_\nu \Theta^{\mu\nu} = \left[\frac{\partial_\mu f_\nu}{2} + \frac{\partial_\nu f_\mu}{2} - \frac{\partial^\alpha f_\alpha \eta_{\mu\nu}}{4}\right]\Theta^{\mu\nu}. \tag{C.9}$$

The result vanishes only if the braket vanishes, which is true for $f^\mu = a^\mu + \omega^{\mu\nu}x_\nu + x^2\beta^\mu - 2x\beta x^\mu + \lambda x^\mu$. The currents

$$\Theta^{\mu\alpha}, \quad x^\mu\Theta^{\nu\alpha} - x^\nu\Theta^{\mu\alpha}, \quad (\eta^{\mu\nu}x^2 - 2x^\mu x^\nu)\Theta^\alpha_\nu, \quad x_\mu\Theta^{\mu\alpha} \tag{C.10}$$

are conserved (the derivative acts on the label α) and their spacial integrals generate the conformal algebra. Notice that the traceless stress tensor already includes a spin dependent part and therefore the second tensor written above denotes the currents of the full Lorentz rotations.

From the algebra it immediately follows $e^{-iD\sigma}P_\mu e^{iD\sigma} = e^{-\sigma}P_\mu$ and thus

$$e^{-iD\sigma}P^2 e^{iD\sigma} = e^{-2\sigma}P^2 \tag{C.11}$$

for any finite parameter σ. The operator P^2 is not a Casimir and therefore the concept of *mass* is not a physical property of the theory. To make these intuitive arguments more formal we act with (C.11) on a momentum eigenstate $|\alpha\rangle$, where $P^2|\alpha\rangle = p^2|\alpha\rangle$. The action of $e^{iD\sigma}$ brings $|\alpha\rangle$ into a new momentum eigenstate $e^{iD\sigma}|\alpha\rangle = |\alpha'\rangle$ with momentum $p'^2 = p^2 e^{-2\sigma}$. Here $|\alpha'\rangle$ can be interpreted as a state with the same physical content of $|\alpha\rangle$ – indeed scale transformations are symmetries by assumption – but seen by another observer. In a scale invariant theory, a physical state can be observed with any possible p^2. In particular, the spectrum must be either continuous or have zero masses.

The realization of the dilatation symmetry on a general operator is the following:

$$\mathcal{O}(x) \to \mathcal{O}'(x) = e^{\lambda\Delta}\mathcal{O}(e^\lambda x)$$

and $\mathcal{O}$ is said to have (scaling) dimension Δ. Primary operators $\mathcal{O}$ are operators that cannot be obtained by differentiating other operators. The descendents are constructed by applying P_μ to the primaries. Notice that from the algebra $\Delta(P_\mu\mathcal{O}) = \Delta + 1$. On the other hand $\Delta(\mathcal{O}_1\mathcal{O}_2) \neq \Delta_1 + \Delta_2$ in general.

The CFT implies that

$$\langle\mathcal{O}(x)\mathcal{O}(0)\rangle = \frac{C_S}{|x|^{2\Delta}}, \tag{C.12}$$

where $C_S > 0$ follows from positivity of $\langle\bar{\mathcal{O}}\bar{\mathcal{O}}\rangle$, where $\bar{\mathcal{O}} = \int_{\mathcal{R}} d^4x\mathcal{O}(x)$ for any $\mathcal{R}$ compact. We can then normalize the operator such that $C_S = 1$. For a gauge singlet primary we generally find a cut in $p^2 \geq 0$. An exception is a free field theory, which has a simple massless pole. This is not a general property of CFTs: interacting theories can have massless fields, provided these are not gauge singlets. For example gauge fields have $\Delta = 1$ by symmetry (see later), and therefore induce a Coulomb force mediated by massless particles. The correlator can also be used to derive unitarity bounds by requiring positivity of the imaginary part. For example: a gauge singlet scalar has $\Delta \geq 1$ (equal if free), a vector has $\Delta \geq 1$ (equal if gauge), a conserved current $\Delta = 3$ and the stress tensor $\Delta = 4$.

All of these CFT results will be changed by the spontaneous breaking of the dilatation symmetry at a scale $\sim f$, and will be recovered at momentum $p^2 \gg f^2$.

C.2 Broken CFTs

The breaking of the conformal group $SO(2,4)$ down to 4D Poincare leads to the appearance of a single NGB, the dilaton, despite the number of broken generators is 5: the generators K^μ of the special conformal transformations do not act independently on the vacuum but generate linear combinations of the very same state generated by D. This directly follows from the algebra. Another way to look at that is by referring to the Weyl symmetry. Since Weyl (local) implies conformal invariance we may suspect that the NGB physics of CFT also follows from Weyl. Hence, since the global Weyl symmetry is a pure dilatation we conclude that the physical field is a dilaton. Below we will give an additional proof.

C.2.1 Coset representation

The nonlinear realization of the dilatations are realized in terms of a coset space element

$$\Sigma(x,\sigma) \equiv e^{iP_\mu x^\mu} e^{iD\sigma}, \tag{C.13}$$

where D is the generator of the scaling transformations, $[D, P_\mu] = -iP_\mu$, and $\sigma = \sigma(x)$ represents the NGB field. Notice that, although translations are not broken, the NGB matrix is taken in the coset (Poincare$\times$Dilatations)/Lorentz. A physical motivation for this choice lies in the fact that the variables x^μ transform nonlinearly under translations and hence are analogous to NGBs.

The Maurer-Cartan one-form for Σ can be derived easily:

$$\begin{aligned} \Sigma^{-1} d\Sigma &= e^{-iD\sigma} iP_\mu dx^\mu e^{iD\sigma} + iDd\sigma \\ &= e^{-\sigma} iP_\mu dx^\mu + iD\partial_\mu\sigma dx^\mu. \end{aligned} \tag{C.14}$$

From the definition

$$\Sigma^{-1} d\Sigma = i(\omega_P^\mu P_\mu + \omega_D D + \omega_J^{\alpha\beta} J_{\alpha\beta}), \tag{C.15}$$

we can read off the covariant derivative for the NGB, $\omega_D = \nabla_\mu \sigma dx^\mu$, and the tetrads $\omega_P^\mu = e_\alpha^\mu dx^\alpha$. The one-form $\omega_J^{\alpha\beta}$ defines the covariant derivative for a general massive field Φ. Our results are

$$\nabla_\mu \sigma = \partial_\mu \sigma \qquad e_\alpha^\mu = e^{-\sigma} \delta_\alpha^\mu \ (g_{\alpha\beta} = e^{-2\sigma} \eta_{\alpha\beta}) \qquad \nabla_\mu \Phi = \partial_\mu \Phi. \tag{C.16}$$

The effective action for σ can be written in terms of $\sigma, \partial_\mu\sigma$ and must be invariant under scalings. The scale invariance is realized in the effective theory as

$$\begin{aligned} \sigma(x) &\to \sigma'(x) = \sigma(e^\lambda x) + \lambda \\ \Phi(x) &\to \Phi'(x) = e^{\Delta\lambda} \Phi(e^\lambda x), \end{aligned} \tag{C.17}$$

where Φ is a general field and Δ its scaling dimension ($\partial_\mu\Phi$ has weight $\Delta+1$). Notice that the lagrangian operator must have weight $\Delta = 4$. Since σ transforms inhomogenuously, the vacuum $\sigma = 0$ is broken. For convenience we introduce the field $\chi \equiv fe^{\sigma/f}$ which is linear in dilatations:

$$\chi(x) \to \chi'(x) = e^{\lambda}\chi(e^{\lambda}x). \tag{C.18}$$

Again, the nonvanishing vacuum is defined as $\chi = f$ and encodes the symmetry breaking.

We can use the same formal machinery introduced for dilatation in the case of the full conformal group, as well. We define the group element

$$\Sigma(x,\sigma,\phi) = e^{ix^\mu P_\mu}e^{i\phi^\alpha K_\alpha}e^{i\sigma D} \tag{C.19}$$

where $\sigma(x), \phi^\alpha(x)$ are the NGBs of dilatations and special conformal transformations respectively. Under a group multiplication by an element g of the conformal group we have the usual definition

$$\Sigma(x',\sigma',\phi') = g\Sigma(x,\sigma,\phi)h^{-1}(x,\sigma,\phi) \tag{C.20}$$

with h an element of the unbroken group (Poincare in our case). By definition the transformations under Lorentz and rotations are those of a scalar and a vector field for σ and ϕ respectively. Let us focus on dilatation first, $g = e^{i\lambda D}$. Using the above rule and the group identities $e^{-i\lambda D}P_\mu e^{i\lambda D} = e^{-\lambda}P_\mu$ and $e^{-i\lambda D}K_\mu e^{i\lambda D} = e^{\lambda}K_\mu$ (consequences of the relations $[D,P] = -iP$ and $[D,K] = iK$) and defining $h = 1$ we find

$$e^{i\phi'^\alpha(x')K_\alpha}e^{i\sigma'(x')D} = e^{i\phi^\alpha e^{-\lambda}K_\alpha}e^{i\lambda D}e^{i\sigma D} \tag{C.21}$$

which is satisfied for $\phi'^\alpha(x') = \phi^\alpha(x)e^{-\lambda}$ and $\sigma'(x') = \sigma(x) + \lambda$.

Performing a similar computation for the case of special transformations we define the transformations of the NGBs as:

$$\begin{aligned} \phi^\alpha(x) \to \phi'^\alpha(x') &= \frac{\partial x'^\alpha}{\partial x^\beta}\left(\phi^\beta(x) - \frac{1}{8}\partial^\beta \log\frac{\partial x'}{\partial x}\right) \\ \sigma(x) \to \sigma'(x') &= \sigma(x) + \frac{1}{4}\log\frac{\partial x'}{\partial x}. \end{aligned} \tag{C.22}$$

We see that ϕ^α transforms as $-\partial^\alpha\sigma/2$. This is crucial, because it tells us that the NGBs of the special conformal transformations are redundant in the construction of an invariant theory.

We can now find the covariant derivatives. For simplicity we work at linear order in the NGBs. We have:

$$\begin{aligned} &\Sigma^{-1}d\Sigma = \\ &i\left[P_\mu dx^\mu(1-\sigma) + K_\alpha d\phi^\alpha + D(d\sigma + 2\eta_{\mu\nu}\phi^\nu dx^\mu) - 2J_{\mu\nu}\phi^\nu dx^\mu\right] + \ldots \end{aligned} \tag{C.23}$$

where we used the commutation relations $[D, P] = -iP$ and $[P_\mu, K_\nu] = 2iJ_{\mu\nu} - 2i\eta_{\mu\nu}D$. This result confirms that the vielbein is $\delta^\mu_\nu(1-\sigma)+\ldots$ and the covariant derivatives are $\nabla\sigma = d\sigma + 2\eta_{\mu\nu}\phi^\nu dx^\mu + \ldots$ and $\nabla\phi^\alpha = d\phi^\alpha + \ldots$. A general field transforming under the group element h as $\Phi'(x') = S\Phi(x)$, where S is by definition a Lorentz rotation with generators $S_{\mu\nu}$, has a covariant derivative defined by $\nabla\Phi = d\Phi - 2iS_{\mu\nu}\phi^\nu dx^\mu\Phi + \ldots$ (notice that by definition this means that Φ has weight $\Delta = 0$).

Once nonlinear terms are included one verifies that the only change is in the covariant derivative for ϕ^α, namely $\nabla\Phi = (d - 2iS_{\mu\nu}\phi^\nu dx^\mu)\Phi$ and $\nabla\sigma = d\sigma + 2\eta_{\mu\nu}\phi^\nu dx^\mu$ are exact ($\nabla\Phi$ has weight $\Delta = 1$).

From a field Φ with general Lorentz quantum numbers we can construct a field $\tilde{\Phi} = \chi^\Delta\Phi$ of arbitrary weight Δ. The above results tell us that the covariant derivative reads

$$\nabla = d - \Delta d\log\chi - 2iS_{\mu\nu}\phi^\nu dx^\mu. \tag{C.24}$$

Notice that $\nabla\chi = 0$: a covariant derivative for χ linear in the field does not exist. The minimal derivative follows from the derivative of ϕ^α substituting $\phi^\alpha \to -\partial^\alpha\sigma/2$ (see next section).

We can now formulate the following rule: the most general Lorentz scalar lagrangian written in terms of the "massive" fields Φ can be made conformally invariant by introducing the dilaton field χ in such a way that the resulting action has zero weight, and by replacing the derivatives with covariant ones.

Notice that the statement "the NGBs relative to spontaneously broken exact symmetries must be derivatively coupled" is true only for such symmetries that leave the lagrangian invariant. In the case of dilatations the lagrangian must have weight $\Delta = 4$, while the invariance is recovered due to the complementary transformation $x \to x' = e^\lambda x$. It follows that an invariant potential for the dilaton can be written, and has the general form $\mathcal{L}_{pot} = -a\chi^4$. This specific potential, however, is not allowed by our assumptions on the vacuum structure. Indeed, such potential is incompatible with the spontaneous breaking of the scaling symmetry: it would imply a vacuum with $f = 0$, *i.e.* with no symmetry breaking. The most general lagrangian for the dilaton can thus be written as:

$$\mathcal{L}_\chi = \frac{(\partial\chi)^2}{2} + c_1\frac{(\partial\chi)^4}{\chi^4} + c_2\frac{(\partial^2\chi)^2}{\chi^2} + \ldots, \tag{C.25}$$

where the third term may be discarded when working at $O(p^4)$.

When the fields Φ are included the number of nonderivative couplings increases. In addition to be formally invariant under conformal transformations, these must be compatible with the vacuum structure.

C.2.2 Phenomenological approach

We now follow Mack and Salam [159] and notice that any representation of the Lorentz group induces a representation of the full conformal group. Indeed, the matrix

$$\Lambda^{\mu}_{\nu}(x) = \frac{\partial x'^{\mu}}{\partial x^{\nu}} |det \frac{\partial x'}{\partial x}|^{-1/4} \tag{C.26}$$

is a spacetime dependent Lorentz matrix due to (C.2). We thus conclude that any field Φ transforming under Lorentz as $\Phi(x) \to \Phi'(x') = D(\Lambda)\Phi(x)$ also transforms under the conformal group as

$$\Phi(x) \to \Phi'(x') = |det \frac{\partial x'}{\partial x}|^{-\Delta/4} D(\Lambda(x))\Phi(x) \tag{C.27}$$

where $\Lambda(x)$ is (C.2) and Δ is the weight of Φ[1]. The matrix D defines the Lorentz representation and, infinitesimally ($\Lambda = 1 + \omega$), reads $D = 1 + i\omega^{\mu\nu} S_{\mu\nu}/2 + O(\omega^2)$.

A conformally invariant lagrangian can thus be derived from an action invariant under Lorentz transformations. However, this is not the end of the story because the derivative is not covariant. In fact we find that $\partial_\mu \Phi(x) \to \partial_{\mu'} \Phi'(x')$, where

$$\begin{aligned} &\partial_{\mu'}\Phi'(x') \\ &= \frac{\partial x^{\nu}}{\partial x'^{\mu}} |det \frac{\partial x'}{\partial x}|^{-\Delta/4} D(\Lambda(x)) \left[\partial_\nu + D^{-1}\partial_\nu D - \frac{\Delta}{4}\partial_\nu \log|det\frac{\partial x'}{\partial x}|\right]\Phi \\ &= \frac{\partial x^{\nu}}{\partial x'^{\mu}} |det \frac{\partial x'}{\partial x}|^{-\Delta/4} D(\Lambda(x)) \left[\partial_\nu + \frac{1}{4}\left(iS_{\nu\lambda} - \Delta\eta_{\nu\lambda}\right)\partial_\lambda \log|det\frac{\partial x'}{\partial x}|\right]\Phi. \end{aligned} \tag{C.28}$$

In the above expressions we used the fact that, by the group definition $D(\Lambda_1)D(\Lambda_2) = D(\Lambda_1\Lambda_2)$, the relation $D^{-1}dD = D(\Lambda^{-1})(D(\Lambda + d\Lambda) - D(\Lambda)) = i(\Lambda^{-1}d\Lambda)S/2$ holds. Finally we used the explicit form (C.2) to replace $\Lambda^{-1}d\Lambda$.

If we introduce a field $\chi = e^{\sigma}$ such that

$$\sigma(x) \to \sigma'(x') = \sigma(x) - \frac{1}{4}|det\frac{\partial x'}{\partial x}| \tag{C.29}$$

we can define the covariant derivative:

$$D_\mu\Phi = \left[\partial_\mu + \left(iS_{\mu\nu} - \Delta\eta_{\mu\nu}\right)\partial_\nu \log\chi\right]\Phi \tag{C.30}$$

Notice that the covariant derivative has weight is $\Delta+1$ and has the correct Lorentz structure:

$$D_\mu\Phi \to \left(D_\mu\Phi\right)'(x') = \left(\Lambda^{-1}\right)_{\mu}^{\ \nu} |det\frac{\partial x'}{\partial x}|^{-(\Delta+1)/4} D(\Lambda(x))D_\nu\Phi. \tag{C.31}$$

[1] Our original definition of scale transformations $\Phi(x) \to \Phi'(x) = e^{\Delta\lambda}\Phi(e^{\lambda}x)$, has now been written as $\Phi(x) \to \Phi'(e^{\lambda}x) = e^{-\Delta\lambda}\Phi(x)$.

The very same field χ can be used to render the action of any Lorentz covariant field Φ also invariant under scalings ($\Lambda = 1$). We conclude that a single physical NGB is required, the dilaton.

An important observation is that the covariant derivative thus defined vanishes when acting on the dilaton (the connection is Levi-Civita). This phenomenon was called inverse Higgs mechanism, see[160] for earlier references. We thus need to find a special covariant derivative for the dilaton. It turns out that there is a minimal two derivative covariant operator which reads (Ricci tensor)

$$\begin{aligned} D_{\mu\nu} &= e^{-2\sigma}\left(\partial_\mu\sigma\partial_\nu\sigma - \partial_\mu\partial_\nu\sigma - \frac{1}{2}\eta_{\mu\nu}(\partial\sigma)^2\right) \\ &= \frac{f^2}{\chi^2}\left(2\frac{\partial_\mu\chi\partial_\nu\chi}{\chi^2} - \frac{\partial_\mu\partial_\nu\chi}{\chi} - \frac{1}{2}\eta_{\mu\nu}\frac{(\partial\chi)^2}{\chi^2}\right), \end{aligned} \tag{C.32}$$

and whose scaling dimension is $\Delta = 0$. From its trace we can derive the leading order derivative term for the dilaton:

$$\mathcal{L}_{kin} = -\frac{1}{2}\chi^4\eta^{\mu\nu}D_{\mu\nu} = -\frac{1}{2}\chi\partial^2\chi. \tag{C.33}$$

The factor χ^4 has been introduced to reproduce the appropriate scaling and $-1/2$ is conventional. The expression evidently shows that the canonical kinetic term $(\partial\chi)^2$ is covariant up to a boundary term.

The special conformal transformations impose additional constraints on the derivative couplings of a scale invariant theory. Let us see the impact of the covariantized derivatives on spin $1/2, 1, 0$ fields, respectively.

Fermions The covariant derivative for a left handed fermion is generally

$$D_\mu\Psi = \left[\partial_\mu + \left(\frac{1}{4}(\sigma_\mu\bar{\sigma}_\nu - \sigma_\nu\bar{\sigma}_\mu) - \Delta\eta_{\mu\nu}\right)\partial_\nu\log\chi\right]\Psi. \tag{C.34}$$

We used

$$S_{\mu\nu} = \frac{i}{4}(\sigma_\mu\bar{\sigma}_\nu - \sigma_\nu\bar{\sigma}_\mu), \tag{C.35}$$

with $\bar{\sigma}_\mu\sigma_\nu + \bar{\sigma}_\nu\sigma_\mu = 2g_{\mu\nu}$. Since such a term must appear as

$$\Psi^\dagger i\bar{\sigma}^\mu D_\mu\Psi = \Psi^\dagger i\bar{\sigma}^\mu\partial_\mu\Psi + \left(\frac{3}{2} - \Delta\right)\Psi^\dagger i\bar{\sigma}^\mu\Psi\partial_\mu\log\chi$$

we see that the interaction with the dilaton is purely immaginary for $Im(\Delta) = 0$. The covariant derivative for a fermion reduces to the ordinary derivative. We can understand this in terms of the Weyl symmetry: for a diagonal metric the covariant derivative of a fermion has no spin-connection term.

Vector fields In this case

$$(S_{\mu\nu})^{\alpha}_{\beta} = -i\left(\eta_{\mu\beta}\delta^{\alpha}_{\nu} - \eta_{\nu\beta}\delta^{\alpha}_{\mu}\right), \tag{C.36}$$

and we find

$$(D_\mu A)_\nu = \partial_\mu A_\nu - \eta_{\mu\nu} A^\alpha \partial_\alpha \log\chi + (1-\Delta)\partial_\mu \log\chi A_\nu. \tag{C.37}$$

For an arbitrary massive spin-1 resonance one finds

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + (1-\Delta)\left(\partial_\mu \log\chi A_\nu - \partial_\nu \log\chi A_\mu\right), \tag{C.38}$$

where $\bar{F}_{\mu\nu}$. The lagrangian contains a non-renormalizable term of the form

$$(\Delta-1)\bar{F}_{\mu\nu}A_\nu \frac{\partial_\mu \bar{\chi}}{f} + O\left(\frac{p^2}{f^2}\right) = -(\Delta-1)\left[gJ_\mu A^\mu + \bar{F}^2_{\mu\nu}\right]\frac{\bar{\chi}}{f} + O\left(\frac{p^2}{f^2}\right). \tag{C.39}$$

Since $(\Delta - 1) = \gamma$ is the anomalous dimension of the vector, we recognize the relation $\gamma = -\beta/g$ which relates the above term to the trace-anomalous couplings.

For a gauge field, however, a covariant field strength can only be constructed if $\Delta = 1$, which is a well known consequence of the CFT algebra.

Scalars The covariant derivative of a scalar H with weight Δ reads:

$$\left(\partial_\mu - \Delta\frac{\partial_\mu\chi}{\chi}\right)H, \tag{C.40}$$

and generally leads to a mixing if the scalar has a non-vanishing vev. The scalar covariant derivative can be understood by identifying a dimension zero scalar $\bar{H} = H(f/\chi)^\Delta$, whose ordinary derivative is covariant by definition. Notice that in terms of $\bar{H}$ and χ both kinetic and potential terms are diagonalized.

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